CBSE Class X Mathematics (Standard) Set 1 (30/4/1) 2024 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section consists of 20 questions of 1 mark each.

Question 1: If $ax + by = a^2 - b^2$ and $bx + ay = 0$, then the value of $x + y$ is:

(A) $a^2 - b^2$

(B) $a + b$

(C) $a - b$

(D) $a^2 + b^2$

Correct Answer: (C) $a - b$

Solution:

We are given two equations:

$$1.\ ax + by = a^2 - b^2$$

$$2.\ bx + ay = 0$$

From equation (2), solve for x or y in terms of the other variable. Assume $ay = -bx$, so:

$$y = -\frac{b}{a}x \quad \text{(substitute this into equation 1).}$$

Substituting $y = -\frac{b}{a}x$ into equation (1):

$$ax + b\left(-\frac{b}{a}x\right) = a^2 - b^2$$

Simplify:

$$ax - \frac{b^2}{a}x = a^2 - b^2$$

Combine terms:

$$\frac{a^2x - b^2x}{a} = a^2 - b^2$$

Factorize x:

$$x(a^2 - b^2) = a(a^2 - b^2)$$

Divide both sides by $a^2 - b^2$ (assuming $a \neq b$):

$$x = a$$

Substitute $x = a$ into $y = -\frac{b}{a}x$:

$$y = -\frac{b}{a}(A) = -b$$

Finally, calculate $x + y$:

$$x + y = a - b$$

—

Conclusion:

The value of $x + y$ is $a - b$.

Quick Tip

When solving simultaneous equations, substitute one variable in terms of the other to simplify and solve step-by-step.

Question 2: The HCF of two numbers 65 and 104 is 13. If LCM of 65 and 104 is $40x$, then the value of x is:

(A) 5

(B) 13

(C) 40

(D) 8

Correct Answer: (B) 13

Solution:

The relationship between the HCF and LCM of two numbers is given by:

$$\text{HCF} \times \text{LCM} = \text{Product of the two numbers.}$$

Here, the two numbers are 65 and 104, with HCF $= 13$ and LCM $= 40x$. Substituting into the equation:

$$13 \times (40x) = 65 \times 104.$$

Calculate 65×104:

$$65 \times 104 = 6760.$$

Now solve for x:

$$520x = 6760 \quad \Rightarrow \quad x = \frac{6760}{520} = 13.$$

—

Conclusion:

The value of x is 13.

Quick Tip

The product of HCF and LCM of two numbers is always equal to the product of the numbers.

Question 3: If a polynomial $p(x)$ is given by $p(x) = x^2 - 5x + 6$, then the value of $p(1) + p(4)$ is:

(A) 0

(B) 4

(C) 2

(D) -4

Correct Answer: (B) 4

Solution:

We are given $p(x) = x^2 - 5x + 6$. First, calculate $p(1)$:

$$p(1) = (1)^2 - 5(1) + 6 = 1 - 5 + 6 = 2.$$

Now calculate $p(4)$:

$$p(4) = (4)^2 - 5(4) + 6 = 16 - 20 + 6 = 2.$$

Add the two values:

$$p(1) + p(4) = 2 + 2 = 4.$$

—

Conclusion:

The value of $p(1) + p(4)$ is 4.

Quick Tip

For polynomial problems, substitute the given values into the expression and simplify step-by-step.

Question 4: If the discriminant of the quadratic equation $3x^2 - 2x + c = 0$ is 16, then the value of c is:

(A) 1

(B) 0

(C) -1

(D) $\sqrt{2}$

Correct Answer: (C) -1

Solution:

The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\Delta = b^2 - 4ac.$$

For the equation $3x^2 - 2x + c = 0$, we have $a = 3$, $b = -2$, and $c = c$. Substituting into the discriminant formula:

$$\Delta = (-2)^2 - 4(3)(c).$$

Simplify:

$$\Delta = 4 - 12c.$$

We are given $\Delta = 16$, so:

$$4 - 12c = 16.$$

Solve for c:

$$-12c = 16 - 4 \quad \Rightarrow \quad -12c = 12 \quad \Rightarrow \quad c = -1.$$

—

Conclusion:

The value of c is -1.

Quick Tip

For quadratic equations, use the discriminant formula $\Delta = b^2 - 4ac$ to determine the nature of the roots or unknown coefficients.

Question 5: If an arc subtends an angle of 90° at the centre of a circle, then the ratio of its length to the circumference of the circle is:

(A) $2 : 3$

(B) $1 : 4$

(C) $4 : 1$

(D) $1 : 3$

Correct Answer: (B) $1 : 4$

Solution:

The length of an arc is proportional to the angle it subtends at the centre of the circle. The total circumference of a circle corresponds to an angle of 360°. Hence, the ratio of the arc length to the circumference is:

$$\text{Ratio} = \frac{\text{Angle subtended by arc}}{\text{Total angle of circle}} = \frac{90^\circ}{360^\circ}.$$

Simplify:

$$\text{Ratio} = \frac{90}{360} = \frac{1}{4}.$$

—

Conclusion:

The ratio of the arc length to the circumference is $1 : 4$.

Quick Tip

The arc length ratio can be determined by dividing the angle subtended by the arc by 360°.

Question 6: The area of the sector of a circle of radius 12 cm is 60π cm^2. The central

angle of this sector is:

(A) 120°

(B) 6°

(C) 75°

(D) 150°

Correct Answer: (D) 150°

Solution:

The formula for the area of a sector of a circle is:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \cdot \pi r^2,$$

where θ is the central angle in degrees and r is the radius of the circle.

Substitute the given values (Area $= 60\pi\,\text{cm}^2$, $r = 12\,\text{cm}$):

$$60\pi = \frac{\theta}{360^\circ} \cdot \pi(12)^2.$$

Simplify:

$$60\pi = \frac{\theta}{360^\circ} \cdot 144\pi.$$

Cancel π and solve for θ:

$$60 = \frac{\theta}{360^\circ} \cdot 144 \quad \Rightarrow \quad \frac{\theta}{360^\circ} = \frac{60}{144}.$$

Simplify:

$$\frac{\theta}{360^\circ} = \frac{5}{12} \quad \Rightarrow \quad \theta = \frac{5}{12} \cdot 360^\circ = 150^\circ.$$

—

Conclusion:

The central angle of the sector is 150°.

Quick Tip

The formula for the area of a sector relates the angle to the total area of the circle. Always substitute known values and simplify systematically.

Question 7: If the difference of mode and median of a data is 24, then the difference of its median and mean is:

(A) 12

(B) 24

(C) 8

(D) 36

Correct Answer: (A) 12

Solution:

For a moderately skewed distribution, the relationship between the mode, median, and mean is approximated by:

$$\text{Mode} - \text{Mean} = 3(\text{Median} - \text{Mean}).$$

We are given:

$$\text{Mode} - \text{Median} = 24.$$

Substitute into the equation:

$$24 = 3(\text{Median} - \text{Mean}).$$

Solve for Median – Mean:

$$\text{Median} - \text{Mean} = \frac{24}{3} = 12.$$

—

Conclusion:

The difference between the median and the mean is 12.

Quick Tip

For skewed distributions, use the empirical relationship between mode, median, and mean to solve for missing values.

Question 8: Two dice are tossed simultaneously. The probability of getting odd numbers on both the dice is:

(A) $6/36$

(B) $3/36$

(C) $12/36$

(D) $9/36$

Correct Answer: (D) $9/36$

Solution:

Each die has 6 faces: $1, 2, 3, 4, 5, 6$. Odd numbers are $1, 3, 5$, so the probability of getting an odd number on one die is:

$$\text{P(odd on one die)} = \frac{3}{6} = \frac{1}{2}.$$

The two dice are independent, so the probability of getting odd numbers on both dice is:

$$\text{P(odd on both dice)} = \text{P(odd on first die)} \cdot \text{P(odd on second die)} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Convert to common fractions:

$$\text{P(odd on both dice)} = \frac{9}{36}.$$

—

Conclusion:

The probability of getting odd numbers on both dice is $9/36$.

Quick Tip

For independent events, multiply individual probabilities to find the combined probability.

Question 9: The ratio of total surface area of a solid hemisphere to the square of its radius is:

(A) $2\pi : 1$

(B) $4\pi : 1$

(C) $3\pi : 1$

(D) $1 : 4\pi$

Correct Answer: (C) $3\pi : 1$

Solution:

The total surface area of a solid hemisphere is the sum of its curved surface area and the area of its circular base:

$$\text{Total Surface Area} = 2\pi r^2 + \pi r^2 = 3\pi r^2.$$

The square of the radius is:

$$\text{Square of radius} = r^2.$$

The ratio of total surface area to the square of the radius is:

$$\text{Ratio} = \frac{3\pi r^2}{r^2} = 3\pi : 1.$$

—

Conclusion:

The ratio of total surface area to the square of the radius is $3\pi : 1$.

Quick Tip

For solid hemispheres, remember that the total surface area includes both the curved surface and the base.

Question 10: If $\sin\theta = 1$, then the value of $\frac{1}{2}\sin\left(\frac{\theta}{2}\right)$ is:

(A) $\frac{1}{2\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2}$

(D) 0

Correct Answer: (A) $\frac{1}{2\sqrt{2}}$

Solution:

We are given $\sin\theta = 1$. This implies:

$$\theta = 90^\circ \quad (\text{since } \sin 90^\circ = 1).$$

Substitute $\theta = 90^\circ$ into the expression $\frac{1}{2}\sin\left(\frac{\theta}{2}\right)$:

$$\frac{1}{2}\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\sin\left(\frac{90^\circ}{2}\right) = \frac{1}{2}\sin(45^\circ).$$

We know $\sin 45^\circ = \frac{1}{\sqrt{2}}$, so:

$$\frac{1}{2}\sin(45^\circ) = \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

—

Conclusion:

The value of $\frac{1}{2}\sin\left(\frac{\theta}{2}\right)$ is $\frac{1}{2\sqrt{2}}$.

Quick Tip

Remember that $\sin 45^\circ = \frac{1}{\sqrt{2}}$. Use trigonometric identities and angle relationships to simplify such expressions.

Question 11: Two lines are given to be parallel. The equation of one of these lines is $5x - 3y = 2$. The equation of the second line can be:

(A) $-15x - 9y = 5$

(B) $15x + 9y = 5$

(C) $9x - 15y = 6$

(D) $-15x + 9y = 5$

Correct Answer: (D) $-15x + 9y = 5$

Solution:

Two lines are parallel if their slopes are equal. The general equation of a line is given by:

$$Ax + By + C = 0.$$

The slope of the line is:

$$\text{Slope} = -\frac{A}{B}.$$

For the given line $5x - 3y = 2$, the slope is:

$$\text{Slope} = -\frac{5}{-3} = \frac{5}{3}.$$

Now check the options for the second line to find which has the same slope: - For $-15x + 9y = 5$:

$$\text{Slope} = -\frac{-15}{9} = \frac{5}{3}.$$

This matches the slope of the given line, so the two lines are parallel.

—

Conclusion:

The equation of the second line is $-15x + 9y = 5$.

Quick Tip

Parallel lines have the same slope. To verify, compare $-\frac{A}{B}$ for both equations.

Question 12: Three numbers in A.P. have the sum 30. What is its middle term?

(A) 4

(B) 10

(C) 16

(D) 8

Correct Answer: (B) 10

Solution:

Let the three numbers in A.P. be $(a - d)$, a, and $(a + d)$. The sum of these numbers is:

$$(a - d) + a + (a + d) = 30.$$

Simplify:

$$3a = 30 \quad \Rightarrow \quad a = 10.$$

Thus, the middle term of the sequence is $a = 10$.

—

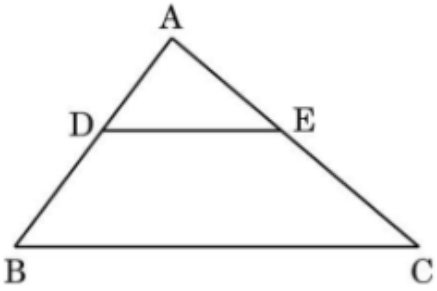
Conclusion:

The middle term of the sequence is 10.

Quick Tip

In an arithmetic progression, the middle term is equal to the average of the sum of all terms.

Question 13: In $\triangle ABC$, $DE \parallel BC$ (as shown in the figure). If $AD = 4$ cm, $AB = 9$ cm, and $AC = 13.5$ cm, then the length of EC is:



(A) 6 cm

(B) 7.5 cm

(C) 9 cm

(D) 5.7 cm

Correct Answer: (B) 7.5 cm

Solution:

Since $DE \parallel BC$, by the basic proportionality theorem (Thales theorem), we have:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Given:

$$AD = 4\,\text{cm}, \quad AB = 9\,\text{cm}, \quad AC = 13.5\,\text{cm}.$$

Calculate DB and AE:

$$DB = AB - AD = 9 - 4 = 5\,\text{cm}.$$

$$AE = AC - EC.$$

From the proportionality relation:

$$\frac{4}{5} = \frac{AE}{EC}.$$

Rewriting $AE = AC - EC$:

$$\frac{4}{5} = \frac{13.5 - EC}{EC}.$$

Cross multiply:

$$4EC = 5(13.5 - EC).$$

Simplify:

$$4EC = 67.5 - 5EC \quad \Rightarrow \quad 9EC = 67.5 \quad \Rightarrow \quad EC = \frac{67.5}{9} = 7.5\,\text{cm}.$$

—

Conclusion:

The length of EC is $7.5\,\text{cm}$.

Quick Tip

Use the basic proportionality theorem to solve problems involving parallel lines in triangles.

Question 14: At some time of the day, the length of the shadow of a tower is equal to its height. Then, the Sun's altitude at that time is:

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Correct Answer: (B) 45°

Solution:

The shadow of the tower is equal to its height. Let the height of the tower be h and the length of its shadow also be h. The altitude of the Sun forms a right triangle, where:

$$\tan(\text{Sun's altitude}) = \frac{\text{Height of tower}}{\text{Length of shadow}}.$$

Substitute:

$$\tan(\text{Sun's altitude}) = \frac{h}{h} = 1.$$

We know:

$$\tan(45^\circ) = 1.$$

Thus, the Sun's altitude is 45°.

—

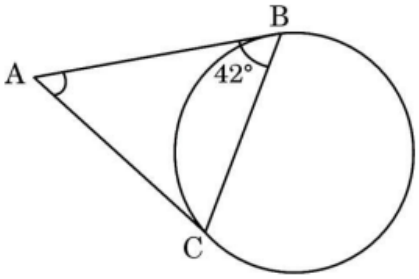
Conclusion:

The Sun's altitude is 45°.

Quick Tip

When the height and shadow of an object are equal, the angle of elevation is always 45°.

Question 15: In the given figure, AB and AC are tangents to the circle. If $\angle ABC = 42^\circ$, then the measure of $\angle BAC$ is:



(A) 96°

(B) 42°

(C) 106°

(D) 86°

Correct Answer: (A) 96°

Solution:

In the given figure, AB and AC are tangents to the circle from point A. The tangents from a common external point to a circle are equal in length, and the angle between the tangents is bisected by the line segment joining the center of the circle to the external point.

Here, $\angle ABC = 42^\circ$ is the angle between the tangent AB and the chord BC. By the

properties of tangents and circles:

$$\angle BAC = 180^\circ - 2 \times \angle ABC.$$

Substitute the given value $\angle ABC = 42^\circ$:

$$\angle BAC = 180^\circ - 2 \times 42^\circ = 180^\circ - 84^\circ = 96^\circ.$$

—

Conclusion:

The measure of $\angle BAC$ is 96°.

Quick Tip

For tangents drawn from a common external point, use the property that the angles formed are related to the supplementary angles of the triangle.

Question 16: The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2, 3)$, $B(6, 7)$, and $C(8, 3)$ is:

(A) $0, 1$

(B) $0, -1$

(C) $-1, 0$

(D) $1, 0$

Correct Answer: (B) $0, -1$

Solution:

In a parallelogram, the diagonals bisect each other. Let the coordinates of the fourth vertex D be (x, y). Using the midpoint formula, the midpoint of diagonal AC must be the same as the midpoint of diagonal BD.

Midpoint of AC:

$$\left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = (3, 3)\,.$$

Midpoint of BD:

$$\left(\frac{6+x}{2}, \frac{7+y}{2}\right).$$

Equating the midpoints:

$$\frac{6+x}{2} = 3 \quad \Rightarrow \quad 6+x = 6 \quad \Rightarrow \quad x = 0,$$

$$\frac{7+y}{2} = 3 \quad \Rightarrow \quad 7+y = 6 \quad \Rightarrow \quad y = -1.$$

Thus, the coordinates of D are $(0, -1)$.

—

Conclusion:

The fourth vertex D is at $(0, -1)$.

Quick Tip

In a parallelogram, use the midpoint formula to find the unknown vertex by equating the midpoints of the diagonals.

Question 17: For an event E, if $P(E) + P(\overline{E}) = q$, then the value of $q^2 - 4$ is:

(A) -3

(B) 3

(C) 5

(D) -5

Correct Answer: (A) -3

Solution:

The probability of an event $P(E)$ and its complement $P(\overline{E})$ always satisfy:

$$P(E) + P(\overline{E}) = 1.$$

Given:

$$P(E) + P(\overline{E}) = q.$$

This implies:

$$q = 1.$$

Now, calculate $q^2 - 4$:

$$q^2 - 4 = 1^2 - 4 = 1 - 4 = -3.$$

—

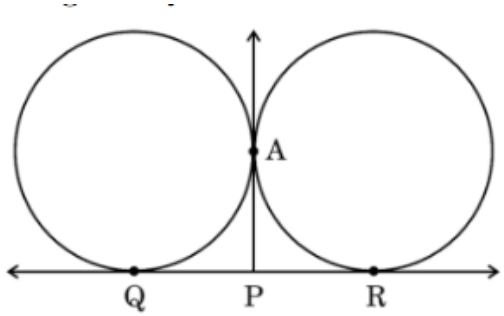
Conclusion:

The value of $q^2 - 4$ is -3.

Quick Tip

The sum of the probabilities of an event and its complement is always 1. Use this relationship to solve related problems.

Question 18: In the given figure, QR is a common tangent to the two given circles touching externally at A. The tangent at A meets QR at P. If $AP = 4.2$ cm, then the length of QR is:



(A) 4.2 cm

(B) 2.1 cm

(C) 8.4 cm

(D) 6.3 cm

Correct Answer: (C) 8.4 cm

Solution:

In the given figure, the tangent QR is divided into two equal segments at P because the circles are symmetric, and the tangent passes through the point of contact A. Thus:

$$QR = 2 \times AP.$$

Substitute the given value $AP = 4.2$ cm:

$$QR = 2 \times 4.2 = 8.4 \text{ cm}.$$

—

Conclusion:

The length of QR is $8.4\,\text{cm}$.

Quick Tip

For tangents between two externally touching circles, the tangent length is twice the distance from the point of contact to the tangent's midpoint.

Question 19: Assertion (A): The mid-point of a line segment divides the line segment in the ratio $1:1$.

Reason (R): The ratio in which the point $(-3, k)$ divides the line segment joining the points $(-5, 4)$ and $(-2, 3)$ is $1:2$.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

1. The mid-point of a line segment always divides the segment into two equal parts, which means the ratio is $1:1$. Therefore, Assertion (A) is true.

2. To verify Reason (R), let the point $(-3, k)$ divide the segment joining $(-5, 4)$ and $(-2, 3)$. Using the section formula:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}.$$

For the x-coordinate:

$$-3 = \frac{1 \cdot (-2) + 2 \cdot (-5)}{1 + 2} = \frac{-2 - 10}{3} = \frac{-12}{3} = -4 \,(\text{which does not match } -3).$$

Thus, the point does not divide the line segment in the ratio $1:2$. Reason (R) is false.

—

Conclusion:

Assertion (A) is true, but Reason (R) is false.

Quick Tip

Use the section formula to determine the ratio in which a point divides a line segment.

Question 20: Assertion (A): If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason (R): Circumference = $2\pi \times$ radius of a circle.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).

Solution:

1. The formula for the circumference of a circle is:

$$\text{Circumference} = 2\pi r.$$

2. Given the circumference is 176 cm, solve for the radius:

$$176 = 2\pi r \quad \Rightarrow \quad r = \frac{176}{2\pi} = \frac{88}{\pi}.$$

Approximating $\pi \approx 3.14$:

$$r = \frac{88}{3.14} = 28 \text{ cm}.$$

3. Both the Assertion (A) and the Reason (R) are true, and Reason (R) correctly explains the Assertion (A).

—

Conclusion:

Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

Quick Tip

For problems involving circles, always use the formula Circumference $= 2\pi r$ and substitute known values to find unknown parameters.

Section - B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

Question 21: Three bells toll at intervals of 9, 12, and 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Solution:

Find the LCM of the intervals 9, 12, and 15:

$$9 = 3^2, \quad 12 = 2^2 \times 3, \quad 15 = 3 \times 5.$$

The LCM is the product of the highest powers of all prime factors:

$$\text{LCM} = 2^2 \times 3^2 \times 5 = 180.$$

Thus, the three bells will toll together after 180 minutes.

—

Conclusion:

The three bells will next toll together after 180 minutes.

Quick Tip

For periodic events, calculate the least common multiple (LCM) of the time intervals to determine when they coincide.

Question 22(A): The minute hand of a clock is 14 cm long. Find the area on the face of the clock described by the minute hand in 5 minutes.

Solution:

The angle subtended by the minute hand in 5 minutes is:

$$\text{Angle} = \frac{360^\circ}{60} \times 5 = 30^\circ.$$

The area described by the minute hand is a sector of a circle, given by:

$$\text{Area} = \frac{\theta}{360^\circ} \cdot \pi r^2,$$

where $\theta = 30^\circ$ and $r = 14\,\text{cm}$.

Substitute the values:

$$\text{Area} = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14.$$

Simplify:

$$\text{Area} = \frac{1}{12} \times \frac{22}{7} \times 196 = \frac{154}{3}\,\text{cm}^2 \approx 51.33\,\text{cm}^2.$$

—

Conclusion:

The area described by the minute hand is approximately $51.33\,\text{cm}^2$.

Quick Tip

The area of a sector is proportional to the angle subtended by it at the center of the circle. Always express the angle in degrees when using the formula.

Question 22(b): Find the length of the arc of a circle that subtends an angle of 60° at the center of the circle of radius 42 cm.

Solution:

The formula for the length of an arc is:

$$\text{Length of arc} = 2\pi r \cdot \frac{\theta}{360},$$

where r is the radius and θ is the central angle in degrees.

Substitute the given values $r = 42\,\text{cm}$ and $\theta = 60^\circ$:

$$\text{Length of arc} = 2 \times \frac{22}{7} \times 42 \times \frac{60}{360}.$$

Simplify step-by-step:

$$\text{Length of arc} = 2 \times \frac{22}{7} \times 42 \times \frac{1}{6}.$$

$$\text{Length of arc} = \frac{2 \times 22 \times 42}{7 \times 6}.$$

$$\text{Length of arc} = \frac{1848}{42} = 44\,\text{cm}.$$

—

Conclusion:

The length of the arc is $44\,\text{cm}$.

Quick Tip

The length of an arc is proportional to the central angle it subtends. Use Length of arc $= 2\pi r \cdot \frac{\theta}{360}$ for quick calculations.

Question 23(A): Evaluate:

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \sin^2 60^\circ}.$$

Solution:

Substitute the trigonometric values:

$$\cos 60^\circ = \frac{1}{2}, \quad \sec 30^\circ = \frac{2}{\sqrt{3}}, \quad \tan 45^\circ = 1, \quad \sin 30^\circ = \frac{1}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}.$$

The numerator is:

$$5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ = 5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1.$$

Simplify:

$$5\left(\frac{1}{4}\right)+4\left(\frac{4}{3}\right)-1=\frac{5}{4}+\frac{16}{3}-1=\frac{15}{12}+\frac{64}{12}-\frac{12}{12}=\frac{67}{12}.$$

The denominator is:

$$\sin^2 30^\circ+\sin^2 60^\circ=\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2=\frac{1}{4}+\frac{3}{4}=1.$$

Thus:

$$\frac{\text{Numerator}}{\text{Denominator}}=\frac{\frac{67}{12}}{1}=\frac{67}{12}.$$

—

Conclusion:

The value of the expression is $\frac{67}{12}$.

Quick Tip

Substitute known trigonometric values into the expression and simplify step-by-step.

Question 23(b): If $\sin(A-B)=\frac{1}{2}$**,** $\cos(A+B)=\frac{1}{2}$**,** $0<A+B\leq 90^\circ$**,** $A>B$**; find** $\angle A$ **and** $\angle B$**.**

Solution:

1. From $\sin(A-B)=\frac{1}{2}$, we know:

$$\sin(A-B)=\sin 30^\circ \quad \Rightarrow \quad A-B=30^\circ \quad \text{(i)}.$$

2. From $\cos(A+B)=\frac{1}{2}$, we know:

$$\cos(A+B)=\cos 60^\circ \quad \Rightarrow \quad A+B=60^\circ \quad \text{(ii)}.$$

3. Solve equations (i) and (ii):

$$A-B=30^\circ, \quad A+B=60^\circ.$$

Add the equations:

$$2A=90^\circ \quad \Rightarrow \quad A=45^\circ.$$

Subtract the equations:

$$2B=30^\circ \quad \Rightarrow \quad B=15^\circ.$$

—

Conclusion:

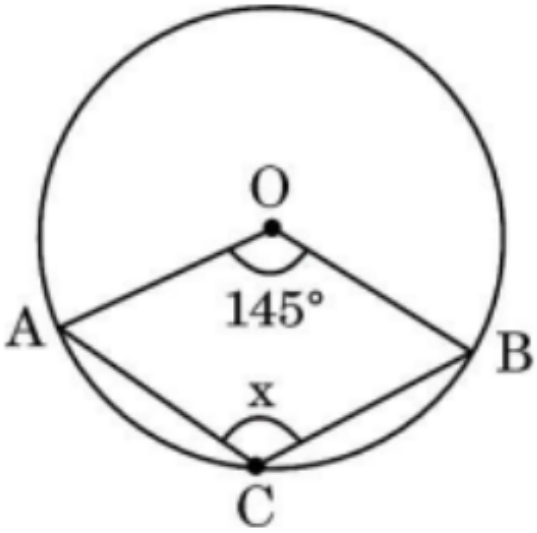
The values of $\angle A$ and $\angle B$ are:

$$A = 45^\circ,\ B = 15^\circ.$$

Quick Tip

For problems involving $\sin$ and $\cos$ equations, use reference angles and solve the resulting linear equations systematically.

Question 24: In the given figure, O is the center of the circle. If $\angle AOB = 145^\circ$, find the value of x.



Solution:

In a circle, the angle subtended by an arc at the center is twice the angle subtended by the same arc at any point on the circumference.

1. Let P be a point on the circumference of the circle. Join AP and BP.
2. The angle subtended by the arc AB at the circumference is:

$$\angle APB = \frac{1}{2} \times \angle AOB = \frac{1}{2} \times 145^\circ = 72.5^\circ.$$

3. In $\triangle APB$, the sum of angles $\angle APB + \angle ACB = 180^\circ$ (angles on a straight line).

4. Solve for $\angle ACB$:

$$\angle ACB = 180^\circ - \angle APB = 180^\circ - 72.5^\circ = 107.5^\circ.$$

Thus:

$$x = \angle ACB = 107.5^\circ.$$

—

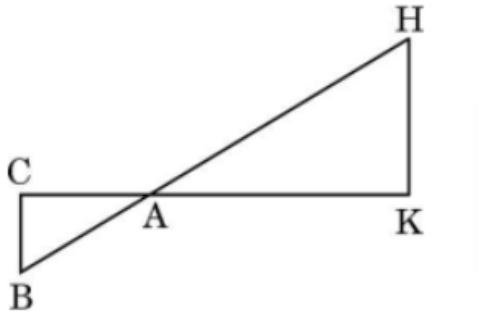
Conclusion:

The value of x is 107.5°.

Quick Tip

In a circle, the angle subtended by an arc at the center is always twice the angle subtended by the same arc at the circumference.

Question 25: In the given figure, $\triangle AHK \sim \triangle ABC$. If $AK = 8$ cm, $BC = 3.2$ cm, and $HK = 6.4$ cm, then find the length of AC.



Solution:

Since $\triangle AHK \sim \triangle ABC$, the corresponding sides are proportional:

$$\frac{AK}{AC} = \frac{HK}{BC}.$$

Substitute the known values:

$$\frac{8}{AC} = \frac{6.4}{3.2}.$$

Simplify:

$$\frac{8}{AC} = 2 \quad \Rightarrow \quad AC = \frac{8}{2} = 4\,\text{cm}.$$

—

Conclusion:

The length of AC is $4\,\text{cm}$.

Quick Tip

For similar triangles, use the property that corresponding sides are proportional to find unknown lengths.

Section - C

This section comprises Short Answer (SA) type questions of 3 marks each.

Question 26: Prove that

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}.$$

Solution:

Start with the L.H.S:

$$\text{L.H.S} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}.$$

Divide both the numerator and denominator by $\cos\theta$:

$$\text{L.H.S} = \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}.$$

Simplify further:

$$\text{L.H.S} = \frac{\tan\theta - 1 + \sec\theta}{(\tan\theta - \sec\theta) + (\sec^2\theta - \tan^2\theta)}.$$

Using the identity $\sec^2\theta - \tan^2\theta = 1$, this becomes:

$$\text{L.H.S} = \frac{\tan\theta - 1 + \sec\theta}{(\sec\theta - \tan\theta)(\tan\theta + \sec\theta - 1)}.$$

Cancel $(\tan\theta + \sec\theta - 1)$ in the numerator and denominator:

$$\text{L.H.S} = \frac{1}{\sec\theta - \tan\theta}.$$

Thus:

$$\text{L.H.S} = \text{R.H.S}.$$

—

Conclusion:

The given identity is proven:

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}.$$

Quick Tip

Divide terms by a common trigonometric function (e.g., $\cos\theta$) to simplify complex expressions and apply trigonometric identities.

Question 27(A): Three coins are tossed simultaneously. What is the probability of getting:

At least one head?

Exactly two tails?

At most one tail?

Solution:

The total number of outcomes when three coins are tossed is:

$$\text{Total outcomes} = 2^3 = 8 \quad \text{(HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)}.$$

(i) Probability of at least one head:

$$P(\text{at least one head}) = 1 - P(\text{no heads}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

(ii) Probability of exactly two tails:

$$P(\text{exactly two tails}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{3}{8} \quad \text{(favorable outcomes: HTT, THT, TTH)}.$$

(iii) Probability of at most one tail:

$$P(\text{at most one tail}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{4}{8} = \frac{1}{2}$$

(favorable outcomes: HHH, HHT, HTH, THH).

—

Conclusion:

- (i) $P(\text{at least one head}) = \frac{7}{8}$.
- (ii) $P(\text{exactly two tails}) = \frac{3}{8}$.
- (iii) $P(\text{at most one tail}) = \frac{1}{2}$.

Quick Tip

For coin toss problems, count all possible outcomes carefully and identify favorable cases for each condition.

Question 27(b): A box contains 90 discs which are numbered 1 to 90. If one disc is drawn at random from the box, find the probability that it bears:
A two-digit number less than 40.
A number divisible by 5 and greater than 50.
A perfect square number.

Solution:

The total number of outcomes is:

$$\text{Total outcomes} = 90.$$

(i) Probability of a two-digit number less than 40: Two-digit numbers less than 40 are 10 to 39 (inclusive). The total count is:

$$\text{Count} = 30 \quad (10, 11, 12, ..., 39).$$

$$P(\text{two-digit number less than } 40) = \frac{30}{90} = \frac{1}{3}.$$

(ii) Probability of a number divisible by 5 and greater than 50: Numbers divisible by 5 and greater than 50 are $55, 60, 65, 70, 75, 80, 85, 90$. The total count is:

$$\text{Count} = 8.$$

$$P(\text{divisible by 5 and greater than 50}) = \frac{8}{90} = \frac{4}{45}.$$

(iii) Probability of a perfect square number: Perfect squares from 1 to 90 are $1, 4, 9, 16, 25, 36, 49, 64, 81$. The total count is:

$$\text{Count} = 9.$$

$$P(\text{perfect square number}) = \frac{9}{90} = \frac{1}{10}.$$

—

Conclusion:

- (i) $P(\text{two-digit number less than 40}) = \frac{1}{3}$.
- (ii) $P(\text{divisible by 5 and greater than 50}) = \frac{4}{45}$.
- (iii) $P(\text{perfect square number}) = \frac{1}{10}$.

Quick Tip

For probability problems, list all favorable outcomes systematically and divide by the total possible outcomes.

Question 28: Rehana went to a bank to withdraw 2000. She asked the cashier to give her 50 and 100 notes only. Rehana got 25 notes in all. Find how many notes of 50 and 100 did she receive.

Solution:

Let the number of 50 notes be x and the number of 100 notes be y.

From the problem, we are given:

$$x + y = 25 \quad \text{(i)}.$$

The total amount is 2000:

$$50x + 100y = 2000.$$

Simplify the second equation:

$$x + 2y = 40 \quad \text{(ii)}.$$

Solve equations (i) and (ii): 1. From equation (i): $x = 25 - y$. 2. Substitute $x = 25 - y$ into equation (ii):

$$25 - y + 2y = 40 \quad \Rightarrow \quad 25 + y = 40 \quad \Rightarrow \quad y = 15.$$

3. Substitute $y = 15$ into equation (i):

$$x + 15 = 25 \quad \Rightarrow \quad x = 10.$$

Thus, Rehana received:

$$x = 10 \quad \text{(50 notes)} \quad \text{and} \quad y = 15 \quad \text{(100 notes)}.$$

—

Conclusion:

Rehana received 10 notes of 50 and 15 notes of 100.

Quick Tip

For such problems, use a system of linear equations to represent the constraints and solve using substitution or elimination methods.

Question 29(A): Find the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial.

Solution:

The given polynomial is:

$$P(x) = 4x^2 + 4x - 3 = (2x + 3)(2x - 1).$$

Zeroes of the polynomial:

$$x = -\frac{3}{2}, \quad x = \frac{1}{2}.$$

1. Verify the sum of the zeroes:

$$\text{Sum of zeroes} = -\frac{3}{2} + \frac{1}{2} = -\frac{4}{2} = -2.$$

This matches:

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}.$$

2. Verify the product of the zeroes:

$$\text{Product of zeroes} = -\frac{3}{2} \times \frac{1}{2} = -\frac{3}{4}.$$

This matches:

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}.$$

—

Conclusion:

The zeroes are $-\frac{3}{2}$ and $\frac{1}{2}$, and they satisfy the relationship between zeroes and coefficients of the polynomial.

Quick Tip

To verify zeroes of a polynomial, use the relationships: Sum of zeroes $= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$ and Product of zeroes $= \frac{\text{constant term}}{\text{coefficient of } x^2}$.

Question 29(b): If α and β are the zeroes of the polynomial $x^2 + x - 2$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Solution:

From the given polynomial:

$$\alpha + \beta = -1, \quad \alpha\beta = -2.$$

We need to find:

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}.$$

1. Using the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$:

$$\alpha^2 + \beta^2 = (-1)^2 - 2(-2) = 1 + 4 = 5.$$

2. Substitute into the expression for $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$:

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{5}{-2} = -\frac{5}{2}.$$

—

Conclusion:

The value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is $-\frac{5}{2}$.

Quick Tip

For problems involving zeroes of polynomials, use the relationships: $\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$ and $\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$.

Question 30: Prove that $\frac{2-\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Solution:

Assume $\frac{2-\sqrt{3}}{5}$ to be a rational number.

Let:

$$\frac{2-\sqrt{3}}{5} = \frac{p}{q},$$

where p and q are integers, and $q \neq 0$.

Rearrange the equation to isolate $\sqrt{3}$:

$$\sqrt{3} = \frac{2q-5p}{q}.$$

Here:

- p and q are integers, so $2q - 5p$ and q are also integers.
- Therefore, $\frac{2q-5p}{q}$ is a rational number.

However, $\sqrt{3}$ is known to be an irrational number. This creates a contradiction.

Thus, our assumption that $\frac{2-\sqrt{3}}{5}$ is rational is false.

—

Conclusion:

$\frac{2-\sqrt{3}}{5}$ is an irrational number.

Quick Tip

To prove a number is irrational, assume it to be rational and show that this assumption leads to a contradiction.

Question 31: Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

From the given figure:

1. $AP = AS$ (i),
2. $BP = BQ$ (ii),
3. $CR = CQ$ (iii),
4. $DR = DS$ (iv).

Adding equations (i), (ii), (iii), and (iv):

$$AP + BP + CR + DR = AS + BQ + CQ + DS.$$

This simplifies to:

$$AB + CD = AD + BC.$$

Since $ABCD$ is a parallelogram, the opposite sides are equal:

$$AB = CD \quad \text{and} \quad AD = BC.$$

Substitute these equalities:

$$2AB = 2AD \quad \Rightarrow \quad AB = AD.$$

Thus, all sides of $ABCD$ are equal, which proves it is a rhombus.

—

Conclusion:

The parallelogram $ABCD$ circumscribing a circle is a rhombus.

Quick Tip

A parallelogram circumscribing a circle always has equal sides due to the tangential property of the circle, making it a rhombus.

Section - D

This section comprises Long Answer (LA) type questions of 5 marks each.

Question 32: Two pillars of equal lengths stand on either side of a road which is $100\,\text{m}$ wide, exactly opposite to each other. At a point on the road between the pillars, the angles of elevation of the tops of the pillars are 60° and 30°. Find the length of each pillar and the distance of the point on the road from the pillars. (Use $\sqrt{3} = 1.732$).

Solution:

Let AB and CD be the two pillars of equal height $h\,\text{m}$, and let P be the point on the road $x\,\text{m}$ away from pillar CD.

In $\triangle CDP$:

$$\tan 60^\circ = \frac{h}{x}, \quad \text{so} \quad h = \sqrt{3}x \quad \text{(i)}.$$

In $\triangle ABP$:

$$\tan 30^\circ = \frac{h}{100 - x}, \quad \text{so} \quad h = \frac{100 - x}{\sqrt{3}} \quad \text{(ii)}.$$

Equate equations (i) and (ii):

$$\sqrt{3}x = \frac{100 - x}{\sqrt{3}}.$$

Multiply through by $\sqrt{3}$:

$$3x = 100 - x.$$

Solve for x:

$$4x = 100, \quad x = 25\,\text{m}.$$

Substitute $x = 25$ into equation (i):

$$h = \sqrt{3}x = 25 \times 1.732 = 43.3\,\text{m}.$$

—

Conclusion:

The height of each pillar is $43.3\,\text{m}$, and the distances of the point P from the pillars are:

$$\text{Distance from } CD = 25\,\text{m}, \quad \text{Distance from } AB = 100 - 25 = 75\,\text{m}.$$

Quick Tip

For such problems, use trigonometric ratios to establish equations for the unknowns and solve them systematically.

Question 33(A): E is a point on the side AD produced of a parallelogram $ABCD$, and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.

Solution:

In $\triangle ABE$ and $\triangle CFB$:

1. $\angle EAB = \angle BCF$ (corresponding angles, since $AB \parallel CD$).
2. $\angle AEB = \angle CBF$ (vertically opposite angles).

Thus, by the AA similarity criterion:

$$\triangle ABE \sim \triangle CFB.$$

—

Conclusion:

$\triangle ABE \sim \triangle CFB$ by the AA similarity criterion.

Quick Tip

To prove triangles similar, identify pairs of equal angles using properties like corresponding angles, vertically opposite angles, or parallel line properties.

Question 33(b): Sides AB, BC, and the median AD of $\triangle ABC$ are respectively proportional to sides PQ, QR, and the median PM of another $\triangle PQR$. Prove that $\triangle ABC \sim \triangle PQR$.

Solution:

From the problem, it is given that:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}.$$

Thus, by the SSS similarity criterion (corresponding sides proportional):

$$\triangle ABD \sim \triangle PQM \quad \text{(i)}.$$

From the property of medians and similarity:

$$\angle B = \angle Q \quad \text{(ii)}.$$

Now, consider $\triangle ABC$ and $\triangle PQR$. The sides are proportional, and the corresponding angles are equal:

$$\frac{AB}{PQ} = \frac{BC}{QR}, \quad \angle B = \angle Q.$$

By the SSS similarity criterion:

$$\triangle ABC \sim \triangle PQR.$$

—

Conclusion:

$\triangle ABC \sim \triangle PQR$ by the SSS similarity criterion.

Quick Tip

To prove two triangles are similar using medians, ensure the sides and medians are proportional and verify one pair of corresponding angles is equal.

Question 34(A): A train travels a distance of 90 km at a constant speed. Had the speed been 15 km/h more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Solution:

Let the original speed of the train be x km/h. The new speed = $(x + 15)$ km/h.

According to the problem:

$$\frac{90}{x} - \frac{90}{x + 15} = \frac{1}{2}.$$

Simplify the equation:

$$90 \left(\frac{1}{x} - \frac{1}{x + 15} \right) = \frac{1}{2},$$

$$\frac{90(x+15)-90x}{x(x+15)} = \frac{1}{2},$$
$$\frac{90 \cdot 15}{x(x+15)} = \frac{1}{2}.$$

Cross-multiply:

$$180x(x+15) = 2700.$$

Simplify:

$$x^2 + 15x - 2700 = 0.$$

Factorize:

$$(x+60)(x-45) = 0.$$

Since speed cannot be negative:

$$x = 45.$$

—

Conclusion:

The original speed of the train is 45 km/h.

Quick Tip

For time-speed-distance problems, set up equations using the relationship: Time $=$ $\frac{\text{Distance}}{\text{Speed}}$, and solve step-by-step.

Question 34(b): Find the value of c for which the quadratic equation

$$(c+1)x^2 - 6(c+1)x + 3(c+9) = 0, \quad c \neq -1,$$

has real and equal roots.

Solution:

For real and equal roots, the discriminant (Δ) of the quadratic equation must be zero:

$$\Delta = b^2 - 4ac = 0.$$

Here:

$$a = c+1, \quad b = -6(c+1), \quad c = 3(c+9).$$

Substitute these into the discriminant formula:

$$[-6(c+1)]^2 - 4(c+1)[3(c+9)] = 0.$$

Simplify:

$$36(c+1)^2 - 12(c+1)(c+9) = 0,$$

$$12(c+1)[3(c+1) - (c+9)] = 0,$$

$$12(c+1)(2c-6) = 0.$$

Since $c \neq -1$, solve:

$$2c - 6 = 0 \quad \Rightarrow \quad c = 3.$$

—

Conclusion:

The value of c is 3.

Quick Tip

For quadratic equations, check the discriminant ($b^2 - 4ac$) to determine the nature of the roots. If $\Delta = 0$, the roots are real and equal.

Question 35: The following table shows the ages of the patients admitted to a hospital during a year:

Age (in years)	Number of patients
$5 - 15$	6
$15 - 25$	11
$25 - 35$	21
$35 - 45$	23
$45 - 55$	14
$55 - 65$	5

Find the mode and mean of the data given above.

Solution:

The table with calculated midpoints (x_i) and $x_i f_i$ is:

Age (in years)	**No. of patients (f_i)**	**Midpoint (x_i)**	$x_i f_i$
$5 - 15$	6	10	60
$15 - 25$	11	20	220
$25 - 35$	21	30	630
$35 - 45$	23	40	920
$45 - 55$	14	50	700
$55 - 65$	5	60	300
Total	**80**		**2830**

1. **Mean Calculation:**

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2830}{80} = 35.375 \,\text{years}.$$

2. **Mode Calculation:**

The modal class is 35–45, as it has the highest frequency ($f = 23$).

Using the formula for mode:

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2} \right) h,$$

where:

- $l = 35$,
- $f_m = 23$,
- $f_1 = 21$,
- $f_2 = 14$,
- $h = 10$.

Substitute the values:

$$\text{Mode} = 35 + \left(\frac{23 - 21}{2(23) - 21 - 14} \right) \times 10 = 35 + \left(\frac{2}{11} \right) \times 10 = 36.81 \,\text{years}.$$

—

Conclusion:

The mode and mean of the data are 36.81 years and 35.375 years, respectively.

Quick Tip

For grouped data, use the formulas for mean and mode:

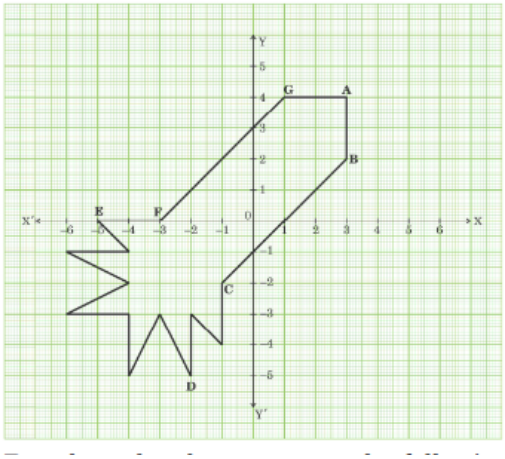
$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}, \quad \text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) h.$$

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Case Study - 1:

Question 36: Ryan, from a very young age, was fascinated by the twinkling of stars and the vastness of space. He always dreamt of becoming an astronaut one day. So he started to sketch his own rocket designs on the graph sheet. One such design is given below:



Based on the above, answer the following questions:

1. Find the mid-point of the segment joining F and G.
2. (A) What is the distance between the points A and C?

   (b) Find the coordinates of the point which divides the line segment joining the points A and B in the ratio $1 : 3$ internally.

3. What are the coordinates of the point D?

—

Solutions:

1. **Mid-point of F and G:**

   The coordinates of F and G are $(-3, 0)$ and $(1, 4)$, respectively.

   Using the mid-point formula:

$$\text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right),$$

$$\text{Mid-point} = \left(\frac{-3+1}{2}, \frac{0+4}{2}\right) = (-1, 2).$$

   Hence, the mid-point is $(-1, 2)$.

2. **(A) Distance between A and C:**

   The coordinates of A and C are $(3, -1)$ and $(-3, -4)$, respectively.

   Using the distance formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

$$\text{Distance} = \sqrt{(-3-3)^2 + (-4-(-1))^2} = \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45}.$$

   Hence, the distance is $\sqrt{52}$ or $2\sqrt{13}$.

   (b) Coordinates of the point dividing A and B in the ratio $1:3$:

   The coordinates of A and B are $(1, 2)$ and $(3, 4)$, respectively.

   Using the section formula:

$$\text{Point} = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right),$$

   where $m_1 = 1$, $m_2 = 3$. Substitute the values:

$$\text{Point} = \left(\frac{1 \cdot 3 + 3 \cdot 1}{1+3}, \frac{1 \cdot 4 + 3 \cdot 2}{1+3}\right) = \left(\frac{3+3}{4}, \frac{4+6}{4}\right) = \left(3, \frac{7}{2}\right).$$

3. **Coordinates of D:**

   From the graph, the coordinates of D are $(-2, -5)$.

—

Conclusion:

- The mid-point of F and G is $(-1, 2)$.
- The distance between A and C is $\sqrt{52}$ or $2\sqrt{13}$.
- The point dividing A and B in the ratio $1 : 3$ is $(3, \frac{7}{2})$.
- The coordinates of D are $(-2, -5)$.

Quick Tip

For solving coordinate geometry problems, use the mid-point formula, distance formula, and section formula systematically to compute the required values.

Case Study - 2:

Question 37: Treasure Hunt is an exciting and adventurous game where participants follow a series of clues/numbers/maps to discover hidden treasures. Players engage in a thrilling quest, solving puzzles and riddles to unveil the location of the coveted prize. While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forming an A.P. If the number on the nth spot is $20 + 4n$, then answer the following questions to help the players in spotting the clues:

1. Which number is on the first spot?
2. (A) Which spot is numbered as 112?

   (b) What is the sum of all the numbers on the first 10 spots?
3. Which number is on the $(n - 2)$th spot?

—

Solutions:

1. **Number on the first spot:**

   Substitute $n = 1$ into the formula $20 + 4n$:

   $$\text{Number on the first spot} = 20 + 4 \cdot 1 = 24.$$

2. **(A) Spot numbered 112:**

   Solve $20 + 4n = 112$:

   $$4n = 112 - 20 = 92 \quad \Rightarrow \quad n = \frac{92}{4} = 23.$$

   Hence, the 23rd spot is numbered 112.

   (b) Sum of the first 10 spots:

   The A.P. has first term $a = 24$ and common difference $d = 4$.

   The sum of the first n terms of an A.P. is:

   $$S_n = \frac{n}{2}[2a + (n-1)d].$$

   Substitute $n = 10$, $a = 24$, $d = 4$:

   $$S_{10} = \frac{10}{2}[2 \cdot 24 + 9 \cdot 4] = 5[48 + 36] = 5 \cdot 84 = 420.$$

   Hence, the sum of the first 10 spots is 420.

3. **Number on the $(n-2)$th spot:**

   Substitute $n - 2$ into the formula $20 + 4n$:

   $$\text{Number on the } (n-2)\text{th spot} = 20 + 4(n-2) = 20 + 4n - 8 = 12 + 4n.$$

   Hence, the number on the $(n-2)$th spot is $12 + 4n$.

—

Conclusion:

- The number on the first spot is 24.
- The spot numbered 112 is the 23rd spot.
- The sum of the first 10 spots is 420.

- The number on the $(n-2)$th spot is $12+4n$.

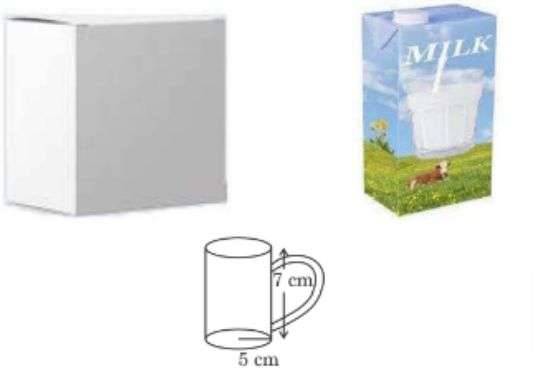
Quick Tip

For arithmetic progression problems, use the formulas for the nth term and sum of n terms:

$$a_n = a + (n-1)d, \quad S_n = \frac{n}{2}[2a + (n-1)d].$$

Case Study - 3:

Question 38: Tamper-proof tetra-packed milk guarantees both freshness and security. This milk ensures uncompromised quality, preserving the nutritional values within and making it a reliable choice for health-conscious individuals.



500 mL milk is packed in a cuboidal container of dimensions 15 cm$\times 8$ cm$\times 5$ cm. These milk packets are then packed in cuboidal cartons of dimensions 30 cm $\times$ 32 cm $\times$ 15 cm.

Based on the above information, answer the following questions:

1. Find the volume of the cuboidal carton.
2. (A) Find the total surface area of a milk packet.

   (b) How many milk packets can be filled in a carton?
3. How much milk can the cup (as shown in the figure) hold?

—

Solutions:

1. **Volume of the cuboidal carton:**

   The dimensions of the carton are 30 cm $\times$ 32 cm $\times$ 15 cm.

Using the volume formula for a cuboid:

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height} = 30 \times 32 \times 15 = 14400\,\text{cm}^3.$$

2. **(A) Total surface area of the milk packet:**

   The dimensions of the milk packet are $15\,\text{cm} \times 8\,\text{cm} \times 5\,\text{cm}$.

   Using the total surface area formula for a cuboid:

$$\text{Total Surface Area} = 2(\text{lb} + \text{bh} + \text{hl}),$$

   $$\text{Total Surface Area} = 2(15 \times 8 + 8 \times 5 + 5 \times 15) = 2(120 + 40 + 75) = 2 \times 235 = 470\,\text{cm}^2.$$

   (b) Number of milk packets in the carton:

   Volume of one milk packet:

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height} = 15 \times 8 \times 5 = 600\,\text{cm}^3.$$

   Number of packets:

$$\text{Number of packets} = \frac{\text{Volume of carton}}{\text{Volume of one packet}} = \frac{14400}{600} = 24.$$

   Hence, 24 packets can be filled in the carton.

3. **Capacity of the cup:**

   The cup is cylindrical with radius $r = 5\,\text{cm}$ and height $h = 7\,\text{cm}$.

   Using the volume formula for a cylinder:

$$\text{Volume} = \pi r^2 h,$$

$$\text{Volume} = \frac{22}{7} \times 5^2 \times 7 = \frac{22}{7} \times 25 \times 7 = 550\,\text{cm}^3.$$

   Hence, the cup can hold $550\,\text{mL}$ of milk.

—

Conclusion:

- The volume of the cuboidal carton is $14400\,\text{cm}^3$.
- The total surface area of the milk packet is $470\,\text{cm}^2$.
- 24 milk packets can be filled in the carton.

- The cup can hold 550 mL of milk.

Quick Tip

For 3D geometry problems, use the volume and surface area formulas specific to the given shape (cuboid or cylinder). Pay attention to unit conversions if required.
