CBSE Class X Mathematics (Standard) Set 1 (30/5/1) Questions with Solutions

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

Question 1: The next (4^{th}) term of the A.P. $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, ... is:
(A) $\sqrt{128}$
(B) $\sqrt{140}$
(C) $\sqrt{162}$
(D) $\sqrt{200}$

Correct Answer: (C) $\sqrt{162}$

Solution:
The given sequence $\sqrt{18},\ \sqrt{50},\ \sqrt{98},\ \ldots$ is in arithmetic progression (A.P.) since the difference between consecutive terms is constant.
Step 1: Calculate the common difference (d):

$$d = \sqrt{50} - \sqrt{18}.$$

Simplify each term:

$$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}, \quad \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}.$$

Thus:

$$d = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}.$$

Step 2: Find the 4^{th} term:
The general term of an A.P. is given by:

$$a_n = a + (n-1)d,$$

where $a = \sqrt{18} = 3\sqrt{2}$, $n = 4$, and $d = 2\sqrt{2}$. Substituting:

$$a_4 = 3\sqrt{2} + (4-1)(2\sqrt{2}) = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}.$$

Simplify:

$$a_4 = \sqrt{81 \cdot 2} = \sqrt{162}.$$

—
Conclusion:
The 4^{th} term is $\sqrt{162}$.

Quick Tip

To find the next term in an A.P., calculate the common difference and apply the formula for the n-th term.

Question 2: If $\frac{x}{3} = 2\sin A, \frac{y}{3} = 2\cos A$, then the value of $x^2 + y^2$ is:
(A) 36
(B) 9
(C) 6
(D) 18

Correct Answer: (A) 36

Solution:
We are given:

$$\frac{x}{3} = 2\sin A \quad \text{and} \quad \frac{y}{3} = 2\cos A.$$

Multiplying through by 3:

$$x = 6\sin A \quad \text{and} \quad y = 6\cos A.$$

The sum of squares is:

$$x^2 + y^2 = (6\sin A)^2 + (6\cos A)^2 = 36(\sin^2 A + \cos^2 A).$$

Using $\sin^2 A + \cos^2 A = 1$:

$$x^2 + y^2 = 36.$$

—

Conclusion:
The value of $x^2 + y^2$ is 36.

Quick Tip

Remember the Pythagorean identity: $\sin^2\theta + \cos^2\theta = 1$, which simplifies problems involving sums of squares.

Question 3: If $4\sec\theta - 5 = 0$, then the value of $\cot\theta$ is:
(A) $\frac{3}{4}$
(B) $\frac{4}{5}$
(C) $\frac{5}{3}$
(D) $\frac{4}{3}$

Correct Answer: (D) $\frac{4}{3}$

Solution:
Given:

$$4\sec\theta - 5 = 0 \quad \Longrightarrow \quad \sec\theta = \frac{5}{4}.$$

We know:

$$\sec^2\theta = 1 + \tan^2\theta.$$

Substitute $\sec\theta = \frac{5}{4}$:

$$\left(\frac{5}{4}\right)^2 = 1 + \tan^2\theta.$$

Simplify:

$$\frac{25}{16} = 1 + \tan^2\theta.$$

$$\tan^2\theta = \frac{25}{16} - 1 = \frac{9}{16}.$$

$$\tan\theta = \frac{3}{4}.$$

The reciprocal of $\tan\theta$ gives $\cot\theta$:

$$\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}.$$

—

Conclusion:
The value of $\cot\theta$ is $\frac{4}{3}$.

Quick Tip

For problems involving trigonometric functions, use known identities like $\sec^2\theta = 1 + \tan^2\theta$ to relate and calculate values.

Question 4: Which out of the following types of straight lines will be represented by the system of equations $3x + 4y = 5$ and $6x + 8y = 7$?
(A) Parallel
(B) Intersecting
(C) Coincident
(D) Perpendicular to each other

Correct Answer: (A) Parallel

Solution:
The given equations are:

$$1.\ 3x + 4y = 5 \quad \text{and} \quad 2.\ 6x + 8y = 7$$

Rewrite Equation 2 to check if it is a multiple of Equation 1:

$$6x + 8y = 7 \quad \Longrightarrow \quad 2(3x + 4y) = 7$$

Since the constant terms (5 and 7) are not in the same ratio as the coefficients, the lines are not coincident. The coefficients of x and y are in the same ratio:

$$\frac{3}{6} = \frac{4}{8}.$$

This implies the lines are parallel.
—
Conclusion:
The lines represented by the equations are parallel.

Quick Tip

For a system of linear equations, lines are parallel if the ratios of the coefficients of x and y are equal but differ in constant terms.

Question 5: The ratio of the sum and product of the roots of the quadratic equation $5x^2 - 6x + 21 = 0$ is:
(A) 5 : 21
(B) 2 : 7
(C) 21 : 5
(D) 7 : 2

Correct Answer: (B) 2 : 7

Solution:
For a quadratic equation of the form:

$$ax^2 + bx + c = 0,$$

the sum of the roots is:

$$-\frac{b}{a}.$$

The product of the roots is:

$$\frac{c}{a}.$$

Substitute the values $a = 5$, $b = -6$, and $c = 21$:

$$\text{Sum of the roots: } -\frac{-6}{5} = \frac{6}{5}.$$

$$\text{Product of the roots: } \frac{21}{5}.$$

Find the ratio of the sum to the product:

$$\text{Ratio} = \frac{\frac{6}{5}}{\frac{21}{5}} = \frac{6}{21} = \frac{2}{7}.$$

—

Conclusion:
The ratio of the sum and product of the roots is $2 : 7$.

Quick Tip

For quadratic equations, use the relationships between coefficients and roots: Sum of roots $= -\frac{b}{a}$, Product of roots $= \frac{c}{a}$.

Question 6: For the data $2, 9, x + 6, 2x + 3, 5, 10, 5$; if the mean is 7, then the value of x is:
(A) 9
(B) 6
(C) 5
(D) 3

Correct Answer: (D) 3

Solution:
The mean of a data set is given by:

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}.$$

Here, the sum of observations is:

$$2 + 9 + (x + 6) + (2x + 3) + 5 + 10 + 5 = 40 + 3x.$$

The number of observations is 7. Substituting into the formula for mean:

$$7 = \frac{40 + 3x}{7}.$$

Multiply through by 7:

$$40 + 3x = 49.$$

Simplify:

$$3x = 9 \quad \Longrightarrow \quad x = 3.$$

—

Conclusion:
The value of x is 3.

Quick Tip

For mean calculations, multiply the given mean by the number of terms to equate it to the sum of the terms.

Question 7: One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 7 is:
(A) $\frac{1}{7}$
(B) $\frac{1}{8}$
(C) $\frac{1}{5}$
(D) $\frac{7}{40}$

Correct Answer: (B) $\frac{1}{8}$

Solution:
The multiples of 7 between 1 and 40 are:

$$7, 14, 21, 28, 35.$$

There are 5 multiples of 7, and the total number of tickets is 40. The probability is:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{5}{40} = \frac{1}{8}.$$

—

Conclusion:
The probability is $\frac{1}{8}$.

Quick Tip

For probability problems, identify the total possible outcomes and favorable outcomes clearly before simplifying the ratio.

Question 8: The perimeter of the sector of a circle of radius 21 cm which subtends an angle of 60° at the center of the circle is:
(A) 22 cm
(B) 43 cm
(C) 64 cm
(D) 462 cm

Correct Answer: (C) 64 cm

Solution:
The perimeter of the sector is given by:

$$\text{Perimeter} = 2r + \text{Arc length}.$$

The arc length is:

$$\text{Arc length} = \frac{\theta}{360^\circ} \cdot 2\pi r.$$

Substitute $\theta = 60^\circ$, $r = 21$, and $\pi = \frac{22}{7}$:

$$\text{Arc length} = \frac{60}{360} \cdot 2 \cdot \frac{22}{7} \cdot 21 = \frac{1}{6} \cdot 2 \cdot \frac{22}{7} \cdot 21 = 22\,\text{cm}.$$

The perimeter is:

$$\text{Perimeter} = 2(21) + 22 = 42 + 22 = 64\,\text{cm}.$$

—

Conclusion:
The perimeter is 64 cm.

> **Quick Tip**
>
> For sector problems, always add the arc length and twice the radius to find the total perimeter.

Question 9: The length of an arc of a circle with radius 12 cm is 10π cm. The angle subtended by the arc at the center of the circle is:
(A) 120°
(B) 6°
(C) 75°
(D) 150°

Correct Answer: (D) 150°

Solution:
The length of an arc is given by:

$$\text{Arc length} = \frac{\theta}{360^\circ} \cdot 2\pi r.$$

Substitute Arc length $= 10\pi$, $r = 12$, and solve for θ:

$$10\pi = \frac{\theta}{360} \cdot 2\pi \cdot 12.$$

Simplify:

$$10 = \frac{\theta}{360} \cdot 24 \quad \Longrightarrow \quad \frac{\theta}{360} = \frac{10}{24}.$$

$$\theta = \frac{10}{24} \cdot 360 = 150^\circ.$$

—

Conclusion:
The angle subtended by the arc is 150°.

Quick Tip

For arc length problems, substitute known values into the formula and solve for the unknown step-by-step.

Question 10: The greatest number which divides 281 and 1249, leaving remainders 5 and 7 respectively, is:
(A) 23
(B) 276
(C) 138
(D) 69

Correct Answer: (C) 138

Solution:
If a number N divides 281 leaving a remainder of 5 and 1249 leaving a remainder of 7, then:

$$281 - 5 = 276 \quad \text{and} \quad 1249 - 7 = 1242$$

must be divisible by N.
Thus, N is the greatest common divisor (GCD) of 276 and 1242.
Step 1: Find the GCD of 276 and 1242. Using the Euclidean algorithm:

$$1242 = 276 \cdot 4 + 138$$

$$276 = 138 \cdot 2 + 0$$

The GCD is 138.

—

Conclusion:
The greatest number which divides 281 and 1249, leaving the specified remainders, is 138.

Quick Tip

For such problems, subtract the remainders from the given numbers and find the GCD of the resulting values.

Question 11: The number of terms in the A.P. $3, 6, 9, 12, \ldots, 111$ **is:**
(A) 36
(B) 40
(C) 37
(D) 30

Correct Answer: (C) 37

Solution:
The general formula for the n-th term of an A.P. is:

$$a_n = a + (n-1)d,$$

where a is the first term, d is the common difference, and n is the number of terms.
Here:

$$a = 3,\ d = 3,\ a_n = 111.$$

Substitute into the formula:

$$111 = 3 + (n-1) \cdot 3.$$

Simplify:

$$111 - 3 = 3(n-1).$$

$$108 = 3(n-1).$$

$$n - 1 = 36 \quad \Longrightarrow \quad n = 37.$$

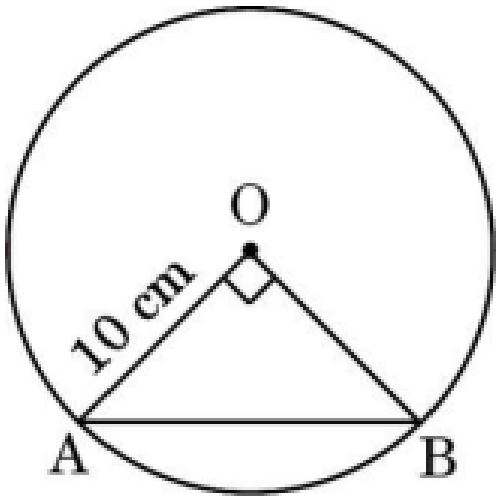
—

Conclusion:
The number of terms in the A.P. is 37.

Quick Tip

For A.P. problems, rearrange the formula for the n-th term to find the total number of terms.

Question 12: A chord of a circle of radius 10 cm subtends a right angle at its center. The length of the chord (in cm) is:



(A) $5\sqrt{2}$
(B) $10\sqrt{2}$
(C) $\frac{5}{\sqrt{2}}$
(D) 5

Correct Answer: (B) $10\sqrt{2}$

Solution:
Given: - Radius $r = 10\,\text{cm}$. - The chord subtends a 90° angle at the center.
In $\triangle OAB$, O is the center, and $\angle AOB = 90^\circ$. Using the Pythagoras theorem:

$$\text{Chord length (AB)} = \sqrt{OA^2 + OB^2}.$$

Substitute $OA = OB = r = 10$:

$$\text{Chord length (AB)} = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}.$$

—

Conclusion:
The length of the chord is $10\sqrt{2}$.

Quick Tip

For chords subtending 90° at the center, use the Pythagoras theorem to calculate the chord length.

Question 13: The LCM of three numbers $28, 44, 132$ **is:**
(A) 258
(B) 231
(C) 462
(D) 924

Correct Answer: (D) 924

Solution:
To find the LCM of $28, 44, 132$: 1. Perform prime factorization:

$$28 = 2^2 \cdot 7, \quad 44 = 2^2 \cdot 11, \quad 132 = 2^2 \cdot 3 \cdot 11.$$

2. Take the highest powers of all prime factors:

$$\text{LCM} = 2^2 \cdot 3 \cdot 7 \cdot 11 = 924.$$

—

Conclusion:
The LCM of $28, 44, 132$ is 924.

Quick Tip

For LCM, take the highest powers of all prime factors common or unique to the given numbers.

Question 14: If the product of two co-prime numbers is 553, then their HCF is:
(A) 1
(B) 553
(C) 7
(D) 79

Correct Answer: (A) 1

Solution:
Co-prime numbers are numbers that have no common factors other than 1.
By definition, the HCF of any two co-prime numbers is always 1.
—
Conclusion:
The HCF of the two co-prime numbers is 1.

Quick Tip

Co-prime numbers always have an HCF of 1, as they share no common factors other than 1.

Question 15: If α and β are the zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$ and $\alpha + \beta = \alpha\beta$, then the value of k is:
(A) $-\frac{2}{3}$
(B) $-\frac{3}{2}$
(C) $\frac{3}{2}$
(D) $\frac{2}{3}$

Correct Answer: (D) $\frac{2}{3}$

Solution:
The polynomial is:

$$p(x) = kx^2 - 30x + 45k.$$

The sum and product of the roots ($\alpha + \beta$ and $\alpha\beta$) are given by:

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{-30}{k} = \frac{30}{k}.$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{45k}{k} = 45.$$

Given:

$$\alpha + \beta = \alpha\beta.$$

Substitute the values:

$$\frac{30}{k} = 45.$$

Solve for k:

$$30 = 45k \quad \Longrightarrow \quad k = \frac{30}{45} = \frac{2}{3}.$$

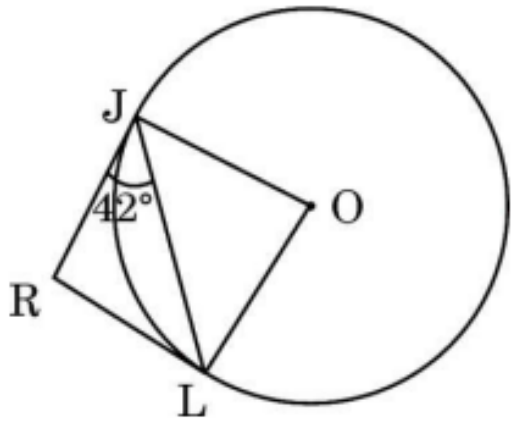
—

Conclusion:
The value of k is $\frac{2}{3}$.

Quick Tip

For problems involving zeroes of polynomials, use the relationships $\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ and $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$.

Question 16: In the given figure, RJ and RL are two tangents to the circle. If $\angle RJL = 42°$, then the measure of $\angle JOL$ is:



(A) 42°
(B) 84°
(C) 96°
(D) 138°

Correct Answer: (B) 84°

Solution:
Given:

$$\angle RJL = 42^\circ, \quad \text{RJ and RL are tangents.}$$

In a circle, the angle formed by the tangents at the external point ($\angle RJL$) is half the angle subtended by the chord at the center ($\angle JOL$):

$$\angle JOL = 2 \cdot \angle RJL.$$

Substitute the given value:

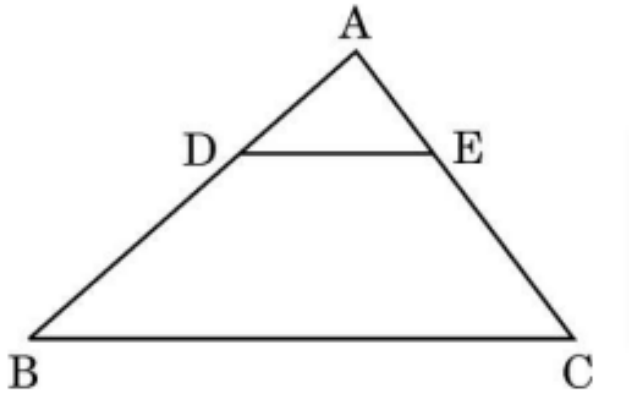
$$\angle JOL = 2 \cdot 42^\circ = 84^\circ.$$

—

Conclusion:
The measure of $\angle JOL$ is 84°.

> **Quick Tip**
>
> For tangents and angles in circles, the angle subtended at the center is twice the angle between the tangents.

Question 17: In the given figure, in $\triangle ABC$, $DE \parallel BC$. If $AD = 2.4\,\text{cm}, DB = 4\,\text{cm}$, and $AE = 2\,\text{cm}$, then the length of AC is:



(A) $10\frac{3}{10}$ cm
(B) $\frac{3}{10}$ cm
(C) $16\frac{3}{3}$ cm
(D) 1.2 cm

Correct Answer: (C) $16\frac{3}{3}$ cm

Solution:
In the given figure, $DE \parallel BC$, so by the Basic Proportionality Theorem (Thales' Theorem):

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 1: Use the property of proportions.
Given:

$$AD = 2.4\,\text{cm}, \quad DB = 4\,\text{cm}, \quad AE = 2\,\text{cm}.$$

Substitute the values:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

$$\frac{2.4}{4} = \frac{2}{EC}.$$

Simplify:

$$EC = \frac{4 \cdot 2}{2.4} = \frac{8}{2.4} = 3.33\,\text{cm}.$$

Step 2: Find the total length of AC.
The total length of AC is:

$$AC = AE + EC.$$

Substitute the values:

$$AC = 2 + 3.33 = 5.33\,\text{cm}.$$

—

Conclusion:
The length of AC is $16\frac{2}{3}\,\text{cm}$.

Quick Tip

For parallel lines in triangles, use the proportionality rule: $\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$.

Question 18: If a vertical pole of length $7.5\,\text{m}$ casts a shadow $5\,\text{m}$ long on the ground and at the same time, a tower casts a shadow $24\,\text{m}$ long, then the height of the tower is:
(A) $20\,\text{m}$
(B) $40\,\text{m}$
(C) $60\,\text{m}$
(D) $36\,\text{m}$

Correct Answer: (D) $36\,\text{m}$

Solution:
The problem involves similar triangles formed by the pole and its shadow, and the tower and its shadow. The ratios of their heights to their respective shadow lengths are equal.
Let the height of the tower be H. Using the property of similar triangles:

$$\frac{\text{Height of the pole}}{\text{Length of the pole's shadow}} = \frac{\text{Height of the tower}}{\text{Length of the tower's shadow}}.$$

Substitute the given values:

$$\frac{7.5}{5} = \frac{H}{24}.$$

Simplify:

$$H = \frac{7.5}{5} \cdot 24 = 36\,\text{m}.$$

—

Conclusion:
The height of the tower is 36 m.

Quick Tip

For shadow problems, use the proportionality property of similar triangles: $\frac{\text{Height}}{\text{Shadow length}}$ is constant for objects under the same light source.

Questions 19 and 20 are Assertion and Reason-based questions.
Two statements are given, one labelled as Assertion (A) and the other as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C), and (D) as given below:
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

—

Question 19:
Assertion (A): ABCD is a trapezium with $DC \parallel AB$. E and F are points on AD and BC, respectively, such that $EF \parallel AB$. Then:

$$\frac{AE}{ED} = \frac{BF}{FC}.$$

Reason (R): Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Solution:
Both Assertion (A) and Reason (R) are true. The line EF, being parallel to the parallel sides of the trapezium (AB and DC), divides the non-parallel sides proportionally. The Reason (R) provides the correct explanation for Assertion (A).

—

Correct Answer: (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

Quick Tip

Use the property of proportionality in trapeziums: A line parallel to the parallel sides divides the non-parallel sides proportionally.

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Question 20:
Assertion (A): The degree of a zero polynomial is not defined.
Reason (R): The degree of a non-zero constant polynomial is 0.

Solution:
Both Assertion (A) and Reason (R) are true. However, Reason (R) is not the correct explanation for Assertion (A). The degree of a zero polynomial is undefined because it does not have any terms, while the degree of a non-zero constant polynomial is defined as 0.

—

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of Assertion (A).

Quick Tip

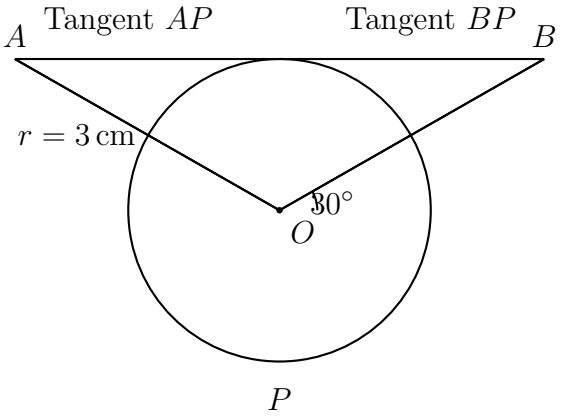
Remember: The degree of a zero polynomial is undefined, while the degree of any non-zero constant polynomial is 0.

Section - B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

Question 21(a): If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

Solution:
Given: - Radius of the circle: $r = 3\,\text{cm}$, - Angle between the tangents: $\angle APB = 60^\circ$.



Step 1: Analyze the geometry.
The tangents form two right triangles with the center of the circle. In $\triangle APO$, where O is the center of the circle:

$$\angle APO = \frac{\angle APB}{2} = \frac{60^\circ}{2} = 30^\circ.$$

Step 2: Use trigonometric ratios.
From $\triangle APO$, using $\tan 30^\circ$:

$$\tan 30^\circ = \frac{\text{Opposite side (radius)}}{\text{Adjacent side (tangent length)}}.$$

Substitute the values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \frac{1}{\sqrt{3}} = \frac{3}{AP}.$$

Solve for AP:

$$AP = 3\sqrt{3}\,\text{cm}.$$

Step 3: Finalize the result.
Since the tangents are symmetrical, the length of each tangent is:

$$AP = 3\sqrt{3}\,\text{cm}.$$

—

Conclusion:
The length of each tangent is $3\sqrt{3}\,\text{cm}$.

> Quick Tip
>
> When two tangents are drawn to a circle, use trigonometric ratios in the formed right triangles to find tangent lengths.

Question 21(b): Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:
Let AB be the diameter of a circle with center O, and let P and Q be the tangents at A and B, respectively.
Step 1: Analyze the geometry.
The tangent to a circle is perpendicular to the radius at the point of tangency. Therefore:

$$\angle OAY = 90^\circ \quad \text{and} \quad \angle OBP = 90^\circ.$$

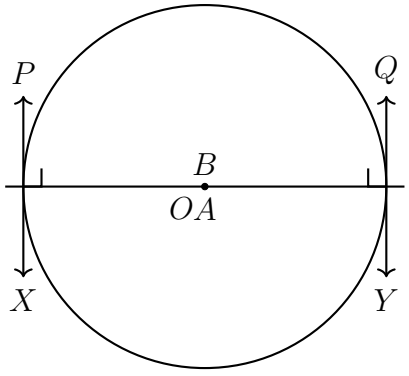
Step 2: Prove parallelism.
The angles $\angle OAY$ and $\angle OBP$ are equal and form alternate interior angles between the lines PQ and XY. Hence, by the property of alternate interior angles:

$$PQ \parallel XY.$$

—

Conclusion:
The tangents drawn at the ends of a diameter of a circle are parallel.

Diagram:



Quick Tip

The tangents at the ends of the diameter are always parallel because they form equal alternate interior angles with the line joining the ends of the diameter.

Question 22: Evaluate:

$$\frac{2\tan 30^\circ \cdot \sec 60^\circ \cdot \tan 45^\circ}{1 - \sin^2 60^\circ}.$$

Solution:

Substitute the trigonometric values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \sec 60^\circ = 2, \quad \tan 45^\circ = 1, \quad \sin^2 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

Simplify the numerator:

$$2 \cdot \frac{1}{\sqrt{3}} \cdot 2 \cdot 1 = \frac{4}{\sqrt{3}}.$$

Simplify the denominator:

$$1 - \sin^2 60^\circ = 1 - \frac{3}{4} = \frac{1}{4}.$$

The entire expression becomes:

$$\frac{\frac{4}{\sqrt{3}}}{\frac{1}{4}} = \frac{4}{\sqrt{3}} \cdot 4 = \frac{16}{\sqrt{3}}.$$

Rationalize the denominator:

$$\frac{16}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{16\sqrt{3}}{3}.$$

—

Conclusion:

The value of the expression is:

$$\frac{16\sqrt{3}}{3}.$$

Quick Tip

Always simplify trigonometric functions step-by-step and rationalize the denominator when necessary.

Question 23: If α, β are zeroes of the polynomial $p(x) = 5x^2 - 6x + 1$, then find the value of $\alpha + \beta + \alpha\beta$.

Solution:

For a polynomial $ax^2 + bx + c$, the sum and product of the roots are:

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{-6}{5} = \frac{6}{5}.$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{1}{5}.$$

Add $\alpha + \beta$ and $\alpha\beta$:

$$\alpha + \beta + \alpha\beta = \frac{6}{5} + \frac{1}{5} = \frac{7}{5}.$$

—

Conclusion:
The value of $\alpha + \beta + \alpha\beta$ is:

$$\frac{7}{5}.$$

Quick Tip

For zeroes of a polynomial, use the relationships:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

Question 24(a): Find the ratio in which the point $P(-4, 6)$ divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$.

Solution:
Let the ratio be $k : 1$. The coordinates of the point dividing a line segment are given by the section formula:

$$\text{Coordinates of } P = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right).$$

Here:

$$P(-4, 6), \quad A(-6, 10), \quad B(3, -8).$$

Equating the x-coordinate of P:

$$-4 = \frac{3k - 6}{k+1}.$$

Simplify:

$$-4(k+1) = 3k - 6.$$

$$-4k - 4 = 3k - 6.$$

$$-4k - 3k = -6 + 4.$$

$$-7k = -2 \quad \Longrightarrow \quad k = \frac{2}{7}.$$

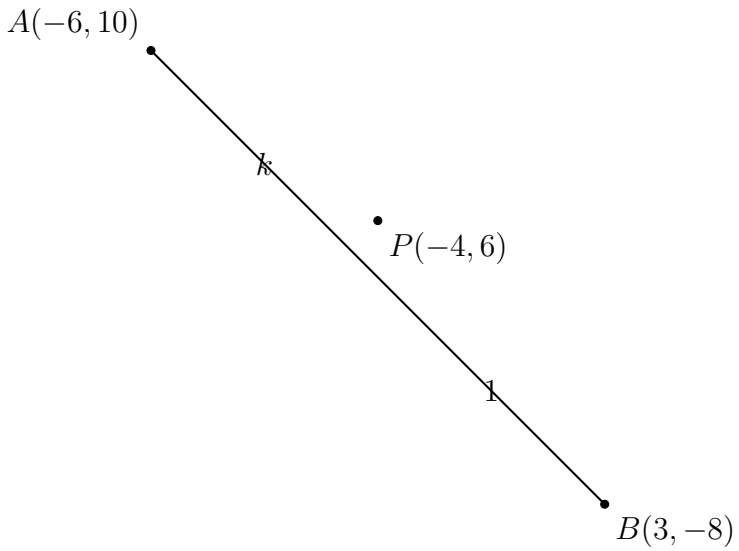
Thus, the required ratio is:

$$k : 1 = 2 : 7.$$

—

Conclusion:
The point $P(-4, 6)$ divides the line segment joining $A(-6, 10)$ and $B(3, -8)$ in the ratio $2 : 7$.

Diagram:



Quick Tip

Use the section formula:

$$(x, y) = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right),$$

and equate the coordinates to find the ratio.

Question 24(b): Prove that the points $A(3, 0)$, $B(6, 4)$, and $C(-1, 3)$ are the vertices of an isosceles triangle.

Solution:
To prove that the given points form an isosceles triangle, we calculate the lengths of the sides using the distance formula:

$$\text{Distance between two points } (x_1, y_1) \text{ and } (x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Step 1: Calculate AB.

$$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Step 2: Calculate BC.

$$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49 + 1} = \sqrt{50}.$$

Step 3: Calculate CA.

$$CA = \sqrt{(3-(-1))^2 + (0-3)^2} = \sqrt{(3+1)^2 + (-3)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5.$$

Step 4: Compare the side lengths.
Here:

$$AB = CA = 5, \quad BC = \sqrt{50}.$$

Since two sides (AB and CA) are equal, the triangle formed by the points $A(3,0)$, $B(6,4)$, and $C(-1,3)$ is an isosceles triangle.

—

Conclusion:
$\triangle ABC$ is an isosceles triangle because $AB = CA$.

Quick Tip

To prove a triangle is isosceles, calculate all three side lengths using the distance formula and check if two of them are equal.

Question 25: A carton consists of 60 shirts, of which 48 are good, 8 have major defects, and 4 have minor defects. Nigam, a trader, will accept the shirts that are good, but Anmol, another trader, will only reject the shirts with major defects. One shirt is drawn at random from the carton. Find the probability that it is acceptable to Anmol.

Solution:
For the shirt to be acceptable to Anmol, it must either be a good shirt or a shirt with minor defects. Therefore, we exclude only the shirts with major defects.
Step 1: Calculate the number of shirts without major defects.
Total shirts without major defects:

$$48\,(\text{good shirts}) + 4\,(\text{minor defects}) = 52.$$

Step 2: Calculate the probability.
Total number of shirts in the carton:

$$60.$$

Probability that the shirt is acceptable to Anmol:

$$P(\text{Acceptable to Anmol}) = \frac{\text{Number of shirts without major defects}}{\text{Total number of shirts}} = \frac{52}{60}.$$

Simplify the fraction:

$$P(\text{Acceptable to Anmol}) = \frac{13}{15}.$$

—

Conclusion:
The probability that the shirt is acceptable to Anmol is:

$$\frac{13}{15}.$$

Quick Tip

When calculating probabilities, carefully exclude only the items explicitly stated as unacceptable.

Section - C

This section comprises Short Answer (SA) type questions of 3 marks each.

Question 26(a): Prove that $\sqrt{3}$ is an irrational number.

Solution:
Let us assume, for the sake of contradiction, that $\sqrt{3}$ is a rational number. Then it can be expressed as:

$$\sqrt{3} = \frac{p}{q},$$

where p and q are integers, $q \neq 0$, and p and q are coprime (have no common factors other than 1).

Step 1: Square both sides.

$$3 = \frac{p^2}{q^2} \quad \Longrightarrow \quad p^2 = 3q^2.$$

Step 2: Analyze divisibility of p.
Since p^2 is divisible by 3, it follows that p must also be divisible by 3 (property of prime numbers). Let:

$$p = 3a, \quad \text{where } a \text{ is an integer.}$$

Step 3: Substitute $p = 3a$ into the equation.

$$p^2 = 3q^2 \quad \Longrightarrow \quad (3a)^2 = 3q^2 \quad \Longrightarrow \quad 9a^2 = 3q^2 \quad \Longrightarrow \quad q^2 = 3a^2.$$

Step 4: Analyze divisibility of q.
Since q^2 is divisible by 3, it follows that q must also be divisible by 3.

Step 5: Contradiction.
If both p and q are divisible by 3, then p and q are not coprime, which contradicts our assumption that $\frac{p}{q}$ is in its simplest form.

—

Conclusion:
The assumption that $\sqrt{3}$ is a rational number leads to a contradiction.
Therefore, $\sqrt{3}$ is an irrational number.

Quick Tip

To prove a number is irrational, assume it is rational and derive a contradiction using properties of divisibility.

Question 26(b): Prove that $(\sqrt{2} + \sqrt{3})^2$ is an irrational number, given that $\sqrt{6}$ is an irrational number.

Solution:
Expand $(\sqrt{2} + \sqrt{3})^2$:

$$(\sqrt{2} + \sqrt{3})^2 = 2 + 3 + 2\sqrt{6} = 5 + 2\sqrt{6}.$$

Step 1: Assume, for contradiction, that $5 + 2\sqrt{6}$ is a rational number.
Let:

$$5 + 2\sqrt{6} = \frac{a}{b},$$

where a, b are integers, and $b \neq 0$.
Step 2: Rearrange to isolate $\sqrt{6}$.

$$2\sqrt{6} = \frac{a}{b} - 5 \quad \Longrightarrow \quad \sqrt{6} = \frac{a - 5b}{2b}.$$

Step 3: Analyze rationality.
Since a and b are integers, $\frac{a-5b}{2b}$ is a rational number. However, it is given that $\sqrt{6}$ is an irrational number. This leads to a contradiction.
Step 4: Conclude irrationality.
The assumption that $5 + 2\sqrt{6}$ is rational is incorrect. Therefore:

$$5 + 2\sqrt{6} = (\sqrt{2} + \sqrt{3})^2$$

is an irrational number.
—
Conclusion:
$(\sqrt{2} + \sqrt{3})^2$ is an irrational number.

Quick Tip

When proving irrationality, assume rationality, isolate the square root term, and demonstrate that it contradicts the given property of irrationality.

Question 27(a): If the sum of the first 14 terms of an A.P. is 1050 and the first term is 10, then find the 20th term and the n-th term.

Solution:
The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2} \left[2a + (n-1)d\right].$$

Here:

$$S_{14} = 1050, \quad a = 10, \quad n = 14.$$

Substitute the values:

$$\frac{14}{2} \left[2(10) + 13d\right] = 1050.$$

Simplify:

$$7 \left[20 + 13d\right] = 1050 \quad \Longrightarrow \quad 20 + 13d = 150 \quad \Longrightarrow \quad d = 10.$$

Find the 20th term (a_{20}):
The n-th term of an A.P. is given by:

$$a_n = a + (n-1)d.$$

For $n = 20$:

$$a_{20} = 10 + (20 - 1)10 = 10 + 190 = 200.$$

Find the general n-th term (a_n):
Substitute $a = 10$ and $d = 10$:

$$a_n = 10 + (n - 1)10 = 10n.$$

—

Conclusion:
The 20th term is 200 and the n-th term is $10n$.

Quick Tip

Use the formula $S_n = \frac{n}{2}[2a+(n-1)d]$ to calculate the sum of n terms and $a_n = a+(n-1)d$ for specific terms.

Question 27(b): The first term of an A.P. is 5, the last term is 45, and the sum of all the terms is 400. Find the number of terms and the common difference of the A.P.

Solution:
The sum of the first n terms is given by:

$$S_n = \frac{n}{2}(a + l),$$

where $a = 5$, $l = 45$, and $S_n = 400$.
Substitute the values:

$$\frac{n}{2}(5 + 45) = 400.$$

Simplify:

$$\frac{n}{2}(50) = 400 \quad \Longrightarrow \quad 25n = 400 \quad \Longrightarrow \quad n = 16.$$

Find the common difference (d):
The last term of an A.P. is given by:

$$a_n = a + (n - 1)d.$$

Substitute $a_n = 45$, $a = 5$, and $n = 16$:

$$45 = 5 + (16 - 1)d \quad \Longrightarrow \quad 45 = 5 + 15d \quad \Longrightarrow \quad 15d = 40 \quad \Longrightarrow \quad d = \frac{40}{15} = \frac{8}{3}.$$

—

Conclusion:
The number of terms is 16 and the common difference is $\frac{8}{3}$.

Quick Tip

For problems involving the sum of an A.P., use $S_n = \frac{n}{2}(a + l)$ when the first and last terms are given.

Question 28: Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:
Let the parallelogram $ABCD$ circumscribe a circle, touching the sides AB, BC, CD, and DA at points P, Q, R, and S, respectively.
Step 1: Use the property of tangents.
The lengths of tangents drawn from an external point to a circle are equal. Therefore:

$$AP = AS, \quad BP = BQ, \quad CR = CQ, \quad DR = DS.$$

Step 2: Add the tangent pairs.
Adding all the equal tangents:

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ).$$

Simplify:

$$AB + CD = AD + BC.$$

Step 3: Use the properties of a parallelogram.
In a parallelogram, opposite sides are equal:

$$AB = CD \quad \text{and} \quad AD = BC.$$

Substitute these values:

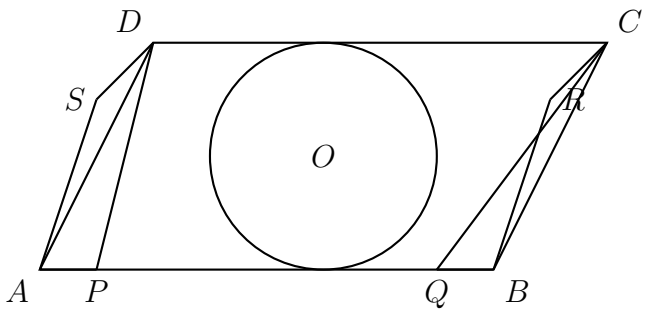
$$AB + AB = AB + AB \quad \Longrightarrow \quad 2AB = 2BC.$$

Divide by 2:

$$AB = BC.$$

Step 4: Conclude that the parallelogram is a rhombus.
Since all sides of the parallelogram are equal ($AB = BC = CD = DA$), $ABCD$ is a rhombus.
—
Conclusion:
The parallelogram circumscribing a circle is a rhombus.

Diagram:



Quick Tip

The key to solving this problem is using the property that the sum of tangents from opposite sides of a circumscribed circle is equal.

Question 29: Prove that:

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\sec A\csc A.$$

Solution:

Step 1: Simplify the Left-Hand Side (LHS).

We start with:

$$\text{LHS}=\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}.$$

Substitute $\tan A=\frac{\sin A}{\cos A}$ and $\cot A=\frac{\cos A}{\sin A}$:

$$\text{LHS}=\frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}+\frac{\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}.$$

Simplify the denominators:

$$1-\frac{\cos A}{\sin A}=\frac{\sin A-\cos A}{\sin A},\quad 1-\frac{\sin A}{\cos A}=\frac{\cos A-\sin A}{\cos A}.$$

Rewrite the fractions:

$$\text{LHS}=\frac{\frac{\sin A}{\cos A}}{\frac{\sin A-\cos A}{\sin A}}+\frac{\frac{\cos A}{\sin A}}{\frac{\cos A-\sin A}{\cos A}}.$$

Simplify each term:

$$\frac{\frac{\sin A}{\cos A}}{\frac{\sin A-\cos A}{\sin A}}=\frac{\sin^2 A}{\cos A(\sin A-\cos A)},$$

$$\frac{\frac{\cos A}{\sin A}}{\frac{\cos A-\sin A}{\cos A}}=\frac{\cos^2 A}{\sin A(\cos A-\sin A)}.$$

Since $(\cos A-\sin A)=-(\sin A-\cos A)$, we combine the terms:

$$\text{LHS}=\frac{\sin^2 A}{\cos A(\sin A-\cos A)}-\frac{\cos^2 A}{\sin A(\sin A-\cos A)}.$$

Step 2: Combine into a single fraction.

$$\text{LHS}=\frac{\sin^3 A-\cos^3 A}{\sin A\cos A(\sin A-\cos A)}.$$

Factorize $\sin^3 A-\cos^3 A$ using the difference of cubes:

$$\sin^3 A-\cos^3 A=(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A).$$

Simplify:

$$\sin^2 A+\cos^2 A=1,\quad \text{so: }\ \sin^2 A+\sin A\cos A+\cos^2 A=1+\sin A\cos A.$$

Substitute back:

$$\text{LHS}=\frac{(\sin A-\cos A)(1+\sin A\cos A)}{\sin A\cos A(\sin A-\cos A)}.$$

Cancel $(\sin A - \cos A)$ in numerator and denominator:

$$\text{LHS} = \frac{1 + \sin A \cos A}{\sin A \cos A}.$$

Split the fraction:

$$\text{LHS} = \frac{1}{\sin A \cos A} + 1.$$

Use identities:

$$\frac{1}{\sin A \cos A} = \sec A \csc A.$$

Thus:

$$\text{LHS} = 1 + \sec A \csc A = \text{RHS}.$$

—

Conclusion:

$$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = 1 + \sec A \csc A.$$

Quick Tip

When dealing with trigonometric proofs, substitute $\tan A$ and $\cot A$ in terms of $\sin A$ and $\cos A$ and simplify step-by-step.

Question 30: Three unbiased coins are tossed simultaneously. Find the probability of getting:

1. At least one head.
2. Exactly one tail.
3. Two heads and one tail.

Solution:
Step 1: Total number of outcomes.
When three coins are tossed, the possible outcomes are:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

Thus, the total number of outcomes is:

$$n(S) = 8.$$

Step 2: Calculate probabilities.

1. **At least one head:**
   Outcomes with at least one head:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH\}.$$

Number of favorable outcomes:

$$n(E) = 7.$$

Probability:

$$P(\text{At least one head}) = \frac{n(E)}{n(S)} = \frac{7}{8}.$$

2. **Exactly one tail:**
   Outcomes with exactly one tail:

$$\{HHT, HTH, THH\}.$$

   Number of favorable outcomes:

$$n(E) = 3.$$

   Probability:

$$P(\text{Exactly one tail}) = \frac{n(E)}{n(S)} = \frac{3}{8}.$$

3. **Two heads and one tail:**
   Outcomes with two heads and one tail:

$$\{HHT, HTH, THH\}.$$

   Number of favorable outcomes:

$$n(E) = 3.$$

   Probability:

$$P(\text{Two heads and one tail}) = \frac{n(E)}{n(S)} = \frac{3}{8}.$$

—

Conclusion:

1. Probability of at least one head: $\frac{7}{8}$.
2. Probability of exactly one tail: $\frac{3}{8}$.
3. Probability of two heads and one tail: $\frac{3}{8}$.

Quick Tip

List all possible outcomes for clarity when calculating probabilities involving multiple events.

Question 31: An arc of a circle of radius 10 cm subtends a right angle at the center of the circle. Find the area of the corresponding major sector. (Use $\pi = 3.14$)

Solution:
Step 1: Calculate the area of the circle.
The area of a circle is given by:

$$\text{Area} = \pi r^2.$$

Substitute $r = 10$ and $\pi = 3.14$:

$$\text{Area of the circle} = 3.14 \times 10^2 = 314\,\text{cm}^2.$$

Step 2: Calculate the area of the minor sector.
The angle subtended by the arc at the center is $90°$. The area of a sector is given by:

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2.$$

Substitute $\theta = 90°$, $r = 10$, and $\pi = 3.14$:

$$\text{Area of minor sector} = \frac{90}{360} \times 3.14 \times 10^2 = \frac{1}{4} \times 314 = 78.5\,\text{cm}^2.$$

Step 3: Calculate the area of the major sector.
The area of the major sector is:

$$\text{Area of major sector} = \text{Area of circle} - \text{Area of minor sector}.$$

Substitute the values:

$$\text{Area of major sector} = 314 - 78.5 = 235.5\,\text{cm}^2.$$

—

Conclusion:
The area of the corresponding major sector is $235.5\,\text{cm}^2$.

Quick Tip

To calculate sector areas, always subtract the minor sector from the total circle area for the major sector.

Section - D

This section comprises Long Answer (LA) type questions of 5 marks each.

Question 32(a): Find the value of k for which the quadratic equation $(k+1)x^2 - 6(k+1)x + 3(k+9) = 0, k \neq -1$ has real and equal roots.

Solution:
For real and equal roots, the discriminant (D) of the quadratic equation must be zero:

$$D = b^2 - 4ac = 0.$$

Here, $a = (k+1)$, $b = -6(k+1)$, and $c = 3(k+9)$.
Substitute these values into the discriminant:

$$D = [-6(k+1)]^2 - 4(k+1)[3(k+9)].$$

Simplify:

$$D = 36(k+1)^2 - 12(k+1)(k+9).$$

Factorize:

$$D = 12(k+1)[3(k+1) - (k+9)].$$

Simplify further:

$$D = 12(k+1)[3k+3-k-9].$$

$$D = 12(k+1)(2k-6).$$

Set $D = 0$:

$$12(k+1)(2k-6) = 0.$$

Solve for k:

$$k+1 = 0 \quad \Longrightarrow \quad k = -1 \quad \text{(not allowed, as } k \neq -1\text{)}.$$

$$2k-6 = 0 \quad \Longrightarrow \quad k = 3.$$

—

Conclusion:
The value of k is $k = 3$.

> **Quick Tip**
>
> For real and equal roots of a quadratic equation, always use the condition $D = b^2 - 4ac = 0$, and solve systematically for unknown parameters.

Question 32(b): The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present ages.

Solution:
Let the present age of the son be x years. Then the present age of the man is $2x^2$ years.
Step 1: Form an equation based on the given condition.
After 8 years, the age of the man will be $2x^2+8$, and the age of the son will be $x+8$. According to the problem:

$$2x^2 + 8 = 4 + 3(x+8).$$

Simplify:

$$2x^2 + 8 = 4 + 3x + 24.$$

$$2x^2 + 8 = 3x + 28.$$

$$2x^2 - 3x - 20 = 0.$$

Step 2: Solve the quadratic equation.
Factorize:

$$2x^2 - 8x + 5x - 20 = 0.$$

$$(2x+5)(x-4) = 0.$$

Solve for x:

$$x = -\frac{5}{2} \quad \text{(not possible, as age cannot be negative)},$$

$$x = 4.$$

Step 3: Calculate the man's age.
Substitute $x = 4$ into $2x^2$:

$$\text{Man's age} = 2(4^2) = 2 \times 16 = 32 \,\text{years}.$$

—

Conclusion:
The son's age is 4 years, and the man's age is 32 years.

> **Quick Tip**
>
> When solving age-related problems, carefully form equations based on given conditions and ensure the solutions are realistic (e.g., positive ages).

Question 33: From a point on a bridge across the river, the angles of depressions of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 4 m from the banks, find the width of the river.

Solution:
Let the width of the river be AB, and let the point P be on the bridge, Q be directly below P, and A and B be on the two banks of the river. Let x and y be the horizontal distances from Q to A and Q to B, respectively.
Step 1: Use tan **in** $\triangle PAQ$**.**
In right-angled $\triangle PAQ$:

$$\tan 30^\circ = \frac{\text{Height}}{\text{Base}} = \frac{4}{x}.$$

$$\frac{1}{\sqrt{3}} = \frac{4}{x} \quad \Longrightarrow \quad x = 4\sqrt{3}.$$

Step 2: Use tan **in** $\triangle PBQ$**.**
In right-angled $\triangle PBQ$:

$$\tan 60^\circ = \frac{\text{Height}}{\text{Base}} = \frac{4}{y}.$$

$$\sqrt{3} = \frac{4}{y} \quad \Longrightarrow \quad y = \frac{4}{\sqrt{3}}.$$

Step 3: Calculate the total width of the river.
The total width of the river is:

$$AB = x + y = 4\sqrt{3} + \frac{4}{\sqrt{3}}.$$

Rationalize the denominator:

$$\frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}.$$

Substitute:

$$AB = 4\sqrt{3} + \frac{4\sqrt{3}}{3}.$$

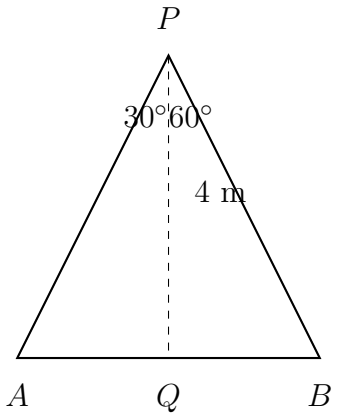
Combine terms:

$$AB = \frac{12\sqrt{3} + 4\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}.$$

—

Conclusion:
The width of the river is $\frac{16\sqrt{3}}{3}$ m.

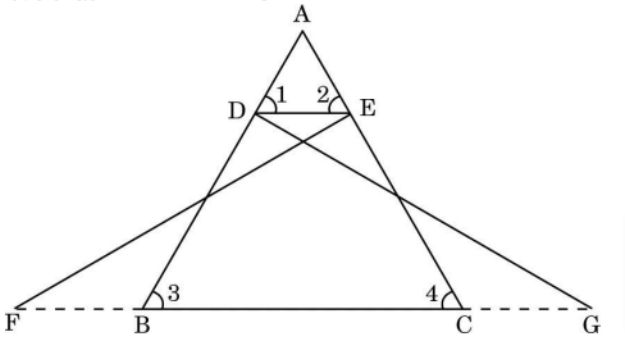
Diagram:



Quick Tip

When solving height and distance problems, use trigonometric ratios like tan to relate angles, heights, and distances, and always rationalize denominators where necessary.

Question 34(a): In the given figure, $\triangle FEC \cong \triangle GDB$ and $\angle 1 = \angle 2$. Prove that $\triangle ADE \sim \triangle ABC$.



Solution:
Step 1: Use congruence to relate angles.
From the given information, $\triangle FEC \cong \triangle GDB$. Therefore:

$$\angle 3 = \angle 4.$$

Step 2: Analyze $\triangle ABC$.
In $\triangle ABC$, since $\angle 3 = \angle 4$, we have:

$$AB = AC \quad \text{(isosceles triangle)} \quad \cdots \text{(i)}.$$

Step 3: Analyze $\triangle ADE$.
In $\triangle ADE$, it is given that $\angle 1 = \angle 2$. Hence:

$$AD = AE \quad \cdots \text{(ii)}.$$

Step 4: Divide the corresponding sides of $\triangle ADE$ and $\triangle ABC$.
From (i) and (ii):

$$\frac{AD}{AB} = \frac{AE}{AC}.$$

Step 5: Use the parallel property.
Since $DE \parallel BC$, the corresponding angles are equal:

$$\angle 1 = \angle 3 \quad \text{and} \quad \angle 2 = \angle 4.$$

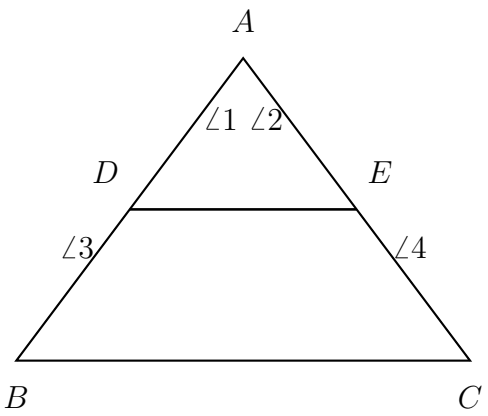
Step 6: Conclude similarity.
By the AA (Angle-Angle) similarity criterion:

$$\triangle ADE \sim \triangle ABC.$$

—

Conclusion:
$\triangle ADE \sim \triangle ABC$.

Diagram:



Quick Tip

For proving similarity in triangles, use the AA (Angle-Angle) criterion and parallel line properties to relate angles and sides effectively.

Question 34(b): Sides AB and AC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

Solution:
Step 1: Extend medians and draw additional constructions.

Produce AD to E such that $AD = DE$ and join EC. Similarly, produce PM to L such that $PM = ML$ and join LR.

Step 2: Prove congruence between auxiliary triangles.

In $\triangle ABD$ and $\triangle ECD$:

$$AD = DE \quad \text{(By construction)},$$
$$\angle 1 = \angle 2 \quad \text{(Vertically opposite angles)},$$
$$BD = DC \quad \text{(Median property)}.$$

Thus, by SAS congruence:

$$\triangle ABD \cong \triangle ECD.$$

This implies:

$$AB = EC.$$

Similarly, in $\triangle PQR$, using the same reasoning:

$$PQ = LR.$$

Step 3: Relate proportionality of sides and medians.

Since $AB : PQ = AC : PR = AD : PM$, we have:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}.$$

Using the properties of the constructions:

$$\frac{EC}{LR} = \frac{AC}{PR} = \frac{2AD}{2PM} = \frac{AE}{PL}.$$

Step 4: Prove similarity.

By the Basic Proportionality Theorem and AA similarity criterion:

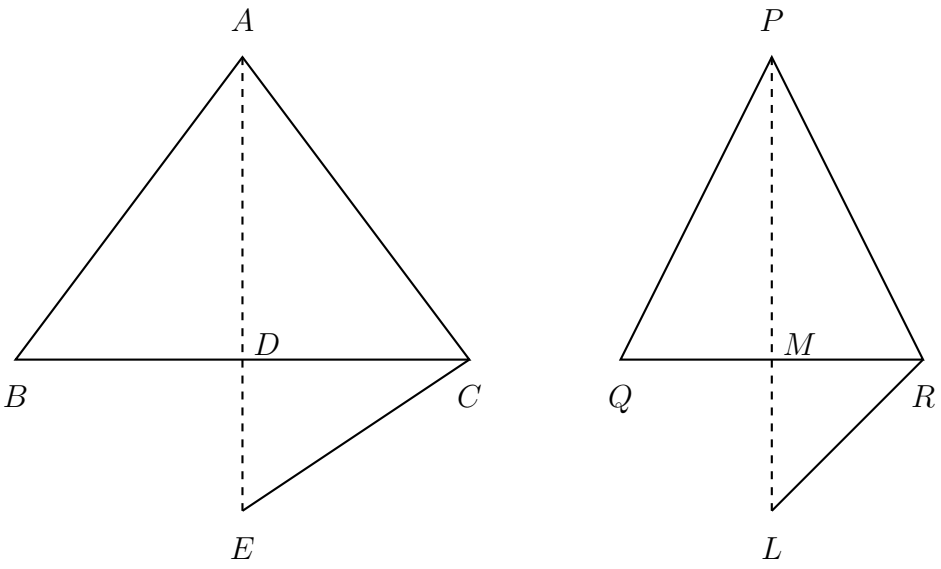
$$\triangle ABC \sim \triangle PQR.$$

—

Conclusion:

$\triangle ABC \sim \triangle PQR$.

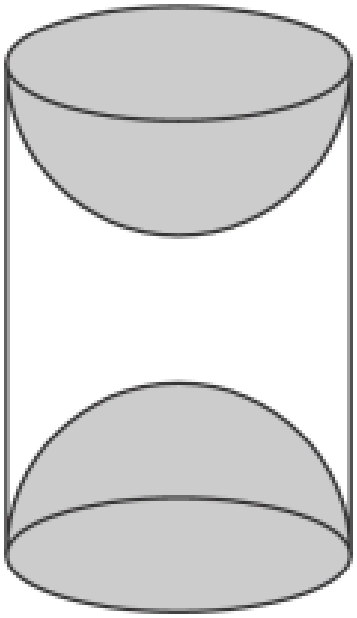
Diagram:



Quick Tip

For proving similarity between triangles, use proportionality of corresponding sides and medians, and utilize constructions for congruence to establish relationships.

Question 35: A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is $5.8\,\text{cm}$ and its base is of radius $2.1\,\text{cm}$, find the total surface area of the article.



Solution:

Step 1: Calculate the Curved Surface Area (CSA) of the cylinder.

The formula for the CSA of a cylinder is:

$$\text{CSA} = 2\pi rh,$$

where $r = 2.1\,\text{cm}$ and $h = 5.8\,\text{cm}$. Substitute the values:

$$\text{CSA of cylinder} = 2 \times \frac{22}{7} \times 2.1 \times 5.8.$$

Simplify:

$$\text{CSA of cylinder} = 76.56\,\text{cm}^2.$$

Step 2: Calculate the CSA of the two hemispheres.

The formula for the CSA of a hemisphere is:

$$\text{CSA of hemisphere} = 2\pi r^2.$$

For two hemispheres:

$$\text{CSA of two hemispheres} = 2 \times 2\pi r^2 = 4\pi r^2.$$

Substitute $r = 2.1\,\text{cm}$:

$$\text{CSA of two hemispheres} = 4 \times \frac{22}{7} \times 2.1 \times 2.1.$$

Simplify:

$$\text{CSA of two hemispheres} = 55.44\,\text{cm}^2.$$

Step 3: Calculate the total surface area of the article.
The total surface area is the sum of the CSA of the cylinder and the CSA of the two hemispheres:

$$\text{Total Surface Area} = 76.56 + 55.44.$$

Simplify:

$$\text{Total Surface Area} = 132\,\text{cm}^2.$$

—

Conclusion:
The total surface area of the article is $132\,\text{cm}^2$.

> **Quick Tip**
>
> To calculate the surface area of composite solids, add the curved surface areas of all components while excluding any overlapping parts.

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: Case Study – 1
Essel World is one of India's largest amusement parks that offers a diverse range of thrilling rides, water attractions, and entertainment options for visitors of all ages. The park is known for its iconic "Water Kingdom" section, making it a popular destination for family outings and fun-filled adventure. The ticket charges for the park are 150 per child and 250 per adult.

On a day, the cashier of the park found that 300 tickets were sold, and an amount of 55,000 was collected.
Based on the above, answer the following questions:

1. If the number of children visited be x and the number of adults visited be y, then write the given situation algebraically.

2. (a) How many children visited the amusement park that day?
   (b) How many adults visited the amusement park that day?

3. How much amount will be collected if 250 children and 100 adults visit the amusement park?

Solution:

(i) Formulate the equations:

Let the number of children be x and the number of adults be y. The given conditions can be written as:

$$x + y = 300 \quad \cdots (i)$$

$$150x + 250y = 55000 \quad \cdots (ii)$$

(ii) Solve for the number of children and adults:

(a) From equations (i) and (ii), solve for x:
Substitute $y = 300 - x$ into equation (ii):

$$150x + 250(300 - x) = 55000.$$

Simplify:

$$150x + 75000 - 250x = 55000.$$

$$-100x + 75000 = 55000 \quad \Longrightarrow \quad -100x = -20000 \quad \Longrightarrow \quad x = 200.$$

Therefore, the number of children is $x = 200$.

(b) Substituting $x = 200$ into equation (i):

$$y = 300 - 200 = 100.$$

Therefore, the number of adults is $y = 100$.

(iii) Calculate the amount collected if 250 children and 100 adults visit the park:

$$\text{Amount collected} = 150 \times 250 + 250 \times 100.$$

$$\text{Amount collected} = 37500 + 25000 = 62500.$$

—

Conclusion:

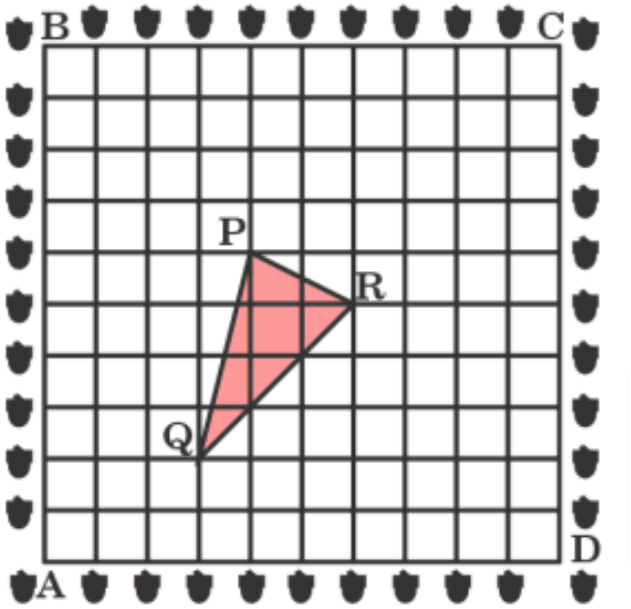
(i) The algebraic equations are $x + y = 300$ and $150x + 250y = 55000$.
(ii) Number of children: 200, Number of adults: 100.
(iii) Total amount collected: 62500.

Quick Tip

Use substitution or elimination methods for solving linear equations systematically. Ensure accurate substitution and simplification in word problems.

Question 37: Case Study – 2

A garden is in the shape of a square. The gardener grew saplings of Ashoka tree on the boundary of the garden at the distance of 1 m from each other. He wants to decorate the garden with rose plants. He chose a triangular region inside the garden to grow rose plants. In the above situation, the gardener took help from the students of class 10. They made a chart for it which looks like the given figure.



Based on the above, answer the following questions:

1. If A is taken as origin, what are the coordinates of the vertices of ΔPQR?

2. (a) Find distances PQ and QR.
   (b) Find the coordinates of the point which divides the line segment joining points P and R in the ratio $2:1$ internally.

3. Find out if ΔPQR is an isosceles triangle.

Solution:

(i) Coordinates of the vertices of ΔPQR:

From the figure, the coordinates are:

$P(4,6),\ Q(3,2),\ R(6,5)$.

(ii) Find distances and coordinates:

(a) Distance between P and Q:

$$PQ = \sqrt{(4-3)^2 + (6-2)^2} = \sqrt{1^2 + 4^2} = \sqrt{1+16} = \sqrt{17}.$$

Distance between Q and R:

$$QR = \sqrt{(3-6)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}.$$

(b) The coordinates of the point dividing PR in the ratio $2:1$:

$$\text{Using section formula: } \left(\frac{2x_2 + 1x_1}{2+1}, \frac{2y_2 + 1y_1}{2+1}\right).$$

Substitute $P(4,6)$ and $R(6,5)$:

$$\left(\frac{2\times 6 + 1\times 4}{3}, \frac{2\times 5 + 1\times 6}{3}\right) = \left(\frac{12+4}{3}, \frac{10+6}{3}\right) = \left(\frac{16}{3}, \frac{16}{3}\right).$$

(iii) Check if ΔPQR is an isosceles triangle:
Distance PR:

$$PR = \sqrt{(4-6)^2 + (6-5)^2} = \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}.$$

Since $PQ \neq QR \neq PR$, ΔPQR is not an isosceles triangle.

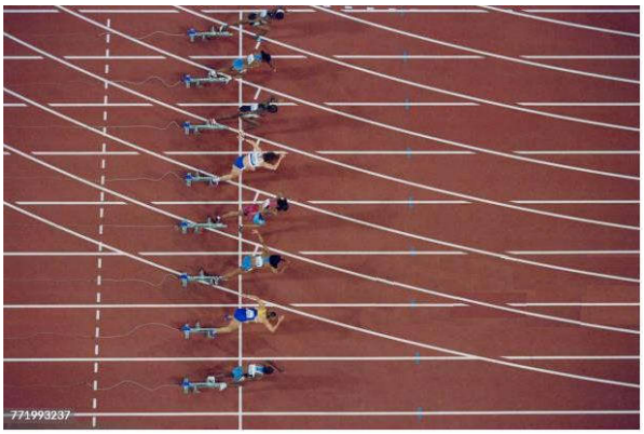
—

Conclusion:
(i) Coordinates of the vertices: $P(4,6), Q(3,2), R(6,5)$.
(ii) (a) $PQ = \sqrt{17}, QR = \sqrt{18}$.
(b) The coordinates of the point dividing PR are $\left(\frac{16}{3}, \frac{16}{3}\right)$.
(iii) ΔPQR is not isosceles.

Quick Tip

To determine the nature of a triangle, compute the lengths of all sides using the distance formula and compare them.

Question 38: Case Study – 3
Activities like running or cycling reduce stress and the risk of mental disorders like depression. Running helps build endurance. Children develop stronger bones and muscles and are less prone to gain weight. The physical education teacher of a school has decided to conduct an inter-school running tournament on his school premises. The time taken by a group of students to run 100 m was noted as follows:



Time (in seconds)	Number of students (f)
$0-20$	8
$20-40$	10
$40-60$	13
$60-80$	6
$80-100$	3

Based on the above, answer the following questions:

1. What is the median class of the above-given data?

2. (a) Find the mean time taken by the students to finish the race.
   (b) Find the mode of the above-given data.

3. How many students took time less than 60 seconds?

Solution:

(i) Median Class:

The cumulative frequency is calculated as follows:

Time (in seconds)	Number of students (f)	Cumulative Frequency (cf)
$0-20$	8	8
$20-40$	10	18
$40-60$	13	31
$60-80$	6	37
$80-100$	3	40

The total number of students is 40. Since $N/2 = 20$, the median class is $40-60$.

(ii) Mean Time:

The table for x_i (midpoints) and $f_i x_i$ is as follows:

Time (in seconds)	Midpoint(x_i)	Number of students (f)	$f_i x_i$
$0-20$	10	8	80
$20-40$	30	10	300
$40-60$	50	13	650
$60-80$	70	6	420
$80-100$	90	3	270

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1720}{40} = 43.$$

(ii) Mode:

The modal class is $40-60$. Using the formula:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where $L = 40$, $f_1 = 13$, $f_0 = 10$, $f_2 = 6$, $h = 20$:

$$\text{Mode} = 40 + \left(\frac{13 - 10}{2(13) - 10 - 6}\right) \times 20 = 40 + \left(\frac{3}{26 - 16}\right) \times 20 = 40 + 6 = 46.$$

(iii) Students Taking Less than 60 Seconds:
From the cumulative frequency table, the number of students taking less than 60 seconds is 31.

—

Conclusion:
(i) Median class: $40 - 60$.
(ii) Mean time: 43, Mode: 46.
(iii) Number of students: 31.

Quick Tip

To calculate the mean, use midpoints and summations. The modal class is identified as the class with the highest frequency.
