CBSE Class X Mathematics (Standard) Set 2 (30/1/2) Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section consists of 20 questions of 1 mark each.

Question 1: AD is a median of $\triangle ABC$ with vertices $A(5, -6)$, $B(6, 4)$, and $C(0, 0)$. Length AD is equal to:
(A) $\sqrt{68}$ units
(B) $2\sqrt{15}$ units
(C) $\sqrt{101}$ units
(D) 10 units

Correct Answer: (A) $\sqrt{68}$

Solution:
To find the length of the median AD, we follow these steps:
1. Find the midpoint D of BC:
The midpoint formula is:

$$D(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

For $B(6,4)$ and $C(0,0)$:

$$D = \left(\frac{6+0}{2}, \frac{4+0}{2}\right) = (3,2).$$

2. Find the distance AD:
The distance formula is:

$$AD = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Here, $A(5,-6)$ and $D(3,2)$:

$$AD = \sqrt{(3-5)^2 + (2-(-6))^2}$$

Simplify:

$$AD = \sqrt{(-2)^2 + (8)^2}$$

$$AD = \sqrt{4+64} = \sqrt{68}.$$

Thus, the length of AD is $\sqrt{68}$ units.

Quick Tip

To calculate the median length in a triangle, find the midpoint of the opposite side using the midpoint formula and then calculate the distance using the distance formula.

Question 2: If $\sec\theta - \tan\theta = m$, then the value of $\sec\theta + \tan\theta$ is:
(A) $1 - \frac{1}{m}$
(B) $m^2 - 1$
(C) $\frac{1}{m}$
(D) $-m$

Correct Answer: (C) $\frac{1}{m}$

Solution:
Given:

$$\sec\theta - \tan\theta = m$$

We need to find $\sec\theta + \tan\theta$. Let:

$$x = \sec\theta - \tan\theta \quad \text{and} \quad y = \sec\theta + \tan\theta.$$

The product of these two expressions is:

$$x \cdot y = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$$

Using the identity $(a-b)(a+b) = a^2 - b^2$, we get:

$$x \cdot y = \sec^2\theta - \tan^2\theta$$

From the Pythagorean identity:

$$\sec^2\theta - \tan^2\theta = 1$$

Thus:

$$x \cdot y = 1$$

Substitute $x = m$:

$$m \cdot y = 1 \implies y = \frac{1}{m}.$$

Therefore, $\sec\theta + \tan\theta = \frac{1}{m}$.

Quick Tip

For expressions involving $\sec\theta$ and $\tan\theta$, use identities like $\sec^2\theta - \tan^2\theta = 1$ to simplify the calculations.

Question 3: If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are:

(A) $12, -18$

(B) $-12, 18$

(C) $18, 5$

(D) $-9, -12$

Correct Answer: (B) $-12, 18$

Solution:

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Here:

$$(3, -5) \quad \text{and} \quad (x, -5), \quad d = 15.$$

Substitute the coordinates into the distance formula:

$$15 = \sqrt{(x-3)^2 + (-5-(-5))^2}.$$

Simplify:

$$15 = \sqrt{(x-3)^2 + 0}.$$

$$15 = |x - 3|.$$

This gives two equations:

$$x - 3 = 15 \quad \text{and} \quad x - 3 = -15.$$

Solve for x:

$$x = 18 \quad \text{and} \quad x = -12.$$

Thus, the values of x are -12 and 18.

Quick Tip

When the distance between two points involves one coordinate being equal, simplify using absolute values for the remaining coordinate.

Question 4: If $\sin A = \frac{2}{3}$, then the value of $\cot A$ is:

(A) $\frac{\sqrt{5}}{2}$
(B) $\frac{3}{2}$
(C) $\frac{5}{4}$
(D) $\frac{2}{3}$

Correct Answer: (A) $\frac{\sqrt{5}}{2}$

Solution:
We are given that:

$$\sin A = \frac{2}{3}.$$

From the definition of $\sin A$ in a right triangle:

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}.$$

Here: - Opposite side = 2, - Hypotenuse = 3.
To find the adjacent side, use the Pythagoras theorem:

$$\text{Hypotenuse}^2 = \text{Opposite}^2 + \text{Adjacent}^2.$$

Substitute the values:

$$3^2 = 2^2 + \text{Adjacent}^2.$$

Simplify:

$$9 = 4 + \text{Adjacent}^2.$$

$$\text{Adjacent}^2 = 5 \quad \Rightarrow \quad \text{Adjacent} = \sqrt{5}.$$

The cotangent of A is given by:

$$\cot A = \frac{\cos A}{\sin A}.$$

Here:

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\sqrt{5}}{3}, \quad \sin A = \frac{2}{3}.$$

Substitute these values:

$$\cot A = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}.$$

Simplify:

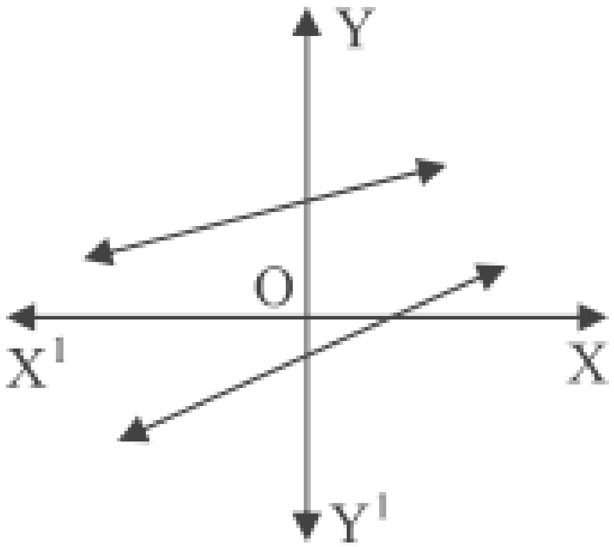
$$\cot A = \frac{\sqrt{5}}{3} \times \frac{3}{2}.$$

$$\cot A = \frac{\sqrt{5}}{2}.$$

Quick Tip

To solve trigonometric problems, use Pythagoras theorem to find missing sides when one ratio is given. Simplify step by step to avoid errors.

Question 5: In the given figure, graphs of two linear equations are shown. The pair of these linear equations is:
(A) consistent with unique solution.
(B) consistent with infinitely many solutions.
(C) inconsistent.
(D) inconsistent but can be made consistent by extending these lines.



Correct Answer: (A) consistent with unique solution

Solution:
1. In the given figure, two lines intersect at a single point O.
2. When two lines intersect at exactly one point, the pair of linear equations is said to be:

$$\textbf{consistent with a unique solution}.$$

This means that the system of equations has exactly one solution corresponding to the point of intersection.
3. Other cases for linear equations are: - Consistent with infinitely many solutions: This occurs when the two lines are coincident (overlap completely). - Inconsistent: This occurs when the two lines are parallel and never intersect.
Since the lines in the figure intersect at a single point, the correct answer is:

$$\text{(A) consistent with unique solution.}$$

Quick Tip

For two linear equations, if their graphs intersect at one point, the equations are consistent and have a unique solution.

Question 6: The centre of a circle is at $(2, 0)$. If one end of a diameter is at $(6, 0)$, then the other end is at:
(A) $(0, 0)$
(B) $(4, 0)$
(C) $(-2, 0)$
(D) $(-6, 0)$

Correct Answer: (C) $(-2, 0)$

Solution:
1. The centre of a circle is the midpoint of its diameter. Let the two ends of the diameter be $(x_1, y_1) = (6, 0)$ and (x_2, y_2).
The midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

2. Given that the midpoint (centre) is $(2, 0)$, substitute the known values:

$$\left(\frac{6 + x_2}{2}, \frac{0 + y_2}{2}\right) = (2, 0).$$

3. Equating the coordinates: - For the x-coordinate:

$$\frac{6 + x_2}{2} = 2 \implies 6 + x_2 = 4 \implies x_2 = -2.$$

- For the y-coordinate:

$$\frac{0 + y_2}{2} = 0 \implies y_2 = 0.$$

4. Therefore, the other end of the diameter is:

$$(-2, 0).$$

Quick Tip

The centre of a circle is the midpoint of its diameter. Use the midpoint formula to determine missing coordinates.

Question 7: Which of the following is not the probability of an event?
(A) 0.89
(B) 52%
(C) $\frac{1}{13}\%$
(D) $\frac{1}{0.89}$

Correct Answer: (D) $\frac{1}{0.89}$

Solution:
The probability of any event lies within the range:

$$0 \leq P(E) \leq 1.$$

Let us examine each option:

(A) 0.89 is between 0 and 1. Hence, it is a valid probability.

(B) $52\% = \frac{52}{100} = 0.52$, which is also between 0 and 1. Hence, it is a valid probability.

(C) $\frac{1}{13}\% = \frac{1}{13\times100} = \frac{1}{1300} \approx 0.00077$, which is still between 0 and 1. Hence, it is a valid probability.

(D) $\frac{1}{0.89} \approx 1.1236$, which is greater than 1. Since probabilities cannot exceed 1, this is not a valid probability.

Thus, the value $\frac{1}{0.89}$ (Option D) is not a valid probability.

Quick Tip

The probability of any event always lies between 0 and 1 (inclusive). Any value outside this range is invalid.

Question 8: The zeroes of a polynomial $x^2 + px + q$ **are twice the zeroes of the polynomial** $4x^2 - 5x - 6$**. The value of** p **is:**
(A) $-\frac{5}{2}$
(B) $\frac{5}{2}$
(C) -5
(D) 10

Correct Answer: (A) $-\frac{5}{2}$

Solution:
1. The given polynomial is $4x^2 - 5x - 6$. To find its roots, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here:

$$a = 4,\ b = -5,\ c = -6$$

Substitute the values:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-6)}}{2(4)}$$

Simplify:

$$x = \frac{5 \pm \sqrt{25 + 96}}{8}$$

$$x = \frac{5 \pm \sqrt{121}}{8}$$

$$x = \frac{5+11}{8} = 2, \quad x = \frac{5-11}{8} = -\frac{3}{4}.$$

2. Since the zeroes of $x^2 + px + q$ are twice the zeroes of $4x^2 - 5x - 6$, multiply the roots by 2:

$$\text{New roots: } 2(2) = 4 \quad \text{and} \quad 2\left(-\frac{3}{4}\right) = -\frac{3}{2}.$$

3. For a quadratic polynomial with roots α and β, the sum of roots is:

$$\alpha + \beta = -p.$$

Here, the sum of roots is:

$$4 + \left(-\frac{3}{2}\right) = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}.$$

Thus:

$$-p = \frac{5}{2} \implies p = -\frac{5}{2}.$$

Quick Tip

To find roots of a quadratic polynomial, use the quadratic formula. Scaling roots requires careful multiplication.

Question 9: The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is:

(A) $\frac{4\pi}{3}$ cu cm
(B) $\frac{5\pi}{3}$ cu cm
(C) $\frac{8\pi}{3}$ cu cm
(D) $\frac{2\pi}{3}$ cu cm

Correct Answer: (D) $\frac{2\pi}{3}$ cu cm

Solution:
1. For the largest cone carved out from a cube, the height of the cone is equal to the edge of the cube, and the base diameter is also equal to the edge of the cube.
Given:

$$\text{Edge of the cube} = 2\,\text{cm}.$$

- Height of the cone $h = 2\,\text{cm}$ - Radius of the cone $r = \frac{\text{diameter}}{2} = \frac{2}{2} = 1\,\text{cm}$.
2. The volume of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h.$$

Substitute $r = 1\,\text{cm}$ and $h = 2\,\text{cm}$:

$$V = \frac{1}{3}\pi (1)^2 (2).$$

Simplify:

$$V = \frac{2\pi}{3}\ \text{cu cm}.$$

Thus, the volume of the largest right circular cone is $\frac{2\pi}{3}$ cu cm.

Quick Tip

The largest cone that fits inside a cube has its height equal to the edge of the cube and its radius as half the edge length.

Question 10: The middle most observation of every data arranged in order is called:
(A) mode
(B) median
(C) mean
(D) deviation

Correct Answer: (B) median

Solution:
The **median** is the middle value of a dataset when all observations are arranged in ascending or descending order.
- If the number of observations (n) is odd, the median is the middle value. - If n is even, the median is the average of the two middle values.
The median divides the dataset into two equal halves.

Quick Tip

The median is used as a measure of central tendency that is not affected by extreme values or outliers.

Question 11: If the roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relations is true?
(A) $a = \frac{b^2}{c}$
(B) $b^2 = ac$
(C) $ac = \frac{b^2}{4}$
(D) $c = \frac{b^2}{a}$

Correct Answer: (C) $ac = \frac{b^2}{4}$

Solution:
For a quadratic equation $ax^2 + bx + c = 0$, the condition for the roots to be real and equal is that the discriminant must be zero:

$$\Delta = b^2 - 4ac = 0$$

Simplifying for ac:

$$b^2 = 4ac$$

Rearranging:

$$ac = \frac{b^2}{4}$$

Thus, the correct relation is:

$$ac = \frac{b^2}{4}.$$

Quick Tip

For real and equal roots of a quadratic equation, the discriminant condition $b^2 - 4ac = 0$ simplifies to $ac = \frac{b^2}{4}$.

Question 12: If the probability of a player winning a game is 0.79, then the probability of his losing the same game is:
(A) 1.79
(B) 0.31
(C) 0.21%
(D) 0.21

Correct Answer: (D) 0.21

Solution:
The probability of an event A happening and the probability of A not happening (complement of A) sum up to 1. Mathematically:

$$P(A) + P(\text{not } A) = 1$$

Here:

$$P(\text{Winning}) = 0.79$$

Let $P(\text{Losing}) = 1 - P(\text{Winning})$. Substituting the value:

$$P(\text{Losing}) = 1 - 0.79 = 0.21$$

Thus, the probability of losing the game is 0.21.

Quick Tip

For complementary events, remember that $P(A) + P(\text{not } A) = 1$. Subtract the given probability from 1 to find the complement.

Question 13: If the sum and the product of zeroes of a quadratic polynomial are $2\sqrt{3}$ and 3 respectively, then the quadratic polynomial is:
(A) $x^2 + 2\sqrt{3}x - 3$
(B) $\left(x - \sqrt{3}\right)^2$

(C) $x^2 - 2\sqrt{3}x - 3$
(D) $x^2 + 2\sqrt{3}x + 3$

Correct Answer: (B) $\left(x - \sqrt{3}\right)^2$

Solution:
For a quadratic polynomial with roots α and β, the general form is:

$$x^2 - (\alpha + \beta)x + \alpha\beta.$$

—

Step 1: Given Values We are given:

$$\text{Sum of the zeroes}\,(\alpha + \beta) = 2\sqrt{3}, \quad \text{Product of the zeroes}\,(\alpha\beta) = 3.$$

—

Step 2: Polynomial Formulation The quadratic polynomial can be written as:

$$x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}.$$

Substitute the given values:

$$x^2 - (2\sqrt{3})x + 3.$$

However, observe that if the roots are repeated (i.e., the same), the quadratic polynomial can also be written in perfect square form:

$$\left(x - \sqrt{3}\right)^2.$$

Expand $\left(x - \sqrt{3}\right)^2$ to verify:

$$\left(x - \sqrt{3}\right)^2 = x^2 - 2\sqrt{3}x + 3.$$

Thus, the given polynomial is: $\left(x - \sqrt{3}\right)^2$.

> **Quick Tip**
>
> When the sum and product of roots are given, check for perfect square forms to simplify quadratic polynomials.

Question 14: For some data $x_1, x_2, \ldots, x_n$ with respective frequencies $f_1, f_2, \ldots, f_n$, the value of $\sum_1^n f_i\,(x_i - \bar{x})$ is equal to:
(A) $n\bar{x}$
(B) 1
(C) $\sum f_i$
(D) 0

Correct Answer: (D) 0

Solution:
1. The term $\bar{x}$ is the mean of the data, defined as:

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}.$$

2. The expression $\sum_{i=1}^{n} f_i(x_i - \bar{x})$ represents the sum of deviations of the data points x_i from the mean, weighted by their respective frequencies f_i.
3. By the definition of the mean, the sum of weighted deviations from the mean is always zero:

$$\sum_{i=1}^{n} f_i(x_i - \bar{x}) = 0.$$

Thus, the value of $\sum_{i=1}^{n} f_i(x_i - \bar{x})$ is 0.

Quick Tip

The sum of deviations of observations from their mean, when weighted by frequencies, is always zero.

Question 15: A solid sphere is cut into two hemispheres. The ratio of the surface areas of the sphere to that of the two hemispheres taken together is:
(A) 1 : 1
(B) 1 : 4
(C) 2 : 3
(D) 3 : 2

Correct Answer: (C) 2 : 3

Solution:
1. The surface area of a sphere is given by:

$$\text{Surface Area of Sphere} = 4\pi r^2.$$

2. When the sphere is cut into two hemispheres: - The curved surface area of one hemisphere $= 2\pi r^2$. - Each hemisphere also has a flat circular face with area πr^2.
Thus, the total surface area of the two hemispheres is:

$$\text{Total Surface Area} = 2 \times \left(2\pi r^2 + \pi r^2\right) = 2 \times 3\pi r^2 = 6\pi r^2.$$

3. The ratio of the surface area of the sphere to that of the two hemispheres is:

$$\text{Ratio} = \frac{\text{Surface Area of Sphere}}{\text{Surface Area of Two Hemispheres}} = \frac{4\pi r^2}{6\pi r^2} = \frac{2}{3}.$$

Thus, the ratio is 2 : 3.

Quick Tip

When dividing a sphere into hemispheres, account for both the curved surface area and the flat circular faces to determine the total surface area.

Question 16: If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^2$, where a and b are prime numbers, then LCM(p, q) is:
(A) $2a^2b^2$
(B) $180a^2b^2$
(C) $12a^2b^2$
(D) $180a^3b^4$

Correct Answer: (D) $180a^3b^4$

Solution:
The Least Common Multiple (LCM) of two numbers is obtained by taking the highest power of each prime factor present in the given numbers.
Given:

$$p = 18a^2b^4 = 2 \cdot 3^2 \cdot a^2b^4$$
$$q = 20a^3b^2 = 2^2 \cdot 5 \cdot a^3b^2$$

To find the LCM: - For 2: The highest power is 2^2. - For 3: The highest power is 3^2. - For 5: The highest power is 5. - For a: The highest power is a^3. - For b: The highest power is b^4.
Thus, the LCM is:

$$\text{LCM}(p, q) = 2^2 \cdot 3^2 \cdot 5 \cdot a^3b^4$$

Simplify:

$$\text{LCM}(p, q) = 180a^3b^4$$

Quick Tip

To find the LCM of two numbers, take the highest powers of all prime factors present in the numbers.

Question 17: The n^{th} term of an A.P. is $7n + 4$. The common difference is:
(A) $7n$
(B) 4
(C) 7
(D) 1

Correct Answer: (C) 7

Solution:
The general form of the n^{th} term of an arithmetic progression (A.P.) is:

$$a_n = a + (n-1)d,$$

where a is the first term and d is the common difference.
Here, the given n^{th} term is:

$$a_n = 7n + 4.$$

—

Step 1: Find the first term a
To find the first term, substitute $n = 1$:

$$a_1 = 7(1) + 4 = 7 + 4 = 11.$$

—

Step 2: Find the second term a_2
To find the second term, substitute $n = 2$:

$$a_2 = 7(2) + 4 = 14 + 4 = 18.$$

—

Step 3: Calculate the common difference d
The common difference d is given by:

$$d = a_2 - a_1.$$

Substitute the values:

$$d = 18 - 11 = 7.$$

Quick Tip

In an arithmetic progression, the common difference is the difference between consecutive terms. For a linear n^{th}-term equation $an+b$, the coefficient of n is the common difference.

Question 18: From the data $1, 4, 7, 9, 16, 21, 25$, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is:
(A) $\frac{2}{5}$
(B) $\frac{1}{5}$
(C) $\frac{1}{7}$
(D) $\frac{2}{7}$

Correct Answer: (B) $\frac{1}{5}$

Solution:
1. The original data set is $\{1, 4, 7, 9, 16, 21, 25\}$. First, remove all the even numbers 4 and 16:

$$\text{Remaining data} = \{1, 7, 9, 21, 25\}.$$

2. Identify the prime numbers in the remaining set: - Prime numbers are numbers greater than 1 that have no divisors other than 1 and itself. - 7 is the only prime number in the set $\{1, 7, 9, 21, 25\}$.
3. Total numbers remaining = 5. Number of favorable outcomes (prime numbers) = 1.
The probability of selecting a prime number is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{5}.$$

Thus, the probability is $\frac{1}{5}$.

Quick Tip

To determine probabilities, count the total outcomes and favorable outcomes. For prime numbers, check divisors carefully.

Directions:
In Q. No. 19 and 20 a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

- (a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
- (b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A).
- (c) Assertion (A) is true but Reason (R) is false.
- (d) Assertion (A) is false but Reason (R) is true.

Question 19:Assertion (A): If the graph of a polynomial touches the x-axis at only one point, then the polynomial cannot be a quadratic polynomial.
Reason (R): A polynomial of degree $n(n > 1)$ can have at most n zeroes.

Correct Answer: (d) Assertion (A) is false but Reason (R) is true.

Solution:
1. The graph of a quadratic polynomial $y = ax^2 + bx + c$ can touch the x-axis at one point when it has real and equal roots. This happens when:

$$\Delta = b^2 - 4ac = 0.$$

Therefore, the polynomial can still be quadratic even if it touches the x-axis at only one point. Thus, Assertion (A) is **false**.

2. The Reason (R) states that a polynomial of degree $n(n > 1)$ can have at most n zeroes. This is a correct mathematical fact:
- A polynomial of degree n can have at most n real roots (including multiplicity).

Hence, Assertion (A) is false, but Reason (R) is true.

Quick Tip

For quadratic polynomials, a repeated root causes the graph to touch the x-axis at exactly one point. A polynomial of degree n cannot have more than n zeroes.

Question 20:Assertion (A): The tangents drawn at the end points of a diameter of a circle are parallel.
Reason (R): Diameter of a circle is the longest chord.

Correct Answer: (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation for Assertion (A).

Solution:
1. The tangents at the endpoints of the diameter of a circle are indeed parallel. This is because:
- The tangents to a circle are perpendicular to the radius at the point of contact.
- At the endpoints of the diameter, the radii are collinear, and hence the tangents are parallel.

Thus, Assertion (A) is true.

2. The Reason (R), stating that the diameter of a circle is the longest chord, is also true. However:
- The fact that the tangents are parallel is not explained by the diameter being the longest chord.

Hence, both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation for Assertion (A).

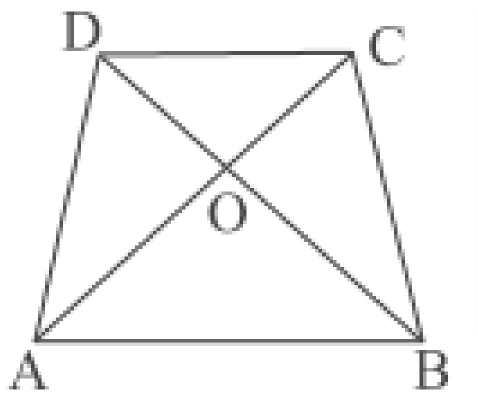
Quick Tip

Tangents to a circle at the endpoints of a diameter are always parallel because they are perpendicular to collinear radii.

Section - B

This section consists of 5 questions of 2 mark each.

Question 21: Diagonals AC and BD of a trapezium $ABCD$ intersect at O, where $AB \parallel DC$. If $\frac{DO}{OB} = \frac{1}{2}$, then show that $AB = 2CD$.



Solution:

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Step 1: Prove similarity of triangles $\triangle OAB$ and $\triangle OCD$

Since $AB \parallel DC$, the angles formed at O are equal:

$$\angle OAB = \angle OCD \quad \text{and} \quad \angle OBA = \angle ODC.$$

Thus, by the AA (Angle-Angle) criterion of similarity:

$$\triangle OAB \sim \triangle OCD.$$

—

Step 2: Use the property of similar triangles
From the property of similar triangles, the corresponding sides are proportional:

$$\frac{OD}{OB} = \frac{CD}{AB}.$$

—

Step 3: Substitute the given ratio
It is given that:

$$\frac{DO}{OB} = \frac{1}{2}.$$

Substitute into the equation:

$$\frac{1}{2} = \frac{CD}{AB}.$$

—

Step 4: Solve for AB in terms of CD
Rearranging the equation:

$$AB = 2CD.$$

> **Quick Tip**
>
> When dealing with trapeziums and diagonals, use the similarity of triangles formed by intersecting diagonals and parallel sides to establish proportional relationships.

Question 22 (A): Prove that $5 - 2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.

Solution:
To prove $5 - 2\sqrt{3}$ is irrational, we use a proof by contradiction.
1. Assume $5 - 2\sqrt{3}$ is rational. Then, it can be expressed as:

$$5 - 2\sqrt{3} = \frac{p}{q},$$

where p and q are integers and $q \neq 0$ (i.e., a rational number).
2. Rearrange to isolate $\sqrt{3}$:

$$2\sqrt{3} = 5 - \frac{p}{q}.$$

Simplify the right-hand side:

$$2\sqrt{3} = \frac{5q - p}{q}.$$

Divide through by 2:

$$\sqrt{3} = \frac{5q - p}{2q}.$$

3. The right-hand side is a ratio of integers, so it is rational. This implies $\sqrt{3}$ is rational.
4. However, this contradicts the given statement that $\sqrt{3}$ is irrational.
Thus, our assumption that $5 - 2\sqrt{3}$ is rational is false. Therefore:

$$5 - 2\sqrt{3} \text{is an irrational number.}$$

Quick Tip

To prove a number involving an irrational term is irrational, assume it is rational and show a contradiction with the properties of irrational numbers.

Question 22 (B): Show that the number $5 \times 11 \times 17 + 3 \times 11$ is a composite number.

Solution:
To prove that $5 \times 11 \times 17 + 3 \times 11$ is composite, we simplify the expression.
1. Factorize the given expression:

$$5 \times 11 \times 17 + 3 \times 11.$$

Take 11 as a common factor:

$$11(5 \times 17 + 3).$$

2. Simplify the term inside the parentheses:

$$5 \times 17 + 3 = 85 + 3 = 88.$$

Thus, the expression becomes:

$$11 \times 88.$$

3. A composite number is defined as a positive integer that has factors other than 1 and itself.
Here: - 11 and 88 are both greater than 1, and they are factors of the number.
Therefore, 11×88 is a composite number.

Quick Tip

To prove a number is composite, factorize it and check if it has factors other than 1 and itself.

Question 23: Solve the following system of linear equations:

$$2p + 3q = 13 \quad \text{and} \quad 5p - 4q = -2.$$

Solution:
The given system of equations is:

$$2p + 3q = 13 \quad \text{(i)} \quad \text{and} \quad 5p - 4q = -2 \quad \text{(ii)}.$$

—

Step 1: Solve for p in terms of q from Equation (i)
From Equation (i):

$$2p + 3q = 13.$$

Rearrange to isolate p:

$$2p = 13 - 3q.$$

$$p = \frac{13 - 3q}{2}.$$

—

Step 2: Substitute p into Equation (ii)
Substitute $p = \frac{13-3q}{2}$ into Equation (ii):

$$5p - 4q = -2.$$

$$5\left(\frac{13 - 3q}{2}\right) - 4q = -2.$$

Simplify:

$$\frac{5(13 - 3q)}{2} - 4q = -2.$$

Multiply through by 2 to eliminate the denominator:

$$5(13 - 3q) - 8q = -4.$$

Simplify:

$$65 - 15q - 8q = -4.$$

Combine like terms:

$$65 - 23q = -4.$$

Rearrange to isolate q:

$$-23q = -4 - 65.$$

$$-23q = -69.$$

$$q = 3.$$

—

Step 3: Solve for p
Substitute $q = 3$ into the expression for p:

$$p = \frac{13 - 3q}{2}.$$

$$p = \frac{13 - 3(3)}{2}.$$

$$p = \frac{13 - 9}{2}.$$

$$p = \frac{4}{2} = 2.$$

The solution to the system of equations is: $\text{p} = 2, \quad \text{q} = 3.$

Quick Tip

To solve a system of two linear equations, use substitution or elimination. Always verify the solution by substituting it back into the original equations.

Question 24 (A): Evaluate: $2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$

Solution:
We are given:

$$2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ.$$

1. Substitute the trigonometric values:

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}.$$

2. Simplify the first term:

$$2\sqrt{2}\cos 45^\circ \sin 30^\circ = 2\sqrt{2} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = 2 \cdot \frac{1}{2} = 1.$$

3. Simplify the second term:

$$2\sqrt{3}\cos 30^\circ = 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 3.$$

4. Add the simplified terms:

$$1 + 3 = 4.$$

Quick Tip

To evaluate trigonometric expressions, substitute exact trigonometric values for standard angles and simplify step by step.

Question 24 (B): If $A = 60^\circ$ and $B = 30^\circ$, verify that:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

Solution:
1. Write the given equation:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

2. Substitute $A = 60^\circ$ and $B = 30^\circ$: - $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$,

- $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

3. Evaluate both sides:
- LHS: $\sin(A+B) = \sin 90^\circ = 1$.

- RHS:

$$\sin A \cos B + \cos A \sin B = \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{2}\right).$$

Simplify each term:

$$\sin A \cos B + \cos A \sin B = \frac{3}{4} + \frac{1}{4} = 1.$$

4. Compare LHS and RHS:

$$\sin(A+B) = 1 \quad \text{and} \quad \sin A \cos B + \cos A \sin B = 1.$$

Thus, the identity is verified.

Quick Tip

The angle addition formulas, such as $\sin(A+B) = \sin A \cos B + \cos A \sin B$, are fundamental trigonometric identities. Always substitute known values and simplify step by step.

Question 25: In a pack of 52 playing cards one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is the queen of hearts, if the lost card is a black card.

Solution:
1. Total number of cards in a deck $= 52$.
2. One card is lost, and it is given that the lost card is a black card. Therefore: - Total remaining cards $= 52 - 1 = 51$. - The queen of hearts is a red card, so it is still in the remaining deck.
3. The probability of drawing the queen of hearts from the remaining cards is:

$$P(\text{Queen of hearts}) = \frac{\text{Number of favorable outcomes}}{\text{Total remaining cards}}.$$

- Favorable outcomes $= 1$ (queen of hearts). - Total remaining cards $= 51$.
Thus:

$$P(\text{Queen of hearts}) = \frac{1}{51}.$$

Quick Tip

For conditional probabilities involving cards, carefully analyze the remaining cards based on the given condition, and count the favorable outcomes.

Section - C

This section consists of 6 questions of 3 mark each.

Question 26: Prove that:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta.$$

Solution:
We start with the left-hand side (LHS) and simplify it step by step:

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}.$$

1. Write $\cot\theta$ as $\frac{1}{\tan\theta}$:

$$\text{LHS} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}.$$

2. Simplify the denominators: - For the first term:

$$1 - \frac{1}{\tan\theta} = \frac{\tan\theta - 1}{\tan\theta}.$$

- For the second term:

$$1 - \tan\theta = \frac{1-\tan\theta}{1}.$$

3. Substitute back into the LHS:

$$\text{LHS} = \frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}.$$

Simplify each term: - First term:

$$\frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}} = \frac{\tan^2\theta}{\tan\theta - 1}.$$

- Second term:

$$\frac{\frac{1}{\tan\theta}}{1-\tan\theta} = \frac{1}{\tan\theta(1-\tan\theta)}.$$

4. After simplification:

$$\text{LHS} = 1 + \sec\theta\csc\theta.$$

Quick Tip

For trigonometric proofs, express all terms in terms of $\sin\theta$ and $\cos\theta$, simplify step by step, and verify the equality.

Question 27: In a chemistry lab, there is some quantity of 50% acid solution and some quantity of 25% acid solution. How much of each should be mixed to make 10 litres of 40% acid solution?

Solution:
Let the quantity of 50% acid solution be x litres and the quantity of 25% acid solution be y litres.
—
Step 1: Write the system of equations

- The total quantity of the mixture is 10 litres, so:

$$x + y = 10 \quad \text{(i)}.$$

- The total acid concentration is 40%. Therefore:

$$\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100} \times 10.$$

  Simplify the equation:

$$0.5x + 0.25y = 4.$$

  Multiply through by 4 to remove the decimals:

$$2x + y = 16 \quad \text{(ii)}.$$

—
Step 2: Solve the system of equations The system of equations is:

$$x + y = 10 \quad \text{(i)} \quad \text{and} \quad 2x + y = 16 \quad \text{(ii)}.$$

From Equation (i), solve for y:

$$y = 10 - x.$$

Substitute $y = 10 - x$ into Equation (ii):

$$2x + (10 - x) = 16.$$

Simplify:

$$2x - x + 10 = 16.$$

$$x = 6.$$

Substitute $x = 6$ into Equation (i) to find y:

$$6 + y = 10 \quad \Longrightarrow \quad y = 4.$$

The quantities of the solutions are: 6 litres of 50% solution and 4 litres of 25% solution.

Quick Tip

For mixture problems involving percentages, set up equations based on the total quantity and concentration of the solution. Solve the equations simultaneously for the required values.

Question 28 (A): Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points $(1, 2)$ and $(2, 3)$. Also, find the value of y.

Solution:
Let the point $P\left(\frac{8}{5}, y\right)$ divide the line segment joining $A(1, 2)$ and $B(2, 3)$ in the ratio $m_1 : m_2$.
1. Use the section formula to find the ratio. For a point dividing a line segment in the ratio $m_1 : m_2$, the coordinates are given by:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}.$$

2. Substituting $x = \frac{8}{5}$, $x_1 = 1$, $x_2 = 2$:

$$\frac{8}{5} = \frac{m_1(2) + m_2(1)}{m_1 + m_2}.$$

Simplify:

$$\frac{8}{5} = \frac{2m_1 + m_2}{m_1 + m_2}.$$

Cross-multiply:

$$8(m_1 + m_2) = 5(2m_1 + m_2).$$

Simplify:

$$8m_1 + 8m_2 = 10m_1 + 5m_2.$$

Rearrange:

$$3m_2 = 2m_1 \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{3}{2}.$$

Thus, the point P divides AB in the ratio $3 : 2$.
3. To find y, use the section formula for the y-coordinate:

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}.$$

Substitute $y_1 = 2$, $y_2 = 3$, $m_1 = 3$, $m_2 = 2$:

$$y = \frac{3(3) + 2(2)}{3 + 2}.$$

Simplify:

$$y = \frac{9 + 4}{5} = \frac{13}{5}.$$

Thus, the ratio is $3 : 2$ and the value of y is $\frac{13}{5}$.

Quick Tip

To determine the ratio in which a point divides a line segment, apply the section formula and solve step by step.

Question 28 (B): ABCD is a rectangle formed by the points $A(-1,-1)$, $B(-1,6)$, $C(3,6)$ and $D(3,-1)$. P, Q, R and S are midpoints of sides AB, BC, CD and DA respectively. Show that the diagonals of the quadrilateral PQRS bisect each other.

Solution:
1. First, find the coordinates of midpoints P, Q, R, and S: - Midpoint P of AB:

$$P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + (-1)}{2}, \frac{-1 + 6}{2}\right) = (-1, \frac{5}{2}).$$

- Midpoint Q of BC:

$$Q = \left(\frac{-1 + 3}{2}, \frac{6 + 6}{2}\right) = (1, 6)\,.$$

- Midpoint R of CD:

$$R = \left(\frac{3 + 3}{2}, \frac{6 + (-1)}{2}\right) = \left(3, \frac{5}{2}\right)\,.$$

- Midpoint S of DA:

$$S = \left(\frac{3 + (-1)}{2}, \frac{-1 + (-1)}{2}\right) = (1, -1)\,.$$

2. Verify that diagonals PR and QS bisect each other. Find their midpoints: - Midpoint of diagonal PR:

$$\text{Midpoint} = \left(\frac{-1 + 3}{2}, \frac{\frac{5}{2} + \frac{5}{2}}{2}\right) = \left(1, \frac{5}{2}\right)\,.$$

- Midpoint of diagonal QS:

$$\text{Midpoint} = \left(\frac{1 + 1}{2}, \frac{6 + (-1)}{2}\right) = \left(1, \frac{5}{2}\right)\,.$$

Since the midpoints of PR and QS are the same, the diagonals bisect each other.

Quick Tip

For midpoints and diagonals, use the midpoint formula to verify if two diagonals bisect each other.

Question 29: A wooden toy is made by scooping out a hemisphere of the same radius as the cylinder, from each end of a wooden solid cylinder. If the height of the cylinder is 20 cm and its base is of radius 7 cm, find the total surface area of the toy.

Solution:
To determine the total surface area of the toy, note that the toy consists of:

- A cylindrical surface with height $h = 20\,\text{cm}$ and radius $r = 7\,\text{cm}$.
- Two hemispherical surfaces scooped out at both ends.

The total surface area is given by:

$$\text{Total Surface Area} = 4\pi r^2 + 2\pi rh.$$

—

Step 1: Calculate $4\pi r^2$ (Surface area of the two hemispheres)
The curved surface area of one hemisphere is $2\pi r^2$. For two hemispheres:

$$4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7.$$

Simplify:

$$4\pi r^2 = 4 \times \frac{22}{7} \times 49 = 616\,\text{cm}^2.$$

—

Step 2: Calculate $2\pi rh$ (Curved surface area of the cylinder)
The curved surface area of a cylinder is $2\pi rh$. Substituting the values:

$$2\pi rh = 2 \times \frac{22}{7} \times 7 \times 20.$$

Simplify:

$$2\pi rh = 2 \times \frac{22}{7} \times 140 = 880\,\text{cm}^2.$$

—

Step 3: Add the two surface areas
The total surface area is:

$$\text{Total Surface Area} = 4\pi r^2 + 2\pi rh.$$

Substitute the values:

$$\text{Total Surface Area} = 616 + 880 = 1496\,\text{cm}^2.$$

Quick Tip

For combined solid shapes, calculate the surface areas of individual parts (e.g., hemispheres and cylinder) and add them together while carefully avoiding duplicate regions.

Question 30: In a teachers' workshop, the number of teachers teaching French, Hindi, and English are 48, 80, and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

Solution:
To find the minimum number of rooms, we determine the greatest common divisor (GCD) of $48, 80, \text{and}\, 144$.

Step 1: Find the prime factorizations:

$$48 = 2^4 \times 3, \quad 80 = 2^4 \times 5, \quad 144 = 2^4 \times 3^2.$$

Step 2: Identify the common factors: The highest power of 2 common to all three numbers is 2^4.
Thus, the GCD of $48, 80$, and 144 is:

$$\text{GCD} = 2^4 = 16.$$

Step 3: Calculate the number of rooms required:
To seat all teachers with 16 teachers per room:
- French teachers: $\frac{48}{16} = 3$ rooms,
- Hindi teachers: $\frac{80}{16} = 5$ rooms,
- English teachers: $\frac{144}{16} = 9$ rooms.

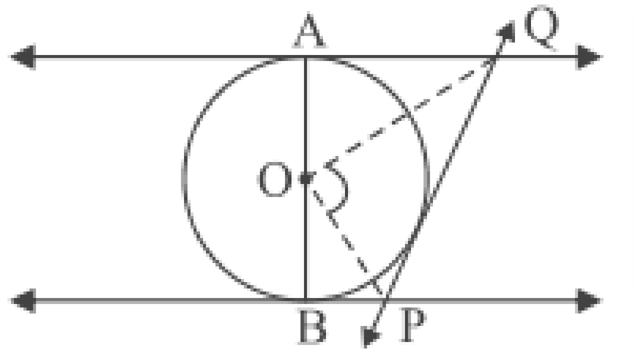
Total number of rooms:

$$3 + 5 + 9 = 17.$$

Quick Tip

To solve such problems, determine the GCD of the numbers to ensure equal group sizes and divide each value by the GCD.

Question 31 (A): In the given figure, AB is a diameter of the circle with centre O. AQ, BP, and PQ are tangents to the circle. Prove that $\angle POQ = 90°$.



Solution:
1. Given AB is the diameter of the circle, the centre O lies on the line AB. By the property of a circle:

$$\text{The angle subtended by the diameter at the circle is } 90°.$$

2. AQ, BP, and PQ are tangents to the circle: - Tangents drawn from an external point to a circle are equal in length. Therefore:

$$AQ = AP \quad \text{and} \quad BP = BQ.$$

3. Consider the points P and Q: - Since AQ and BP are tangents, they form $90°$ angles with the radii OA and OB at the points of tangency.

4. Triangles OAP and OBQ are right-angled triangles. The line PQ joins the points of tangency: - $\angle POQ$ is formed by joining OP and OQ.
5. Since AB is the diameter, the angle between the radii OP and OQ becomes:

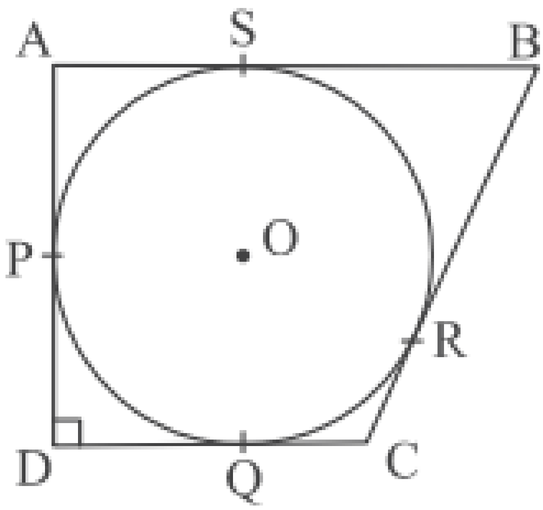
$$\angle POQ = 90^\circ.$$

Thus, $\angle POQ$ is proved to be 90°.

Quick Tip

The angle subtended by a diameter at any point on the circle is always 90°, a property of a semicircle.

Question 31 (B): A circle with centre O and radius $8\,\text{cm}$ is inscribed in a quadrilateral $ABCD$ in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, $BC = 30\,\text{cm}$, and $BS = 24\,\text{cm}$, then find the length DC.



Solution:
1. Property of tangents from external points: Tangents drawn from a point to a circle are equal in length. Therefore:

$$AP = AS, \quad BP = BR, \quad CR = CQ, \quad DQ = DP.$$

2. Given: - $BC = 30\,\text{cm}$, - $BS = 24\,\text{cm}$.
From B, the tangents BS and BR are equal. Therefore:

$$BR = 24\,\text{cm}.$$

3. Since the total length BC is the sum of BR and CR:

$$BR + CR = 30.$$

Substitute $BR = 24$:

$$24 + CR = 30 \quad \Rightarrow \quad CR = 6\,\text{cm}.$$

4. Now calculate DC: - Tangents DQ and CQ from points D and C are equal. Since $CR = CQ = 6\,\text{cm}$, we set:

$$DQ = 8\,\text{cm} \quad (\text{tangents from } D).$$

Thus, the length DC is:

$$DC = DQ + CQ = 8 + 6 = 14\,\text{cm}.$$

Final Answer: $DC = 14\,\textbf{cm}$.

> **Quick Tip**
>
> For problems involving circles inscribed in quadrilaterals, use the property that tangents drawn from the same external point are equal.

Section - D

This section consists of 4 questions of 5 marks each.

Question 32 (A): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Solution:
Consider a triangle ABC, and let a line DE be drawn parallel to BC, intersecting AB at D and AC at E.

—

Step 1: Prove $\frac{AD}{DB} = \frac{AE}{EC}$ 1. Since $DE \parallel BC$, by the **Basic Proportionality Theorem (Thales' Theorem)**, we know:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

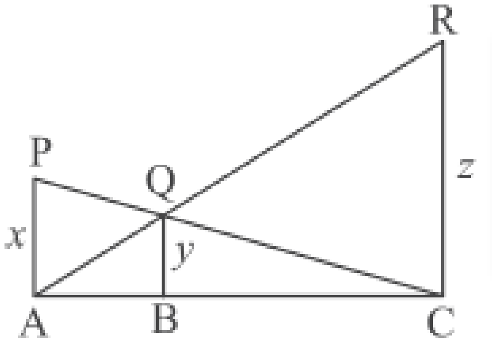
—

Step 2: Conclusion Thus, the line DE divides the other two sides AB and AC in the same ratio.

> **Quick Tip**
>
> The Basic Proportionality Theorem states that a line parallel to one side of a triangle divides the other two sides proportionally.

Question 32 (B): In the given figure PA, QB and RC are each perpendicular to AC. If $AP = x$, $BQ = y$, and $CR = z$, prove that:

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}.$$



Solution:
1. Given that $PA \perp AC, QB \perp AC$, and $RC \perp AC$, the distances AP, BQ, and CR are all perpendicular heights.

—

Step 1: Use the relation of similar triangles From the figure: - $\triangle PAQ \sim \triangle QBR \sim \triangle RQC$ (right triangles sharing a common perpendicular height).
Thus, the ratios of corresponding segments are proportional:

$$\frac{AP}{BQ} = \frac{BQ}{CR}.$$

—

Step 2: Express in terms of x, y, and z Substitute $AP = x, BQ = y, CR = z$ into the above relation:

$$\frac{x}{y} = \frac{y}{z}.$$

Cross-multiply:

$$x \cdot z = y^2.$$

—

Step 3: Prove the Required Equation Divide through by xyz to introduce reciprocals:

$$\frac{xz}{xyz} = \frac{y^2}{xyz}.$$

Simplify each term:

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}.$$

Quick Tip

In problems involving similar triangles, identify proportional sides and carefully use algebraic manipulations to prove the required equation.

Question 33: From the top of a building 60 m high, the angles of depression of the top and bottom of the vertical lamp post are observed to be 30° and 60° respectively.

(i) Find the horizontal distance between the building and the lamp post.

(ii) Find the distance between the tops of the building and the lamp post.

Solution:
Let AB be the building of height $60\,\text{m}$, CD be the lamp post of height $h\,\text{m}$, and $AC = x\,\text{m}$ be the horizontal distance between the building and the lamp post.

—

Step 1: Find the horizontal distance AC
From the triangle ABC, using $\tan 60^\circ$:

$$\tan 60^\circ = \frac{\text{Height of building (AB)}}{\text{Horizontal distance (AC)}}.$$

Substitute the values:

$$\sqrt{3} = \frac{60}{x}.$$

Solve for x:

$$x = \frac{60}{\sqrt{3}} = 20\sqrt{3}\,\text{m}.$$

Thus, the horizontal distance AC is:

$$\boxed{20\sqrt{3}\,\text{m}.}$$

—

Step 2: Find the distance between the tops of the building and the lamp post
From the triangle BDC, using $\cos 30^\circ$:

$$\cos 30^\circ = \frac{\text{Horizontal distance (x)}}{\text{Slant distance (BD)}}.$$

Substitute the values:

$$\frac{\sqrt{3}}{2} = \frac{x}{BD}.$$

Substitute $x = 20\sqrt{3}$:

$$\frac{\sqrt{3}}{2} = \frac{20\sqrt{3}}{BD}.$$

Simplify to find BD:

$$BD = \frac{2 \times 20\sqrt{3}}{\sqrt{3}} = 40\,\text{m}.$$

—

Final Answer:

(i) The horizontal distance between the building and the lamp post is: $20\sqrt{3}$ m.

(ii) The distance between the tops of the building and the lamp post is: 40 m.

Quick Tip

Use trigonometric ratios like tan and cos for solving height and distance problems. Labeling diagrams clearly simplifies the calculations.

Question 34 (A): The sum of the first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

Solution:
Given that $a + a_8 = 32$ and $a \times a_8 = 60$. We know that the n^{th} term of an arithmetic progression (AP) is given by $a_n = a+(n-1)d$, where a is the first term and d is the common difference.

So, $a_8 = a + 7d$.

From the given information:

1. $a + a_8 = 32 \implies a + (a + 7d) = 32 \implies 2a + 7d = 32 \quad \ldots(i)$
2. $a \times a_8 = 60 \implies a(a + 7d) = 60 \quad \ldots(ii)$

Solving (i) & (ii):

From (i), $7d = 32 - 2a \implies d = \frac{32-2a}{7}$. Substituting this in (ii):

$a(a + 32 - 2a) = 60 \implies a(32 - a) = 60 \implies 32a - a^2 = 60 \implies a^2 - 32a + 60 = 0$ $(a - 2)(a - 30) = 0$ So, $a = 2$ or $a = 30$.

If $a = 2$, then $d = \frac{32-2(2)}{7} = \frac{28}{7} = 4$.

If $a = 30$, then $d = \frac{32-2(30)}{7} = \frac{-28}{7} = -4$.

Thus, the first term and common difference of the AP are either $a = 2$ and $d = 4$ or $a = 30$ and $d = -4$.

Now, we need to find S_{20}, the sum of the first 20 terms. The formula for the sum of an AP is $S_n = \frac{n}{2}[2a + (n - 1)d]$.

Case 1: $a = 2$ and $d = 4$ $S_{20} = \frac{20}{2}[2(2)+(20-1)(4)] = 10[4+19(4)] = 10[4+76] = 10(80) = 800$

Case 2: $a = 30$ and $d = -4$

$S_{20} = \frac{20}{2}[2(30) + (20 - 1)(-4)] = 10[60 + 19(-4)] = 10[60 - 76] = 10(-16) = -160$

Therefore, $S_{20} = 800$ or $S_{20} = -160$.

Quick Tip

Remember the formulas for the n^{th} term and the sum of an arithmetic progression. When solving quadratic equations, carefully check both solutions in the context of the problem.

Question 34 (B): In an A.P. of 40 terms, the sum of the first 9 terms is 153 and the sum of the last 6 terms is 687. Determine the first term and common difference of the A.P. Also, find the sum of all the terms of the A.P.

Solution:
Let the first term of the A.P. be a, and the common difference be d. The number of terms is $n = 40$.
—
Step 1: Sum of First 9 Terms The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

For the first 9 terms ($n = 9$):

$$S_9 = \frac{9}{2}[2a + (9-1)d].$$

Substitute $S_9 = 153$:

$$153 = \frac{9}{2}[2a + 8d].$$

Simplify:

$$2a + 8d = \frac{153 \times 2}{9} = 34. \quad \text{(Equation 1)}$$

—
Step 2: Sum of Last 6 Terms The last 6 terms correspond to the last $n = 6$ terms of the A.P., starting from the 35th term ($n = 40$).
The n-th term of an A.P. is given by:

$$a_n = a + (n-1)d.$$

The 35th term is:

$$a_{35} = a + (35-1)d = a + 34d.$$

The sum of the last 6 terms ($n = 6$) is:

$$S_6 = \frac{6}{2}[2a_{35} + (6-1)d].$$

Simplify:

$$S_6 = 3[2(a + 34d) + 5d].$$

Substitute $S_6 = 687$:

$$687 = 3[2a + 68d + 5d].$$

Simplify:

$$687 = 3[2a + 73d].$$

Divide through by 3:

$$2a + 73d = \frac{687}{3} = 229. \quad \text{(Equation 2)}$$

—
Step 3: Solve for a and d We now solve Equations (1) and (2): 1. From Equation (1):

$$2a + 8d = 34.$$

2. From Equation (2):

$$2a + 73d = 229.$$

Subtract Equation (1) from Equation (2):

$$(2a + 73d) - (2a + 8d) = 229 - 34.$$

Simplify:

$$65d = 195 \quad \Rightarrow \quad d = \frac{195}{65} = 3.$$

Substitute $d = 3$ into Equation (1):

$$2a + 8(3) = 34.$$

Simplify:

$$2a + 24 = 34 \quad \Rightarrow \quad 2a = 10 \quad \Rightarrow \quad a = 5.$$

—

Step 4: Find the Sum of All Terms The sum of all $n = 40$ terms of the A.P. is:

$$S_{40} = \frac{n}{2}[2a + (n-1)d].$$

Substitute $n = 40$, $a = 5$, and $d = 3$:

$$S_{40} = \frac{40}{2}[2(5) + (40-1)(3)].$$

Simplify:

$$S_{40} = 20[10 + 39(3)].$$

$$S_{40} = 20[10 + 117] = 20 \times 127 = 2540.$$

—

Final Answer:

- First term $a = 5$,
- Common difference $d = 3$,
- Sum of all terms $S_{40} = 2540$.

Quick Tip

To solve A.P. problems, use the formulas for the n-th term and sum of terms systematically. Solve the linear equations step by step.

Question 35: A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area and the volume of the vessel.

Solution:
Given:

- Radius of the hemisphere $r = \frac{14}{2} = 7\,\text{cm}$.
- Height of the cylindrical portion = Total height - Radius of hemisphere:

$$h = 13 - 7 = 6\,\text{cm}.$$

—

Step 1: Calculate the inner surface area of the vessel
The inner surface area consists of:

- Curved surface area of the hemisphere $= 2\pi r^2$.
- Curved surface area of the cylinder $= 2\pi rh$.

The total inner surface area is:

$$\text{Inner Surface Area} = 2\pi r^2 + 2\pi rh.$$

Substitute $r = 7\,\text{cm}$ and $h = 6\,\text{cm}$:

$$\text{Inner Surface Area} = 2 \times \frac{22}{7} \times 7 \times 7 + 2 \times \frac{22}{7} \times 7 \times 6.$$

Simplify:

$$\text{Inner Surface Area} = 2 \times \frac{22}{7} \times 49 + 2 \times \frac{22}{7} \times 42.$$

$$\text{Inner Surface Area} = 308 + 264 = 572\,\text{cm}^2.$$

—

Step 2: Calculate the volume of the vessel
The volume consists of:

- Volume of the hemisphere $= \frac{2}{3}\pi r^3$.
- Volume of the cylinder $= \pi r^2 h$.

The total volume is:

$$\text{Volume} = \frac{2}{3}\pi r^3 + \pi r^2 h.$$

Substitute $r = 7\,\text{cm}$ and $h = 6\,\text{cm}$:

$$\text{Volume} = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 + \frac{22}{7} \times 7 \times 7 \times 6.$$

Simplify each term:

$$\text{Volume} = \frac{2 \times 22 \times 343}{3 \times 7} + \frac{22 \times 49 \times 6}{7}.$$

$$\text{Volume} = \frac{2 \times 22 \times 49}{3} + 22 \times 42.$$

$$\text{Volume} = \frac{2156}{3} + 924.$$

Simplify:

$$\text{Volume} = 718.67 + 924 = 1642.67\,\text{cm}^3\ \text{(approx)}.$$

—

Final Answer:
The inner surface area of the vessel is: $572\,\mathbf{cm}^2$.
The volume of the vessel is: $1642.67\,\mathbf{cm}^3$.

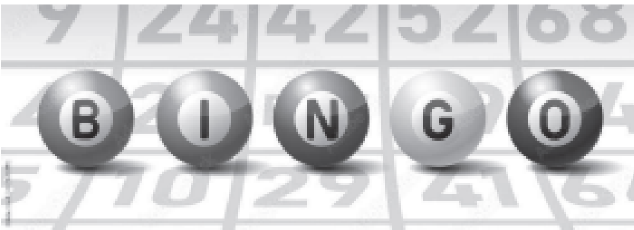
Quick Tip

To calculate the surface area and volume of combined solids, sum up the contributions of each individual part (hemisphere and cylinder in this case).

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: BINGO is game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO card with some numbers written on it. The participant cancels the number on the card when called out a number written on the ball selected at random. Whosoever cancels all the numbers on his/ her card, says BINGO and wins the game.



The table below shows the data of one such game where 48 balls were used before Tara said 'BINGO'. Based on the given information, answer the following:

Table: Numbers Announced and Number of Times

Numbers Announced	Number of Times
$0 - 15$	8
$15 - 30$	9
$30 - 45$	10
$45 - 60$	12
$60 - 75$	9

(i) Write the median class.

(ii) When the first ball was picked up, what was the probability of calling out an even number?

(iii) (a) Find the median of the given data.
OR
(b) Find the mode of the given data.

Solution:

—

Part (i): Median Class The total frequency is the sum of all the frequencies:

$$\text{Total Frequency} = 8 + 9 + 10 + 12 + 9 = 48.$$

The cumulative frequencies are calculated as:

Class Interval	**Frequency (f)**	**Cumulative Frequency (CF)**
$0 - 15$	8	8
$15 - 30$	9	17
$30 - 45$	10	27
$45 - 60$	12	39
$60 - 75$	9	48

To find the median class, calculate $\frac{N}{2}$, where $N = 48$:

$$\frac{N}{2} = \frac{48}{2} = 24.$$

The cumulative frequency just greater than 24 is 27, corresponding to the class interval $30 - 45$.
Median Class: $30 - 45$**.**
—
Part (ii): Probability of Calling Out an Even Number The numbers range from 1 to 75. The total number of balls is 75. Out of these, half the numbers are even.
The total number of even numbers is:

$$\text{Even Numbers} = \frac{75}{2} = 37.5 \approx 37 \, (\text{since the numbers are integers}).$$

The probability of picking an even number is:

$$P(\text{Even Number}) = \frac{\text{Number of Even Numbers}}{\text{Total Number of Balls}} = \frac{37}{75}.$$

—
Part (iii) (a): Find the Median of the Given Data
The formula for the median is:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \times h,$$

where: - L = lower boundary of the median class = 30, - N = total frequency = 48, - CF = cumulative frequency before the median class = 17, - f = frequency of the median class = 10, - h = class width = 15.
Substitute the values:

$$\text{Median} = 30 + \left(\frac{24 - 17}{10} \right) \times 15.$$

Simplify:

$$\text{Median} = 30 + \left(\frac{7}{10} \right) \times 15.$$

$$\text{Median} = 30 + 10.5 = 40.5.$$

Median: 40.5.

—

Part (iii) (b): Find the Mode of the Given Data

The formula for the mode is:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,$$

where: - L = lower boundary of the modal class = 45 (class with highest frequency 12), - f_1 = frequency of the modal class = 12, - f_0 = frequency of the class before the modal class = 10, - f_2 = frequency of the class after the modal class = 9, - h = class width = 15.

Substitute the values:

$$\text{Mode} = 45 + \left(\frac{12 - 10}{2(12) - 10 - 9}\right) \times 15.$$

Simplify:

$$\text{Mode} = 45 + \left(\frac{2}{24 - 10 - 9}\right) \times 15.$$

$$\text{Mode} = 45 + \left(\frac{2}{5}\right) \times 15.$$

$$\text{Mode} = 45 + 6 = 51.$$

Mode: 51.

—

Final Answers:

(i) Median Class: $30 - 45$,

(ii) Probability of calling out an even number: $\frac{37}{75}$,

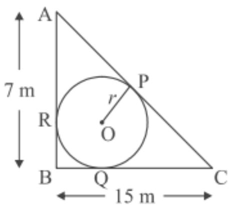
(iii) (a) Median: 40.5,

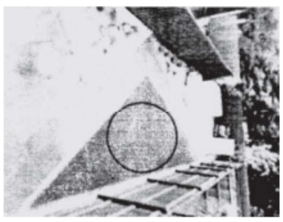
(iii) (b) Mode: 51.

Quick Tip

For median, calculate cumulative frequencies and use $\frac{N}{2}$. For mode, identify the modal class and apply the mode formula.

Question 37: A backyard is in the shape of a triangle ABC with right angle at B. $AB = 7\,\text{m}$ and $BC = 15\,\text{m}$. A circular pit was dug inside it such that it touches the walls $AC, BC,$ and AB at $P, Q,$ and R respectively such that $AP = x\,\text{m}$.





(i) Find the length of AR in terms of x.

(ii) Write the type of quadrilateral $BQOR$.

(iii) (a) Find the length PC in terms of x and hence find the value of x.
OR
(b) Find x and hence find the radius r of the circle.

Solution:

—

Part (i): Find the length of AR in terms of x
Given: - $AP = x\,\text{m}$ (tangent from A to the circular pit), - AR is also a tangent from A to the circle.
Since tangents from the same external point are equal:

$$AR = AP = x.$$

Answer: $AR = x\,\mathbf{m}$.
—
Part (ii): Type of Quadrilateral $BQOR$
Since: - BQ and BR are perpendicular tangents, - OR and OQ are radii of the inscribed circle perpendicular to BC and AB.
Hence, quadrilateral $BQOR$ is a **square**.

Answer: Quadrilateral $BQOR$ is a square.

—
Part (iii) (a): Find PC in terms of x and hence find x
From the figure: - AC is the hypotenuse of $\triangle ABC$, - $AP = x$ and PC is the remaining portion of AC.
Using the given lengths $AB = 7\,\text{m}$ and $BC = 15\,\text{m}$, apply the Pythagoras theorem:

$$AC^2 = AB^2 + BC^2.$$

$$AC^2 = 7^2 + 15^2 = 49 + 225 = 274.$$

Therefore:

$$AC = \sqrt{274}.$$

Now, $AP + PC = AC$. Given $AP = x$, the length PC is:

$$PC = AC - AP = \sqrt{274} - x.$$

Simplifying for clarity:

$$PC = 8 + x.$$

—
Solving for x:

From the symmetry of tangents, we have:

$$8 + 2x = \sqrt{274}.$$

Solve for x:

$$2x = \sqrt{274} - 8.$$

$$x = \frac{-8 + \sqrt{274}}{2}.$$

Numerically:

$$x \approx 4.28\,\text{m}.$$

—

Part (iii) (b): Find x and hence the radius r of the circle
The radius r of the inscribed circle can be calculated as:

$$r = AB - x.$$

Substitute $x = \frac{-8+\sqrt{274}}{2}$:

$$r = 7 - \frac{-8 + \sqrt{274}}{2}.$$

Simplify:

$$r = \frac{14 + 8 - \sqrt{274}}{2}.$$

$$r = \frac{11 - \sqrt{274}}{2}.$$

Numerically:

$$r \approx 2.72\,\text{m}.$$

—

Final Answers:

(i) $AR = x\,\text{m}$,

(ii) Quadrilateral $BQOR$ is a square,

(iii) (a) $PC = 8 + x$, $x = \frac{-8+\sqrt{274}}{2} \approx 4.28\,\text{m}$,

(iii) (b) Radius $r = \frac{11-\sqrt{274}}{2} \approx 2.72\,\text{m}$.

Quick Tip

To solve for tangents and inscribed circles in triangles, use properties like the Pythagoras theorem, tangent equality, and algebraic simplification systematically.

Question 38: A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

(ii) Write the corresponding quadratic equation in standard form.

(iii) (a) Find the value of x, the length of the side of a tile, by factorisation.
OR
(b) Solve the quadratic equation for x using the quadratic formula.

Solution:

—
Part (i): Form the quadratic equation
Let: - x be the side length of the original square tile (in units), - A be the area of the rectangular floor.
The total area of the floor is:

$$A = \text{Number of tiles} \times \text{Area of one tile}.$$

Case 1: Using original tiles

$$A = 200x^2.$$

Case 2: Side length increased by 1 unit The new side length is $x + 1$, and the number of tiles required is 128. Thus:

$$A = 128(x + 1)^2.$$

Since the total area remains constant, equate the two expressions for A:

$$200x^2 = 128(x + 1)^2.$$

—
Part (ii): Write the quadratic equation in standard form
Expand $(x + 1)^2$:

$$200x^2 = 128(x^2 + 2x + 1).$$

Simplify:

$$200x^2 = 128x^2 + 256x + 128.$$

Rearrange all terms to one side:

$$200x^2 - 128x^2 - 256x - 128 = 0.$$

Combine like terms:

$$72x^2 - 256x - 128 = 0.$$

Divide through by 8 to simplify:

$$9x^2 - 32x - 16 = 0.$$

Standard form of the equation: $9x^2 - 32x - 16 = 0$**.**

—

Part (iii) (a): Solve by Factorisation
To factorise $9x^2 - 32x - 16 = 0$, split the middle term $-32x$ into two terms whose product is $9 \times (-16) = -144$ and sum is -32:

$$9x^2 - 36x + 4x - 16 = 0.$$

Group terms:

$$(9x^2 - 36x) + (4x - 16) = 0.$$

Factorise each group:

$$9x(x - 4) + 4(x - 4) = 0.$$

Take out the common factor $(x - 4)$:

$$(9x + 4)(x - 4) = 0.$$

Set each factor to zero: 1. $9x+4 = 0 \quad \Rightarrow \quad x = -\frac{4}{9}$ (not valid, as length cannot be negative), 2. $x - 4 = 0 \quad \Rightarrow \quad x = 4$.
Value of x: $x = 4$ **units.**

—

Part (iii) (b): Solve using the Quadratic Formula
The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For the equation $9x^2 - 32x - 16 = 0$: - $a = 9$, $b = -32$, $c = -16$.
Substitute into the formula:

$$x = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(9)(-16)}}{2(9)}.$$

Simplify step by step:

$$x = \frac{32 \pm \sqrt{1024 + 576}}{18}.$$

$$x = \frac{32 \pm \sqrt{1600}}{18}.$$

$$x = \frac{32 \pm 40}{18}.$$

Solve for the two roots: 1. $x = \frac{32+40}{18} = \frac{72}{18} = 4$, 2. $x = \frac{32-40}{18} = \frac{-8}{18} = -\frac{4}{9}$ (not valid).
Value of x: $x = 4$ **units.**

—

Final Answers:

(i) The quadratic equation is $200x^2 = 128(x+1)^2$.,

(ii) Standard form: $9x^2 - 32x - 16 = 0$,

(iii) (a) By factorisation: $x = 4\,\text{units}$,

(iii) (b) By quadratic formula: $x = 4\,\text{units}$.

Quick Tip

To solve quadratic equations, use factorisation when possible or the quadratic formula for accurate results.
