

CBSE Class X Mathematics (Standard) Set 3 (30/3/3) Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections** – Section A, B, C, D, and E.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice**. However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed**.

Section - A

This section consists of 20 questions of 1 mark each.

Question 1: A bag contains 3 red balls, 5 white balls, and 7 black balls. The probability that a ball drawn from the bag at random will be neither red nor black is:

- (A) $\frac{1}{3}$
- (B) $\frac{1}{5}$
- (C) $\frac{7}{15}$
- (D) $\frac{8}{15}$

Correct Answer: (A) $\frac{1}{3}$

Solution:

The total number of balls is:

$$3 + 5 + 7 = 15.$$

The number of balls that are neither red nor black (white balls) is:

$$5.$$

The probability is:

$$P(\text{neither red nor black}) = \frac{\text{Number of white balls}}{\text{Total number of balls}} = \frac{5}{15} = \frac{1}{3}.$$

Conclusion:

The probability that the ball will be neither red nor black is $\frac{1}{3}$.

Quick Tip

To calculate probabilities, divide the favorable outcomes by the total outcomes and simplify.

Question 2: The probability of getting a bad egg in a lot of 400 eggs is 0.045. The number of good eggs in the lot is:

- (A) 18
- (B) 180
- (C) 382
- (D) 220

Correct Answer: (C) 382

Solution:

The probability of getting a bad egg is:

$$P(\text{bad egg}) = 0.045.$$

The total number of eggs is 400, so the number of bad eggs is:

$$\text{Number of bad eggs} = 0.045 \times 400 = 18.$$

The number of good eggs is:

$$\text{Number of good eggs} = 400 - 18 = 382.$$

Conclusion:

The number of good eggs in the lot is 382.

Quick Tip

For probability problems, multiply the total outcomes by the given probability to find specific counts.

Question 3: Perimeter of a sector of a circle whose central angle is 90° and radius 7 cm is:

- (A) 35 cm
- (B) 11 cm
- (C) 22 cm
- (D) 25 cm

Correct Answer: (D) 25 cm

Solution:

The perimeter of a sector of a circle is the sum of: 1. The length of the arc, and 2. Twice the radius of the circle.

The formula for the arc length is:

$$\text{Arc length} = 2\pi r \cdot \frac{\theta}{360^\circ}.$$

Here:

$$r = 7 \text{ cm}, \quad \theta = 90^\circ.$$

Substitute the values:

$$\text{Arc length} = 2 \cdot \frac{22}{7} \cdot 7 \cdot \frac{90}{360}.$$

Simplify step-by-step:

$$\text{Arc length} = \frac{2 \cdot 22 \cdot 7 \cdot 90}{7 \cdot 360} = \frac{22 \cdot 90}{180} = 11 \text{ cm}.$$

Now, calculate the perimeter:

$$\text{Perimeter} = \text{Arc length} + 2r = 11 + 2 \cdot 7 = 11 + 14 = 25 \text{ cm}.$$

Conclusion:

The perimeter of the sector is 25 cm.

Quick Tip

To calculate the perimeter of a sector, add the arc length to twice the radius.

Question 4: A pair of irrational numbers whose product is a rational number is:

- (A) $(\sqrt{16}, \sqrt{4})$
- (B) $(\sqrt{5}, \sqrt{2})$

- (C) $(\sqrt{3}, \sqrt{27})$
(D) $(\sqrt{36}, \sqrt{2})$

Correct Answer: (C) $(\sqrt{3}, \sqrt{27})$

Solution:

The product of two irrational numbers can be rational if their product simplifies to a perfect square or rational number.

Here:

$$\sqrt{3} \cdot \sqrt{27} = \sqrt{3 \times 27} = \sqrt{81} = 9,$$

which is a rational number.

For the other pairs: - $(\sqrt{16}, \sqrt{4})$ simplifies to $\sqrt{64}$, but $\sqrt{16}$ and $\sqrt{4}$ are not irrational. - $(\sqrt{5}, \sqrt{2})$ results in $\sqrt{10}$, which is irrational. - $(\sqrt{36}, \sqrt{2})$ results in $\sqrt{72}$, which is also irrational.

Conclusion:

The correct pair is $(\sqrt{3}, \sqrt{27})$.

Quick Tip

The product of two irrational numbers is rational if the resulting value simplifies to a perfect square or a rational number.

Question 5: Maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is:

- (A) 4
(B) 3
(C) 2
(D) 1

Correct Answer: (C) 2

Solution:

When two circles intersect at two distinct points, the following cases arise:

- **Direct tangents:** These are tangents that do not pass through the region of intersection. There are 2 such tangents.

- **Common internal tangents:** These are tangents that pass through the region of intersection. In this case, there are no internal tangents, as the circles intersect.

Thus, the maximum number of common tangents is:

$$2 \text{ (direct tangents only).}$$

Conclusion:

The maximum number of common tangents for two circles intersecting at two distinct points is 2.

Quick Tip

The number of common tangents between two circles depends on their relative positions (disjoint, intersecting, or touching).

Question 6: From a point on the ground, which is 30 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is found to be 60° . The height (in metres) of the tower is:

- (A) $10\sqrt{3}$
- (B) $30\sqrt{3}$
- (C) 60
- (D) 30

Correct Answer: (B) $30\sqrt{3}$

Solution:

Let the height of the tower be h . Using trigonometry, we have:

$$\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}}.$$

Here, $\theta = 60^\circ$, and the adjacent side is 30 m. Substitute:

$$\tan 60^\circ = \frac{h}{30}.$$

The value of $\tan 60^\circ$ is $\sqrt{3}$:

$$\sqrt{3} = \frac{h}{30}.$$

Solve for h :

$$h = 30\sqrt{3}.$$

Conclusion:

The height of the tower is $30\sqrt{3}$ m.

Quick Tip

In problems involving angles of elevation, use trigonometric ratios like $\tan \theta$ to find unknown heights or distances.

Question 7: Given $\text{HCF}(2520, 6600) = 40$, $\text{LCM}(2520, 6600) = 252 \times k$, then the value of k is:

- (A) 1650
- (B) 1600
- (C) 165
- (D) 1625

Correct Answer: (A) 1650

Solution:

The product of HCF and LCM of two numbers is equal to the product of the numbers. This can be expressed as:

$$\text{HCF} \times \text{LCM} = \text{Number 1} \times \text{Number 2}.$$

Substitute the given values:

$$40 \times (252 \times k) = 2520 \times 6600.$$

Simplify:

$$252 \times k = \frac{2520 \times 6600}{40}.$$

Calculate:

$$252 \times k = 415800.$$

Solve for k :

$$k = \frac{415800}{252} = 1650.$$

Conclusion:

The value of k is 1650.

Quick Tip

For any two numbers, the product of their HCF and LCM equals the product of the numbers.

Question 8: The mean of five numbers is 15. If we include one more number, the mean of six numbers becomes 17. The included number is:

- (A) 27
- (B) 37
- (C) 17
- (D) 25

Correct Answer: (A) 27

Solution:

The total of five numbers is given by:

$$\text{Total of 5 numbers} = \text{Mean} \times \text{Count} = 15 \times 5 = 75.$$

When one more number is included, the mean becomes 17. Therefore, the total of six numbers is:

$$\text{Total of 6 numbers} = \text{Mean} \times \text{Count} = 17 \times 6 = 102.$$

The included number is:

$$\text{Included number} = \text{Total of 6 numbers} - \text{Total of 5 numbers} = 102 - 75 = 27.$$

Conclusion:

The included number is 27.

Quick Tip

For mean-related problems, calculate the total sum using $\text{Total} = \text{Mean} \times \text{Count}$. Use this relationship to find missing values.

Question 9: The pair of linear equations $x + 2y + 5 = 0$ and $-3x = 6y - 1$ has

- (A) unique solution
- (B) exactly two solutions
- (C) infinitely many solutions
- (D) no solution

Correct Answer: (D) no solution

Solution:

The given pair of equations are:

$$1. x + 2y + 5 = 0 \quad \text{or} \quad x + 2y = -5,$$

$$2. -3x = 6y - 1 \quad \text{or} \quad 3x + 6y = 1.$$

Convert both equations into the standard form $ax + by + c = 0$:

$$1. x + 2y + 5 = 0,$$

$$2. 3x + 6y - 1 = 0.$$

Find the ratios of coefficients:

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{5}{-1} = -5.$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel and do not intersect. Hence, the system of equations has no solution.

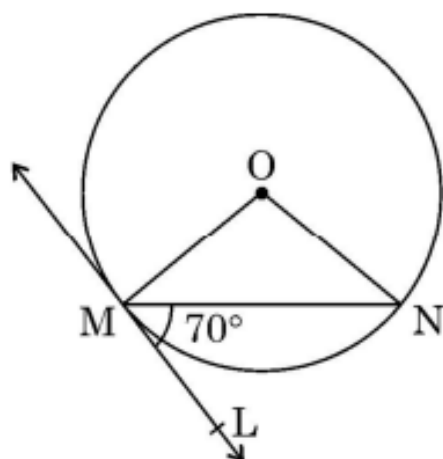
Conclusion:

The given pair of equations is inconsistent, and there is no solution.

Quick Tip

For a pair of linear equations, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel, and there is no solution.

Question 10: In the given figure, O is the centre of the circle. MN is the chord and the tangent ML at point M makes an angle of 70° with MN . The measure of $\angle MON$ is:



- (A) 120°
- (B) 140°
- (C) 70°
- (D) 90°

Correct Answer: (B) 140°

Solution:

The angle between a tangent and a chord drawn at the point of tangency is equal to the angle subtended by the chord at the centre of the circle.

From the figure:

$$\angle TML = \angle MON.$$

Here:

$$\angle TML = 70^\circ.$$

Since $\angle MON$ is subtended at the centre of the circle, it is twice the angle at the tangent:

$$\angle MON = 2 \cdot \angle TML = 2 \cdot 70^\circ = 140^\circ.$$

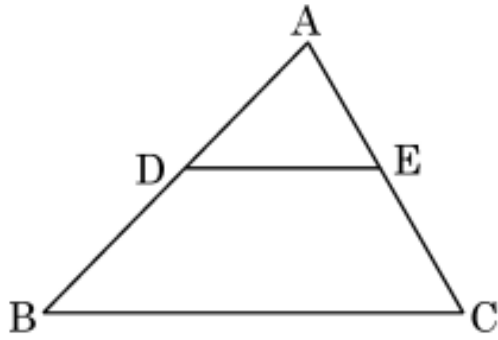
Conclusion:

The measure of $\angle MON$ is 140° .

Quick Tip

The angle subtended by a chord at the centre of a circle is twice the angle subtended at the tangent.

Question 11: In $\triangle ABC$, $DE \parallel BC$ (as shown in the figure). If $AD = 2$ cm, $BD = 3$ cm, $BC = 7.5$ cm, then the length of DE (in cm) is:



- (A) 2.5
- (B) 3
- (C) 5
- (D) 6

Correct Answer: (B) 3

Solution:

By the Basic Proportionality Theorem (Thales' theorem), if a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally. Hence:

$$\frac{AD}{AB} = \frac{DE}{BC}.$$

First, find AB (the total length of side AB):

$$AB = AD + BD = 2 + 3 = 5 \text{ cm.}$$

Now substitute the known values:

$$\frac{AD}{AB} = \frac{DE}{BC}.$$

$$\frac{2}{5} = \frac{DE}{7.5}.$$

Solve for DE :

$$DE = \frac{2}{5} \times 7.5 = 3 \text{ cm.}$$

Conclusion:

The length of DE is 3 cm.

Quick Tip

The Basic Proportionality Theorem states that if a line is parallel to one side of a triangle, it divides the other sides proportionally.

Question 12: Two dice are thrown together. The probability that they show different numbers is:

- (A) $\frac{1}{6}$
- (B) $\frac{5}{6}$

- (C) $\frac{1}{3}$
 (D) $\frac{2}{3}$

Correct Answer: (B) $\frac{5}{6}$

Solution:

When two dice are thrown, the total number of outcomes is:

$$6 \times 6 = 36.$$

For the two dice to show the same number (e.g., 1, 1, 2, 2, ..., 6, 6), there are 6 favorable outcomes. The probability of the same number is:

$$P(\text{same}) = \frac{6}{36} = \frac{1}{6}.$$

The probability of showing different numbers is the complement:

$$P(\text{different}) = 1 - P(\text{same}) = 1 - \frac{1}{6} = \frac{5}{6}.$$

—

Conclusion:

The probability that the two dice show different numbers is $\frac{5}{6}$.

Quick Tip

The complement rule states that $P(A') = 1 - P(A)$. Use it for quick probability calculations.

Question 13: If $\sin \alpha = \frac{\sqrt{3}}{2}$, $\cos \beta = \frac{\sqrt{3}}{2}$, then $\tan \alpha \cdot \tan \beta$ is:

- (A) $\sqrt{3}$
 (B) $\frac{1}{\sqrt{3}}$
 (C) 1
 (D) 0

Correct Answer: (C) 1

Solution:

1. For $\sin \alpha = \frac{\sqrt{3}}{2}$:

$$\alpha = 60^\circ \implies \tan \alpha = \sqrt{3}.$$

2. For $\cos \beta = \frac{\sqrt{3}}{2}$:

$$\beta = 30^\circ \implies \tan \beta = \frac{1}{\sqrt{3}}.$$

3. Multiply $\tan \alpha$ and $\tan \beta$:

$$\tan \alpha \cdot \tan \beta = \sqrt{3} \cdot \frac{1}{\sqrt{3}} = 1.$$

—

Conclusion:

The value of $\tan \alpha \cdot \tan \beta$ is 1.

Quick Tip

Remember the trigonometric values for standard angles 30° , 45° , and 60° . They simplify calculations involving sine, cosine, and tangent.

Question 14: What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial?

- (A) 1
- (B) 2
- (C) 4
- (D) 5

Correct Answer: (B) 2

Solution:

For 3 to be a zero of the resulting polynomial, substituting $x = 3$ in the polynomial $x^2 - 5x + 4 + k$ should give:

$$3^2 - 5(3) + 4 + k = 0.$$

Simplify:

$$9 - 15 + 4 + k = 0 \quad \implies \quad -2 + k = 0 \quad \implies \quad k = 2.$$

Conclusion:

The value that should be added to the polynomial is 2.

Quick Tip

To make a specific number a zero of a polynomial, substitute it into the polynomial and solve for the unknown constant.

Question 15: The smallest irrational number by which $\sqrt{20}$ should be multiplied so as to get a rational number is:

- (A) $\sqrt{20}$
- (B) $\sqrt{2}$
- (C) 5
- (D) $\sqrt{5}$

Correct Answer: (D) $\sqrt{5}$

Solution:

To make $\sqrt{20}$ a rational number, it should be multiplied by an irrational number that removes

the square root. The simplest way is to simplify:

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}.$$

Multiplying $\sqrt{20}$ by $\sqrt{5}$:

$$\sqrt{20} \times \sqrt{5} = 2\sqrt{5} \times \sqrt{5} = 2 \times 5 = 10.$$

Thus, $\sqrt{5}$ is the smallest irrational number that makes $\sqrt{20}$ rational.

Conclusion:

The smallest irrational number is $\sqrt{5}$.

Quick Tip

For square root simplifications, multiply by a term that completes the square under the radical to eliminate it.

Question 16: If the diagonals of a quadrilateral divide each other proportionally, then it is a:

- (A) parallelogram
- (B) rectangle
- (C) square
- (D) trapezium

Correct Answer: (D) trapezium

Solution:

For a quadrilateral: - If the diagonals divide each other proportionally, it is a **trapezium**.

- This property arises because the triangles formed by the diagonals are similar, which is a characteristic feature of trapeziums.

In other types of quadrilaterals like parallelograms, rectangles, or squares, the diagonals either bisect each other or have specific geometric constraints, but they do not divide each other proportionally.

Conclusion:

If the diagonals of a quadrilateral divide each other proportionally, it is a trapezium.

Quick Tip

A trapezium's diagonals divide each other proportionally because it forms pairs of similar triangles.

Question 17: The common difference of the A.P.

$$\frac{1}{2x}, \frac{1-4x}{2x}, \frac{1-8x}{2x}, \dots \text{ is:}$$

- (A) $-2x$
- (B) -2
- (C) 2
- (D) $2x$

Correct Answer: (B) -2

Solution:

The terms of the A.P. are:

$$T_1 = \frac{1}{2x}, \quad T_2 = \frac{1-4x}{2x}, \quad T_3 = \frac{1-8x}{2x}.$$

The common difference d is:

$$d = T_2 - T_1 = \frac{1-4x}{2x} - \frac{1}{2x}.$$

Combine terms:

$$d = \frac{(1-4x) - 1}{2x} = \frac{-4x}{2x} = -2.$$

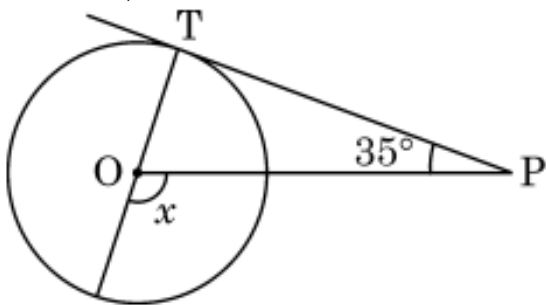
Conclusion:

The common difference of the A.P. is -2 .

Quick Tip

To find the common difference of an A.P., subtract any term from the preceding term.

Question 18: In the given figure, if PT is a tangent to a circle with centre O and $\angle TPO = 35^\circ$, then the measure of $\angle x$ is:



- (A) 110°
- (B) 115°
- (C) 120°
- (D) 125°

Correct Answer: (D) 125°

Solution:

In the given figure: - PT is a tangent to the circle at T . - $\angle TPO = 35^\circ$.

The angle $\angle x$ is an exterior angle of the triangle $\triangle OPT$. The exterior angle of a triangle is equal to the sum of the two opposite interior angles. Thus:

$$\angle x = \angle O + \angle TPO.$$

Since $\angle O = 90^\circ$ (radius OT is perpendicular to the tangent PT):

$$\angle x = 90^\circ + 35^\circ = 125^\circ.$$

Conclusion:

The measure of $\angle x$ is 125° .

Quick Tip

For a tangent to a circle, the angle between the radius and the tangent is always 90° .

Directions: Questions number 19 and 20 are Assertion and Reason-based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below:

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

Question 19: Assertion (A): The point which divides the line segment joining the points $A(1, 2)$ and $B(-1, 1)$ internally in the ratio $1 : 2$ is $(-\frac{1}{3}, \frac{5}{3})$.

Reason (R): The coordinates of the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1 : m_2$ are:

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right).$$

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:

To find the point dividing the line segment joining $A(1, 2)$ and $B(-1, 1)$ in the ratio $1 : 2$, we use the formula:

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right).$$

Substitute $m_1 = 1, m_2 = 2, A(1, 2), B(-1, 1)$:

$$x = \frac{(1)(-1) + (2)(1)}{1 + 2} = \frac{-1 + 2}{3} = \frac{1}{3}, \quad y = \frac{(1)(1) + (2)(2)}{1 + 2} = \frac{1 + 4}{3} = \frac{5}{3}.$$

The point is $(\frac{1}{3}, \frac{5}{3})$, not $(-\frac{1}{3}, \frac{5}{3})$.

Therefore, Assertion (A) is false, but Reason (R) is correct.

Quick Tip

When dividing a line segment in a ratio, use the section formula:

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right).$$

Make sure the ratio and coordinates are substituted correctly.

Question 20: Assertion (A): In a cricket match, a batsman hits a boundary 9 times out of 45 balls he plays. The probability that in a given ball, he does not hit the boundary is $\frac{4}{5}$.

Reason (R): $P(E) + P(\text{not } E) = 1$.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

The probability of hitting a boundary is:

$$P(E) = \frac{\text{Number of times boundary is hit}}{\text{Total number of balls}} = \frac{9}{45} = \frac{1}{5}.$$

The probability of not hitting a boundary is:

$$P(\text{not } E) = 1 - P(E) = 1 - \frac{1}{5} = \frac{4}{5}.$$

The Reason (R) correctly states that $P(E) + P(\text{not } E) = 1$, which is used to calculate $P(\text{not } E)$. Therefore, both Assertion (A) and Reason (R) are true, and Reason (R) explains Assertion (A).

Quick Tip

For probabilities, the sum of the probability of an event and its complement is always 1:

$$P(E) + P(\text{not } E) = 1.$$

Use this rule to find the complement probability quickly.

Section - B

This section consists of 5 questions of 2 mark each.

Question 21(a): If $2x + y = 13$ and $4x - y = 17$, find the value of $(x - y)$.

Solution:

The given equations are:

$$2x + y = 13 \quad (\text{i}),$$

$$4x - y = 17 \quad (\text{ii}).$$

Add equations (i) and (ii) to eliminate y :

$$(2x + y) + (4x - y) = 13 + 17.$$

Simplify:

$$6x = 30 \quad \implies \quad x = 5.$$

Substitute $x = 5$ into equation (i):

$$2(5) + y = 13 \quad \implies \quad 10 + y = 13 \quad \implies \quad y = 3.$$

The value of $x - y$ is:

$$x - y = 5 - 3 = 2.$$

—

Conclusion:

The value of $x - y$ is 2.

Quick Tip

For solving linear equations, try eliminating one variable by adding or subtracting equations, then solve for the other variable.

Question 21(b): Sum of two numbers is 105 and their difference is 45. Find the numbers.

Solution:

Let the two numbers be x and y , where $x > y$.

The given equations are:

$$x + y = 105 \quad (\text{i}),$$

$$x - y = 45 \quad (\text{ii}).$$

Add equations (i) and (ii) to eliminate y :

$$(x + y) + (x - y) = 105 + 45.$$

Simplify:

$$2x = 150 \quad \implies \quad x = 75.$$

Substitute $x = 75$ into equation (i):

$$75 + y = 105 \quad \implies \quad y = 105 - 75 = 30.$$

Thus, the two numbers are:

$$x = 75, y = 30.$$

—

Conclusion:

The numbers are 75 and 30.

Quick Tip

To solve problems involving the sum and difference of two numbers, add and subtract the equations to eliminate one variable and solve for the other.

Question 22: Evaluate:

$$\frac{5 \tan 60^\circ}{(\sin^2 60^\circ + \cos^2 60^\circ) \tan 30^\circ}.$$

Correct Answer: 20**Solution:**

Step 1: Simplify the denominator. Using the Pythagorean identity:

$$\sin^2 60^\circ + \cos^2 60^\circ = 1.$$

Step 2: Substitute values of $\tan 60^\circ$ and $\tan 30^\circ$:

$$\tan 60^\circ = \sqrt{3}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

Step 3: Substitute into the expression:

$$\frac{5 \tan 60^\circ}{(\sin^2 60^\circ + \cos^2 60^\circ) \tan 30^\circ} = \frac{5 \cdot \sqrt{3}}{1 \cdot \frac{1}{\sqrt{3}}}.$$

Step 4: Simplify the fraction:

$$\frac{5 \cdot \sqrt{3}}{\frac{1}{\sqrt{3}}} = 5 \cdot \sqrt{3} \cdot \sqrt{3} = 5 \cdot 3 = 15.$$

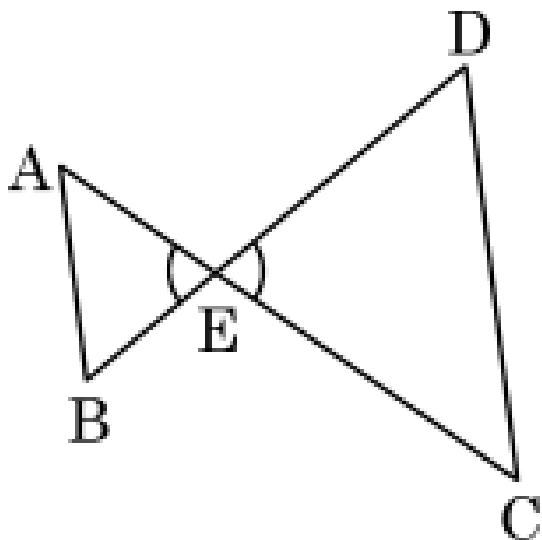
Conclusion:

The value of the expression is 15.

Quick Tip

For trigonometric simplifications, use the Pythagorean identities $\sin^2 x + \cos^2 x = 1$ and standard angle values for quick calculations.

Question 23: In the given figure, $\frac{EA}{EC} = \frac{EB}{ED}$, prove that $\triangle EAB \sim \triangle ECD$.



Solution:

In $\triangle EAB$ and $\triangle ECD$, we are given:

$$\frac{EA}{EC} = \frac{EB}{ED}.$$

Also, $\angle AEB = \angle CED$ (vertically opposite angles).

By the Side-Angle-Side (SAS) similarity criterion:

$$\triangle EAB \sim \triangle ECD.$$

Conclusion:

It is proven that $\triangle EAB \sim \triangle ECD$.

Quick Tip

For two triangles to be similar by SAS similarity, the ratios of two corresponding sides must be equal, and the included angles must be equal.

Question 24: One card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that the card drawn:

- (i) is the queen of hearts;
- (ii) is not a jack.

Solution:

Total number of outcomes = 52.

- (i) The probability of drawing the queen of hearts is:

$$P(\text{queen of hearts}) = \frac{1}{52}.$$

- (ii) There are 4 jacks in the deck, so the number of cards that are not jacks is:

$$52 - 4 = 48.$$

The probability of not drawing a jack is:

$$P(\text{not a jack}) = \frac{48}{52} = \frac{12}{13}.$$

Conclusion:

(i) $P(\text{queen of hearts}) = \frac{1}{52}.$

(ii) $P(\text{not a jack}) = \frac{12}{13}.$

Quick Tip

In probability, $P(\text{not } A) = 1 - P(A)$. For cards, ensure you account for the total number of specific cards (e.g., 4 jacks, 4 queens).

Question 25(a): Find a relation between x and y such that the point $P(x, y)$ is equidistant from the points $A(7, 1)$ and $B(3, 5)$.

Solution:

The condition for equidistance is:

$$PA = PB \implies PA^2 = PB^2.$$

Substitute the coordinates:

$$(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2.$$

Expand both sides:

$$(x^2 - 14x + 49) + (y^2 - 2y + 1) = (x^2 - 6x + 9) + (y^2 - 10y + 25).$$

Simplify:

$$-14x - 2y + 50 = -6x - 10y + 34.$$

Rearrange:

$$-8x + 8y + 16 = 0 \implies -8(x - y - 2) = 0.$$

Therefore, the relation is:

$$x - y - 2 = 0.$$

Conclusion:

The required relation between x and y is:

$$x - y - 2 = 0.$$

Quick Tip

To find the relation for equidistant points, equate the squared distance from each point and simplify the equation.

Question 25(b): Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$ such that AB is a diameter of the circle. Find the value of y . Also, find the radius of the circle.

Solution:

The centre $O(2, -3y)$ is the midpoint of AB . Using the midpoint formula:

$$\left(\frac{-1 + 5}{2}, \frac{y + 7}{2} \right) = (2, -3y).$$

Equating the x -coordinates:

$$\frac{-1 + 5}{2} = 2 \quad \implies \quad \frac{4}{2} = 2 \quad (\text{satisfied}).$$

Equating the y -coordinates:

$$\frac{y + 7}{2} = -3y.$$

Simplify:

$$y + 7 = -6y \quad \implies \quad 7y = -7 \quad \implies \quad y = -1.$$

Now calculate the radius of the circle. Since AB is the diameter, the radius is half the length of AB .

The distance AB is:

$$AB = \sqrt{(5 - (-1))^2 + (7 - (-1))^2}.$$

Simplify:

$$AB = \sqrt{(5 + 1)^2 + (7 + 1)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10.$$

The radius is:

$$\text{Radius} = \frac{AB}{2} = \frac{10}{2} = 5.$$

Conclusion:

The value of y is -1 , and the radius of the circle is 5 .

Quick Tip

When the diameter is given, the radius is half its length. Use the distance formula to calculate the diameter and divide by 2 for the radius.

Section - C

This section consists of 6 questions of 3 mark each.

Question 26(a): Find the ratio in which the line segment joining the points $(5, 3)$ and $(-1, 6)$ is divided by the Y -axis.

Solution:

Let the line segment divide the Y-axis at the point $(0, y)$.

Let the required ratio be $k : 1$. Using the section formula, the x -coordinate of the point dividing the line segment is:

$$x = \frac{k(-1) + 1(5)}{k + 1}.$$

Since the point lies on the Y-axis, $x = 0$. Substitute $x = 0$:

$$\frac{k(-1) + 5}{k + 1} = 0.$$

Simplify:

$$-k + 5 = 0 \quad \implies \quad k = 5.$$

Hence, the required ratio is:

$$5 : 1.$$

Conclusion:

The line segment is divided by the Y-axis in the ratio $5 : 1$.

Quick Tip

For a line segment divided by an axis, use the section formula and set the relevant coordinate (x for the Y-axis, y for the X-axis) to 0.

Question 26(b): $P(-2, 5)$ and $Q(3, 2)$ are two points. Find the coordinates of the point R on line segment PQ such that $PR = 2QR$.

Solution:

Let the coordinates of R be (x, y) .

Given:

$$PR : RQ = 2 : 1.$$

Using the section formula:

$$x = \frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \quad y = \frac{m_2y_1 + m_1y_2}{m_1 + m_2}.$$

Here:

$$m_1 = 2, m_2 = 1, P(-2, 5), Q(3, 2).$$

Substitute into the formulas for x and y :

$$x = \frac{1(-2) + 2(3)}{2 + 1} = \frac{-2 + 6}{3} = \frac{4}{3}.$$

$$y = \frac{1(5) + 2(2)}{2 + 1} = \frac{5 + 4}{3} = 3.$$

Thus, the coordinates of R are:

$$\left(\frac{4}{3}, 3\right).$$

Conclusion:

The coordinates of the point R are:

$$\left(\frac{4}{3}, 3\right).$$

Quick Tip

Use the section formula for dividing a line segment in a given ratio:

$$\left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right).$$

Ensure the correct substitution of ratios and coordinates.

Question 27(a): If the sum of the first 7 terms of an A.P. is 49 and that of the first 17 terms is 289, find the sum of its first 20 terms.

Solution:

Let a be the first term and d be the common difference.

The sum of n terms of an A.P. is given by:

$$S_n = \frac{n}{2}[2a + (n - 1)d].$$

For the first 7 terms ($S_7 = 49$):

$$\frac{7}{2}[2a + 6d] = 49.$$

Simplify:

$$7a + 21d = 49 \implies a + 3d = 7 \quad (\text{i}).$$

For the first 17 terms ($S_{17} = 289$):

$$\frac{17}{2}[2a + 16d] = 289.$$

Simplify:

$$17a + 136d = 289 \implies a + 8d = 17 \quad (\text{ii}).$$

Solve equations (i) and (ii):

$$\begin{aligned} a + 3d &= 7, \\ a + 8d &= 17. \end{aligned}$$

Subtract (i) from (ii):

$$(8d - 3d) = (17 - 7) \implies 5d = 10 \implies d = 2.$$

Substitute $d = 2$ into (i):

$$a + 3(2) = 7 \implies a = 1.$$

Now, find the sum of the first 20 terms (S_{20}):

$$S_{20} = \frac{20}{2}[2a + 19d].$$

Substitute $a = 1$ and $d = 2$:

$$S_{20} = 10[2(1) + 19(2)] = 10[2 + 38] = 10 \cdot 40 = 400.$$

Conclusion:

The sum of the first 20 terms of the A.P. is 400.

Quick Tip

For sums of A.P., use the formula $S_n = \frac{n}{2}[2a + (n-1)d]$ and solve equations systematically to find a and d .

Question 27(b): The ratio of the 10th term to its 30th term of an A.P. is $1 : 3$, and the sum of its first six terms is 42. Find the first term and the common difference of the A.P.

Solution:

Let a be the first term and d be the common difference.

The general term of an A.P. is given by:

$$T_n = a + (n - 1)d.$$

For the 10th term:

$$T_{10} = a + 9d.$$

For the 30th term:

$$T_{30} = a + 29d.$$

Given:

$$\frac{T_{10}}{T_{30}} = \frac{1}{3}.$$

Substitute T_{10} and T_{30} :

$$\frac{a + 9d}{a + 29d} = \frac{1}{3}.$$

Cross-multiply:

$$3(a + 9d) = (a + 29d).$$

Simplify:

$$3a + 27d = a + 29d \quad \implies \quad 2a = 2d \quad \implies \quad a = d.$$

The sum of the first 6 terms is given as $S_6 = 42$. The formula for the sum of the first n terms is:

$$S_n = \frac{n}{2}[2a + (n - 1)d].$$

For S_6 :

$$\frac{6}{2}[2a + 5d] = 42.$$

Simplify:

$$3(2a + 5d) = 42 \quad \implies \quad 2a + 5d = 14 \quad (\text{ii}).$$

Substitute $a = d$ into (ii):

$$2d + 5d = 14 \implies 7d = 14 \implies d = 2.$$

Since $a = d$, we have:

$$a = 2.$$

Conclusion:

The first term (a) is 2, and the common difference (d) is 2.

Quick Tip

When ratios of terms are given, equate their formula and simplify. Use the sum formula to find unknowns like a and d .

Question 28: Solve the following system of linear equations graphically:

$$x - y + 1 = 0$$

$$x + y = 5$$

Solution:

The given equations are: 1. $x - y + 1 = 0$, or equivalently $x - y = -1$ (i). 2. $x + y = 5$ (ii).

Step 1: Find points for each line.

For equation (i) ($x - y = -1$):

$$\text{If } x = 0, -y = -1 \implies y = 1.$$

$$\text{If } x = -1, -y = -2 \implies y = 2.$$

$$\text{If } x = 1, -y = 0 \implies y = 0.$$

Thus, the points for $x - y = -1$ are:

$$(0, 1), (-1, 2), (1, 0).$$

For equation (ii) ($x + y = 5$):

$$\text{If } x = 0, y = 5.$$

$$\text{If } x = 5, y = 0.$$

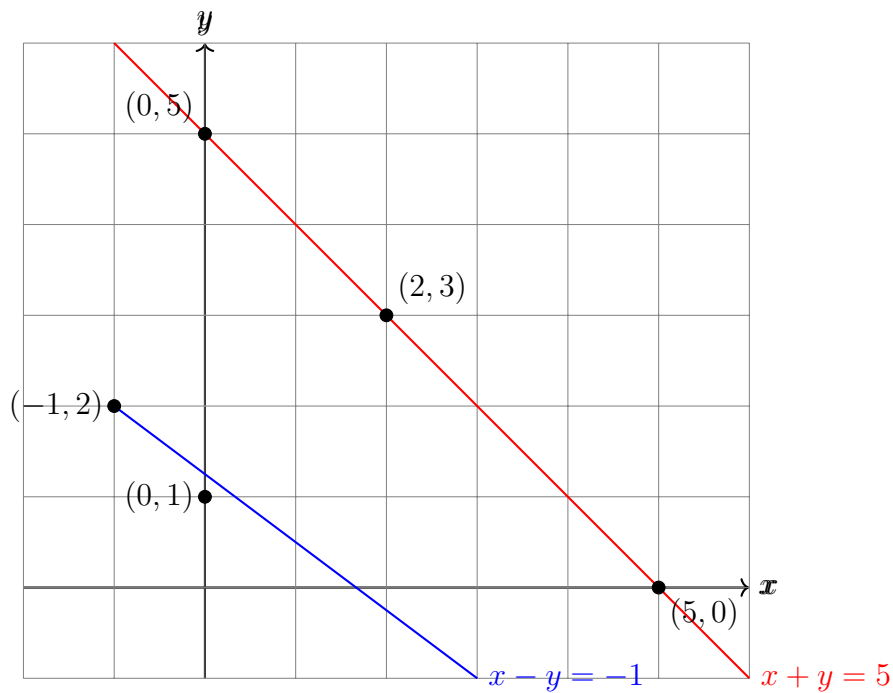
$$\text{If } x = 2, y = 3.$$

Thus, the points for $x + y = 5$ are:

$$(0, 5), (5, 0), (2, 3).$$

Step 2: Plot the lines on the graph.

- Plot the line $x - y = -1$ using the points $(0, 1), (-1, 2), (1, 0)$. - Plot the line $x + y = 5$ using the points $(0, 5), (5, 0), (2, 3)$.



Step 3: Find the point of intersection.

The two lines intersect at the point $(2, 3)$. This is the solution to the system of equations.

Conclusion:

The solution to the given system of equations is:

$$x = 2, y = 3.$$

Quick Tip

To solve linear equations graphically, plot each equation as a straight line and find their point of intersection. This point gives the solution.

Question 29: Find the zeroes of the quadratic polynomial $x^2 - 15$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Solution:

Let $P(x) = x^2 - 15$.

Factorize $P(x)$:

$$P(x) = (x - \sqrt{15})(x + \sqrt{15}).$$

Thus, the zeroes of $P(x)$ are:

$$-\sqrt{15} \quad \text{and} \quad \sqrt{15}.$$

Verification:

1. Sum of zeroes:

$$-\sqrt{15} + \sqrt{15} = 0.$$

Compare with:

$$\text{Sum of zeroes} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = \frac{0}{1} = 0.$$

2. Product of zeroes:

$$(-\sqrt{15}) \times (\sqrt{15}) = -15.$$

Compare with:

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{-15}{1} = -15.$$

Conclusion:

The sum and product of the zeroes are verified to match the relationships:

$$\text{Sum of zeroes} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}, \quad \text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}.$$

Quick Tip

For a quadratic polynomial $ax^2 + bx + c$, the sum of zeroes is $-\frac{b}{a}$ and the product of zeroes is $\frac{c}{a}$.

Question 30: From an external point P , two tangents PA and PB are drawn to the circle with centre O . Prove that OP is the perpendicular bisector of chord AB .

Solution:

1. Given: A circle with centre O , and two tangents PA and PB drawn from an external point P . A and B are points of tangency, and AB is a chord.

2. To Prove: OP is the perpendicular bisector of AB .

3. Proof:

- Join OA , OB , and OP . - In $\triangle OPA$ and $\triangle OPB$: - $OP = OP$ (common side), - $PA = PB$ (tangents from an external point are equal), - $OA = OB$ (radii of the same circle).

Thus, $\triangle OPA \cong \triangle OPB$ (by SSS congruence).

- Since $\triangle OPA \cong \triangle OPB$, $\angle OPA = \angle OPB$.

- Line OP is perpendicular to AB because the tangents PA and PB are symmetrical about OP .

4. Hence, OP is the perpendicular bisector of AB .

Conclusion:

The line OP bisects AB at right angles.

Quick Tip

When two tangents are drawn from an external point, they are equal, and the line joining the external point to the circle's center is always perpendicular to the chord joining the tangency points.

Question 31: Prove that:

$$(\csc \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta) = 1.$$

Solution:

To prove the given identity, simplify step by step:

1. Simplify $\csc \theta - \sin \theta$:

$$\csc \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta}.$$

Using $1 - \sin^2 \theta = \cos^2 \theta$, we get:

$$\csc \theta - \sin \theta = \frac{\cos^2 \theta}{\sin \theta}.$$

2. Simplify $\sec \theta - \cos \theta$:

$$\sec \theta - \cos \theta = \frac{1}{\cos \theta} - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta}.$$

Using $1 - \cos^2 \theta = \sin^2 \theta$, we get:

$$\sec \theta - \cos \theta = \frac{\sin^2 \theta}{\cos \theta}.$$

3. Simplify $\tan \theta + \cot \theta$:

$$\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}.$$

Taking the LCM:

$$\tan \theta + \cot \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}.$$

Using $\sin^2 \theta + \cos^2 \theta = 1$, we get:

$$\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}.$$

4. Multiply the three expressions:

$$(\csc \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta) = \frac{\cos^2 \theta}{\sin \theta} \cdot \frac{\sin^2 \theta}{\cos \theta} \cdot \frac{1}{\sin \theta \cos \theta}.$$

Simplify:

$$= \frac{\cos^2 \theta \sin^2 \theta}{\sin \theta \cos \theta} \cdot \frac{1}{\sin \theta \cos \theta} = \frac{\cos^2 \theta \sin^2 \theta}{\sin^2 \theta \cos^2 \theta} = 1.$$

Conclusion:

The given expression is equal to 1.

Quick Tip

Use trigonometric identities like $1 - \sin^2 \theta = \cos^2 \theta$ and $\sin^2 \theta + \cos^2 \theta = 1$ for simplifications. Simplify each term before multiplying.

Section - D

This section consists of 4 questions of 5 marks each.

Question 32: A man on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60° . Find the time taken by the boat from here to reach the shore.

Solution:

1. Let the height of the cliff be h , the distance of the boat from the shore at the first observation be x , and the distance moved by the boat in 6 minutes be y .

2. From $\triangle QAB$:

$$\tan 60^\circ = \frac{h}{x} \implies h = \sqrt{3}x. \quad (1)$$

3. From $\triangle PAB$:

$$\tan 30^\circ = \frac{h}{y+x} \implies \frac{1}{\sqrt{3}} = \frac{h}{y+x} \implies y+x = \sqrt{3}h. \quad (2)$$

4. Substitute $h = \sqrt{3}x$ from (1) into (2):

$$y+x = \sqrt{3}(\sqrt{3}x) \implies y+x = 3x \implies y = 2x.$$

5. The time taken by the boat to cover distance y is 6 minutes. Since $y = 2x$, the time taken by the boat to travel distance x is:

$$\frac{1}{2} \times 6 = 3 \text{ minutes.}$$

Conclusion:

The time taken by the boat to reach the shore is 3 minutes.

Quick Tip

Use trigonometric ratios like $\tan$ in right triangles to relate height and distances. Substitution simplifies calculations when multiple triangles share common parameters.

Question 33(a): A solid iron pole consists of a solid cylinder of height 200 cm and base diameter 28 cm, which is surmounted by another cylinder of height 50 cm and radius 7 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass.

Solution:

The pole consists of two cylindrical parts. Let us calculate the volume of each cylinder separately and then find the total mass.

—
1. Volume of the lower cylinder:

Height (h_1) = 200 cm,

Radius (r_1) = $\frac{\text{diameter}}{2} = \frac{28}{2} = 14 \text{ cm}$.

The volume of a cylinder is given by:

$$V = \pi r^2 h.$$

For the lower cylinder:

$$V_1 = \pi r_1^2 h_1 = \pi(14)^2(200).$$

Simplify:

$$V_1 = \pi(196)(200) = 39200\pi \text{ cm}^3.$$

—
2. Volume of the upper cylinder:

Height (h_2) = 50 cm,

Radius (r_2) = 7 cm.

For the upper cylinder:

$$V_2 = \pi r_2^2 h_2 = \pi(7)^2(50).$$

Simplify:

$$V_2 = \pi(49)(50) = 2450\pi \text{ cm}^3.$$

—
3. Total volume of the pole:

$$V_{\text{total}} = V_1 + V_2 = 39200\pi + 2450\pi = 41650\pi \text{ cm}^3.$$

Substitute $\pi \approx 3.1416$:

$$V_{\text{total}} = 41650 \times 3.1416 = 130881.55 \text{ cm}^3.$$

—
4. Mass of the pole:

Given that 1 cm^3 of iron has a mass of 8 g:

$$\text{Mass} = V_{\text{total}} \times 8 = 130881.55 \times 8 = 1047052.4 \text{ g}.$$

Convert to kilograms:

$$\text{Mass} = \frac{1047052.4}{1000} = 1047.05 \text{ kg}.$$

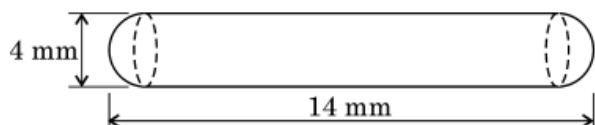
—
Conclusion:

The mass of the iron pole is approximately 1047.05 kg.

Quick Tip

To find the mass of a solid object, calculate its volume using geometric formulas, then multiply by the given density (mass per unit volume).

Question 33(b): A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm, and the diameter of the capsule is 4 mm. Find its surface area and volume.



Solution:

The capsule consists of: 1. A cylindrical part with two hemispheres attached at the ends. 2. Radius (r) of the hemispheres and the cylinder:

$$r = \frac{\text{diameter}}{2} = \frac{4}{2} = 2 \text{ mm.}$$

3. Length of the cylindrical part:

$$\text{Length of cylinder} = \text{Total length of capsule} - 2(\text{Radius of hemispheres}) = 14 - 4 = 10 \text{ mm.}$$

1. Surface Area of the Capsule:

The surface area consists of: 1. Curved surface area (CSA) of the cylinder:

$$\text{CSA of cylinder} = 2\pi rh = 2 \times \frac{22}{7} \times 2 \times 10 = 40 \times \frac{22}{7} = 125.71 \text{ mm}^2.$$

2. CSA of the two hemispheres:

$$\text{CSA of one hemisphere} = 2\pi r^2 = 2 \times \frac{22}{7} \times 2^2 = 2 \times \frac{22}{7} \times 4 = \frac{176}{7} = 25.14 \text{ mm}^2.$$

For two hemispheres:

$$\text{CSA of both hemispheres} = 2 \times 25.14 = 50.28 \text{ mm}^2.$$

Total surface area:

$$\text{Surface Area} = \text{CSA of cylinder} + \text{CSA of hemispheres} = 125.71 + 50.28 = 176 \text{ mm}^2.$$

2. Volume of the Capsule:

The volume consists of: 1. Volume of the cylinder:

$$\text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 2^2 \times 10 = \frac{22}{7} \times 4 \times 10 = \frac{880}{7} = 125.71 \text{ mm}^3.$$

2. Volume of the two hemispheres:

$$\text{Volume of one hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 2^3 = \frac{2}{3} \times \frac{22}{7} \times 8 = \frac{352}{21} = 16.76 \text{ mm}^3.$$

For two hemispheres:

$$\text{Volume of both hemispheres} = 2 \times 16.76 = 33.52 \text{ mm}^3.$$

Total volume:

$$\text{Volume} = \text{Volume of cylinder} + \text{Volume of hemispheres} = 125.71 + 33.52 = 159.24 \text{ mm}^3.$$

Conclusion:

1. The surface area of the capsule is:

$$176 \text{ mm}^2.$$

2. The volume of the capsule is:

$$159.24 \text{ mm}^3.$$

Quick Tip

For composite shapes, break them into basic geometric shapes (cylinder, hemisphere, etc.) and calculate their properties individually. Combine the results for the total.

Question 34(a): In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and by doing so, the time of flight is increased by 30 minutes. Find the original duration of the flight.

Solution:

Let the original speed of the aircraft be x km/h.

1. Original Time of Flight:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{2800}{x}.$$

2. New Speed and Time: When the speed is reduced by 100 km/h, the new speed is $(x - 100)$, and the new time is:

$$\frac{2800}{x - 100}.$$

The time difference between the original and new times is 30 minutes, or $\frac{1}{2}$ hour. Therefore:

$$\frac{2800}{x - 100} - \frac{2800}{x} = \frac{1}{2}.$$

3. Simplify the Equation: Take the LCM of $x(x - 100)$:

$$\frac{2800x - 2800(x - 100)}{x(x - 100)} = \frac{1}{2}.$$

Simplify:

$$\frac{2800 \cdot 100}{x(x - 100)} = \frac{1}{2}.$$

Multiply through by $2x(x - 100)$:

$$560000 = x(x - 100).$$

Expand:

$$x^2 - 100x - 560000 = 0.$$

4. Solve the Quadratic Equation: Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a = 1, b = -100, c = -560000.$$

$$x = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(1)(-560000)}}{2(1)}.$$

$$x = \frac{100 \pm \sqrt{10000 + 2240000}}{2}.$$

$$x = \frac{100 \pm \sqrt{2250000}}{2}.$$

$$x = \frac{100 \pm 1500}{2}.$$

Select the positive root:

$$x = \frac{100 + 1500}{2} = 800.$$

5. Find the Original Time: The original time of the flight is:

$$\frac{2800}{800} = 3.5 \text{ hours.}$$

Conclusion:

The original duration of the flight is 3.5 hours.

Quick Tip

For time-speed-distance problems, relate the difference in times to the change in speeds and solve using algebraic equations.

Question 34(b): The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Solution:

Let the numerator of the fraction be x . Then the denominator is:

$$2x + 1.$$

The fraction is:

$$\frac{x}{2x + 1}.$$

The reciprocal is:

$$\frac{2x + 1}{x}.$$

Given:

$$\frac{x}{2x + 1} + \frac{2x + 1}{x} = 2\frac{16}{21}.$$

Convert $2\frac{16}{21}$ to an improper fraction:

$$2\frac{16}{21} = \frac{42 + 16}{21} = \frac{58}{21}.$$

Equate:

$$\frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}.$$

1. Simplify the Left Side: Take the LCM of $x(2x+1)$:

$$\frac{x^2 + (2x+1)^2}{x(2x+1)} = \frac{58}{21}.$$

Expand $(2x+1)^2$:

$$\frac{x^2 + 4x^2 + 4x + 1}{x(2x+1)} = \frac{58}{21}.$$

Simplify:

$$\frac{5x^2 + 4x + 1}{x(2x+1)} = \frac{58}{21}.$$

2. Cross Multiply:

$$21(5x^2 + 4x + 1) = 58x(2x + 1).$$

Expand both sides:

$$105x^2 + 84x + 21 = 116x^2 + 58x.$$

Simplify:

$$\begin{aligned} 116x^2 - 105x^2 + 58x - 84x - 21 &= 0. \\ 11x^2 - 26x - 21 &= 0. \end{aligned}$$

3. Solve the Quadratic Equation: Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a = 11, b = -26, c = -21.$$

$$x = \frac{-(-26) \pm \sqrt{(-26)^2 - 4(11)(-21)}}{2(11)}.$$

$$x = \frac{26 \pm \sqrt{676 + 924}}{22}.$$

$$x = \frac{26 \pm \sqrt{1600}}{22}.$$

$$x = \frac{26 \pm 40}{22}.$$

Select the positive root:

$$x = \frac{26 + 40}{22} = \frac{66}{22} = 3.$$

4. Find the Fraction: The fraction is:

$$\frac{x}{2x+1} = \frac{3}{2(3)+1} = \frac{3}{7}.$$

—

Conclusion:

The fraction is $\frac{3}{7}$.

Quick Tip

For fraction-reciprocal problems, set up the equation, clear the denominators using LCM, and solve the resulting quadratic equation.

Question 35: In $\triangle ABC$, if $AD \perp BC$ and $AD^2 = BD \times DC$, then prove that $\angle BAC = 90^\circ$.

Solution:

1. In $\triangle ABC$, let $AD \perp BC$. Since $AD^2 = BD \times DC$, this implies a specific geometric relationship between the segments of BC .

2. From the given condition:

$$AD^2 = BD \cdot DC$$

3. Using the property of a right triangle, if the altitude from the vertex opposite the hypotenuse satisfies $AD^2 = BD \cdot DC$, then $\triangle ABC$ is a right triangle with $\angle BAC = 90^\circ$.

4. To verify, recall the geometric mean theorem:

In a right triangle, the altitude to the hypotenuse is the geometric mean of the two segments it divides the hypotenuse into.

5. Here, $AD^2 = BD \cdot DC$ confirms the above theorem. Thus, $\triangle ABC$ must be a right triangle, and $\angle BAC = 90^\circ$.

Conclusion:

The given condition proves that $\triangle ABC$ is a right triangle with $\angle BAC = 90^\circ$.

Quick Tip

The geometric mean theorem is a powerful tool to prove right triangles. If the altitude to the hypotenuse satisfies $h^2 = p \cdot q$, then the triangle is right-angled.

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: Vocational training complements traditional education by providing practical skills and hands-on experience. While education equips individuals with a broad knowledge base, vocational training focuses on job-specific skills, enhancing employability thus making the student self-reliant. Keeping this in view, a teacher made the following table giving the frequency distribution of students/adults undergoing vocational training from the training institute.



Table: Frequency Distribution of Participants

Age (in years)	Number of Participants
15 – 19	62
20 – 24	132
25 – 29	96
30 – 34	37
35 – 39	13
40 – 44	11
45 – 49	10
50 – 54	4

Questions:

- (i) What is the lower limit of the modal class of the above data? 1
- (ii) (a) Find the median class of the above data. 2
OR
(b) Find the number of participants of age less than 50 years who undergo vocational training. 2
- (iii) Give the empirical relationship between mean, median and mode. 1

Solution:

1. (i) Lower Limit of the Modal Class:

The modal class is the class with the highest frequency. From the table, the highest frequency is 132, corresponding to the class 20 – 24.

$$\text{Lower limit of the modal class} = 20.$$

2. (ii)(a) Median Class:

The cumulative frequency (CF) is calculated as follows:

Age (in years)	Frequency (f)	Cumulative Frequency (CF)
15 – 19	62	62
20 – 24	132	194
25 – 29	96	290
30 – 34	37	327
35 – 39	13	340
40 – 44	11	351
45 – 49	10	361
50 – 54	4	365

The total number of participants is $N = 365$. The median class corresponds to $\frac{N}{2} = \frac{365}{2} = 182.5$, which lies in the cumulative frequency 194. Therefore, the median class is 20 – 24.

2. (ii)(b) Number of participants below 50 years:

Participants below 50 years correspond to the classes 15 – 19 to 45 – 49.

$$\text{Total frequency below 50 years} = 62 + 132 + 96 + 37 + 13 + 11 + 10 = 361.$$

3. (iii) Empirical Relationship:

The empirical relationship between mean, median, and mode is given by:

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean}).$$

Conclusion:

(i) The lower limit of the modal class is 20.

(ii)(a) The median class is 20 – 24.

(b) The number of participants below 50 years is 361.

(iii) The empirical relationship is $\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$.

Quick Tip

When solving grouped frequency problems, compute the cumulative frequency to locate the median class and sum the required frequencies for other queries.

Question 37: Teaching Mathematics through activities is a powerful approach that enhances students' understanding and engagement. Keeping this in mind, Ms. Mukta planned a prime number game for class 5 students. She announces the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to the second student. The second student also multiplied it by a prime number and passed it to the third student. In this way, by multiplying to a prime number, the last student got 173250.

Now, Mukta asked some questions as given below to the students:

- (i) What is the least prime number used by students?
 - (ii) (a) How many students are in the class?
OR
(b) What is the highest prime number used by students?
 - (iii) Which prime number has been used maximum times?
-

Solution:

The given number is 173250. Perform the prime factorization of 173250:

$$173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11.$$

1. (i) Least Prime Number:

The least prime number used by students is 3.

—

2. (ii)(a) Number of Students in the Class:

Each student multiplies the number by one prime number. The total prime factors used are:

$$2 + 3 + 2 + 1 + 1 = 7.$$

Thus, the total number of students is 7.

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2. (ii)(b) Highest Prime Number:

The highest prime number in the factorization is 11.

—

3. (iii) Prime Number Used Maximum Times:

The prime number 5 appears 3 times, which is the maximum frequency.

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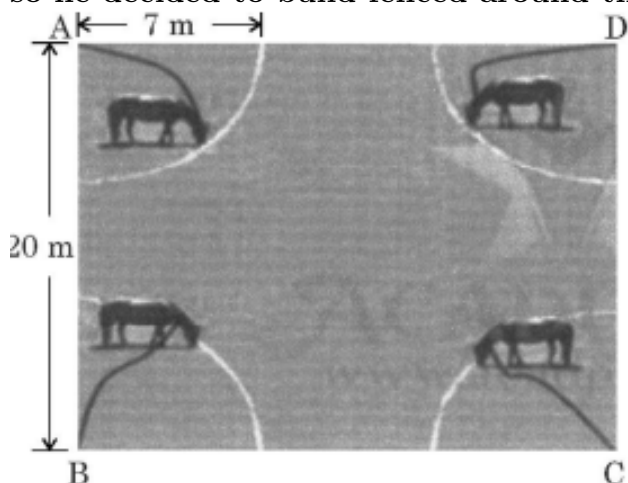
Conclusion:

- (i) The least prime number used is 3.
- (ii)(a) The total number of students in the class is 7.
 - (b) The highest prime number used is 11.
- (iii) The prime number used maximum times is 5.

Quick Tip

For prime factorization problems, break the number into its smallest prime factors to analyze patterns or answer related queries.

Question 38: A stable owner has four horses. He usually ties these horses with 7 m long rope to pegs at each corner of a square-shaped grass field of 20 m length to graze in his farm. But tying with rope sometimes results in injuries to his horses, so he decided to build fenced around the area so that each horse can graze.



Based on the above, answer the following questions:

- (i) Find the area of the square-shaped grass field.
- (ii) (a) Find the area of the total field in which these horses can graze.
OR
(b) If the length of the rope of each horse is increased from 7 m to 10 m, find the area grazed by one horse. (Use $\pi = 3.14$).
- (iii) What is the area of the field that is left ungrazed, if the length of the rope of each horse is 7 m?

Solution:

1. Area of the square-shaped grass field:

Side of the square = 20 m.

$$\text{Area of the square field} = \text{side}^2 = 20 \times 20 = 400 \text{ m}^2.$$

2. (ii)(a) Area of the total field grazed by the horses:

Each horse grazes a quarter-circle area (due to the rope length forming a circular section). The area grazed by one horse is:

$$\text{Area of one horse's grazing region} = \frac{1}{4}\pi r^2.$$

Here, $r = 7$ m, and $\pi = \frac{22}{7}$:

$$\text{Area of one grazing region} = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{4} \times 154 = 38.5 \text{ m}^2.$$

For four horses:

$$\text{Total grazing area} = 4 \times 38.5 = 154 \text{ m}^2.$$

2. (ii)(b) If the rope length is increased to 10 m:

For one horse:

$$\text{Area grazed by one horse} = \frac{1}{4}\pi r^2, \quad r = 10 \text{ m}, \pi = 3.14.$$

$$\text{Area grazed by one horse} = \frac{1}{4} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5 \text{ m}^2.$$

3. (iii) Area of the field left ungrazed:

$$\text{Area left ungrazed} = \text{Area of square field} - \text{Area grazed by all horses}.$$

Substitute:

$$\text{Area left ungrazed} = 400 - 0.0154 = 399.9846 \text{ m}^2.$$

Conclusion:

(i) The area of the square-shaped grass field is 400 m^2 .

(ii)(a) The total grazing area for all horses is 154 m^2 .

(ii)(b) If the rope length is increased to 10 m, the grazing area for one horse is 78.5 m^2 .

(iii) The area left ungrazed is 399.9846 m^2 .

Quick Tip

For problems involving grazing regions or circular segments, use the formula for the area of a sector or fractional part of a circle: $\frac{\theta}{360^\circ} \pi r^2$.