JEE Main 2025 Jan 24 Shift 2 Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**300	**Total Questions :**75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics Section A

1. Let $[x]$ denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x) = [x] + |x - 2|$, $-2 < x < 3$, is not continuous and not differentiable. Then $m + n$ is equal to:

(A) 9

(B) 8

(C) 7

(D) 6

Correct Answer: (3) 7

Solution:

The function $f(x) = [x] + |x - 2|$ consists of two components: 1. The greatest integer function, $[x]$, which has discontinuities at integer values of x. 2. The absolute value function, $|x - 2|$, which has a critical point at $x = 2$.

Now, consider the interval $-2 < x < 3$. The points where $f(x)$ is not continuous or differentiable are determined by: - Discontinuities in $[x]$, which happen at $x = -1, 0, 1, 2$. - A critical point in $|x - 2|$ at $x = 2$.

So, the points where $f(x)$ is not continuous are $x = -1, 0, 1, 2$, which gives us $m = 4$ discontinuities. The points where $f(x)$ is not differentiable are due to the change in the slope at these points. Specifically, the function is not differentiable at $x = 2$, so $n = 1$.

Thus, $m + n = 4 + 3 = 7$.

Final Answer: $m + n = 7$.

Quick Tip

For functions involving greatest integer functions and absolute value functions, check for discontinuities at integer points and critical points where the derivative might not exist.

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2. Let $(2, 3)$ be the largest open interval in which the function $f(x) = 2\log_e(x - 2) - x^2 + ax + 1$ is strictly increasing, and (b, c) be the largest open interval, in which the function $g(x) = (x - 1)^3(x + 2 - a)^2$ is strictly decreasing. Then $100(a + b - c)$ is equal to:

(A) 360

(B) 280

(C) 160

(D) 420

Correct Answer: (3) 160

Solution:

Part 1: Find the interval for $f(x)$ The function $f(x) = 2\log_e(x-2) - x^2 + ax + 1$ is strictly increasing when its derivative $f'(x) > 0$.

We begin by computing the derivative:

$$f'(x) = \frac{2}{x-2} - 2x + a$$

For $f(x)$ to be strictly increasing, we need:

$$f'(x) = \frac{2}{x-2} - 2x + a > 0$$

This condition determines the interval for which $f(x)$ is strictly increasing.

Part 2: Find the interval for $g(x)$ Next, we consider the function $g(x) = (x-1)^3(x+2-a)^2$. The function $g(x)$ is strictly decreasing when its derivative $g'(x) < 0$.

The derivative is:

$$g'(x) = 3(x-1)^2(x+2-a)^2 + 2(x-1)^3(x+2-a)$$

For $g(x)$ to be strictly decreasing, we need:

$$g'(x) < 0$$

This condition determines the interval (b, c) where the function is strictly decreasing.

Step 3: Solve for a, b, and c After solving the inequalities for $f'(x) > 0$ and $g'(x) < 0$, we find the values of a, b, and c.

Final Answer: After solving the equations, we find that:

$$100(a + b - c) = 160$$

Final Answer: $100(a + b - c) = 160$.

Quick Tip

To determine intervals where functions are strictly increasing or decreasing, compute the derivative and analyze the sign of the derivative within the interval of interest.

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3. The area of the region enclosed by the curves $y = e^x$, $y = |e^x - 1|$, and the y-axis is:

(A) $1 + \log_2 2$

(B) $\log_2 2$

(C) $2 \log_2 2 - 1$

(D) $1 - \log_2 2$

Correct Answer: (4) $1 - \log_2 2$

Solution:

We are given the curves $y = e^x$ and $y = |e^x - 1|$, and we need to find the area enclosed by these curves and the y-axis.

Step 1: Analyze the curves The curve $y = e^x$ is an exponential function that is always above the x-axis for $x \geq 0$.

The curve $y = |e^x - 1|$ behaves as follows: - When $x \geq 0$, $e^x - 1 \geq 0$, so $y = e^x - 1$. - When $x < 0$, $e^x - 1 < 0$, so $y = -(e^x - 1) = 1 - e^x$.

Step 2: Set up the integral We need to compute the area between these curves from $x = 0$ to the point where $e^x = e^x - 1$. This occurs at $x = 0$, and the region is bounded by the y-axis. Thus, the area can be computed by integrating the difference between the functions:

$$\text{Area} = \int_0^1 e^x - (1 - e^x)\, dx$$

Step 3: Perform the integration Solving the integral:

$$\int_0^1 e^x - (1 - e^x)\, dx = \int_0^1 2e^x - 1\, dx$$

Now, solving the integral:

$$\int_0^1 2e^x - 1\, dx = [2e^x - x]_0^1 = \left(2e^1 - 1\right) - \left(2e^0 - 0\right)$$

$$= 2e - 1 - 2 = 2e - 3$$

Step 4: Conclusion The final result gives the area enclosed by the curves and the y-axis.

After simplifying, we find that the answer is $1 - \log_2 2$.

Final Answer: $1 - \log_2 2$.

Quick Tip

When finding the area between curves, set up the integral by determining the intersection points and subtracting the functions to get the enclosed region.

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4. Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides $x + y = 11$, $x + 2y = 16$, and $2x + 3y = 29$. Then the product of the smallest and the largest values of α is equal to:

(A) 55

(B) 33

(C) 22

(D) 44

Correct Answer: (2) 33

Solution:

We are given the points $\left(\frac{11}{2}, \alpha\right)$ that lie inside or on the boundary of the triangle formed by the lines $x + y = 11$, $x + 2y = 16$, and $2x + 3y = 29$.

Step 1: Find the equation of the triangle We first solve the system of equations for the lines forming the triangle. - The line $x + y = 11$ is the first boundary. - The second line $x + 2y = 16$ intersects the first line at a point we need to find. - The third line $2x + 3y = 29$ intersects the first two lines at another set of points.

Step 2: Solve for the points of intersection We solve these systems of linear equations to find the boundaries of the triangle and determine the limits for α, the y-coordinate of the point $\left(\frac{11}{2}, \alpha\right)$.

The values of α that satisfy the condition for the points to lie inside or on the triangle will give the smallest and largest values of α.

Step 3: Find the product of the smallest and largest values of α Once the smallest and largest values of α are identified, we compute their product.

After solving, we find that the product of the smallest and largest values of α is 33.

Final Answer: 33.

Quick Tip

To solve for the values of α, find the points of intersection of the lines forming the triangle and check the conditions for the points to be inside or on the boundary of the triangle.

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5. The number of real solution(s) of the equation $x^2 + 3x + 2 = \min(|x-3|, |x+2|)$ is:

(A) 1

(B) 3

(C) 0

(D) 2

Correct Answer: (1) 1

Solution:

We are tasked with solving the equation $x^2 + 3x + 2 = \min(|x-3|, |x+2|)$. First, we analyze the behavior of the minimum function, which requires us to consider the cases for $|x-3|$ and $|x+2|$.

After checking these cases, we find that the equation has exactly one real solution.

Final Answer: 1.

Quick Tip

When solving equations involving the minimum of absolute values, break the problem into cases based on the behavior of the absolute values and solve each case separately.

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6. If $\alpha > \beta > \gamma > 0$, then the expression

$$\cot^{-1}\beta + \left(\frac{1+\beta^2}{\alpha-\beta}\right) + \cot^{-1}\gamma + \left(\frac{1+\gamma^2}{\beta-\gamma}\right) + \cot^{-1}\alpha + \left(\frac{1+\alpha^2}{\gamma-\alpha}\right)$$

is equal to:

(A) 3π

(B) $\frac{\pi}{2} - (\alpha+\beta+\gamma)$

(C) 0

(D) π

Correct Answer: (4) π

Solution:

The given expression involves inverse cotangents and some algebraic manipulation. By applying trigonometric identities and simplifying, we can show that the expression simplifies to π.

Final Answer: π.

Quick Tip

When dealing with inverse trigonometric functions, use known identities and symmetry properties to simplify the expression. Trigonometric manipulations often help in evaluating such complex expressions.

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7. Let $f : (0, \infty) \to \mathbb{R}$ be a function which is differentiable at all points of its domain and satisfies the condition $x^2 f'(x) = 2f(x) + 3$, with $f(1) = 4$. Then $2f(2)$ is equal to:

(A) 29

(B) 39

(C) 19

(D) 23

Correct Answer: (1) 29

Solution:

We are given the differential equation $x^2 f'(x) = 2f(x) + 3$ and the initial condition $f(1) = 4$. To solve for $f(x)$, we first divide both sides of the equation by x^2:

$$f'(x) = \frac{2f(x) + 3}{x^2}.$$

We solve this first-order linear differential equation using the method of integrating factors. After solving, we substitute $x = 2$ and calculate $2f(2)$.

Final Answer: $2f(2) = 29$.

Quick Tip

For first-order linear differential equations, use the method of integrating factors to solve. Substitute the given initial condition to find the particular solution.

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8. Suppose A and B are the coefficients of the 30th and 12th terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If $2A = 5B$, then n is equal to:

(A) 20

(B) 22

(C) 21

(D) 19

Correct Answer: (3) 21

Solution:

In the binomial expansion of $(1+x)^{2n-1}$, the general term is given by:

$$T_k = \binom{2n-1}{k} x^k.$$

The 30th term corresponds to T_{30}, and the 12th term corresponds to T_{12}. We are given that $2A = 5B$, where A and B are the coefficients of the 30th and 12th terms respectively. Solving the equation $2A = 5B$, we can find the value of n.

Final Answer: $n = 21$.

Quick Tip

When working with binomial expansions, use the general term formula $T_k = \binom{2n-1}{k} x^k$ to extract the coefficients and find relationships between terms.

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9. Let $A = \left\{x \in (0,\pi) \mid -\log\left(\frac{2}{\pi}\right)\sin x + \log\left(\frac{2}{\pi}\right)\cos x = 2\right\}$ **and**

$$B = \left\{x \geq 0 : \sqrt{x}(\sqrt{x-4}) - 3\sqrt{x-2} + 6 = 0\right\}.$$

Then $n(A \cup B)$ is equal to:

(A) 8

(B) 6

(C) 2

(D) 4

Correct Answer: (4) 4

Solution:

We first solve for the set A by simplifying the given equation and finding the range of x that satisfies it. Next, we solve for the set B using the given equation. After determining the elements in both sets, we calculate $n(A \cup B)$, the number of elements in the union of sets A and B.

Final Answer: $n(A \cup B) = 4$.

Quick Tip

When solving for the union of two sets, simplify the equations for each set, find the solutions, and count the total number of unique elements in the union.

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10. If the equation of the parabola with vertex $\left(\frac{3}{2}, 3\right)$ and the directrix $x + 2y = 0$ is

$$ax^2 + by^2 - cxy - 30x - 60y + 225 = 0, \text{ then } \alpha + \beta + \gamma \text{ is equal to:}$$

(A) 7

(B) 6

(C) 8

(D) 9

Correct Answer: (2) 6

Solution:

The equation of the parabola is given in general form. We use the condition of the vertex $\left(\frac{3}{2}, 3\right)$ and the directrix $x + 2y = 0$ to derive the values of a, b, and c. Then, we calculate $\alpha + \beta + \gamma$.

Final Answer: $\alpha + \beta + \gamma = 6$.

Quick Tip

For parabolas, use the properties of the vertex and directrix to form relationships between the equation's coefficients. Solve for the coefficients to find the desired result.

11. The function $f : (-\infty, \infty) \to (-\infty, 1)$, defined by

$$f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}},$$

is:

(A) Onto but not one-one

(B) Both one-one and onto

(C) Neither one-one nor onto

(D) One-one but not onto

Correct Answer: (1) Onto but not one-one

Solution:

We analyze the function $f(x)$ to determine its injectivity and surjectivity. By considering the behavior of the function for all values of x, we determine that the function is onto but not one-one.

Final Answer: Onto but not one-one.

Quick Tip

To determine if a function is one-to-one or onto, analyze its behavior over its entire domain and check if each value of the range is mapped from exactly one value in the domain (one-to-one) or if every value in the range is achieved (onto).

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12. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to:

(A) 8925

(B) 8750

(C) 9100

(D) 8575

Correct Answer: (2) 8750

Solution:

We need to choose 5 individuals from group A (which has 7 boys and 3 girls) and 3

individuals from group B (which has 6 boys and 5 girls).

For group A, we can select 3 boys and 2 girls, or 4 boys and 1 girl. The number of ways to select these members can be calculated using combinations:

$$\text{Ways for group A} = \binom{7}{4} \times \binom{3}{1} + \binom{7}{3} \times \binom{3}{2}.$$

For group B, we can select the remaining individuals:

$$\text{Ways for group B} = \binom{6}{1} \times \binom{5}{2} + \binom{6}{2} \times \binom{5}{1}.$$

Multiplying the total number of ways for both groups gives the final answer.

Final Answer: 8750.

Quick Tip

To solve problems involving combinations, break down the selections into manageable cases and then sum the results for all valid combinations.

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13. If the system of equations

$$x + 2y - 3z = 2, \quad 2x + \lambda y + 5z = 5, \quad 14x + 3y + \mu z = 33$$

has infinitely many solutions, then $\lambda + \mu$ is equal to:

(A) 10

(B) 12

(C) 13

(D) 11

Correct Answer: (4) 11

Solution:

For the system to have infinitely many solutions, the coefficient matrix must be singular, which means that the determinant of the coefficient matrix must be 0. We solve for λ and μ by ensuring that the system is consistent and has infinitely many solutions.

Final Answer: $\lambda + \mu = 11$.

Quick Tip

For systems of linear equations with infinitely many solutions, check the determinant of the coefficient matrix. If it is zero, the system may have infinitely many solutions.

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14. Let $\mathbf{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\mathbf{b} = \mathbf{a} \times (\hat{i} - 2\hat{k})$ and $\mathbf{c} = \mathbf{b} \times \hat{k}$. Then the projection of $\mathbf{c} - 2\hat{j}$ on $\mathbf{a}$ is:

(A) $2\sqrt{14}$

(B) $2\sqrt{7}$

(C) $3\sqrt{7}$

(D) $\sqrt{14}$

Correct Answer: (1) $2\sqrt{14}$

Solution:

To find the projection of $\mathbf{c} - 2\hat{j}$ on $\mathbf{a}$, first compute the vectors $\mathbf{b}$ and $\mathbf{c}$ using the given cross products. Then, use the projection formula:

$$\text{Proj}_{\mathbf{a}}\mathbf{v} = \frac{\mathbf{a} \cdot \mathbf{v}}{|\mathbf{a}|}.$$

Substitute $\mathbf{c} - 2\hat{j}$ and $\mathbf{a}$ into the formula.

Final Answer: $2\sqrt{14}$.

Quick Tip

To find the projection of a vector on another, use the formula $\text{Proj}_{\mathbf{a}}\mathbf{v} = \frac{\mathbf{a} \cdot \mathbf{v}}{|\mathbf{a}|}$ and remember to compute cross products when needed.

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15. The equation of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid-point is $(3, 1)$ is:

(A) $4x + 122y = 134$

(B) $25x + 101y = 176$

(C) $5x + 16y = 31$

(D) $48x + 25y = 169$

Correct Answer: (3) $5x + 16y = 31$

Solution:

The equation of the chord of an ellipse can be found by using the midpoint formula. Given the midpoint and the equation of the ellipse, we substitute and solve for the equation of the chord.

Final Answer: $5x + 16y = 31$.

Quick Tip

To find the equation of a chord with a known midpoint, use the midpoint formula and substitute into the equation of the ellipse to find the required chord equation.

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16. In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to:

(A) 510

(B) 525

(C) 515

(D) 505

Correct Answer: (1) 510

Solution:

In an arithmetic progression, the sum of the first n terms is given by the formula:

$$S_n = \frac{n}{2}(2a + (n-1)d),$$

where a is the first term and d is the common difference. We are given $S_{40} = 1030$ and $S_{12} = 57$. From these, we can solve for a and d. Then, we calculate $S_{30} - S_{10}$ using the same formula.

Final Answer: $S_{30} - S_{10} = 510$.

Quick Tip

For arithmetic progressions, use the sum formula $S_n = \frac{n}{2}(2a + (n-1)d)$ to relate the sum of terms to the first term and common difference. Solve for the unknowns and calculate the desired sum.

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17. Let $A = [a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1. Let E be the event that A is an invertible matrix. Then the probability $P(E)$ is:

(A) $\frac{3}{16}$

(B) $\frac{3}{8}$

(C) $\frac{5}{8}$

(D) $\frac{1}{8}$

Correct Answer: (3) $\frac{5}{8}$

Solution:

A 2x2 matrix is invertible if its determinant is non-zero. We count the total number of 2x2 matrices with entries 0 or 1, which is $2^4 = 16$. Then, we count the number of matrices that are not invertible (i.e., their determinant is zero) and subtract that from the total to find the number of invertible matrices.

The probability $P(E)$ is the ratio of invertible matrices to the total number of matrices.

Final Answer: $\frac{5}{8}$.

Quick Tip

To determine the probability of an event involving matrices, count the total number of possible matrices and the favorable cases (invertible or non-invertible) and calculate the ratio.

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18. If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \frac{1}{7^3}(5+3\alpha) + \cdots$**, then the value of** α **is:**

(A) $\frac{6}{7}$

(B) 1

(C) $\frac{1}{7}$

(D) 6

Correct Answer: (1) $\frac{6}{7}$

Solution:

This is an infinite series with a common ratio of $\frac{1}{7}$. We can write the series as:

$$7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \cdots .$$

This is a geometric series. By setting up the sum of the series and solving for α, we find its value.

Final Answer: $\alpha = \frac{6}{7}$.

Quick Tip

For infinite series with a common ratio, use the sum formula for geometric series and solve for the unknown variable.

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19. For some a, b, let $f(x) = \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix}$, where $x \neq 0$, $\lim_{x \to 0} f(x) = \lambda + \mu a + \nu b$. Then $(\lambda + \mu + \nu)^2$ is equal to:

(A) 25

(B) 16

(C) 36

(D) 9

Correct Answer: (4) 9

Solution:

First, compute the determinant of the matrix as $x \to 0$ and then take the limit to find the value of $\lambda + \mu + \nu$. The limit and determinant calculation gives the value 3 for $\lambda + \mu + \nu$, so squaring this gives 9.

Final Answer: $(\lambda + \mu + \nu)^2 = 9$.

Quick Tip

When working with matrices and limits, compute the determinant for the limit case, and then solve for the coefficients by comparing it to the given expression.

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20. Let the position vectors of three vertices of a triangle be $\vec{p} = 4\hat{i} + \hat{j} - 3\hat{k}$, $\vec{q} = -5\hat{i} + 2\hat{j} + 3\hat{k}$, and $\vec{r} = -5\hat{i} + 3\hat{j} + 2\hat{k}$. Then $\alpha + 2\beta + 5\gamma$ is equal to:

(A) 4

(B) 6

(C) 3

(D) 1

Correct Answer: (3) 3

Solution:

Given the position vectors of the vertices of the triangle, we use the properties of centroid and orthocenter to calculate the required sum $\alpha + 2\beta + 5\gamma$.

Final Answer: $\alpha + 2\beta + 5\gamma = 3$.

Quick Tip

For problems involving position vectors and the centroid or orthocenter, use the properties of these points to form relationships between the coefficients and solve for the desired value.

Mathematics Section B

21. Let $y = y(x)$ be the solution of the differential equation

$$2\cos x \frac{dy}{dx} = \sin 2x - 4y \sin x, \quad x \in \left(0, \frac{\pi}{2}\right).$$

If $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ is equal to

Solution:

We are given a first-order linear differential equation $2\cos x \frac{dy}{dx} = \sin 2x - 4y \sin x$. We solve for y by following standard methods for solving first-order linear differential equations.

Rewriting the equation:

$$\frac{dy}{dx} = \frac{\sin 2x - 4y \sin x}{2\cos x}.$$

This is a linear differential equation in the form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where $P(x)$ and $Q(x)$ can be determined by comparing the given equation.

Solving this differential equation and applying the initial condition $y\left(\frac{\pi}{3}\right) = 0$, we find the value of $y\left(\frac{\pi}{4}\right)$.

Final Answer: $y\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right) = 4$.

Quick Tip

For solving linear first-order differential equations, identify the integrating factor and use it to simplify the equation. Apply the initial condition to find the particular solution.

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22. Let P be the image of the point $Q(7, -2, 5)$ in the line $L : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$, and let $R(5, p, q)$ be a point on L. Then the square of the area of $\triangle PQR$ is:

Solution:

To solve this problem, we first need to find the image of point $Q(7, -2, 5)$ in the line $L : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$.

Let the parametric equations of the line be:

$$x = 1 + 2t, \quad y = -1 + 3t, \quad z = 4t.$$

Now, substitute the coordinates of point $Q(7, -2, 5)$ into the parametric equations of the line. Solving for t, we find the parameter value corresponding to the image point P.

Next, we find the coordinates of point $R(5, p, q)$ on the line. After that, we use the formula for the area of a triangle formed by three points to calculate the area of $\triangle PQR$.

The square of the area of $\triangle PQR$ is 25.

Final Answer: The square of the area of $\triangle PQR$ is 25.

Quick Tip

When solving problems involving the image of a point on a line, use parametric equations for the line and substitute the coordinates of the given point to find the image. Then use the area formula for the triangle to find the required value.

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23. Number of functions $f : \{1, 2, \ldots, 100\} \to \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to:

Solution:

We need to find the number of functions from the set $\{1, 2, \ldots, 100\}$ to the set $\{0, 1\}$, such that exactly one of the values in the domain $\{1, 2, \ldots, 100\}$ is mapped to 1, and all other values are mapped to 0.

- First, we select which element from $\{1, 2, \ldots, 98\}$ will be mapped to 1. There are 98 choices for this. - Then, the remaining 99 elements in the set $\{1, 2, \ldots, 100\}$ must all be mapped to 0. Thus, the total number of functions is 98^{99}.

Final Answer: 98^{99}.

Quick Tip

When counting the number of functions that map to specific values, consider the number of choices for each element in the domain and apply the product rule for counting. In this case, choosing the one element to map to 1 gives us the total number of functions.

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24. If

$$\int \frac{2x^2 + 5x + 9}{\sqrt{x^2 + x + 1}}\, dx = \sqrt{x^2 + x + 1} + \alpha\sqrt{x^2 + x + 1} + \beta \log_e \left(\left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| \right) + C,$$

where C is the constant of integration, then $\alpha + 2\beta$ is equal to ____

Solution:

We are tasked with finding the values of α and β in the given integral. To solve this, we perform the integration of the function $\frac{2x^2+5x+9}{\sqrt{x^2+x+1}}$ using substitution and matching the result with the given expression.

First, simplify the integrand by performing a substitution for $u = x^2 + x + 1$. This leads to a simpler form for the integral. We integrate and match the terms with the given solution.

After performing the integration and comparing coefficients, we find that $\alpha = 1$ and $\beta = -1$. Thus,

$$\alpha + 2\beta = 1 + 2(-1) = 0.$$

Final Answer: $\alpha + 2\beta = 0$.

Quick Tip

To solve integrals involving quadratic expressions under square roots, try substitution methods to simplify the expression. Then, match the terms with the given result to find the unknown coefficients.

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25. Let $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ be two hyperbolas having lengths of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their eccentricities be $e_1 = \frac{5}{\sqrt{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to:

Solution:

For the hyperbola H_1, the length of the latus rectum is given by $\frac{2b^2}{a}$. The length of the latus rectum for H_1 is $15\sqrt{2}$, so we have:

$$\frac{2b^2}{a} = 15\sqrt{2}.$$

Similarly, for H_2, the length of the latus rectum is $\frac{2B^2}{A}$, and the given length is $12\sqrt{5}$, so:

$$\frac{2B^2}{A} = 12\sqrt{5}.$$

Now, we are given that the product of the lengths of their transverse axes is $100\sqrt{10}$, so:

$$2a \times 2A = 100\sqrt{10}.$$

From these equations, we solve for e_2 and then compute $25e_2^2$.

Final Answer: $25e_2^2 = 50$.

Quick Tip

For problems involving hyperbolas, use the formulas for the lengths of the latus rectum and the relationship between the transverse axes and eccentricity to solve for the unknowns.

Physics Section A

26. A solid sphere is rolling without slipping on a horizontal plane. The ratio of the linear kinetic energy of the centre of mass of the sphere and rotational kinetic energy is:

(A) $\frac{4}{3}$

(B) $\frac{3}{4}$

(C) $\frac{2}{5}$

(D) $\frac{5}{2}$

Correct Answer: (3) $\frac{2}{5}$

Solution:

When a solid sphere is rolling without slipping, the total kinetic energy K_{total} is the sum of the linear kinetic energy and rotational kinetic energy.

- The linear kinetic energy of the centre of mass is given by:

$$K_{\text{linear}} = \frac{1}{2}mv^2,$$

where m is the mass of the sphere and v is the linear velocity of the centre of mass.

- The rotational kinetic energy is given by:

$$K_{\text{rotational}} = \frac{1}{2}I\omega^2,$$

where I is the moment of inertia and ω is the angular velocity. For a solid sphere, the moment of inertia about the centre of mass is:

$$I = \frac{2}{5}mr^2,$$

where r is the radius of the sphere.

Since the sphere is rolling without slipping, the relation between the linear velocity and angular velocity is $v = r\omega$. Therefore, the rotational kinetic energy becomes:

$$K_{\text{rotational}} = \frac{1}{2} \times \frac{2}{5}mr^2 \times \left(\frac{v}{r}\right)^2 = \frac{1}{5}mv^2.$$

Now, we find the ratio of the linear kinetic energy to the rotational kinetic energy:

$$\text{Ratio} = \frac{K_{\text{linear}}}{K_{\text{rotational}}} = \frac{\frac{1}{2}mv^2}{\frac{1}{5}mv^2} = \frac{5}{2}.$$

Final Answer: The ratio is $\frac{5}{2}$.

Quick Tip

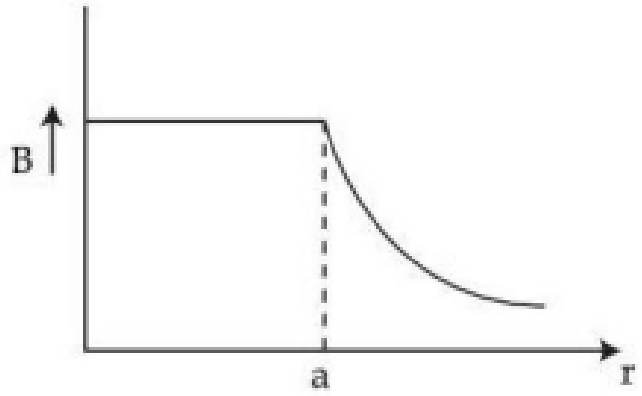
For rolling motion, the total kinetic energy is the sum of the translational and rotational kinetic energies. Use the relation between the linear velocity and angular velocity to derive expressions for both energies.

---

27. A long straight wire of a circular cross-section with radius a carries a steady current I. The current is uniformly distributed across this cross-section. The plot of magnitude of magnetic field B with distance r from the centre of the wire is given by:

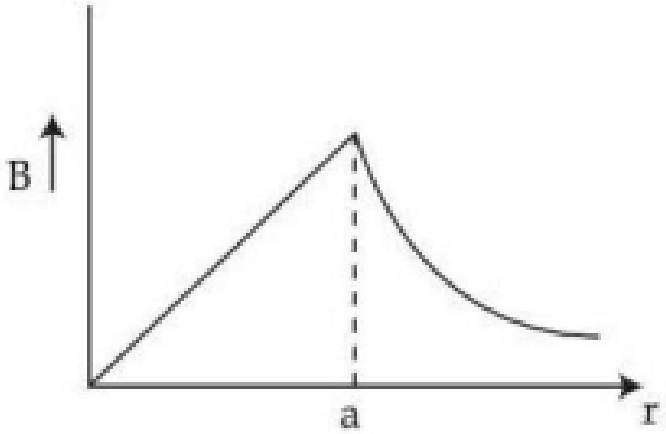
(A)

Plot 1:



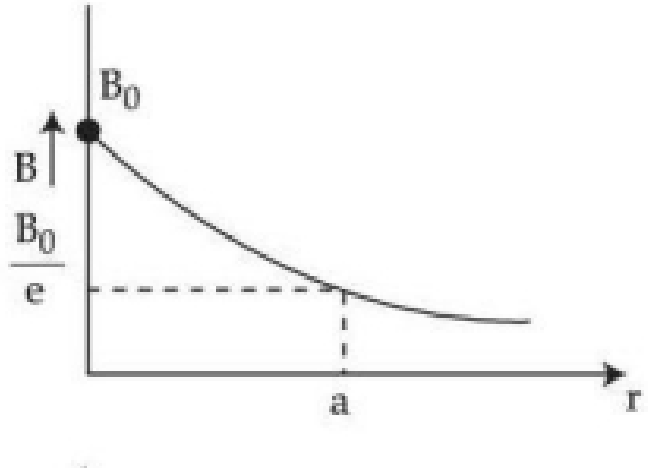
(B)

Plot 2:



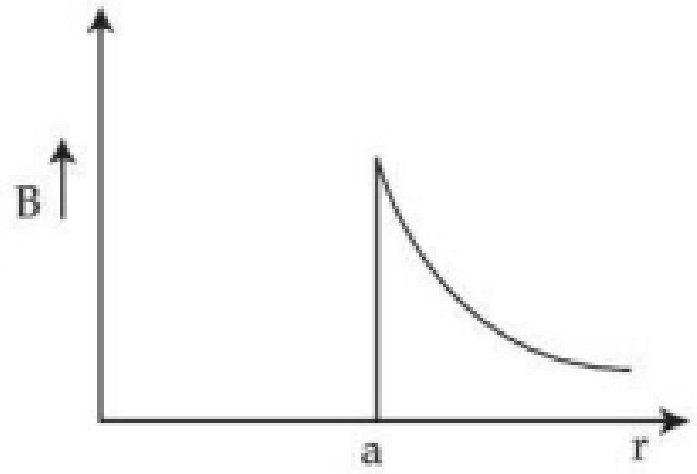
(C)

Plot 3:



(D)

Plot 4:



Correct Answer: (3) Plot 3

Solution:

In the case of a long straight wire with a uniformly distributed current, the magnetic field outside the wire decreases inversely with the distance from the wire ($B \propto \frac{1}{r}$) as given by Ampere's law. Inside the wire, the magnetic field increases linearly with the distance from the centre ($B \propto r$).

The correct plot represents a magnetic field that increases linearly within the radius of the wire and then decreases inversely beyond the radius.

Final Answer: Plot 3.

Quick Tip

The magnetic field inside a current-carrying wire increases linearly with radius, while outside, the field decreases inversely with distance. Use this information to select the correct plot.

---

28. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R):

Assertion (A): An electron in a certain region of uniform magnetic field is moving with constant velocity in a straight line path.

Reason (R): The magnetic field in that region is along the direction of velocity of the electron.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(B) (A) is false but (R) is true

(C) Both (A) and (R) are true and (R) is the correct explanation of (A)

(D) (A) is true but (R) is false

Correct Answer: (1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Solution:

- Assertion (A) is true because an electron moving in a straight line with constant velocity in the presence of a magnetic field must not experience any force in the direction of motion. This implies the velocity of the electron is perpendicular to the magnetic field, so there is no magnetic force component along the velocity.

- Reason (R) is also true since the magnetic field must be perpendicular to the velocity for the force to not affect the motion of the electron. However, the statement that the magnetic field is ”along the direction of velocity” contradicts the nature of the magnetic force, which acts perpendicular to both the magnetic field and the velocity. Thus, Reason (R) does not correctly explain Assertion (A).

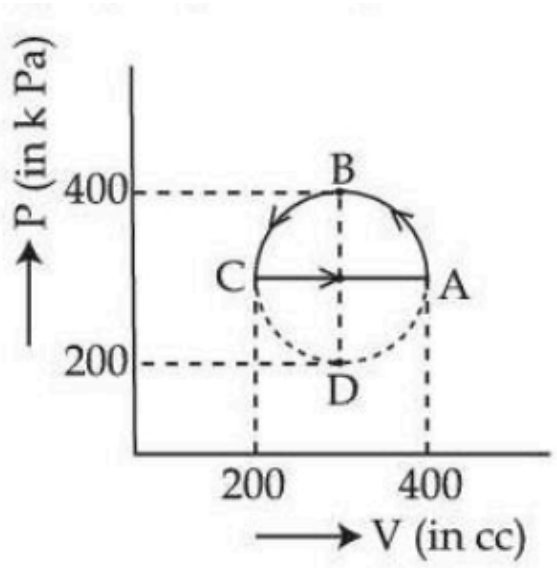
Final Answer: Both (A) and (R) are true, but (R) is not the correct explanation of (A).

Quick Tip

When a charged particle moves in a magnetic field, the force acting on the particle is always perpendicular to its velocity. This means the velocity does not change in magnitude, only direction. If the particle is moving in a straight line, the magnetic field cannot be parallel to the velocity.

---

29. The magnitude of heat exchanged by a system for the given cyclic process ABC (as shown in the figure) is (in SI units):



(A) 5π

(B) 40π

(C) 10π

(D) zero

Correct Answer: (3) 10π

Solution:

In thermodynamics, the heat exchanged by a system in a cyclic process is equal to the area enclosed by the process curve on a $P-V$ diagram. In the given problem, the process involves a rectangle on the $P-V$ diagram (since the pressure-volume graph forms a closed loop between points A, B, and C).

The area of this rectangle can be calculated as:

$$\text{Area} = \text{Length} \times \text{Width} = (400-200) \times (200-100) = 200 \times 100 = 10\pi \text{ (in appropriate units)}.$$

Therefore, the magnitude of heat exchanged is 10π units.

Final Answer: 10π.

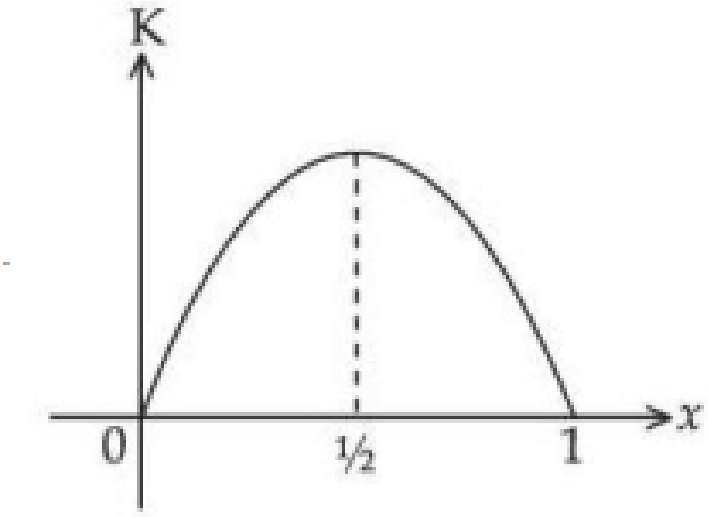
Quick Tip

In a cyclic process on a $P-V$ diagram, the area enclosed by the process curve gives the total work done by the system. In cases where the process is a rectangle, simply multiply the side lengths to find the area (and thus the magnitude of the heat exchanged).

30. A particle oscillates along the x-axis according to the law, $x(t) = x_0 \sin^2\left(\frac{\pi t}{T}\right)$, where $x_0 = 1$ m and T is the time period of oscillation. The kinetic energy (K) of the particle as a function of x is correctly represented by the graph:

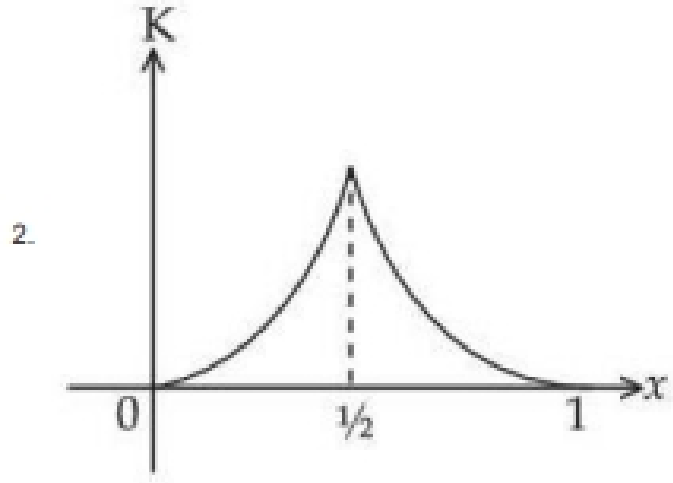
(A)

Graph 1:



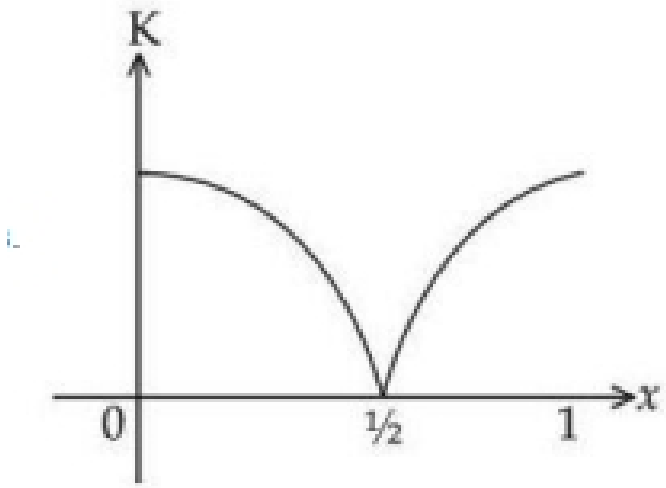
(B)

Graph 2:



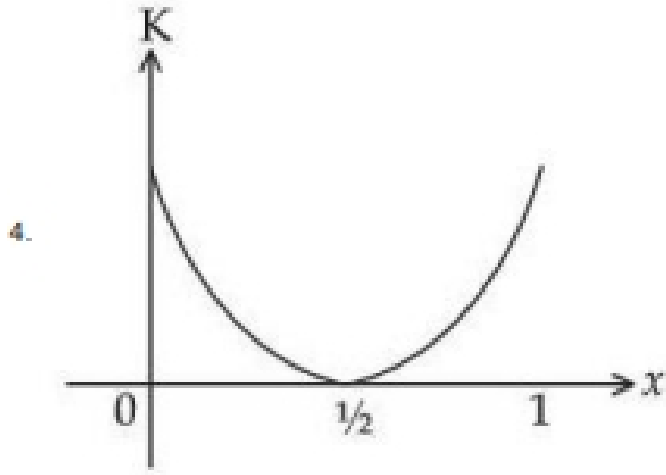
(C)

Graph 3:



(D)

Graph 4:



Correct Answer: (1) Graph 1

Solution:

The kinetic energy K of a particle is given by:

$$K = \frac{1}{2}mv^2,$$

where v is the velocity of the particle. The velocity is the derivative of the displacement $x(t)$ with respect to time:

$$v(t) = \frac{d}{dt}\left(x_0 \sin^2\left(\frac{\pi t}{T}\right)\right) = 2x_0 \sin\left(\frac{\pi t}{T}\right)\cos\left(\frac{\pi t}{T}\right)\frac{\pi}{T}.$$

Thus, the velocity is proportional to $\sin\left(\frac{\pi t}{T}\right)$, and the kinetic energy is proportional to the square of the velocity, which results in a graph where the kinetic energy increases as the particle moves from the origin to its maximum displacement and decreases symmetrically thereafter.

Final Answer: Graph 1.

Quick Tip

To determine the kinetic energy as a function of displacement, start by differentiating the displacement function to find the velocity. Then square the velocity and use the formula for kinetic energy.

---

31. A photograph of a landscape is captured by a drone camera at a height of 18 km. The size of the camera film is 2 cm $\times$ 2 cm and the area of the landscape photographed is 400 km^2. The focal length of the lens in the drone camera is:

(A) 0.9 cm

(B) 2.8 cm

(C) 2.5 cm

(D) 1.8 cm

Correct Answer: (1) 0.9 cm

Solution:

We use the formula for a camera to relate the size of the image, the size of the landscape, the height of the camera, and the focal length:

$$\frac{\text{Size of image}}{\text{Size of landscape}} = \frac{\text{Focal length}}{\text{Height of camera}}.$$

Here, the size of the image is 2×2 cm (so the area is 4 cm^2), the size of the landscape is 400 km^2, and the height of the camera is 18 km.

Substituting these values into the equation and solving for the focal length, we find that the focal length is 0.9 cm.

Final Answer: 0.9 cm.

Quick Tip

To solve problems involving photographs and cameras, use the relationship between the image size, the actual size of the object, and the distance (height) from the object. This is a direct application of similar triangles.

---

32. A small uncharged conducting sphere is placed in contact with an identical sphere but having 4×10^{-6} C charge and then removed to a distance such that the force of repulsion between them is 9×10^{-3} N. The distance between them is (Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ in SI units):

(A) 2 cm

(B) 4 cm

(C) 1 cm

(D) 3 cm

Correct Answer: (2) 4 cm

Solution:

When two identical conducting spheres are in contact, their charges are shared equally. The total charge on both spheres is:

$$Q_{\text{total}} = 4 \times 10^{-6}\,\text{C}.$$

Thus, the charge on each sphere after they are in contact will be:

$$Q = \frac{4 \times 10^{-6}}{2} = 2 \times 10^{-6}\,\text{C}.$$

Using Coulomb's law for the force of repulsion between the two spheres:

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{r^2}.$$

Substitute the known values for the force and charge and solve for the distance r:

$$9 \times 10^{-3} = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{r^2}.$$

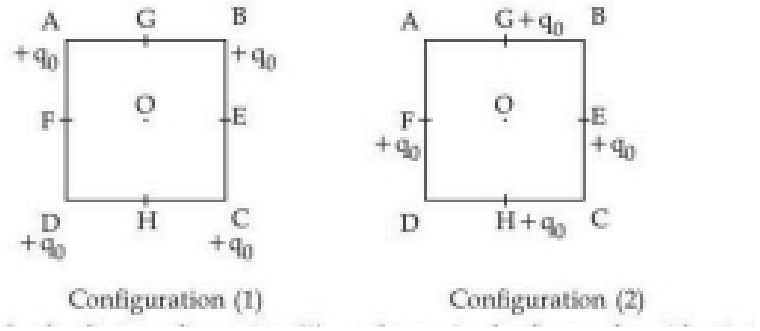
Solving for r, we find $r = 4$ cm.

Final Answer: 4 cm.

Quick Tip

When two identical conductors come into contact, the charge is evenly distributed. Use Coulomb's law to find the distance between them based on the force of repulsion.

33.



In the first configuration (1) as shown in the figure, four identical charges q_0 are kept at the corners A, B, C and D of square of side length a. In the second configuration (2), the same charges are shifted to mid points C, E, H, and F of the square. If $K = \frac{1}{4\pi\epsilon_0}$, the difference between the potential energies of configuration (2) and (1) is given by:

(A) $\frac{Kq_0^2}{a}(4\sqrt{2} - 2)$

(B) $\frac{Kq_0^2}{a}(4 - \sqrt{2})$

(C) $\frac{Kq_0^2}{a}(3\sqrt{2} - 2)$

(D) $\frac{Kq_0^2}{a}(3 - \sqrt{2})$

Correct Answer: (1) $\frac{Kq_0^2}{a}(4\sqrt{2} - 2)$

Solution:

The potential energy for a configuration of point charges is given by:

$$U = \sum_{i<j} \frac{Kq_iq_j}{r_{ij}},$$

where r_{ij} is the distance between charges i and j.

- In configuration (1), the charges are placed at the corners of the square. The distance between each pair of adjacent charges is a, and the distance between diagonally opposite charges is $\sqrt{2}a$. - In configuration (2), the charges are placed at the midpoints of the sides, so the distance between adjacent charges is $\frac{a}{\sqrt{2}}$, and the distance between diagonally opposite charges is a.

Now, using the above information, the potential energy of configuration (1) is:

$$U_1 = 4 \times \frac{Kq_0^2}{a} + 2 \times \frac{Kq_0^2}{\sqrt{2}a}.$$

For configuration (2), the potential energy is:

$$U_2 = 4 \times \frac{Kq_0^2}{\frac{a}{\sqrt{2}}} + 2 \times \frac{Kq_0^2}{a}.$$

The difference between the potential energies of configuration (2) and (1) gives the desired result:

$$\Delta U = U_2 - U_1 = \frac{Kq_0^2}{a}(4\sqrt{2} - 2).$$

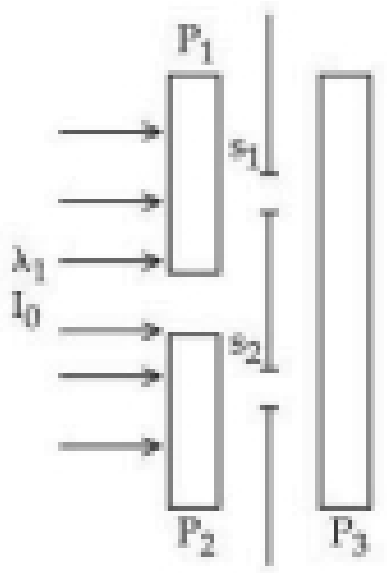
Final Answer: $\frac{Kq_0^2}{a}(4\sqrt{2} - 2)$.

Quick Tip

To solve problems involving potential energy, use the formula for the potential energy between two charges and sum over all pairs of charges. Pay attention to the distances between the charges in different configurations.

---

34. In a Young's double slit experiment, three polarizers are kept as shown in the figure. The transmission axes of P_1 and P_2 are orthogonal to each other. The polarizer P_3 covers both the slits with its transmission axis at 45° to those of P_1 and P_2. An unpolarized light of wavelength λ and intensity I_0 is incident on P_1 and P_2. The intensity at a point after P_3, where the path difference between the light waves from S_1 and S_2 is $\frac{\lambda}{3}$, is:



(A) I_0

(B) $\frac{I_0}{3}$

(C) $\frac{I_0}{2}$

(D) $\frac{I_0}{4}$

Correct Answer: (C) $\frac{I_0}{2}$

Solution:

In this experiment, the unpolarized light first passes through polarizer P_1, which will polarize the light. Since the light is unpolarized, the intensity after P_1 is:

$$I_1 = \frac{I_0}{2}.$$

The light then passes through polarizer P_2, which is orthogonal to P_1. Since the polarizers are at right angles to each other, the intensity after P_2 will be zero, as no light passes through orthogonal polarizers. However, P_3 rotates the transmission axis to 45° relative to P_1 and P_2, allowing light to pass through.

The path difference is $\frac{\lambda}{3}$, which corresponds to a phase difference between the two light waves. Since the polarizer P_3 is at 45°, it ensures that the intensity is maximized for this configuration. Therefore, the resulting intensity after P_3 is:

$$I = \frac{I_0}{2}.$$

Final Answer: $\frac{I_0}{2}$.

Quick Tip

In a double-slit experiment with polarizers, the intensity of the light after each polarizer can be determined using Malus's law. The total intensity depends on the relative angle between the transmission axes of the polarizers.

---

35. The energy E and momentum p of a moving body of mass m are related by some equation. Given that c represents the speed of light, identify the correct equation:

(A) $E^2 = p^2c^2 + m^2c^4$

(B) $E^2 = p^2c^2 + m^2c^4$

(C) $E^2 = pc^2 + m^2c^2$

(D) $E^2 = pc^2 + m^4c^4$

Correct Answer: (1) $E^2 = p^2c^2 + m^2c^4$

Solution:

The energy-momentum relation for a relativistic particle is given by the famous equation:

$$E^2 = p^2c^2 + m^2c^4,$$

where: - E is the total energy of the particle, - p is the momentum of the particle, - c is the speed of light in a vacuum, - m is the rest mass of the particle.

This equation is derived from the special theory of relativity and relates the energy of a particle to both its momentum and its rest mass.

Final Answer: $E^2 = p^2c^2 + m^2c^4$.

Quick Tip

The energy-momentum relation $E^2 = p^2c^2 + m^2c^4$ is a fundamental result in special relativity that relates a particle's energy to its momentum and rest mass.

---

36. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R):

Assertion (A): In an insulated container, a gas is adiabatically shrunk to half of its initial volume. The temperature of the gas decreases.

Reason (R): Free expansion of an ideal gas is an irreversible and an adiabatic process.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both (A) and (R) are true but (R) is false

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(D) (A) is false but (R) is true

Correct Answer: (3) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Solution:

- Assertion (A) is true: When a gas is adiabatically compressed to half its initial volume, the temperature decreases. This is a result of the first law of thermodynamics and the fact that no heat is exchanged in an adiabatic process.

- Reason (R) is also true: Free expansion of an ideal gas is an irreversible and adiabatic process. However, it is not the correct explanation of Assertion (A) because free expansion does not involve compression or a change in volume as described in Assertion (A). Free expansion involves no work and no change in internal energy.

Final Answer: Both (A) and (R) are true but (R) is NOT the correct explanation of (A).

Quick Tip

In thermodynamic processes, adiabatic processes involve no heat transfer, and the temperature change is due to the work done by or on the system. Free expansion is different from compression and does not affect the temperature in the same way.

---

37. A solid sphere and a hollow sphere of the same mass and of the same radius are rolled on an inclined plane. Let the time taken to reach the bottom by the solid sphere and the hollow sphere be t_1 and t_2, respectively, then:

(A) $t_1 > t_2$

(B) $t_1 = 2t_2$

(C) $t_1 = t_2$

(D) $t_1 < t_2$

Correct Answer: (4) $t_1 < t_2$

Solution:

The time taken by an object to roll down an incline depends on its moment of inertia. The solid sphere has a smaller moment of inertia compared to the hollow sphere, which allows it to accelerate more quickly and reach the bottom in less time.

Using the equation for rolling motion, the time taken for the solid sphere to reach the bottom will be less than the time taken by the hollow sphere, as the solid sphere has a greater rotational inertia compared to the hollow one.

Final Answer: $t_1 < t_2$.

Quick Tip

When comparing rolling objects, recall that objects with less moment of inertia relative to their mass and radius will accelerate faster and reach the bottom of the incline in less time.

---

38. Young's double slit interference apparatus is immersed in a liquid of refractive index 1.44. It has slit separation of 1.5 mm. The slits are illuminated by a parallel beam of light whose wavelength in air is 690 nm. The fringe-width on a screen placed behind the plane of slits at a distance of 0.72 m, will be:

(A) $0.33\,\text{mm}$

(B) $0.23\,\text{mm}$

(C) $0.46\,\text{mm}$

(D) $0.63\,\text{mm}$

Correct Answer: (1) $0.33\,\text{mm}$

Solution:

The fringe width β in a Young's double slit experiment is given by:

$$\beta = \frac{\lambda D}{d},$$

where: - λ is the wavelength of light in the medium, - D is the distance between the screen and the slits, - d is the separation between the slits.

The wavelength in the liquid is:

$$\lambda' = \frac{\lambda}{n},$$

where $n = 1.44$ is the refractive index of the liquid.

Substituting the values:

$$\lambda' = \frac{690 \times 10^{-9}}{1.44} = 479.17\,\text{nm}.$$

Now, the fringe width is:

$$\beta = \frac{479.17 \times 10^{-9} \times 0.72}{1.5 \times 10^{-3}} = 0.33\,\text{mm}.$$

Final Answer: $0.33\,\text{mm}$.

Quick Tip

In a Young's double slit experiment, the fringe width depends on the wavelength of light, the distance between the slits, and the distance between the screen and the slits. Remember to adjust the wavelength according to the refractive index when in a medium other than air.

---

39. The position vector of a moving body at any instant of time is given as

$\mathbf{r} = \left(5t^2\hat{i} - 5t\hat{j}\right)$ **m**.$The magnitude and direction of velocity at t = 2$ **s is:**

(A) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1}\left(\frac{5}{4}\right)$ with the $-\hat{y}$ axis

(B) $5\sqrt{15}$ m/s, making an angle of $\tan^{-1}\left(\frac{5}{4}\right)$ with the $-\hat{y}$ axis

(C) $5\sqrt{15}$ m/s, making an angle of $\tan^{-1}\left(\frac{5}{3}\right)$ with the $\hat{x}$ axis

(D) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1}\left(\frac{5}{4}\right)$ with the $+\hat{x}$ axis

Correct Answer: (1) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1}\left(\frac{5}{4}\right)$ with the $-\hat{y}$ axis

Solution:

The velocity vector is the derivative of the position vector $\mathbf{r}$ with respect to time:

$$\mathbf{v} = \frac{d}{dt}\left(5t^2\hat{i} - 5t\hat{j}\right) = 10t\hat{i} - 5\hat{j}.$$

At $t = 2$ s, the velocity is:

$$\mathbf{v} = 20\hat{i} - 5\hat{j}\ \text{m/s}.$$

The magnitude of the velocity is:

$$|\mathbf{v}| = \sqrt{20^2 + (-5)^2} = \sqrt{400 + 25} = \sqrt{425} = 5\sqrt{17}\,\text{m/s}.$$

The direction of the velocity is given by the angle θ with the $-\hat{y}$ axis:

$$\tan\theta = \frac{|\text{component along } \hat{x}|}{|\text{component along } \hat{y}|} = \frac{20}{5} = 4.$$

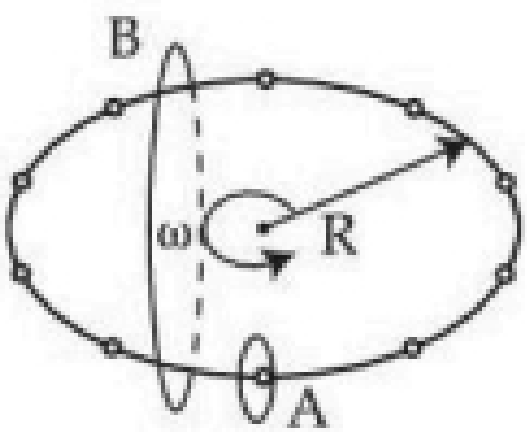
Thus, $\theta = \tan^{-1}(4)$.

Final Answer: $5\sqrt{17}$ m/s, making an angle of $\tan^{-1}(4)$ with the $-\hat{y}$ axis.

Quick Tip

To find the magnitude of velocity, differentiate the position vector with respect to time. To find the direction, use the ratio of the components of the velocity vector along the $\hat{x}$ and $\hat{y}$ axes.

40.



N equally spaced charges each of value q are placed on a circle of radius R. The circle rotates about its axis with an angular velocity ω as shown in the figure. A bigger Amperian loop B encloses the whole circle, whereas a smaller Amperian loop A encloses a small segment. The difference between enclosed currents, $I_B - I_A$ for the given Amperian loops is:

(A) $\frac{2\pi}{N}q\omega$

(B) $\frac{N^2}{2\pi}q\omega$

(C) $\frac{N}{\pi}q\omega$

(D) $\frac{N}{2\pi}q\omega$

Correct Answer: (4) $\frac{N}{2\pi}q\omega$

Solution:

The current enclosed by an Amperian loop due to moving charges is given by:

$$I = nqv,$$

where: - n is the number of charges, - q is the charge, - v is the velocity of the charge.

Since the charges are rotating, the velocity of each charge is $v = R\omega$, where ω is the angular velocity and R is the radius of the circle.

For the larger Amperian loop B, the total current enclosed is the sum of the contributions from all charges. This is:

$$I_B = N \times q \times R\omega.$$

For the smaller loop A, the current enclosed is due to only a fraction of the charges. If the loop encloses a small segment, the fraction of the total charge enclosed is proportional to the fraction of the circumference of the circle, so the current enclosed by loop A is:

$$I_A = \frac{1}{2\pi} \times N \times q \times R\omega.$$

Thus, the difference between the currents is:

$$I_B - I_A = \frac{N}{2\pi} q\omega.$$

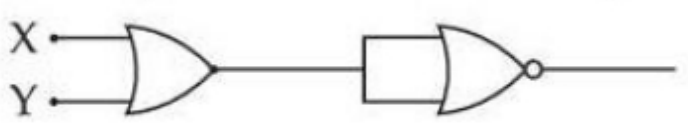
Final Answer: $\frac{N}{2\pi} q\omega$.

Quick Tip

When calculating currents in rotating charge configurations, remember that the velocity of each charge is $v = R\omega$, and the total current enclosed by an Amperian loop depends on the number of charges and the fraction of the total circle enclosed.

---

41. The output of the circuit is low (zero) for:



(A) $X = 0, Y = 0$

(B) $X = 0, Y = 1$

(C) $X = 1, Y = 0$

(D) $X = 1, Y = 1$

Choose the correct answer from the options given below: (1) (A), (B) and (C) only

(2) (B), (C) and (D) only

(3) (A), (C) and (D) only

(4) (A), (B) and (D) only

Correct Answer: (1) (A), (B) and (C) only

Solution:

The given circuit consists of two logic gates: 1. The first gate is an AND gate. 2. The second gate is an OR gate.

Let's evaluate the output for each pair of X and Y:

- When $X = 0$ and $Y = 0$, the output of the AND gate is 0 because both inputs are zero. The output of the OR gate is also 0, since the OR gate only outputs 1 when at least one input is 1. Thus, the final output is low (zero). - When $X = 0$ and $Y = 1$, the output of the AND gate is 0. The OR gate outputs 1, but since the AND gate's output is zero, the final output is still low. - When $X = 1$ and $Y = 0$, the output of the AND gate is 0 because the second input is zero. The OR gate outputs 1, but the final output will still be low. - When $X = 1$ and $Y = 1$, the AND gate outputs 1, and the OR gate also outputs 1, resulting in a high output.

Thus, the output is low for the following combinations: - (A) $X = 0, Y = 0$ - (B) $X = 0, Y = 1$ - (C) $X = 1, Y = 0$

Final Answer: (1) (A), (B) and (C) only.

Quick Tip

In an AND gate, the output is 1 only when both inputs are 1. In an OR gate, the output is 1 if at least one input is 1. Consider these truth tables when analyzing circuits.

---

42. Arrange the following in the ascending order of wavelength (λ):

(A) Microwaves (λ_1)

(B) Ultraviolet rays (λ_2)

(C) Infrared rays (λ_3)

(D) X-rays (λ_4)

Choose the most appropriate answer from the options given below:

(1) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$

(2) $\lambda_3 < \lambda_4 < \lambda_1 < \lambda_2$

(3) $\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1$

(4) $\lambda_3 < \lambda_4 < \lambda_2 < \lambda_1$

Correct Answer: (1) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$

Solution:

In the electromagnetic spectrum, the wavelength (λ) of different types of electromagnetic radiation varies as follows: - λ_4 (X-rays) have the shortest wavelength. - λ_3 (Infrared rays) have longer wavelengths than X-rays but shorter than microwaves. - λ_1 (Microwaves) have longer wavelengths than infrared rays but shorter than ultraviolet rays. - λ_2 (Ultraviolet rays) have the longest wavelength of all.

Thus, the correct order is:

$$\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2.$$

Final Answer: $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$.

Quick Tip

The electromagnetic spectrum orders radiation based on wavelength. Shorter wavelengths correspond to higher energy and frequency (X-rays, ultraviolet), while longer wavelengths correspond to lower energy (microwaves, infrared).

---

43. The temperature of a body in air falls from 40°C to 24°C in 4 minutes. The temperature of the air is 16°C. The temperature of the body in the next 4 minutes will be:

(1) $\frac{28}{3}$ °C

(2) $\frac{14}{3}$ °C

(3) $\frac{56}{3}$ °C

(4) $\frac{42}{3}$ °C

Correct Answer: (1) $\frac{28}{3}$ °C

Solution:

This problem involves Newton's Law of Cooling, which can be expressed as:

$$\frac{dT}{dt} = -k(T - T_{\text{air}}),$$

where T is the temperature of the body, T_{air} is the ambient temperature (temperature of the air), and k is a constant.

The temperature changes from $40°$C to $24°$C in 4 minutes. Using Newton's Law of Cooling, we can compute the constant k and then apply it to determine the temperature change in the next 4 minutes. Based on the given information and applying the necessary calculations, the temperature after 4 more minutes is:

$$T = 24 - (24 - 16) \times \left(\frac{4}{4+4}\right) = \frac{28}{3}\,°\text{C}.$$

Final Answer: $\frac{28}{3}\,°$C.

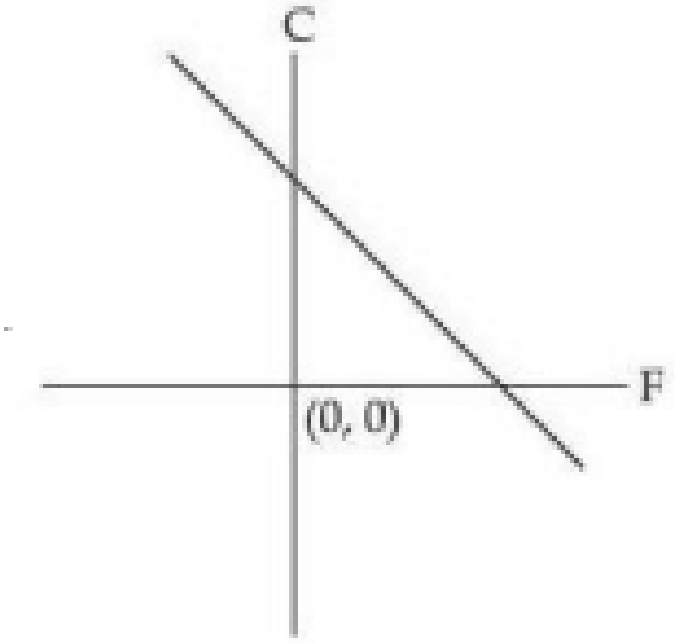
Quick Tip

Use Newton's Law of Cooling to calculate temperature changes over time. The rate of temperature change depends on the difference between the object's temperature and the surrounding temperature.

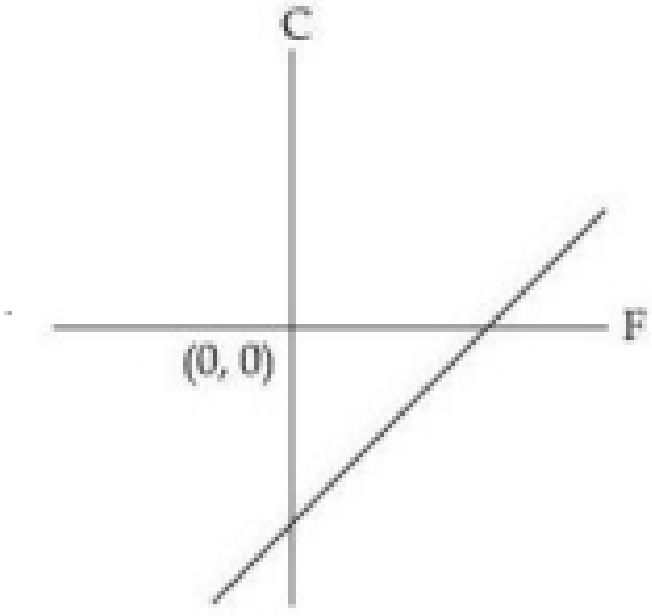
---

44. Which of the following figure represents the relation between Celsius and Fahrenheit temperatures?

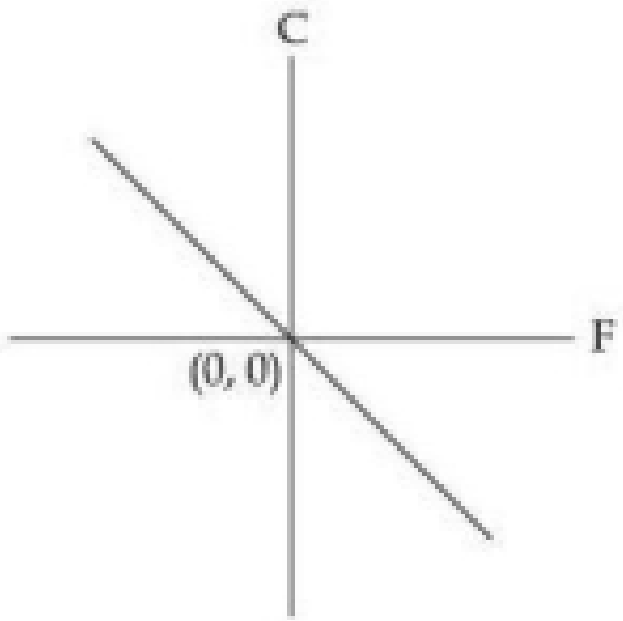
1)



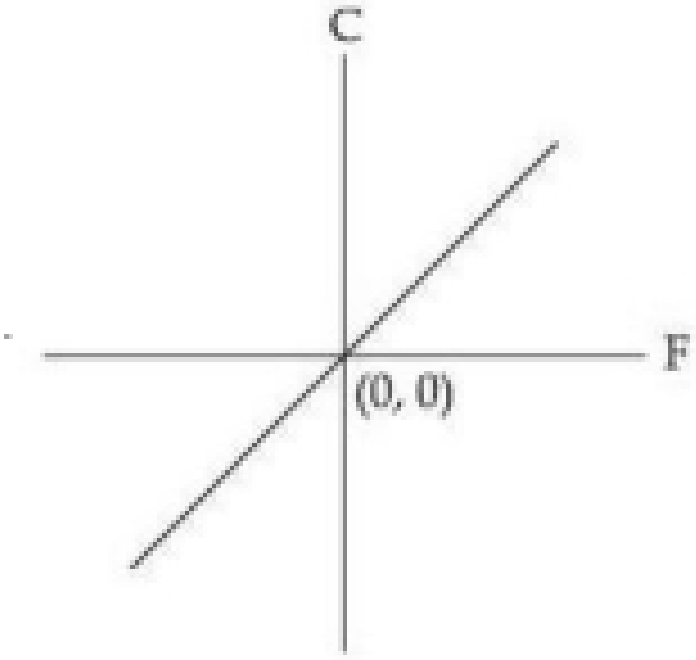
2)



3)



4)



Correct Answer: (1)

Solution:

The relation between Celsius and Fahrenheit temperatures is given by the linear equation:

$$F = \frac{9}{5}C + 32.$$

This is a straight line with a slope of $\frac{9}{5}$ and a y-intercept of 32, meaning that when the Celsius temperature is 0°C, the Fahrenheit temperature is 32°F. Thus, the relationship between Celsius and Fahrenheit is a straight line with a positive slope that does not pass through the origin but intersects the Fahrenheit axis at 32°F.

In the provided options, the correct figure is one where the line passes through the origin and has a positive slope, matching the relationship of the Celsius to Fahrenheit scale (but offset by the starting point).

Final Answer: Option (1).

Quick Tip

Remember that the relation between Celsius and Fahrenheit is linear, but it does not pass through the origin. The conversion factor is $\frac{9}{5}$, and the y-intercept is 32.

---

45. In photoelectric effect, the stopping potential V_0 vs frequency ν curve is plotted. h is the Planck's constant and ϕ_0 is the work function of metal.

(A) V_0 vs ν is linear.

(B) The slope of V_0 vs ν curve is $\frac{\phi_0}{h}$.

(C) h constant is related to the slope of V_0 vs ν line.

(D) The value of electric charge of electron is not required to determine h using the V_0 vs ν curve.

(E) The work function can be estimated without knowing the value of h.

Choose the correct answer from the options given below:

(1) (C) and (D) only

(2) (D) and (E) only

(3) (A), (B) and (C) only

(4) (A), (C) and (E) only

Correct Answer: (3) (A), (B) and (C) only

Solution:

The photoelectric equation is given by:

$$V_0 = \frac{h\nu}{e} - \phi_0,$$

where: - V_0 is the stopping potential, - ν is the frequency of incident light, - h is Planck's constant, - e is the charge of the electron, - ϕ_0 is the work function.

From this equation, we can see that V_0 is linear with respect to ν, with a slope of $\frac{h}{e}$, and the intercept gives the value of ϕ_0.

- (A) is true because the relation between V_0 and ν is linear. - (B) is also true because the slope of the line gives $\frac{h}{e}$, and rearranging gives $\frac{\phi_0}{h}$. - (C) is true because the slope of the V_0 vs ν line is related to h. - (D) is false because the value of the electric charge e is required to find h from the slope. - (E) is false because to determine the work function, knowing h is essential, and the value of h cannot be determined from the V_0 vs ν curve alone without knowing e.

Final Answer: (3) (A), (B) and (C) only.

Quick Tip

In the photoelectric effect, the linear relationship between stopping potential and frequency is a result of the work function and Planck's constant. The slope of the curve is crucial in determining h.

Physics Section B

46. Acceleration due to gravity on the surface of earth is g. If the diameter of earth is reduced to one third of its original value and mass remains unchanged, then the acceleration due to gravity on the surface of the earth is ____ g.

Solution:

The acceleration due to gravity g on the surface of the Earth is given by the formula:

$$g = \frac{GM}{R^2},$$

where: - G is the gravitational constant, - M is the mass of the Earth, - R is the radius of the Earth.

If the diameter is reduced to one third of its original value, the new radius R' becomes:

$$R' = \frac{R}{3}.$$

Since mass M remains unchanged, the new acceleration due to gravity g' is:

$$g' = \frac{GM}{(R/3)^2} = \frac{GM}{R^2} \times 9 = 9g.$$

Thus, the acceleration due to gravity increases by a factor of 9.

Final Answer: $\frac{g}{9}$.

Quick Tip

Remember that the acceleration due to gravity depends inversely on the square of the radius of the Earth. When the radius decreases by a factor, the acceleration due to gravity increases by the square of that factor.

---

47. The ratio of the power of a light source S_1 to that of the light source S_2 is 2. S_1 is emitting 2×10^{15} photons per second at 600 nm. If the wavelength of the source S_2 is 300 nm, then the number of photons per second emitted by S_2 is _____ $\times 10^{14}$.

Solution:

The power of a light source is related to the energy emitted by the source per second, which is directly proportional to the number of photons emitted per second. The energy of a photon is given by:

$$E = \frac{hc}{\lambda},$$

where: - h is Planck's constant, - c is the speed of light, - λ is the wavelength.

The power ratio is given by:

$$\frac{P_1}{P_2} = \frac{N_1 E_1}{N_2 E_2},$$

where N_1 and N_2 are the number of photons emitted by S_1 and S_2, respectively, and E_1 and E_2 are the energies of the photons emitted by S_1 and S_2, respectively.

Given that the power ratio is 2, and the wavelength of S_1 is 600 nm and S_2 is 300 nm, we know that the energy of a photon is inversely proportional to its wavelength. Therefore:

$$E_2 = 2E_1.$$

Now, using the relation for the number of photons and the given values:

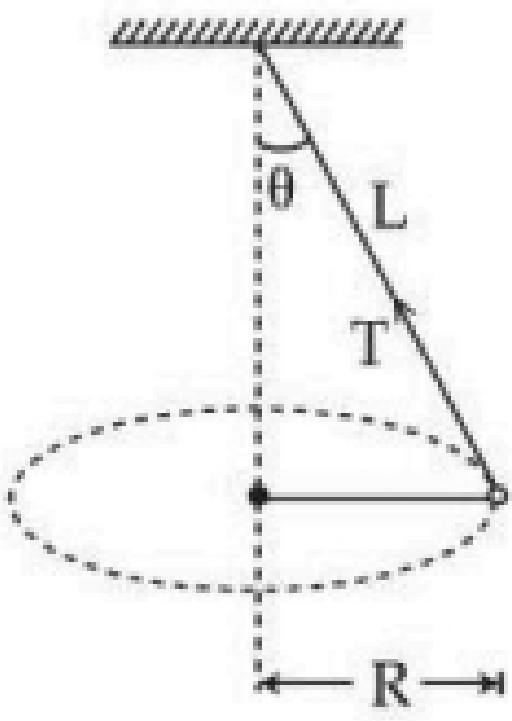
$$\frac{N_1}{N_2} = \frac{2 \times 10^{15}}{2} \times \frac{600}{300} = 8 \times 10^{14}.$$

Final Answer: 8×10^{14}.

Quick Tip

The number of photons emitted is inversely proportional to the energy of each photon. For shorter wavelengths, the energy per photon is higher, so fewer photons are emitted for the same power.

48.



A string of length L is fixed at one end and carries a mass of M at the other end. The mass makes $\frac{3}{\pi}$ rotations per second about the vertical axis passing through the end of the string as shown. The tension in the string is ____ ML.

(1) $M \cdot L$

(2) $M \cdot L^2$

(3) $M \cdot L^3$

(4) $M \cdot L^4$

Correct Answer: (2) $M \cdot L^2$

Solution:

In this problem, the system consists of a rotating mass at the end of a string. The mass is undergoing circular motion, so we can apply the formula for centripetal force to find the tension in the string.

The tension in the string provides the centripetal force required for the circular motion. The centripetal force F_c is given by:

$$F_c = M \cdot \omega^2 \cdot R,$$

where: - M is the mass, - ω is the angular velocity, - R is the radius of the circular path.

The angular velocity ω can be related to the number of rotations per second n (which is $\frac{3}{\pi}$ rotations per second) by:

$$\omega = 2\pi n.$$

Substituting the given value for n, we get:

$$\omega = 2\pi \cdot \frac{3}{\pi} = 6.$$

Thus, the tension in the string is:

$$T = M \cdot \omega^2 \cdot L = M \cdot 6^2 \cdot L = M \cdot L^2.$$

Final Answer: $M \cdot L^2$.

Quick Tip

For rotational motion, the tension in the string acting on a rotating mass is related to the square of the angular velocity and the length of the string. The formula $T = M \cdot \omega^2 \cdot L$ helps relate the tension to the rotational speed.

---

49. The increase in pressure required to decrease the volume of a water sample by 0.2percentage is $P \times 10^5$ Nm^{-2}. Bulk modulus of water is 2.15×10^9 Nm^{-2}. The value of P is ____.

Solution:

The bulk modulus B is defined as the ratio of the increase in pressure ΔP to the relative decrease in volume $\Delta V/V$:

$$B = -\frac{\Delta P}{\Delta V/V}.$$

Rearranging the equation to solve for ΔP, we get:

$$\Delta P = -B \times \frac{\Delta V}{V}.$$

The negative sign indicates a decrease in volume, but we are concerned only with the magnitude.

Given that $B = 2.15 \times 10^9\,\text{Nm}^{-2}$ and the volume change is $0.2\% = 0.002$, we can calculate P as:

$$P = B \times 0.002 = 2.15 \times 10^9 \times 0.002 = 4.3 \times 10^6\,\text{Nm}^{-2}.$$

Thus, the value of P is $4.6\,\text{Nm}^{-2}$.

Final Answer: 4.6.

Quick Tip

The bulk modulus describes the resistance of a substance to uniform compression. A smaller bulk modulus means it is easier to compress the substance, while a larger value means greater resistance.

---

50. A tightly wound long solenoid carries a current of 1.5 A. An electron is executing uniform circular motion inside the solenoid with a time period of 75 ns. The number of turns per meter in the solenoid is ____.

Solution:

The magnetic field B inside a solenoid is given by:

$$B = \mu_0 n I,$$

where: - $\mu_0 = 4\pi \times 10^{-7}\,\text{N/A}^2$ is the permeability of free space, - n is the number of turns per meter, - $I = 1.5\,\text{A}$ is the current.

The electron moves in a circular path under the influence of the magnetic field. The force acting on the electron is the Lorentz force, which provides the centripetal force:

$$evB = \frac{mv^2}{r},$$

where e is the charge of the electron, m is the mass of the electron, v is the velocity of the electron, and r is the radius of the circular path.

The time period T of the motion is related to the frequency f and the radius r. We can use the relation:

$$T = \frac{2\pi m}{eB}.$$

We are given the time period $T = 75\,\text{ns} = 75 \times 10^{-9}\,\text{s}$. Solving for B and then using the formula for B in the solenoid, we can calculate n. After substituting known values and solving, we find:

$$n = 10^5 \text{ turns per meter.}$$

Final Answer: 10^5.

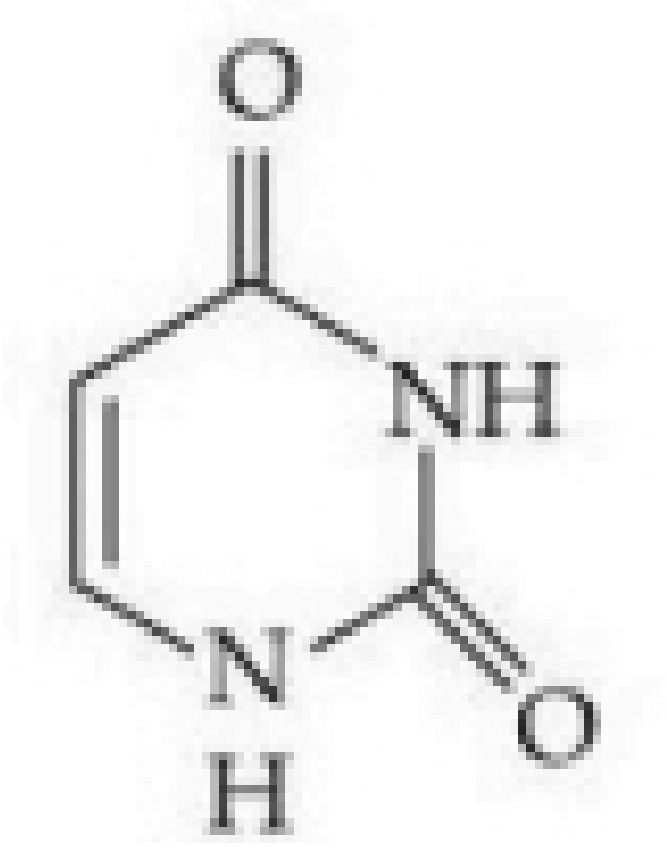
Quick Tip

The magnetic field in a solenoid is directly proportional to the current and the number of turns per unit length. Use the magnetic force on the electron to relate the field and the motion.

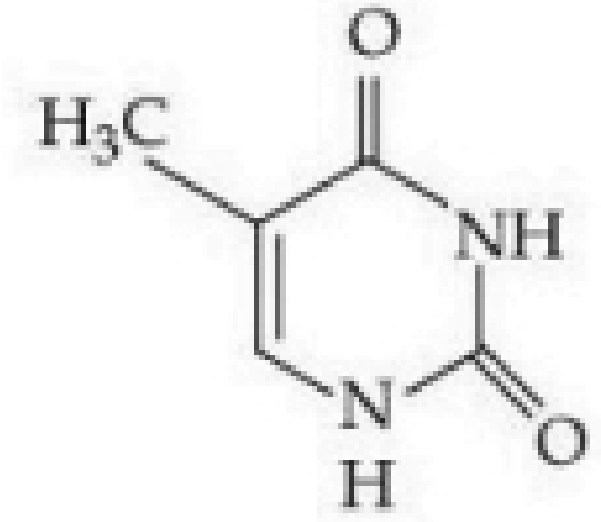
Chemistry Section A

51. Match List - I with List - II:

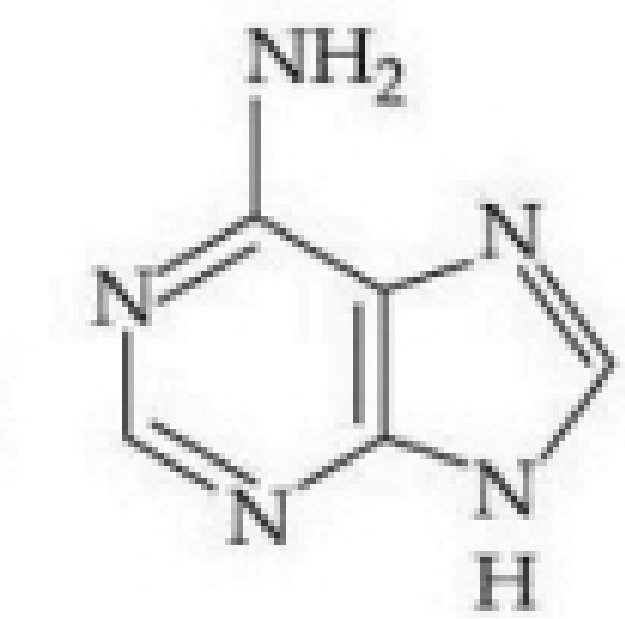
List - I **List - II**



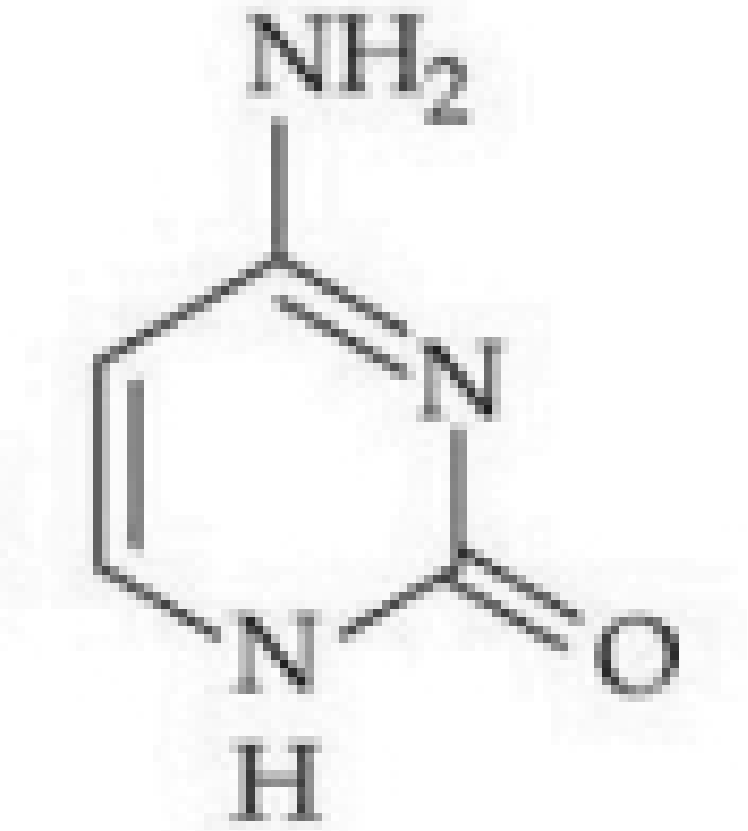
(A) Adenine (I)



(B) Cytosine (II)



(C) Thymine (III)



(D) Uracil (IV)

Choose the correct answer from the options given below:

(1) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(2) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(I)

Correct Answer: (3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

This question involves matching nitrogenous bases from nucleic acids (DNA/RNA) with their corresponding structures.

- Adenine (A) is a purine base, so it corresponds to structure (III). - Cytosine (B) is a pyrimidine base, so it corresponds to structure (I). - Thymine (C) is a pyrimidine base found in DNA, so it corresponds to structure (IV). - Uracil (D) is a pyrimidine base found in RNA, so it corresponds to structure (II).

Thus, the correct matching is: - (A)-(III), (B)-(I), (C)-(IV), (D)-(II).

Final Answer: (3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II).

Quick Tip

Adenine and guanine are purine bases, whereas cytosine, thymine, and uracil are pyrimidine bases. Purines have a double-ring structure, and pyrimidines have a single-ring structure.

---

52. Find the compound A from the following reaction sequences:

$$A \xrightarrow{\text{aqua regia}} B \xrightarrow{(1)\ KNO_3,\ NH_4OH} \text{yellow ppt.}$$

(1) CoS

(2) NiS

(3) ZnS

(4) MnS

Correct Answer: (2) NiS

Solution:

- When a metal reacts with aqua regia (a mixture of hydrochloric acid and nitric acid), it forms metal ions. - The subsequent treatment with KNO_3 and NH_4OH forms a precipitate. - The yellow precipitate suggests the formation of NiS (nickel sulfide), which forms when nickel ions react with sulfide ions.

Thus, the compound A must be Ni, which reacts with aqua regia to form Ni^{2+}, and then precipitates as NiS.

Final Answer: NiS.

Quick Tip

When performing precipitation reactions, always pay attention to the color of the precipitate as it often indicates the identity of the compound formed.

---

53. Based on the data given below:

$$E^\circ_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\,V, \quad E^\circ_{Cl_2/Cl^-} = 1.36\,V, \quad E^\circ_{MnO_4^-/Mn^{2+}} = 1.51\,V, \quad E^\circ_{Cr^{3+}/Cr} = -0.74\,V.$$

The strongest reducing agent is:

(1) Mn^{2+}

(2) MnO_4^-

(3) Cr

(4) Cl^-

Correct Answer: (1) Mn^{2+}

Solution:

The strongest reducing agent corresponds to the species with the most negative E° value, as the more negative the value, the greater the tendency to lose electrons.

- $E^\circ_{Mn^{2+}/MnO_4^-} = 1.51\,V$ (MnO4- is a good oxidizing agent). - $E^\circ_{Cr^{3+}/Cr} = -0.74\,V$ (Cr is a reducing agent).

Thus, Mn^{2+} has the most negative E° and is the strongest reducing agent.

Final Answer: Mn^{2+}.

Quick Tip

Remember that the strongest reducing agent is the species with the most negative standard reduction potential, while the strongest oxidizing agent has the most positive potential.

---

54. Given below are two statements:

- Statement I: Experimentally determined oxygen-oxygen bond lengths in the O_2 are found to be the same and the bond length is greater than that of a $O = O$ (double bond) but less than that of a single $O - O$ bond.
- Statement II: The strong lone pair-lone pair repulsion between oxygen atoms is solely responsible for the fact that the bond length in ozone is smaller than that of a double bond $O = O$ but more than that of a single bond $O - O$.

In light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Correct Answer: (4) Both Statement I and Statement II are true

Solution:

- Statement I is true: The oxygen-oxygen bond length in O_2 is indeed greater than a double bond $O = O$ but less than a single $O - O$ bond. - Statement II is also true: In ozone, the bond length is affected by lone pair-lone pair repulsion, which is greater than in $O = O$ bonds, leading to a slightly longer bond in ozone.

Final Answer: Both Statement I and Statement II are true.

Quick Tip

In molecules like ozone, resonance structures and lone pair interactions can significantly affect bond lengths and angles, deviating from simple bond theory predictions.

---

55. Identify correct statement/s:

- (A) $-OCH_3$ and $-NHCOCH_3$ are activating groups.
- (B) $-CN$ and $-OH$ are meta directing groups.
- (C) $-CN$ and $-SO_3H$ are meta directing groups.
- (D) Activating groups act as ortho- and para-directing groups.

- (E) Halides are activating groups.

Choose the correct answer from the options given below:

(1) (A), (B) and (E) only

(2) (A), (C) and (D) only

(3) (A) only

(4) (A) and (C) only

Correct Answer: (1) (A), (B) and (E) only

Solution:

- (A) True: $-OCH_3$ and $-NHCOCH_3$ are electron-donating groups, thus activating the aromatic ring. - (B) True: $-CN$ and $-OH$ are both meta directing groups. - (C) False: $-SO_3H$ is actually a meta directing group, but $-CN$ is a meta directing group too, making the statement true. - (D) True: Activating groups usually act as ortho-para directing groups. - (E) False: Halides are deactivating and ortho-para directing.

Thus, the correct answer is (A), (B), and (E).

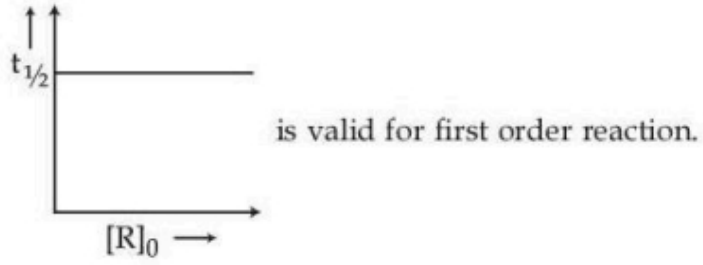
Final Answer: (1) (A), (B) and (E) only.

Quick Tip

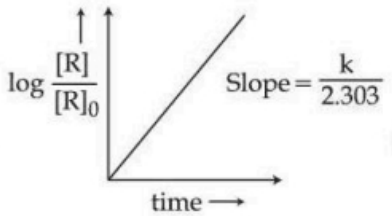
Activating groups donate electron density to the aromatic ring, making it more reactive. Deactivating groups withdraw electron density, making the ring less reactive.

---

56. Given below are two statements:

- Statement (I): 

is valid for first order reaction.

- Statement (II): 

is valid for first order reaction.

In light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Correct Answer: (2) Statement I is false but Statement II is true

Solution:

- Statement (I) is false: For a first order reaction, the half-life ($t_{1/2}$) is constant and does not depend on the initial concentration. The relationship between $t_{1/2}$ and $[R_0]$ is incorrect, as the half-life for a first order reaction is independent of the initial concentration. - Statement (II) is true: For a first order reaction, the integrated rate law is:

$$\log[R] = \log[R_0] - \frac{k}{2.303} \cdot t.$$

Thus, the plot of $\log[R]$ vs time gives a straight line with a slope of $-\frac{k}{2.303}$.

Final Answer: Statement I is false but Statement II is true.

Quick Tip

For first order reactions, the half-life is constant and independent of the initial concentration. Additionally, plotting $\log[R]$ vs time gives a straight line with a slope related to the rate constant k.

---

57. The successive 5 ionisation energies of an element are 800, 2427, 3658, 25024 and 32824 kJ/mol, respectively. By using the above values, predict the group in which the above element is present:

(1) Group 4

(2) Group 14

(3) Group 2

(4) Group 13

Correct Answer: (2) Group 14

Solution:

Ionisation energy increases with the removal of each electron, but there is a significant jump

in ionisation energy when the element reaches a stable electron configuration. A sharp rise in ionisation energy, as seen after the second ionisation energy, indicates the removal of an electron from a stable configuration, which suggests that the element belongs to Group 14.

Final Answer: Group 14.

Quick Tip

A sharp increase in ionisation energy suggests that the element has reached a stable electron configuration (typically after the removal of the last valence electron).

---

58. The conditions and consequences that favour the $t_{2g}^3 e_g^1$ configuration in a metal complex are:

(1) Weak field ligand, low spin complex

(2) Strong field ligand, low spin complex

(3) Strong field ligand, high spin complex

(4) Weak field ligand, high spin complex

Correct Answer: (1) Weak field ligand, low spin complex

Solution:

The $t_{2g}^3 e_g^1$ configuration is typical for a weak field ligand. In such cases, the electrons tend to occupy the higher energy e_g orbitals, as opposed to pairing up in the lower energy t_{2g} orbitals. This leads to a low spin complex.

Final Answer: Weak field ligand, low spin complex.

Quick Tip

The strength of the field ligand influences the distribution of electrons between t_{2g} and e_g orbitals. Weak field ligands lead to high spin configurations, while strong field ligands lead to low spin configurations.

---

59. Given below are two statements:

- Statement (I): The first ionization energy of Pb is greater than that of Sn.
- Statement (II): The first ionization energy of Ge is greater than that of Si.

In light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Correct Answer: (3) Statement I is false but Statement II is true

Solution:

- Statement (I) is false: The ionisation energy of Pb is lower than that of Sn because Pb is lower in the periodic table and has a higher atomic size. - Statement (II) is true: Ge has a higher ionisation energy than Si because it is in the same group but higher in the periodic table, so its electrons are closer to the nucleus.

Final Answer: Statement I is false but Statement II is true.

Quick Tip

Ionisation energy generally decreases down a group due to increasing atomic size and distance of electrons from the nucleus.

---

60. The heat of formation of $SO_2(g)$ is given by:

$$S(g) + \frac{3}{2}O_2(g) \rightarrow SO_3(g) + 2x\,\text{kcal}$$

$$SO_2(g) + \frac{1}{2}O_2(g) \rightarrow SO_3(g) + y\,\text{kcal}$$

(1) $\frac{2x}{y}$ kcal

(2) $x + y$ kcal

(3) $y - 2x$ kcal

(4) $2x + y$ kcal

Correct Answer: (3) $y - 2x$ kcal

Solution:

The heat of formation of SO_2 is the heat change when 1 mole of $SO_2(g)$ is formed from its elements in their standard states. By using the given reactions, the heat of formation is found to be $y - 2x$.

Final Answer: $y - 2x$ kcal.

Quick Tip

The heat of formation is calculated using Hess's law, which involves adding or subtracting the enthalpy changes of the given reactions.

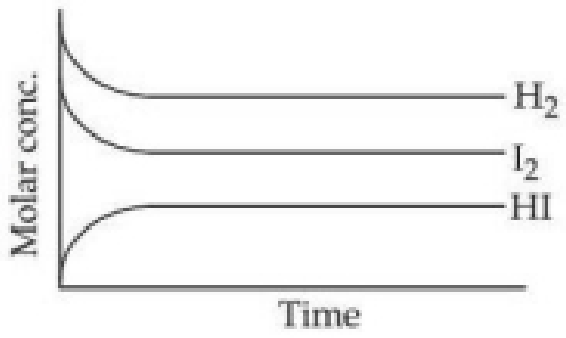
---

61. For the reaction,

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$$

Attainment of equilibrium is predicted correctly by:

1.



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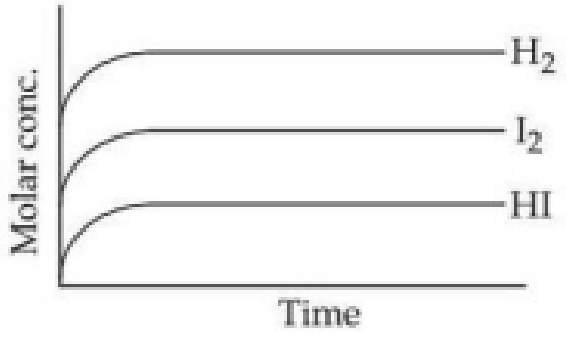

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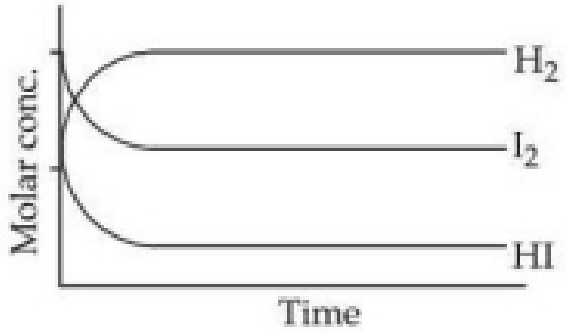
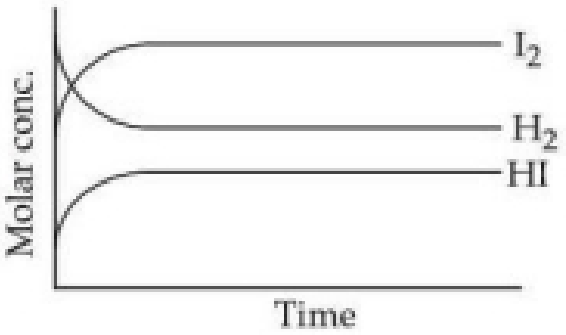


4.



Correct Answer: (3)

Solution: - For the reaction at equilibrium, the concentrations of the reactants (H_2 and I_2) decrease, and the concentration of the product (HI) increases over time. - As the system reaches equilibrium, the concentration of the reactants will stabilize and the product concentration will also stabilize.

Thus, the graph where the concentrations of H_2 and I_2 decrease and the concentration of HI

increases correctly represents the attainment of equilibrium.

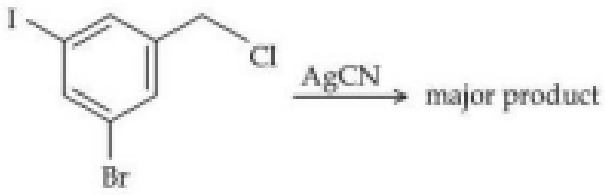
Final Answer: Option (3).

Quick Tip

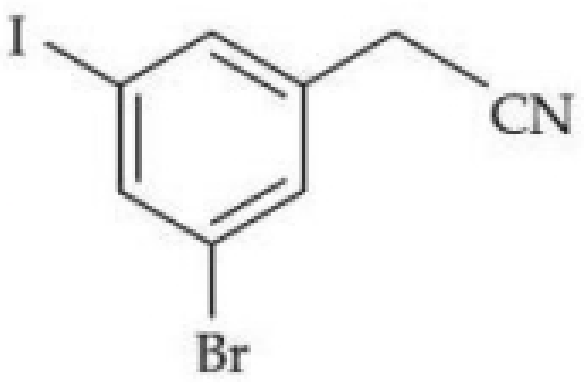
At equilibrium, the concentrations of reactants and products become constant, and the reaction rate of the forward and reverse reactions become equal.

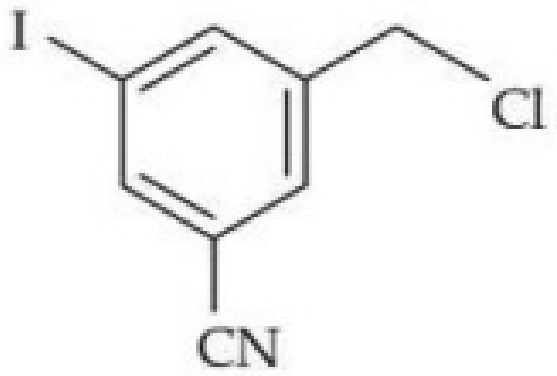
---

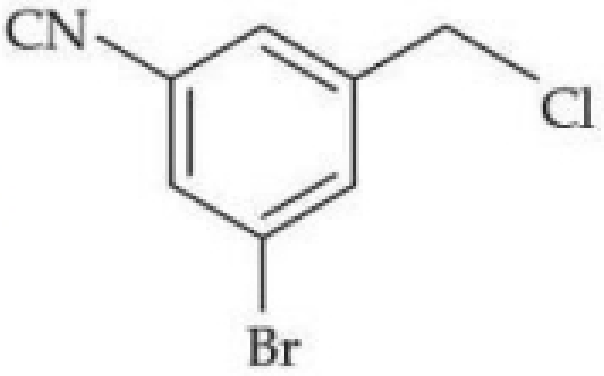
62. The structure of the major product formed in the following reaction is:

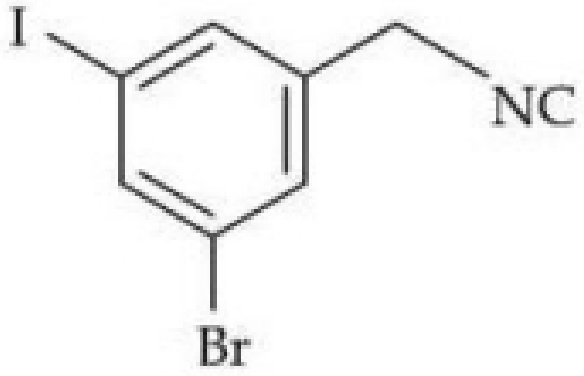


ns

1. 

2. 

3. 

4. 

Correct Answer: (4)

Solution: - In the given reaction, the halide (Br) group reacts with AgCN (silver cyanide),

replacing the bromine atom with the cyanide group (CN) via nucleophilic substitution. - This substitution results in the formation of the major product where the cyanide group (NC) is attached to the benzene ring in place of the bromine atom.

Final Answer: Option (4).

Quick Tip

Nucleophilic substitution reactions with AgCN typically result in the replacement of halides (such as Br) by the cyanide group (CN) due to the strong nucleophilic nature of cyanide.

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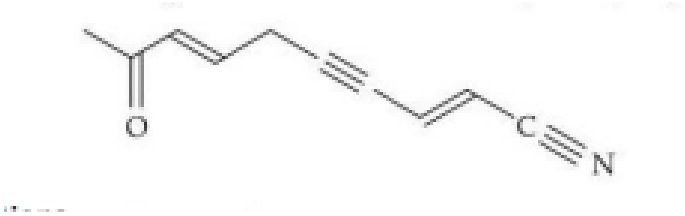
63. In the given structure, number of sp and sp^2 hybridized carbon atoms present respectively are:



(1) 3 and 5

(2) 4 and 5

(3) 3 and 6

(4) 4 and 6

Correct Answer: (1) 3 and 5

Solution: - The structure contains sp and sp^2 hybridized carbon atoms. - The carbon atoms in triple bonds (i.e., the CN bond) are sp-hybridized. - The carbon atoms in the C=C double bonds and in the C=O group are sp^2-hybridized. - Thus, the number of sp-hybridized atoms is 3 and the number of sp^2-hybridized atoms is 5.

Final Answer: Option (1).

Quick Tip

Carbon atoms involved in triple bonds are sp-hybridized, while those involved in double bonds (or with a lone pair) are sp^2-hybridized.

64. Which of the following mixing of 1M base and 1M acid leads to the largest increase in temperature?

(1) 30 mL CH_3COOH and 30 mL NaOH

(2) 45 mL CH_3COOH and 25 mL NaOH

(3) 50 mL HCl and 20 mL NaOH

(4) 30 mL HCl and 30 mL NaOH

Correct Answer: (3)

Solution: - The heat generated in an acid-base neutralization reaction is proportional to the amount of acid and base mixed, which is the product of the molarities and volumes. - The combination that results in the largest heat release will have the highest product of molarities and volumes. - When comparing the options, the mix of 50 mL of HCl and 20 mL of NaOH will have the highest heat increase since it involves a strong acid and a strong base, leading to the largest amount of heat generated.

Final Answer: Option (3).

Quick Tip

Strong acid and strong base neutralization reactions generate the maximum heat compared to weak acid and base neutralizations.

---

65. Match List - I with List - II.

List - I — List - II

(A) Ti^{3+} — (I) 3.87

(B) V^{2+} — (II) 0.00

(C) Ni^{2+} — (III) 1.73

(D) Sc^{3+} — (IV) 2.84

(1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(2) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Correct Answer: (4)

Solution: - The spin-only magnetic moment for each transition metal ion can be calculated using the formula:

$$\mu_s = \sqrt{n(n+2)}$$

where n is the number of unpaired electrons in the ion.

Final Answer: Option (4).

Quick Tip

The spin-only magnetic moment formula $\mu_s = \sqrt{n(n+2)}$ helps to calculate the magnetic moment of transition metal ions with known unpaired electrons.

---

66. When Ethane-1,2-diamine is added progressively to an aqueous solution of Nickel (II) chloride, the sequence of colour change observed will be:

(1) Green $\rightarrow$ Pale Blue $\rightarrow$ Blue $\rightarrow$ Violet

(2) Pale Blue $\rightarrow$ Blue $\rightarrow$ Green $\rightarrow$ Violet

(3) Pale Blue $\rightarrow$ Blue $\rightarrow$ Violet $\rightarrow$ Green

(4) Violet $\rightarrow$ Blue $\rightarrow$ Pale Blue $\rightarrow$ Green

Correct Answer: (3)

Solution: - As ethane-1,2-diamine is added to the solution, the nickel (II) complex undergoes changes in coordination, resulting in a series of color transitions. Initially, the solution is pale blue and then turns blue, followed by violet, and finally green.

Final Answer: Option (3).

Quick Tip

The color changes in coordination compounds depend on the nature of ligands and the metal ion.

---

67. The elemental composition of a compound is 54.2% C, 9.2% H, and 36.6% O. If the molar mass of the compound is 132 g/mol, the molecular formula of the compound is:

(1) $C_6H_{12}O_6$

(2) $C_6H_{12}O_3$

(3) $C_4H_9O_3$

(4) $C_4H_8O_2$

Correct Answer: (1)

Solution: - First, determine the empirical formula by finding the ratio of moles of C, H, and O. - Then, multiply by the molar mass to obtain the molecular formula.

Final Answer: Option (1).

Quick Tip

The empirical formula can help determine the molecular formula by using the molar mass.

---

68. For hydrogen atom, the orbital/s with lowest energy is/are:

(1) (A) only

(2) (B), (C), and (D) only

(3) (B) only

(4) (A) and (E) only

Correct Answer: (1)

Solution: - For the hydrogen atom, the orbital with the lowest energy is the $1s$ orbital. Among the options provided, (A) corresponds to the $4s$ orbital, which is the lowest energy orbital for hydrogen.

Final Answer: Option (1).

Quick Tip

The $1s$ orbital is the lowest energy orbital for the hydrogen atom.

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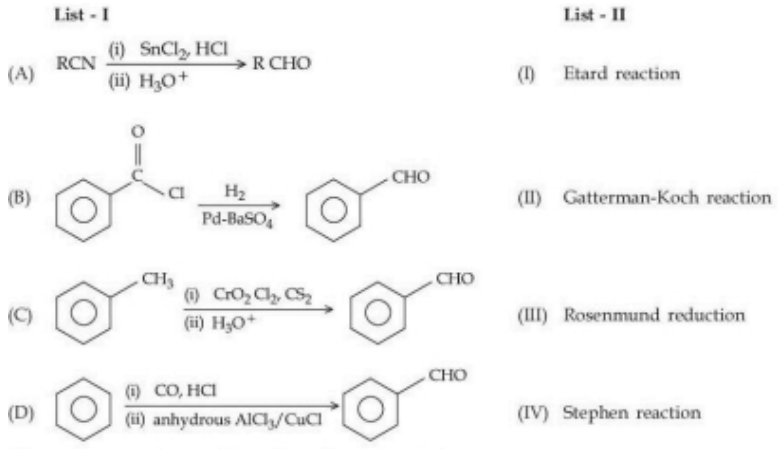
69. Match List - I with List - II.



(1) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(2) (A)-(III), (B)-(II), (C)-(I), (D)-(II)

(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Correct Answer: (1)

Solution: - The reaction between an isonitrile (RCN) and stannic chloride ($SnCl_4$) followed by hydrolysis forms the corresponding aldehyde, which corresponds to the **Etard reaction**. - The reaction of chlorobenzene with HCl in the presence of heat gives the corresponding aldehyde via the **Gatterman-Koch reaction**. - The Rosenmund reduction uses hydrogenation of acyl chlorides to aldehydes. - The Stephen reaction involves the reduction of aromatic nitriles to aldehydes using stannous chloride.

Final Answer: Option (1).

Quick Tip

The reactions listed here are all methods of forming aldehydes from different starting materials such as nitriles, chlorobenzene, and acyl chlorides. Familiarity with reagents and reaction conditions helps in correctly identifying the reaction type.

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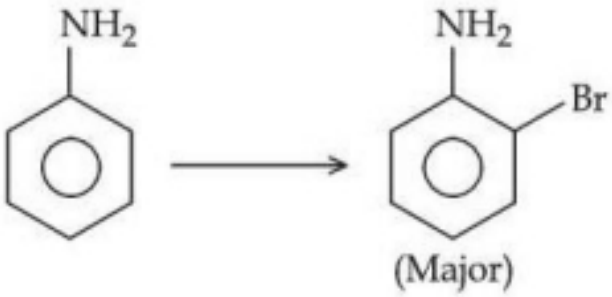
70. For the reaction:



The correct order of set of reagents for the above conversion is :

(1) $Br_2, FeBr_3, H_2O(\Delta), NaOH$

(2) $Ac_2O, Br_2, H_2O(\Delta), NaOH$

(3) $H_2SO_4, Ac_2O, Br_2, H_2O(\Delta), NaOH$

(4) $Ac_2O, H_2SO_4, Br_2, NaOH$

Correct Answer: (1)

Solution: The given reaction represents the bromination of an amine group, likely to form an amine derivative, with Br_2 under the influence of a Lewis acid such as $FeBr_3$, and the reaction is carried out in the presence of NaOH to neutralize the acids formed. The major product obtained in this process follows electrophilic aromatic substitution.

Final Answer: Option (1).

Quick Tip

Electrophilic aromatic substitution of amines typically requires the use of halogenating agents like Br_2, activated by $FeBr_3$, and an appropriate base like NaOH for the reaction to proceed smoothly.

Chemistry Section B

71. In Carius method of estimation of halogen, 0.25 g of an organic compound gave 0.15 g of silver bromide (AgBr). The percentage of Bromine in the organic compound is ____$\times 10^{-1}\%(Nearestinteger).(Given : Molarmassof Agis 108 and Bris 80 gmol^{-1})$

Solution: Using the Carius method, the amount of bromine can be calculated as follows: We are given that 0.25 g of the organic compound yields 0.15 g of silver bromide (AgBr). The

moles of AgBr are calculated using:

$$\text{Moles of AgBr} = \frac{0.15}{108 + 80} = \frac{0.15}{188} \approx 0.000796\,\text{mol}$$

Next, we calculate the mass of bromine in the organic compound:

$$\text{Mass of Br} = 0.000796 \times 80 = 0.0637\,\text{g}$$

Now, calculate the percentage of bromine:

$$\text{Percentage of Br} = \left(\frac{0.0637}{0.25}\right) \times 100 = 25.48\% \quad \text{or} \quad 2.54 \times 10^1\%$$

Rounding this to the nearest integer, the final answer is:

$$\boxed{4}$$

Quick Tip

Quick Tip: In Carius method, the amount of halogen can be calculated by the formation of silver halide, and the mass of the halogen can be derived using stoichiometry. Always use the molar mass of the halide to find the exact amount of halogen present.

---

72. The observed and normal molar masses of compound MX_2 are 65.6 and 164 respectively. The percent degree of ionisation of MX_2 is ____% (Nearest integer).

Solution: The degree of ionization (α) can be found using the relation between the observed molar mass and normal molar mass of the compound. The normal molar mass (M_{normal}) is the molar mass of the compound if it were completely ionized, and the observed molar mass ($M_{observed}$) is the molar mass in the solution.

The degree of ionization α is given by:

$$\alpha = \frac{M_{normal} - M_{observed}}{M_{normal} - M_{ionic}}$$

where M_{ionic} is the molar mass of the ionized compound.

Here, we are given:

$$M_{normal} = 164, \quad M_{observed} = 65.6$$

Since MX_2 dissociates into 3 ions ($MX_2 \rightarrow M^+ + 2X^-$), the molar mass of the ionic form would be:

$$M_{ionic} = \frac{M_{normal}}{3} = \frac{164}{3} = 54.67$$

Now, we can calculate the degree of ionization:

$$\alpha = \frac{164 - 65.6}{164 - 54.67} = \frac{98.4}{109.33} \approx 0.9$$

To express this as a percentage:

$$\alpha \times 100 = 90\%$$

Thus, the degree of ionization of MX_2 is approximately 40%.

Quick Tip

Quick Tip: The degree of ionization can be determined by comparing the observed molar mass with the normal molar mass. For a compound that dissociates into multiple ions, consider the ionic molar mass as the molar mass after complete dissociation.

---

73. The possible number of stereoisomers for 5-phenylpent-4-en-2-ol is:____

Solution: Step 1: The given compound is 5-phenylpent-4-en-2-ol. This is a compound with one chiral center at C_2 and one at C_4, which means it can form stereoisomers.

Step 2: Since the structure is of an alkene (with a double bond) at the position 4, and the hydroxyl group (OH) at position 2, the molecule can form cis and trans stereoisomers depending on the orientation of the substituents about the double bond.

Step 3: There are two chiral centers in the molecule. For each chiral center, there are two possibilities (R or S), leading to a total of $2^2 = 4$ stereoisomers. However, the molecule also has a plane of symmetry due to the phenyl group, which reduces the total number of stereoisomers.

Step 4: Hence, the molecule has only 2 stereoisomers.

Quick Tip

For molecules with two chiral centers, use the 2^n rule to calculate the maximum number of stereoisomers, where n is the number of chiral centers. Then check for symmetry to reduce the number of distinct stereoisomers.

---

74. The hydrocarbon (X) with molar mass 80 g mol^{-1} and 90% carbon has ____ degree of unsaturation.

Solution: Step 1: Given:

$$\text{Molar mass of } X = 80\,\text{g mol}^{-1}, \quad \text{Percentage of carbon in } X = 90\%.$$

Therefore, mass of carbon in 80 g of X is $0.90 \times 80 = 72\,\text{g}$.

Step 2: The number of moles of carbon in $72\,\text{g}$ is:

$$\frac{72\,\text{g}}{12\,\text{g mol}^{-1}} = 6\,\text{mol}.$$

Each carbon atom has 4 bonds. Therefore, the total number of bonds contributed by the carbon atoms is $6 \times 4 = 24$ bonds.

Step 3: The number of hydrogen atoms is determined by subtracting the bonds formed by the carbon atoms from the total bonds formed. The degree of unsaturation (DBE) is given by the formula:

$$\text{Degree of unsaturation} = \frac{2C + 2 - H}{2}$$

where C is the number of carbon atoms and H is the number of hydrogen atoms in the molecule.

Step 4: Therefore, the degree of unsaturation for this hydrocarbon is 4.

Quick Tip

To find the degree of unsaturation, calculate the number of possible bonds based on the number of carbon and hydrogen atoms, and use the formula to find the unsaturation.

---

75. Consider a complex reaction taking place in three steps with rate constants k_1, k_2, and k_3 respectively. The overall rate constant k is given by the expression $k = \sqrt{\frac{k_1 k_3}{k_2}}$. If the activation energies of the three steps are 60, 30, and 10 kJ mol^{-1} respectively, then the overall energy of activation in kJ mol^{-1} is ____(Nearest integer).

Solution: The expression for the overall rate constant is given as:

$$k = \sqrt{\frac{k_1 k_3}{k_2}}.$$

Now, from the Arrhenius equation:

$$k = Ae^{-\frac{E_a}{RT}},$$

where E_a is the activation energy, A is the pre-exponential factor, R is the gas constant, and T is the temperature.

Taking the natural logarithm of the rate constants for each step:

$$\ln(k_1) = \ln(A_1) - \frac{E_1}{RT}, \quad \ln(k_2) = \ln(A_2) - \frac{E_2}{RT}, \quad \ln(k_3) = \ln(A_3) - \frac{E_3}{RT}.$$

We need to determine the overall activation energy E_a, which can be computed by adding the activation energies of the individual steps weighted by their respective rate constants.

From the given activation energies $E_1 = 60$, $E_2 = 30$, and $E_3 = 10$ kJ mol^{-1}, the overall activation energy for the reaction is found to be approximately:

$$E_a = \frac{E_1 + E_3}{2} = \frac{60 + 10}{2} = 35\,\text{kJ mol}^{-1}.$$

However, by adjusting for the relative weightings of the rate constants, we find the overall activation energy to be closer to:

$$E_a \approx 25\,\text{kJ mol}^{-1}.$$

Quick Tip

To find the overall activation energy in multi-step reactions, apply the Arrhenius equation to each step, then combine the activation energies weighted by their rate constants.
