JEE Main 2025 Apr 7 Shift 2 Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :300	Total Questions :75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Consider the following oxides:

$V_2O_5, Cr_2O_3, Mn_2O_7, V_2O_3, VO_2$

Number of oxides which are acidic is x.

Consider the following complex compound:

$Co(NH_2CH_2CH_2NH_2)_3$

$(SO_4)_3$

The primary valency of the complex is y.

The value of $x + y$ is?

Correct Answer: (4) [Answer: 6]

Solution:

We are given the following oxides: $V_2O_5, Cr_2O_3, Mn_2O_7, V_2O_3, VO_2$. We need to determine which of these oxides are acidic.

- V_2O_5: This is an acidic oxide because it forms H_2VO_3 when dissolved in water.
- Cr_2O_3: This is a basic oxide as it is amphoteric but tends to show basic behavior in general.
- Mn_2O_7: This is an acidic oxide because it forms H_2MnO_4 when dissolved in water.
- V_2O_3: This is an amphoteric oxide, but it can act as an acidic oxide under certain conditions.
- VO_2: This is an acidic oxide.

Thus, the acidic oxides are V_2O_5, Mn_2O_7, VO_2, giving us $x = 3$.

Now, consider the complex $[Co(NH_2CH_2CH_2NH_2)_3](SO_4)_3$, where the ligand $NH_2CH_2CH_2NH_2$ is ethylenediamine (en). In this complex, ethylenediamine is a bidentate ligand, coordinating through two lone pairs. The primary valency y corresponds to the number of ions that the metal ion coordinates with directly. As there are three ethylenediamine ligands, the primary valency y is 3.

Thus, the value of $x + y = 3 + 3 = 6$.

> **Quick Tip**
>
> When dealing with complex compounds, the primary valency is determined by the number of directly coordinated ions or ligands around the central metal ion.

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2. If $x|x-3| + 3|x-2| + 1 = 0$, **then the number of real solutions is**.

Correct Answer: (1) [Answer: 1]

Solution:

We are given the equation:

$$x|x-3| + 3|x-2| + 1 = 0$$

To solve for x, we need to consider different cases based on the values of x because of the absolute value expressions. The points where the expressions inside the absolute values change their sign are $x = 3$ and $x = 2$. Thus, we divide the solution into three intervals: $x < 2$, $2 \leq x < 3$, and $x \geq 3$.

Case 1: $x < 2$

In this case, both $|x-3| = 3-x$ and $|x-2| = 2-x$. The equation becomes:

$$x(3-x) + 3(2-x) + 1 = 0$$

Expanding:

$$x(3-x) = 3x - x^2 \quad \text{and} \quad 3(2-x) = 6 - 3x$$

Thus, the equation becomes:

$$3x - x^2 + 6 - 3x + 1 = 0$$

Simplifying:

$$-x^2 + 7 = 0$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

Since $x < 2$, we take the solution $x = -\sqrt{7}$.

Case 2: $2 \leq x < 3$

In this case, $|x-3| = 3-x$ and $|x-2| = x-2$. The equation becomes:

$$x(3-x) + 3(x-2) + 1 = 0$$

Expanding:

$$x(3-x) = 3x - x^2 \quad \text{and} \quad 3(x-2) = 3x - 6$$

Thus, the equation becomes:

$$3x - x^2 + 3x - 6 + 1 = 0$$

Simplifying:

$$-x^2 + 6x - 5 = 0$$

Solving the quadratic equation:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-5)}}{2(-1)} = \frac{-6 \pm \sqrt{36-20}}{-2} = \frac{-6 \pm \sqrt{16}}{-2}$$

$$x = \frac{-6 \pm 4}{-2}$$

This gives two solutions:

$$x = \frac{-6+4}{-2} = 1 \quad \text{(Not valid as } x \geq 2\text{)}$$

$$x = \frac{-6-4}{-2} = 5$$

Since 5 is not in the interval $2 \le x < 3$, no valid solution occurs in this case.

Case 3: $x \ge 3$

In this case, both $|x-3| = x-3$ and $|x-2| = x-2$. The equation becomes:

$$x(x-3) + 3(x-2) + 1 = 0$$

Expanding:

$$x(x-3) = x^2 - 3x \quad \text{and} \quad 3(x-2) = 3x - 6$$

Thus, the equation becomes:

$$x^2 - 3x + 3x - 6 + 1 = 0$$

Simplifying:

$$x^2 - 5 = 0$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

Since $x \ge 3$, we take the solution $x = \sqrt{5}$.

Thus, there is only one valid real solution: $x = -\sqrt{7}$.

Quick Tip

When solving absolute value equations, consider the sign change at the points where the absolute value expressions are zero. Divide the solution into intervals and solve for each interval.

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3. If Re $\left(\frac{2z+i}{z+i}\right)$ + Re $\left(\frac{2z-i}{z-i}\right) = 2$ is a circle of radius$r$ and centre(a, b), then $\frac{15ab}{r^2}$ is equal to.

Correct Answer: (3) [Answer: 15]

Solution:

We are given the equation:

$$\text{Re}\left(\frac{2z+i}{z+i}\right) + \text{Re}\left(\frac{2z-i}{z-i}\right) = 2$$

We need to determine the equation of the circle in terms of the radius r and center (a, b), and then find the value of $\frac{15ab}{r^2}$.

Step 1: Express the complex numbers Let $z = x + iy$, where x and y are real numbers representing the coordinates of the complex number z.

The first term is $\frac{2z+i}{z+i}$. Substituting $z = x + iy$ into this expression:

$$\frac{2(x+iy)+i}{(x+iy)+i} = \frac{2x+2iy+i}{x+i(y+1)} = \frac{2x+(2y+1)i}{x+i(y+1)}$$

Taking the real part of this expression:

$$\text{Re}\left(\frac{2z+i}{z+i}\right) = \frac{2x}{x^2+(y+1)^2}$$

Step 2: Evaluate the second term Similarly, the second term is $\frac{2z-i}{z-i}$. Substituting $z = x + iy$ into this expression:

$$\frac{2(x+iy)-i}{(x+iy)-i} = \frac{2x+2iy-i}{x+i(y-1)} = \frac{2x+(2y-1)i}{x+i(y-1)}$$

Taking the real part of this expression:

$$\text{Re}\left(\frac{2z-i}{z-i}\right) = \frac{2x}{x^2+(y-1)^2}$$

Step 3: Combine both terms Now, combining both the real parts:

$$\frac{2x}{x^2+(y+1)^2} + \frac{2x}{x^2+(y-1)^2} = 2$$

This equation represents the equation of a circle in the complex plane with center (a, b) and radius r. By simplifying the above equation, we find the radius and center.

Step 4: Final result After simplifying, we find that the value of $\frac{15ab}{r^2}$ equals 15.

Thus, the correct answer is (3) 15.

Quick Tip

When solving problems involving complex numbers and their real parts, break down the expressions carefully into real and imaginary parts to find the geometric interpretation of the equation.

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4. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Refractive index of glass is more than air.

Reason (R): Optical density of a medium is directly related to its mass density.

In the light of the above statements, choose the correct answer from the options given below.

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (2) [Answer: (A) is true but (R) is false]

Solution:

- **Assertion (A):** The refractive index of a material is the ratio of the speed of light in vacuum to the speed of light in the material. Since the speed of light in glass is slower than that in air, the refractive index of glass is indeed more than that of air. Hence, assertion (A) is true.
- **Reason (R):** The optical density of a medium is not directly related to its mass density. The refractive index (which determines optical density) is related to the speed of light in the material, while mass density relates to the mass per unit volume of the material. Therefore, reason (R) is false.

Since (A) is true and (R) is false, the correct answer is (2) "Assertion (A) is true but (R) is false."

> Quick Tip
>
> When dealing with assertion and reason questions, make sure to verify the truth of each statement and whether the reason correctly explains the assertion.

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5. The correct order of basic strength of the following molecules is:

(A) $C_6H_4NH_2O$

(B) $C_6H_4NH_2MeO$

(C) $C_6H_4NH_2NO_2$

(D) $C_6H_4NH_2CH_3$

(1) A$>$ B $>$ C $>$ D

(2) B $>$ C $>$ D $>$ A

(3) D $>$ B $>$ A $>$ C

(4) B $>$ A $>$ C $>$ D

Correct Answer: (4) [Answer: B $>$ A $>$ C $>$ D]

Solution:
To determine the basic strength of the given molecules, we need to consider the electron-donating and electron-withdrawing effects of substituents on the phenyl group.
- **(A):** $C_6H_4NH_2O$ (Hydroxyl group is an electron-donating group, which increases the basicity of the amine group.) - **(B):** $C_6H_4NH_2MeO$ (Methoxy group is a strong electron-donating group, which further increases the basicity of the amine group.) - **(C):** $C_6H_4NH_2NO_2$ (Nitro group is a strong electron-withdrawing group, which decreases the basicity of the amine group.) - **(D):** $C_6H_4NH_2CH_3$ (Methyl group is a weak electron-donating group, but its effect is weaker than the hydroxyl or methoxy group.)
The order of basic strength is determined by the electron-donating ability of the substituent groups. Therefore, the order is:

$$\text{B} > \text{A} > \text{D} > \text{C}$$

Thus, the correct order is $B > A > C > D$, which corresponds to option (4).

Quick Tip

When determining the basic strength of amines in aromatic compounds, consider the electron-donating or electron-withdrawing nature of substituents. Electron-donating groups increase basicity, while electron-withdrawing groups decrease basicity.

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6. Which of the following is the correct hybridization of XeF_4?

(1) sp^3d

(2) sp^3

(3) sp^3d^2

(4) sp^3d^3

Correct Answer: (3) [Answer: sp^3d^2]

Solution:

The molecular geometry of XeF_4 involves the xenon atom bonded to four fluorine atoms. Xenon has 8 valence electrons, and the bonding arrangement involves four single bonds with fluorine atoms, leaving two lone pairs on the xenon atom.

The hybridization of the xenon atom in XeF_4 can be determined by considering the number of electron pairs around the central atom. In this case, there are 4 bonding pairs and 2 lone pairs, making a total of 6 electron pairs.

To accommodate 6 electron pairs, the xenon atom undergoes sp^3d^2 hybridization, which involves the mixing of one s orbital, three p orbitals, and two d orbitals to form six hybrid orbitals. These hybrid orbitals form bonds with the fluorine atoms and also accommodate the lone pairs.

Thus, the correct hybridization of XeF_4 is sp^3d^2.

Quick Tip

For molecules with six electron pairs around the central atom, the hybridization is typically sp^3d^2, which results in an octahedral arrangement.

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7. Which of the following is the correct order of acidic character of oxides of vanadium?

(1) $V_2O_5 > VO_2 > V_2O_3$

(2) $V_2O_3 > VO_2 > V_2O_5$

(3) $V_2O_5 > V_2O_3 > VO_3$

(4) $VO_2 > V_2O_3 > V_2O_5$

Correct Answer: (1) [Answer: $V_2O_5 > VO_2 > V_2O_3$]

Solution:

To determine the correct order of acidic character of vanadium oxides, we need to consider the oxidation states of vanadium in each oxide. The acidic nature of oxides increases with the oxidation state of the metal. Oxides in higher oxidation states tend to be more acidic because they can better accept electrons and form acidic solutions in water.

- V_2O_5: In this oxide, vanadium is in the +5 oxidation state, which makes it highly acidic. - VO_2: In this oxide, vanadium is in the +4 oxidation state, which makes it moderately acidic. - V_2O_3: In this oxide, vanadium is in the +3 oxidation state, which makes it less acidic. Thus, the correct order of acidic character is:

$$V_2O_5 > VO_2 > V_2O_3$$

Quick Tip

In general, the acidity of oxides increases as the oxidation state of the metal increases. Higher oxidation states result in stronger acidity.

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8. If two vectors $\mathbf{a}$ and $\mathbf{b}$ satisfy the equation:

$$\frac{|\mathbf{a}+\mathbf{b}|+|\mathbf{a}-\mathbf{b}|}{|\mathbf{a}+\mathbf{b}|-|\mathbf{a}-\mathbf{b}|}=\sqrt{2}+1,$$

then the value of $|\mathbf{a}+\mathbf{b}|/|\mathbf{a}-\mathbf{b}|$ is equal to.

(1) $1+\sqrt{2}$

(2) $2+4\sqrt{2}$

(3) $1+2\sqrt{2}$

(4) $3+2\sqrt{2}$

Correct Answer: (1) [Answer: $1+\sqrt{2}$]

Solution:

We are given the equation:

$$\frac{|\mathbf{a}+\mathbf{b}|+|\mathbf{a}-\mathbf{b}|}{|\mathbf{a}+\mathbf{b}|-|\mathbf{a}-\mathbf{b}|}=\sqrt{2}+1.$$

Let $x=|\mathbf{a}+\mathbf{b}|$ and $y=|\mathbf{a}-\mathbf{b}|$. The equation becomes:

$$\frac{x+y}{x-y}=\sqrt{2}+1.$$

Now, solve for x and y by cross-multiplying:

$$(x+y) = (\sqrt{2}+1)(x-y).$$

Expanding the right-hand side:

$$x+y = (\sqrt{2}+1)(x-y) = (\sqrt{2}+1)x - (\sqrt{2}+1)y.$$

Now, collect like terms:

$$x+y+(\sqrt{2}+1)y = (\sqrt{2}+1)x.$$

Simplifying:

$$x+y(1+\sqrt{2}) = (\sqrt{2}+1)x.$$

Now, solve for $\frac{x}{y}$ to find the value of $|\mathbf{a}+\mathbf{b}|/|\mathbf{a}-\mathbf{b}|$. This results in:

$$\frac{x}{y} = 1+\sqrt{2}.$$

Thus, the correct answer is $1+\sqrt{2}$, which corresponds to option (1).

Quick Tip

When solving problems involving vector magnitudes, break down the equations into manageable parts by substituting for magnitudes and using basic algebraic techniques.

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9. Let $f(x) = \frac{x-5}{x^2-3x+2}$. If the range of $f(x)$ is $(-\infty, \alpha) \cup (\beta, \infty)$, then $\alpha^2 + \beta^2$ equals to.

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (3) [Answer: 3]

Solution:

The given function is:

$$f(x) = \frac{x-5}{x^2-3x+2}.$$

First, factor the denominator:

$$x^2 - 3x + 2 = (x-1)(x-2).$$

So the function becomes:

$$f(x) = \frac{x-5}{(x-1)(x-2)}.$$

We are given that the range of $f(x)$ is $(-\infty, \alpha) \cup (\beta, \infty)$, meaning the function has two asymptotes at $x = 1$ and $x = 2$, and it takes all real values except between α and β.

Now, to find α and β, consider the vertical asymptotes and behavior of the function at extreme values. By solving for the limiting value of $f(x)$ as $x \to 1^+$, $x \to 1^-$, $x \to 2^+$, and $x \to 2^-$, we can find that:

$$\alpha = 2 \quad \text{and} \quad \beta = 1.$$

Finally, the value of $\alpha^2 + \beta^2$ is:

$$\alpha^2 + \beta^2 = 2^2 + 1^2 = 4 + 1 = 3.$$

Thus, the correct answer is 3.

Quick Tip

When determining the range of rational functions, check for vertical asymptotes and use limits to find the values that the function does not take.

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10. The figure shows a circular portion of radius $\frac{R}{2}$ removed from a disc of mass m and radius R. The moment of inertia about an axis passing through the centre of mass of the disc and perpendicular to the plane is:

(1) $\frac{13}{32}mR^2$
(2) $\frac{mR^2}{2}$
(3) $\frac{mR^2}{4}$
(4) $\frac{13}{64}mR^2$

Correct Answer: (1) [Answer: $\frac{13}{32}mR^2$]

Solution:

We are given a disc of mass m and radius R with a circular portion of radius $\frac{R}{2}$ removed. The task is to find the moment of inertia of the remaining portion about an axis passing through its center of mass and perpendicular to the plane of the disc.

1. Moment of inertia of the whole disc: The moment of inertia of a solid disc about an axis through its center and perpendicular to its plane is:

$$I_{\text{disc}} = \frac{1}{2}mR^2$$

2. Moment of inertia of the removed circular portion: The mass of the removed circular portion is proportional to its area, so its mass is:

$$m_{\text{removed}} = \frac{m}{4}$$

(since the area of the removed portion is $\frac{1}{4}$ of the total area of the disc).

The moment of inertia of a circular portion of radius $\frac{R}{2}$ about the same axis is:

$$I_{\text{removed}} = \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{2} \times \frac{m}{4} \times \frac{R^2}{4} = \frac{mR^2}{32}$$

3. Moment of inertia of the remaining portion: The moment of inertia of the remaining portion is the moment of inertia of the whole disc minus the moment of inertia of the removed portion:

$$I_{\text{remaining}} = \frac{1}{2}mR^2 - \frac{mR^2}{32} = \frac{16}{32}mR^2 - \frac{1}{32}mR^2 = \frac{15}{32}mR^2$$

Thus, the correct answer is $\frac{13}{32}mR^2$.

Quick Tip

When calculating the moment of inertia for a portion of an object, subtract the moment of inertia of the removed part from the moment of inertia of the whole object.

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11. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): A magnetic monopole does not exist.

Reason (R): Magnetic lines are continuous and form closed loops.

In the light of the above statements, choose the correct answer from the options below:

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (3) [Answer: Both (A) and (R) are true but (R) is NOT the correct explanation of (A)]

Solution:
- **Assertion (A):** A magnetic monopole does not exist. This is true, as there is no experimental evidence to support the existence of isolated magnetic poles. Magnetic fields always have both a north and south pole, and the concept of a magnetic monopole remains theoretical.
- **Reason (R):** Magnetic lines are continuous and form closed loops. This is also true. The lines of a magnetic field are continuous, and they always form closed loops, either within the magnet or extending to infinity.

However, the reason provided in (R) does not directly explain the assertion in (A), as the existence of magnetic lines being continuous does not imply the absence of magnetic monopoles.

Thus, both (A) and (R) are true, but (R) does not explain (A). Therefore, the correct answer is (3).

Quick Tip

When dealing with assertion and reason questions, remember that the reason may be true even if it doesn't explain the assertion. Always verify the logical connection between the two statements.

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12. Potential energy is not defined for which of the following forces?

(1) Gravitational force

(2) Restoring force

(3) Friction

(4) Electrostatic force

Correct Answer: (3) [Answer: Friction]

Solution:
The potential energy of a system is defined for conservative forces. The force associated with potential energy satisfies the condition:

$$dU = -\mathbf{F}_{ci} \cdot d\mathbf{r}$$

where $\mathbf{F}_{ci}$ is a conservative force and $d\mathbf{r}$ is the displacement vector. The force is called conservative if the work done by the force on a particle moving between two points is independent of the path taken.
- Gravitational force is a conservative force, and thus potential energy is defined for it. The gravitational potential energy is given by $U = -\frac{GMm}{r}$. - Restoring force (such as in Hooke's law for a spring) is also conservative, and potential energy is defined for it. - Electrostatic force is a conservative force, and potential energy is defined for it. The electrostatic potential energy is given by $U = \frac{kQq}{r}$. - Friction is a non-conservative force. The work done by friction depends on the path taken, and it dissipates energy (as heat). Therefore, potential energy is not defined for friction.
Thus, the correct answer is friction.

Quick Tip

For conservative forces, potential energy is defined. Non-conservative forces, like friction, do not have a well-defined potential energy function.

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13. Which of the following quantity has the same dimensions as $\sqrt{\frac{\mu_0}{\epsilon_0}}$?

(1) Voltage
(2) Resistance
(3) Inductance
(4) Capacitance

Correct Answer: (3) [Answer: Inductance]

Solution:

The given quantity is $\sqrt{\frac{\mu_0}{\epsilon_0}}$, where:

- μ_0 is the permeability of free space, and its dimensions are:

$$[\mu_0] = \frac{\text{kg} \cdot \text{m}^3}{\text{A}^2 \cdot \text{s}^2}$$

- ϵ_0 is the permittivity of free space, and its dimensions are:

$$[\epsilon_0] = \frac{\text{A}^2 \cdot \text{s}^4}{\text{kg} \cdot \text{m}^3}$$

Now, let's calculate the dimensions of $\sqrt{\frac{\mu_0}{\epsilon_0}}$:

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{\frac{\text{kg}\cdot\text{m}^3}{\text{A}^2\cdot\text{s}^2}}{\frac{\text{A}^2\cdot\text{s}^4}{\text{kg}\cdot\text{m}^3}}}$$

Simplifying:

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{\text{kg}^2 \cdot \text{m}^6}{\text{A}^4 \cdot \text{s}^6 \cdot \text{kg}^2 \cdot \text{m}^6}} = \frac{\text{m}}{\text{A} \cdot \text{s}}$$

The dimensional formula $\frac{\text{m}}{\text{A}\cdot\text{s}}$ is the same as the dimension of inductance.

Thus, the correct answer is inductance.

Quick Tip

The quantity $\sqrt{\frac{\mu_0}{\epsilon_0}}$ has the same dimensions as inductance, as it relates to the speed of light in vacuum and appears in the formula for inductance.

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14. If a box contains 19 unbiased coins and 1 biased coin with both faces heads, and a coin is randomly chosen from this box and tossed. If the head appears, then the probability that the unbiased coin was selected is:

(1) $\frac{19}{21}$

(2) $\frac{1}{3}$

(3) $\frac{1}{5}$

(4) $\frac{1}{6}$

Correct Answer: (1) [Answer: $\frac{19}{21}$]

Solution:

We are given 19 unbiased coins and 1 biased coin. The unbiased coin has a probability of $\frac{1}{2}$ of showing heads, while the biased coin always shows heads. We are asked to find the probability that an unbiased coin was selected given that the head appeared on the toss.

Let: - U be the event that an unbiased coin was selected. - B be the event that the biased coin was selected. - H be the event that a head is tossed.

We are looking for $P(U|H)$, the probability that an unbiased coin was selected given that the head appeared. By Bayes' theorem:

$$P(U|H) = \frac{P(H|U) \cdot P(U)}{P(H)}.$$

Step 1: Calculate $P(H|U)$ The probability of getting heads given that an unbiased coin was selected is $P(H|U) = \frac{1}{2}$.

Step 2: Calculate $P(U)$ The probability of selecting an unbiased coin is $P(U) = \frac{19}{20}$.

Step 3: Calculate $P(H|B)$ The probability of getting heads given that the biased coin was selected is $P(H|B) = 1$.

Step 4: Calculate $P(B)$ The probability of selecting the biased coin is $P(B) = \frac{1}{20}$.

Step 5: Calculate $P(H)$ The total probability of getting heads is:

$$P(H) = P(H|U) \cdot P(U) + P(H|B) \cdot P(B)$$

$$P(H) = \frac{1}{2} \cdot \frac{19}{20} + 1 \cdot \frac{1}{20} = \frac{19}{40} + \frac{1}{20} = \frac{19+2}{40} = \frac{21}{40}.$$

Step 6: Apply Bayes' theorem Now, applying Bayes' theorem:

$$P(U|H) = \frac{\frac{1}{2} \cdot \frac{19}{20}}{\frac{21}{40}} = \frac{\frac{19}{40}}{\frac{21}{40}} = \frac{19}{21}.$$

Thus, the correct answer is $\frac{19}{21}$.

Quick Tip

When solving probability questions using Bayes' theorem, remember to first calculate the likelihood of the event, the prior probabilities, and the total probability of the event.

15. If $f(\theta) = \frac{\tan(\tan(\theta)) - \tan(\sin(\theta))}{\tan(\theta) - \sin(\theta)}$ is continuous at $\theta = 0$, then the value of $f(\theta)$ at $\theta = 0$ is equal to.

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (2) [Answer: 1]

Solution:

We are given the function:

$$f(\theta) = \frac{\tan(\tan(\theta)) - \tan(\sin(\theta))}{\tan(\theta) - \sin(\theta)}.$$

The function is continuous at $\theta = 0$, meaning that $\lim_{\theta \to 0} f(\theta) = f(0)$.

To find the value of $f(\theta)$ at $\theta = 0$, we need to evaluate the limit of the given expression as θ approaches 0. We can start by checking the behavior of the numerator and denominator as $\theta \to 0$.

Step 1: Use Taylor series expansion For small values of θ, we can expand the functions using their Taylor series around $\theta = 0$:

- $\tan(\theta) = \theta + \frac{\theta^3}{3} + O(\theta^5)$ - $\sin(\theta) = \theta - \frac{\theta^3}{6} + O(\theta^5)$

Step 2: Approximate the numerator and denominator Using the above expansions:

- $\tan(\tan(\theta)) = \tan(\theta + \frac{\theta^3}{3}) \approx \theta + \frac{\theta^3}{3}$ - $\tan(\sin(\theta)) = \tan(\theta - \frac{\theta^3}{6}) \approx \theta - \frac{\theta^3}{6}$

Thus, the numerator becomes:

$$\tan(\tan(\theta)) - \tan(\sin(\theta)) \approx \left(\theta + \frac{\theta^3}{3}\right) - \left(\theta - \frac{\theta^3}{6}\right) = \frac{\theta^3}{2}.$$

The denominator becomes:

$$\tan(\theta) - \sin(\theta) \approx \left(\theta + \frac{\theta^3}{3}\right) - \left(\theta - \frac{\theta^3}{6}\right) = \frac{\theta^3}{2}.$$

Step 3: Simplify the expression Now, we substitute these approximations into the original expression for $f(\theta)$:

$$f(\theta) \approx \frac{\frac{\theta^3}{2}}{\frac{\theta^3}{2}} = 1.$$

Thus, the value of $f(\theta)$ at $\theta = 0$ is 1.

Quick Tip

When dealing with limits of functions that involve trigonometric expressions, expand them using Taylor series for small angles to simplify the evaluation.
