CBSE Class X Mathematics (Standard) Set 1 (30/2/1) 2024 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section consists of 20 questions of 1 mark each.

Question 1: The value of k for which the system of equations $3x - y + 8 = 0$ and $6x - ky + 16 = 0$ has infinitely many solutions, is:

(A) -2

(B) 2

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Correct Answer: (B) 2

Solution:

For a system of two linear equations to have infinitely many solutions, the ratios of the coefficients of x, y, and the constant terms must be equal. Mathematically:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

Here, the given equations are:

$$3x - y + 8 = 0 \quad \text{and} \quad 6x - ky + 16 = 0.$$

Compare the coefficients with the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:

$$a_1 = 3,\ b_1 = -1,\ c_1 = 8 \quad \text{and} \quad a_2 = 6,\ b_2 = -k,\ c_2 = 16.$$

From the condition for infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

Substitute the values:

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}.$$

Simplify each ratio:

$$\frac{3}{6} = \frac{1}{2} \quad \text{and} \quad \frac{8}{16} = \frac{1}{2}.$$

Thus:

$$\frac{-1}{-k} = \frac{1}{2}.$$

Solve for k:

$$\frac{1}{k} = \frac{1}{2} \implies k = 2.$$

Quick Tip

For two equations to have infinitely many solutions, ensure that the ratios of corresponding coefficients $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Question 2: Point P divides the line segment joining the points $A(4, -5)$ and $B(1, 2)$ in the ratio $5 : 2$. Co-ordinates of point P are:

(A) $\left(\frac{5}{2}, -\frac{3}{2}\right)$

(B) $\left(\frac{11}{7}, 0\right)$

(C) $\left(\frac{13}{7}, 0\right)$

(D) $\left(0, \frac{13}{7}\right)$

Correct Answer: (C) $\left(\frac{13}{7}, 0\right)$

Solution:

The formula to find the coordinates of a point $P(x, y)$ dividing the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ is:

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

Here:

$$x_1 = 4,\ y_1 = -5,\ x_2 = 1,\ y_2 = 2,\ m = 5,\ n = 2.$$

—

Step 1: Calculate the x-coordinate of P

Substitute the values in the formula:

$$x = \frac{m \cdot x_2 + n \cdot x_1}{m+n}.$$

$$x = \frac{5 \cdot 1 + 2 \cdot 4}{5+2}.$$

Simplify:

$$x = \frac{5+8}{7} = \frac{13}{7}.$$

—

Step 2: Calculate the y-coordinate of P

Substitute the values in the formula:

$$y = \frac{m \cdot y_2 + n \cdot y_1}{m + n}.$$

$$y = \frac{5 \cdot 2 + 2 \cdot (-5)}{5 + 2}.$$

Simplify:

$$y = \frac{10 - 10}{7} = 0.$$

—

Step 3: Conclusion

The coordinates of P are:

$$\left(\frac{13}{7}, 0\right).$$

Quick Tip

To find the coordinates of a point dividing a line segment, apply the section formula $P(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

Question 3: The common difference of an A.P. in which $a_{15} - a_{11} = 48$, is:

(A) 12

(B) 16

(C) -12

(D) -16

Correct Answer: (A) 12

Solution:

The formula for the n-th term of an arithmetic progression is:

$$a_n = a_1 + (n - 1)d,$$

where d is the common difference.

Given:

$$a_{15} - a_{11} = 48.$$

Using the formula for a_n:

$$a_{15} = a_1 + 14d \quad \text{and} \quad a_{11} = a_1 + 10d.$$

Subtract a_{11} from a_{15}:

$$a_{15} - a_{11} = (a_1 + 14d) - (a_1 + 10d).$$

Simplify:

$$a_{15} - a_{11} = 4d.$$

Given $a_{15} - a_{11} = 48$, we get:

$$4d = 48 \implies d = 12.$$

—

Quick Tip

In an arithmetic progression, the difference between two terms depends on the number of terms between them multiplied by the common difference d.

Question 4: The quadratic equation $x^2 + x + 1 = 0$ has ____ roots:

(A) real and equal

(B) irrational

(C) real and distinct

(D) not-real

Correct Answer: (D) not-real

Solution:

To determine the nature of roots of a quadratic equation $ax^2 + bx + c = 0$, calculate the discriminant:

$$D = b^2 - 4ac.$$

Here:

$$a = 1,\ b = 1,\ c = 1.$$

Substitute into the discriminant:

$$D = (1)^2 - 4(1)(1).$$

$$D = 1 - 4 = -3.$$

Since $D < 0$, the roots are not real.

—

Quick Tip

If the discriminant $D < 0$, the roots of a quadratic equation are imaginary (not-real).

Question 5: If the HCF (2520, 6600) = 40 and LCM (2520, 6600) = $252 \times k$**, then the value of** k **is:**

(A) 1650

(B) 1600

(C) 165

(D) 1625

Correct Answer: (A) 1650

Solution:

The relationship between HCF, LCM, and two numbers a and b is:

$$\text{HCF} \times \text{LCM} = a \times b.$$

Here:

$$\text{HCF} = 40,\ \text{LCM} = 252 \times k,\ a = 2520,\ b = 6600.$$

Substitute into the equation:

$$40 \times (252 \times k) = 2520 \times 6600.$$

Simplify 2520×6600:

$$2520 \times 6600 = 16632000.$$

Simplify $40 \times 252 \times k$:

$$40 \times 252 = 10080.$$

Thus:

$$10080 \times k = 16632000.$$

Solve for k:

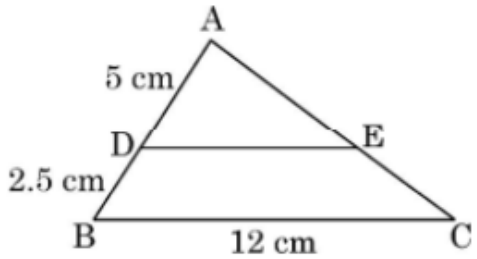
$$k = \frac{16632000}{10080} = 1650.$$

—

Quick Tip

Use the relationship HCF $\times$ LCM $=$ Product of the two numbers to solve problems involving LCM and HCF.

Question 6: In the given figure $\triangle ABC$ is shown. DE is parallel to BC. If $AD = 5$ cm, $DB = 2.5$ cm and $BC = 12$ cm, then DE is equal to:



(A) 10 cm

(B) 6 cm

(C) 8 cm

(D) 7.5 cm

Correct Answer: (C) 8 cm

Solution:

Since $DE \parallel BC$, by the **Basic Proportionality Theorem (Thales' Theorem)**, the ratio of corresponding segments is equal. Therefore:

$$\frac{AD}{AB} = \frac{DE}{BC}.$$

—

Step 1: Find AB (total length of $AD + DB$).

Given:

$$AD = 5\,\text{cm},\ DB = 2.5\,\text{cm}.$$

Thus:

$$AB = AD + DB = 5 + 2.5 = 7.5\,\text{cm}.$$

—

Step 2: Substitute the known values into the formula.

We now substitute into the proportionality equation:

$$\frac{AD}{AB} = \frac{DE}{BC}.$$

Substitute $AD = 5$, $AB = 7.5$, and $BC = 12$:

$$\frac{5}{7.5} = \frac{DE}{12}.$$

—

Step 3: Simplify and solve for DE.

Simplify $\frac{5}{7.5}$:

$$\frac{5}{7.5} = \frac{2}{3}.$$

Thus:

$$\frac{2}{3} = \frac{DE}{12}.$$

Solve for DE by cross-multiplying:

$$DE = \frac{2}{3} \times 12 = 8\,\text{cm}.$$

—

Step 4: Conclusion

The length of DE is:

$$8\,\text{cm}.$$

Quick Tip

When a line is parallel to one side of a triangle, the Basic Proportionality Theorem divides the other two sides in the same ratio. Use proportions to find unknown lengths.

Question 7: If $\sin\theta = \cos\theta$ $(0^\circ < \theta < 90^\circ)$, then the value of $(\sec\theta \cdot \sin\theta)$ is:

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 1

(D) 0

Correct Answer: (C) 1

Solution:

Given $\sin\theta = \cos\theta$, we know:

$$\sin^2\theta + \cos^2\theta = 1.$$

Since $\sin\theta = \cos\theta$, substitute:

$$2\sin^2\theta = 1 \implies \sin^2\theta = \frac{1}{2}.$$

Thus:

$$\sin\theta = \frac{1}{\sqrt{2}} \quad \text{and} \quad \cos\theta = \frac{1}{\sqrt{2}}.$$

The secant of θ is:

$$\sec\theta = \frac{1}{\cos\theta} = \sqrt{2}.$$

Now calculate $\sec\theta \cdot \sin\theta$:

$$\sec\theta \cdot \sin\theta = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1.$$

—

Quick Tip

If $\sin\theta = \cos\theta$, the angle θ is 45° and trigonometric identities simplify calculations.

Question 8: Two dice are rolled together. The probability of getting the sum of the two numbers to be more than 10, is:

(A) $\frac{1}{9}$

(B) $\frac{1}{6}$

(C) $\frac{7}{12}$

(D) $\frac{1}{12}$

Correct Answer: (D) $\frac{1}{12}$

Solution:

To calculate the probability, note that the sum of two dice can range from 2 to 12. We are tasked with finding the probability of the sum being more than 10.

The possible outcomes for a sum > 10 are:

$$\text{(5,6), (6,5), (6,6).}$$

There are 3 favorable outcomes.

The total number of outcomes when two dice are rolled is:

$$6 \times 6 = 36.$$

Thus, the probability is:

$$P(\text{Sum} > 10) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{3}{36} = \frac{1}{12}.$$

—

Quick Tip

To solve dice probability problems, count all favorable outcomes and divide by the total outcomes $6 \times 6 = 36$.

Question 9: If α and β are zeroes of the polynomial $5x^2 + 3x - 7$, the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is:

(A) $-\frac{3}{7}$

(B) $\frac{3}{5}$

(C) $\frac{3}{7}$

(D) $-\frac{5}{7}$

Correct Answer: (C) $\frac{3}{7}$

Solution:

The polynomial is:

$$5x^2 + 3x - 7.$$

For any quadratic polynomial $ax^2 + bx + c$, the sum and product of the roots are:

$$\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}.$$

Here:

$$a = 5,\ b = 3,\ c = -7.$$

Calculate the sum and product:

$$\alpha + \beta = -\frac{3}{5}, \quad \alpha\beta = \frac{-7}{5}.$$

We are tasked to find $\frac{1}{\alpha} + \frac{1}{\beta}$, which simplifies as:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}.$$

Substitute the values:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-\frac{3}{5}}{\frac{-7}{5}}.$$

Simplify:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-3}{-7} = \frac{3}{7}.$$

—

Quick Tip

For quadratic polynomials, use the relations $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ to simplify root-related expressions.

Question 10: The perimeters of two similar triangles ABC and PQR are 56 cm and 48 cm respectively. PQ/AB is equal to:

(A) $\frac{7}{8}$

(B) $\frac{6}{7}$

(C) $\frac{7}{6}$

(D) $\frac{8}{7}$

Correct Answer: (B) $\frac{6}{7}$

Solution:

For two similar triangles, the ratio of their corresponding sides is equal to the ratio of their perimeters.

Given:

$$\frac{\text{Perimeter of } PQR}{\text{Perimeter of } ABC} = \frac{PQ}{AB}.$$

Substitute the values:

$$\frac{PQ}{AB} = \frac{\text{Perimeter of } PQR}{\text{Perimeter of } ABC} = \frac{48}{56}.$$

Simplify the ratio:

$$\frac{PQ}{AB} = \frac{48 \div 8}{56 \div 8} = \frac{6}{7}.$$

—

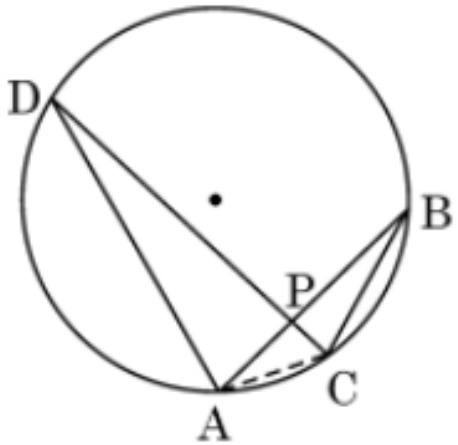
Conclusion:

The value of $\frac{PQ}{AB}$ is: $\frac{6}{7}$.

Quick Tip

For similar triangles, the ratio of corresponding sides is equal to the ratio of their perimeters.

Question 11: AB and CD are two chords of a circle intersecting at P. Choose the correct statement from the following:



(A) $\triangle ADP \sim \triangle CBA$

(B) $\triangle ADP \sim \triangle BPC$

(C) $\triangle ADP \sim \triangle BCP$

(D) $\triangle ADP \sim \triangle CBP$

Correct Answer: (D) $\triangle ADP \sim \triangle CBP$

Solution:

When two chords AB and CD intersect at a point P within a circle, the triangles formed, $\triangle ADP$ and $\triangle CBP$, are similar.

—

Reason: 1. The angles opposite the intersecting chords are equal because they are subtended by the same arc:

$$\angle ADP = \angle CBP.$$

2. The vertical angles at P are equal:

$$\angle DPA = \angle BPC.$$

By the AA (Angle-Angle) Criterion, the two triangles are similar:

$$\triangle ADP \sim \triangle CBP.$$

Quick Tip

When two chords of a circle intersect, the triangles formed are similar by the AA criterion of similarity.

Question 12: If the value of each observation in a data is increased by 2, then the median of the new data:

(A) increases by 2

(B) increases by $2n$

(C) remains same

(D) decreases by 2

Correct Answer: (A) increases by 2

Solution:

The median of a data set is the middle value when the data is arranged in ascending order.

—

Step 1: Effect of Adding a Constant

If each observation in the data is increased by 2, all values shift upward by the same amount. This shift does not alter the position of the median but increases its value by 2.

—

Conclusion:

The new median is:

$$\text{New Median} = \text{Old Median} + 2.$$

Quick Tip

When a constant value is added to all observations in a data set, the mean, median, and mode all increase by the same constant value.

Question 13: A box contains cards numbered 6 to 55. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square, is:

(A) $\frac{7}{50}$

(B) $\frac{7}{55}$

(C) $\frac{1}{10}$

(D) $\frac{5}{49}$

Correct Answer: (C) $\frac{1}{10}$

Solution:

The total number of cards is:

$$55 - 6 + 1 = 50 \,\text{cards}.$$

—

Step 1: Identify the Perfect Squares between 6 and 55

The perfect squares in this range are:

$$9, 16, 25, 36, 49.$$

Count the total number of perfect squares:

$$5 \text{ perfect squares.}$$

—

Step 2: Probability of Drawing a Perfect Square

The probability is given by:

$$P(\text{perfect square}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}.$$

Substitute the values:

$$P(\text{perfect square}) = \frac{5}{50} = \frac{1}{10}.$$

—

Conclusion:

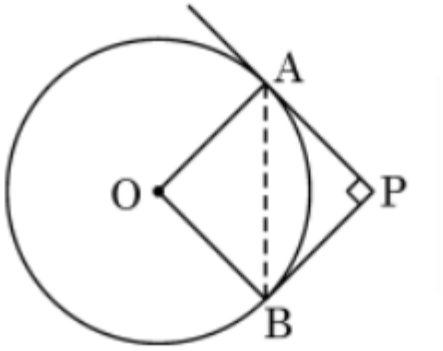
The probability that the drawn card has a number which is a perfect square is:

$$\frac{1}{10}.$$

Quick Tip

To find probabilities, identify all favorable outcomes and divide by the total number of possible outcomes.

Question 14: In the given figure, tangents PA and PB to the circle centered at O, from point P are perpendicular to each other. If $PA = 5$ cm, then length of AB is equal to:



(A) $5\,\text{cm}$

(B) $5\sqrt{2}\,\text{cm}$

(C) $2\sqrt{5}\,\text{cm}$

(D) $10\,\text{cm}$

Correct Answer: (B) $5\sqrt{2}\,\text{cm}$

Solution:

In the given figure, PA and PB are tangents to the circle and are perpendicular at point P.

Thus, $\triangle OAP$ forms a right-angled triangle at A, where:

$$OP\,(\text{hypotenuse}) = AB \quad \text{and} \quad PA = PB = 5\,\text{cm}.$$

—

Step 1: Use Pythagoras Theorem

For right-angled triangle OPA:

$$OP^2 = PA^2 + PB^2.$$

Substitute the values:

$$OP^2 = 5^2 + 5^2.$$

$$OP^2 = 25 + 25 = 50.$$

Taking square root:

$$OP = \sqrt{50} = 5\sqrt{2}.$$

—

Conclusion:

The length of AB is:

$$5\sqrt{2}\,\text{cm}.$$

Quick Tip

In right-angled triangles involving tangents, always apply the Pythagoras theorem to calculate the hypotenuse or missing sides.

Question 15: XOYZ is a rectangle with vertices $X(-3,0), O(0,0), Y(0,4)$ and $Z(x,y)$. The length of its diagonal is:

(A) 5 units

(B) $\sqrt{5}$ units

(C) $x^2 + y^2$ units

(D) 4 units

Correct Answer: (A) 5 units

Solution:

The length of the diagonal of a rectangle can be calculated using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

—

Step 1: Identify Coordinates of $XOYZ$

Given vertices are:

$$X(-3,0),\ O(0,0),\ Y(0,4).$$

The diagonal XY can be found as:

$$XY = \sqrt{(0 - (-3))^2 + (4 - 0)^2}.$$

—

Step 2: Apply the Distance Formula

Substitute the coordinates:

$$XY = \sqrt{(3)^2 + (4)^2}.$$

Simplify:

$$XY = \sqrt{9 + 16} = \sqrt{25} = 5.$$

—

Conclusion:

The length of the diagonal is:

$$5 \text{ units}.$$

Quick Tip

The diagonal of a rectangle can be directly calculated using the distance formula between opposite vertices.

Question 16: Which term of the A.P. $-29, -26, -23, \ldots, 61$ **is** 16**?**

(A) 11^{th}

(B) 16^{th}

(C) 10^{th}

(D) 31^{st}

Correct Answer: (B) 16^{th}

Solution:

The general term of an A.P. is given by:

$$a_n = a + (n-1)d,$$

where a is the first term, d is the common difference, and n is the term number.

—

Step 1: Identify a**,** d**, and** a_n

Given:

$$a = -29, \quad d = -26 - (-29) = 3, \quad a_n = 61.$$

—

Step 2: Substitute in the Formula

$$a_n = a + (n-1)d.$$

Substitute the known values:

$$61 = -29 + (n-1) \cdot 3.$$

Simplify:

$$61 + 29 = (n-1) \cdot 3.$$

$$90 = (n-1) \cdot 3.$$

Divide by 3:

$$n - 1 = 30 \implies n = 31.$$

—

Conclusion:

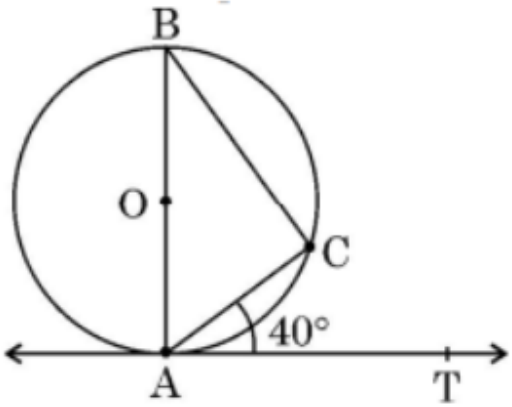
The term number is:

$$16^{\text{th}} \text{ term}.$$

Quick Tip

To find a specific term in an arithmetic progression, always use the general formula $a_n = a + (n-1)d$ and substitute known values.

Question 17: In the given figure, AT is tangent to a circle centered at O. If $\angle CAT = 40^\circ$, then $\angle CBA$ is equal to:



(A) 70°

(B) 50°

(C) 65°

(D) 40°

Correct Answer: (D) 40°

Solution:

In the figure: - AT is a tangent to the circle at A. - The property of tangents states that the angle between the tangent and a chord through the point of contact is equal to the angle subtended

by the chord at the opposite arc.

—

Step 1: Tangent-Chord Property

Given:

$$\angle CAT = 40^\circ.$$

By the tangent-chord property:

$$\angle CBA = \angle CAT.$$

—

Conclusion:

$$\angle CBA = 40^\circ.$$

Quick Tip

The tangent-chord property states that the angle between a tangent and a chord is equal to the angle subtended by the chord at the opposite arc.

Question 18: After an examination, a teacher wants to know the marks obtained by the maximum number of students in her class. She requires to calculate ___ of marks.

(A) median

(B) mode

(C) mean

(D) range

Correct Answer: (B) mode

Solution:

The **mode** of a data set refers to the value that occurs most frequently. Since the teacher wants to know the marks obtained by the maximum number of students, she needs to calculate the **mode** of marks.

—

Explanation:

- Median: Represents the middle value of a sorted data set.
- Mean: The average of all values.
- Range: The difference between the highest and lowest values.
- Mode: The most frequently occurring value in a data set.

Here, the mode is the most appropriate measure since it highlights the most common marks.

Quick Tip

The mode is useful in identifying the most frequently occurring value, which is ideal for determining trends or patterns in a data set.

Directions:

In Q. No. 19 and 20 a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

- (A) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
- (b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A).
- (c) Assertion (A) is true but Reason (R) is false.
- (d) Assertion (A) is false but Reason (R) is true.

Question 19: If $\sin A = \frac{1}{3}$ **(where** $0^\circ < A < 90^\circ$**), then the value of** $\cos A$ **is** $\frac{2\sqrt{2}}{3}$**.**

Assertion (A): If $\sin A = \frac{1}{3}$, then $\cos A = \frac{2\sqrt{2}}{3}$.

Reason (R): For every angle θ, $\sin^2\theta + \cos^2\theta = 1$.

Correct Answer: (A) Both Assertion (A) and (R) are true. Reason (R) is the correct explanation of Assertion (A).

Solution:

Using the Pythagorean identity:

$$\sin^2 A + \cos^2 A = 1.$$

Given $\sin A = \frac{1}{3}$, substitute the value:

$$\left(\frac{1}{3}\right)^2 + \cos^2 A = 1.$$

$$\frac{1}{9} + \cos^2 A = 1.$$

Simplify:

$$\cos^2 A = 1 - \frac{1}{9} = \frac{8}{9}.$$

Taking the square root:

$$\cos A = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}.$$

Thus, Assertion (A) is true, and Reason (R) correctly explains the result.

Quick Tip

The Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ is a fundamental trigonometric property.

Question 20: Two cubes, each with edge length 10 cm, are joined together. The total surface area of the newly formed cuboid is 1200 cm^2.

Assertion (A): The total surface area of the newly formed cuboid is $1200\,\text{cm}^2$.

Reason (R): The area of each surface of a cube with a side 10 cm is $100\,\text{cm}^2$.

Correct Answer: (D) Assertion (A) is not true but Reason (R) is true.

Solution:

To find the total surface area of a newly formed cuboid, note: 1. Each cube has an edge length of 10 cm.

2. When two cubes are joined along one face, the resulting cuboid has dimensions: - Length = 20 cm (combined), - Width = 10 cm, - Height = 10 cm.

—

Step 1: Surface Area of the Cuboid The formula for the total surface area of a cuboid is:

$$\text{Surface Area} = 2\left(l \cdot w + w \cdot h + h \cdot l\right).$$

Substitute $l = 20\,\text{cm}, w = 10\,\text{cm}, h = 10\,\text{cm}$:

$$\text{Surface Area} = 2\left(20 \cdot 10 + 10 \cdot 10 + 10 \cdot 20\right).$$

Simplify:

$$\text{Surface Area} = 2\,(200 + 100 + 200) = 2 \cdot 500 = 1000\,\text{cm}^2.$$

—

Step 2: Conclusion - The actual total surface area is $1000\,\text{cm}^2$, not $1200\,\text{cm}^2$. - The Reason (R) is correct as each surface area of the cube is $100\,\text{cm}^2$.

Thus, Assertion (A) is false, but Reason (R) is true.

Quick Tip

When two cubes are joined, the surface area decreases because one face of each cube is no longer exposed.

Section - B

This section consists of 5 questions of 2 mark each.

Question 21: Can the number $(15)^n$, n being a natural number, end with the digit 0? Give reasons.

Solution:

The number 15^n can be expressed as:

$$15^n = 5^n \times 3^n.$$

To end with the digit 0, a number must have both 2 and 5 as its prime factors (since $2 \times 5 = 10$).

—

Step 1: Prime Factorization - $15 = 5 \times 3$. Thus, 15^n only includes prime factors 5 and 3.

—

Step 2: Absence of Factor 2 Since 15^n does not include 2 as a prime factor, it cannot end with the digit 0.

—

Conclusion: A number can only end with 0 if it has 2 and 5 as factors. Therefore, 15^n cannot end with 0.

Quick Tip

To check if a number ends with 0, ensure its prime factorization includes both 2 and 5.

Question 22: Find the type of triangle ABC formed whose vertices are $A(1, 0)$, $B(-5, 0)$, and $C(-2, 5)$.

Solution:

To determine the type of triangle, calculate the distances between the vertices AB, BC, and CA using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

—

Step 1: Calculate AB

$$AB = \sqrt{(-5 - 1)^2 + (0 - 0)^2} = \sqrt{(-6)^2} = \sqrt{36} = 6.$$

—

Step 2: Calculate BC

$$BC = \sqrt{(-5 + 2)^2 + (0 - 5)^2} = \sqrt{(-3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}.$$

—

Step 3: Calculate CA

$$CA = \sqrt{(1 + 2)^2 + (0 - 5)^2} = \sqrt{(3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}.$$

—

Step 4: Compare the Sides - $AB = 6$, - $BC = \sqrt{34}$, - $CA = \sqrt{34}$.

Since $BC = CA$, the triangle ABC has two equal sides.

—

Conclusion: The triangle ABC is **isosceles**.

Quick Tip

An isosceles triangle has two sides of equal length. Use the distance formula to compare side lengths.

Question 23(A): Evaluate $2\sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ$.

Solution:

We need to evaluate $2\sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ$.

—

Step 1: Substitute the Values - $\sin 30^\circ = \frac{1}{2}$, - $\sec 60^\circ = 2$, - $\tan 60^\circ = \sqrt{3}$.

Substitute these values into the given expression:

$$2\sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ = 2\left(\frac{1}{2}\right)^2 \times 2 + (\sqrt{3})^2.$$

—

Step 2: Simplify the Expression

$$2\left(\frac{1}{2}\right)^2 \times 2 = 2 \times \frac{1}{4} \times 2 = 1.$$

$$(\sqrt{3})^2 = 3.$$

Add the two results:

$$1 + 3 = 4.$$

—

Conclusion: The value of $2\sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ$ is 4.

Quick Tip

Memorize the standard trigonometric values for 30°, 60°, and 45° to solve such problems quickly.

Question 23(b): If $2\sin(A+B) = \sqrt{3}$ and $\cos(A-B) = 1$, then find the measures of angles A and B such that $0 \leq A, B, (A+B) \leq 90^\circ$.

Solution:

Given:

$$2\sin(A+B) = \sqrt{3} \quad \text{and} \quad \cos(A-B) = 1.$$

—

Step 1: Solve for $A+B$ Divide both sides of $2\sin(A+B) = \sqrt{3}$ by 2:

$$\sin(A+B) = \frac{\sqrt{3}}{2}.$$

From the standard trigonometric values:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}.$$

Thus:

$$A + B = 60^\circ.$$

—

Step 2: Solve for $A-B$ Given $\cos(A-B) = 1$, we know:

$$\cos 0^\circ = 1.$$

Thus:

$$A - B = 0^\circ \implies A = B.$$

—

Step 3: Find A **and** B From $A+B = 60^\circ$ and $A = B$, solve for A and B:

$$A + A = 60^\circ \implies 2A = 60^\circ \implies A = 30^\circ.$$

Since $A = B$, we get:

$$B = 30^\circ.$$

—

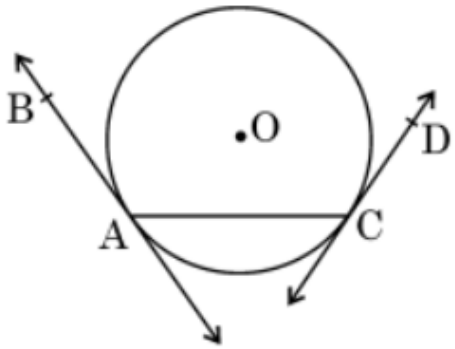
Conclusion: The measures of angles A and B are:

$$A = 30^\circ \quad \text{and} \quad B = 30^\circ.$$

Quick Tip

Use the properties of trigonometric functions and standard values to solve for unknown angles efficiently.

Question 24: In the given figure, AB and CD are tangents to a circle centered at O. Is $\angle BAC = \angle DCA$? Justify your answer.



Solution:

To prove: $\angle BAC = \angle DCA$.

—

Step 1: Join OA and OC

Since AB and CD are tangents to the circle at points A and C, the radii OA and OC are perpendicular to the tangents:

$$OA = OC \quad \text{(radii of the same circle)}.$$

—

Step 2: Analyze the angles The angles $\angle OAC$ and $\angle OCA$ are equal because they are opposite equal radii:

$$\angle OAC = \angle OCA.$$

Also, the angles $\angle OAB$ and $\angle OCD$ are equal because the tangents AB and CD subtend equal angles with the radii:

$$\angle OAB = \angle OCD.$$

—

Step 3: Combine the results Now, add the angles:

$$\angle OAC + \angle OAB = \angle OCA + \angle OCD.$$

This gives:

$$\angle BAC = \angle DCA.$$

—

Conclusion: Hence, $\angle BAC$ is equal to $\angle DCA$.

Quick Tip

The tangents drawn to a circle from an external point are equal in length and subtend equal angles with the radii of the circle.

Question 25(A): In what ratio is the line segment joining the points $(3, -5)$ and $(-1, 6)$ divided by the line $y = x$?

Solution:

Let the required ratio be $K : 1$.

—

Step 1: Coordinates of the point of division P:

The coordinates of P dividing the line segment joining $(3, -5)$ and $(-1, 6)$ in the ratio $K : 1$ are:

$$P\left(\frac{-K+3}{K+1}, \frac{6K-5}{K+1}\right).$$

—

Step 2: Point lies on the line $y = x$

Since $y = x$, the x-coordinate and y-coordinate of P are equal:

$$\frac{-K+3}{K+1} = \frac{6K-5}{K+1}.$$

Simplify:

$$-K + 3 = 6K - 5.$$

$$8K = 8 \implies K = 1.$$

Thus, the ratio is:

$$\text{Ratio} = 8 : 7.$$

—

Final Answer: The required ratio is $8 : 7$.

—

Question 25(b): $A(3,0)$**,** $B(6,4)$**, and** $C(-1,3)$ **are vertices of a triangle** ABC**. Find the length of its median** BE**.**

Solution:

Step 1: Midpoint of AC

The coordinates of the midpoint E of AC are:

$$E = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + (-1)}{2}, \frac{0 + 3}{2}\right) = \left(1, \frac{3}{2}\right).$$

—

Step 2: Length of median BE

The length of BE is given by:

$$BE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substitute $B(6,4)$ and $E\left(1, \frac{3}{2}\right)$:

$$BE = \sqrt{(6-1)^2 + \left(4 - \frac{3}{2}\right)^2}.$$

Simplify:

$$BE = \sqrt{5^2 + \left(\frac{8-3}{2}\right)^2} = \sqrt{25 + \left(\frac{5}{2}\right)^2}.$$

$$BE = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{100}{4} + \frac{25}{4}} = \sqrt{\frac{125}{4}}.$$

$$BE = \frac{\sqrt{125}}{2} = \frac{5\sqrt{5}}{2}.$$

—

Final Answer: The length of the median BE is $\frac{5\sqrt{5}}{2}$.

Section - C

This section consists of 6 questions of 3 mark each.

Question 26(A): If the sum of first m **terms of an A.P. is same as the sum of its first** n **terms (**$m \neq n$**), then show that the sum of its first** $(m+n)$ **terms is zero.**

Solution:

The sum of the first m terms of an A.P. is:

$$S_m = \frac{m}{2}[2a + (m-1)d].$$

Similarly, the sum of the first n terms is:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

—

Step 1: Equating the two sums

Since $S_m = S_n$, we have:

$$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d].$$

Simplify by multiplying both sides by 2:

$$m[2a + (m-1)d] = n[2a + (n-1)d].$$

—

Step 2: Solving for $2a$

Expand and simplify:

$$2am + m(m-1)d = 2an + n(n-1)d.$$

Rearranging terms gives:

$$2a(m-n) = d[n^2 - m^2 - (n-m)].$$

Simplifying further:

$$2a = -d(m+n-1).$$

—

Step 3: Sum of $(m+n)$ terms

The sum of the first $(m+n)$ terms is:

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d].$$

Substitute $2a = -d(m+n-1)$ into the equation:

$$S_{m+n} = \frac{m+n}{2}[-d(m+n-1) + (m+n-1)d].$$

Simplify:

$$S_{m+n} = \frac{m+n}{2}[0] = 0.$$

—

Final Answer: Thus, the sum of the first $(m+n)$ terms is zero.

—

Question 26(b): In an A.P., the sum of three consecutive terms is 24 and the sum of their squares is 194. Find the numbers.

Solution:

Let the three consecutive terms in the A.P. be $a-d, a, a+d$.

—

Step 1: Sum of three terms

The sum of the terms is given as:

$$(a-d) + a + (a+d) = 24.$$

Simplify:

$$3a = 24 \implies a = 8.$$

—

Step 2: Sum of squares of the terms

The sum of the squares of the terms is:

$$(a-d)^2 + a^2 + (a+d)^2 = 194.$$

Substitute $a = 8$:

$$(8-d)^2 + 8^2 + (8+d)^2 = 194.$$

Simplify:

$$(64 - 16d + d^2) + 64 + (64 + 16d + d^2) = 194.$$

Combine like terms:

$$2d^2 + 192 = 194.$$

$$2d^2 = 2 \implies d^2 = 1 \implies d = \pm 1.$$

—

Step 3: Find the terms

If $d = 1$, the terms are:

$$a - d, a, a + d = 7, 8, 9.$$

If $d = -1$, the terms are:

$$a - d, a, a + d = 9, 8, 7.$$

—

Final Answer: The numbers are $7, 8, 9$ or $9, 8, 7$.

Question 27: Prove that $\sqrt{5}$ is an irrational number.

Solution:

Assume $\sqrt{5}$ is a rational number. Then it can be expressed as:

$$\sqrt{5} = \frac{p}{q},$$

where p and q are co-prime integers and $q \neq 0$.

—

Step 1: Square both sides

Squaring both sides, we get:

$$5q^2 = p^2.$$

This implies p^2 is divisible by 5. Hence, p must also be divisible by 5 (as 5 is prime). Let:

$$p = 5a, \text{ where } a \text{ is an integer.}$$

—

Step 2: Substitute $p = 5a$

Substitute $p = 5a$ into $5q^2 = p^2$:

$$5q^2 = (5a)^2.$$

Simplify:

$$5q^2 = 25a^2 \implies q^2 = 5a^2.$$

Thus, q^2 is divisible by 5, which means q is also divisible by 5.

—

Step 3: Contradiction

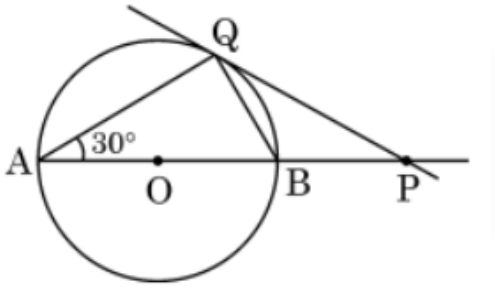
From steps 1 and 2, we conclude that both p and q are divisible by 5, which contradicts the assumption that p and q are co-prime.

—

Conclusion:

Our assumption that $\sqrt{5}$ is rational is incorrect. Hence, $\sqrt{5}$ is an irrational number.

Question 28(A): In the given figure, PQ is tangent to a circle centred at O and $\angle BAQ = 30°$; show that $BP = BQ$.



Solution:

Step 1: Join OQ

Since OQ and OA are radii of the circle:

$$OQ = OA \implies \angle 2 = 30°.$$

—

Step 2: Using tangent properties and angle sum

At point Q, the tangent PQ is perpendicular to the radius OQ:

$$\angle 3 = 90° - \angle 2 = 90° - 30° = 60°.$$

At point B, the angle $\angle 4$ is:

$$\angle 4 = 90° - \angle 3 = 90° - 60° = 30°.$$

—

Step 3: Angle relationship and equality

From the figure:

$$\angle 6 = \angle 1 + \angle 2 = 60°.$$

Hence:

$$\angle 5 = 90^\circ - \angle 6 = 90^\circ - 60^\circ = 30^\circ = \angle 4.$$

—

Step 4: Conclusion

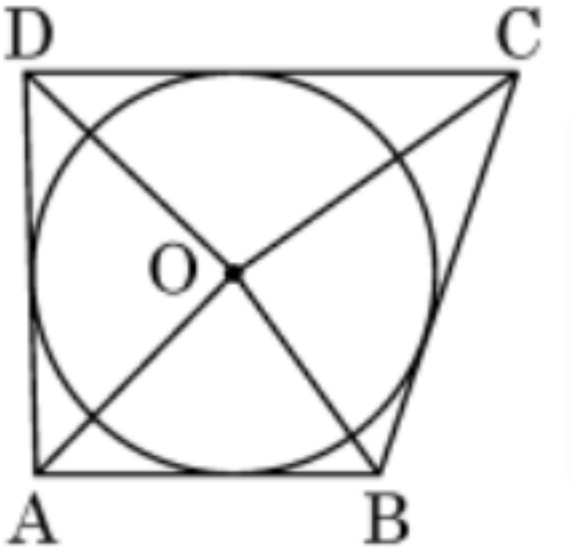
Since $\angle BPQ = \angle BQP$, the two tangents BP and BQ are equal:

$$BP = BQ.$$

Quick Tip

The tangents drawn to a circle from an external point are always equal in length.

Question 28(b): In the given figure, $AB, BC, CD,$ and DA are tangents to the circle with centre O forming a quadrilateral $ABCD$. Show that $\angle AOB + \angle COD = 180^\circ$.



Solution:

Step 1: Join the radii and tangents

Join $OP, OQ, OR,$ and OS as shown in the figure. Since tangents from a point to a circle are equal, we observe the following:

$$\triangle POB \cong \triangle QOB \quad \text{(by symmetry)} \implies \angle 1 = \angle 2.$$

—

Step 2: Angle relationships

Similarly:

$$\angle 3 = \angle 4,\ \angle 5 = \angle 6, \text{ and } \angle 7 = \angle 8.$$

The sum of all angles at O is 360°, so:

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ.$$

—

Step 3: Grouping angles

Group the angles symmetrically:

$$2(\angle 1 + \angle 3 + \angle 5) = 360^\circ.$$

Simplify:

$$\angle AOB + \angle COD = 180^\circ.$$

—

Conclusion:

Thus, it is proved that:

$$\angle AOB + \angle COD = 180^\circ.$$

Quick Tip

The tangents to a circle from external points form symmetrical triangles, which simplifies angle calculations.

Question 29: Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \cos\theta \cdot \frac{1-\sin\theta}{\cos\theta}$.

Solution:

Step 1: Simplify the Left-Hand Side (LHS)

$$\text{LHS} = \frac{(\sec^2\theta - \tan^2\theta) + (\sec\theta - \tan\theta)}{1 + \sec\theta + \tan\theta}.$$

Using the identity $\sec^2\theta - \tan^2\theta = 1$, we get:

$$\text{LHS} = \frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta}.$$

—

Step 2: Factorize the denominator and numerator

Using the form $(a-b)(a+b) = a^2 - b^2$, we write:

$$1 + \sec\theta - \tan\theta = \frac{\cos\theta(1 - \sin\theta)}{\cos\theta}.$$

Thus:

$$\text{LHS} = \frac{1 - \sin\theta}{\cos\theta} \cdot \cos\theta = \text{RHS}.$$

—

Conclusion:

Hence, it is proved that:

$$\frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \cos\theta \cdot \frac{1 - \sin\theta}{\cos\theta}.$$

Quick Tip

Use trigonometric identities like $\sec^2\theta - \tan^2\theta = 1$ and factorize carefully to simplify complex expressions.

Question 30: In a test, the marks obtained by 100 students (out of 50) are given below:

Marks Obtained	Number of Students (f_i)	x_i	$f_i x_i$
$0 - 10$	12	5	60
$10 - 20$	23	15	345
$20 - 30$	34	25	850
$30 - 40$	25	35	875
$40 - 50$	6	45	270
Total	100		2400

Solution:

Step 1: Recall the formula for Mean:

The formula for the mean is:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}.$$

—

Step 2: Substitute the given values:

Here:

$$\sum f_i x_i = 2400 \quad \text{and} \quad \sum f_i = 100.$$

Thus:

$$\text{Mean} = \frac{2400}{100} = 24.$$

—

Conclusion:

The mean marks of the students is: 24

Quick Tip

The mean is calculated by dividing the sum of all values ($\sum f_i x_i$) by the total number of observations ($\sum f_i$).

Question 31: In a 2-digit number, the digit at the unit's place is 5 less than the digit at the ten's place. The product of the digits is 36. Find the number.

Solution:

Step 1: Let the digit at the ten's place be x.

Since the digit at the unit's place is 5 less than the ten's place, we have:

$$\text{Unit's place digit} = x - 5.$$

—

Step 2: Form the equation.

The product of the digits is given as 36:

$$x(x - 5) = 36.$$

Simplify:

$$x^2 - 5x - 36 = 0.$$

—

Step 3: Solve the quadratic equation.

Factorize $x^2 - 5x - 36 = 0$:

$$(x - 9)(x + 4) = 0.$$

Thus:

$$x = 9 \quad \text{or} \quad x = -4.$$

Since x represents a digit, it must be positive:

$$x = 9.$$

—

Step 4: Find the number.

If $x = 9$, the digit at the unit's place is:

$$x - 5 = 9 - 5 = 4.$$

Thus, the required number is:

$$94.$$

Conclusion:

The required number is 94.

Quick Tip

When solving quadratic equations, ensure the solution aligns with the context of the problem, such as digits being non-negative and single-digit numbers.

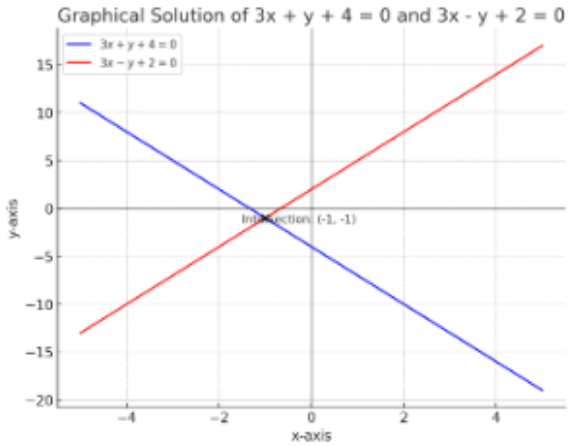
Section - D

This section consists of 4 questions of 5 marks each.

Question 32(A): Using graphical method, solve the following system of equations:

$$3x + y + 4 = 0 \quad \text{and} \quad 3x - y + 2 = 0$$

Solution:



Quick Tip

The intersection point of the two lines represents the solution of the system of equations.

Question 32 (b): Tara scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks deducted for each wrong answer, then Tara would have scored 50 marks. Assuming Tara attempted all questions, find the total number of questions in the test.

Solution:

Step 1: Define variables.

Let the number of right answers be x and the number of wrong answers be y. The total number of questions is:

$$x + y = N \quad \text{(where } N \text{ is the total number of questions).}$$

—

Step 2: Form equations based on given conditions.

1. From the first condition (3 marks for each correct and -1 for each wrong answer), Tara's score is:

$$3x - y = 40 \quad \text{(Equation 1)}.$$

2. From the second condition (4 marks for correct and -2 for wrong answers), Tara's score is:

$$4x - 2y = 50 \quad \text{(Equation 2)}.$$

—

Step 3: Solve the equations.

From Equation 1:

$$3x - y = 40 \implies y = 3x - 40.$$

Substitute $y = 3x - 40$ into Equation 2:

$$4x - 2(3x - 40) = 50.$$

Simplify:

$$4x - 6x + 80 = 50.$$

$$-2x + 80 = 50.$$

$$-2x = -30 \implies x = 15.$$

—

Step 4: Find y.

Substitute $x = 15$ into $y = 3x - 40$:

$$y = 3(15) - 40 = 45 - 40 = 5.$$

—

Step 5: Total number of questions.

The total number of questions is:

$$N = x + y = 15 + 5 = 20.$$

—

Conclusion:

The total number of questions in the test is 20.

Quick Tip

Always carefully frame equations based on the conditions given in the question and check the consistency of your solution.

Question 33(A): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Solution:

Let $\triangle ABC$ be a triangle, and let a line DE be drawn parallel to BC, intersecting AB at D and AC at E.

—

Step 1: Prove that $\frac{AD}{DB} = \frac{AE}{EC}$

1. Since $DE \parallel BC$, by the **Basic Proportionality Theorem (Thales' Theorem)**, we have:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

2. The theorem states that if a line is drawn parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

—

Step 2: Conclusion

Thus, the line DE divides the sides AB and AC in the same ratio:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Quick Tip

The Basic Proportionality Theorem (Thales' Theorem) is a fundamental result in geometry used to prove proportional divisions of triangle sides.

Question 33(b): Sides AB and AC and median AD to $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another triangle $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

Solution:

Given two triangles $\triangle ABC$ and $\triangle PQR$:

1. It is given that:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}.$$

2. In $\triangle ABC$, AD is the median, which divides BC into two equal parts. Similarly, in $\triangle PQR$, PM is the median, which divides QR into two equal parts.

—

Step 1: Prove similarity of triangles using the proportionality condition

- Since:

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad \text{and} \quad \frac{AD}{PM},$$

the corresponding sides of triangles $\triangle ABC$ and $\triangle PQR$ are proportional.

- By the **Side-Side-Side (SSS) Similarity Criterion**, two triangles are similar if their corresponding sides are proportional.

—

Step 2: Conclusion

From the given condition and the SSS similarity criterion, we conclude that:

$$\triangle ABC \sim \triangle PQR.$$

Quick Tip

To prove two triangles are similar, use the SSS Similarity Criterion when the ratios of corresponding sides are equal.

Question 34: From the top of a 45 m high lighthouse, the angles of depression of two ships, on the opposite side of it, are observed to be 30° and 60°. If the line joining the

ships passes through the foot of the lighthouse, find the distance between the ships. (Use $\sqrt{3} = 1.73$)

Solution:

Let $AB = 45\,\text{m}$ be the height of the lighthouse. Let the ships be positioned at C and D, on opposite sides of the lighthouse.

- $\angle ACB = 30^\circ$ (depression angle of ship C). - $\angle ADB = 60^\circ$ (depression angle of ship D).

Let $BC = x$ and $BD = y$ be the horizontal distances of ships C and D from the base of the lighthouse.

—

Step 1: Use trigonometric relations

From $\triangle ABC$, using $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$:

$$\tan 30^\circ = \frac{AB}{BC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{45}{x}.$$

Solve for x:

$$x = 45\sqrt{3} = 45 \times 1.73 = 77.85\,\text{m}.$$

From $\triangle ABD$, using $\tan 60^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$:

$$\tan 60^\circ = \frac{AB}{BD} \quad \Rightarrow \quad \sqrt{3} = \frac{45}{y}.$$

Solve for y:

$$y = \frac{45}{\sqrt{3}} = \frac{45}{1.73} = 25.98\,\text{m}.$$

—

Step 2: Find the distance between the ships CD

Since the ships C and D are on opposite sides of the lighthouse, the total distance between them is:

$$CD = BC + BD = x + y.$$

Substitute the values of x and y:

$$CD = 77.85 + 25.98 = 103.83\,\text{m}.$$

—

Final Answer:

The distance between the ships is approximately $103.83\,\text{m}$.

Quick Tip

To solve angle of depression problems, always use trigonometric ratios like $\tan\theta$ and carefully analyze the triangle formed with horizontal distances.

Question 35: The perimeter of a certain sector of a circle of radius $5.6\,\text{m}$ is $20.0\,\text{m}$. Find the area of the sector.

Solution:

The perimeter of a sector is given by:

$$\text{Perimeter} = 2r + \frac{2\pi r\theta}{360}.$$

Substitute the known values $r = 5.6\,\text{m}$ and Perimeter $= 20\,\text{m}$:

$$2(5.6) + \frac{2 \times \frac{22}{7} \times 5.6 \times \theta}{360} = 20.$$

Simplify:

$$11.2 + \frac{2 \times 22 \times 5.6 \times \theta}{7 \times 360} = 20.$$

$$11.2 + \frac{246.4\theta}{2520} = 20.$$

Simplify further:

$$\frac{246.4\theta}{2520} = 8.8.$$

Solve for θ:

$$\theta = \frac{8.8 \times 2520}{246.4} = 90^\circ.$$

—

Step 2: Find the area of the sector

The area of a sector is given by:

$$\text{Area} = \frac{\pi r^2 \theta}{360}.$$

Substitute $r = 5.6\,\text{m}$, $\theta = 90^\circ$, and $\pi = \frac{22}{7}$:

$$\text{Area} = \frac{22}{7} \times 5.6 \times 5.6 \times \frac{90}{360}.$$

Simplify:

$$\text{Area} = \frac{22}{7} \times 5.6 \times 5.6 \times \frac{1}{4}.$$

$$\text{Area} = \frac{22 \times 31.36}{28}.$$

$$\text{Area} = 24.64\,\text{m}^2.$$

—

Final Answer:

The area of the sector is $24.64\,\text{m}^2$.

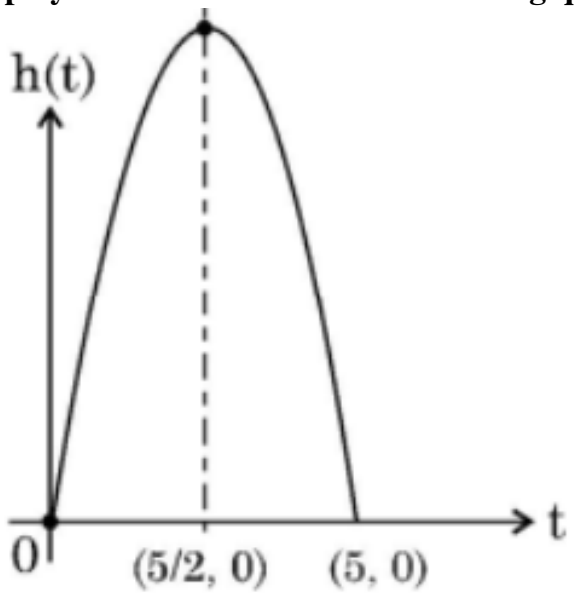
Quick Tip

The perimeter of a sector combines the arc length and the two radii. Always simplify step by step to find the unknown angle or area.

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$. Observe the graph of the polynomial and answer the following questions:



(i) Write zeroes of the given polynomial.

(ii) Find the maximum height achieved by the ball.

(iii) (A) After throwing upward, how much time did the ball take to reach the height of $30\,\text{m}$?

OR

(iii) (b) Find the two different values of t when the height of the ball was $20\,\text{m}$.

Solution:

(i) Write zeroes of the given polynomial:

The given polynomial is:

$$h = 25t - 5t^2 \quad \Rightarrow \quad h = -5t(t-5).$$

Setting $h = 0$:

$$-5t(t-5) = 0 \implies t = 0 \,\text{and}\, t = 5.$$

Thus, the zeroes of the polynomial are $t = 0$ and $t = 5$.

—

(ii) Maximum height achieved by the ball:

The height is given as:

$$h = 25t - 5t^2.$$

This is a quadratic equation in the form $h = at^2 + bt + c$ where $a = -5$, $b = 25$, and $c = 0$. The time at which the maximum height occurs is:

$$t = -\frac{b}{2a}.$$

Substitute $a = -5$ and $b = 25$:

$$t = -\frac{25}{2(-5)} = \frac{25}{10} = \frac{5}{2}.$$

At $t = \frac{5}{2}$, the height is:

$$h = 25\left(\frac{5}{2}\right) - 5\left(\frac{5}{2}\right)^2.$$

Simplify:

$$h = 25 \times \frac{5}{2} - 5 \times \frac{25}{4}.$$

$$h = \frac{125}{2} - \frac{125}{4}.$$

Take the LCM of 4:

$$h = \frac{250}{4} - \frac{125}{4} = \frac{125}{4}.$$

$$h = 31.25\,\text{m}.$$

Thus, the maximum height achieved by the ball is $31.25\,\text{m}$.

—

(iii) (A) Time taken to reach the height of $30\,$m:

Given:

$$h = 25t - 5t^2 \quad \text{and} \quad h = 30.$$

Substitute:

$$30 = 25t - 5t^2.$$

Rearrange:

$$-5t^2 + 25t - 30 = 0.$$

Divide through by -5:

$$t^2 - 5t + 6 = 0.$$

Factorize:

$$(t - 2)(t - 3) = 0.$$

Thus:

$$t = 2 \text{ and } t = 3.$$

Therefore, the ball takes $t = 2$ seconds and $t = 3$ seconds to reach the height of 30 m.

—

(iii) (b) Time when height was 20 m:

Given $h = 20$:

$$20 = 25t - 5t^2.$$

Rearrange:

$$-5t^2 + 25t - 20 = 0.$$

Divide through by -5:

$$t^2 - 5t + 4 = 0.$$

Factorize:

$$(t - 4)(t - 1) = 0.$$

Thus:

$$t = 4 \text{ and } t = 1.$$

—

Final Answers:

- (i) Zeroes are $t = 0$ and $t = 5$.
- (ii) Maximum height is 31.25 m.
- (iii) (A) Time to reach 30 m is $t = 2$ seconds and $t = 3$ seconds.
- (iii) (b) Time when height was 20 m is $t = 1$ second and $t = 4$ seconds.

Quick Tip

For quadratic motion problems, identify the maximum height using $t = -\frac{b}{2a}$ and solve for specific heights using factorization or the quadratic formula.

Question 37: The word 'circus' has the same root as 'circle'. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd.

A circus tent is cylindrical up to a height of 8 m and conical above it. The diameter of the base is 28 m and the total height of the tent is 18.5 m.

Based on the above, answer the following questions:

(i) Find slant height of the conical part.

(ii) Determine the floor area of the tent.

(iii) (A) Find area of the cloth used for making the tent.

OR

(iii) (b) Find total volume of air inside an empty tent.

Solution:

(i) Find slant height of the conical part:

The height of the conical part is:

$$\text{Height of cone} = 18.5 - 8 = 10.5\,\text{m}.$$

The radius of the base is:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{28}{2} = 14\,\text{m}.$$

Using the Pythagoras theorem, the slant height l is:

$$l = \sqrt{(\text{Height})^2 + (\text{Radius})^2}.$$

$$l = \sqrt{(10.5)^2 + (14)^2}.$$

Simplify:

$$l = \sqrt{110.25 + 196} = \sqrt{306.25} = 17.5\,\text{m}.$$

—

(ii) Determine the floor area of the tent:

The floor area is the area of the circular base:

$$\text{Floor area} = \pi r^2.$$

Substitute $r = 14\,\text{m}$ and $\pi = \frac{22}{7}$:

$$\text{Floor area} = \frac{22}{7} \times 14 \times 14.$$

Simplify:

$$\text{Floor area} = 616\,\text{m}^2.$$

—

(iii) (A) Find area of the cloth used for making the tent:

The area of cloth includes the curved surface area of the cylinder and cone.

1. Curved surface area of cylinder:

$$\text{CSA of cylinder} = 2\pi r h.$$

Substitute $r = 14\,\text{m}$ and $h = 8\,\text{m}$:

$$\text{CSA of cylinder} = 2 \times \frac{22}{7} \times 14 \times 8.$$

$$\text{CSA of cylinder} = 704\,\text{m}^2.$$

2. Curved surface area of cone:

$$\text{CSA of cone} = \pi r l.$$

Substitute $r = 14\,\text{m}$ and $l = 17.5\,\text{m}$:

$$\text{CSA of cone} = \frac{22}{7} \times 14 \times 17.5.$$

$$\text{CSA of cone} = 770\,\text{m}^2.$$

Total area of cloth:

$$\text{Total area} = \text{CSA of cylinder} + \text{CSA of cone}.$$

$$\text{Total area} = 704 + 770 = 1474\,\text{m}^2.$$

—

(iii) (b) Find total volume of air inside the empty tent:

The volume of the tent includes the volume of the cylinder and cone.

1. Volume of cylinder:

$$\text{Volume of cylinder} = \pi r^2 h.$$

Substitute $r = 14\,\text{m}$ and $h = 8\,\text{m}$:

$$\text{Volume of cylinder} = \frac{22}{7} \times 14 \times 14 \times 8.$$

$$\text{Volume of cylinder} = 4928\,\text{m}^3.$$

2. Volume of cone:

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h.$$

Substitute $r = 14\,\text{m}$ and $h = 10.5\,\text{m}$:

$$\text{Volume of cone} = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 10.5.$$

$$\text{Volume of cone} = 2156\,\text{m}^3.$$

Total volume:

$$\text{Total volume} = \text{Volume of cylinder} + \text{Volume of cone}.$$

$$\text{Total volume} = 4928 + 2156 = 7084\,\text{m}^3.$$

—

Final Answers:

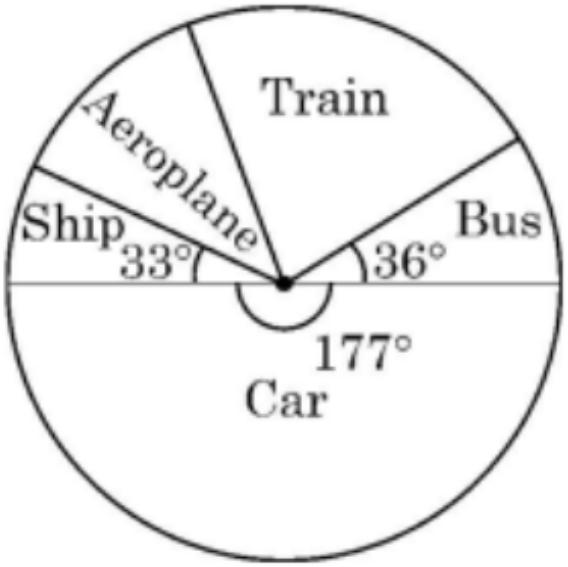
- (i) Slant height of conical part = $17.5\,\text{m}$.
- (ii) Floor area of the tent = $616\,\text{m}^2$.
- (iii) (A) Area of cloth used = $1474\,\text{m}^2$.
- (iii) (b) Total volume of air = $7084\,\text{m}^3$.

Quick Tip

To calculate the total area and volume of structures combining a cylinder and cone, solve for each part separately and add the results.

Question 38: In a survey on holidays, 120 people were asked to state which type of transport they used on their last holiday. The following pie chart shows the results of the survey.

Observe the pie chart and answer the following questions:



(i) If one person is selected at random, find the probability that he/she travelled by bus or ship.

(ii) Which is the most favourite mode of transport and how many people used it?

(iii) (A) A person is selected at random. If the probability that he did not use train is $\frac{4}{5}$, find the number of people who used train.

OR

(iii) (b) The probability that randomly selected person used aeroplane is $\frac{7}{60}$. Find the revenue collected by air company at the rate of 5000 per person.

Solution:

(i) Find the probability of travelling by bus or ship:

The angle representing:

$$\text{Bus} = 36^\circ \quad \text{and} \quad \text{Ship} = 33^\circ.$$

The total angle of the pie chart is 360°. Therefore, the probability is:

$$P(\text{travelling by bus or ship}) = \frac{\text{Bus angle} + \text{Ship angle}}{\text{Total angle}}.$$

$$P = \frac{36^\circ + 33^\circ}{360^\circ} = \frac{69}{360} = \frac{23}{120}.$$

—

(ii) Find the most favourite mode of transport and the number of people:

The angle for **Car** is 177°, which is the largest angle. The number of people who used car is:

$$\text{Number of people} = \frac{\text{Angle for Car}}{\text{Total angle}} \times \text{Total surveyed people}.$$

$$\text{Number of people} = \frac{177}{360} \times 120.$$

$$\text{Number of people} = 59.$$

—

(iii) (A) Find the number of people who used train:

The probability that a person did not use train is:

$$P(\text{Not using train}) = \frac{4}{5}.$$

The probability that a person used train is:

$$P(\text{Using train}) = 1 - \frac{4}{5} = \frac{1}{5}.$$

The number of people who used train is:

$$\text{Number of people} = P(\text{Using train}) \times \text{Total surveyed people}.$$

$$\text{Number of people} = \frac{1}{5} \times 120 = 24.$$

—

(iii) (b) Find the revenue collected for aeroplane users:

The probability of using an aeroplane is $\frac{7}{60}$. The number of people who used an aeroplane is:

$$\text{Number of people} = P(\text{Using aeroplane}) \times \text{Total surveyed people}.$$

$$\text{Number of people} = \frac{7}{60} \times 120.$$

$$\text{Number of people} = 14.$$

The revenue collected at the rate of 5000 per person is:

$$\text{Revenue} = \text{Number of people} \times \text{Rate per person}.$$

$$\text{Revenue} = 14 \times 5000 = 70,000\,.$$

—

Final Answers:

(i) Probability of travelling by bus or ship = $\frac{23}{120}$.

(ii) Most favourite mode of transport: Car, with 59 people.

(iii) (A) Number of people who used train = 24.

(b) Revenue collected = $70,000$.

Quick Tip

To calculate probabilities using a pie chart, divide the specific angle by the total $360°$.

For revenue-related problems, multiply the number of users by the rate per person.