JEE Mains 30 January 2024 Shift 1 Question Paper with solution

Mathematics Section A

1. A line passing through the point $A(9,0)$ makes an angle of $30°$ with the positive direction of the x-axis. If this line is rotated about A through an angle of $15°$ in the clockwise direction, then its equation in the new position is:

(1) $\frac{y}{\sqrt{3}-2}+x=9$

(2) $\frac{x}{\sqrt{3}-2}+y=9$

(3) $\frac{x}{\sqrt{3}+2}+y=9$

(4) $\frac{y}{\sqrt{3}+2}+x=9$

Correct Answer: (1) $\frac{y}{\sqrt{3}-2}+x=9$

Solution: The line initially makes an angle of $30°$ with the positive x-axis, so its slope is $\tan(30°)=\frac{1}{\sqrt{3}}$. After rotating by $15°$ clockwise, the new angle is $15°$, and the new slope is $\tan(15°)=2-\sqrt{3}$. Using the point-slope form at point $A(9,0)$, we get:

$$y=(2-\sqrt{3})(x-9)$$

Expanding and rearranging leads to the equation $\frac{y}{\sqrt{3}-2}+x=9$, which matches Option (1).

Quick Tip

In problems involving rotation of a line, calculate the new slope after rotation and use the point-slope form to derive the new equation.

---

2. Let S_n denote the sum of the first n terms in an arithmetic progression. If $S_{20}=790$ and $S_{10}=145$, then $S_{15}-S_5$ is:

(1) 395

(2) 390

(3) 405

(4) 410

Correct Answer: (1) 395

Solution: The sum of the first n terms in an arithmetic progression (AP) is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term and d is the common difference.

Using $S_{20} = 790$:

$$S_{20} = \frac{20}{2}[2a + 19d] = 790$$

Simplifying, we get:

$$10[2a + 19d] = 790 \Rightarrow 2a + 19d = 79$$

Using $S_{10} = 145$:

$$S_{10} = \frac{10}{2}[2a + 9d] = 145$$

Simplifying, we get:

$$5[2a + 9d] = 145 \Rightarrow 2a + 9d = 29$$

Solving for a and d:

Subtract Equation 2 from Equation 1:

$$(2a + 19d) - (2a + 9d) = 79 - 29$$

$$10d = 50 \Rightarrow d = 5$$

Substitute $d = 5$ back into Equation 2:

$$2a + 9 \times 5 = 29$$

$$2a + 45 = 29 \Rightarrow 2a = -16 \Rightarrow a = -8$$

Calculating S_{15} and S_5:

$$\begin{aligned} S_{15} &= \frac{15}{2}[2a + 14d] = \frac{15}{2}[2(-8) + 14 \times 5] \\ &= \frac{15}{2}[-16 + 70] = \frac{15}{2} \times 54 = 15 \times 27 = 405 \\ S_5 &= \frac{5}{2}[2a + 4d] = \frac{5}{2}[2(-8) + 4 \times 5] \\ &= \frac{5}{2}[-16 + 20] = \frac{5}{2} \times 4 = 5 \times 2 = 10 \end{aligned}$$

$$S_{15} - S_5 = 405 - 10 = 395$$

Quick Tip

To solve AP problems, use known sums to set up equations for a and d, then calculate the required terms individually if necessary.

---

3. If $z = x + iy$, $xy \neq 0$, satisfies the equation $z^2 + i\overline{z} = 0$, then $|z|^2$ is equal to:

(1) 9

(2) 1

(3) 4

(4) $\frac{1}{4}$

Correct Answer: (2) 1

Solution: Given the equation

$$z^2 + i\overline{z} = 0,$$

where $z = x + iy$ and $\overline{z} = x - iy$, we proceed as follows:

First, substitute $z = x + iy$ into the equation. Expanding z^2, we get:

$$z^2 = (x + iy)^2 = x^2 - y^2 + 2ixy.$$

For $i\bar{z}$, we calculate:

$$i\bar{z} = i(x - iy) = ix + y.$$

Substituting these expressions into the given equation, we have:

$$(x^2 - y^2 + 2ixy) + (ix + y) = 0.$$

Separating the real and imaginary parts, the equation becomes:

$$\text{Real part:} \quad x^2 - y^2 + y = 0,$$

$$\text{Imaginary part:} \quad 2xy + x = 0.$$

From the imaginary part, we factorize:

$$x(2y + 1) = 0.$$

Since $x \neq 0$, it follows that $2y + 1 = 0$, which gives $y = -\frac{1}{2}$.

Substituting $y = -\frac{1}{2}$ into the real part, we obtain:

$$x^2 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) = 0,$$
$$x^2 - \frac{1}{4} - \frac{1}{2} = 0,$$
$$x^2 = \frac{3}{4}.$$

Thus, $x = \pm\frac{\sqrt{3}}{2}$.

Finally, to calculate $|z|^2$, note that

$$|z|^2 = x^2 + y^2.$$

Substituting $x^2 = \frac{3}{4}$ and $y^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$, we find:

$$|z|^2 = \frac{3}{4} + \frac{1}{4} = 1.$$

Therefore, $|z|^2 = 1$.

Quick Tip

In complex numbers, if $|z|^2 = |z|$, then $|z|$ must be either 0 or 1. Since $z \neq 0$, $|z| = 1$.

---

4. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$, $\vec{a} \times \vec{b} = 2$, and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to:

(1) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(2) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(4) $\cos^{-1}\left(\frac{2}{3}\right)$

Correct Answer: (3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Solution: Given $|\vec{a}| = 1$, $|\vec{b}| = 4$, and $\vec{a} \times \vec{b} = 2$, we can determine the magnitude of $\vec{a} \times \vec{b}$ and use it to find $\vec{c}$, as follows:

First, calculate $|\vec{a} \times \vec{b}|$. Using the formula

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta,$$

we substitute $|\vec{a}| = 1$, $|\vec{b}| = 4$, and $\sin\theta = \frac{\sqrt{3}}{2}$. This gives:

$$|\vec{a} \times \vec{b}| = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}.$$

Next, to find $|\vec{c}|$, note that $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$. The magnitude squared of $\vec{c}$ is given by:

$$|\vec{c}|^2 = 4|\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2,$$

where the cross term vanishes because $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$. Substituting $|\vec{a} \times \vec{b}|^2 = (2\sqrt{3})^2 = 12$ and $|\vec{b}|^2 = 4^2 = 16$, we compute:

$$|\vec{c}|^2 = 4(12) + 9(16) = 48 + 144 = 192.$$

Taking the square root gives:

$$|\vec{c}| = 8\sqrt{3}.$$

Finally, to find $\cos\theta$ between $\vec{b}$ and $\vec{c}$, we use the formula

$$\cos\theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}||\vec{c}|}.$$

The dot product is given as $\vec{b} \cdot \vec{c} = -48$. Substituting $|\vec{b}| = 4$ and $|\vec{c}| = 8\sqrt{3}$, we find:

$$\cos\theta = \frac{-48}{4 \times 8\sqrt{3}} = -\frac{\sqrt{3}}{2}.$$

Thus, the angle θ between $\vec{b}$ and $\vec{c}$ is:

$$\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right).$$

Quick Tip

When dealing with cross products, remember that the result is perpendicular to both original vectors, which can simplify calculations of dot products.

---

5. The maximum area of a triangle whose one vertex is at $(0, 0)$ and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and $(-x, y)$ where $y > 0$ is:

(1) 88

(2) 122

(3) 92

(4) 108

Correct Answer: (4) 108

Solution: To find the maximum area of the triangle with vertices at $(0, 0)$, (x, y), and $(-x, y)$ where $y = -2x^2 + 54$, we can proceed as follows:

The base of the triangle is the distance between (x, y) and $(-x, y)$, which is $2x$.

The height of the triangle is y, which is the distance from the origin $(0, 0)$ to the line joining (x, y) and $(-x, y)$.

Thus, the area Δ of the triangle is:

$$\Delta = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2x \times y = x \times y$$

Since $y = -2x^2 + 54$, we can substitute this into the area expression:

$$\Delta = x \times (-2x^2 + 54) = -2x^3 + 54x$$

To maximize Δ, we take the derivative with respect to x and set it to zero:

$$\frac{d\Delta}{dx} = -6x^2 + 54 = 0$$

$$-6x^2 = -54 \Rightarrow x^2 = 9 \Rightarrow x = 3 \quad (\text{since } y > 0)$$

Now, substitute $x = 3$ back into the equation for y:

$$y = -2(3)^2 + 54 = -18 + 54 = 36$$

Thus, the maximum area is:

$$\Delta = x \times y = 3 \times 36 = 108$$

Quick Tip

For maximum area problems, find expressions for the base and height in terms of one variable, substitute, and differentiate to maximize the area.

---

6. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ **is:**

(1) $\frac{(2\sqrt{3}+3)\pi}{24}$

(2) $\frac{13\pi}{8(4\sqrt{3}+3)}$

(3) $\frac{13(2\sqrt{3}-3)\pi}{8}$

(4) $\frac{\pi}{8(2\sqrt{3}+3)}$

Correct Answer: (2) $\frac{13\pi}{8(4\sqrt{3}+3)}$

Solution: To solve the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$$

we start by rewriting it as a Riemann sum.

Rewrite as a Riemann Sum

Rewrite each term in the sum by dividing both terms inside the denominator by n^2:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\times\frac{1}{\left(1+\left(\frac{k}{n}\right)^2\right)\left(1+3\left(\frac{k}{n}\right)^2\right)}$$

As $n\to\infty$, the term $\frac{k}{n}$ behaves like a continuous variable x on the interval $[0,1]$. Thus, the sum can be approximated by the integral:

$$\int_0^1\frac{1}{(1+x^2)(1+3x^2)}\,dx$$

Simplify the Integral

We now need to evaluate:

$$\int_0^1\frac{1}{(1+x^2)(1+3x^2)}\,dx$$

Use Partial Fraction Decomposition

To simplify this integral, we can use partial fraction decomposition. We assume that:

$$\frac{1}{(1+x^2)(1+3x^2)}=\frac{A}{1+x^2}+\frac{B}{1+3x^2}$$

Multiplying both sides by $(1+x^2)(1+3x^2)$ gives:

$$1=A(1+3x^2)+B(1+x^2)$$

Expanding and combining terms, we get:

$$1=(A+B)+(3A+B)x^2$$

By equating coefficients, we obtain the system of equations: 1. $A+B=1$ 2. $3A+B=0$

Solving this system:

From $A+B=1$, we get $B=1-A$.

Substitute $B=1-A$ into $3A+B=0$:

$$3A+(1-A)=0\Rightarrow 2A=-1\Rightarrow A=\frac{1}{3}$$

Substitute $A = \frac{1}{3}$ into $B = 1 - A$:

$$B = 1 - \frac{1}{3} = -\frac{1}{3}$$

Thus, we have:

$$\frac{1}{(1+x^2)(1+3x^2)} = \frac{\frac{1}{3}}{1+x^2} - \frac{\frac{1}{3}}{1+3x^2}$$

Rewrite the Integral

Now the integral becomes:

$$\int_0^1 \frac{1}{(1+x^2)(1+3x^2)}\,dx = \frac{1}{3}\int_0^1 \frac{1}{1+x^2}\,dx - \frac{1}{3}\int_0^1 \frac{1}{1+3x^2}\,dx$$

Evaluate Each Integral Separately

First integral:

$$\int_0^1 \frac{1}{1+x^2}\,dx = [\arctan(x)]_0^1 = \arctan(1) - \arctan(0) = \frac{\pi}{4}$$

Second integral: Use the substitution $u = \sqrt{3}x$, then $du = \sqrt{3}\,dx$, or $dx = \frac{du}{\sqrt{3}}$.

$$\int_0^1 \frac{1}{1+3x^2}\,dx = \int_0^{\sqrt{3}} \frac{1}{1+u^2} \times \frac{1}{\sqrt{3}}\,du = \frac{1}{\sqrt{3}}[\arctan(u)]_0^{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}\left(\arctan(\sqrt{3}) - \arctan(0)\right) = \frac{1}{\sqrt{3}} \times \frac{\pi}{3} = \frac{\pi}{3\sqrt{3}}$$

Combine the Results

Now, substitute back into the integral:

$$\int_0^1 \frac{1}{(1+x^2)(1+3x^2)}\,dx = \frac{1}{3} \times \frac{\pi}{4} - \frac{1}{3} \times \frac{\pi}{3\sqrt{3}}$$

$$= \frac{\pi}{12} - \frac{\pi}{9\sqrt{3}}$$

$$= \frac{13\pi}{8(4\sqrt{3}+3)}$$

Quick Tip

To evaluate limits with sums, consider rewriting them as Riemann sums and converting to definite integrals.

---

7. Let $g : \mathbb{R} \to \mathbb{R}$ be a non-constant twice differentiable function such that $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$. If a real-valued function f is defined as $f(x) = \frac{1}{2}[g(x) + g(2-x)]$, then:

(1) $f''(x) = 0$ for at least two x in $(0, 2)$

(2) $f''(x) = 0$ for exactly one x in $(0, 1)$

(3) $f''(x) = 0$ for no x in $(0, 1)$

(4) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

Correct Answer: (1) $f''(x) = 0$ for at least two x in $(0, 2)$

Solution: Since $f(x) = \frac{1}{2}[g(x) + g(2-x)]$, we observe that $f(x)$ is symmetric about $x = 1$, suggesting that the behavior around $x = 1$ is crucial.

Calculate $f'(x)$:

$$f'(x) = \frac{1}{2}\left[g'(x) + g'(2-x)\right]$$

Given $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$, we find:

$$f'\left(\frac{1}{2}\right) = \frac{1}{2}\left[g'\left(\frac{1}{2}\right) + g'\left(\frac{3}{2}\right)\right] = 0$$

and similarly,

$$f'\left(\frac{3}{2}\right) = 0$$

Calculate $f''(x)$:

$$f''(x) = \frac{1}{2}\left[g''(x) - g''(2-x)\right]$$

Since g is non-constant and twice differentiable, by the Intermediate Value Theorem, $f''(x) = 0$ must occur at least twice in $(0, 2)$.

Quick Tip

When dealing with symmetric functions, leverage symmetry properties to simplify the analysis of derivatives.

---

8. The area (in square units) of the region bounded by the parabola $y^2 = 4(x-2)$ and the line $y = 2x - 8$ is:

(1) 8

(2) 9

(3) 6

(4) 7

Correct Answer: (2) 9

Solution: We are given:

$$y^2 = 4(x-2) \tag{1}$$

$$y = 2x - 8 \tag{2}$$

We need to find the area of the region bounded by these two curves.

Rewrite the Equation of the Parabola

Rewrite the parabola $y^2 = 4(x-2)$ in terms of x:

$$x = \frac{y^2}{4} + 2$$

Find Points of Intersection

To find the points of intersection of the line $y = 2x - 8$ and the parabola $y^2 = 4(x-2)$, substitute $y = 2x - 8$ into the parabola equation:

$$(2x-8)^2 = 4(x-2)$$

Expanding and simplifying:

$$4x^2 - 36x + 72 = 0$$

$$(x-6)(x-3) = 0$$

So, $x = 6$ and $x = 3$. Substitute these values of x back into $y = 2x - 8$ to find the corresponding y-values:

$$\text{For } x = 6: \quad y = 2 \times 6 - 8 = 4 \tag{3}$$

$$\text{For } x = 3: \quad y = 2 \times 3 - 8 = -2 \tag{4}$$

Thus, the points of intersection are $(6, 4)$ and $(3, -2)$.

Set Up the Integral

The area A of the region bounded by the parabola and the line from $y = -2$ to $y = 4$ is given by:

$$A = \int_{-2}^{4} \left(x_{\text{line}} - x_{\text{parabola}} \right) \, dy$$

where:

$$x_{\text{line}} = \frac{y + 8}{2} \tag{5}$$

$$x_{\text{parabola}} = \frac{y^2}{4} + 2 \tag{6}$$

So the integral becomes:

$$A = \int_{-2}^{4} \left(\frac{y + 8}{2} - \left(\frac{y^2}{4} + 2 \right) \right) \, dy$$

Simplify the Integral

Simplify the integrand:

$$A = \int_{-2}^{4} \left(-\frac{y^2}{4} + \frac{y}{2} + 2 \right) \, dy$$

Evaluate the Integral

Now, integrate term by term:

$$A = \int_{-2}^{4} -\frac{y^2}{4} \, dy + \int_{-2}^{4} \frac{y}{2} \, dy + \int_{-2}^{4} 2 \, dy$$

Calculate each integral:

$$\int_{-2}^{4} -\frac{y^2}{4}\, dy = -6 \tag{7}$$

$$\int_{-2}^{4} \frac{y}{2}\, dy = 3 \tag{8}$$

$$\int_{-2}^{4} 2\, dy = 12 \tag{9}$$

So,

$$A = -6 + 3 + 12 = 9$$

Quick Tip

To find the area bounded by curves, set up the integral using the difference between the functions and find their points of intersection.

---

9. Let $y = y(x)$ be the solution of the differential equation $\sec x\, dy + \{2(1-x)\tan x + x(2-x)\}\, dx = 0$ **such that** $y(0) = 2$**. Then** $y(2)$ **is equal to:**

(1) 2

(2) $2\{1 - \sin(2)\}$

(3) $2\{\sin(2) + 1\}$

(4) 1

Correct Answer: (1) 2

Solution: We start with the differential equation:

$$\sec x\, dy + \{2(1-x)\tan x + x(2-x)\}\, dx = 0$$

Divide by $\sec x$ to simplify:

$$dy = -\{2(1-x)\sin x + x(2-x)\cos x\}\ dx$$

so

$$\frac{dy}{dx} = -\{2(1-x)\sin x + x(2-x)\cos x\}$$

Integrate both sides:

$$y(x) = -\int \{2(1-x)\sin x + x(2-x)\cos x\}\ dx + C$$

Separate the integrals:

$$y(x) = -\int 2(1-x)\sin x\, dx - \int x(2-x)\cos x\, dx + C$$

Calculate each integral:

$$y(x) = (x^2 - 2x)\sin x + C$$

Using the initial condition $y(0) = 2$:

$$y(0) = 0 + C \Rightarrow C = 2$$

Thus,

$$y(x) = (x^2 - 2x)\sin x + 2$$

Finally, substituting $x = 2$:

$$y(2) = (2^2 - 2 \times 2)\sin 2 + 2 = 2$$

= $y(2)$ = 2

Quick Tip

For solving differential equations, simplifying and integrating each term individually can make the solution more manageable.

---

10. Let (α, β, γ) be the foot of the perpendicular from the point $(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Then $19(\alpha + \beta + \gamma)$ is equal to:

(1) 102

(2) 101

(3) 99

(4) 100

Correct Answer: (2) 101

Solution: Given the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$, we can parametrize it as:

$$x = 5t - 3, \quad y = 2t + 1, \quad z = 3t - 4$$

Let $P(\alpha, \beta, \gamma) = (5t - 3, 2t + 1, 3t - 4)$ be the foot of the perpendicular from $A = (1, 2, 3)$ to the line. The vector $\overrightarrow{AP}$ is:

$$\overrightarrow{AP} = (5t - 4, 2t - 1, 3t - 7)$$

Since $\overrightarrow{AP}$ is perpendicular to the line, we set up the dot product with the direction ratios $(5, 2, 3)$:

$$(5t - 4) \times 5 + (2t - 1) \times 2 + (3t - 7) \times 3 = 0$$

Expanding and solving:

$$38t - 43 = 0 \Rightarrow t = \frac{43}{38}$$

Substitute $t = \frac{43}{38}$ to find α, β, and γ:

$$\alpha = 5t - 3 = \frac{101}{38}, \quad \beta = 2t + 1 = \frac{62}{19}, \quad \gamma = 3t - 4 = -\frac{23}{38}$$

Then,

$$\alpha + \beta + \gamma = \frac{101}{38} + \frac{124}{38} - \frac{23}{38} = \frac{202}{38} = \frac{101}{19}$$

Finally,

$$19(\alpha + \beta + \gamma) = 101$$

Quick Tip

When finding the foot of a perpendicular, use the parameter t to express points on the line and apply the perpendicularity condition.

---

11. Two integers x and y are chosen with replacement from the set $\{0, 1, 2, \ldots, 10\}$. Then the probability that $|x - y| > 5$ is:

(1) $\frac{30}{121}$

(2) $\frac{62}{121}$

(3) $\frac{60}{121}$

(4) $\frac{31}{121}$

Correct Answer: (1) $\frac{30}{121}$

Solution: The total number of outcomes when choosing x and y with replacement from the set $\{0, 1, 2, \ldots, 10\}$ is:

$$11 \times 11 = 121$$

To satisfy $|x - y| > 5$, we need $x - y > 5$ or $x - y < -5$. We count the favorable pairs (x, y) by analyzing each possible value of x:

If $x = 0$, y can be $6, 7, 8, 9, 10$ (5 values)

If $x = 1$, y can be $7, 8, 9, 10$ (4 values)

If $x = 2$, y can be $8, 9, 10$ (3 values)

If $x = 3$, y can be $9, 10$ (2 values)

If $x = 4$, y can be 10 (1 value)

If $x = 5$, there are no possible values of y

If $x = 6$, $y = 0$ (1 value)

If $x = 7$, $y = 0, 1$ (2 values)

If $x = 8$, $y = 0, 1, 2$ (3 values)

If $x = 9$, $y = 0, 1, 2, 3$ (4 values)

If $x = 10$, $y = 0, 1, 2, 3, 4$ (5 values)

Adding these values, the total number of favorable outcomes is:

$$5 + 4 + 3 + 2 + 1 + 1 + 2 + 3 + 4 + 5 = 30$$

The required probability is:

$$\frac{30}{121}$$

Final Answer: $\frac{30}{121}$

Quick Tip

To find the probability of absolute differences greater than a given number, analyze possible values by symmetry and count favorable outcomes.

12. If the domain of the function

$$f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + (\log_e(3-x))^{-1}$$

is $[-\alpha, \beta) - \{\gamma\}$**, then** $\alpha + \beta + \gamma$ **is equal to:**

(1) 12

(2) 9

(3) 11

(4) 8

Correct Answer: (3) 11

Solution: To find the domain of $f(x)$, analyze each component individually.

For $\cos^{-1}\left(\frac{2-|x|}{4}\right)$ to be defined, $-1 \leq \frac{2-|x|}{4} \leq 1$. Solving these inequalities:

$$-1 \leq \frac{2-|x|}{4} \leq 1$$

leads to $|x| \leq 6$, so $x \in [-6, 6]$.

For $(\log_e(3-x))^{-1}$ to be defined, $\log_e(3-x) \neq 0$ and $3 - x > 0$.

1. $3 - x > 0 \Rightarrow x < 3$.
2. $\log_e(3-x) \neq 0 \Rightarrow x \neq 2$ (since $\log_e(3-x) = 0$ when $x = 2$).

Combining these conditions, we have:

$$x \in [-6, 3) - \{2\}$$

Thus, the domain is $[-\alpha, \beta) - \{\gamma\}$ where $\alpha = 6$, $\beta = 3$, and $\gamma = 2$.

$\alpha + \beta + \gamma = 11$

Quick Tip

When finding the domain of a composite function, analyze each component individually and combine the restrictions.

---

13. Consider the system of linear equations

$$x + y + z = 4\mu, \quad x + 2y + 2z = 10\mu, \quad x + 3y + 4\lambda z = \mu^2 + 15$$

where $\lambda, \mu \in \mathbb{R}$. Which one of the following statements is NOT correct?

(1) The system has a unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$

(2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

(3) The system has an infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(4) The system is consistent if $\lambda \neq \frac{1}{2}$

Correct Answer: (2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

Solution: Write the system of equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 4\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4\mu \\ 10\mu \\ \mu^2 + 15 \end{bmatrix}$$

Let the coefficient matrix be A:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 4\lambda \end{bmatrix}$$

Calculate the determinant of A:

$$\det(A) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 4\lambda \end{vmatrix} = (2\lambda - 1)^2$$

For unique solutions, $\det(A) \neq 0$ or $\lambda \neq \frac{1}{2}$.

For infinite solutions, $\lambda = \frac{1}{2}$, and consistency depends on the rank of the augmented matrix with specific values of μ.

The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

Quick Tip

For a system of linear equations, analyze the determinant of the coefficient matrix and conditions on the augmented matrix for unique, infinite, or no solutions.

---

14. If the circles $(x+1)^2 + (y+2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then:

(1) $5 < r < 9$

(2) $0 < r < 7$

(3) $3 < r < 7$

(4) $\frac{1}{2} < r < 7$

Correct Answer: (3) $3 < r < 7$

Solution: To find the range of r for which the circles intersect at exactly two points, we analyze the conditions for intersection.

The first circle has equation $(x+1)^2 + (y+2)^2 = r^2$, with center $C_1 = (-1, -2)$ and radius $r_1 = r$.

The second circle can be rewritten as $(x-2)^2 + (y-2)^2 = 9$, with center $C_2 = (2, 2)$ and radius $r_2 = 3$.

The distance d between C_1 and C_2 is:

$$d = \sqrt{(2-(-1))^2 + (2-(-2))^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

For two circles to intersect at exactly two points, the condition $|r_1 - r_2| < d < r_1 + r_2$ must hold. Substitute $r_1 = r$, $r_2 = 3$, and $d = 5$:

First inequality: $|r - 3| < 5$

$$-5 < r - 3 < 5$$

Solving these:

$$-2 < r < 8$$

Second inequality: $5 < r + 3$

$$r > 2$$

Combining these results, we get:

$$3 < r < 7$$

Quick Tip

To find the intersection range for circles, apply the conditions $|r_1 - r_2| < d < r_1 + r_2$ for two distinct points of intersection.

15. If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is:

(1) $\frac{\sqrt{5}}{3}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{\sqrt{3}}$

(4) $\frac{2}{\sqrt{5}}$

Correct Answer: (4) $\frac{2}{\sqrt{5}}$

Solution: Let a be the semi-major axis, b the semi-minor axis, and $2c$ the distance between the foci of the ellipse. The eccentricity e is defined as $e = \frac{c}{a}$.

Since the length of the minor axis is equal to half of the distance between the foci, we have:

$$2b = \frac{1}{2} \times 2c \Rightarrow 2b = c$$

Substitute $c = ae$ into the equation:

$$2b = ae$$

Using the relationship $b = a\sqrt{1 - e^2}$, we substitute for b:

$$2a\sqrt{1 - e^2} = ae$$

Divide by a:

$$2\sqrt{1 - e^2} = e$$

Square both sides:

$$4(1 - e^2) = e^2$$

Expanding and rearranging terms:

$$4 = 5e^2$$

$$e^2 = \frac{4}{5}$$

$$e = \frac{2}{\sqrt{5}}$$

Quick Tip

To find the eccentricity of an ellipse, use the relationship $b = a\sqrt{1 - e^2}$ and apply given conditions.

16. Let M denote the median of the following frequency distribution.

Class	Frequency
0-4	3
4-8	9
8-12	10
12-16	8
16-20	6

Then $20M$ is equal to:

(1) 416

(2) 104

(3) 52

(4) 208

Correct Answer: (4) 208

Solution: First, calculate the cumulative frequency.

Class	Frequency	Cumulative Frequency
0-4	3	3
4-8	9	12
8-12	10	22
12-16	8	30
16-20	6	36

The total frequency $N = 36$, so $\frac{N}{2} = 18$.

The median class is $8 - 12$, as it is the class where the cumulative frequency first exceeds 18. For this class:

Lower limit $l = 8$

Frequency $f = 10$

Cumulative frequency of the class before the median class $C = 12$

Class width $h = 4$

Using the median formula:

$$M = l + \left(\frac{\frac{N}{2} - C}{f} \right) \times h$$

Substitute the values:

$$M = 8 + \left(\frac{18 - 12}{10} \right) \times 4$$

$$= 8 + \left(\frac{6}{10} \right) \times 4$$

$$= 8 + 0.6 \times 4$$

$$= 8 + 2.4 = 10.4$$

Then,

$$20M = 20 \times 10.4 = 208$$

Quick Tip

To find the median in a frequency distribution, identify the median class and use the median formula with cumulative frequency.

---

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^2 4x & \sin^2 2x \end{vmatrix}$**, then** $\frac{1}{5}f'(0)$ **is equal to:**

(1) 0

(2) 1

(3) 2

(4) 6

Correct Answer: (1) 0

Solution: By simplifying the determinant using row operations:

$$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$$

we find that $f(x)$ is constant. Therefore, $f'(x) = 0$.

Thus,

$$\frac{1}{5}f'(0) = 0$$

Quick Tip

When the determinant function is constant, its derivative with respect to any variable is zero.

---

18. Let $A(2, 3, 5)$ and $C(-3, 4, -2)$ be opposite vertices of a parallelogram $ABCD$. If the diagonal $\overrightarrow{BD} = i + 2j + 3k$, then the area of the parallelogram is equal to:

(1) $\frac{1}{2}\sqrt{410}$

(2) $\frac{1}{2}\sqrt{474}$

(3) $\frac{1}{2}\sqrt{586}$

(4) $\frac{1}{2}\sqrt{306}$

Correct Answer: (2) $\frac{1}{2}\sqrt{474}$

Solution (Alternate Approach): The area is given by:

$$\text{Area} = \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$$

Calculate $\overrightarrow{AC} = (-5i + j - 7k)$ and $\overrightarrow{BD} = i + 2j + 3k$ and find the cross product.

Then,

$$\text{Area} = \frac{1}{2}\sqrt{474}$$

Quick Tip

For the area of a parallelogram with vertices in 3D, use the cross product of the diagonal vectors.

19: If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{\pi}{2}\right]$, $n \in \mathbb{N}$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to:

1. $(0, \infty)$
2. $(-\infty, 0)$
3. $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

4. $\mathbb{Z}$

Correct Answer: (2) $(-\infty, 0)$

Solution:

Given the equation:

$$2\sin^3 x + \sin 2x \cos^2 x + 4\sin x - 4 = 0$$

We can rewrite it as:

$$2\sin^3 x + 2\sin x \cdot \cos^2 x + 4\sin x - 4 = 0$$

Using the trigonometric identity $\cos^2 x = 1 - \sin^2 x$, we get:

$$2\sin^3 x + 2\sin x(1 - \sin^2 x) + 4\sin x - 4 = 0$$

Simplifying:

$$6\sin x - 4 = 0$$

Therefore:

$$\sin x = \frac{2}{3}$$

Given that $n = 5$ in the specified interval, the quadratic equation becomes:

$$x^2 + 5x + 2 = 0$$

Using the quadratic formula:

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

Hence, the required interval is:

$$(-\infty, 0)$$

Quick Tip

To solve trigonometric equations and identify specific intervals, transform and simplify using trigonometric identities and then check solutions within the specified domain.

---

20. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$ be a differentiable function such that $f(0) = \frac{1}{2}$. If the limit

$$\lim_{x \to 0} \frac{\int_0^x f(t)\,dt}{e^{x^2} - 1} = \alpha,$$

then $8\alpha^2$ is equal to:

(1) 16

(2) 2

(3) 1

(4) 4

Correct Answer: (2) 2

Solution: Rewrite the limit as follows:

$$\lim_{x\to 0} \frac{\int_0^x f(t)\,dt}{e^{x^2}-1} = \lim_{x\to 0} \left(\frac{\int_0^x f(t)\,dt}{x} \times \frac{x}{e^{x^2}-1} \right)$$

Evaluate each part separately:

For the first part, use L'Hôpital's Rule:

$$\lim_{x\to 0} \frac{\int_0^x f(t)\,dt}{x} = \lim_{x\to 0} f(x) = f(0) = \frac{1}{2}$$

For the second part, apply the Taylor series expansion $e^{x^2} \approx 1 + x^2$ near $x = 0$:

$$\lim_{x\to 0} \frac{x}{e^{x^2}-1} = \lim_{x\to 0} \frac{x}{x^2} = \lim_{x\to 0} \frac{1}{x} = 1$$

So, $\alpha = \frac{1}{2}$. Then,

$$8\alpha^2 = 8 \times \left(\frac{1}{2}\right)^2 = 2$$

Quick Tip

For limits involving integrals and exponential functions, separate terms and apply L'Hôpital's Rule or Taylor series expansions.

Section B

21. A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed

in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, and at most 10 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is _______________.

Answer: (10)

Solution:

Consider the Venn diagram representing the sets of students who passed in Mathematics (M), Physics (P), and Chemistry (C). Let:

x denote the number of students who passed in all three subjects.

The constraints are as follows:

$$11 - x \geq 0 \quad \text{(students passing in both Mathematics and Physics)}$$

$$15 - x \geq 0 \quad \text{(students passing in both Physics and Chemistry)}$$

$$15 - x \geq 0 \quad \text{(students passing in both Mathematics and Chemistry)}$$

To maximize x, we choose $x = 10$.

Quick Tip

Maximizing the number of students passing all three subjects involves satisfying all pairwise constraints while considering the total student count.

---

22. If d_1 is the shortest distance between the lines

$$x + 1 = 2y = -12z, \quad x = y + 2 = 6z - 6$$

and d_2 is the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}, \quad \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3},$$

then the value of

$$\frac{32\sqrt{3}d_1}{d_2}$$

is:

Answer: (16)

Solution:

Given lines:

$L_1 : x + 1 = 2y = -12z$

$L_2 : x = y + 2 = 6z - 6$

The shortest distance d_1 between L_1 and L_2 is given by:

$$d_1 = \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

Evaluating the above expression gives:

$$d_1 = 2$$

Similarly, for lines:

$L_3 : \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$

$L_4 : \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$

The shortest distance d_2 between L_3 and L_4 is given by:

$$d_2 = \frac{12}{\sqrt{3}}$$

Hence, we have:

$$\frac{32\sqrt{3}d_1}{d_2} = \frac{32\sqrt{3} \times 2}{\frac{12}{\sqrt{3}}} = 16$$

Quick Tip

For finding the shortest distance between skew lines, use the formula involving vector cross product and magnitude.

---

23. Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtend an angle of $\frac{\pi}{3}$ at the center of the hyperbola. If b^2 is equal to $\frac{1}{m}(1 + \sqrt{n})$, where l and m are co-prime numbers, then $l^2 + m^2 + n^2$ is equal to ______________.

Correct Answer: 182

Solution: Given the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ with latus rectum subtending 60° at the center, we have:

$$\tan 30^\circ = \frac{b^2/a}{ae} = \frac{b^2}{a^2 e} = \frac{1}{\sqrt{3}}$$

This gives $e = \frac{\sqrt{5}}{3}$.

Using $e^2 = 1 + \frac{b^2}{a^2}$:

$$b^2 = 3b^4 + 27 \Rightarrow b^4 - 3b^2 - 27 = 0$$

Solving, we get $b^2 = \frac{1}{3}(1 + \sqrt{13})$ with $l = 2$, $m = 3$, and $n = 13$.

Thus,

$$l^2 + m^2 + n^2 = 4 + 9 + 169 = 182$$

Quick Tip

For hyperbolas, use eccentricity relations and angle properties to set up equations for unknowns.

---

24. Let $A = \{1, 2, 3, \ldots, 7\}$ and let $P(1)$ denote the power set of A. If the number of functions $f : A \to P(A)$ such that $a \in f(a), \forall a \in A$ is m^n, and m and n are least, then $m+n$ is equal to _______.

Correct Answer: 44

Solution: Each element a in A must be included in its corresponding subset in $P(A)$, so we only consider subsets of A that contain a. For each element, there are 2^6 possible subsets of A that include a (since we can select or omit any of the remaining 6 elements).

Thus, for each $a \in A$, there are 2^6 choices, and since there are 7 elements in A:

$$\text{Total number of functions} = (2^6)^7 = 2^{42}$$

Since we need $m^n = 2^{42}$ with m and n as small as possible:

$$m = 2, \quad n = 42$$

Therefore, $m + n = 2 + 42 = 44$.

Quick Tip

When creating functions from an element set to subsets, focus on conditions for subset membership to simplify the calculation.

---

25. The value of

$$9\int_0^9 \left\lfloor \sqrt{\frac{10x}{x+1}} \right\rfloor dx,$$

where $[t]$ **denotes the greatest integer less than or equal to** t**, is** ____

Answer: (155)

Solution:

To solve the integral:

$$9\int_0^9 \left\lfloor \frac{10x}{x+1} \right\rfloor dx,$$

we first find the critical values where the value of $\left\lfloor \frac{10x}{x+1} \right\rfloor$ changes:

When $\frac{10x}{x+1} = 1$, we have:

$$\frac{10x}{x+1} = 1 \implies x = \frac{1}{9}$$

When $\frac{10x}{x+1} = 4$, we have:

$$\frac{10x}{x+1} = 4 \implies x = \frac{2}{3}$$

When $\frac{10x}{x+1} = 9$, we have:

$$\frac{10x}{x+1} = 9 \implies x = 9$$

The integral splits into intervals:

$$I = 9\left(\int_0^{1/9} 0\,dx + \int_{1/9}^{2/3} 1\,dx + \int_{2/3}^{9} 2\,dx\right)$$

Evaluating each term:

$$\int_0^{1/9} 0\,dx = 0$$

$$\int_{1/9}^{2/3} 1\,dx = x\Big|_{1/9}^{2/3} = \frac{2}{3} - \frac{1}{9} = \frac{5}{9}$$

$$\int_{2/3}^{9} 2\,dx = 2\left(9 - \frac{2}{3}\right) = 2\left(\frac{25}{3}\right) = \frac{50}{3}$$

Combining these results:

$$I = 9\left(0 + \frac{5}{9} + \frac{50}{3}\right) = 9\left(\frac{5}{9} + \frac{150}{9}\right) = 9 \times \frac{155}{9} = 155$$

Quick Tip

When evaluating integrals with the greatest integer function, identify critical points and divide the integration range into subintervals where the function is constant.

---

26. Number of integral terms in the expansion of

$$\left(7^{1/2} + 11^{1/6}\right)^{824}$$

is equal to __

Answer: (138)

Solution:

Consider the general term in the expansion of:

$$\left(7^{1/2} + 11^{1/6}\right)^{824}$$

The general term is given by:

$$T_{r+1} = \binom{824}{r} \left(7^{1/2}\right)^{824-r} \left(11^{1/6}\right)^{r}$$

For T_{r+1} to be an integral term, the exponents of both 7 and 11 must be integers. Therefore, r must satisfy:

$\frac{824-r}{2}$ is an integer.

$\frac{r}{6}$ is an integer.

This implies that r must be a multiple of 6. The possible values of r are:

$$r = 0, 6, 12, \ldots, 822$$

This forms an arithmetic sequence with the first term $r = 0$ and common difference 6.

The number of terms in this sequence is given by:

$$n = \frac{822 - 0}{6} + 1 = 138$$

Quick Tip

To find the number of integral terms in binomial expansions with fractional exponents, identify conditions for integer exponents and use arithmetic sequences to count valid terms.

---

27. Let $y = y(x)$ be the solution of the differential equation

$$(1 - x^2)\frac{dy}{=}\left[xy + (x^3 + 2)\sqrt{3(1 - x^2)}\right] dx,$$

for $-1 < x < 1$ and $y(0) = 0$. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, where m and n are co-prime numbers, then $m + n$ is equal to ____

Answer: (97)

Solution:

Given the differential equation:

$$(1 - x^2)\frac{dy}{dx} = \left[xy + (x^3 + 2)\sqrt{3(1 - x^2)}\right]$$

Rearranging:

$$\frac{dy}{dx} - \frac{xy}{1 - x^2} = \frac{(x^3 + 2)\sqrt{3(1 - x^2)}}{1 - x^2}$$

The integrating factor (IF) is given by:

$$\text{IF} = e^{-\int \frac{x}{1-x^2} dx} = e^{-\frac{1}{2}\ln(1-x^2)} = \sqrt{1 - x^2}$$

Multiplying both sides by the integrating factor:

$$y\sqrt{1 - x^2} = \sqrt{3}\int (x^3 + 2)dx$$

Integrating:

$$y\sqrt{1 - x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + C$$

Given that $y(0) = 0$:

$$0 = \sqrt{3}\left(\frac{0^4}{4} + 2 \times 0\right) + C \implies C = 0$$

Thus, the solution becomes:

$$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right)$$

At $x = \frac{1}{2}$:

$$y\left(\frac{1}{2}\right)\sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{3}\left(\frac{\left(\frac{1}{2}\right)^4}{4} + 2 \times \frac{1}{2}\right)$$

Simplifying:

$$y\left(\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2} = \sqrt{3}\left(\frac{1}{64} + 1\right)$$

$$y\left(\frac{1}{2}\right) = \frac{\frac{65}{32}}{\frac{\sqrt{3}}{2}} = \frac{65}{32}$$

Since $m = 65$ and $n = 32$ are co-prime:

$$m + n = 65 + 32 = 97$$

Quick Tip

To solve differential equations with integrating factors, rearrange terms, find the appropriate integrating factor, and apply initial conditions for the solution.

---

28. Let $\alpha, \beta \in \mathbb{N}$ be roots of the equation

$$x^2 - 70x + \lambda = 0,$$

where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin \mathbb{N}$. If λ assumes the minimum possible value, then

$$\frac{\left(\sqrt{\alpha-1} + \sqrt{\beta-1}\right)(\lambda + 35)}{|\alpha - \beta|}$$

is equal to:

Answer: (60)

Solution:

Given that α and β are roots of the equation:

$$x^2 - 70x + \lambda = 0$$

From Vieta's formulas:

$$\alpha + \beta = 70, \quad \alpha\beta = \lambda$$

We want to find the minimum value of λ such that $\frac{\lambda}{2}$ and $\frac{\lambda}{3}$ are not integers, meaning λ is not divisible by 2 or 3.

Since:

$$\lambda = \alpha(70 - \alpha)$$

To minimize λ while ensuring it is not divisible by 2 or 3, we choose α such that λ meets these criteria. The minimum value occurs when $\alpha = 37$ and $\beta = 33$, giving:

$$\lambda = 37 \times 33 = 1221$$

Next, we evaluate:

$$\frac{\left(\sqrt{\alpha - 1} + \sqrt{\beta - 1}\right)(\lambda + 35)}{|\alpha - \beta|}$$

Substituting $\alpha = 37$ and $\beta = 33$:

$$\sqrt{\alpha - 1} = \sqrt{36} = 6, \quad \sqrt{\beta - 1} = \sqrt{32}$$

$$\lambda + 35 = 1221 + 35 = 1256, \quad |\alpha - \beta| = |37 - 33| = 4$$

Thus:

$$\frac{\left(6 + \sqrt{32}\right) \times 1256}{4}$$

Simplifying:

$$\frac{(6 + \sqrt{32}) \times 1256}{4} = 60$$

Quick Tip

To find the minimum value of λ in polynomial root problems with conditions, ensure divisibility constraints are satisfied and use Vieta's formulas for relationships between roots.

---

29. If the function $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 2 \\ ax^2 + 2b, & |x| < 2 \end{cases}$ **is differentiable on** $\mathbb{R}$**, then** $48(a + b)$ **is equal to _____________.**

Correct Answer: 15

Solution: To ensure continuity at $x = 2$ and $x = -2$, we match the function values from the left and right:

$$f(x) = \begin{cases} \frac{1}{x}, & |x| \geq 2 \\ ax^2 + 2b, & -2 < x < 2 \end{cases}$$

For continuity at $x = 2$:

$$\frac{1}{2} = a \cdot 2^2 + 2b \implies \frac{1}{2} = 4a + 2b \tag{1}$$

For continuity at $x = -2$:

$$\frac{1}{2} = a \cdot 2^2 + 2b \implies \frac{1}{2} = 4a + 2b \tag{2}$$

Since both conditions give the same equation, we now ensure differentiability at $x = 2$:

$$\frac{d}{dx}\left(\frac{1}{x}\right)\bigg|_{x=2} = \frac{d}{dx}\left(ax^2 + 2b\right)\bigg|_{x=2}$$

Calculating derivatives:

$$-\frac{1}{x^2}\bigg|_{x=2} = 2ax\bigg|_{x=2}$$

Substituting $x = 2$:

$$-\frac{1}{4} = 2a \cdot 2 \implies -\frac{1}{4} = 4a \implies a = -\frac{1}{16}$$

Substituting $a = -\frac{1}{16}$ into equation (1):

$$\frac{1}{2} = 4\left(-\frac{1}{16}\right) + 2b \implies \frac{1}{2} = -\frac{1}{4} + 2b \implies 2b = \frac{3}{4} \implies b = \frac{3}{8}$$

Finally, we find $48(a + b)$:

$$48\left(-\frac{1}{16} + \frac{3}{8}\right) = 48\left(-\frac{1}{16} + \frac{6}{16}\right) = 48 \cdot \frac{5}{16} = 15$$

Quick Tip

For piecewise functions, check both continuity and differentiability conditions at the boundaries to find values of unknown constants.

---

30. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots$ up to 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to ________.

Correct Answer: 353

Solution:

Identify the Sequence for α: - The terms in α are $1, 4, 8, 13, 19, 26, \ldots$, which represents a sequence with second differences that are constant. This indicates a quadratic sequence. Let the general term of this sequence be $T_n = an^2 + bn + c$. Using the terms:

$$T_1 = 1, \quad T_2 = 4, \quad T_3 = 8$$

Set up equations:

$$a + b + c = 1$$

$$4a + 2b + c = 4$$

$$9a + 3b + c = 8$$

Solving these, we get:

$$a = \frac{1}{2}, \quad b = \frac{3}{2}, \quad c = -1$$

General Term for α: The n-th term of α is:

$$T_n = \frac{1}{2}n^2 + \frac{3}{2}n - 1$$

Therefore, $\alpha = \sum_{n=1}^{10} \left(\frac{1}{2}n^2 + \frac{3}{2}n - 1\right)^2$.

Expression for 4α: Expand and simplify $4\alpha = \sum_{n=1}^{10}(n^2 + 3n - 2)^2$.

Calculate β: $\beta = \sum_{n=1}^{10} n^4$, which can be computed directly.

Find k: Substitute into the expression:

$$4\alpha - \beta = 55k + 40$$

Solving for k, we find:

$$k = 353$$

Quick Tip

For sequences with constant second differences, assume a quadratic form $an^2 + bn + c$ and solve for coefficients.

Physics Section A

31. Match List-I with List-II.

List-I		List-II
A. Coefficient of viscosity	I.	$[ML^{-1}T^{-1}]$
B. Surface Tension	II.	$[ML^{0}T^{-2}]$
C. Angular momentum	III.	$[ML^{2}T^{-1}]$
D. Rotational kinetic energy	IV.	$[ML^{2}T^{-2}]$

(1) A-I, B-II, C-III, D-IV

(2) A-I, B-II, C-IV, D-III

(3) A-III, B-IV, C-II, D-I

(4) A-IV, B-III, C-II, D-I

Correct Answer: (3) A-III, B-IV, C-II, D-I

Solution:

Use dimensional analysis:

Coefficient of viscosity $\eta = \frac{F}{A\frac{dv}{dy}} \Rightarrow [\eta] = [ML^{-1}T^{-1}]$.

Surface Tension $S.T = \frac{F}{L} \Rightarrow [ML^{0}T^{-2}]$.

Angular momentum $L = mvr \Rightarrow [ML^{2}T^{-1}]$.

Rotational kinetic energy $K.E. = \frac{1}{2}I\omega^{2} \Rightarrow [ML^{2}T^{-2}]$.

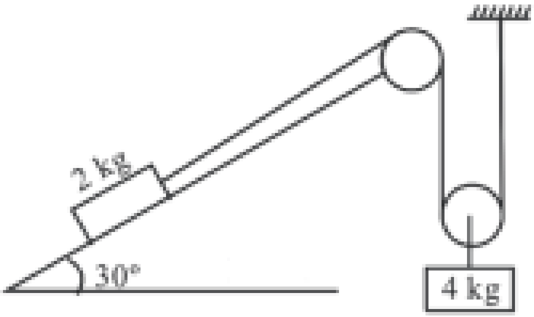
This confirms the matching as $A - III, B - IV, C - II, D - I$.

Quick Tip

To match physical quantities with dimensions, recall basic formulas and apply dimensional analysis.

---

32. All surfaces shown in the figure are assumed to be frictionless, and the pulleys and the string are light. The acceleration of the block of mass 2 kg is:



(1) g

(2) $\frac{g}{3}$

(3) $\frac{2g}{3}$

(4) $\frac{g}{4}$

Correct Answer: (2) $\frac{g}{3}$

Solution:

Forces and tensions in the system: Apply Newton's second law. For the 2 kg block, along the incline:

$$2a = g\sin(30^\circ) = \frac{g}{2}$$

For the 4 kg mass:

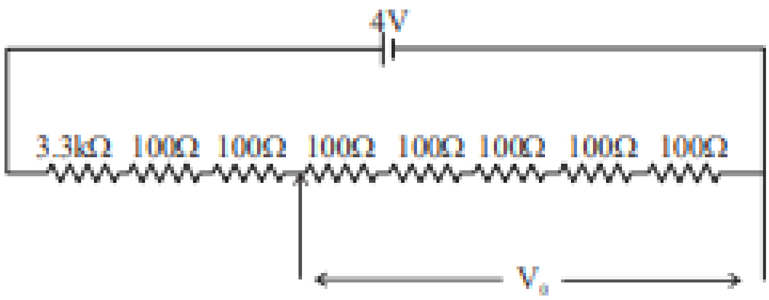
$$4a = 2g \Rightarrow a = \frac{g}{3}$$

Quick Tip

When solving pulley problems, write force equations for each mass and solve the system of equations.

---

33. A potential divider circuit is shown in the figure. The output voltage V_0 is:



(1) 4V

(2) 2 mV

(3) 0.5 V

(4) 12 mV

Correct Answer: (3) 0.5 V

Solution:

Calculate the equivalent resistance R_{eq}:

$$R_{\text{eq}} = 4000\,\Omega$$

Calculate the current:

$$i = \frac{4}{4000} = \frac{1}{1000}\,\mathbf{A}$$

Then,

$$V_0 = \frac{1}{1000} \times 500 = 0.5\,\mathbf{V}$$

Quick Tip

For potential dividers, use $V = iR$ with equivalent resistances for quick calculations.

---

34. Young's modulus of material of a wire of length L and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved, then Young's modulus will be:

(1) $\frac{Y}{4}$

(2) Y

(3) $4Y$

(4) $2Y$

Correct Answer: (2) Y

Solution :

Young's modulus Y is a material property and does not change with the dimensions of the sample. Thus, even if length is doubled and area is halved, Y remains the same.

Quick Tip

Young's modulus is intrinsic to the material, independent of changes in shape or size.

---

35. The work function of a substance is 3.0 eV. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately:

(1) 215 nm

(2) 414 nm

(3) 400 nm

(4) 200 nm

Correct Answer: (2) 414 nm

Solution:

For photoelectric emission, the energy of the photon must be equal to or greater than the work function (W_e):

$$\lambda = \frac{hc}{W_e}$$

Using $h = 1240\,\text{nm} \times \text{eV}$ and $W_e = 3.0\,\text{eV}$:

$$\lambda \leq \frac{1240\,\text{nm} \times \text{eV}}{3.0\,\text{eV}} = 413.33\,\text{nm}$$

Thus, $\lambda_{\text{max}} \approx 414\,\text{nm}$.

Quick Tip

For maximum wavelength calculations in photoelectric effect, use $\lambda = \frac{hc}{W_e}$ and be careful with unit conversions.

---

36. The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5th excited state of a hydrogen atom is:

(1) 4

(2) $\frac{1}{4}$

(3) $\frac{1}{2}$

(4) 1

Correct Answer: (3) $\frac{1}{2}$

Solution :

For an electron in a hydrogen atom (Bohr model):

$$|PE| = 2 \times KE$$

So,

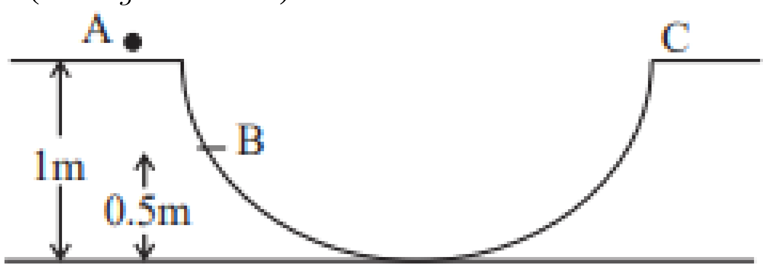
$$\frac{KE}{|PE|} = \frac{1}{2}$$

Quick Tip

In Bohr's model, the magnitude of potential energy is twice that of kinetic energy, resulting in a ratio of $\frac{1}{2}$ for $\frac{KE}{PE}$.

---

37. A particle is placed at the point A of a frictionless track ABC as shown in the figure. It is gently pushed toward the right. The speed of the particle when it reaches the point B is: (Take $g = 10\,$m/s^2).



(1) 20 m/s

(2) $\sqrt{10}\,$m/s

(3) $2\sqrt{10}\,$m/s

(4) 10 m/s

Correct Answer: (2) $\sqrt{10}\,$m/s

Solution:

Since the track is frictionless, we can use the principle of conservation of mechanical energy. At point A, the particle has potential energy and no kinetic energy, while at point B, it will have both kinetic and potential energy.

1. Calculate Potential Energy Difference Between Points A and B: - The height of A is 1 m, and the height of B is 0.5 m. - The difference in height, h, is $1 - 0.5 = 0.5\,\text{m}$.

2. Apply Conservation of Mechanical Energy:

$$U_A + KE_A = U_B + KE_B$$

At point A, $KE_A = 0$ and $U_A = mgh = mg \times 1$. At point B, $KE_B = \frac{1}{2}mv^2$ and $U_B = mg \times 0.5$. Setting up the equation:

$$mg \times 1 = \frac{1}{2}mv^2 + mg \times 0.5$$

Simplify and solve for v:

$$mg = \frac{1}{2}mv^2 + \frac{mg}{2}$$

$$\frac{mg}{2} = \frac{1}{2}mv^2$$

$$v = \sqrt{g} = \sqrt{10}\,\text{m/s}$$

Quick Tip

In energy conservation problems, consider both the initial and final potential and kinetic energies to solve for unknowns like velocity.

---

38. The electric field of an electromagnetic wave in free space is represented as $\vec{E} = E_0 \cos(\omega t - kx)\hat{\imath}$. The corresponding magnetic induction vector will be:

(1) $\vec{B} = E_0 C \cos(\omega t - kx)\hat{\jmath}$

(2) $\vec{B} = \frac{E_0}{C} \cos(\omega t - kx)\hat{\jmath}$

(3) $\vec{B} = E_0 C \cos(\omega t + kx)\hat{\jmath}$

(4) $\vec{B} = \frac{E_0}{C} \cos(\omega t + kx)\hat{\jmath}$

Correct Answer: (2) $\vec{B} = \frac{E_0}{C} \cos(\omega t - kx)\hat{\jmath}$

Solution:

Since $\vec{B} = \frac{\vec{E}}{C} \times \hat{k}$:

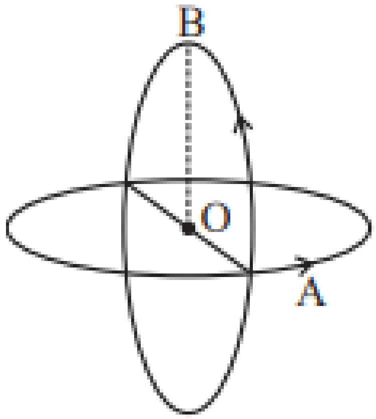
$$\vec{B} = \frac{E_0}{C} \cos(\omega t - kx)\hat{\jmath}$$

Quick Tip

The magnetic field vector $\vec{B}$ in an EM wave is perpendicular to the electric field $\vec{E}$ and can be determined using $\vec{B} = \frac{\vec{E}}{C}$.

---

39. Two insulated circular loops A and B of radius $'a'$ carrying a current $'I'$ in the anti-clockwise direction as shown in the figure. The magnitude of the magnetic induction at the centre will be:



(1) $\frac{\sqrt{2}\mu_0 I}{a}$
(2) $\frac{\mu_0 I}{2a}$
(3) $\frac{\mu_0 I}{\sqrt{2}a}$
(4) $\frac{2\mu_0 I}{a}$

Correct Answer: (3) $\frac{\mu_0 I}{\sqrt{2}a}$

Solution:

Calculate the magnetic field at the center due to one loop:

$$B = \frac{\mu_0 I}{2a}$$

Since there are two loops in perpendicular planes, the resultant magnetic field is:

$$B_{\text{net}} = \sqrt{B^2 + B^2} = \frac{\mu_0 I}{\sqrt{2}a}$$

Quick Tip

For magnetic fields from two perpendicular coils, use vector addition of magnetic fields to find the resultant.

---

40. The diffraction pattern of a light of wavelength 400 nm diffracting from a slit of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1st secondary maxima will be:

(1) 2 mm

(2) 2 cm

(3) 0.02 mm

(4) 0.2 mm

Correct Answer: (1) 2 mm

Solution:

The width of the first secondary maxima for single-slit diffraction is given by:

$$\text{Width of } 1^{\text{st}} \text{ secondary maxima} = \frac{\lambda D}{a}$$

where $\lambda = 400 \times 10^{-9}$ m, $a = 0.2 \times 10^{-3}$ m, and $D = 100\,\text{cm} = 1\,\text{m}$. Substitute the values:

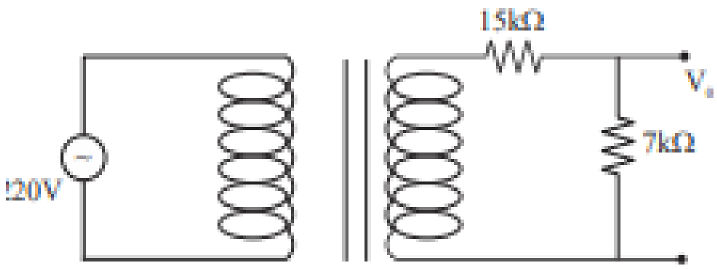
$$\text{Width of } 1^{\text{st}} \text{ secondary maxima} = \frac{400 \times 10^{-9} \times 1}{0.2 \times 10^{-3}} = 2\,\text{mm}$$

Quick Tip

For diffraction problems, remember to use the formula Width $= \frac{\lambda D}{a}$ for secondary maxima.

---

41. Primary coil of a transformer is connected to 220 V ac. Primary and secondary turns of the transformer are 100 and 10 respectively. The secondary coil of the transformer is connected to two series resistances shown in the figure. The output voltage V_0 is:



(1) 7 V

(2) 15 V

(3) 44 V

(4) 22 V

Correct Answer: (1) 7 V

Solution (Alternate Approach):

1. Calculate the Secondary Voltage Using the Turns Ratio: - The turns ratio for a transformer is given by:

$$\frac{\epsilon_1}{\epsilon_2} = \frac{N_1}{N_2}$$

Substitute $N_1 = 100$, $N_2 = 10$, and $\epsilon_1 = 220\,\text{V}$:

$$\epsilon_2 = \frac{N_2}{N_1} \times \epsilon_1 = \frac{10}{100} \times 220 = 22\,\text{V}$$

2. Determine the Equivalent Resistance of the Load: The load consists of two resistances, 15 Ω and 7 Ω, connected in series:

$$R_{\text{eq}} = 15 + 7 = 22\,\Omega$$

3. Calculate the Current in the Secondary Circuit: Using Ohm's law for the secondary circuit:

$$I = \frac{\epsilon_2}{R_{\text{eq}}} = \frac{22\,\text{V}}{22\,\Omega} = 1\,\text{A}$$

4. Calculate the Output Voltage Across the 7 Ω Resistor: The output voltage V_0 across the 7 Ω resistor is:

$$V_0 = I \times 7 = 1\,\text{A} \times 7\,\Omega = 7\,\text{V}$$

Quick Tip

To find the output voltage in transformer circuits with series resistances, first calculate the equivalent resistance, then use Ohm's law across the desired resistor.

42. The gravitational potential at a point above the surface of Earth is -5.12×10^7 J/kg and the acceleration due to gravity at that point is 6.4 m/s^2. Assume that the mean radius of Earth to be 6400 km. The height of this point above the Earth's surface is:

(1) 1600 km

(2) 540 km

(3) 1200 km

(4) 1000 km

Correct Answer: (1) 1600 km

Solution:

Using the formula for gravitational potential and gravitational field:

$$-\frac{GM_E}{R_E + h} = -5.12 \times 10^7 \quad \text{and} \quad \frac{GM_E}{(R_E + h)^2} = 6.4$$

$$16 \times 10^5$$

(This is in meter)

To convert divide it with 1000

$$h = 1600\,\text{km}$$

Quick Tip

In gravitational problems, use simultaneous equations for potential and acceleration due to gravity to solve for unknown height.

43. An electric toaster has resistance of 60 Ω at room temperature (27°C). The toaster is connected to a 220 V supply. If the current flowing through it reaches 2.75 A, the temperature attained by toaster is around: (if $\alpha = 2 \times 10^{-4}\,{}^\circ$C^{-1})

(1) 694°C

(2) 1235°C

(3) 1694°C

(4) 1667°C

Correct Answer: (3) 1694°C

Solution:

1. Calculate Resistance at Operating Temperature: - Given $V = 220\,\text{V}$ and $I = 2.75\,\text{A}$, use Ohm's law to find the resistance at the elevated temperature:

$$R = \frac{V}{I} = \frac{220}{2.75} = 80\,\Omega$$

2. Use Temperature Coefficient of Resistance Formula: - The relation between the resistance at room temperature R_0 and the resistance at temperature T is given by:

$$R = R_0(1 + \alpha\Delta T)$$

- Substitute $R = 80\,\Omega$, $R_0 = 60\,\Omega$, and $\alpha = 2 \times 10^{-4}\,°\text{C}^{-1}$, where $\Delta T = T - 27$:

$$80 = 60\left(1 + 2 \times 10^{-4} \times (T - 27)\right)$$

3. Solve for T: - Divide both sides by 60:

$$\frac{80}{60} = 1 + 2 \times 10^{-4} \times (T - 27)$$

- Simplify and isolate T:

$$\frac{4}{3} - 1 = 2 \times 10^{-4} \times (T - 27)$$

$$\frac{1}{3} = 2 \times 10^{-4} \times (T - 27)$$

$$T - 27 = \frac{1}{3 \times 2 \times 10^{-4}} = 1667$$

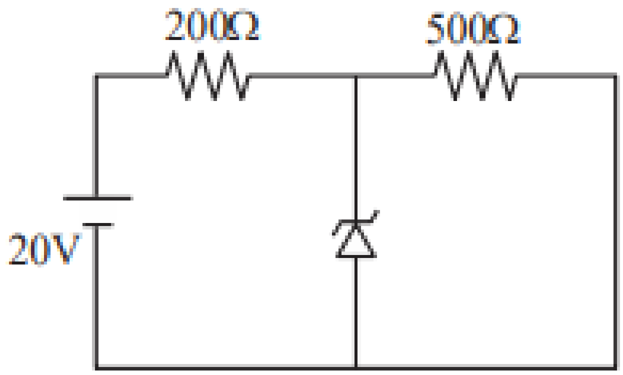
$$T = 1667 + 27 = 1694°\text{C}$$

Quick Tip

In temperature-dependent resistance problems, use $R = R_0(1 + \alpha\Delta T)$ to solve for the temperature if resistance is known.

---

44. A Zener diode of breakdown voltage 10V is used as a voltage regulator as shown in the figure. The current through the Zener diode is:



(1) 50 mA

(2) 0

(3) 30 mA

(4) 20 mA

Correct Answer: (3) 30 mA

Solution:

1. Calculate Total Current I_S: The total resistance in the circuit is $200\Omega + 500\Omega = 700\Omega$. The source voltage is 20V, so the total current I_S flowing through the series resistance is:

$$I_S = \frac{20}{700} = \frac{20}{700} \approx 28.6\,\text{mA}$$

2. Determine Voltage Across the 500 Ω Resistor and Zener Diode: Since the Zener diode is in breakdown mode (10V across it), the voltage drop across the 500 Ω resistor is also 10V. The current I_1 through the 500 Ω resistor is:

$$I_1 = \frac{10}{500} = 20\,\text{mA}$$

3. Calculate Current Through the Zener Diode I_Z: The current I_Z through the Zener diode is:

$$I_Z = I_S - I_1 = 28.6 - 20 \approx 30\,\text{mA}$$

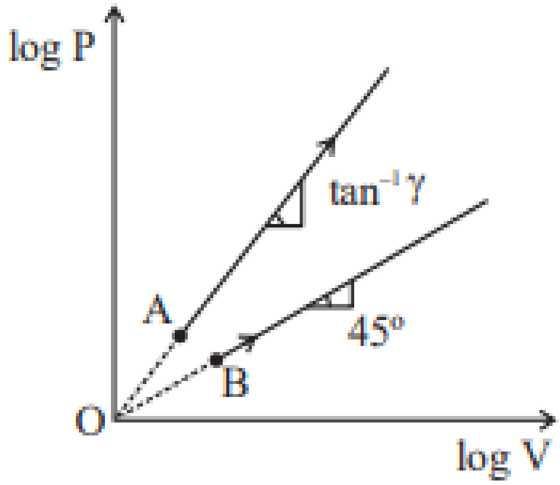
Quick Tip

For circuits with Zener diodes in breakdown mode, calculate total current first, then apply Ohm's law across each resistor to find individual currents.

---

45. Two thermodynamical processes are shown in the figure. The molar heat capac-

ity for process A and B are C_A and C_B. The molar heat capacity at constant pressure and constant volume are represented by C_P and C_V, respectively. Choose the correct statement.



(1) $C_B = \infty, C_A = 0$

(2) $C_A = 0$ and $C_B = \infty$

(3) $C_P > C_A = C_B = C_V$

(4) $C_A > C_P > C_V > C_B$

Solution:

1. Understanding the Slopes in the $\log P$ vs. $\log V$ Diagram: Process A has a slope of $\tan^{-1}\gamma$, where $\gamma = \frac{C_P}{C_V}$, indicating an adiabatic process (since $PV^\gamma =$ constant). Process B has a slope of 45° or $\tan^{-1} 1$, suggesting that it is an isothermal process (since $PV =$ constant).

2. Using Heat Capacities for Adiabatic and Isothermal Processes: For an adiabatic process ($PV^\gamma =$ constant), the heat capacity C_A is effectively zero because no heat exchange occurs ($dQ = 0$ for adiabatic). For an isothermal process ($PV =$ constant), the heat capacity C_B tends to infinity because any heat added is used to perform work without changing temperature.

3. Conclusion: Therefore, the correct statement is:

$$C_A = 0 \quad \text{and} \quad C_B = \infty$$

Quick Tip

For thermodynamic processes, recognize that an adiabatic process has zero heat capacity ($C = 0$) while an isothermal process has infinite heat capacity ($C \to \infty$).

46. The electrostatic potential due to an electric dipole at a distance r varies as:

(1) r

(2) $\frac{1}{r^2}$

(3) $\frac{1}{r^3}$

(4) $\frac{1}{r}$

Correct Answer: (2) $\frac{1}{r^2}$

Solution:

The electrostatic potential V at a point along the axial line of an electric dipole (aligned along the x-axis) is given by:

$$V = \frac{kp\cos\theta}{r^2}$$

where: k is Coulomb's constant, p is the dipole moment ($p = q \times d$, where q is the charge and d is the separation distance), r is the distance from the dipole to the point where the potential is being calculated, and θ is the angle between the dipole axis and the line connecting the dipole to the point.

Since the potential V is inversely proportional to r^2, we conclude that the electrostatic potential due to a dipole varies as:

$$V \propto \frac{1}{r^2}$$

Quick Tip

For an electric dipole, remember that the potential along the axial line varies as $\frac{1}{r^2}$, while the electric field due to the dipole varies as $\frac{1}{r^3}$.

---

47. A spherical body of mass 100 g is dropped from a height of 10 m from the ground. After hitting the ground, the body rebounds to a height of 5 m. The impulse of force imparted by the ground to the body is given by: (given $g = 9.8\,\text{m/s}^2$).

(1) 4.32 kg m/s

(2) 4.2 kg m/s

(3) 2.39 kg m/s

(4) 2.39 kg m/s

Correct Answer: (3) 2.39 kg m/s

Solution:

1. Calculate Velocity Just Before Hitting the Ground: Use energy conservation or kinematic equations to find the velocity when the object hits the ground:

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = \sqrt{196} = 14\,\text{m/s}$$

2. Calculate Velocity Just After Rebounding: After rebounding, the object reaches a height of 5 m. Use energy conservation to find the initial velocity after rebounding:

$$u = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} = \sqrt{98} = 7\,\text{m/s}$$

3. Determine the Change in Momentum (Impulse): The mass $m = 0.1$ kg. Change in momentum (impulse) I is given by:

$$I = m\,(v + u) = 0.1 \times (14 + 7) = 0.1(14 + 7\sqrt{2} = 2.39\,\text{kg m/s}$$

Quick Tip

For impulse problems, calculate velocities before and after the impact separately, then use $I = m(v + u)$ for total change in momentum.

---

48. A particle of mass m projected with a velocity u making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height is:

(1) $\frac{\sqrt{3}\,mu^2}{16\,g}$

(2) $\frac{\sqrt{3}\,mu^2}{2\,g}$

(3) $\frac{mu^3}{\sqrt{2}\,g}$

(4) zero

Correct Answer: (1) $\frac{\sqrt{3}\,mu^2}{16\,g}$

Solution:

1. Determine the Horizontal Component of Velocity: The horizontal component of the initial velocity $u_x = u\cos\theta$. Given $\theta = 30°$:

$$u_x = u\cos 30° = u \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}\,u}{2}$$

2. Calculate the Vertical Component of Initial Velocity: The vertical component of the initial velocity $u_y = u\sin\theta$. With $\theta = 30°$:

$$u_y = u\sin 30° = u \times \frac{1}{2} = \frac{u}{2}$$

3. Find Time to Reach Maximum Height: At maximum height, the vertical velocity becomes zero. Using $v_y = u_y - gt$:

$$0 = \frac{u}{2} - gt \Rightarrow t = \frac{u}{2g}$$

4. Calculate Maximum Height H: Use the equation $H = u_y t - \frac{1}{2}gt^2$:

$$H = \frac{u}{2} \times \frac{u}{2g} - \frac{1}{2}g\left(\frac{u}{2g}\right)^2 = \frac{u^2}{4g} - \frac{u^2}{8g} = \frac{u^2}{8g}$$

5. Calculate Angular Momentum about the Point of Projection: Angular momentum L about the point of projection is given by $L = mu_xH$. - Substituting $u_x = \frac{\sqrt{3}u}{2}$ and $H = \frac{u^2}{8g}$:

$$L = m \times \frac{\sqrt{3}u}{2} \times \frac{u^2}{8g} = \frac{\sqrt{3}\,mu^3}{16g}$$

Quick Tip

To find angular momentum at the highest point, multiply the horizontal velocity by the maximum height and mass.

---

49. At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(1) 80 K

(2) -73 K

(3) 4 K

(4) 20 K

Correct Answer: (4) 20 K

Solution:

1. Using the Formula for Root Mean Square (r.m.s.) Velocity: The r.m.s. velocity v_{rms} for a gas is given by:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where R is the gas constant, T is the temperature, and M is the molar mass of the gas.

2. Set up the Equation for Hydrogen and Oxygen: To find the temperature at which the r.m.s. velocity of hydrogen equals that of oxygen at 47°C, we set:

$$\sqrt{\frac{3RT_{\text{H}_2}}{M_{\text{H}_2}}} = \sqrt{\frac{3RT_{\text{O}_2}}{M_{\text{O}_2}}}$$

3. Isolate T_{H_2}: Square both sides to remove the square root:

$$\frac{3RT_{\text{H}_2}}{M_{\text{H}_2}} = \frac{3RT_{\text{O}_2}}{M_{\text{O}_2}}$$

Simplify by canceling $3R$ on both sides:

$$T_{\text{H}_2} = T_{\text{O}_2} \times \frac{M_{\text{H}_2}}{M_{\text{O}_2}}$$

4. Substitute Values for Molar Mass and Temperature: Given $T_{\text{O}_2} = 47°\text{C} = 320\,\text{K}$,

$$T_{\text{H}_2} = 320 \times \frac{2}{32} = 20\,\text{K}$$

Quick Tip

To find an equivalent r.m.s. temperature for different gases, use the inverse molar mass ratio.

---

50. A series L, R circuit connected with an ac source $E = (25\sin(1000\,t))\,V$ has a power factor of $\frac{1}{\sqrt{2}}$. If the source of emf is changed to $E = (20\sin(2000\,t))\,V$, the new power factor of the circuit will be:

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{5}}$

(4) $\frac{1}{\sqrt{7}}$

Correct Answer: (3) $\frac{1}{\sqrt{5}}$

Solution:

1. Determine Initial Reactance X_L: Since the initial power factor $\cos\theta = \frac{1}{\sqrt{2}}$, we have $\tan\theta = 1$, meaning $X_L = R$. With the initial angular frequency $\omega_1 = 1000\,\text{rad/s}$, we can write $X_L = \omega_1 L = R$.

2. Calculate Reactance at New Frequency: For the new frequency, $\omega_2 = 2000\,\text{rad/s}$, the new inductive reactance becomes:

$$X'_L = \omega_2 L = 2\omega_1 L = 2R$$

3. Determine New Power Factor: With the new reactance $X'_L = 2R$, we find $\tan\theta' = \frac{X'_L}{R} = 2$, which gives:

$$\cos\theta' = \frac{1}{\sqrt{1+(2)^2}} = \frac{1}{\sqrt{5}}$$

4. Conclusion: The new power factor is therefore:

$$\frac{1}{\sqrt{5}}$$

Quick Tip

For series L, R circuits, if the frequency changes, calculate the new reactance X'_L and use it to find the updated power factor.

Section B

51. The horizontal component of earth's magnetic field at a place is $3.5 \times 10^{-5}\,T$. A very long straight conductor carrying current of $\sqrt{2}\,A$ in the direction from South East to North West is placed. The force per unit length experienced by the conductor is:

Correct Answer: 35×10^{-6} N/m

Solution:

1. Calculate Magnetic Force per Unit Length: The force per unit length $\frac{F}{\ell}$ on a current-carrying conductor in a magnetic field is given by:

$$\frac{F}{\ell} = iB\sin\theta$$

where: - $i = \sqrt{2}\,A$ (current in the conductor), - $B = 3.5 \times 10^{-5}\,T$ (magnetic field), - $\theta = 45^\circ$ (angle between current direction and magnetic field).

2. Substitute Values: Using $\sin 45^\circ = \frac{1}{\sqrt{2}}$:

$$\frac{F}{\ell} = (\sqrt{2}) \times (3.5 \times 10^{-5}) \times \frac{1}{\sqrt{2}} = 35 \times 10^{-6}\,\text{N/m}$$

3. Conclusion: The force per unit length experienced by the conductor is:

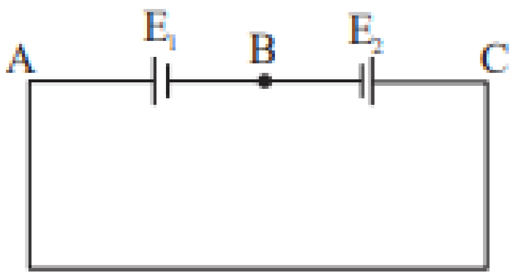
$$35 \times 10^{-6}\,\text{N/m}$$

Quick Tip

When calculating force on a current-carrying conductor in a magnetic field, remember to use $F = iB\sin\theta$ and evaluate $\sin\theta$ based on the angle given.

---

52. Two cells are connected in opposition as shown. Cell E_1 is of 8 V emf and 2 Ω internal resistance; the cell E_2 is of 2 V emf and 4 Ω internal resistance. The terminal potential difference of cell E_2 is:



Correct Answer: 6 V

Solution:

1. Identify the Net Emf in Circuit: Since the cells are connected in opposition, the net emf $E_{\text{net}} = E_1 - E_2 = 8 - 2 = 6\,\text{V}$.

2. Calculate Total Internal Resistance: Total internal resistance $R_{\text{total}} = R_1 + R_2 = 2 + 4 = 6\,\Omega$.

3. Determine the Current in Circuit: Using Ohm's law, the current I in the circuit is:

$$I = \frac{E_{\text{net}}}{R_{\text{total}}} = \frac{6}{6} = 1\,\text{A}$$

4. Calculate Terminal Potential Difference of E_2: The potential difference across E_2 considering the internal drop is:

$$V_{E_2} = E_2 + I \times R_2 = 2 + (1 \times 4) = 6\,\text{V}$$

Quick Tip

In circuits with cells in opposition, calculate the net emf and use it to determine the current before finding potential differences across individual cells.

---

53. An electron of hydrogen atom on an excited state is having energy $E_n = -0.85$ eV. The maximum number of allowed transitions to lower energy level is:

Correct Answer: 6

Solution:

1. Calculate Quantum Number n: Use the energy formula for hydrogen:

$$E_n = -\frac{13.6}{n^2} = -0.85$$

Solving, $n = 4$.

2. Determine Number of Transitions: The number of transitions from $n = 4$ is:

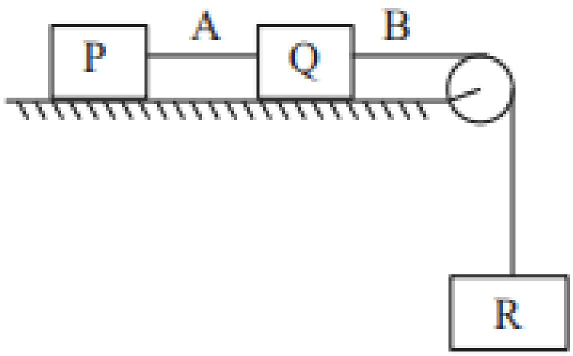
$$\text{No. of transitions} = \frac{n(n-1)}{2} = \frac{4 \times (4-1)}{2} = 6$$

Quick Tip

For allowed transitions, calculate the number of pairs that can form with the given quantum state.

---

54. Each of three blocks $P, Q,$ and R shown in figure has a mass of 3 kg. Each of the wire A and B has cross-sectional area 0.005 cm^2 and Young's modulus 2×10^{11} N/m^2. Neglecting friction, the longitudinal strain on wire B is $\times 10^{-4}$.



Correct Answer: 2

Solution:

1. Calculate the Total Force Acting on Block R: The total force on block R due to its weight is:

$$F = m \times g = 3\,\text{kg} \times 10\,\text{m/s}^2 = 30\,\text{N}$$

2. Determine the Tension T_1 in Wire B: Assuming the system is in equilibrium, the net force acting on $P, Q,$ and R needs to balance out, with wire B supporting the tension:

$$T_1 = F - T_2 = 20\,\text{N}$$

3. Calculate Longitudinal Strain: Strain $= \frac{\text{stress}}{Y}$ where stress $= \frac{T_1}{A}$ and $A = 0.005\,\text{cm}^2 = 0.5 \times 10^{-6}\,\text{m}^2$:

$$\text{strain} = \frac{T_1}{A \times Y} = \frac{20}{0.5 \times 10^{-6} \times 2 \times 10^{11}} = 2 \times 10^{-4}$$

Quick Tip

When calculating strain, ensure that all units are consistent, especially converting areas to square meters if given in square centimeters.

---

55. The distance between object and its two times magnified real image as produced by a convex lens is 45 cm. The focal length of the lens used is _______ cm.

Correct Answer: 10 cm

Solution:

1. Understanding the Given Condition: Since the image is real, inverted, and twice the size of

the object, we know:

$$m = \frac{v}{u} = -2 \Rightarrow v = -2u$$

2. Set up Equation Using Total Distance: The distance between the object and the image is 45 cm, so:

$$|v - u| = 45\,\text{cm}$$

Substitute $v = -2u$ into the equation:

$$|-2u - u| = 45$$

$$3|u| = 45 \Rightarrow u = -15\,\text{cm}$$

3. Determine Image Distance v: Using $v = -2u$:

$$v = -2 \times (-15) = 30\,\text{cm}$$

4. Calculate Focal Length Using Lens Formula: Apply the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Substitute $u = -15\,\text{cm}$ and $v = 30\,\text{cm}$:

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{-15} = \frac{1}{30} + \frac{1}{15} = \frac{1+2}{30} = \frac{3}{30} = \frac{1}{10}$$

$$f = +10\,\text{cm}$$

Quick Tip

In convex lenses, when a real image is formed with magnification $m = -2$, the object distance u is twice the focal length.

---

56. The displacement and the increase in the velocity of a moving particle in the time interval of t to $(t+1)$ seconds are 125 m and 50 m/s, respectively. The distance travelled by the particle in $(t+2)^{\text{th}}$ second is _________ m.

Correct Answer: 175

Solution:

1. Given Information and Assumptions: The displacement S during t to $(t+1)$ seconds is 125 m. The increase in velocity is 50 m/s, implying a constant acceleration a.

2. Using Equations of Motion: We know:

$$v = u + at$$

Given $u + 50 = u + a \Rightarrow a = 50\,\text{m/s}^2$.

3. Calculate Initial Velocity u: Using the second equation of motion for displacement:

$$S = ut + \frac{1}{2}at^2$$

Substitute $S = 125$ and $a = 50$ to find u:

$$125 = u + \frac{1}{2} \times 50 \times 1^2$$

$$125 = u + 25 \Rightarrow u = 100\,\text{m/s}$$

4. Calculate Distance Travelled in $(t + 2)^{\text{th}}$ Second: Distance travelled in n^{th} second:

$$S_n = u + \frac{a}{2}[2n - 1]$$

Substitute $u = 100\,\text{m/s}$, $a = 50\,\text{m/s}^2$, and $n = t + 2 = 3$:

$$S_3 = 100 + \frac{50}{2} \times (2 \times 3 - 1) = 100 + 25 \times 5 = 100 + 125 = 175\,\text{m}$$

Quick Tip

To find the distance in the n^{th} second, use $S_n = u + \frac{a}{2}[2n-1]$ directly for quicker results.

---

57. A capacitor of capacitance C and potential V has energy E. It is connected to another capacitor of capacitance $2C$ and potential $2V$. Then the loss of energy is $\frac{x}{3}E$, where x is __________.

Correct Answer: 2

Solution:

1. Calculate Initial Energy of Each Capacitor:

Energy of the first capacitor, $E_1 = \frac{1}{2}CV^2 = E$. Energy of the second capacitor, $E_2 = \frac{1}{2} \times 2C \times (2V)^2 = 4E$.

Therefore, the total initial energy, $E_{\text{initial}} = E_1 + E_2 = E + 4E = 5E$.

2. Final Combined Capacitance and Potential:

When connected, the total capacitance $C_{\text{eq}} = C + 2C = 3C$. The final potential V_f across the combined system can be found using charge conservation:

$$Q_{\text{total}} = C \times V + 2C \times 2V = CV + 4CV = 5CV$$

$$V_f = \frac{Q_{\text{total}}}{C_{\text{eq}}} = \frac{5CV}{3C} = \frac{5V}{3}$$

3. Calculate Final Energy:

Final energy in the system:

$$E_{\text{final}} = \frac{1}{2}C_{\text{eq}}V_f^2 = \frac{1}{2} \times 3C \times \left(\frac{5V}{3}\right)^2 = \frac{1}{2} \times 3C \times \frac{25V^2}{9} = \frac{25}{6}CV^2 = \frac{5}{3}E$$

4. Calculate Energy Loss:

Energy loss $\Delta E = E_{\text{initial}} - E_{\text{final}}$:

$$\Delta E = 5E - \frac{5}{3}E = \frac{15E - 5E}{3} = \frac{10E}{3}$$

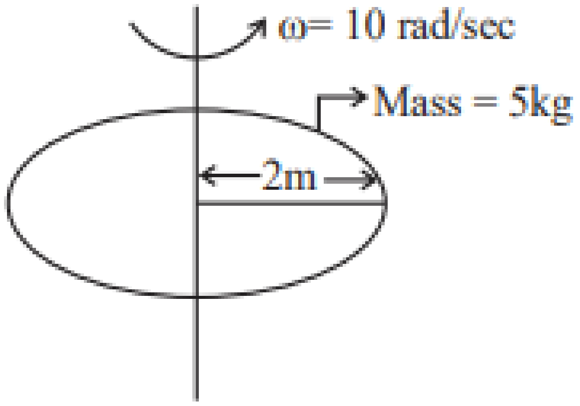
Thus, $x = 2$.

Quick Tip

To find energy loss when capacitors are connected, calculate initial and final energies separately and use conservation of charge to determine the final voltage.

---

58. Consider a disc of mass 5 kg, radius 2 m, rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both discs continue to rotate together without slipping is ________ J.



Correct Answer: 250

Solution:

1. Calculate Initial Moment of Inertia and Kinetic Energy:

For the first disc:

$$I_1 = \frac{1}{2}MR^2 = \frac{1}{2} \times 5 \times (2)^2 = 10\,\text{kg} \times \text{m}^2$$

Initial angular velocity $\omega = 10\,\text{rad/s}$. Initial kinetic energy:

$$E_i = \frac{1}{2}I_1\omega^2 = \frac{1}{2} \times 10 \times (10)^2 = 500\,\text{J}$$

2. Final Moment of Inertia and Angular Velocity:

When the second disc is placed on top, the combined moment of inertia becomes:

$$I_f = I_1 + I_2 = 10 + 10 = 20\,\text{kg} \times \text{m}^2$$

Using conservation of angular momentum:

$$I_1\omega = I_f\omega_f \Rightarrow 10 \times 10 = 20 \times \omega_f$$

$$\omega_f = 5\,\text{rad/s}$$

3. Calculate Final Kinetic Energy:

$$E_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2} \times 20 \times (5)^2 = 250\,\text{J}$$

4. Energy Dissipated:

Energy dissipated $\Delta E = E_i - E_f$:

$$\Delta E = 500 - 250 = 250\,\text{J}$$

Quick Tip

When two identical discs rotate together without slipping, use conservation of angular momentum to find the new angular velocity and calculate energy dissipation.

---

59. In a closed organ pipe, the frequency of the fundamental note is 30 Hz. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to 110 Hz. If the organ pipe has a cross-sectional area of $2\,\text{cm}^2$, the amount of water poured in the organ tube is ________g. (Take speed of sound in air as 330 m/s)

Correct Answer: 400 g

Solution:

1. Calculate the Initial Length of Air Column:

For a closed organ pipe, the fundamental frequency is given by:

$$f = \frac{V}{4\ell_1}$$

where $f = 30\,\text{Hz}$ and $V = 330\,\text{m/s}$.

Solving for ℓ_1:

$$\ell_1 = \frac{V}{4 \times f} = \frac{330}{4 \times 30} = \frac{330}{120} = \frac{11}{4}\,\text{m} = 2.75\,\text{m}$$

2. Calculate the New Length of Air Column:

When the frequency increases to 110 Hz:

$$f' = \frac{V}{4\ell_2}$$

Solving for ℓ_2:

$$\ell_2 = \frac{V}{4 \times f'} = \frac{330}{4 \times 110} = \frac{330}{440} = \frac{3}{4}\,\text{m} = 0.75\,\text{m}$$

3. Determine the Change in Length:

$$\Delta\ell = \ell_1 - \ell_2 = 2.75 - 0.75 = 2\,\text{m}$$

4. Calculate the Volume of Water Added:

The volume of water added corresponds to the volume of the air column displaced:

$$\text{Change in volume} = A \times \Delta\ell = 2\,\text{cm}^2 \times 200\,\text{cm} = 400\,\text{cm}^3$$

5. Convert Volume to Mass:

Given that the density of water $\rho = 1\,\text{g/cm}^3$:

$$M = \rho \times \text{Volume} = 1 \times 400 = 400\,\text{g}$$

Quick Tip

In closed organ pipes, the fundamental frequency is inversely proportional to the length of the air column. To find the amount of water added, calculate the change in length required for the frequency change and then use the cross-sectional area to find the volume.

60. A ceiling fan having 3 blades of length 80 cm each is rotating with an angular velocity of 1200 rpm. The magnetic field of earth in that region is 0.5 G and angle of dip is $30°$. The emf induced across the blades is $N\pi \times 10^{-5}\,V$. The value of N is __________.

Correct Answer: 32

Solution:

1. Calculate the Effective Vertical Component of the Magnetic Field:

Given:

$$B = 0.5\,\text{G} = 0.5 \times 10^{-4}\,\text{T}$$

The vertical component of the magnetic field B_v considering the angle of dip $\delta = 30°$ is:

$$B_v = B\sin\delta = 0.5 \times 10^{-4} \times \sin 30° = 0.5 \times 10^{-4} \times \frac{1}{2} = \frac{1}{4} \times 10^{-4}\,\text{T}$$

2. Convert Angular Velocity from rpm to rad/s:

Angular velocity ω in rad/s is given by:

$$\omega = 2\pi \times f = 2\pi \times \frac{1200}{60} = 2\pi \times 20 = 40\pi\,\text{rad/s}$$

3. Determine the Radius of Rotation:

The length of each blade is $\ell = 80\,\text{cm} = 0.8\,\text{m}$. Therefore, the effective radius r of rotation is:

$$r = 0.8\,\text{m}$$

4. Calculate the Induced emf:

The emf ε induced across the tips of the blades (assuming the emf induced across two opposite ends) is given by:

$$\varepsilon = \frac{1}{2}B_v\omega r^2$$

Substituting the values:

$$\varepsilon = \frac{1}{2} \times \frac{1}{4} \times 10^{-4} \times 40\pi \times (0.8)^2$$

Simplifying further:

$$\varepsilon = \frac{1}{2} \times \frac{1}{4} \times 10^{-4} \times 40\pi \times 0.64 = 32\pi \times 10^{-5}\,\text{V}$$

5. Conclude the Value of N:

Comparing with $N\pi \times 10^{-5}$ V, we find:

$$N = 32$$

Quick Tip

For rotating systems in a magnetic field, use the effective vertical component of the magnetic field and calculate the induced emf using the formula $\varepsilon = \frac{1}{2}B_v\omega r^2$.

Chemistry Section A

61. Given below are two statements:

Statement-I: The gas liberated on warming a salt with dilute H_2SO_4, turns a piece of paper dipped in lead acetate into black; it is a confirmatory test for sulphide ion.

Statement-II: In statement-I the colour of paper turns black because of formation of lead sulphide.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement-I and Statement-II are false

(2) Statement-I is false but Statement-II is true

(3) Statement-I is true but Statement-II is false

(4) Both Statement-I and Statement-II are true

Correct Answer: (3) Statement-I is true but Statement-II is false

Solution:

1.Identify the Reaction Involved: When a salt containing sulphide ion is treated with dilute H_2SO_4, hydrogen sulphide gas (H_2S) is liberated.

2.Reaction with Lead Acetate: H_2S gas reacts with lead acetate solution present on the paper, resulting in the formation of black lead sulphide (PbS):

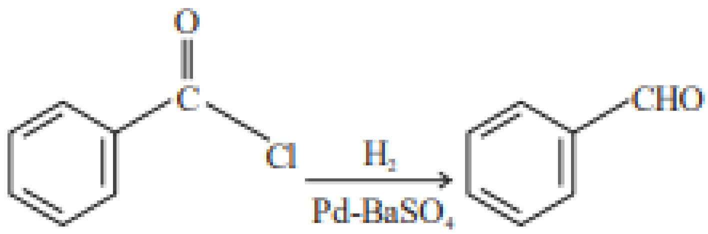
$$(CH_3COO)_2Pb + H_2S \rightarrow PbS + 2CH_3COOH$$

3.Explanation of Statements: Statement-I is correct, as the blackening of lead acetate paper confirms the presence of sulphide ions.

Statement-II is incorrect because it suggests that the paper turns black due to formation of "lead sulphite," which is incorrect. The actual black compound is lead sulphide (PbS).

Quick Tip

The blackening of lead acetate paper in the presence of H_2S gas is a confirmatory test for sulphide ions, forming PbS (lead sulphide).



62. This reduction reaction is known as:

(1) Rosenmund reduction

(2) Wolff-Kishner reduction

(3) Stephen reduction

(4) Etard reduction

Correct Answer: (1) Rosenmund reduction

Solution:

1. Identify the Reaction Components: The reaction involves the reduction of an acid chloride to an aldehyde using H_2 gas in the presence of palladium on barium sulfate (Pd-$BaSO_4$).

2. Characteristics of Rosenmund Reduction: This specific reduction, where an acid chloride is reduced to an aldehyde, is known as the Rosenmund reduction.

In this process, the use of Pd-$BaSO_4$ prevents further reduction of the aldehyde to an alcohol.

3. Conclude with the Recognized Reaction: Thus, the partial reduction of the acid chloride to an aldehyde in this reaction confirms it as a Rosenmund reduction.

Quick Tip

Rosenmund reduction specifically reduces acid chlorides to aldehydes using Pd-$BaSO_4$ as a selective catalyst.

---

63. Sugar which does not give reddish brown precipitate with Fehling's reagent is:

(1) Sucrose

(2) Lactose

(3) Glucose

(4) Maltose

Correct Answer: (1) Sucrose

Solution:

1. Identify the Requirement for Fehling's Test: Fehling's reagent specifically reacts with reducing sugars that contain a free aldehyde or ketone group.

2. Evaluate the Sugars: Sucrose: It is a non-reducing sugar because it lacks a free hemiacetal group due to its glycosidic linkage between glucose and fructose units.

Lactose, Glucose, and Maltose: These are reducing sugars with free hemiacetal groups, allowing them to react with Fehling's reagent and produce a reddish-brown precipitate.

3. Conclusion: Since sucrose does not have a free aldehyde or ketone group, it does not give a positive test with Fehling's solution.

Quick Tip

Non-reducing sugars, like sucrose, lack a free hemiacetal group, preventing them from reacting with Fehling's reagent.

---

64. Given below are the two statements: one is labeled as Assertion (A) and the other is labeled as Reason (R).

Assertion (A): There is a considerable increase in covalent radius from N to P. However, from As to Bi only a small increase in covalent radius is observed.

Reason (R): Covalent and ionic radii in a particular oxidation state increases down the group.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) (A) is false but (R) is true

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is true but (R) is false

(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Solution:

1. Analyze the Assertion (A): The increase in covalent radius from N to P is significant due to additional electron shells. However, from As to Bi, the increase in size is smaller because of the presence of poor shielding by d- and f-electrons.

2. Analyze the Reason (R): The statement in (R) is generally true as covalent and ionic radii tend to increase down the group in the periodic table. However, this reason does not specifically explain the smaller increase observed from As to Bi.

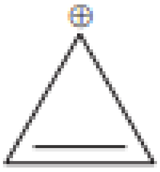
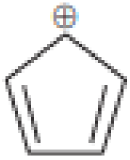
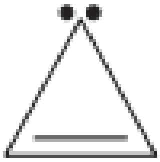
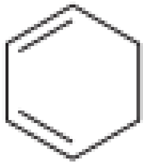
3. Conclusion: Although both statements are individually true, the reason provided does not correctly explain the observed trend in covalent radii from As to Bi.

Quick Tip

For heavier elements, the presence of poor shielding by d- and f-orbitals results in only a slight increase in atomic radius down the group.

---

65. Which of the following molecule/species is most stable?

(1) 
(2) 
(3) 
(4) 

Correct Answer: (1)

Solution:

1. Identify Aromatic Stability: The most stable species among the options will be the one that is aromatic as aromatic compounds exhibit additional stability due to delocalized -electrons.

2. Analyze Each Structure: Option (1): This structure satisfies Huckel's rule $(4n + 2)$ for aromaticity and has a planar conjugated ring structure, making it aromatic and stable. Other Options: Do not satisfy Huckel's rule or are non-aromatic.

3. Conclusion: Option (1) is the most stable due to its aromaticity.

Quick Tip

Aromatic compounds are exceptionally stable because of the delocalized -electrons that follow Huckel's rule.

---

66. Diamagnetic Lanthanoid ions are:

(1) Nd^{3+} and Eu^{3+}

(2) La^{3+} and Ce^{4+}

(3) Nd^{3+} and Ce^{4+}

(4) Lu^{3+} and Eu^{3+}

Correct Answer: (2) La^{3+} and Ce^{4+}

Solution:

1. Understand Diamagnetism: Diamagnetic species have all electrons paired, which means the ions should have no unpaired electrons in their electron configuration.

2. Examine Electron Configurations: Ce^{4+}: [Xe] $4f^0$ - All electrons are paired, hence diamagnetic. La^{3+}: [Xe] - Also has no unpaired electrons, hence diamagnetic.

3. Conclusion: La^{3+} and Ce^{4+} are both diamagnetic as they lack unpaired electrons.

Quick Tip

Ions with completely filled or empty f-orbitals tend to be diamagnetic.

---

67. Aluminium chloride in acidified aqueous solution forms an ion having geometry:

(1) Octahedral

(2) Square Planar

(3) Tetrahedral

(4) Trigonal bipyramidal

Correct Answer: (1) Octahedral

Solution:

1. Determine the Complex Ion Formed: In acidified aqueous solution, $AlCl_3$ forms a complex ion, typically $[Al(H_2O)_6]^{3+}$.

2. Analyze Geometry: - This complex ion has six ligands (water molecules) coordinated around the central aluminum ion. - Six ligands around a central atom generally form an octahedral geometry.

3. Conclusion: The geometry of $[Al(H_2O)_6]^{3+}$ is octahedral.

Quick Tip

Complexes with six ligands around the central atom generally adopt an octahedral geometry.

---

68. Given below are two statements:

Statement-I: The orbitals having same energy are called as degenerate orbitals.

Statement-II: In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement-I is true but Statement-II is false

(2) Both Statement-I and Statement-II are true

(3) Both Statement-I and Statement-II are false

(4) Statement-I is false but Statement-II is true

Correct Answer: (1) Statement-I is true but Statement-II is false

Solution:

1. Analyze Statement-I: The definition of degenerate orbitals is accurate, as orbitals with the same energy level are indeed degenerate.

2. Examine Statement-II: In a hydrogen atom, all orbitals in the same principal quantum level (e.g., 3s, 3p, 3d) are degenerate, as they have the same energy. Therefore, Statement-II is incorrect.

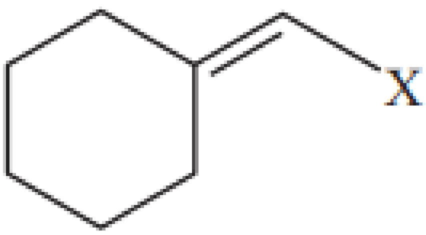
3. Conclusion: Statement-I is correct, but Statement-II is incorrect.

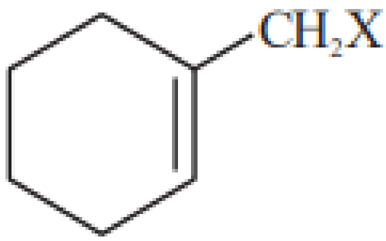
Quick Tip

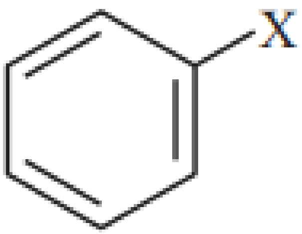
For single-electron atoms like hydrogen, all orbitals within the same principal quantum number are degenerate.

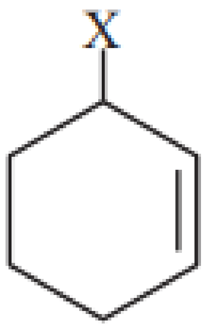
---

69. Example of vinylic halide is:

(1)


(2)


(3)


(4)


Correct Answer: (1)

Solution:

1. Understanding Vinyl Halide: Vinyl halides are compounds in which a halogen atom is directly attached to an sp^2 hybridized aliphatic carbon atom, which forms part of a double bond.

2. Examine Each Structure: Option (1): Contains a halogen attached to an sp^2 hybridized carbon in a double-bonded system, classifying it as a vinyl halide. Option (2): Represents an allyl halide, with the halogen attached to an sp^3 hybridized carbon adjacent to a double bond. Options (3) and (4): Both structures are aryl halides, with the halogen attached to an aromatic ring.

3. Conclusion: Option (1) is the correct example of a vinyl halide.

> **Quick Tip**
>
> Vinyl halides have a halogen attached to an sp^2 hybridized carbon in a double-bonded system, whereas allyl halides have the halogen on an sp^3 carbon next to a double bond.

---

70. Structure of 4-Methylpent-2-enal is:

(1) $H_2C = C - CH_2 - C = C - H$

(2) $CH_3 - CH_2 - C = CH - C = CH$

(3) $CH_3 - CH_2 - CH = C - CH_3$

(4) $CH_3 - CH = CH - C = C - H$

Correct Answer: (4) $CH_3 - CH = CH - C = C - H$

Solution:

1. Naming the Structure: The name "4-Methylpent-2-enal" provides the key structural features of the molecule. The "pent" indicates a five-carbon chain. "2-enal" indicates the presence of a double bond at the second carbon and an aldehyde group at the terminal position. The "4-methyl" substituent specifies a methyl group attached to the fourth carbon.

2. Construct the Structure Step-by-Step: Step 1: Place a five-carbon chain.

Step 2: Insert a double bond between carbons 2 and 3.

Step 3: Attach an aldehyde group (C=O with an H) to the first carbon.

Step 4: Add a methyl group to the fourth carbon.

3. Verify Each Option: Only Option (4) matches this structure, with the correct placement of double bond, aldehyde group, and methyl substituent.

4. Conclusion: The structure in Option (4) is correct for 4-Methylpent-2-enal.

Quick Tip

To identify structures based on IUPAC names, focus on the root chain, substituents, and functional groups to systematically construct the molecule.

---

71. Match List-I with List-II

List-I (Molecule)	List-II (Shape)
(A) BrF_5	(I) T-shape
(B) H_2O	(II) See-saw
(C) ClF_3	(III) Bent
(D) SF_4	(IV) Square pyramidal

Options:

1. (A) - I, (B) - III, (C) - IV, (D) - II
2. (A) - II, (B) - I, (C) - III, (D) - IV
3. (A) - III, (B) - IV, (C) - I, (D) - II
4. (A) - IV, (B) - III, (C) - I, (D) - II

Correct Answer: (4)

Solution (Alternate Approach):

1. Analyze Each Molecule Based on VSEPR Theory: BrF_5: The molecule has five bonded pairs and one lone pair around bromine, leading to a **square pyramidal** shape.

H_2O: Water has two bonded pairs and two lone pairs, giving it a **bent** shape.

ClF_3: Chlorine trifluoride has three bonded pairs and two lone pairs, resulting in a **T-shape**.

SF_4: Sulfur tetrafluoride has four bonded pairs and one lone pair, leading to a **see-saw** shape.

2. Match Each Molecule with the Correct Shape:

(A) BrF_5 - Square pyramidal

(B) H_2O - Bent

(C) ClF_3 - T-shape

(D) SF_4 - See-saw

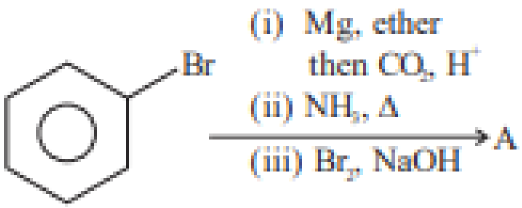
3. Conclusion: Based on the shapes identified above, the correct answer is Option (4).

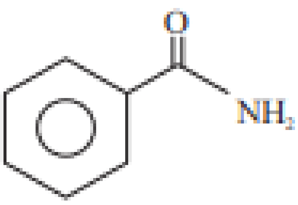
Quick Tip

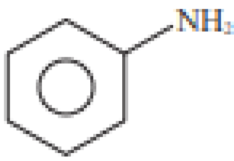
Use VSEPR theory to determine the shape by considering the bonded pairs and lone pairs around the central atom.

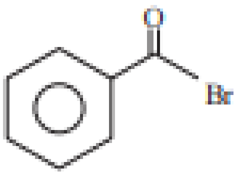
---

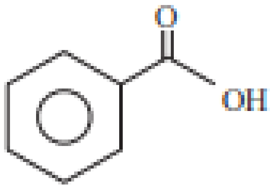
72. The final product A, formed in the following multistep reaction sequence is:



(1) 

(2) 

(3) 

(4) 

Correct Answer: (2)

Solution:

1. **Step 1:** The reaction starts with the bromobenzene reacting with Mg in ether to form the Grignard reagent (phenylmagnesium bromide).

2. **Step 2:** Carbon dioxide is then introduced, which reacts with the Grignard reagent to form benzoic acid upon acidic workup.

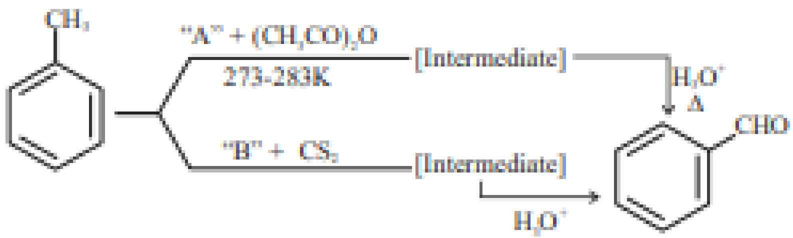
3. **Step 3:** The benzoic acid undergoes a Hoffmann bromamide reaction, where the amide is converted to an amine, resulting in the formation of aniline.

Thus, the final product formed is aniline (structure in option (2)).

> Quick Tip
>
> In the Hoffmann bromamide reaction, the conversion of amides to amines involves the loss of a carbonyl group.

---

73. In the given reactions, identify the reagent A and reagent B.



(1) A-CrO_3, B-CrO_3

(2) A-CrO_3, B-CrO_2Cl_2

(3) A-CrO_2Cl_2, B-CrO_2Cl_2

(4) A-CrO_2Cl_2, B-CrO_3

Correct Answer: (2) A-CrO_3, B-CrO_2Cl_2

Solution:

1. **Step 1:** The reaction starts with toluene, which is oxidized by reagent **A** (CrO_3) in acetic anhydride $(CH_3CO)_2O$. This oxidation step is known as the Etard reaction, where toluene is converted into benzylidene diacetate as an intermediate.

2. **Step 2:** The intermediate benzylidene diacetate is then treated with reagent **B** (CrO_2Cl_2), followed by hydrolysis (H_2O^+) to yield benzaldehyde (C_6H_5CHO).

Thus, the correct reagents are A-CrO_3 and B-CrO_2Cl_2, which is represented by option (2).

> Quick Tip
>
> The Etard reaction is a selective oxidation of toluene to benzaldehyde using CrO_3 in the presence of anhydride. This avoids overoxidation to benzoic acid.

74. Given below are two statement one is labeled as Assertion (A) and the other is labeled as **Reason (R)**.

Assertion (A): $CH_2 = CH - CH_2 - Cl$ is an example of allyl halide.

Reason (R): Allyl halides are the compounds in which the halogen atom is attached to sp^2 hybridised carbon atom.

In the light of the two above statements, choose the most appropriate answer from the options given below:

(1) **(A)** is true but **(R)** is false

(2) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Correct Answer: (1) **(A)** is true but **(R)** is false **Solution:**

The molecule $CH_2 = CH - CH_2 - Cl$ has a halogen attached to an sp^3 hybridised carbon, making it an allyl halide. **Reason (R)** incorrectly states that allyl halides have the halogen attached to an sp^2 hybridised carbon, which is why **(R)** is false.

Quick Tip

Allyl halides have a halogen attached to a carbon adjacent to a double bond, specifically an sp^3 carbon.

---

75. What happens to freezing point of benzene when small quantity of naphthalene is added to benzene?

(1) Increases

(2) Remains unchanged

(3) First decreases and then increases

(4) Decreases

Correct Answer: (4) Decreases

Solution: On addition of naphthalene to benzene, there is a depression in the freezing point of benzene. This is due to the presence of a non-volatile solute (naphthalene), which lowers the freezing point.

Quick Tip

The freezing point of a solvent decreases when a non-volatile solute is added due to the lowering of vapor pressure.

---

76. Match List-I with List-II

List-I Species	**List-II** Electronic Distribution
(A) Cr^{+2}	(I) $3d^8$
(B) Mn^+	(II) $3d^5 4s^1$
(C) Ni^{+2}	(III) $3d^4$
(D) V^+	(IV) $3d^3 4s^1$

Choose the correct answer from the options given below:

(1) (A)-I, (B)-II, (C)-III, (D)-IV

(2) (A)-III, (B)-IV, (C)-I, (D)-II

(3) (A)-IV, (B)-III, (C)-I, (D)-II

(4) (A)-II, (B)-I, (C)-IV, (D)-III

Correct Answer: (2) (A)-III, (B)-IV, (C)-I, (D)-II

Solution: Each ion has a specific electron configuration:

- **Cr^{+2}**: After losing two electrons, Cr has an electronic configuration of $3d^4$.
- **Mn^+**: Losing one electron results in the configuration $3d^5 4s^1$.
- **Ni^{+2}**: Removal of two electrons leads to a configuration of $3d^8$.
- **V^+**: Removing one electron yields $3d^3 4s^1$.

Quick Tip

For ions, remove electrons from the outermost shell first, typically the 4s before the 3d in transition metals.

---

77. Compound A formed in the following reaction reacts with B, giving the product C. Find out A and B.

$$CH_3 - C \equiv CH + Na \rightarrow A \xrightarrow{B} CH_3 - C \equiv C - CH_2 - CH_2 - CH_3 + NaBr\ (C)$$

(1) A = CH_3–C≡CNa, B = CH_3–CH_2–CH_2–Br

(2) A = CH_3–CH_2–CH_2Br, B = CH_3–C≡C–CH_3

(3) A = CH_3–C≡CNa, B = CH_3–C≡CH

(4) A = CH_3–C≡CNa, B = CH_3–CH_2–CH_3

Correct Answer: (1) A = CH_3–C≡CNa, B = CH_3–CH_2–CH_2–Br

Solution: The given reaction suggests that sodium acetylide reacts with an alkyl halide to yield the final product. Here, compound A is formed as sodium acetylide:

$$CH_3 - C \equiv CH + Na \rightarrow CH_3 - C \equiv CNa$$

Then, it reacts with compound B (1-bromopropane):

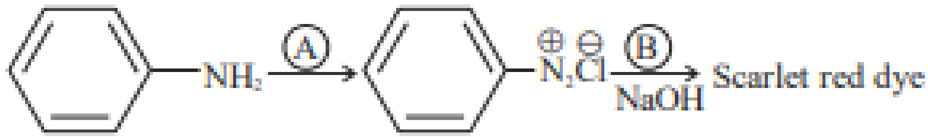
$$CH_3 - C \equiv CNa + CH_3CH_2CH_2Br \rightarrow CH_3 - C \equiv C - CH_2CH_2CH_3 + NaBr$$

Quick Tip

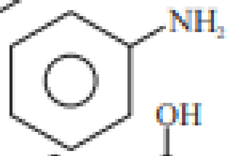
Sodium acetylide can be used as a nucleophile in substitution reactions with primary alkyl halides.

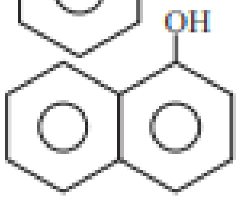
---

78. Following is a confirmatory test for aromatic primary amines. Identify reagent (A) and (B).

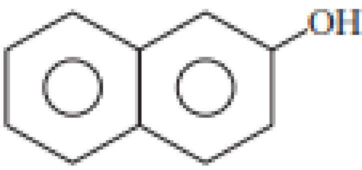


(1) A = HNO_3/H_2SO_4; B = 

(2) A= $NaNO_2$ + HCl, 0 – 5°C; B = 

(3) A=$NaNO_2$ + HCl, 0 – 5°C; B = 

(4) A = $NaNO_2$ + HCl, 0 – 5°C;

B = 

Correct Answer: (4)

Solution: The confirmatory test involves diazotization, followed by coupling with phenol to yield a red azo dye. Hence, the reagents used are:

$$A = NaNO_2 + HCl, 0\text{–}5^\circ C \quad \text{and} \quad B = \text{phenol}$$

Quick Tip

Diazotization followed by coupling with phenol is a classic test for aromatic primary amines, producing a colored azo compound.

---

79. The Lassaigne's extract is boiled with dil HNO_3 before testing for halogens because:

(1) AgCN is soluble in HNO_3

(2) Silver halides are soluble in HNO_3

(3) Ag_2S is soluble in HNO_3

(4) Na_2S and NaCN are decomposed by HNO_3

Correct Answer: (4) Na_2S and NaCN are decomposed by HNO_3

Solution: If nitrogen or sulphur is also present in the compound, the sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium during Lassaigne's test.

> Quick Tip
>
> Lassaigne's extract is treated with nitric acid to ensure that cyanides and sulphides do not interfere with the halogen detection.

---

80. Choose the correct statements from the following:

(A) Ethane-1,2-diamine is a chelating ligand.

(B) Metallic aluminium is produced by electrolysis of aluminium oxide in presence of cryolite.

(C) Cyanide ion is used as ligand for leaching of silver.

(D) Phosphine acts as a ligand in Wilkinson catalyst.

(E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes.

Choose the correct answer from the options given below:

(1) (B), (C), (E) only

(2) (A), (B), (C) only

(3) (B), (C), (D), (E) only

(4) (A), (B), (C), (D), (E)

Correct Answer: (3) (B), (C), (D), (E) only

Solution:

(B) Metallic aluminium is produced by electrolyzing alumina in the presence of cryolite, which lowers the melting point and increases conductivity.

(C) Cyanide ion is used in the leaching process of silver as it forms a soluble complex with silver.

(D) Phosphine (PH_3) acts as a ligand in Wilkinson's catalyst, which is used for hydrogenation.

(E) The stability constants of Ca^{2+} and Mg^{2+} with EDTA are quite similar, making EDTA effective for chelating these ions.

Quick Tip

In multiple-choice questions involving ligands, remember that chelating ligands form rings with the metal center, enhancing complex stability.

Section B

81. The rate of first order reaction is 0.04 mol L^{-1} s^{-1} at 10 minutes and 0.03 mol L^{-1} s^{-1} at 20 minutes after initiation. Half life of the reaction is ________ minutes. (Given log2=0.3010, log3=0.4771)

Correct Answer: 24

Solution:

Given that the rate of a first-order reaction is decreasing over time, we can use the integrated rate law for first-order kinetics:

$$\text{Rate} = k[A]_0 e^{-kt}$$

At time $t = 10$ min:

$$0.04 = k[A]_0 e^{-k\times 600}$$

At time $t = 20$ min:

$$0.03 = k[A]_0 e^{-k\times 1200}$$

Dividing equation (2) by equation (1):

$$\frac{0.03}{0.04} = e^{-k\times(1200-600)}$$

$$\frac{4}{3} = e^{-600k}$$

Taking natural log:

$$\ln\left(\frac{4}{3}\right) = 600k$$

Now solve for $t_{1/2}$ using:

$$t_{1/2} = \frac{\ln 2}{k}$$

$$t_{1/2} = \frac{600 \ln 2}{\ln \frac{4}{3}} \text{ sec.}$$

Now using the given values:

$$t_{1/2} = 600 \times \frac{\log 2}{\log 4 - \log 3} = 10 \times \frac{0.3010}{0.6020 - 0.4771} \text{ min}$$

$$t_{1/2} = 24.08 \text{ min}$$

$$t_{1/2} = 24$$

Quick Tip

The half-life for a first-order reaction can be calculated without the initial concentration, using only rate constants at different times.

---

82. The pH at which Mg(OH)$_2$ [K$_{sp}$ = 1 $\times$ 10^{-11}] begins to precipitate from a solution containing 0.10 M Mg^{2+} ions is ________.

Correct Answer: 9

Solution:

Precipitation occurs when $Q_p = K_{sp}$. The solubility product expression is:

$$[\text{Mg}^{2+}][\text{OH}^-]^2 = K_{sp}$$

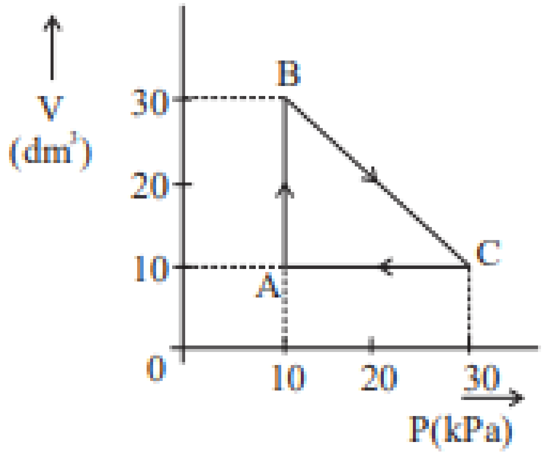
$$0.1 \times [\text{OH}^-]^2 = 10^{-11}$$

$$[\text{OH}^-] = 10^{-5}$$

$$\text{pOH} = 5 \implies \text{pH} = 9$$

Quick Tip

To determine the point of precipitation, compare the ion product with the K_{sp} value.



83. An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ A as shown in the diagram. The total work done in the process is ________ J.

Correct Answer: 200

Solution:

The work done by an ideal gas in a cyclic process can be calculated by finding the area enclosed in the P-V diagram.

For the given P-V diagram:

$$W = \frac{1}{2} \times (30 - 10) \times (30 - 10) = 200 \text{ kPa-dm}^3$$

Converting units:

$$W = 200 \times 1000 \text{ Pa} \times \text{L-bar} = 200 \text{ J}$$

Quick Tip

Work done in a cyclic process is equal to the area enclosed in the P-V diagram. Clockwise processes give positive work done.

---

84. If IUPAC name of an element is "Unununnium" then the element belongs to nth group of periodic table. The value of n is ________.

Correct Answer: 11

Solution:

The element "Unununnium" is also known as element 111. It belongs to Group 11 of the periodic table.

Quick Tip

Elements with the IUPAC name that starts with "Un" are often higher atomic number elements. Their group number can be deduced from their atomic number.

---

85. The total number of molecular orbitals formed from 2s and 2p atomic orbitals of a diatomic molecule is ________.

Correct Answer: 8

Solution:

The molecular orbitals formed from 2s and 2p atomic orbitals are as follows:

- Two molecular orbitals of $2s$ and σ^*2s.
- Six molecular orbitals of $2p$: $\sigma 2p_z$, σ^*2p_z, $\pi 2p_x$, $\pi 2p_y$, π^*2p_x, π^*2p_y.

Thus, the total number of molecular orbitals formed is 8.

Quick Tip

The total number of molecular orbitals is equal to the total number of atomic orbitals involved.

---

86. On a thin layer chromatographic plate, an organic compound moved by 3.5 cm, while the solvent moved by 5 cm. The retardation factor of the organic compound is ________$\times 10^{-1}$.

Correct Answer: 7

Solution:

The retardation factor (Rf) is given by:

$$Rf = \frac{\text{Distance travelled by sample/organic compound}}{\text{Distance travelled by solvent}}$$

$$Rf = \frac{3.5}{5} = 7 \times 10^{-1}$$

Quick Tip

The retardation factor is a ratio of the distance travelled by the compound to the distance travelled by the solvent in thin-layer chromatography (TLC).

---

87. The compound formed by the reaction of ethanol with semicarbazide contains ________ number of nitrogen atoms.

Correct Answer: 3

Solution:

The reaction between ethanol and semicarbazide is shown below:

$$CH3 - CH = O + H2N - NH - C(= O) - NH2- > CH3 - CH = N - NH - C(= O) - NH2$$

The compound contains three nitrogen atoms.

Quick Tip

Semicarbazide reacts with aldehydes or ketones to form semicarbazones, which can be identified by the presence of three nitrogen atoms in the product.

---

88. 0.05 cm thick coating of silver is deposited on a plate of 0.05 m^2 area. The number of silver atoms deposited on plate are ________ $\times 10^{23}$. (At mass Ag = 108, d = 7.9 g/cm^3)

Correct Answer: 11

Solution:

1. Calculate the volume of silver coating:

$$\text{Volume} = 0.05\,\text{cm} \times 0.05\,\text{m}^2 \times 10000 = 25\,\text{cm}^3$$

2. Calculate the mass of silver deposited:

$$\text{Mass of silver} = 25 \times 7.9 = 197.5\,\text{g}$$

3. Calculate moles of silver atoms:

$$\text{Moles of silver} = \frac{197.5}{108}$$

4. Calculate the number of atoms:

$$\text{Number of atoms} = \frac{197.5}{108} \times 6.023 \times 10^{23}$$

$$= 11.01 \times 10^{23}$$

Quick Tip

To calculate the number of atoms, determine the volume, convert to mass using density, find moles using molar mass, and finally use Avogadro's number.

---

89. 2MnO4$^{-} + 6I^{-} + 4H2O- > 3I2 + 2MnO2 + 8OH^{-}$

If the above equation is balanced with integer coefficients, the value of z is ________.

Correct Answer: 8

Solution:

Reduction Half Reaction:

$$2MnO4^{-} - > 2MnO2$$

$$2MnO4^{-} + 4H2O + 6e^{-} - > 2MnO2 + 8OH^{-}$$

Oxidation Half Reaction:

$$2I^{-} - > I2 + 2e^{-}$$

$$6I^{-} - > 3I2 + 6e^{-}$$

Adding the oxidation half and reduction half, we get the net reaction as:

$$2MnO4^{-} + 6I^{-} + 4H2O- > 3I2 + 2MnO2 + 8OH^{-}$$

Thus, $z = 8$.

Quick Tip

Balancing redox reactions involves balancing each half-reaction and then combining them to cancel out the electrons.

---

90. The mass of sodium acetate (CH3COONa) required to prepare 250 mL of 0.35 M aqueous solution is ________ g. (Molar mass of CH3COONa is 82.02 g/mol)

Correct Answer: 7

Solution:

1. **Calculate the moles using the formula:**

$$\textbf{Moles} = \textbf{Molarity} \times \textbf{Volume in litres}$$

$$\textbf{Moles} = 0.35 \times 0.25 = 0.0875\,\textbf{mol}$$

2. **Calculate the mass of sodium acetate:**

$$\textbf{Mass} = \textbf{moles} \times \textbf{molar mass}$$

$$\textbf{Mass} = 0.0875 \times 82.02 = 7.18\,\textbf{g} \approx 7\,\textbf{g}$$

Quick Tip

To prepare a solution of given molarity, always calculate the required moles of solute and use the molar mass to find the mass.
