CBSE Class X Mathematics (Standard) Set 2 (30/5/2) Question Paper with Solutions

Time Allowed : 3 Hours	**Maximum Marks :** 80	**Total Questions :** 38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

Question 1: If α and β are the zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$ and $\alpha + \beta = \alpha\beta$, then the value of k is:

(A) $-\frac{2}{3}$
(B) $-\frac{3}{2}$
(C) $\frac{3}{2}$
(D) $\frac{2}{3}$

Correct Answer: (D) $\frac{2}{3}$

Solution:
The polynomial is:

$$p(x) = kx^2 - 30x + 45k.$$

The sum and product of the roots ($\alpha + \beta$ and $\alpha\beta$) are given by:

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{-30}{k} = \frac{30}{k}.$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{45k}{k} = 45.$$

Given:

$$\alpha + \beta = \alpha\beta.$$

Substitute the values:

$$\frac{30}{k} = 45.$$

Solve for k:

$$30 = 45k \quad \Longrightarrow \quad k = \frac{30}{45} = \frac{2}{3}.$$

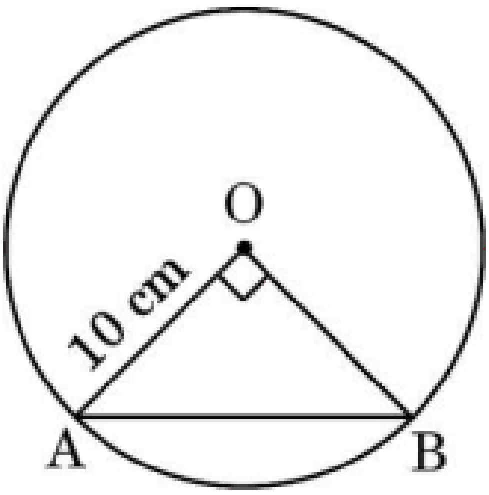
—

Conclusion:
The value of k is $\frac{2}{3}$.

Quick Tip

For problems involving zeroes of polynomials, use the relationships $\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ and $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$.

Question 2: A chord of a circle of radius 10 cm subtends a right angle at its center. The length of the chord (in cm) is:



(A) $5\sqrt{2}$
(B) $10\sqrt{2}$

(C) $\frac{5}{\sqrt{2}}$
(D) 5

Correct Answer: (B) $10\sqrt{2}$

Solution:
Given: - Radius $r = 10\,\text{cm}$. - The chord subtends a 90° angle at the center.
In $\triangle OAB$, O is the center, and $\angle AOB = 90^\circ$. Using the Pythagoras theorem:

$$\text{Chord length (AB)} = \sqrt{OA^2 + OB^2}.$$

Substitute $OA = OB = r = 10$:

$$\text{Chord length (AB)} = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}.$$

—

Conclusion:
The length of the chord is $10\sqrt{2}$.

Quick Tip

For chords subtending 90° at the center, use the Pythagoras theorem to calculate the chord length.

Question 3: The next (4^{th}) term of the A.P. $\sqrt{18},\ \sqrt{50},\ \sqrt{98},\ \ldots$ **is:**
(A) $\sqrt{128}$
(B) $\sqrt{140}$
(C) $\sqrt{162}$
(D) $\sqrt{200}$

Correct Answer: (C) $\sqrt{162}$

Solution:
The given sequence $\sqrt{18},\ \sqrt{50},\ \sqrt{98},\ \ldots$ is in arithmetic progression (A.P.) since the difference between consecutive terms is constant.
Step 1: Calculate the common difference (d):

$$d = \sqrt{50} - \sqrt{18}.$$

Simplify each term:

$$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}, \quad \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}.$$

Thus:

$$d = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}.$$

Step 2: Find the 4^{th} term:
The general term of an A.P. is given by:

$$a_n = a + (n-1)d,$$

where $a = \sqrt{18} = 3\sqrt{2}$, $n = 4$, and $d = 2\sqrt{2}$. Substituting:

$$a_4 = 3\sqrt{2} + (4-1)(2\sqrt{2}) = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}.$$

Simplify:

$$a_4 = \sqrt{81 \cdot 2} = \sqrt{162}.$$

—

Conclusion:
The 4^{th} term is $\sqrt{162}$.

Quick Tip

To find the next term in an A.P., calculate the common difference and apply the formula for the n-th term.

Question 4: If the product of two co-prime numbers is 553, then their HCF is:
(A) 1
(B) 553
(C) 7
(D) 79

Correct Answer: (A) 1

Solution:
Co-prime numbers are numbers that have no common factors other than 1.
By definition, the HCF of any two co-prime numbers is always 1.
—
Conclusion:
The HCF of the two co-prime numbers is 1.

Quick Tip

Co-prime numbers always have an HCF of 1, as they share no common factors other than 1.

Question 5: If $x = a\cos\theta$ and $y = b\sin\theta$, then the value of $b^2x^2 + a^2y^2$ is:
(A) a^2b^2
(B) ab
(C) a^4b^4
(D) $a^2 + b^2$

Correct Answer: (A) a^2b^2

Solution:
We are given:

$$x = a\cos\theta \quad \text{and} \quad y = b\sin\theta.$$

Substitute these values into $b^2x^2 + a^2y^2$:

$$b^2x^2 + a^2y^2 = b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2.$$

Simplify each term:

$$b^2x^2 = b^2a^2\cos^2\theta, \quad a^2y^2 = a^2b^2\sin^2\theta.$$

Thus:

$$b^2x^2 + a^2y^2 = b^2a^2\cos^2\theta + a^2b^2\sin^2\theta.$$

Factor out a^2b^2:

$$b^2x^2 + a^2y^2 = a^2b^2(\cos^2\theta + \sin^2\theta).$$

Using the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$:

$$b^2x^2 + a^2y^2 = a^2b^2 \cdot 1 = a^2b^2.$$

—

Conclusion:
The value of $b^2x^2 + a^2y^2$ is a^2b^2.

Quick Tip

When solving expressions involving trigonometric identities, always check for common factors and simplify using $\cos^2\theta + \sin^2\theta = 1$.

Question 6: If the quadratic equation $ax^2 + bx + c = 0$ has real and equal roots, then the value of c is:

(A) $\frac{b}{2a}$
(B) $-\frac{b}{2a}$
(C) $\frac{b^2}{4a}$
(D) $-\frac{b^2}{4a}$

Correct Answer: (C) $\frac{b^2}{4a}$

Solution:
The given quadratic equation is:

$$ax^2 + bx + c = 0.$$

For real and equal roots, the discriminant D is zero:

$$D = b^2 - 4ac = 0.$$

Step 1: Simplify the discriminant:

$$b^2 = 4ac.$$

Step 2: Express c in terms of a and b:

$$c = \frac{b^2}{4a}.$$

—

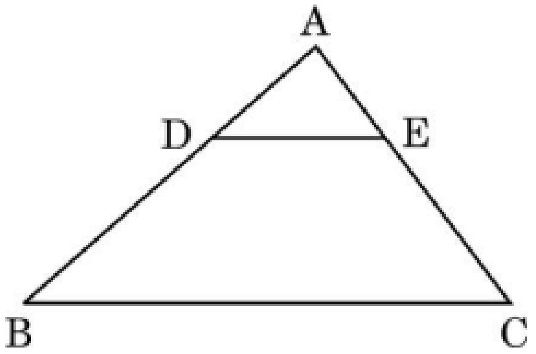
Conclusion:
The value of c for the quadratic equation to have real and equal roots is:

$$c = \frac{b^2}{4a}.$$

Quick Tip

For real and equal roots of a quadratic equation, use $b^2 - 4ac = 0$ to relate c to a and b.

Question 7: In the given figure, in $\triangle ABC$, $DE \parallel BC$. If $AD = 2.4\,\text{cm}$, $DB = 4\,\text{cm}$, and $AE = 2\,\text{cm}$, then the length of AC is:



(A) $\frac{10}{3}$ cm
(B) $\frac{3}{10}$ cm
(C) $\frac{16}{3}$ cm
(D) 1.2 cm

Correct Answer: (C) $\frac{16}{3}$ cm

Solution:
Given $DE \parallel BC$, by the Basic Proportionality Theorem (Thales' theorem), we have:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 1: Express EC in terms of AE, AD, and DB:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Substitute $AD = 2.4\,\text{cm}, DB = 4\,\text{cm}, AE = 2\,\text{cm}$:

$$\frac{2.4}{4} = \frac{2}{EC}.$$

Step 2: Solve for EC:

$$EC = \frac{2 \times 4}{2.4} = \frac{8}{2.4} = \frac{10}{3}\,\text{cm}.$$

Step 3: Find AC:

$$AC = AE + EC = 2 + \frac{10}{3} = \frac{6}{3} + \frac{10}{3} = \frac{16}{3}\,\text{cm}.$$

—

Conclusion:
The length of AC is:

$$AC = \frac{16}{3}\,\text{cm}.$$

Quick Tip

Use the Basic Proportionality Theorem ($\frac{AD}{DB} = \frac{AE}{EC}$) to solve problems involving parallel lines in triangles.

Question 8: The length of an arc of a circle with radius 12 cm is 10π cm. The angle subtended by the arc at the center of the circle is:
(A) $120°$
(B) $6°$
(C) $75°$
(D) $150°$

Correct Answer: (D) $150°$

Solution:
The length of an arc is given by:

$$\text{Arc length} = \frac{\theta}{360°} \cdot 2\pi r.$$

Substitute Arc length $= 10\pi$, $r = 12$, and solve for θ:

$$10\pi = \frac{\theta}{360} \cdot 2\pi \cdot 12.$$

Simplify:

$$10 = \frac{\theta}{360} \cdot 24 \quad \Longrightarrow \quad \frac{\theta}{360} = \frac{10}{24}.$$

$$\theta = \frac{10}{24} \cdot 360 = 150°.$$

—

Conclusion:
The angle subtended by the arc is $150°$.

Quick Tip

For arc length problems, substitute known values into the formula and solve for the unknown step-by-step.

Question 9: If $4\sec\theta - 5 = 0$, then the value of $\cot\theta$ is:
(A) $\frac{3}{4}$
(B) $\frac{4}{5}$
(C) $\frac{5}{3}$
(D) $\frac{4}{3}$

Correct Answer: (D) $\frac{4}{3}$

Solution:
The given equation is:
$$4\sec\theta - 5 = 0.$$

Step 1: Solve for $\sec\theta$:
$$\sec\theta = \frac{5}{4}.$$

Step 2: Use the trigonometric identity:
$$\sec^2\theta = 1 + \tan^2\theta.$$

Substitute $\sec\theta = \frac{5}{4}$:
$$\left(\frac{5}{4}\right)^2 = 1 + \tan^2\theta.$$
$$\frac{25}{16} = 1 + \tan^2\theta.$$
$$\tan^2\theta = \frac{25}{16} - 1 = \frac{25}{16} - \frac{16}{16} = \frac{9}{16}.$$
$$\tan\theta = \pm\frac{3}{4}.$$

Step 3: Use the reciprocal identity:
$$\cot\theta = \frac{1}{\tan\theta}.$$

Substitute $\tan\theta = \frac{3}{4}$ (taking the positive value as standard in this case):
$$\cot\theta = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$$

—

Conclusion:
The value of $\cot\theta$ is:
$$\cot\theta = \frac{4}{3}.$$

Quick Tip

Always check the signs of trigonometric functions based on the quadrant when solving such equations.

Question 10: The perimeter of the sector of a circle of radius 21 cm which subtends an angle of 60° at the center of the circle is:
(A) 22 cm
(B) 43 cm
(C) 64 cm
(D) 462 cm

Correct Answer: (C) 64 cm

Solution:
The perimeter of the sector is given by:

$$\text{Perimeter} = 2r + \text{Arc length}.$$

The arc length is:

$$\text{Arc length} = \frac{\theta}{360^\circ} \cdot 2\pi r.$$

Substitute $\theta = 60^\circ$, $r = 21$, and $\pi = \frac{22}{7}$:

$$\text{Arc length} = \frac{60}{360} \cdot 2 \cdot \frac{22}{7} \cdot 21 = \frac{1}{6} \cdot 2 \cdot \frac{22}{7} \cdot 21 = 22\,\text{cm}.$$

The perimeter is:

$$\text{Perimeter} = 2(21) + 22 = 42 + 22 = 64\,\text{cm}.$$

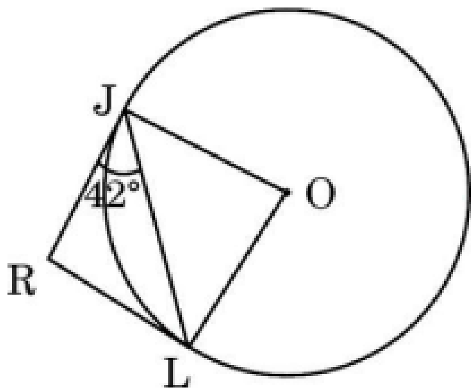
—

Conclusion:
The perimeter is 64 cm.

Quick Tip

For sector problems, always add the arc length and twice the radius to find the total perimeter.

Question 11: In the given figure, RJ and RL are two tangents to the circle. If $\angle RJL = 42^\circ$, then the measure of $\angle JOL$ is:



(A) 42°
(B) 84°
(C) 96°
(D) 138°

Correct Answer: (B) 84°

Solution:
Given:

$$\angle RJL = 42^\circ, \quad \text{RJ and RL are tangents.}$$

In a circle, the angle formed by the tangents at the external point ($\angle RJL$) is half the angle subtended by the chord at the center ($\angle JOL$):

$$\angle JOL = 2 \cdot \angle RJL.$$

Substitute the given value:

$$\angle JOL = 2 \cdot 42^\circ = 84^\circ.$$

—

Conclusion:
The measure of $\angle JOL$ is 84°.

Quick Tip

For tangents and angles in circles, the angle subtended at the center is twice the angle between the tangents.

Question 12: If the prime factorisation of 2520 is $2^3 \times 3^a \times b \times 7$, then the value of $a + 2b$ is:
(A) 12
(B) 10
(C) 9
(D) 7

Correct Answer: (A) 12

Solution:
The prime factorisation of 2520 can be done as follows:

$$2520 = 2^3 \times 3^2 \times 5 \times 7.$$

Comparing this with the given form $2^3 \times 3^a \times b \times 7$:

$$a = 2 \quad \text{and} \quad b = 5.$$

Now calculate $a + 2b$:

$$a + 2b = 2 + 2 \times 5 = 2 + 10 = 12.$$

—

Conclusion:
The value of $a + 2b$ is 12.

Quick Tip

When comparing prime factorizations, match the powers of each prime number carefully to find the corresponding coefficients.

Question 13: Which out of the following types of straight lines will be represented by the system of equations $3x + 4y = 5$ and $6x + 8y = 7$?
(A) Parallel
(B) Intersecting
(C) Coincident
(D) Perpendicular to each other

Correct Answer: (A) Parallel

Solution:
The given equations are:

$$1.\ 3x + 4y = 5 \quad \text{and} \quad 2.\ 6x + 8y = 7$$

Rewrite Equation 2 to check if it is a multiple of Equation 1:

$$6x + 8y = 7 \quad \Longrightarrow \quad 2(3x + 4y) = 7$$

Since the constant terms (5 and 7) are not in the same ratio as the coefficients, the lines are not coincident. The coefficients of x and y are in the same ratio:

$$\frac{3}{6} = \frac{4}{8}.$$

This implies the lines are parallel.
—
Conclusion:
The lines represented by the equations are parallel.

Quick Tip

For a system of linear equations, lines are parallel if the ratios of the coefficients of x and y are equal but differ in constant terms.

Question 14: One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 7 is:
(A) $\frac{1}{7}$
(B) $\frac{1}{8}$

(C) $\frac{1}{5}$
(D) $\frac{7}{40}$

Correct Answer: (B) $\frac{1}{8}$

Solution:
The multiples of 7 between 1 and 40 are:

$$7, 14, 21, 28, 35.$$

There are 5 multiples of 7, and the total number of tickets is 40. The probability is:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{5}{40} = \frac{1}{8}.$$

—

Conclusion:
The probability is $\frac{1}{8}$.

Quick Tip

For probability problems, identify the total possible outcomes and favorable outcomes clearly before simplifying the ratio.

Question 15: The LCM of three numbers $28, 44, 132$ is:
(A) 258
(B) 231
(C) 462
(D) 924

Correct Answer: (D) 924

Solution:
To find the LCM of $28, 44, 132$: 1. Perform prime factorization:

$$28 = 2^2 \cdot 7, \quad 44 = 2^2 \cdot 11, \quad 132 = 2^2 \cdot 3 \cdot 11.$$

2. Take the highest powers of all prime factors:

$$\text{LCM} = 2^2 \cdot 3 \cdot 7 \cdot 11 = 924.$$

—

Conclusion:
The LCM of $28, 44, 132$ is 924.

Quick Tip

For LCM, take the highest powers of all prime factors common or unique to the given numbers.

Question 16: The number of terms in the A.P. $3, 6, 9, 12, \ldots, 111$ **is:**
(A) 36
(B) 40
(C) 37
(D) 30

Correct Answer: (C) 37

Solution:
The general formula for the n-th term of an A.P. is:

$$a_n = a + (n-1)d,$$

where a is the first term, d is the common difference, and n is the number of terms.
Here:

$$a = 3,\ d = 3,\ a_n = 111.$$

Substitute into the formula:

$$111 = 3 + (n-1) \cdot 3.$$

Simplify:

$$111 - 3 = 3(n-1).$$

$$108 = 3(n-1).$$

$$n - 1 = 36 \quad \Longrightarrow \quad n = 37.$$

—

Conclusion:
The number of terms in the A.P. is 37.

> Quick Tip
>
> For A.P. problems, rearrange the formula for the n-th term to find the total number of terms.

Question 17: The ratio of the length of a pole and its shadow on the ground is $1 : \sqrt{3}$**. The angle of elevation of the Sun is:**
(A) $90°$
(B) $60°$
(C) $45°$
(D) $30°$

Correct Answer: (D) $30°$

Solution:
Let the height of the pole be h and the length of the shadow be x. Given:

$$\frac{h}{x} = \frac{1}{\sqrt{3}}.$$

From trigonometry:

$$\tan\theta = \frac{h}{x}.$$

Substitute $\tan\theta = \frac{1}{\sqrt{3}}$:

$$\theta = 30^\circ.$$

—

Conclusion:
The angle of elevation of the Sun is 30°.

> **Quick Tip**
>
> For elevation problems, use the tangent ratio $\tan\theta = \frac{\text{height of object}}{\text{length of shadow}}$.

Question 18: If the mean and mode of a data are 24 and 12 respectively, then its median is:
(A) 25
(B) 18
(C) 20
(D) 22

Correct Answer: (C) 20

Solution:
From the empirical relationship between mean, median, and mode:

$$\text{Mean} - \text{Median} = 3(\text{Mean} - \text{Mode}).$$

Substitute the given values:

$$24 - \text{Median} = 3(24 - 12).$$

Simplify:

$$24 - \text{Median} = 36.$$

$$\text{Median} = 24 - 12 = 20.$$

—

Conclusion:
The median of the data is 20.

> **Quick Tip**
>
> The empirical formula $\text{Mean} - \text{Median} = 3(\text{Mean} - \text{Mode})$ is helpful in solving problems involving mean, median, and mode.

Questions 19 and 20 are Assertion and Reason-based questions.
Two statements are given, one labelled as Assertion (A) and the other as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C), and (D) as given below:
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

—

Question 19:
Assertion (A): ABCD is a trapezium with $DC \parallel AB$. E and F are points on AD and BC, respectively, such that $EF \parallel AB$. Then:

$$\frac{AE}{ED} = \frac{BF}{FC}.$$

Reason (R): Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

Solution:

Both Assertion (A) and Reason (R) are true. The line EF, being parallel to the parallel sides of the trapezium (AB and DC), divides the non-parallel sides proportionally. The Reason (R) provides the correct explanation for Assertion (A).

Quick Tip

Use the property of proportionality in trapeziums: A line parallel to the parallel sides divides the non-parallel sides proportionally.

Question 20:
Assertion (A): The degree of a zero polynomial is not defined.
Reason (R): The degree of a non-zero constant polynomial is 0.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of Assertion (A).

Solution:

Both Assertion (A) and Reason (R) are true. However, Reason (R) is not the correct explanation for Assertion (A). The degree of a zero polynomial is undefined because it does not have any terms, while the degree of a non-zero constant polynomial is defined as 0.

Quick Tip

Remember: The degree of a zero polynomial is undefined, while the degree of any non-zero constant polynomial is 0.

Section - B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

Question 21: If α and β are zeroes of the quadratic polynomial $p(x) = x^2 - 5x + 4$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution:
The given polynomial is $p(x) = x^2 - 5x + 4$.
For a quadratic polynomial $ax^2 + bx + c$, the sum and product of the roots are:

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{-5}{1} = 5,$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{4}{1} = 4.$$

Using the relationship for reciprocals of roots:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}.$$

Substitute the values:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{5}{4}.$$

Now calculate the required expression:

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{5}{4} - 2 \cdot 4.$$

Simplify:

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{5}{4} - \frac{32}{4} = \frac{-27}{4}.$$

—

Conclusion:
The value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is:

$$\frac{-27}{4}.$$

Quick Tip

For quadratic polynomials, remember:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}.$$

Use this property to simplify expressions involving reciprocals of roots.

Question 22(a): Find the ratio in which the point $P(-4, 6)$ divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$.

Solution:
Let the ratio in which P divides the line AB be $k : 1$. Using the section formula, the coordinates of the dividing point $P(x, y)$ are:

$$x = \frac{kx_2 + x_1}{k+1}, \quad y = \frac{ky_2 + y_1}{k+1}.$$

Substitute the coordinates of $P(-4, 6)$, $A(-6, 10)$, and $B(3, -8)$:

$$-4 = \frac{3k - 6}{k+1}, \quad 6 = \frac{-8k + 10}{k+1}.$$

Solve the first equation:

$$-4(k+1) = 3k - 6 \implies -4k - 4 = 3k - 6 \implies -7k = -2 \implies k = \frac{2}{7}.$$

Thus, the ratio is $k : 1 = 2 : 7$.

—

Conclusion:
The required ratio in which $P(-4, 6)$ divides AB is:

$$2 : 7.$$

Quick Tip

For points dividing a line segment, use the section formula:

$$x = \frac{kx_2 + x_1}{k+1}, \quad y = \frac{ky_2 + y_1}{k+1}.$$

Question 22(b): Prove that the points $(3, 0)$, $(6, 4)$, and $(-1, 3)$ are the vertices of an isosceles triangle.

Solution:
Let the points be $A(3, 0)$, $B(6, 4)$, and $C(-1, 3)$. Calculate the lengths of the sides using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

1. Length of AB:

$$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

2. Length of BC:

$$BC = \sqrt{(6-(-1))^2 + (4-3)^2} = \sqrt{(6+1)^2 + 1^2} = \sqrt{7^2 + 1^2} = \sqrt{49 + 1} = \sqrt{50}.$$

3. Length of CA:

$$CA = \sqrt{(3-(-1))^2 + (0-3)^2} = \sqrt{(3+1)^2 + (-3)^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5.$$

Since $AB = CA$, the triangle is isosceles.

—

Conclusion:
The points $A(3,0)$, $B(6,4)$, and $C(-1,3)$ form an isosceles triangle as:

$$AB = CA = 5.$$

Quick Tip

To prove a triangle is isosceles, calculate the lengths of all sides using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Check if any two sides are equal.

Question 23: Evaluate:

$$\frac{2\tan 30^\circ \cdot \sec 60^\circ \cdot \tan 45^\circ}{1 - \sin^2 60^\circ}.$$

Solution:
Substitute the trigonometric values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \sec 60^\circ = 2, \quad \tan 45^\circ = 1, \quad \sin^2 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

Simplify the numerator:

$$2 \cdot \frac{1}{\sqrt{3}} \cdot 2 \cdot 1 = \frac{4}{\sqrt{3}}.$$

Simplify the denominator:

$$1 - \sin^2 60^\circ = 1 - \frac{3}{4} = \frac{1}{4}.$$

The entire expression becomes:

$$\frac{\frac{4}{\sqrt{3}}}{\frac{1}{4}} = \frac{4}{\sqrt{3}} \cdot 4 = \frac{16}{\sqrt{3}}.$$

Rationalize the denominator:

$$\frac{16}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{16\sqrt{3}}{3}.$$

—

Conclusion:
The value of the expression is:

$$\frac{16\sqrt{3}}{3}.$$

Quick Tip

Always simplify trigonometric functions step-by-step and rationalize the denominator when necessary.

Question 24: A carton consists of 60 shirts, of which 48 are good, 8 have major defects, and 4 have minor defects. Nigam, a trader, will accept the shirts that are good, but Anmol, another trader, will only reject the shirts with major defects. One shirt is drawn at random from the carton. Find the probability that it is acceptable to Anmol.

Solution:
For the shirt to be acceptable to Anmol, it must either be a good shirt or a shirt with minor defects. Therefore, we exclude only the shirts with major defects.
Step 1: Calculate the number of shirts without major defects.
Total shirts without major defects:

$$48\,(\text{good shirts}) + 4\,(\text{minor defects}) = 52.$$

Step 2: Calculate the probability.
Total number of shirts in the carton:

$$60.$$

Probability that the shirt is acceptable to Anmol:

$$P(\text{Acceptable to Anmol}) = \frac{\text{Number of shirts without major defects}}{\text{Total number of shirts}} = \frac{52}{60}.$$

Simplify the fraction:

$$P(\text{Acceptable to Anmol}) = \frac{13}{15}.$$

—

Conclusion:
The probability that the shirt is acceptable to Anmol is:

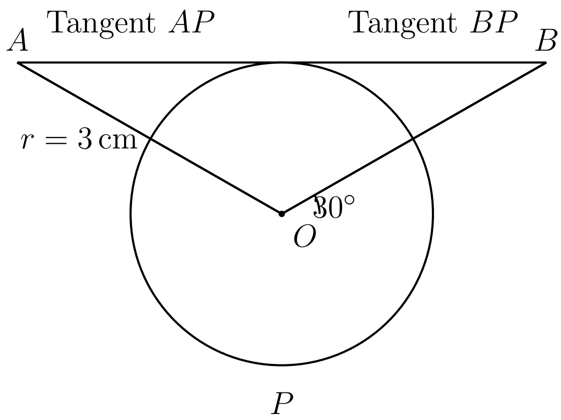
$$\frac{13}{15}.$$

Quick Tip

When calculating probabilities, carefully exclude only the items explicitly stated as unacceptable.

Question 25(a): If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

Solution:
Given: - Radius of the circle: $r = 3\,\text{cm}$, - Angle between the tangents: $\angle APB = 60^\circ$.



Step 1: Analyze the geometry.
The tangents form two right triangles with the center of the circle. In $\triangle APO$, where O is the center of the circle:

$$\angle APO = \frac{\angle APB}{2} = \frac{60^\circ}{2} = 30^\circ.$$

Step 2: Use trigonometric ratios.
From $\triangle APO$, using $\tan 30^\circ$:

$$\tan 30^\circ = \frac{\text{Opposite side (radius)}}{\text{Adjacent side (tangent length)}}.$$

Substitute the values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \frac{1}{\sqrt{3}} = \frac{3}{AP}.$$

Solve for AP:

$$AP = 3\sqrt{3}\,\text{cm}.$$

Step 3: Finalize the result.
Since the tangents are symmetrical, the length of each tangent is:

$$AP = 3\sqrt{3}\,\text{cm}.$$

—

Conclusion:
The length of each tangent is $3\sqrt{3}\,\text{cm}$.

Quick Tip

When two tangents are drawn to a circle, use trigonometric ratios in the formed right triangles to find tangent lengths.

Question 25(b): Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:
Let AB be the diameter of a circle with center O, and let P and Q be the tangents at A and B, respectively.

Step 1: Analyze the geometry.
The tangent to a circle is perpendicular to the radius at the point of tangency. Therefore:

$$\angle OAY = 90^\circ \quad \text{and} \quad \angle OBP = 90^\circ.$$

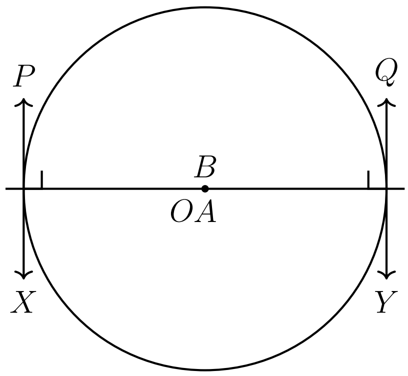
Step 2: Prove parallelism.
The angles $\angle OAY$ and $\angle OBP$ are equal and form alternate interior angles between the lines PQ and XY. Hence, by the property of alternate interior angles:

$$PQ \parallel XY.$$

—

Conclusion:
The tangents drawn at the ends of a diameter of a circle are parallel.

Diagram:



Quick Tip

The tangents at the ends of the diameter are always parallel because they form equal alternate interior angles with the line joining the ends of the diameter.

Section - C

This section comprises Short Answer (SA) type questions of 3 marks each.

Question 26: An arc of a circle of radius 10 cm subtends a right angle at the center of the circle. Find the area of the corresponding major sector. (Use $\pi = 3.14$)

Solution:
Step 1: Calculate the area of the circle.
The area of a circle is given by:

$$\text{Area} = \pi r^2.$$

Substitute $r = 10$ and $\pi = 3.14$:

$$\text{Area of the circle} = 3.14 \times 10^2 = 314\,\text{cm}^2.$$

Step 2: Calculate the area of the minor sector.
The angle subtended by the arc at the center is 90°. The area of a sector is given by:

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2.$$

Substitute $\theta = 90^\circ$, $r = 10$, and $\pi = 3.14$:

$$\text{Area of minor sector} = \frac{90}{360} \times 3.14 \times 10^2 = \frac{1}{4} \times 314 = 78.5\,\text{cm}^2.$$

Step 3: Calculate the area of the major sector.
The area of the major sector is:

$$\text{Area of major sector} = \text{Area of circle} - \text{Area of minor sector}.$$

Substitute the values:

$$\text{Area of major sector} = 314 - 78.5 = 235.5\,\text{cm}^2.$$

—

Conclusion:
The area of the corresponding major sector is $235.5\,\text{cm}^2$.

Quick Tip

To calculate sector areas, always subtract the minor sector from the total circle area for the major sector.

Question 27: Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:
Let the parallelogram $ABCD$ circumscribe a circle, touching the sides AB, BC, CD, and DA at points P, Q, R, and S, respectively.
Step 1: Use the property of tangents.
The lengths of tangents drawn from an external point to a circle are equal. Therefore:

$$AP = AS, \quad BP = BQ, \quad CR = CQ, \quad DR = DS.$$

Step 2: Add the tangent pairs.
Adding all the equal tangents:

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ).$$

Simplify:

$$AB + CD = AD + BC.$$

Step 3: Use the properties of a parallelogram.
In a parallelogram, opposite sides are equal:

$$AB = CD \quad \text{and} \quad AD = BC.$$

Substitute these values:

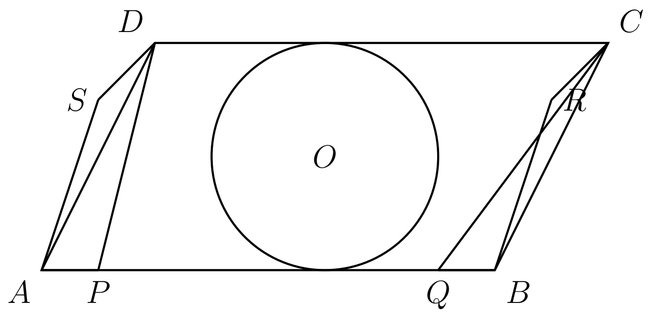
$$AB + AB = AB + AB \quad \Longrightarrow \quad 2AB = 2BC.$$

Divide by 2:

$$AB = BC.$$

Step 4: Conclude that the parallelogram is a rhombus.
Since all sides of the parallelogram are equal ($AB = BC = CD = DA$), $ABCD$ is a rhombus.
—
Conclusion:
The parallelogram circumscribing a circle is a rhombus.

Diagram:



Quick Tip

The key to solving this problem is using the property that the sum of tangents from opposite sides of a circumscribed circle is equal.

Question 28(a): Prove that $\sqrt{3}$ is an irrational number.

Solution:
Let us assume, for the sake of contradiction, that $\sqrt{3}$ is a rational number. Then it can be expressed as:

$$\sqrt{3} = \frac{p}{q},$$

where p and q are integers, $q \neq 0$, and p and q are coprime (have no common factors other than 1).
Step 1: Square both sides.

$$3 = \frac{p^2}{q^2} \quad \Longrightarrow \quad p^2 = 3q^2.$$

Step 2: Analyze divisibility of p.
Since p^2 is divisible by 3, it follows that p must also be divisible by 3 (property of prime numbers). Let:

$$p = 3a, \quad \text{where } a \text{ is an integer.}$$

Step 3: Substitute $p = 3a$ into the equation.

$$p^2 = 3q^2 \implies (3a)^2 = 3q^2 \implies 9a^2 = 3q^2 \implies q^2 = 3a^2.$$

Step 4: Analyze divisibility of q.
Since q^2 is divisible by 3, it follows that q must also be divisible by 3.
Step 5: Contradiction.
If both p and q are divisible by 3, then p and q are not coprime, which contradicts our assumption that $\frac{p}{q}$ is in its simplest form.
—

Conclusion:
The assumption that $\sqrt{3}$ is a rational number leads to a contradiction.
Therefore, $\sqrt{3}$ is an irrational number.

> **Quick Tip**
>
> To prove a number is irrational, assume it is rational and derive a contradiction using properties of divisibility.

Question 28(b): Prove that $(\sqrt{2} + \sqrt{3})^2$ is an irrational number, given that $\sqrt{6}$ is an irrational number.

Solution:
Expand $(\sqrt{2} + \sqrt{3})^2$:

$$(\sqrt{2} + \sqrt{3})^2 = 2 + 3 + 2\sqrt{6} = 5 + 2\sqrt{6}.$$

Step 1: Assume, for contradiction, that $5 + 2\sqrt{6}$ is a rational number.
Let:

$$5 + 2\sqrt{6} = \frac{a}{b},$$

where a, b are integers, and $b \neq 0$.
Step 2: Rearrange to isolate $\sqrt{6}$.

$$2\sqrt{6} = \frac{a}{b} - 5 \implies \sqrt{6} = \frac{a - 5b}{2b}.$$

Step 3: Analyze rationality.
Since a and b are integers, $\frac{a-5b}{2b}$ is a rational number. However, it is given that $\sqrt{6}$ is an irrational number. This leads to a contradiction.
Step 4: Conclude irrationality.
The assumption that $5 + 2\sqrt{6}$ is rational is incorrect. Therefore:

$$5 + 2\sqrt{6} = (\sqrt{2} + \sqrt{3})^2$$

is an irrational number.
Conclusion:
$(\sqrt{2} + \sqrt{3})^2$ is an irrational number.

Quick Tip

When proving irrationality, assume rationality, isolate the square root term, and demonstrate that it contradicts the given property of irrationality.

Question 29(a): If the sum of the first 14 terms of an A.P. is 1050 and the first term is 10, then find the 20th term and the n-th term.

Solution:
The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$

Here:

$$S_{14} = 1050, \quad a = 10, \quad n = 14.$$

Substitute the values:

$$\frac{14}{2}\left[2(10) + 13d\right] = 1050.$$

Simplify:

$$7\left[20 + 13d\right] = 1050 \implies 20 + 13d = 150 \implies d = 10.$$

Find the 20th term (a_{20}):
The n-th term of an A.P. is given by:

$$a_n = a + (n-1)d.$$

For $n = 20$:

$$a_{20} = 10 + (20-1)10 = 10 + 190 = 200.$$

Find the general n-th term (a_n):
Substitute $a = 10$ and $d = 10$:

$$a_n = 10 + (n-1)10 = 10n.$$

—

Conclusion:
The 20th term is 200 and the n-th term is $10n$.

Quick Tip

Use the formula $S_n = \frac{n}{2}[2a+(n-1)d]$ to calculate the sum of n terms and $a_n = a+(n-1)d$ for specific terms.

Question 29(b): The first term of an A.P. is 5, the last term is 45, and the sum of all the terms is 400. Find the number of terms and the common difference of the A.P.

Solution:
The sum of the first n terms is given by:

$$S_n = \frac{n}{2}(a + l),$$

where $a = 5$, $l = 45$, and $S_n = 400$.
Substitute the values:

$$\frac{n}{2}(5 + 45) = 400.$$

Simplify:

$$\frac{n}{2}(50) = 400 \quad \Longrightarrow \quad 25n = 400 \quad \Longrightarrow \quad n = 16.$$

Find the common difference (d):
The last term of an A.P. is given by:

$$a_n = a + (n - 1)d.$$

Substitute $a_n = 45$, $a = 5$, and $n = 16$:

$$45 = 5 + (16 - 1)d \quad \Longrightarrow \quad 45 = 5 + 15d \quad \Longrightarrow \quad 15d = 40 \quad \Longrightarrow \quad d = \frac{40}{15} = \frac{8}{3}.$$

—
Conclusion:
The number of terms is 16 and the common difference is $\frac{8}{3}$.

Quick Tip

For problems involving the sum of an A.P., use $S_n = \frac{n}{2}(a + l)$ when the first and last terms are given.

Question 30: Prove that:

$$\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = 2\csc\theta.$$

Solution:
Let us simplify the left-hand side (LHS):

$$\text{LHS} = \frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta}.$$

Take the LCM of the denominators:

$$\text{LHS} = \frac{\sin^2\theta + (1 + \cos\theta)^2}{\sin\theta(1 + \cos\theta)}.$$

Simplify the numerator:

$$(1 + \cos\theta)^2 = 1 + 2\cos\theta + \cos^2\theta.$$

Thus:

$$\text{LHS} = \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1 + \cos\theta)}.$$

Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$:

$$\text{LHS} = \frac{1 + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}.$$

Simplify further:

$$\text{LHS} = \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}.$$

Factor 2 from the numerator:

$$\text{LHS} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}.$$

Cancel $1+\cos\theta$ from the numerator and denominator (valid as $1+\cos\theta \neq 0$):

$$\text{LHS} = \frac{2}{\sin\theta}.$$

Using the reciprocal identity $\csc\theta = \frac{1}{\sin\theta}$:

$$\text{LHS} = 2\csc\theta.$$

Thus, the left-hand side equals the right-hand side:

$$\text{LHS} = \text{RHS}.$$

—

Conclusion:
The given identity is proven:

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\csc\theta.$$

Quick Tip

For trigonometric proofs:

- Simplify each term separately before combining them.
- Use identities like $\sin^2\theta + \cos^2\theta = 1$ and $\csc\theta = \frac{1}{\sin\theta}$.
- Factor and cancel common terms carefully.

Question 31: A jar contains 54 marbles, each of which is blue, green, or white. The probability of selecting a blue marble at random from the jar is $\frac{1}{3}$, and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does this jar contain?

Solution:
Let the number of white marbles in the jar be x. The total number of marbles is 54. The probability of selecting a white marble is given by:

$$P(\text{white marbles}) = \frac{x}{54}.$$

The total probability of selecting a marble is 1 (since every marble must be blue, green, or white). Therefore:

$$P(\text{blue}) + P(\text{green}) + P(\text{white}) = 1.$$

Substitute the given probabilities for blue and green marbles:

$$\frac{1}{3} + \frac{4}{9} + \frac{x}{54} = 1.$$

Simplify the fractions:

$$\frac{1}{3} = \frac{18}{54}, \quad \frac{4}{9} = \frac{24}{54}.$$

Substitute these into the equation:

$$\frac{18}{54} + \frac{24}{54} + \frac{x}{54} = 1.$$

Combine the fractions:

$$\frac{18 + 24 + x}{54} = 1.$$

Simplify:

$$\frac{42 + x}{54} = 1.$$

Multiply through by 54:

$$42 + x = 54.$$

Solve for x:

$$x = 54 - 42 = 12.$$

Conclusion:
The number of white marbles in the jar is: 12.

Quick Tip

When solving probability problems:

$$P(\text{Total}) = 1, \text{ and fractions must align with the total sum.}$$

Section - D

This section comprises Long Answer (LA) type questions of 5 marks each.

Question 32: From a point on a bridge across the river, the angles of depressions of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 4 m from the banks, find the width of the river.

Solution:
Let the width of the river be AB, and let the point P be on the bridge, Q be directly below P, and A and B be on the two banks of the river. Let x and y be the horizontal distances from Q to A and Q to B, respectively.

Step 1: Use tan in $\triangle PAQ$.
In right-angled $\triangle PAQ$:

$$\tan 30^\circ = \frac{\text{Height}}{\text{Base}} = \frac{4}{x}.$$
$$\frac{1}{\sqrt{3}} = \frac{4}{x} \implies x = 4\sqrt{3}.$$

Step 2: Use tan in $\triangle PBQ$.
In right-angled $\triangle PBQ$:

$$\tan 60^\circ = \frac{\text{Height}}{\text{Base}} = \frac{4}{y}.$$
$$\sqrt{3} = \frac{4}{y} \implies y = \frac{4}{\sqrt{3}}.$$

Step 3: Calculate the total width of the river.
The total width of the river is:

$$AB = x + y = 4\sqrt{3} + \frac{4}{\sqrt{3}}.$$

Rationalize the denominator:

$$\frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}.$$

Substitute:

$$AB = 4\sqrt{3} + \frac{4\sqrt{3}}{3}.$$

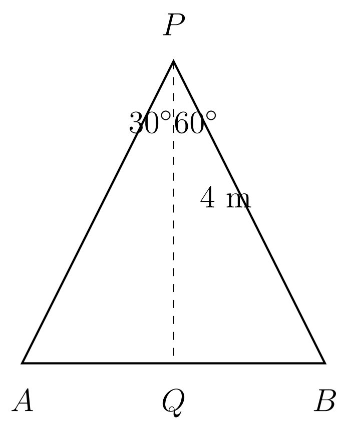
Combine terms:

$$AB = \frac{12\sqrt{3} + 4\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}.$$

—

Conclusion:
The width of the river is $\frac{16\sqrt{3}}{3}$ m.

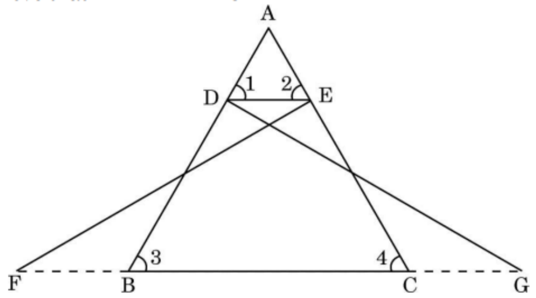
Diagram:



Quick Tip

When solving height and distance problems, use trigonometric ratios like tan to relate angles, heights, and distances, and always rationalize denominators where necessary.

Question 33(a): In the given figure, $\triangle FEC \cong \triangle GDB$ and $\angle 1 = \angle 2$. Prove that $\triangle ADE \sim \triangle ABC$.



Solution:
From the given information:

$$\triangle FEC \cong \triangle GDB.$$

This implies:

$$\angle 3 = \angle 4.$$

In $\triangle ABC$:

$$\angle 3 = \angle 4 \quad \text{(as given)}.$$

Thus:

$$AB = AC \quad \text{(i)}.$$

In $\triangle ADE$:

$$\angle 1 = \angle 2 \quad \text{(as given)}.$$

Thus:

$$AD = AE \quad \text{(ii)}.$$

Now, divide equation (ii) by equation (i):

$$\frac{AD}{AB} = \frac{AE}{AC}.$$

This implies:

$$DE \parallel BC \quad \text{(By Basic Proportionality Theorem)}.$$

Since:

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4,$$

it follows that:

$$\triangle ADE \sim \triangle ABC \quad \text{(by the AA similarity criterion)}.$$

—

Conclusion:

$$\triangle ADE \sim \triangle ABC.$$

Quick Tip

To prove similarity between triangles, look for proportional sides and equal angles. The AA similarity criterion is one of the simplest methods.

Question 33(b): Sides AB and AC and median AD of $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

Solution:
To prove $\triangle ABC \sim \triangle PQR$, extend AD to E such that $AD = DE$, and join EC. Similarly, extend PM to L such that $PM = ML$, and join LR.
1. Since $\triangle ABD \cong \triangle ECD$ (by construction), we have:

$$AB = EC.$$

2. Similarly, in $\triangle PQR$, extend PM, and by symmetry, we have:

$$PQ = LR.$$

Now, by the given proportionality condition:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}.$$

Since $AD = DE$ and $PM = ML$, the proportionality extends:

$$\frac{EC}{LR} = \frac{AC}{PR} = \frac{AD}{PM}.$$

Thus:

$$\triangle AEC \sim \triangle PLR \quad \text{(by the proportionality criterion).}$$

3. Since $\triangle AEC \sim \triangle PLR$, we have:

$$\angle BAC = \angle QPR.$$

4. Similarly, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$, proving $\triangle ABC \sim \triangle PQR$ by the AA similarity criterion.

—

Conclusion:

$$\triangle ABC \sim \triangle PQR.$$

Quick Tip

To prove triangle similarity using medians:

- Extend the median symmetrically and use congruence or similarity of smaller triangles.
- Check proportionality and equal angles to apply the AA similarity criterion.

Question 34: A tent is in the shape of a cylinder, surmounted by a conical top. If the height and diameter of the cylindrical part are 3.5 m and 6 m, and the slant height of the top is 4.2 m, find the area of canvas used for making the tent. Also, find the cost of canvas of the tent at the rate of 500 per m^2.

Solution:
The radius of the base of the cylinder is:

$$r = \frac{\text{diameter}}{2} = \frac{6}{2} = 3\,\text{m}.$$

The curved surface area (CSA) of the cylindrical part is given by:

$$\text{CSA of cylinder} = 2\pi rh.$$

Substitute $r = 3\,\text{m}$ and $h = 3.5\,\text{m}$:

$$\text{CSA of cylinder} = 2 \times \frac{22}{7} \times 3 \times 3.5 = 66\,\text{m}^2.$$

The CSA of the conical part is given by:

$$\text{CSA of cone} = \pi rl.$$

Substitute $r = 3\,\text{m}$ and $l = 4.2\,\text{m}$:

$$\text{CSA of cone} = \frac{22}{7} \times 3 \times 4.2 = 39.6\,\text{m}^2.$$

The total area of canvas required is:

$$\text{Total area} = \text{CSA of cylinder} + \text{CSA of cone}.$$

Substitute the values:

$$\text{Total area} = 66 + 39.6 = 105.6\,\text{m}^2.$$

The cost of the canvas is calculated as:

$$\text{Cost} = \text{Total area} \times \text{Rate per m}^2.$$

Substitute the values:

$$\text{Cost} = 105.6 \times 500 = 52,800\,.$$

—

Conclusion:
The total area of the canvas required is:

$$105.6\,\text{m}^2.$$

The cost of the canvas is:

$$52,800.$$

Quick Tip

For structures combining cylinders and cones:

- Use the CSA formula for each shape.
- Ensure the radius is consistent across both parts.
- Add the areas to find the total surface area.

Question 35(a): A 2-digit number is such that the product of the digits is 14. When 45 is added to the number, the digits are reversed. Find the number.

Solution:
Let the two-digit number be $10x + y$, where x is the tens digit and y is the units digit.
From the problem, the product of the digits is:

$$xy = 14 \quad \text{(i)}.$$

When 45 is added to the number, the digits are reversed:

$$10x + y + 45 = 10y + x.$$

Simplify:

$$9x - 9y = -45 \implies x - y = -5 \quad \text{(ii)}.$$

From equations (i) and (ii):

$$y - x = 5 \quad \text{(rewrite equation (ii))}.$$

Substitute $y = x + 5$ into equation (i):

$$x(x + 5) = 14.$$

Simplify:

$$x^2 + 5x - 14 = 0.$$

Solve the quadratic equation using factorization:

$$x^2 + 7x - 2x - 14 = 0 \implies (x + 7)(x - 2) = 0.$$

Thus:

$$x = -7 \quad \text{(not valid as } x \text{ is a digit)}, \quad x = 2.$$

Substitute $x = 2$ into $y = x + 5$:

$$y = 2 + 5 = 7.$$

The two-digit number is:

$$10x + y = 10 \times 2 + 7 = 27.$$

—

Conclusion:
The required number is:

$$27.$$

Quick Tip

For problems involving two-digit numbers:

- Represent the number as $10x + y$, where x and y are the digits.
- Use the given conditions to set up equations.
- Solve the equations systematically to find the digits.

Question 35(b): The side of a square exceeds the side of another square by 4 cm, and the sum of the areas of the two squares is 400 cm^2. Find the sides of the squares.

Solution:
Let the side of the first square be x cm.
Then, the side of the second square is:

$$x + 4\,\text{cm}.$$

The area of the first square is:

$$x^2.$$

The area of the second square is:

$$(x+4)^2.$$

The sum of the areas of the two squares is given as 400 cm^2:

$$x^2 + (x+4)^2 = 400.$$

Simplify:

$$x^2 + x^2 + 8x + 16 = 400.$$

Combine like terms:

$$2x^2 + 8x + 16 = 400.$$

Simplify further:

$$2x^2 + 8x - 384 = 0.$$

Divide through by 2:

$$x^2 + 4x - 192 = 0.$$

Factorize the quadratic equation:

$$(x+16)(x-12) = 0.$$

Thus:

$$x = -16 \quad \text{(not valid as side length cannot be negative)}, \quad x = 12.$$

If $x = 12$, the sides of the squares are:

$$\text{First square: } x = 12\,\text{cm}, \quad \text{Second square: } x + 4 = 16\,\text{cm}.$$

—

Conclusion:
The sides of the squares are:

$$12\,\text{cm and } 16\,\text{cm}.$$

Quick Tip

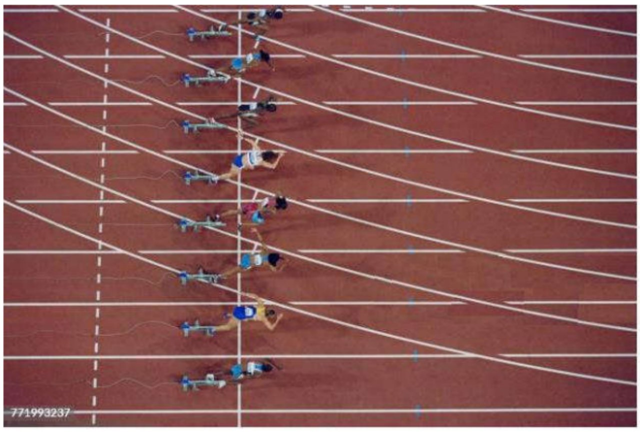
For problems involving squares:

- Use the formula for the area of a square: $\text{Area} = \text{Side}^2$.
- Set up the equation based on the given relationships and solve systematically.
- Discard negative solutions, as side lengths cannot be negative.

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: Case Study – 1
Activities like running or cycling reduce stress and the risk of mental disorders like depression. Running helps build endurance. Children develop stronger bones and muscles and are less prone to gain weight. The physical education teacher of a school has decided to conduct an inter-school running tournament on his school premises. The time taken by a group of students to run 100 m was noted as follows:



Time (in seconds)	Number of students (f)
$0 - 20$	8
$20 - 40$	10
$40 - 60$	13
$60 - 80$	6
$80 - 100$	3

Based on the above, answer the following questions:

1. What is the median class of the above-given data?

2. (a) Find the mean time taken by the students to finish the race.
   (b) Find the mode of the above-given data.

3. How many students took time less than 60 seconds?

Solution:
(i) Median Class:
The cumulative frequency is calculated as follows:

Time (in seconds)	Number of students (f)	Cumulative Frequency (cf)
$0 - 20$	8	8
$20 - 40$	10	18
$40 - 60$	13	31
$60 - 80$	6	37
$80 - 100$	3	40

The total number of students is 40. Since $N/2 = 20$, the median class is $40 - 60$.

(ii) Mean Time:

The table for x_i (midpoints) and $f_i x_i$ is as follows:

Time (in seconds)	Midpoint(x_i)	Number of students (f)	$f_i x_i$
$0 - 20$	10	8	80
$20 - 40$	30	10	300
$40 - 60$	50	13	650
$60 - 80$	70	6	420
$80 - 100$	90	3	270

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1720}{40} = 43.$$

(ii) Mode:

The modal class is $40 - 60$. Using the formula:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where $L = 40,\ f_1 = 13,\ f_0 = 10,\ f_2 = 6,\ h = 20$:

$$\text{Mode} = 40 + \left(\frac{13 - 10}{2(13) - 10 - 6}\right) \times 20 = 40 + \left(\frac{3}{26 - 16}\right) \times 20 = 40 + 6 = 46.$$

(iii) Students Taking Less than 60 Seconds:

From the cumulative frequency table, the number of students taking less than 60 seconds is 31.

—

Conclusion:

(i) Median class: $40 - 60$.

(ii) Mean time: 43, Mode: 46.

(iii) Number of students: 31.

Quick Tip

To calculate the mean, use midpoints and summations. The modal class is identified as the class with the highest frequency.

Question 37: Case Study – 2

Essel World is one of India's largest amusement parks that offers a diverse range of thrilling rides, water attractions, and entertainment options for visitors of all ages.

The park is known for its iconic "Water Kingdom" section, making it a popular destination for family outings and fun-filled adventure. The ticket charges for the park are 150 per child and 250 per adult.

On a day, the cashier of the park found that 300 tickets were sold, and an amount of 55,000 was collected.
Based on the above, answer the following questions:

1. If the number of children visited be x and the number of adults visited be y, then write the given situation algebraically.

2. (a) How many children visited the amusement park that day?
   (b) How many adults visited the amusement park that day?

3. How much amount will be collected if 250 children and 100 adults visit the amusement park?

Solution:
(i) Formulate the equations:
Let the number of children be x and the number of adults be y. The given conditions can be written as:

$$x + y = 300 \quad \cdots (i)$$
$$150x + 250y = 55000 \quad \cdots (ii)$$

(ii) Solve for the number of children and adults:
(a) From equations (i) and (ii), solve for x:
Substitute $y = 300 - x$ into equation (ii):

$$150x + 250(300 - x) = 55000.$$

Simplify:

$$150x + 75000 - 250x = 55000.$$
$$-100x + 75000 = 55000 \quad \Longrightarrow \quad -100x = -20000 \quad \Longrightarrow \quad x = 200.$$

Therefore, the number of children is $x = 200$.

(b) Substituting $x = 200$ into equation (i):

$$y = 300 - 200 = 100.$$

Therefore, the number of adults is $y = 100$.

(iii) Calculate the amount collected if 250 children and 100 adults visit the park:

$$\text{Amount collected} = 150 \times 250 + 250 \times 100.$$

$$\text{Amount collected} = 37500 + 25000 = 62500.$$

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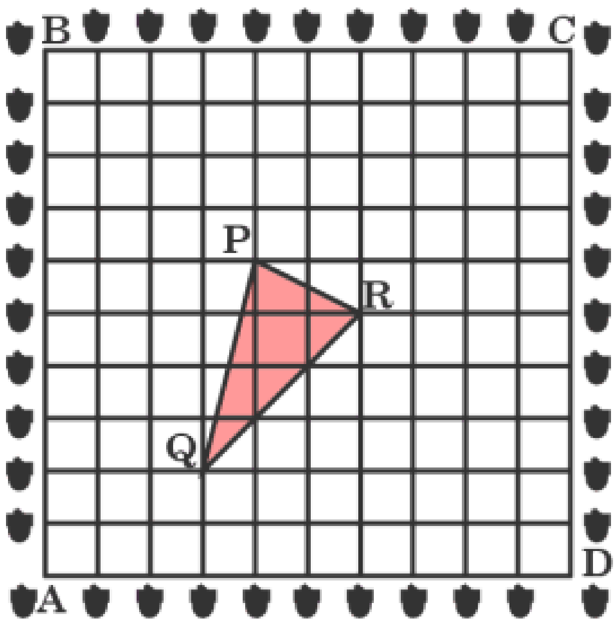
Conclusion:
(i) The algebraic equations are $x + y = 300$ and $150x + 250y = 55000$.
(ii) Number of children: 200, Number of adults: 100.
(iii) Total amount collected: 62500.

Quick Tip

Use substitution or elimination methods for solving linear equations systematically. Ensure accurate substitution and simplification in word problems.

Question 38: Case Study – 3
A garden is in the shape of a square. The gardener grew saplings of Ashoka tree on the boundary of the garden at the distance of 1 m from each other. He wants to decorate the garden with rose plants. He chose a triangular region inside the garden to grow rose plants. In the above situation, the gardener took help from the students of class 10. They made a chart for it which looks like the given figure.



Based on the above, answer the following questions:

1. If A is taken as origin, what are the coordinates of the vertices of ΔPQR?

2. (a) Find distances PQ and QR.
   (b) Find the coordinates of the point which divides the line segment joining points P and R in the ratio $2:1$ internally.

3. Find out if ΔPQR is an isosceles triangle.

Solution:
(i) Coordinates of the vertices of ΔPQR:
From the figure, the coordinates are:
$P(4,6),\ Q(3,2),\ R(6,5).$
(ii) Find distances and coordinates:
(a) Distance between P and Q:

$$PQ = \sqrt{(4-3)^2 + (6-2)^2} = \sqrt{1^2 + 4^2} = \sqrt{1+16} = \sqrt{17}.$$

Distance between Q and R:

$$QR = \sqrt{(3-6)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}.$$

(b) The coordinates of the point dividing PR in the ratio $2:1$:

$$\text{Using section formula: } \left(\frac{2x_2 + 1x_1}{2+1}, \frac{2y_2 + 1y_1}{2+1}\right).$$

Substitute $P(4,6)$ and $R(6,5)$:

$$\left(\frac{2\times 6 + 1\times 4}{3}, \frac{2\times 5 + 1\times 6}{3}\right) = \left(\frac{12+4}{3}, \frac{10+6}{3}\right) = \left(\frac{16}{3}, \frac{16}{3}\right).$$

(iii) Check if ΔPQR is an isosceles triangle:
Distance PR:

$$PR = \sqrt{(4-6)^2 + (6-5)^2} = \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}.$$

Since $PQ \neq QR \neq PR$, ΔPQR is not an isosceles triangle.

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Conclusion:
(i) Coordinates of the vertices: $P(4,6), Q(3,2), R(6,5)$.
(ii) (a) $PQ = \sqrt{17}, QR = \sqrt{18}$.
(b) The coordinates of the point dividing PR are $\left(\frac{16}{3}, \frac{16}{3}\right)$.
(iii) ΔPQR is not isosceles.

Quick Tip

To determine the nature of a triangle, compute the lengths of all sides using the distance formula and compare them.