CBSE Class X Mathematics (Standard) Set 2 (30/4/2) Questions with Solutions

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section consists of 20 questions of 1 mark each.

Question 1: If $\sin\theta = 1$**, then the value of** $\frac{1}{2}\sin\left(\frac{\theta}{2}\right)$ **is:**
(A) $\frac{1}{2\sqrt{2}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{2}$
(D) 0

Correct Answer: (A) $\frac{1}{2\sqrt{2}}$

Solution:
We are given $\sin\theta = 1$. This implies:

$$\theta = 90^\circ \quad (\text{since } \sin 90^\circ = 1).$$

Substitute $\theta = 90^\circ$ into the expression $\frac{1}{2}\sin\left(\frac{\theta}{2}\right)$:

$$\frac{1}{2}\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\sin\left(\frac{90^\circ}{2}\right) = \frac{1}{2}\sin(45^\circ).$$

We know $\sin 45^\circ = \frac{1}{\sqrt{2}}$, so:

$$\frac{1}{2}\sin(45^\circ) = \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

—

Conclusion:
The value of $\frac{1}{2}\sin\left(\frac{\theta}{2}\right)$ is $\frac{1}{2\sqrt{2}}$.

Quick Tip

Remember that $\sin 45^\circ = \frac{1}{\sqrt{2}}$. Use trigonometric identities and angle relationships to simplify such expressions.

Question 2: The ratio of total surface area of a solid hemisphere to the square of its radius is:
(A) $2\pi : 1$
(B) $4\pi : 1$
(C) $3\pi : 1$
(D) $1 : 4\pi$

Correct Answer: (C) $3\pi : 1$

Solution:
The total surface area of a solid hemisphere is the sum of its curved surface area and the area of its circular base:

$$\text{Total Surface Area} = 2\pi r^2 + \pi r^2 = 3\pi r^2.$$

The square of the radius is:

$$\text{Square of radius} = r^2.$$

The ratio of total surface area to the square of the radius is:

$$\text{Ratio} = \frac{3\pi r^2}{r^2} = 3\pi : 1.$$

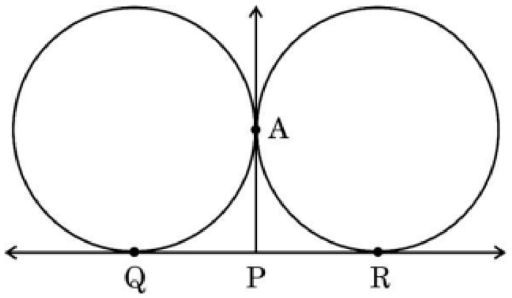
—

Conclusion:
The ratio of total surface area to the square of the radius is $3\pi : 1$.

Quick Tip

For solid hemispheres, remember that the total surface area includes both the curved surface and the base.

Question 3: In the given figure, QR is a common tangent to the two given circles touching externally at A. The tangent at A meets QR at P. If $AP = 4.2$ cm, then the length of QR is:



(A) 4.2 cm
(B) 2.1 cm
(C) 8.4 cm
(D) 6.3 cm

Correct Answer: (C) 8.4 cm

Solution:
In the given figure, the tangent QR is divided into two equal segments at P because the circles are symmetric, and the tangent passes through the point of contact A. Thus:

$$QR = 2 \times AP.$$

Substitute the given value $AP = 4.2$ cm:

$$QR = 2 \times 4.2 = 8.4 \text{ cm}.$$

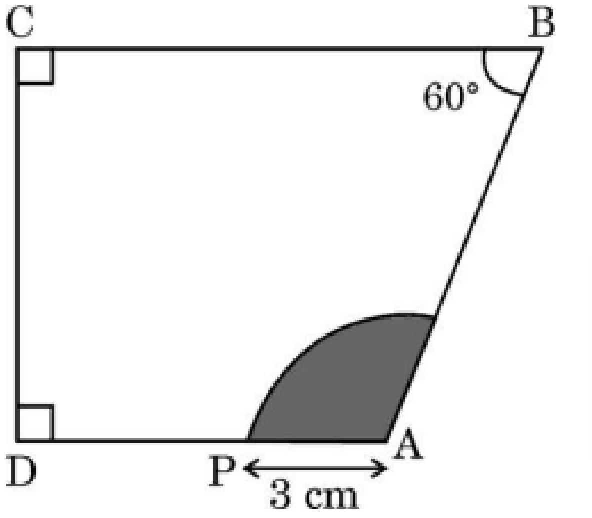
—

Conclusion:
The length of QR is 8.4 cm.

Quick Tip

For tangents between two externally touching circles, the tangent length is twice the distance from the point of contact to the tangent's midpoint.

Question 4: If in the given figure, $\angle C = \angle D = 90°$, $\angle B = 60°$, and $AP = 3$ cm, then the area of the shaded region is:



(A) $3\pi\,\text{cm}^2$
(B) $6\pi\,\text{cm}^2$
(C) $7\pi\,\text{cm}^2$
(D) $9\pi\,\text{cm}^2$

Correct Answer: (A) 3π **cm**^{2}

Solution:
The shaded region represents a sector of a circle with a central angle of $60°$. The radius of the sector is $3\,\text{cm}$ (distance AP). The area of a sector is calculated as:

$$\text{Area of sector} = \frac{\theta}{360°}\pi r^2$$

Substituting the values:

$$\text{Area of sector} = \frac{60°}{360°}\pi(3)^2 = \frac{1}{6}\pi(9) = 3\pi\,\text{cm}^2$$

Thus, the area of the shaded region is $3\pi\,\text{cm}^2$.

Quick Tip

For sector problems, remember to use $\frac{\theta}{360°}\pi r^2$ for the area, where θ is the central angle in degrees and r is the radius.

Question 5: If the mean of $6, 7, p, 8, q, 14$ is 9, then:

(A) $p - q = 19$
(B) $p + q = 19$
(C) $p - q = 21$
(D) $p + q = 21$

Correct Answer: (B) $p + q = 19$

Solution:
The formula for mean is:

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Substituting the given values:

$$9 = \frac{6 + 7 + p + 8 + q + 14}{6}$$

Simplify:

$$9 \times 6 = 6 + 7 + p + 8 + q + 14$$

$$54 = 35 + p + q$$

$$p + q = 19$$

Thus, the correct answer is $p + q = 19$.

> Quick Tip
>
> For mean calculations, always multiply the mean by the number of observations to find the total sum.

Question 6: At some time of the day, the length of the shadow of a tower is equal to its height. Then, the Sun's altitude at that time is:
(A) 30°
(B) 45°
(C) 60°
(D) 90°

Correct Answer: (B) 45°

Solution:
The shadow of the tower is equal to its height. Let the height of the tower be h and the length of its shadow also be h. The altitude of the Sun forms a right triangle, where:

$$\tan(\text{Sun's altitude}) = \frac{\text{Height of tower}}{\text{Length of shadow}}.$$

Substitute:

$$\tan(\text{Sun's altitude}) = \frac{h}{h} = 1.$$

We know:

$$\tan(45^\circ) = 1.$$

Thus, the Sun's altitude is 45°.
—
Conclusion:
The Sun's altitude is 45°.

Quick Tip

When the height and shadow of an object are equal, the angle of elevation is always 45°.

Question 7: If an arc subtends an angle of 90° at the centre of a circle, then the ratio of its length to the circumference of the circle is:
(A) 2 : 3
(B) 1 : 4
(C) 4 : 1
(D) 1 : 3

Correct Answer: (B) 1 : 4

Solution:
The length of an arc is proportional to the angle it subtends at the centre of the circle. The total circumference of a circle corresponds to an angle of 360°. Hence, the ratio of the arc length to the circumference is:

$$\text{Ratio} = \frac{\text{Angle subtended by arc}}{\text{Total angle of circle}} = \frac{90^\circ}{360^\circ}.$$

Simplify:

$$\text{Ratio} = \frac{90}{360} = \frac{1}{4}.$$

—

Conclusion:
The ratio of the arc length to the circumference is 1 : 4.

Quick Tip

The arc length ratio can be determined by dividing the angle subtended by the arc by 360°.

Question 8: If $ax + by = a^2 - b^2$ and $bx + ay = 0$, then the value of $x + y$ is:
(A) $a^2 - b^2$
(B) $a + b$
(C) $a - b$
(D) $a^2 + b^2$

Correct Answer: (C) $a - b$

Solution:
We are given two equations:

1. $ax + by = a^2 - b^2$

2. $bx + ay = 0$

From equation (2), solve for x or y in terms of the other variable. Assume $ay = -bx$, so:

$$y = -\frac{b}{a}x \quad \text{(substitute this into equation 1).}$$

Substituting $y = -\frac{b}{a}x$ into equation (1):

$$ax + b\left(-\frac{b}{a}x\right) = a^2 - b^2$$

Simplify:

$$ax - \frac{b^2}{a}x = a^2 - b^2$$

Combine terms:

$$\frac{a^2x - b^2x}{a} = a^2 - b^2$$

Factorize x:

$$x(a^2 - b^2) = a(a^2 - b^2)$$

Divide both sides by $a^2 - b^2$ (assuming $a \neq b$):

$$x = a$$

Substitute $x = a$ into $y = -\frac{b}{a}x$:

$$y = -\frac{b}{a}(a) = -b$$

Finally, calculate $x + y$:

$$x + y = a - b$$

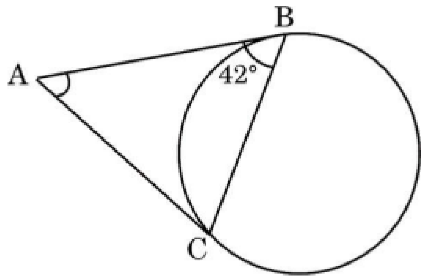
—

Conclusion:
The value of $x + y$ is $a - b$.

Quick Tip

When solving simultaneous equations, substitute one variable in terms of the other to simplify and solve step-by-step.

Question 9: In the given figure, AB and AC are tangents to the circle. If $\angle ABC = 42°$, then the measure of $\angle BAC$ is:



(A) $96°$
(B) $42°$
(C) $106°$
(D) $86°$

Correct Answer: (A) $96°$

Solution:
The tangents AB and AC meet at point A, and the angle between the tangents at the external point is calculated as follows:

$$\angle BAC = 180° - 2 \times \angle ABC$$

Substituting the values:

$$\angle BAC = 180° - 2 \times 42° = 180° - 84° = 96°$$

Thus, the measure of $\angle BAC$ is $96°$.

Quick Tip

For angles formed by tangents and circles, use the formula $\angle BAC = 180° - 2 \times \angle ABC$ to find the desired angle.

Question 10: If the discriminant of the quadratic equation $3x^2 - 2x + c = 0$ is 16, then the value of c is:

(A) 1
(B) 0
(C) -1
(D) $\sqrt{2}$

Correct Answer: (C) -1

Solution:
The discriminant (Δ) of a quadratic equation is given by:

$$\Delta = b^2 - 4ac$$

For the equation $3x^2 - 2x + c = 0$, $a = 3$, $b = -2$, and c is unknown. Substituting the given discriminant:

$$16 = (-2)^2 - 4 \cdot 3 \cdot c$$

Simplify:

$$16 = 4 - 12c$$

$$12c = 4 - 16 = -12$$

$$c = -1$$

Thus, the value of c is -1.

> **Quick Tip**
>
> Remember, for quadratic equations, $b^2 - 4ac$ determines the nature of roots: $\Delta > 0$ for real and distinct roots, $\Delta = 0$ for real and equal roots, and $\Delta < 0$ for complex roots.

Question 11: The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2,3)$, $B(6,7)$, and $C(8,3)$ is:
(A) $0, 1$
(B) $0, -1$
(C) $-1, 0$
(D) $1, 0$

Correct Answer: (B) $0, -1$

Solution:
In a parallelogram, the diagonals bisect each other. Let the coordinates of the fourth vertex D be (x, y). Using the midpoint formula, the midpoint of diagonal AC must be the same as the midpoint of diagonal BD.
Midpoint of AC:

$$\left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = (3,3).$$

Midpoint of BD:

$$\left(\frac{6+x}{2}, \frac{7+y}{2}\right).$$

Equating the midpoints:

$$\frac{6+x}{2} = 3 \quad \Rightarrow \quad 6+x=6 \quad \Rightarrow \quad x=0,$$

$$\frac{7+y}{2} = 3 \quad \Rightarrow \quad 7+y=6 \quad \Rightarrow \quad y=-1.$$

Thus, the coordinates of D are $(0, -1)$.
—
Conclusion:
The fourth vertex D is at $(0, -1)$.

> **Quick Tip**
>
> In a parallelogram, use the midpoint formula to find the unknown vertex by equating the midpoints of the diagonals.

Question 12: Two dice are tossed simultaneously. The probability of getting odd numbers on both the dice is:

(A) 6/36
(B) 3/36
(C) 12/36
(D) 9/36

Correct Answer: (D) 9/36

Solution:
Each die has 6 faces: $1, 2, 3, 4, 5, 6$. Odd numbers are $1, 3, 5$, so the probability of getting an odd number on one die is:

$$\text{P(odd on one die)} = \frac{3}{6} = \frac{1}{2}.$$

The two dice are independent, so the probability of getting odd numbers on both dice is:

$$\text{P(odd on both dice)} = \text{P(odd on first die)} \cdot \text{P(odd on second die)} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Convert to common fractions:

$$\text{P(odd on both dice)} = \frac{9}{36}.$$

—

Conclusion:
The probability of getting odd numbers on both dice is 9/36.

Quick Tip

For independent events, multiply individual probabilities to find the combined probability.

Question 13: Two lines are given to be parallel. The equation of one of these lines is $5x - 3y = 2$. The equation of the second line can be:
(A) $-15x - 9y = 5$
(B) $15x + 9y = 5$
(C) $9x - 15y = 6$
(D) $-15x + 9y = 5$

Correct Answer: (D) $-15x + 9y = 5$

Solution:
Two lines are parallel if their slopes are equal. The general equation of a line is given by:

$$Ax + By + C = 0.$$

The slope of the line is:

$$\text{Slope} = -\frac{A}{B}.$$

For the given line $5x - 3y = 2$, the slope is:

$$\text{Slope} = -\frac{5}{-3} = \frac{5}{3}.$$

Now check the options for the second line to find which has the same slope: - For $-15x+9y=5$:

$$\text{Slope} = -\frac{-15}{9} = \frac{5}{3}.$$

This matches the slope of the given line, so the two lines are parallel.

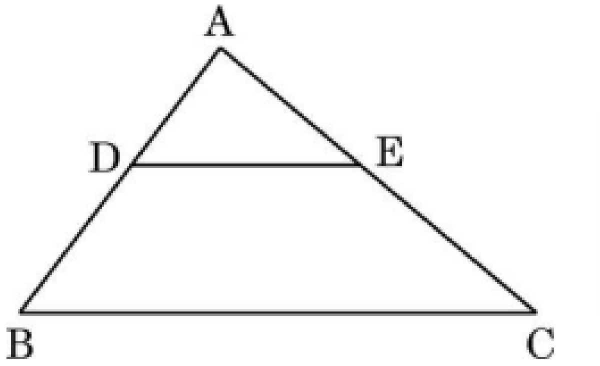
—

Conclusion:
The equation of the second line is $-15x+9y=5$.

Quick Tip

Parallel lines have the same slope. To verify, compare $-\frac{A}{B}$ for both equations.

Question 14: In $\triangle ABC$, $DE \parallel BC$ (as shown in the figure). If $AD = 4\,\text{cm}$, $AB = 9\,\text{cm}$, and $AC = 13.5\,\text{cm}$, then the length of EC is:



(A) 6 cm
(B) 7.5 cm
(C) 9 cm
(D) 5.7 cm

Correct Answer: (B) 7.5 cm

Solution:
Using the Basic Proportionality Theorem (Thales Theorem), which states that if a line is parallel to one side of a triangle and intersects the other two sides, it divides them in the same ratio:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given:

$$AD = 4\,\text{cm},\ AB = 9\,\text{cm},\ AC = 13.5\,\text{cm}$$

Calculate DB:

$$DB = AB - AD = 9 - 4 = 5\,\text{cm}$$

Substituting into the proportion:

$$\frac{4}{5} = \frac{AE}{EC}$$

Let $AE = x$ and $EC = y$. Since $AE + EC = AC$:

$$x + y = 13.5\,\text{cm}$$

Substitute $\frac{4}{5} = \frac{x}{y}$:

$$x = \frac{4}{5}y$$

Substitute $x = \frac{4}{5}y$ into $x + y = 13.5$:

$$\frac{4}{5}y + y = 13.5$$

Simplify:

$$\frac{9}{5}y = 13.5$$

$$y = \frac{13.5 \times 5}{9} = 7.5\,\text{cm}$$

Thus, $EC = 7.5\,\text{cm}$.

Quick Tip

For questions involving parallel lines in triangles, use the Basic Proportionality Theorem to establish the ratios of corresponding segments.

Question 15: From the letters of the word "MOBILE", a letter is selected at random. The probability that the selected letter is a vowel, is:

(A) $\frac{3}{7}$
(B) $\frac{1}{6}$
(C) $\frac{1}{2}$
(D) $\frac{1}{3}$

Correct Answer: (C) $\frac{1}{2}$

Solution:
The word "MOBILE" consists of 6 letters: M, O, B, I, L, E. Among these, the vowels are O, I, and E (3 vowels).
The total number of letters is 6. The probability of selecting a vowel is:

$$\text{Probability} = \frac{\text{Number of vowels}}{\text{Total letters}} = \frac{3}{6} = \frac{1}{2}.$$

Quick Tip

Always count the vowels (A, E, I, O, U) carefully, and divide by the total number of letters for probability questions.

Question 16: If $3825 = 3^x \times 5^y \times 17^z$, then the value of $x + y - 2z$ is:

(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (C) 2

Solution:
Factorize 3825:

$$3825 \div 3 = 1275, \quad 1275 \div 3 = 425 \quad \Rightarrow 3^2$$
$$425 \div 5 = 85, \quad 85 \div 5 = 17 \quad \Rightarrow 5^2 \text{ and } 17^1$$

Thus:

$$3825 = 3^2 \times 5^2 \times 17^1$$

Here:

$$x = 2,\ y = 2,\ z = 1$$

Now calculate:

$$x + y - 2z = 2 + 2 - 2 \times 1 = 2.$$

Quick Tip

For prime factorization questions, divide the number repeatedly by primes until it reduces to 1.

Question 17: A quadratic polynomial, one of whose zeroes is $2 + \sqrt{5}$ and the sum of whose zeroes is 4, is:

(A) $x^2 + 4x - 1$
(B) $x^2 - 4x - 1$
(C) $x^2 - 4x + 1$
(D) $x^2 + 4x + 1$

Correct Answer: (B) $x^2 - 4x - 1$

Solution:
Let the zeroes of the polynomial be $2 + \sqrt{5}$ and $2 - \sqrt{5}$. Using the sum and product of roots:

$$\text{Sum of roots} = (2 + \sqrt{5}) + (2 - \sqrt{5}) = 4,$$

$$\text{Product of roots} = (2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1.$$

The quadratic polynomial is given by:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = x^2 - 4x - 1.$$

Thus, the required polynomial is $x^2 - 4x - 1$.

Quick Tip

For quadratic equations, use the sum and product of roots formulas:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}).$$

Question 18: If the first three terms of an A.P. are $3p - 1$, $3p + 5$, and $5p + 1$ respectively; then the value of p is:

(A) 2
(B) -3
(C) 4
(D) 5

Correct Answer: (D) 5

Solution:
In an A.P., the difference between consecutive terms is constant. Hence:

$$(3p + 5) - (3p - 1) = (5p + 1) - (3p + 5).$$

Simplify both sides:

$$3p + 5 - 3p + 1 = 5p + 1 - 3p - 5,$$
$$6 = 2p - 4.$$

Solve for p:

$$2p = 6 + 4 = 10, \quad p = 5.$$

Thus, the value of p is 5.

Quick Tip

In arithmetic progressions, verify the common difference by equating consecutive term differences.

Question 19: Assertion (A): If the circumference of a circle is $176\,\text{cm}$, then its radius is $28\,\text{cm}$.
Reason (R): Circumference $= 2\pi \times$ radius of a circle.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).

Solution:
1. The formula for the circumference of a circle is:

$$\text{Circumference} = 2\pi r.$$

2. Given the circumference is 176 cm, solve for the radius:

$$176 = 2\pi r \quad \Rightarrow \quad r = \frac{176}{2\pi} = \frac{88}{\pi}.$$

Approximating $\pi \approx 3.14$:

$$r = \frac{88}{3.14} = 28\,\text{cm}.$$

3. Both the Assertion (A) and the Reason (R) are true, and Reason (R) correctly explains the Assertion (A).
—
Conclusion:
Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

Quick Tip

For problems involving circles, always use the formula Circumference $= 2\pi r$ and substitute known values to find unknown parameters.

Question 20: Assertion (A): The mid-point of a line segment divides the line segment in the ratio $1 : 1$.
Reason (R): The ratio in which the point $(-3, k)$ divides the line segment joining the points $(-5, 4)$ and $(-2, 3)$ is $1 : 2$.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:
1. The mid-point of a line segment always divides the segment into two equal parts, which means the ratio is $1 : 1$. Therefore, Assertion (A) is true.

2. To verify Reason (R), let the point $(-3, k)$ divide the segment joining $(-5, 4)$ and $(-2, 3)$. Using the section formula:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}.$$

For the x-coordinate:

$$-3 = \frac{1 \cdot (-2) + 2 \cdot (-5)}{1 + 2} = \frac{-2 - 10}{3} = \frac{-12}{3} = -4 \, (\text{which does not match } -3).$$

Thus, the point does not divide the line segment in the ratio $1 : 2$. Reason (R) is false.

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Conclusion:
Assertion (A) is true, but Reason (R) is false.

> **Quick Tip**
>
> Use the section formula to determine the ratio in which a point divides a line segment.

Section - B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

Question 21 (a): Evaluate:

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \sin^2 60^\circ}$$

Solution:
Substitute the trigonometric values:

$$\cos 60^\circ = \frac{1}{2}, \quad \sec 30^\circ = \frac{2}{\sqrt{3}}, \quad \tan 45^\circ = 1, \quad \sin 30^\circ = \frac{1}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}.$$

The expression becomes:

$$\frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}.$$

Simplify the numerator:

$$5 \cdot \frac{1}{4} + 4 \cdot \frac{4}{3} - 1 = \frac{5}{4} + \frac{16}{3} - 1.$$

Take the LCM:

$$\frac{15}{12} + \frac{64}{12} - \frac{12}{12} = \frac{67}{12}.$$

Simplify the denominator:

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1.$$

Thus, the value is:

$$\frac{\frac{67}{12}}{1} = \frac{67}{12}.$$

Question 21 (b): If $\sin(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}$, $0^\circ < A + B \leq 90^\circ$, $A > B$; find $\angle A$ and $\angle B$.

Solution:
We know:

$$\sin(A - B) = \sin 30^\circ \implies A - B = 30^\circ \quad (1)$$

$$\cos(A + B) = \cos 60^\circ \implies A + B = 60^\circ \quad (2)$$

Adding and subtracting (1) and (2):

$$(A + B) + (A - B) = 60^\circ + 30^\circ \implies 2A = 90^\circ \implies A = 45^\circ.$$

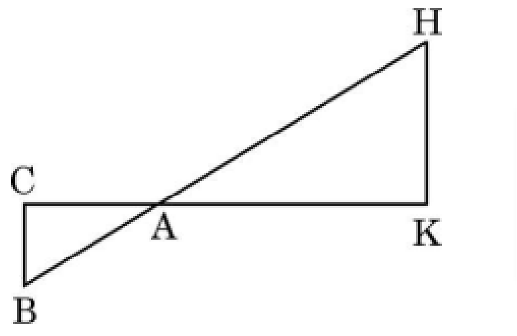
$$(A + B) - (A - B) = 60^\circ - 30^\circ \implies 2B = 30^\circ \implies B = 15^\circ.$$

Thus, $\angle A = 45^\circ$ and $\angle B = 15^\circ$.

Quick Tip

To solve trigonometric equations involving angles, use known values of trigonometric ratios and logical constraints on angles.

Question 22: In the given figure, $\triangle AHK \sim \triangle ABC$. If $AK = 8$ cm, $BC = 3.2$ cm, and $HK = 6.4$ cm, then find the length of AC.



Solution:
Since $\triangle AHK \sim \triangle ABC$, the corresponding sides are proportional:

$$\frac{AK}{AC} = \frac{HK}{BC}.$$

Substitute the known values:

$$\frac{8}{AC} = \frac{6.4}{3.2}.$$

Simplify:

$$\frac{8}{AC} = 2 \quad \Rightarrow \quad AC = \frac{8}{2} = 4\,\text{cm}.$$

—

Conclusion:
The length of AC is $4\,\text{cm}$.

Quick Tip

For similar triangles, use the property that corresponding sides are proportional to find unknown lengths.

Question 23: In a school, there are two sections of class X. There are 40 students in the first section and 48 students in the second section. Determine the minimum number of books required for their class library so that they can be distributed equally among students of both sections.

Solution:
To find the minimum number of books, calculate the LCM of the number of students in both sections.

$$40 = 2^3 \times 5, \quad 48 = 2^4 \times 3$$

$$\text{L.C.M.}(40, 48) = 2^4 \times 3 \times 5 = 240$$

Thus, the minimum number of books required in the library is:

$$\boxed{240}.$$

Quick Tip

To find the minimum items required for equal distribution, always calculate the LCM of the given quantities.

Question 24(a): The minute hand of a clock is $14\,\text{cm}$ long. Find the area on the face of the clock described by the minute hand in 5 minutes.

Solution:
The angle subtended by the minute hand in 5 minutes is:

$$\text{Angle} = \frac{360^\circ}{60} \times 5 = 30^\circ.$$

The area described by the minute hand is a sector of a circle, given by:

$$\text{Area} = \frac{\theta}{360^\circ} \cdot \pi r^2,$$

where $\theta = 30^\circ$ and $r = 14\,\text{cm}$.
Substitute the values:

$$\text{Area} = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14.$$

Simplify:

$$\text{Area} = \frac{1}{12} \times \frac{22}{7} \times 196 = \frac{154}{3}\,\text{cm}^2 \approx 51.33\,\text{cm}^2.$$

—

Conclusion:
The area described by the minute hand is approximately $51.33\,\text{cm}^2$.

> Quick Tip
>
> The area of a sector is proportional to the angle subtended by it at the center of the circle. Always express the angle in degrees when using the formula.

Question 24(b): Find the length of the arc of a circle that subtends an angle of $60°$ at the center of the circle of radius $42\,\text{cm}$.

Solution:
The formula for the length of an arc is:

$$\text{Length of arc} = 2\pi r \cdot \frac{\theta}{360},$$

where r is the radius and θ is the central angle in degrees.
Substitute the given values $r = 42\,\text{cm}$ and $\theta = 60°$:

$$\text{Length of arc} = 2 \times \frac{22}{7} \times 42 \times \frac{60}{360}.$$

Simplify step-by-step:

$$\text{Length of arc} = 2 \times \frac{22}{7} \times 42 \times \frac{1}{6}.$$

$$\text{Length of arc} = \frac{2 \times 22 \times 42}{7 \times 6}.$$

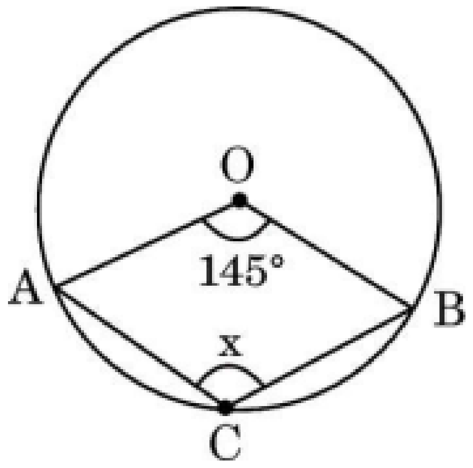
$$\text{Length of arc} = \frac{1848}{42} = 44\,\text{cm}.$$

—

Conclusion:
The length of the arc is $44\,\text{cm}$.

> Quick Tip
>
> The length of an arc is proportional to the central angle it subtends. Use Length of arc $= 2\pi r \cdot \frac{\theta}{360}$ for quick calculations.

Question 25: In the given figure, O is the center of the circle. If $\angle AOB = 145°$, find the value of x.



Solution:
In a circle, the angle subtended by an arc at the center is twice the angle subtended by the same arc at any point on the circumference.
1. Let P be a point on the circumference of the circle. Join AP and BP.
2. The angle subtended by the arc AB at the circumference is:

$$\angle APB = \frac{1}{2} \times \angle AOB = \frac{1}{2} \times 145^\circ = 72.5^\circ.$$

3. In $\triangle APB$, the sum of angles $\angle APB + \angle ACB = 180^\circ$ (angles on a straight line).
4. Solve for $\angle ACB$:

$$\angle ACB = 180^\circ - \angle APB = 180^\circ - 72.5^\circ = 107.5^\circ.$$

Thus:

$$x = \angle ACB = 107.5^\circ.$$

—

Conclusion:
The value of x is 107.5°.

> **Quick Tip**
>
> In a circle, the angle subtended by an arc at the center is always twice the angle subtended by the same arc at the circumference.

Section - C

This section comprises Short Answer (SA) type questions of 3 marks each.

Question 26(a): Three coins are tossed simultaneously. What is the probability of getting:

At least one head?
Exactly two tails?
At most one tail?

Solution:
The total number of outcomes when three coins are tossed is:

$$\text{Total outcomes} = 2^3 = 8 \quad (\text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}).$$

(i) Probability of at least one head:

$$P(\text{at least one head}) = 1 - P(\text{no heads}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

(ii) Probability of exactly two tails:

$$P(\text{exactly two tails}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{3}{8} \quad (\text{favorable outcomes: HTT, THT, TTH}).$$

(iii) Probability of at most one tail:

$$P(\text{at most one tail}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{4}{8} = \frac{1}{2}$$

(favorable outcomes: HHH, HHT, HTH, THH).

—

Conclusion:

- (i) $P(\text{at least one head}) = \frac{7}{8}$.
- (ii) $P(\text{exactly two tails}) = \frac{3}{8}$.
- (iii) $P(\text{at most one tail}) = \frac{1}{2}$.

Quick Tip

For coin toss problems, count all possible outcomes carefully and identify favorable cases for each condition.

Question 26(b): A box contains 90 discs which are numbered 1 to 90. If one disc is drawn at random from the box, find the probability that it bears:
A two-digit number less than 40.
A number divisible by 5 and greater than 50.
A perfect square number.

Solution:
The total number of outcomes is:

$$\text{Total outcomes} = 90.$$

(i) Probability of a two-digit number less than 40: Two-digit numbers less than 40 are 10 to 39 (inclusive). The total count is:

$$\text{Count} = 30 \quad (10,\ 11,\ 12,\ ...,\ 39).$$

$$P(\text{two-digit number less than 40}) = \frac{30}{90} = \frac{1}{3}.$$

(ii) Probability of a number divisible by 5 and greater than 50: Numbers divisible by 5 and greater than 50 are $55, 60, 65, 70, 75, 80, 85, 90$. The total count is:

$$\text{Count} = 8.$$

$$P(\text{divisible by 5 and greater than 50}) = \frac{8}{90} = \frac{4}{45}.$$

(iii) Probability of a perfect square number: Perfect squares from 1 to 90 are $1, 4, 9, 16, 25, 36, 49, 64, 81$. The total count is:

$$\text{Count} = 9.$$

$$P(\text{perfect square number}) = \frac{9}{90} = \frac{1}{10}.$$

—

Conclusion:

- (i) $P(\text{two-digit number less than 40}) = \frac{1}{3}$.
- (ii) $P(\text{divisible by 5 and greater than 50}) = \frac{4}{45}$.
- (iii) $P(\text{perfect square number}) = \frac{1}{10}$.

Quick Tip

For probability problems, list all favorable outcomes systematically and divide by the total possible outcomes.

Question 27: Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:
From the given figure:

1. $AP = AS \quad \text{(i)},$
2. $BP = BQ \quad \text{(ii)},$
3. $CR = CQ \quad \text{(iii)},$
4. $DR = DS \quad \text{(iv)}.$

Adding equations (i), (ii), (iii), and (iv):

$$AP + BP + CR + DR = AS + BQ + CQ + DS.$$

This simplifies to:

$$AB + CD = AD + BC.$$

Since $ABCD$ is a parallelogram, the opposite sides are equal:

$$AB = CD \quad \text{and} \quad AD = BC.$$

Substitute these equalities:

$$2AB = 2AD \quad \Rightarrow \quad AB = AD.$$

Thus, all sides of $ABCD$ are equal, which proves it is a rhombus.

—

Conclusion:
The parallelogram $ABCD$ circumscribing a circle is a rhombus.

Quick Tip

A parallelogram circumscribing a circle always has equal sides due to the tangential property of the circle, making it a rhombus.

Question 28: Prove that $\frac{2-\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Solution:
Assume $\frac{2-\sqrt{3}}{5}$ to be a rational number.
Let:

$$\frac{2-\sqrt{3}}{5} = \frac{p}{q},$$

where p and q are integers, and $q \neq 0$.
Rearrange the equation to isolate $\sqrt{3}$:

$$\sqrt{3} = \frac{2q - 5p}{q}.$$

Here:

- p and q are integers, so $2q - 5p$ and q are also integers.
- Therefore, $\frac{2q-5p}{q}$ is a rational number.

However, $\sqrt{3}$ is known to be an irrational number. This creates a contradiction.
Thus, our assumption that $\frac{2-\sqrt{3}}{5}$ is rational is false.

—

Conclusion:
$\frac{2-\sqrt{3}}{5}$ is an irrational number.

Quick Tip

To prove a number is irrational, assume it to be rational and show that this assumption leads to a contradiction.

Question 29: If $\sin\theta + \cos\theta = p$ and $\sec\theta + \csc\theta = q$, then prove that $q(p^2 - 1) = 2p$.

Solution:

$$\begin{aligned}
\text{L.H.S.} &= q(p^2 - 1) \\
&= (\sec\theta + \csc\theta)\left[(\sin\theta + \cos\theta)^2 - 1\right] \\
&= \left[\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right]\left[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\right] \\
&= \left[\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right]\left[1 + 2\sin\theta\cos\theta - 1\right] \\
&= 2(\sin\theta + \cos\theta) \\
&= 2p \\
&= \text{R.H.S.}
\end{aligned}$$

Quick Tip

Using trigonometric identities such as $\sin^2\theta + \cos^2\theta = 1$ and factoring common terms simplifies proofs like this efficiently.

Question 30(a): Find the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial.

Solution:
The given polynomial is:

$$P(x) = 4x^2 + 4x - 3 = (2x + 3)(2x - 1).$$

Zeroes of the polynomial:

$$x = -\frac{3}{2}, \quad x = \frac{1}{2}.$$

1. Verify the sum of the zeroes:

$$\text{Sum of zeroes} = -\frac{3}{2} + \frac{1}{2} = -\frac{4}{2} = -2.$$

This matches:

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}.$$

2. Verify the product of the zeroes:

$$\text{Product of zeroes} = -\frac{3}{2} \times \frac{1}{2} = -\frac{3}{4}.$$

This matches:

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}.$$

—

Conclusion:
The zeroes are $-\frac{3}{2}$ and $\frac{1}{2}$, and they satisfy the relationship between zeroes and coefficients of the polynomial.

> **Quick Tip**
>
> To verify zeroes of a polynomial, use the relationships: Sum of zeroes $= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$ and Product of zeroes $= \frac{\text{constant term}}{\text{coefficient of } x^2}$.

Question 30(b): If α and β are the zeroes of the polynomial x^2+x-2, find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

Solution:
From the given polynomial:

$$\alpha+\beta=-1, \quad \alpha\beta=-2.$$

We need to find:

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}.$$

1. Using the identity $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$:

$$\alpha^2+\beta^2=(-1)^2-2(-2)=1+4=5.$$

2. Substitute into the expression for $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$:

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{5}{-2}=-\frac{5}{2}.$$

—

Conclusion:
The value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is $-\frac{5}{2}$.

> **Quick Tip**
>
> For problems involving zeroes of polynomials, use the relationships: $\alpha+\beta=-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$ and $\alpha\beta=\frac{\text{constant term}}{\text{coefficient of } x^2}$.

Question 31: A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was 15,000 after 4 years of service and 18,000 after 10 years of service, what was his starting salary and what was the annual increment?

Solution:
Let his starting salary be a and annual increment be d.

From the question:

A.T.Q.

$$a + 3d = 15000 \quad \text{———(i)}$$

$$a + 9d = 18000 \quad \text{———(ii)}$$

These equations represent the salary progression where:

- $a + 3d$ is the salary after 4 years of service.
- $a + 9d$ is the salary after 10 years of service.

To find a (starting salary) and d (annual increment), we solve the equations simultaneously.
Step 1: Subtract Equation (i) from Equation (ii):

$$(a + 9d) - (a + 3d) = 18000 - 15000$$

$$6d = 3000 \quad \implies \quad d = 500$$

Step 2: Substitute $d = 500$ into Equation (i):

$$a + 3(500) = 15000$$

$$a + 1500 = 15000 \quad \implies \quad a = 13500$$

Final Answer:

- Starting salary = 13,500
- Annual increment = 500

Explanation: The problem uses the concept of arithmetic progression, where the salary increases each year by a fixed amount d. By forming equations for the salary after a specific number of years and solving them, we find both the initial salary and the annual increment.

Quick Tip

For problems involving salary increments, use arithmetic progression formulas and solve equations step-by-step for accurate results.

Section - D

This section comprises Long Answer (LA) type questions of 5 marks each.

Question 32(a): Find the value of k for which the quadratic equation $(k+1)x^2 - 2(3k+1)x + (8k+1) = 0$ has real and equal roots.

Solution:
For real and equal roots, the discriminant of the quadratic equation must be zero:

$$\Delta = [-2(3k+1)]^2 - 4(k+1)(8k+1) = 0$$

Simplify:

$$[-2(3k+1)]^2 = 4(3k+1)^2$$

$$4(k+1)(8k+1) = 4(8k^2+8k+k+1) = 32k^2+36k+4$$

Equating the discriminant to zero:

$$4(3k+1)^2 - 4(8k^2+9k+1) = 0$$

$$k^2 - 3k = 0$$

Factorize:

$$k(k-3) = 0$$

$$k = 0 \quad \text{or} \quad k = 3$$

Question 32(b): A 2-digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Solution:
Let the required number be $10x + y$, where x and y are the digits of the number.
Step 1: Write the given conditions as equations.

$$\text{Condition 1: } xy = 18 \quad \text{———(i)}$$

$$\text{Condition 2: } (10x+y) - 63 = 10y + x \quad \text{———(ii)}$$

Simplify Equation (ii):

$$10x + y - 63 = 10y + x$$

$$9x - 9y = 63$$

$$x - y = 7 \quad \text{———(iii)}$$

Step 2: Solve Equations (i) and (iii). From Equation (iii):

$$x = y + 7$$

Substitute $x = y + 7$ into Equation (i):

$$(y+7)y = 18$$

$$y^2 + 7y - 18 = 0$$

Factorize:

$$(y+9)(y-2) = 0$$

$$y = 2 \quad (\text{since } y > 0).$$

Substitute $y = 2$ into Equation (iii):

$$x - 2 = 7 \quad \Longrightarrow \quad x = 9$$

Step 3: Find the number. The required number is:

$$10x + y = 10(9) + 2 = 92$$

Final Answer: The required number is 92.

> Quick Tip
>
> For digit-based problems, represent the number as $10x + y$ and systematically use the given conditions to form equations.

Question 33: The following table shows the ages of the patients admitted to a hospital during a year:

Age (in years)	Number of patients
5 – 15	6
15 – 25	11
25 – 35	21
35 – 45	23
45 – 55	14
55 – 65	5

Find the mode and mean of the data given above.

Solution:
The table with calculated midpoints (x_i) and $x_i f_i$ is:

Age (in years)	No. of patients (f_i)	Midpoint (x_i)	$x_i f_i$
5 – 15	6	10	60
15 – 25	11	20	220
25 – 35	21	30	630
35 – 45	23	40	920
45 – 55	14	50	700
55 – 65	5	60	300
Total	**80**		**2830**

1. **Mean Calculation:**

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2830}{80} = 35.375\,\text{years}.$$

2. **Mode Calculation:**
The modal class is 35–45, as it has the highest frequency ($f = 23$).
Using the formula for mode:

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) h,$$

where:

- $l = 35$,
- $f_m = 23$,
- $f_1 = 21$,

- $f_2 = 14$,
- $h = 10$.

Substitute the values:

$$\text{Mode} = 35 + \left(\frac{23 - 21}{2(23) - 21 - 14} \right) \times 10 = 35 + \left(\frac{2}{11} \right) \times 10 = 36.81 \, \text{years}.$$

—

Conclusion:
The mode and mean of the data are 36.81 years and 35.375 years, respectively.

Quick Tip

For grouped data, use the formulas for mean and mode:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}, \quad \text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2} \right) h.$$

Question 34(a): E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.

Solution:
In $\triangle ABE$ and $\triangle CFB$:

- $\angle EAB = \angle BCF$ (corresponding angles of $\triangle ABE$ and $\triangle CFB$).
- $\angle AEB = \angle CBF$ (vertically opposite angles).

By the AA (Angle-Angle) similarity criterion:

$$\triangle ABE \sim \triangle CFB$$

Quick Tip

In proving triangle similarity, use the AA criterion, which requires two pairs of corresponding angles to be equal.

Question 34(b): Sides AB, BC and the median AD of $\triangle ABC$ are respectively proportional to sides PQ, QR and the median PM of another $\triangle PQR$. Prove that $\triangle ABC \sim \triangle PQR$.

Solution:
Given:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

Step 1: Show proportionality of medians.
Since the medians AD and PM divide the opposite sides proportionally:

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad \text{and} \quad \frac{AD}{PM} \text{ confirms proportionality.}$$

Step 2: Use the SSS similarity criterion.
From the given proportionality:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

By the SSS (Side-Side-Side) similarity criterion:

$$\triangle ABC \sim \triangle PQR$$

Conclusion: $\triangle ABC \sim \triangle PQR$.

Quick Tip

For proving similarity using SSS, show that the corresponding sides of the two triangles are proportional.

Question 35: The angles of depression of the top and the bottom of a 50 m high building from the top of a tower are 45° and 60°, respectively. Find the height of the tower. (Use $\sqrt{3} = 1.73$)

Solution:
Let AB be the tower of height H meters and CD be the building.
Step 1: Apply trigonometry in $\triangle ABD$.

$$\text{In } \triangle ABD, \quad \tan 60^\circ = \sqrt{3} = \frac{H}{x}$$

$$\implies H = \sqrt{3} \cdot x \quad \text{——(i)}$$

Step 2: Apply trigonometry in $\triangle AEC$.

$$\text{In } \triangle AEC, \quad \tan 45^\circ = 1 = \frac{H - 50}{x}$$

$$\implies H - 50 = x \quad \text{——(ii)}$$

Step 3: Solve equations (i) and (ii) to find H. From Equation (ii):

$$x = H - 50$$

Substitute $x = H - 50$ into Equation (i):

$$H = \sqrt{3} \cdot (H - 50)$$

$$H = \sqrt{3} \cdot H - 50\sqrt{3}$$

$$H - \sqrt{3} \cdot H = -50\sqrt{3}$$

$$H(1 - \sqrt{3}) = -50\sqrt{3}$$

$$H = \frac{50\sqrt{3}}{\sqrt{3}-1}$$

Step 4: Rationalize the denominator.

$$H = \frac{50\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$H = \frac{50\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$H = 25\sqrt{3}(\sqrt{3}+1)$$

$$H = 25(3+\sqrt{3}) = 75 + 25\sqrt{3}$$

Substitute $\sqrt{3} = 1.73$:

$$H = 75 + 25(1.73)$$

$$H = 75 + 43.25 = 118.25$$

Final Answer: The height of the tower is $118.25\,\text{m}$.

Quick Tip

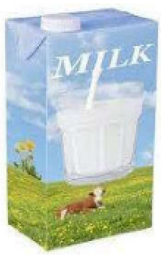
Use trigonometric ratios such as tan for angle of depression problems, and ensure all calculations are consistent with given values like $\sqrt{3}$.

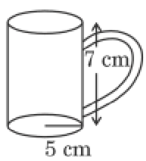
Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Case Study - 1:

Question 36: Tamper-proof tetra-packed milk guarantees both freshness and security. This milk ensures uncompromised quality, preserving the nutritional values within and making it a reliable choice for health-conscious individuals.





500 mL milk is packed in a cuboidal container of dimensions $15\,\text{cm} \times 8\,\text{cm} \times 5\,\text{cm}$. These milk packets are then packed in cuboidal cartons of dimensions $30\,\text{cm} \times 32\,\text{cm} \times 15\,\text{cm}$.

Based on the above information, answer the following questions:

1. Find the volume of the cuboidal carton.
2. (a) Find the total surface area of a milk packet.
   (b) How many milk packets can be filled in a carton?
3. How much milk can the cup (as shown in the figure) hold?

—

Solutions:

1. **Volume of the cuboidal carton:**
   The dimensions of the carton are $30\,\text{cm} \times 32\,\text{cm} \times 15\,\text{cm}$.
   Using the volume formula for a cuboid:
   $$\text{Volume} = \text{length} \times \text{breadth} \times \text{height} = 30 \times 32 \times 15 = 14400\,\text{cm}^3.$$
2. **(a) Total surface area of the milk packet:**
   The dimensions of the milk packet are $15\,\text{cm} \times 8\,\text{cm} \times 5\,\text{cm}$.
   Using the total surface area formula for a cuboid:
   $$\text{Total Surface Area} = 2(\text{lb} + \text{bh} + \text{hl}),$$
   $$\text{Total Surface Area} = 2(15 \times 8 + 8 \times 5 + 5 \times 15) = 2(120 + 40 + 75) = 2 \times 235 = 470\,\text{cm}^2.$$
   (b) Number of milk packets in the carton:
   Volume of one milk packet:
   $$\text{Volume} = \text{length} \times \text{breadth} \times \text{height} = 15 \times 8 \times 5 = 600\,\text{cm}^3.$$
   Number of packets:
   $$\text{Number of packets} = \frac{\text{Volume of carton}}{\text{Volume of one packet}} = \frac{14400}{600} = 24.$$
   Hence, 24 packets can be filled in the carton.
3. **Capacity of the cup:**
   The cup is cylindrical with radius $r = 5\,\text{cm}$ and height $h = 7\,\text{cm}$.
   Using the volume formula for a cylinder:
   $$\text{Volume} = \pi r^2 h,$$
   $$\text{Volume} = \frac{22}{7} \times 5^2 \times 7 = \frac{22}{7} \times 25 \times 7 = 550\,\text{cm}^3.$$
   Hence, the cup can hold 550 mL of milk.

—

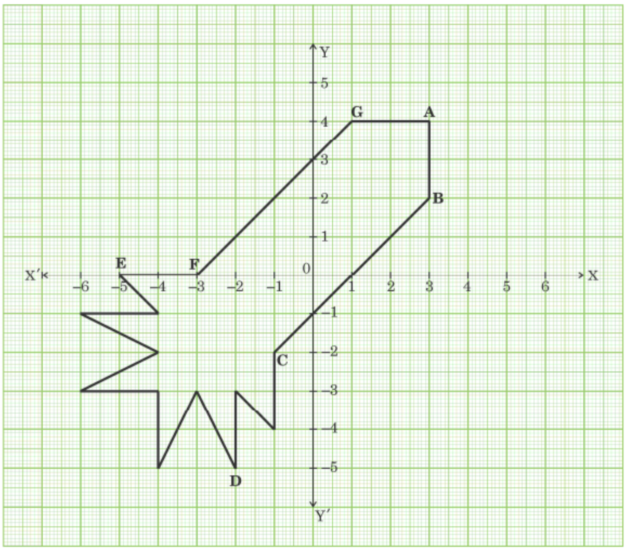
Conclusion:

- The volume of the cuboidal carton is $14400\,\text{cm}^3$.
- The total surface area of the milk packet is $470\,\text{cm}^2$.
- 24 milk packets can be filled in the carton.
- The cup can hold 550 mL of milk.

Quick Tip

For 3D geometry problems, use the volume and surface area formulas specific to the given shape (cuboid or cylinder). Pay attention to unit conversions if required.

Case Study - 2:
Question 37: Ryan, from a very young age, was fascinated by the twinkling of stars and the vastness of space. He always dreamt of becoming an astronaut one day. So he started to sketch his own rocket designs on the graph sheet. One such design is given below:



Based on the above, answer the following questions:

1. Find the mid-point of the segment joining F and G.
2. (a) What is the distance between the points A and C?
   (b) Find the coordinates of the point which divides the line segment joining the points A and B in the ratio 1 : 3 internally.
3. What are the coordinates of the point D?

—

Solutions:

1. **Mid-point of F and G:**
   The coordinates of F and G are $(-3, 0)$ and $(1, 4)$, respectively.
   Using the mid-point formula:

$$\text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right),$$

$$\text{Mid-point} = \left(\frac{-3 + 1}{2}, \frac{0 + 4}{2}\right) = (-1, 2).$$

   Hence, the mid-point is $(-1, 2)$.

2. **(a) Distance between A and C:**
The coordinates of A and C are $(3, -1)$ and $(-3, -4)$, respectively.
Using the distance formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

$$\text{Distance} = \sqrt{(-3-3)^2 + (-4-(-1))^2} = \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45}.$$

Hence, the distance is $\sqrt{52}$ or $2\sqrt{13}$.
(b) Coordinates of the point dividing A and B in the ratio $1:3$:
The coordinates of A and B are $(1, 2)$ and $(3, 4)$, respectively.
Using the section formula:

$$\text{Point} = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right),$$

where $m_1 = 1$, $m_2 = 3$. Substitute the values:

$$\text{Point} = \left(\frac{1\cdot 3 + 3\cdot 1}{1+3}, \frac{1\cdot 4 + 3\cdot 2}{1+3}\right) = \left(\frac{3+3}{4}, \frac{4+6}{4}\right) = \left(3, \frac{7}{2}\right).$$

3. **Coordinates of D:**
From the graph, the coordinates of D are $(-2, -5)$.

—

Conclusion:

- The mid-point of F and G is $(-1, 2)$.
- The distance between A and C is $\sqrt{52}$ or $2\sqrt{13}$.
- The point dividing A and B in the ratio $1:3$ is $(3, \frac{7}{2})$.
- The coordinates of D are $(-2, -5)$.

Quick Tip

For solving coordinate geometry problems, use the mid-point formula, distance formula, and section formula systematically to compute the required values.

Case Study - 3:
Question 38: Treasure Hunt is an exciting and adventurous game where participants follow a series of clues/numbers/maps to discover hidden treasures. Players engage in a thrilling quest, solving puzzles and riddles to unveil the location of the coveted prize.
While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forming an A.P. If the number on the nth spot is $20 + 4n$, then answer the following questions to help the players in spotting the clues:

1. Which number is on the first spot?
2. (a) Which spot is numbered as 112?
   (b) What is the sum of all the numbers on the first 10 spots?
3. Which number is on the $(n-2)$th spot?

—

Solutions:

1. **Number on the first spot:**
   Substitute $n = 1$ into the formula $20 + 4n$:
   $$\text{Number on the first spot} = 20 + 4 \cdot 1 = 24.$$
2. **(a) Spot numbered 112:**
   Solve $20 + 4n = 112$:
   $$4n = 112 - 20 = 92 \quad \Rightarrow \quad n = \frac{92}{4} = 23.$$
   Hence, the 23rd spot is numbered 112.
   (b) Sum of the first 10 spots:
   The A.P. has first term $a = 24$ and common difference $d = 4$.
   The sum of the first n terms of an A.P. is:
   $$S_n = \frac{n}{2}[2a + (n-1)d].$$
   Substitute $n = 10$, $a = 24$, $d = 4$:
   $$S_{10} = \frac{10}{2}[2 \cdot 24 + 9 \cdot 4] = 5[48 + 36] = 5 \cdot 84 = 420.$$
   Hence, the sum of the first 10 spots is 420.
3. **Number on the $(n-2)$th spot:**
   Substitute $n - 2$ into the formula $20 + 4n$:
   $$\text{Number on the } (n-2)\text{th spot} = 20 + 4(n-2) = 20 + 4n - 8 = 12 + 4n.$$
   Hence, the number on the $(n-2)$th spot is $12 + 4n$.

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Conclusion:

- The number on the first spot is 24.
- The spot numbered 112 is the 23rd spot.
- The sum of the first 10 spots is 420.
- The number on the $(n-2)$th spot is $12+4n$.

Quick Tip

For arithmetic progression problems, use the formulas for the nth term and sum of n terms:

$$a_n = a + (n-1)d, \quad S_n = \frac{n}{2}[2a + (n-1)d].$$
