CBSE Class X Mathematics (Standard) Set 2 (30/3/2) Question Paper with Solutions

Time Allowed : 3 Hours	**Maximum Marks :** 80	**Total Questions :** 38

General Instructions

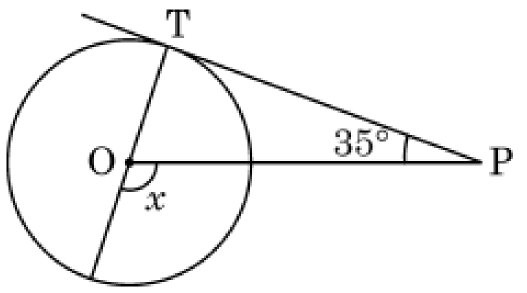
Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section consists of 20 questions of 1 mark each.

Question 1: In the given figure, if PT is a tangent to a circle with centre O and $\angle TPO = 35°$, then the measure of $\angle x$ is:



(A) 110°
(B) 115°
(C) 120°
(D) 125°

Correct Answer: (D) 125°

Solution:
In the given figure: - PT is a tangent to the circle at T. - $\angle TPO = 35°$.
The angle $\angle x$ is an exterior angle of the triangle $\triangle OPT$. The exterior angle of a triangle is equal to the sum of the two opposite interior angles. Thus:

$$\angle x = \angle O + \angle TPO.$$

Since $\angle O = 90°$ (radius OT is perpendicular to the tangent PT):

$$\angle x = 90° + 35° = 125°.$$

—

Conclusion:
The measure of $\angle x$ is 125°.

Quick Tip

For a tangent to a circle, the angle between the radius and the tangent is always 90°.

Question 2: The probability of guessing the correct answer to a certain test question is $\frac{x}{6}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then the value of x is:
(A) 2
(B) 3
(C) 4
(D) 6

Correct Answer: (A) 2

Solution:
The total probability for an event and its complement is:

$$P(\text{correct answer}) + P(\text{not correct answer}) = 1.$$

Substitute the given probabilities:

$$\frac{x}{6} + \frac{2}{3} = 1.$$

Simplify the equation:

$$\frac{x}{6} = 1 - \frac{2}{3}.$$

Convert $1 - \frac{2}{3}$ to a common denominator:

$$\frac{x}{6} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}.$$

Multiply through by 6:

$$x = 6 \times \frac{1}{3} = 2.$$

—

Conclusion:
The value of x is 2.

> **Quick Tip**
>
> The sum of probabilities for an event and its complement is always 1. Use this relationship to solve such questions.

Question 3: From a point on the ground, which is $30\,\text{m}$ away from the foot of a vertical tower, the angle of elevation of the top of the tower is found to be 60°. The height (in metres) of the tower is:

(A) $10\sqrt{3}$
(B) $30\sqrt{3}$
(C) 60
(D) 30

Correct Answer: (B) $30\sqrt{3}$

Solution:
Let the height of the tower be h. Using trigonometry, we have:

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}.$$

Here, $\theta = 60^\circ$, and the adjacent side is $30\,\text{m}$. Substitute:

$$\tan 60^\circ = \frac{h}{30}.$$

The value of $\tan 60^\circ$ is $\sqrt{3}$:

$$\sqrt{3} = \frac{h}{30}.$$

Solve for h:

$$h = 30\sqrt{3}.$$

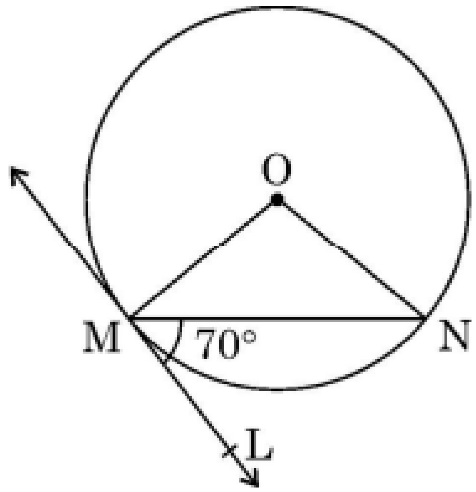
—

Conclusion:
The height of the tower is $30\sqrt{3}\,\text{m}$.

Quick Tip

In problems involving angles of elevation, use trigonometric ratios like $\tan\theta$ to find unknown heights or distances.

Question 4: In the given figure, O is the centre of the circle. MN is the chord and the tangent ML at point M makes an angle of $70°$ with MN. The measure of $\angle MON$ is:



(A) $120°$
(B) $140°$
(C) $70°$
(D) $90°$

Correct Answer: (B) $140°$

Solution:
The angle between a tangent and a chord drawn at the point of tangency is equal to the angle subtended by the chord at the centre of the circle.
From the figure:

$$\angle TML = \angle MON.$$

Here:

$$\angle TML = 70°.$$

Since $\angle MON$ is subtended at the centre of the circle, it is twice the angle at the tangent:

$$\angle MON = 2 \cdot \angle TML = 2 \cdot 70° = 140°.$$

—

Conclusion:
The measure of $\angle MON$ is $140°$.

Quick Tip

The angle subtended by a chord at the centre of a circle is twice the angle subtended at the tangent.

Question 5: If a pair of linear equations in two variables is consistent, then the lines represented by the two equations are:
(A) always intersecting
(B) parallel
(C) always coincident
(D) intersecting or coincident

Correct Answer: (D) intersecting or coincident

Solution:
For a consistent pair of linear equations, there is always at least one solution. This can occur in the following cases:

1. Intersecting Lines: The lines intersect at a single point, giving a unique solution.

2. Coincident Lines: The lines overlap entirely, resulting in infinitely many solutions.

Thus, the lines represented by the equations are either intersecting or coincident.

—
Conclusion:
The lines represented by the equations are either intersecting or coincident, making the correct answer (D).

Quick Tip

For consistent systems of linear equations, either the lines intersect at one point (unique solution) or they coincide (infinitely many solutions).

Question 6: If the area of a sector of a circle is $\frac{7}{20}$ of the area of the circle, then the angle at the centre is equal to:
(A) $110°$
(B) $130°$
(C) $100°$
(D) $126°$

Correct Answer: (D) $126°$

Solution:

The area of a sector of a circle is given by:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \text{Area of circle},$$

where θ is the angle at the center.
It is given that the area of the sector is $\frac{7}{20}$ of the area of the circle:

$$\frac{\theta}{360^\circ} = \frac{7}{20}.$$

Simplify to find θ:

$$\theta = \frac{7}{20} \times 360^\circ.$$

Calculate:

$$\theta = \frac{7 \times 360}{20} = \frac{2520}{20} = 126^\circ.$$

—

Conclusion:
The angle at the center is 126°.

Quick Tip

For sector problems, use the formula Area of sector $= \frac{\theta}{360^\circ} \times$ Area of circle to relate the fraction of the area to the central angle.

Question 7: If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; then the probability that this digit is an odd prime number is:
(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{4}{9}$
(D) $\frac{5}{9}$

Correct Answer: (A) $\frac{1}{3}$

Solution:
The given digits are:

$$1, 2, 3, 4, 5, 6, 7, 8, 9.$$

The odd prime numbers from this set are:

$$3, 5, 7.$$

The total number of digits is 9, and the number of odd prime numbers is 3. Hence, the probability is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{9} = \frac{1}{3}.$$

—

Conclusion:
The probability that the chosen digit is an odd prime number is $\frac{1}{3}$.

Quick Tip

Prime numbers are numbers greater than 1 that have no divisors other than 1 and themselves.

Question 8: If the diagonals of a quadrilateral divide each other proportionally, then it is a:
(A) parallelogram
(B) rectangle
(C) square
(D) trapezium

Correct Answer: (D) trapezium

Solution:
For a quadrilateral: - If the diagonals divide each other proportionally, it is a **trapezium.**

- This property arises because the triangles formed by the diagonals are similar, which is a characteristic feature of trapeziums.

In other types of quadrilaterals like parallelograms, rectangles, or squares, the diagonals either bisect each other or have specific geometric constraints, but they do not divide each other proportionally.
—
Conclusion:
If the diagonals of a quadrilateral divide each other proportionally, it is a trapezium.

Quick Tip

A trapezium's diagonals divide each other proportionally because it forms pairs of similar triangles.

Question 9: If $a = 2^2 \times 3^x$, $b = 2^2 \times 3 \times 5$, $c = 2^2 \times 3 \times 7$, and LCM$(a, b, c) = 3780$, then x is equal to:
(A) 1
(B) 2
(C) 3
(D) 0

Correct Answer: (C) 3

Solution:
The LCM is obtained by taking the highest powers of all prime factors across a, b, and c. From the given LCM:

$$3780 = 2^2 \times 3^3 \times 5 \times 7.$$

In $a = 2^2 \times 3^x$, the power of 3 must match the highest power in the LCM, which is 3. Therefore:

$$x = 3.$$

—

Conclusion:
The value of x is 3.

Question 10: Two coins are tossed simultaneously. The probability of getting at most one tail is:
(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{3}{4}$
(D) 1

Correct Answer: (C) $\frac{3}{4}$

Solution:
The sample space for tossing two coins is:

$$\{HH, HT, TH, TT\}.$$

The outcomes with at most one tail are:

$$\{HH, HT, TH\}.$$

The total number of favorable outcomes is 3, and the total number of outcomes is 4. Therefore, the probability is:

$$P(\text{at most one tail}) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{3}{4}.$$

—

Conclusion:
The probability of getting at most one tail is $\frac{3}{4}$.

Question 11: If the mean of five observations $x, x+2, x+4, x+6, x+8$ is 11, then the value of x is:
(A) 4
(B) 7
(C) 11
(D) 6

Correct Answer: (B) 7

Solution:
The mean of five observations is given by:

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}.$$

Substitute the given values:

$$11 = \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5}.$$

Simplify the numerator:

$$11 = \frac{5x + 20}{5}.$$

Multiply through by 5:

$$55 = 5x + 20.$$

Solve for x:

$$5x = 55 - 20 = 35 \quad \implies \quad x = 7.$$

—

Conclusion:
The value of x is 7.

Quick Tip

To find the zeroes of a quadratic polynomial, use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Question 12: The zeroes of the quadratic polynomial $2x^2 - 3x - 9$ are:
(A) $3, -\frac{3}{2}$
(B) $-3, -\frac{3}{2}$
(C) $-3, \frac{3}{2}$
(D) $3, \frac{3}{2}$

Correct Answer: (A) $3, -\frac{3}{2}$

Solution:
The given quadratic polynomial is:

$$2x^2 - 3x - 9.$$

The formula for finding the roots of a quadratic equation $ax^2 + bx + c = 0$ is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Here, $a = 2$, $b = -3$, and $c = -9$. Substitute into the formula:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-9)}}{2(2)}.$$

Simplify:

$$x = \frac{3 \pm \sqrt{9 + 72}}{4} = \frac{3 \pm \sqrt{81}}{4}.$$

$$x = \frac{3 \pm 9}{4}.$$

The roots are:

$$x = \frac{3 + 9}{4} = 3 \quad \text{and} \quad x = \frac{3 - 9}{4} = -\frac{3}{2}.$$

—

Conclusion:
The zeroes of the quadratic polynomial are 3 and $-\frac{3}{2}$.

Quick Tip

To find the zeroes of a quadratic polynomial, use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Question 13: Maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is:
(A) 4
(B) 3
(C) 2
(D) 1

Correct Answer: (C) 2

Solution:
When two circles intersect at two distinct points, the following cases arise:
- **Direct tangents:** These are tangents that do not pass through the region of intersection. There are 2 such tangents.
- **Common internal tangents:** These are tangents that pass through the region of intersection. In this case, there are no internal tangents, as the circles intersect.

Thus, the maximum number of common tangents is:

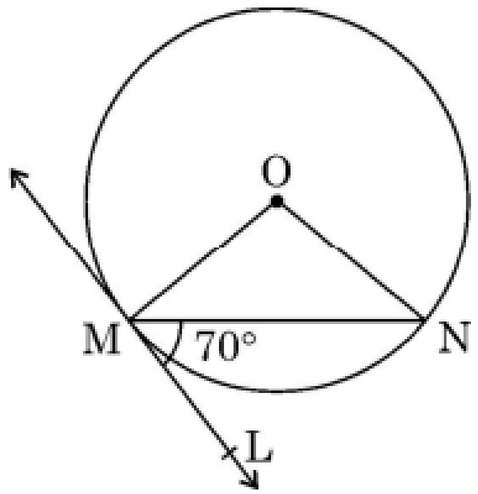
$$2 \quad \text{(direct tangents only)}.$$

—
Conclusion:
The maximum number of common tangents for two circles intersecting at two distinct points is 2.

Quick Tip

The number of common tangents between two circles depends on their relative positions (disjoint, intersecting, or touching).

Question 14: In $\triangle ABC$, $DE \parallel BC$ (as shown in the figure). If $AD = 2\,\text{cm}$, $BD = 3\,\text{cm}$, $BC = 7.5\,\text{cm}$, then the length of DE (in cm) is:



(A) 2.5
(B) 3
(C) 5
(D) 6

Correct Answer: (B) 3

Solution:
By the Basic Proportionality Theorem (Thales' theorem), if a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally. Hence:

$$\frac{AD}{AB} = \frac{DE}{BC}.$$

First, find AB (the total length of side AB):

$$AB = AD + BD = 2 + 3 = 5\,\text{cm}.$$

Now substitute the known values:

$$\frac{AD}{AB} = \frac{DE}{BC}.$$

$$\frac{2}{5} = \frac{DE}{7.5}.$$

Solve for DE:

$$DE = \frac{2}{5} \times 7.5 = 3\,\text{cm}.$$

—

Conclusion:
The length of DE is $3\,\text{cm}$.

Quick Tip

The Basic Proportionality Theorem states that if a line is parallel to one side of a triangle, it divides the other sides proportionally.

Question 15: If $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\phi = \frac{1}{2}$, then $\tan(\theta+\phi)$ is:
(A) $\sqrt{3}$
(B) $\frac{1}{\sqrt{3}}$
(C) 1
(D) not defined

Correct Answer: (A) $\sqrt{3}$

Solution:
We are given:

$$\cos\theta = \frac{\sqrt{3}}{2}, \quad \sin\phi = \frac{1}{2}.$$

From trigonometric identities:

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}} = \frac{1}{2},$$

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$

The formula for $\tan(\theta+\phi)$ is:

$$\tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi}.$$

Compute $\tan\theta$ and $\tan\phi$:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}, \quad \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}.$$

Substitute into the formula:

$$\tan(\theta+\phi) = \frac{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}.$$

—

Conclusion:
The value of $\tan(\theta+\phi)$ is $\sqrt{3}$.

Quick Tip

To compute $\tan(\theta+\phi)$, use the formula $\tan(\theta+\phi) = \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$ and substitute the values.

Question 16: Given HCF(2520, 6600) = 40, LCM(2520, 6600) = $252 \times k$, then the value of k is:
(A) 1650
(B) 1600

(C) 165
(D) 1625

Correct Answer: (A) 1650

Solution:
The product of HCF and LCM of two numbers is equal to the product of the numbers. This can be expressed as:

$$\text{HCF} \times \text{LCM} = \text{Number 1} \times \text{Number 2}.$$

Substitute the given values:

$$40 \times (252 \times k) = 2520 \times 6600.$$

Simplify:

$$252 \times k = \frac{2520 \times 6600}{40}.$$

Calculate:

$$252 \times k = 415800.$$

Solve for k:

$$k = \frac{415800}{252} = 1650.$$

—

Conclusion:
The value of k is 1650.

Quick Tip

For any two numbers, the product of their HCF and LCM equals the product of the numbers.

Question 17: If the sum of the first n terms of an A.P. is $3n^2 + 4n$ and its common difference is 6, then its first term is:
(A) 7
(B) 4
(C) 6
(D) 3

Correct Answer: (A) 7

Solution:
The sum of the first n terms of an A.P. is given as:

$$S_n = 3n^2 + 4n.$$

The first term a_1 is:

$$a_1 = S_1 = 3(1)^2 + 4(1) = 3 + 4 = 7.$$

The common difference d is:

$$d = a_2 - a_1.$$

Find a_2 by calculating $S_2 - S_1$:

$$S_2 = 3(2)^2 + 4(2) = 3(4) + 8 = 12 + 8 = 20.$$

Thus:

$$a_2 = S_2 - S_1 = 20 - 7 = 13, \quad d = a_2 - a_1 = 13 - 7 = 6.$$

The first term $a_1 = 7$.

—

Conclusion:
The first term of the A.P. is 7.

> **Quick Tip**
>
> To find the zeroes of a quadratic polynomial, use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Question 18: What should be subtracted from the polynomial $x^2 - 16x + 30$, so that 15 is the zero of the resulting polynomial?

(A) 30
(B) 14
(C) 15
(D) 16

Correct Answer: (C) 15

Solution:
The polynomial is:

$$P(x) = x^2 - 16x + 30.$$

For $x = 15$ to be a zero of the resulting polynomial:

$$P(15) = 0.$$

Substitute $x = 15$ in $P(x)$:

$$P(15) = (15)^2 - 16(15) + 30.$$

Simplify:

$$P(15) = 225 - 240 + 30 = 225 - 210 = 15.$$

To make $P(15) = 0$, we need to subtract 15 from the polynomial.

—

Conclusion:
The value to be subtracted is 15.

> **Quick Tip**
>
> To find the zeroes of a quadratic polynomial, use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Directions: Questions number 19 and 20 are Assertion and Reason-based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below:
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Question 19: Assertion (A): In a cricket match, a batsman hits a boundary 9 times out of 45 balls he plays. The probability that in a given ball, he does not hit the boundary is $\frac{4}{5}$.
Reason (R): $P(E) + P(\text{not } E) = 1$.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:
The probability of hitting a boundary is:

$$P(E) = \frac{\text{Number of times boundary is hit}}{\text{Total number of balls}} = \frac{9}{45} = \frac{1}{5}.$$

The probability of not hitting a boundary is:

$$P(\text{not } E) = 1 - P(E) = 1 - \frac{1}{5} = \frac{4}{5}.$$

The Reason (R) correctly states that $P(E)+P(\text{not } E) = 1$, which is used to calculate $P(\text{not } E)$. Therefore, both Assertion (A) and Reason (R) are true, and Reason (R) explains Assertion (A).

Quick Tip

For probabilities, the sum of the probability of an event and its complement is always 1:

$$P(E) + P(\text{not } E) = 1.$$

Use this rule to find the complement probability quickly.

Question 20: Assertion (A): The point which divides the line segment joining the points $A(1, 2)$ and $B(-1, 1)$ internally in the ratio $1 : 2$ is $\left(-\frac{1}{3}, \frac{5}{3}\right)$.
Reason (R): The coordinates of the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1 : m_2$ are:

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:
To find the point dividing the line segment joining $A(1,2)$ and $B(-1,1)$ in the ratio $1:2$, we use the formula:

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

Substitute $m_1 = 1, m_2 = 2, A(1,2), B(-1,1)$:

$$x = \frac{(1)(-1) + (2)(1)}{1+2} = \frac{-1+2}{3} = \frac{1}{3}, \quad y = \frac{(1)(1) + (2)(2)}{1+2} = \frac{1+4}{3} = \frac{5}{3}.$$

The point is $\left(\frac{1}{3}, \frac{5}{3}\right)$, not $\left(-\frac{1}{3}, \frac{5}{3}\right)$.

Therefore, Assertion (A) is false, but Reason (R) is correct.

Quick Tip

When dividing a line segment in a ratio, use the section formula:

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

Make sure the ratio and coordinates are substituted correctly.

Section - B

This section consists of 5 questions of 2 mark each.

Question 21: Evaluate:

$$\frac{\sec^2 45^\circ - \tan^2 45^\circ}{\sin^2 45^\circ}.$$

Solution:
We start with the given expression:

$$\frac{\sec^2 45^\circ - \tan^2 45^\circ}{\sin^2 45^\circ}.$$

1. Use the trigonometric identities: - $\sec^2\theta - \tan^2\theta = 1$, - $\sin 45^\circ = \frac{1}{\sqrt{2}}$.
Substitute these values:

$$\frac{\sec^2 45^\circ - \tan^2 45^\circ}{\sin^2 45^\circ} = \frac{1}{\sin^2 45^\circ}.$$

2. Calculate $\sin^2 45^\circ$:

$$\sin^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}.$$

Substitute $\sin^2 45^\circ = \frac{1}{2}$ into the expression:

$$\frac{1}{\sin^2 45^\circ} = \frac{1}{\frac{1}{2}} = 2.$$

—

Conclusion:
The value of the given expression is 2.

> **Quick Tip**
>
> Remember that $\sec^2\theta - \tan^2\theta = 1$ and use the square of trigonometric values for accurate simplifications.

Question 22(a): Find a relation between x and y such that the point $P(x, y)$ is equidistant from the points $A(7, 1)$ and $B(3, 5)$.

Solution:
The condition for equidistance is:

$$PA = PB \quad \Longrightarrow \quad PA^2 = PB^2.$$

Substitute the coordinates:

$$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2.$$

Expand both sides:

$$(x^2 - 14x + 49) + (y^2 - 2y + 1) = (x^2 - 6x + 9) + (y^2 - 10y + 25).$$

Simplify:

$$-14x - 2y + 50 = -6x - 10y + 34.$$

Rearrange:

$$-8x + 8y + 16 = 0 \quad \Longrightarrow \quad -8(x - y - 2) = 0.$$

Therefore, the relation is:

$$x - y - 2 = 0.$$

—

Conclusion:
The required relation between x and y is:

$$x - y - 2 = 0.$$

> **Quick Tip**
>
> To find the relation for equidistant points, equate the squared distance from each point and simplify the equation.

Question 22(b): Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$ such that AB is a diameter of the circle. Find the value of y. Also, find the radius of the circle.

Solution:
The centre $O(2, -3y)$ is the midpoint of AB. Using the midpoint formula:

$$\left(\frac{-1+5}{2}, \frac{y+7}{2}\right) = (2, -3y).$$

Equating the x-coordinates:

$$\frac{-1+5}{2} = 2 \implies \frac{4}{2} = 2 \quad \text{(satisfied)}.$$

Equating the y-coordinates:

$$\frac{y+7}{2} = -3y.$$

Simplify:

$$y + 7 = -6y \implies 7y = -7 \implies y = -1.$$

Now calculate the radius of the circle. Since AB is the diameter, the radius is half the length of AB.
The distance AB is:

$$AB = \sqrt{(5-(-1))^2 + (7-(-1))^2}.$$

Simplify:

$$AB = \sqrt{(5+1)^2 + (7+1)^2} = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10.$$

The radius is:

$$\text{Radius} = \frac{AB}{2} = \frac{10}{2} = 5.$$

—

Conclusion:
The value of y is -1, and the radius of the circle is 5.

> **Quick Tip**
>
> When the diameter is given, the radius is half its length. Use the distance formula to calculate the diameter and divide by 2 for the radius.

Question 23: One card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that the card drawn:
(i) is the queen of hearts;
(ii) is not a jack.

Solution:
Total number of outcomes = 52.
(i) The probability of drawing the queen of hearts is:

$$P(\text{queen of hearts}) = \frac{1}{52}.$$

(ii) There are 4 jacks in the deck, so the number of cards that are not jacks is:

$$52 - 4 = 48.$$

The probability of not drawing a jack is:

$$P(\text{not a jack}) = \frac{48}{52} = \frac{12}{13}.$$

—

Conclusion:
(i) $P(\text{queen of hearts}) = \frac{1}{52}$.
(ii) $P(\text{not a jack}) = \frac{12}{13}$.

Quick Tip

In probability, $P(\text{not A}) = 1 - P(A)$. For cards, ensure you account for the total number of specific cards (e.g., 4 jacks, 4 queens).

Question 24(a): If $2x + y = 13$ and $4x - y = 17$, find the value of $(x - y)$.

Solution:
The given equations are:

$$2x + y = 13 \quad \text{(i)},$$
$$4x - y = 17 \quad \text{(ii)}.$$

Add equations (i) and (ii) to eliminate y:

$$(2x + y) + (4x - y) = 13 + 17.$$

Simplify:

$$6x = 30 \quad \implies \quad x = 5.$$

Substitute $x = 5$ into equation (i):

$$2(5) + y = 13 \quad \implies \quad 10 + y = 13 \quad \implies \quad y = 3.$$

The value of $x - y$ is:

$$x - y = 5 - 3 = 2.$$

—

Conclusion:
The value of $x - y$ is 2.

Quick Tip

For solving linear equations, try eliminating one variable by adding or subtracting equations, then solve for the other variable.

Question 24(b): Sum of two numbers is 105 and their difference is 45. Find the numbers.

Solution:
Let the two numbers be x and y, where $x > y$.

The given equations are:

$$x + y = 105 \quad \text{(i)},$$
$$x - y = 45 \quad \text{(ii)}.$$

Add equations (i) and (ii) to eliminate y:

$$(x + y) + (x - y) = 105 + 45.$$

Simplify:

$$2x = 150 \quad \implies \quad x = 75.$$

Substitute $x = 75$ into equation (i):

$$75 + y = 105 \quad \implies \quad y = 105 - 75 = 30.$$

Thus, the two numbers are:

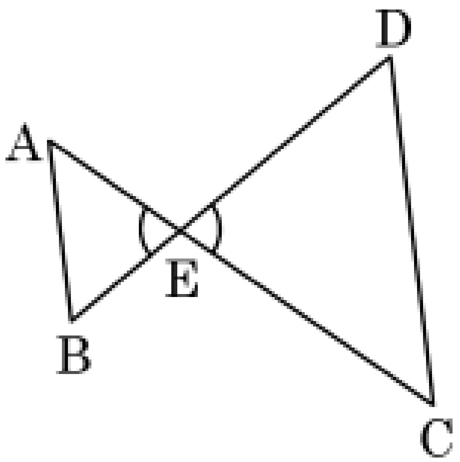
$$x = 75,\ y = 30.$$

—

Conclusion:
The numbers are 75 and 30.

> **Quick Tip**
>
> To solve problems involving the sum and difference of two numbers, add and subtract the equations to eliminate one variable and solve for the other.

Question 25: In the given figure, $\frac{EA}{EC} = \frac{EB}{ED}$, prove that $\triangle EAB \sim \triangle ECD$.



Solution:
In $\triangle EAB$ and $\triangle ECD$, we are given:

$$\frac{EA}{EC} = \frac{EB}{ED}.$$

Also, $\angle AEB = \angle CED$ (vertically opposite angles).

By the Side-Angle-Side (SAS) similarity criterion:

$$\triangle EAB \sim \triangle ECD.$$

—

Conclusion:
It is proven that $\triangle EAB \sim \triangle ECD$.

Quick Tip

For two triangles to be similar by SAS similarity, the ratios of two corresponding sides must be equal, and the included angles must be equal.

Section - C

This section consists of 6 questions of 3 mark each.

Question 26: Solve the following system of linear equations graphically:

$$x - y + 1 = 0$$
$$x + y = 5$$

Solution:
The given equations are: 1. $x-y+1=0$, or equivalently $x-y=-1 \quad$ (i). 2. $x+y=5 \quad$ (ii).
—
Step 1: Find points for each line.
For equation (i) $(x - y = -1)$:

$$\text{If } x = 0,\ -y = -1 \implies y = 1.$$
$$\text{If } x = -1,\ -y = -2 \implies y = 2.$$
$$\text{If } x = 1,\ -y = 0 \implies y = 0.$$

Thus, the points for $x - y = -1$ are:

$$(0,1),\ (-1,2),\ (1,0).$$

For equation (ii) $(x + y = 5)$:

$$\text{If } x = 0,\ y = 5.$$
$$\text{If } x = 5,\ y = 0.$$
$$\text{If } x = 2,\ y = 3.$$

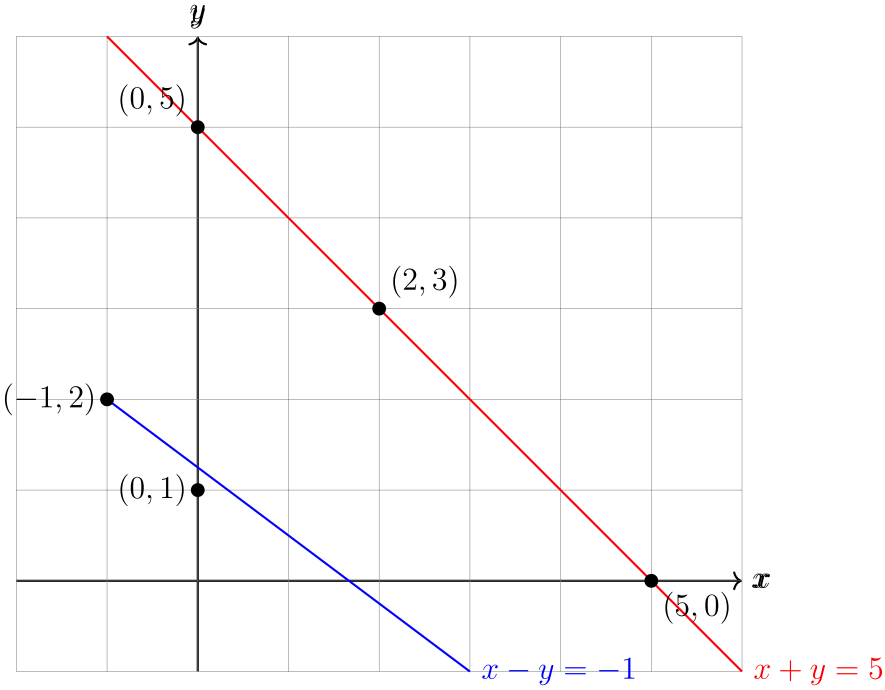
Thus, the points for $x + y = 5$ are:

$$(0,5),\ (5,0),\ (2,3).$$

—

Step 2: Plot the lines on the graph.

- Plot the line $x - y = -1$ using the points $(0,1), (-1,2), (1,0)$. - Plot the line $x + y = 5$ using the points $(0,5), (5,0), (2,3)$.



—

Step 3: Find the point of intersection.

The two lines intersect at the point $(2,3)$. This is the solution to the system of equations.

—

Conclusion:

The solution to the given system of equations is:

$$x = 2,\ y = 3.$$

Quick Tip

To solve linear equations graphically, plot each equation as a straight line and find their point of intersection. This point gives the solution.

Question 27: Prove that

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}.$$

Solution:

Let the given expression be:

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}.$$

Take a common denominator:

$$\text{Expression} = \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}.$$

Simplify the numerator using the identity $(x + y)^2 + (x - y)^2 = 2x^2 + 2y^2$:

$$(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2\sin^2 A + 2\cos^2 A.$$

Using $\sin^2 A + \cos^2 A = 1$, the numerator becomes:

$$2\sin^2 A + 2\cos^2 A = 2(1) = 2.$$

Simplify the denominator:

$$(\sin A - \cos A)(\sin A + \cos A) = \sin^2 A - \cos^2 A.$$

The expression now becomes:

$$\frac{2}{\sin^2 A - \cos^2 A}.$$

Using the identity $\sin^2 A - \cos^2 A = 2\sin^2 A - 1$, the denominator is:

$$\sin^2 A - \cos^2 A = 2\sin^2 A - 1.$$

Thus, the expression becomes:

$$\frac{2}{2\sin^2 A - 1}.$$

—

Conclusion:

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}.$$

Quick Tip

For trigonometric proofs, simplify each term step by step and use standard identities like $\sin^2 A + \cos^2 A = 1$ and $\sin^2 A - \cos^2 A = 2\sin^2 A - 1$.

Question 28: (a) In what ratio does the X-axis divide the line segment joining the points $(2, -3)$ and $(5, 6)$? Also, find the coordinates of the point of intersection.
OR
(b) Find the length of the median AD of ΔABC having vertices $A(0, -1)$, $B(2, 1)$, and $C(0, 3)$.
—

Solution (a):
The X-axis divides the line segment at the point where $y = 0$. Let the point of intersection be $(x, 0)$. Using the section formula:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}.$$

Substitute $y = 0$:

$$\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = 0.$$

Substitute $y_1 = -3$, $y_2 = 6$:

$$m_1(6) + m_2(-3) = 0 \implies 6m_1 - 3m_2 = 0 \implies 2m_1 = m_2.$$

Thus, the ratio is:

$$m_1 : m_2 = 1 : 2.$$

Find x using the section formula:

$$x = \frac{(1)(5) + (2)(2)}{1 + 2} = \frac{5 + 4}{3} = 3.$$

The point of intersection is:

$$(3, 0).$$

—

Conclusion:
The X-axis divides the line segment in the ratio $1 : 2$, and the point of intersection is $(3, 0)$.
—

Solution (b):
The median AD divides the triangle into two equal areas. The midpoint D of BC is:

$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right).$$

Substitute $B(2, 1)$, $C(0, 3)$:

$$D = \left(\frac{2 + 0}{2}, \frac{1 + 3}{2}\right) = (1, 2).$$

The length of AD is:

$$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substitute $A(0, -1)$, $D(1, 2)$:

$$AD = \sqrt{(1 - 0)^2 + (2 - (-1))^2} = \sqrt{1^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10}.$$

—

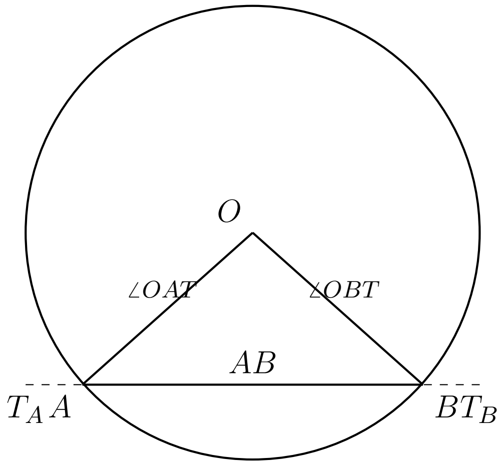
Conclusion:
The length of the median AD is $\sqrt{10}$.

Quick Tip

Use the section formula $x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$ and $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$ to find points of division. For medians, the midpoint formula helps simplify the calculations.

Question 29: Prove that the tangents drawn at the endpoints of a chord of a circle make equal angles with the chord.

Solution:
Let the chord AB of a circle have endpoints A and B, and let O be the center of the circle. Draw tangents at A and B, and let the chord AB intersect the tangents at angles $\angle OAT$ and $\angle OBT$.



1. Properties of the circle:
- The radius OA is perpendicular to the tangent at A.
- The radius OB is perpendicular to the tangent at B.

2. Triangles involved:
- In $\triangle OAT$, $\angle OAT$ is the angle between the tangent at A and the chord AB.
- In $\triangle OBT$, $\angle OBT$ is the angle between the tangent at B and the chord AB.

3. Prove equality:
Since the chord AB subtends equal angles at the center ($\angle OAB = \angle OBA$) and the radii OA and OB are equal, the triangles $\triangle OAT$ and $\triangle OBT$ are congruent (by RHS criterion).
Thus:
$$\angle OAT = \angle OBT.$$

—

Conclusion:
The tangents drawn at the endpoints of a chord of a circle make equal angles with the chord.

> **Quick Tip**
>
> For chord-tangent problems, remember that the radius is perpendicular to the tangent, and congruence of triangles can be used to prove angle relationships.

Question 30: Find the zeroes of the quadratic polynomial $x^2 - 15$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Solution:
Let $P(x) = x^2 - 15$.
Factorize $P(x)$:
$$P(x) = (x - \sqrt{15})(x + \sqrt{15}).$$

Thus, the zeroes of $P(x)$ are:

$$-\sqrt{15} \quad \text{and} \quad \sqrt{15}.$$

—

Verification:
1. Sum of zeroes:

$$-\sqrt{15} + \sqrt{15} = 0.$$

Compare with:

$$\text{Sum of zeroes} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = \frac{0}{1} = 0.$$

2. Product of zeroes:

$$(-\sqrt{15}) \times (\sqrt{15}) = -15.$$

Compare with:

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{-15}{1} = -15.$$

—

Conclusion:
The sum and product of the zeroes are verified to match the relationships:

$$\text{Sum of zeroes} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}, \quad \text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}.$$

> **Quick Tip**
>
> For a quadratic polynomial $ax^2 + bx + c$, the sum of zeroes is $-\frac{b}{a}$ and the product of zeroes is $\frac{c}{a}$.

Question 31(a): If the sum of the first 7 terms of an A.P. is 49 and that of the first 17 terms is 289, find the sum of its first 20 terms.

Solution:
Let a be the first term and d be the common difference.
The sum of n terms of an A.P. is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

For the first 7 terms ($S_7 = 49$):

$$\frac{7}{2}[2a + 6d] = 49.$$

Simplify:

$$7a + 21d = 49 \quad \Longrightarrow \quad a + 3d = 7 \quad \text{(i)}.$$

For the first 17 terms ($S_{17} = 289$):

$$\frac{17}{2}[2a + 16d] = 289.$$

Simplify:

$$17a + 136d = 289 \quad \Longrightarrow \quad a + 8d = 17 \quad \text{(ii)}.$$

Solve equations (i) and (ii):

$$a + 3d = 7,$$

$$a + 8d = 17.$$

Subtract (i) from (ii):

$$(8d - 3d) = (17 - 7) \implies 5d = 10 \implies d = 2.$$

Substitute $d = 2$ into (i):

$$a + 3(2) = 7 \implies a = 1.$$

Now, find the sum of the first 20 terms (S_{20}):

$$S_{20} = \frac{20}{2}[2a + 19d].$$

Substitute $a = 1$ and $d = 2$:

$$S_{20} = 10[2(1) + 19(2)] = 10[2 + 38] = 10 \cdot 40 = 400.$$

—

Conclusion:
The sum of the first 20 terms of the A.P. is 400.

Quick Tip

For sums of A.P., use the formula $S_n = \frac{n}{2}[2a+(n-1)d]$ and solve equations systematically to find a and d.

Question 31(b): The ratio of the 10th term to its 30th term of an A.P. is $1:3$, and the sum of its first six terms is 42. Find the first term and the common difference of the A.P.

Solution:
Let a be the first term and d be the common difference.
The general term of an A.P. is given by:

$$T_n = a + (n - 1)d.$$

For the 10th term:

$$T_{10} = a + 9d.$$

For the 30th term:

$$T_{30} = a + 29d.$$

Given:

$$\frac{T_{10}}{T_{30}} = \frac{1}{3}.$$

Substitute T_{10} and T_{30}:

$$\frac{a + 9d}{a + 29d} = \frac{1}{3}.$$

Cross-multiply:

$$3(a+9d) = (a+29d).$$

Simplify:

$$3a + 27d = a + 29d \implies 2a = 2d \implies a = d.$$

—

The sum of the first 6 terms is given as $S_6 = 42$. The formula for the sum of the first n terms is:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

For S_6:

$$\frac{6}{2}[2a + 5d] = 42.$$

Simplify:

$$3(2a+5d) = 42 \implies 2a + 5d = 14 \quad \text{(ii)}.$$

Substitute $a = d$ into (ii):

$$2d + 5d = 14 \implies 7d = 14 \implies d = 2.$$

Since $a = d$, we have:

$$a = 2.$$

—

Conclusion:
The first term (a) is 2, and the common difference (d) is 2.

Quick Tip

When ratios of terms are given, equate their formula and simplify. Use the sum formula to find unknowns like a and d.

Section - D

This section consists of 4 questions of 5 marks each.

Question 32(a): A solid iron pole consists of a solid cylinder of height 200 cm and base diameter 28 cm, which is surmounted by another cylinder of height 50 cm and radius 7 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass.

Solution:
The pole consists of two cylindrical parts. Let us calculate the volume of each cylinder separately and then find the total mass.

—

1. Volume of the lower cylinder:
Height $(h_1) = 200$ cm,
Radius $(r_1) = \frac{\text{diameter}}{2} = \frac{28}{2} = 14\,\text{cm}.$

The volume of a cylinder is given by:

$$V = \pi r^2 h.$$

For the lower cylinder:

$$V_1 = \pi r_1^2 h_1 = \pi(14)^2(200).$$

Simplify:

$$V_1 = \pi(196)(200) = 39200\pi \, \text{cm}^3.$$

—

2. Volume of the upper cylinder:
Height $(h_2) = 50$ cm,
Radius $(r_2) = 7$ cm.
For the upper cylinder:

$$V_2 = \pi r_2^2 h_2 = \pi(7)^2(50).$$

Simplify:

$$V_2 = \pi(49)(50) = 2450\pi \, \text{cm}^3.$$

—

3. Total volume of the pole:

$$V_{\text{total}} = V_1 + V_2 = 39200\pi + 2450\pi = 41650\pi \, \text{cm}^3.$$

Substitute $\pi \approx 3.1416$:

$$V_{\text{total}} = 41650 \times 3.1416 = 130881.55 \, \text{cm}^3.$$

—

4. Mass of the pole:
Given that 1 cm^3 of iron has a mass of 8 g:

$$\text{Mass} = V_{\text{total}} \times 8 = 130881.55 \times 8 = 1047052.4 \, \text{g}.$$

Convert to kilograms:

$$\text{Mass} = \frac{1047052.4}{1000} = 1047.05 \, \text{kg}.$$

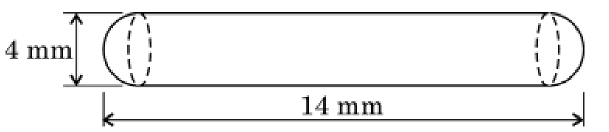
—

Conclusion:
The mass of the iron pole is approximately $1047.05 \, \text{kg}$.

Quick Tip

To find the mass of a solid object, calculate its volume using geometric formulas, then multiply by the given density (mass per unit volume).

Question 32(b): A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm, and the diameter of the capsule is 4 mm. Find its surface area and volume.



Solution:
The capsule consists of: 1. A cylindrical part with two hemispheres attached at the ends. 2. Radius (r) of the hemispheres and the cylinder:

$$r = \frac{\text{diameter}}{2} = \frac{4}{2} = 2\,\text{mm}.$$

3. Length of the cylindrical part:

$$\text{Length of cylinder} = \text{Total length of capsule} - 2(\text{Radius of hemispheres}) = 14 - 4 = 10\,\text{mm}.$$

—

1. Surface Area of the Capsule:
The surface area consists of: 1. Curved surface area (CSA) of the cylinder:

$$\text{CSA of cylinder} = 2\pi rh = 2 \times \frac{22}{7} \times 2 \times 10 = 40 \times \frac{22}{7} = 125.71\,\text{mm}^2.$$

2. CSA of the two hemispheres:

$$\text{CSA of one hemisphere} = 2\pi r^2 = 2 \times \frac{22}{7} \times 2^2 = 2 \times \frac{22}{7} \times 4 = \frac{176}{7} = 25.14\,\text{mm}^2.$$

For two hemispheres:

$$\text{CSA of both hemispheres} = 2 \times 25.14 = 50.28\,\text{mm}^2.$$

Total surface area:

$$\text{Surface Area} = \text{CSA of cylinder} + \text{CSA of hemispheres} = 125.71 + 50.28 = 176\,\text{mm}^2.$$

—

2. Volume of the Capsule:
The volume consists of: 1. Volume of the cylinder:

$$\text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 2^2 \times 10 = \frac{22}{7} \times 4 \times 10 = \frac{880}{7} = 125.71\,\text{mm}^3.$$

2. Volume of the two hemispheres:

$$\text{Volume of one hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 2^3 = \frac{2}{3} \times \frac{22}{7} \times 8 = \frac{352}{21} = 16.76\,\text{mm}^3.$$

For two hemispheres:

$$\text{Volume of both hemispheres} = 2 \times 16.76 = 33.52\,\text{mm}^3.$$

Total volume:

$$\text{Volume} = \text{Volume of cylinder} + \text{Volume of hemispheres} = 125.71 + 33.52 = 159.24\,\text{mm}^3.$$

—

Conclusion:
1. The surface area of the capsule is:
$$176\,\text{mm}^2.$$
2. The volume of the capsule is:
$$159.24\,\text{mm}^3.$$

—

Quick Tip

For composite shapes, break them into basic geometric shapes (cylinder, hemisphere, etc.) and calculate their properties individually. Combine the results for the total.

Question 33(a): In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and by doing so, the time of flight is increased by 30 minutes. Find the original duration of the flight.

Solution:
Let the original speed of the aircraft be x km/h.
1. Original Time of Flight:
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{2800}{x}.$$
2. New Speed and Time: When the speed is reduced by 100 km/h, the new speed is $(x - 100)$, and the new time is:
$$\frac{2800}{x-100}.$$
The time difference between the original and new times is 30 minutes, or $\frac{1}{2}$ hour. Therefore:
$$\frac{2800}{x-100} - \frac{2800}{x} = \frac{1}{2}.$$
3. Simplify the Equation: Take the LCM of $x(x - 100)$:
$$\frac{2800x - 2800(x-100)}{x(x-100)} = \frac{1}{2}.$$
Simplify:
$$\frac{2800 \cdot 100}{x(x-100)} = \frac{1}{2}.$$
Multiply through by $2x(x - 100)$:
$$560000 = x(x - 100).$$
Expand:
$$x^2 - 100x - 560000 = 0.$$
4. Solve the Quadratic Equation: Using the quadratic formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a = 1,\ b = -100,\ c = -560000.$$

$$x = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(1)(-560000)}}{2(1)}.$$

$$x = \frac{100 \pm \sqrt{10000 + 2240000}}{2}.$$

$$x = \frac{100 \pm \sqrt{2250000}}{2}.$$

$$x = \frac{100 \pm 1500}{2}.$$

Select the positive root:

$$x = \frac{100 + 1500}{2} = 800.$$

5. Find the Original Time: The original time of the flight is:

$$\frac{2800}{800} = 3.5\,\text{hours}.$$

—

Conclusion:
The original duration of the flight is 3.5 hours.

> **Quick Tip**
>
> For time-speed-distance problems, relate the difference in times to the change in speeds and solve using algebraic equations.

Question 33(b): The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Solution:
Let the numerator of the fraction be x. Then the denominator is:

$$2x + 1.$$

The fraction is:

$$\frac{x}{2x+1}.$$

The reciprocal is:

$$\frac{2x+1}{x}.$$

Given:

$$\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21}.$$

Convert $2\frac{16}{21}$ to an improper fraction:

$$2\frac{16}{21} = \frac{42+16}{21} = \frac{58}{21}.$$

Equate:

$$\frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}.$$

1. Simplify the Left Side: Take the LCM of $x(2x+1)$:

$$\frac{x^2+(2x+1)^2}{x(2x+1)}=\frac{58}{21}.$$

Expand $(2x+1)^2$:

$$\frac{x^2+4x^2+4x+1}{x(2x+1)}=\frac{58}{21}.$$

Simplify:

$$\frac{5x^2+4x+1}{x(2x+1)}=\frac{58}{21}.$$

2. Cross Multiply:

$$21(5x^2+4x+1)=58x(2x+1).$$

Expand both sides:

$$105x^2+84x+21=116x^2+58x.$$

Simplify:

$$116x^2-105x^2+58x-84x-21=0.$$

$$11x^2-26x-21=0.$$

3. Solve the Quadratic Equation: Using the quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a},\quad a=11,\ b=-26,\ c=-21.$$

$$x=\frac{-(-26)\pm\sqrt{(-26)^2-4(11)(-21)}}{2(11)}.$$

$$x=\frac{26\pm\sqrt{676+924}}{22}.$$

$$x=\frac{26\pm\sqrt{1600}}{22}.$$

$$x=\frac{26\pm 40}{22}.$$

Select the positive root:

$$x=\frac{26+40}{22}=\frac{66}{22}=3.$$

4. Find the Fraction: The fraction is:

$$\frac{x}{2x+1}=\frac{3}{2(3)+1}=\frac{3}{7}.$$

—

Conclusion:
The fraction is $\frac{3}{7}$.

Quick Tip

For fraction-reciprocal problems, set up the equation, clear the denominators using LCM, and solve the resulting quadratic equation.

Question 34: Through the mid-point M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC in L and AD produced in E. Prove that $EL = 2BL$.

—

Solution:

Let $ABCD$ be a parallelogram with M as the mid-point of CD. The diagonals of a parallelogram bisect each other, so AC is divided equally by the point of intersection O of the diagonals.

1. **Step 1:** Using the properties of a parallelogram: - Since M is the mid-point of CD, BM divides AC in the ratio $2:1$ at L (from the mid-point theorem).

2. **Step 2:** Extend BM to meet AD produced at E. Using the concept of proportionality: - The triangle ΔBLM and ΔELM are similar because they share the same base LM and are cut by parallel lines.

3. **Step 3:** From similarity, $EL = 2BL$: - Since L divides AC in the ratio $2:1$, the extended line EL satisfies $EL = 2BL$.

—

Conclusion:

Thus, it is proved that $EL = 2BL$ using the mid-point theorem and similarity of triangles.

> **Quick Tip**
>
> Use the mid-point theorem to determine proportional relationships in parallelograms, and apply similarity of triangles to solve extended line segment problems.

Question 35: The angle of elevation of a jet plane from a point A on the ground is $60°$. After a flight of 30 seconds, the angle of elevation changes to $30°$. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane.

—

Solution:

Refer to the diagram for clarity. Let the point P represent the jet plane's position initially, and Q represent its position after 30 seconds. The height of the plane is constant at $3600\sqrt{3}$ m.
Step 1: Calculate x in $\triangle APB$ Using $\tan 60° = \sqrt{3}$, we have:

$$\tan 60° = \frac{\text{Height of plane}}{\text{Base (AB)}} = \frac{3600\sqrt{3}}{x}.$$

Simplify:

$$x = 3600\,\text{m}.$$

Step 2: Calculate $x + y$ in $\triangle AQC$ Using $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we have:

$$\tan 30^\circ = \frac{\text{Height of plane}}{\text{Base (AC)}} = \frac{3600\sqrt{3}}{x + y}.$$

Substitute $x = 3600\,\text{m}$:

$$\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{3600 + y}.$$

Simplify:

$$3600 + y = 3 \times 3600 = 10800 \quad \implies \quad y = 7200\,\text{m}.$$

Step 3: Calculate the speed of the jet plane The total horizontal distance covered by the plane in 30 seconds is:

$$y = 7200\,\text{m}.$$

The speed of the plane is:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{7200}{30} = 240\,\text{m/s}.$$

—

Conclusion:
The speed of the jet plane is $240\,\text{m/s}$.

Quick Tip

Remember that for height and angle of elevation problems, use trigonometric identities $\tan\theta = \frac{\text{Height}}{\text{Base}}$. Calculate distances step-by-step and substitute known values carefully.

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: Teaching Mathematics through activities is a powerful approach that enhances students' understanding and engagement. Keeping this in mind, Ms. Mukta planned a prime number game for class 5 students. She announces the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to the second student. The second student also multiplied it by a prime number and passed it to the third student. In this way, by multiplying to a prime number, the last student got 173250.
Now, Mukta asked some questions as given below to the students:

(i) What is the least prime number used by students?

(ii) (a) How many students are in the class?
OR
(b) What is the highest prime number used by students?

(iii) Which prime number has been used maximum times?

—

Solution:

The given number is 173250. Perform the prime factorization of 173250:

$$173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11.$$

1. (i) Least Prime Number:
The least prime number used by students is 3.
—
2. (ii)(a) Number of Students in the Class:
Each student multiplies the number by one prime number. The total prime factors used are:

$$2 + 3 + 2 + 1 + 1 = 7.$$

Thus, the total number of students is 7.
—
2. (ii)(b) Highest Prime Number:
The highest prime number in the factorization is 11.
—
3. (iii) Prime Number Used Maximum Times:
The prime number 5 appears 3 times, which is the maximum frequency.
—
Conclusion:

(i) The least prime number used is 3.

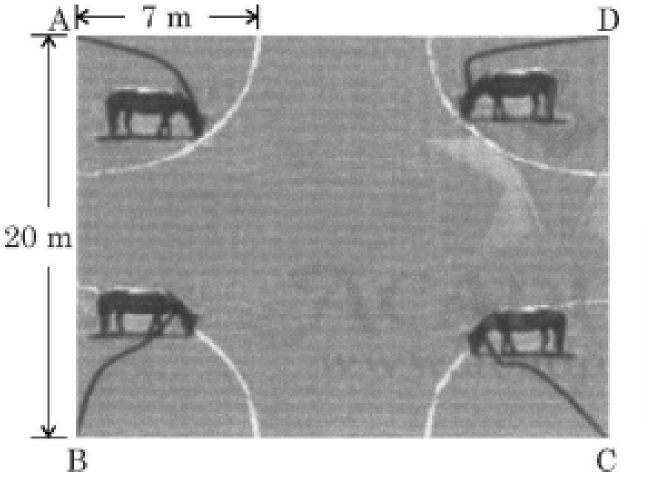
(ii)(a) The total number of students in the class is 7.
(b) The highest prime number used is 11.

(iii) The prime number used maximum times is 5.

Quick Tip

For prime factorization problems, break the number into its smallest prime factors to analyze patterns or answer related queries.

Question 37: A stable owner has four horses. He usually ties these horses with 7 m long rope to pegs at each corner of a square-shaped grass field of 20 m length to graze in his farm. But tying with rope sometimes results in injuries to his horses, so he decided to build fenced around the area so that each horse can graze.



Based on the above, answer the following questions:

(i) Find the area of the square-shaped grass field.

(ii) (a) Find the area of the total field in which these horses can graze.
OR
(b) If the length of the rope of each horse is increased from 7 m to 10 m, find the area grazed by one horse. (Use $\pi = 3.14$).

(iii) What is the area of the field that is left ungrazed, if the length of the rope of each horse is 7 m?

—

Solution:

1. Area of the square-shaped grass field:

$$\text{Side of the square} = 20\,\text{m}.$$
$$\text{Area of the square field} = \text{side}^2 = 20 \times 20 = 400\,\text{m}^2.$$

—

2. (ii)(a) Area of the total field grazed by the horses:
Each horse grazes a quarter-circle area (due to the rope length forming a circular section). The area grazed by one horse is:

$$\text{Area of one horse's grazing region} = \frac{1}{4}\pi r^2.$$

Here, $r = 7\,\text{m}$, and $\pi = \frac{22}{7}$:

$$\text{Area of one grazing region} = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{4} \times 154 = 38.5\,\text{m}^2.$$

For four horses:
$$\text{Total grazing area} = 4 \times 38.5 = 154\,\text{m}^2.$$

—

2. (ii)(b) If the rope length is increased to 10 m:

For one horse:

$$\text{Area grazed by one horse} = \frac{1}{4}\pi r^2, \quad r = 10\,\text{m},\ \pi = 3.14.$$

$$\text{Area grazed by one horse} = \frac{1}{4} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5\,\text{m}^2.$$

—

3. (iii) Area of the field left ungrazed:

$$\text{Area left ungrazed} = \text{Area of square field} - \text{Area grazed by all horses}.$$

Substitute:

$$\text{Area left ungrazed} = 400 - 0.0154 = 399.9846\,\text{m}^2.$$

—

Conclusion:

(i) The area of the square-shaped grass field is $400\,\text{m}^2$.

(ii)(a) The total grazing area for all horses is $154\,\text{m}^2$.

(ii)(b) If the rope length is increased to 10 m, the grazing area for one horse is $78.5\,\text{m}^2$.

(iii) The area left ungrazed is $399.9846\,\text{m}^2$.

Quick Tip

For problems involving grazing regions or circular segments, use the formula for the area of a sector or fractional part of a circle: $\frac{\theta}{360^\circ}\pi r^2$.

Question 38: Vocational training complements traditional education by providing practical skills and hands-on experience. While education equips individuals with a broad knowledge base, vocational training focuses on job-specific skills, enhancing employability thus making the student self-reliant. Keeping this in view, a teacher made the following table giving the frequency distribution of students/adults undergoing vocational training from the training institute.

—

Table: Frequency Distribution of Participants

Age (in years)	Number of Participants
$15-19$	62
$20-24$	132
$25-29$	96
$30-34$	37
$35-39$	13
$40-44$	11
$45-49$	10
$50-54$	4

—

Questions:

(i) What is the lower limit of the modal class of the above data? 1

(ii) (a) Find the median class of the above data. 2
OR
(b) Find the number of participants of age less than 50 years who undergo vocational training. 2

(iii) Give the empirical relationship between mean, median and mode. 1

—

Solution:

1. (i) Lower Limit of the Modal Class:
The modal class is the class with the highest frequency. From the table, the highest frequency is 132, corresponding to the class $20-24$.

$$\text{Lower limit of the modal class} = 20.$$

—

2. (ii)(a) Median Class:
The cumulative frequency (CF) is calculated as follows:

Age (in years)	Frequency (f)	Cumulative Frequency (CF)
$15-19$	62	62
$20-24$	132	194
$25-29$	96	290
$30-34$	37	327
$35-39$	13	340
$40-44$	11	351
$45-49$	10	361
$50-54$	4	365

The total number of participants is $N = 365$. The median class corresponds to $\frac{N}{2} = \frac{365}{2} = 182.5$, which lies in the cumulative frequency 194. Therefore, the median class is $20-24$.
—

2. (ii)(b) Number of participants below 50 years:
Participants below 50 years correspond to the classes $15 - 19$ to $45 - 49$.

$$\text{Total frequency below 50 years} = 62 + 132 + 96 + 37 + 13 + 11 + 10 = 361.$$

—
3. (iii) Empirical Relationship:
The empirical relationship between mean, median, and mode is given by:

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean}).$$

—
Conclusion:

(i) The lower limit of the modal class is 20.

(ii)(a) The median class is $20 - 24$.
(b) The number of participants below 50 years is 361.

(iii) The empirical relationship is $\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$.

Quick Tip

When solving grouped frequency problems, compute the cumulative frequency to locate the median class and sum the required frequencies for other queries.
