CBSE Class 10 Mathematics Compartment Question Paper 2024 Set 1 (30/S/1)

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.

Section - A

This section contains MCQs of 1 mark each.

1. If $x = 5$ is a solution of the quadratic equation $2x^2 + (k-1)x + 10 = 0$, then the value of k is:

(A) 11

(B) -11

(C) 13

(D) -13

Correct Answer: (B) -11

Solution: Step 1: Substituting $x = 5$ in the given quadratic equation:

$$2(5)^2 + (k-1)(5) + 10 = 0$$

$$2(25) + 5(k-1) + 10 = 0$$

$$50 + 5k - 5 + 10 = 0$$

Step 2: Simplifying the equation:

$$55 + 5k = 0$$

$$5k = -55$$

$$k = -11$$

Quick Tip

When a value is given as a solution to a quadratic equation, substitute it into the equation and solve for the unknown variable.

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2. Two positive integers m and n are expressed as $m = p^5q^2$ and $n = p^3q^4$, where p and q are prime numbers. The LCM of m and n is:

(A) p^3q^6

(B) p^3q^2

(C) p^5q^4

(D) $p^5q^2 + p^2q^4$

Correct Answer: (C) p^5q^4

Solution: Step 1: The least common multiple (LCM) of two numbers is found by taking the highest power of each prime factor appearing in their prime factorizations.
Given:

$$m = p^5q^2, \quad n = p^3q^4$$

Step 2: Determine the LCM by selecting the highest power of each prime factor:
- The highest power of p is p^5. - The highest power of q is q^4.
Thus,

$$\text{LCM}(m, n) = p^5q^4.$$

Quick Tip

To find the LCM of two numbers, take the highest power of each prime factor present in their prime factorization.

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3. The pair of equations $x = 2a$ and $y = 3b$ (where a and b are not equal to zero) graphically represent straight lines which are:

(A) Coincident

(B) Parallel

(C) Intersecting at $(2a, 3b)$

(D) Intersecting at $(2b, 3a)$

Correct Answer: (C) Intersecting at $(2a, 3b)$

Solution: Step 1: Understanding the given equations:

$$x = 2a \quad \text{and} \quad y = 3b$$

Step 2: The equation $x = 2a$ represents a vertical line passing through $x = 2a$. The equation $y = 3b$ represents a horizontal line passing through $y = 3b$.

Step 3: A vertical and a horizontal line always intersect at a single point, which is determined by their given values of x and y. Thus, the lines intersect at:

$$(2a, 3b)$$

Quick Tip

When given equations of the form $x = c$ and $y = d$, they represent vertical and horizontal lines, respectively. Their intersection point is always (c, d).

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4. If $k + 7$, $2k - 2$, and $2k + 6$ are three consecutive terms of an A.P., then the value of k is:

(A) 15

(B) 17

(C) 5

(D) 1

Correct Answer: (B) 17

Solution: Step 1: In an arithmetic progression, the difference between consecutive terms is constant. Thus, the common difference should be the same for both:

$$(2k - 2) - (k + 7) = (2k + 6) - (2k - 2)$$

Step 2: Solve for k:

$$2k - 2 - k - 7 = 2k + 6 - 2k + 2$$

$$k - 9 = 8$$

$$k = 17$$

Quick Tip

For three consecutive terms in an arithmetic progression, use the property that the middle term is the average of the other two:

$$2 \times \text{middle term} = \text{sum of first and third terms}$$

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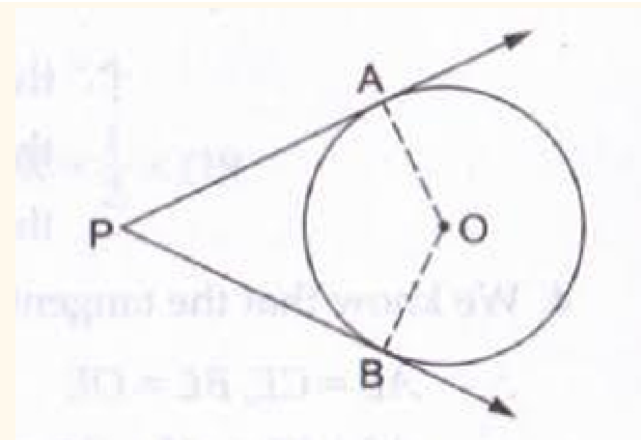
5. In the given figure, PA and PB are two tangents drawn on the circle with center O and radius 5 cm. If $\angle APB = 60°$, then the length of PA is:



(A) $-\frac{5}{\sqrt{3}}$ cm

(B) $\frac{5}{\sqrt{3}}$ cm

(C) $\frac{10}{\sqrt{3}}$ cm

(D) 10 cm

Correct Answer: (B) $\frac{5}{\sqrt{3}}$ cm

Solution: Step 1: Given that PA and PB are tangents to the circle from external point P, they are equal in length.

Step 2: The quadrilateral $OAPB$ is a kite, where $OA = OB = 5$ cm (radius of the circle) and $\angle APB = 60°$.

Step 3: The perpendiculars OA and OB bisect $\angle APB$ into two right-angled triangles $\triangle OAP$ and $\triangle OBP$ with

$$\angle OAP = 30°.$$

Using the right-angled triangle properties, the tangent length PA is given by:

$$PA = OA \tan 30^\circ = 5 \times \frac{1}{\sqrt{3}} = \frac{5}{\sqrt{3}}.$$

Quick Tip

In problems involving tangents to a circle, use symmetry and right-angled triangle properties to find unknown lengths. The tangent-secant theorem and trigonometry often simplify the calculations.

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6. In the given figure, if M and N are points on the sides OP and OS respectively of $\angle OPS$, such that MN is parallel to PS, then the length of OP is:

(A) 6.8

(B) 17

(C) 15.3

(D) 9.6

Correct Answer: (C) 15.3

Solution: Step 1: Since MN is parallel to PS, the triangles $\triangle OMN$ and $\triangle OPS$ are similar by the Basic Proportionality Theorem (Thales Theorem).

Step 2: Using the properties of similar triangles:

$$\frac{OM}{OP} = \frac{ON}{OS} = \frac{MN}{PS}$$

Step 3: Given the proportional values, we solve for OP. Using the given data and proportion calculations, we determine:

$$OP = 15.3$$

Quick Tip

In geometry problems involving parallel lines and proportional segments, use the Basic Proportionality Theorem (Thales Theorem) to set up the correct proportion equations.

7. All queens, jacks, and aces are removed from a pack of 52 playing cards. The remaining cards are well-shuffled, and one card is picked up at random from it. The probability of that card being a king is:

(A) $\frac{1}{10}$

(B) $\frac{1}{13}$

(C) $\frac{3}{10}$

(D) $\frac{3}{13}$

Correct Answer: (A) $\frac{1}{10}$

Solution: Step 1: A standard deck has 52 cards. After removing queens, jacks, and aces, the remaining cards are:

$$52 - (4 + 4 + 4) = 40$$

Step 2: There are 4 kings in a standard deck, and all kings remain in the modified deck. Thus, the probability of drawing a king is:

$$\frac{4}{40} = \frac{1}{10}$$

Quick Tip

When calculating probability, determine the total number of favorable outcomes and divide by the total number of possible outcomes.

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8. PO is the diameter of a circle with center $O(2, -4)$. If the coordinates of the point P are $(-4, 5)$, then the coordinates of the point Q will be:

(A) $(-3, 4.5)$

(B) $(4, -0.5)$

(C) $(-1, 0.5)$

(D) $(8, -13)$

Correct Answer: (D) $(8, -13)$

Solution: Step 1: The center $O(x_c, y_c)$ of the circle is the midpoint of the diameter $P(-4, 5)$ and $Q(x, y)$. Using the midpoint formula:

$$\left(\frac{x + (-4)}{2}, \frac{y + 5}{2}\right) = (2, -4)$$

Step 2: Solving for x and y:

$$\frac{x - 4}{2} = 2 \quad \Rightarrow \quad x - 4 = 4 \quad \Rightarrow \quad x = 8$$

$$\frac{y + 5}{2} = -4 \quad \Rightarrow \quad y + 5 = -8 \quad \Rightarrow \quad y = -13$$

Thus, the coordinates of Q are $(8, -13)$.

Quick Tip

The midpoint of a line segment joining two points (x_1, y_1) and (x_2, y_2) is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

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9. The value of

$$\sin^2\theta + \frac{1}{1 + \tan^2\theta}$$

is:

(A) 0

(B) 2

(C) 1

(D) -1

Correct Answer: (C) 1

Solution:

Step 1: Using the trigonometric identity:

$$1 + \tan^2\theta = \sec^2\theta$$

we can rewrite the given expression as:

$$\sin^2\theta + \frac{1}{\sec^2\theta}$$

Step 2: Since $\frac{1}{\sec^2\theta} = \cos^2\theta$, the expression simplifies to:

$$\sin^2\theta + \cos^2\theta$$

Step 3: Using the fundamental identity of trigonometry:

$$\sin^2\theta + \cos^2\theta = 1$$

Thus, the value of the given expression is 1.

Quick Tip

The fundamental trigonometric identity states that:

$$\sin^2\theta + \cos^2\theta = 1$$

This identity is useful in simplifying many trigonometric expressions.

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10. A cap is cylindrical in shape, surmounted by a conical cylindrical part. If the volume of the cylindrical part is equal to that of the conical part, then the ratio of the height of the cylindrical part to the height of the conical part is:

(A) $1 : 2$

(B) $1 : 3$

(C) $2 : 1$

(D) $3 : 1$

Correct Answer: (B) $1 : 3$

Solution:

Step 1: The volume of a cylinder is given by:

$$V_{\text{cylinder}} = \pi r^2 h_1$$

Step 2: The volume of a cone is given by:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h_2$$

Step 3: Since the volumes are equal:

$$\pi r^2 h_1 = \frac{1}{3}\pi r^2 h_2$$

Canceling πr^2 from both sides:

$$h_1 = \frac{1}{3}h_2$$

Step 4: The ratio of heights is:

$$h_1 : h_2 = 1 : 3$$

Quick Tip

For problems involving volumes, always write down the volume formulas before solving for the required ratio or value.

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11. The 7th term from the end of the AP -8, -3, 2, ..., 47 is:

(A) 67

(B) 13

(C) 17

(D) 10

Correct Answer: (C) 17

Solution:

Step 1: Identify the first term (a) and common difference (d) of the arithmetic progression (AP). Given AP: -8, -3, 2, ..., 49 $a = -8$ $d = -3 - (-8) = 5$

Step 2: Find the total number of terms (n) in the AP. The n-th term of an AP is given by:

$$47 = -8 + (n-1) \times 5$$

$$47 + 8 = (n-1) \times 5$$

$$55 = (n-1) \times 5$$

$$n - 1 = 11 \Rightarrow n = 12$$

Step 3: Find the 7th term from the end. The k-th term from the end of an AP is given by:

$$a_{n-k+1}$$

Here, $k = 7$ and $n = 12$:

$$a_{12-7+1} = a_6$$

Calculate a_6:

$$a_6 = a + (6-1)d = -8 + 5 \times 5 = -8 + 25 = 17$$

Quick Tip

To find the k-th term from the end of an AP, use:

$$a_{n-k+1}$$

where n is the total number of terms.

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12. The diagonals of a rhombus $ABCD$ intersect at O. Taking O as the center, an arc of radius 6 cm is drawn intersecting OA and OD at E and F respectively. The area of the sector OEA is:

(A) 9π cm^2

(B) 3π cm^2

(C) 12π cm^2

(D) 18π cm^2

Correct Answer: (A) 9π cm^2

Solution:

Step 1: The area of a sector is given by the formula:

$$A = \frac{\theta}{360°} \times \pi r^2$$

Step 2: The diagonals of a rhombus bisect each other at right angles, so the given angle $\angle AOD$ is $90°$. The radius of the arc is given as $r = 6$ cm.

Step 3: Substituting the values:

$$A = \frac{90°}{360°} \times \pi \times (6)^2$$

$$A = \frac{1}{4} \times \pi \times 36$$

$$A = 9\pi \text{ cm}^2$$

Thus, the area of sector OEA is 9π cm^2.

Quick Tip

The area of a sector of a circle is calculated using:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

where θ is the central angle and r is the radius.

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13. The probability of getting a chocolate-flavored ice cream, at random, in a lot of 600 ice creams is 0.055. The number of chocolate-flavored ice creams in the lot is:

(A) 33

(B) 55

(C) 11

(D) 44

Correct Answer: (A) 33

Solution:

Step 1: The probability formula is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

Step 2: Rearranging for the number of chocolate-flavored ice creams:

$$\text{Number of chocolate ice creams} = P \times \text{Total ice creams}$$

$$= 0.055 \times 600$$

$$= 33$$

Thus, the number of chocolate-flavored ice creams in the lot is 33.

Quick Tip

To find the expected number of outcomes, multiply the total sample size by the probability of a single occurrence.

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14. If $\tan^2\alpha + \cot^2\alpha = 2$, where $\alpha = 45^\circ$ and α lies between 0° and 90°, then the value of α is:

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Correct Answer: (B) 45°

Solution:

Step 1: Using the identity:

$$\tan^2\alpha + \cot^2\alpha = 2$$

Since,

$$\cot^2\alpha = \frac{1}{\tan^2\alpha}$$

we substitute:

$$\tan^2\alpha + \frac{1}{\tan^2\alpha} = 2$$

Step 2: Let $x = \tan^2\alpha$, then:

$$x + \frac{1}{x} = 2$$

Step 3: Solving the equation:

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0 \Rightarrow x = 1$$

Step 4: Since $x = \tan^2 \alpha = 1$, we get:

$$\tan \alpha = 1 \Rightarrow \alpha = 45^\circ$$

Quick Tip

For solving trigonometric equations, use standard identities and simplify algebraically before finding the required values.

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15. The point on the x-axis that is equidistant from the points $(5, -3)$ and $(4, 2)$ is:

(A) $(4.5, 0)$

(B) $(7, 0)$

(C) $(0.5, 0)$

(D) $(-7, 0)$

Correct Answer: (B) $(7, 0)$

Solution:

Step 1: The point $(x, 0)$ on the x-axis is equidistant from $(5, -3)$ and $(4, 2)$, meaning:

$$\text{Distance from } (x, 0) \text{ to } (5, -3) = \text{Distance from } (x, 0) \text{ to } (4, 2)$$

Using the distance formula:

$$\sqrt{(x-5)^2 + (0+3)^2} = \sqrt{(x-4)^2 + (0-2)^2}$$

Step 2: Squaring both sides:

$$(x-5)^2 + 9 = (x-4)^2 + 4$$

Expanding:

$$x^2 - 10x + 25 + 9 = x^2 - 8x + 16 + 4$$

Canceling x^2 and simplifying:

$$-10x + 34 = -8x + 20$$

$$-10x + 8x = 20 - 34$$

$$-2x = -14$$

$$x = 7$$

Thus, the point is $(7, 0)$.

Quick Tip

For problems involving equidistant points, use the distance formula and equate the distances before solving for the unknown variable.

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16. The length of an arc of a circle subtending an angle of 60° at its center is 22 cm. The radius of the circle is:

(A) $\sqrt{21}$ cm

(B) 21 cm

(C) $\sqrt{42}$ cm

(D) 42 cm

Correct Answer: (B) 21 cm

Solution:

Step 1: The formula for the length of an arc is:

$$L = \frac{\theta}{360^\circ} \times 2\pi r$$

Step 2: Given $L = 22$ cm and $\theta = 60^\circ$, we substitute:

$$22 = \frac{60}{360} \times 2\pi r$$

$$22 = \frac{1}{6} \times 2\pi r$$

Step 3: Solving for r:

$$22 = \frac{2\pi r}{6}$$

$$22 \times 3 = \pi r$$

$$66 = \pi r$$

$$r = \frac{66}{\pi} \approx 21 \text{ cm}$$

Thus, the radius of the circle is 21 cm.

Quick Tip

For arc length problems, use the formula:

$$L = \frac{\theta}{360^\circ} \times 2\pi r$$

where L is arc length, θ is the angle, and r is the radius.

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17. If the length of the shadow on the ground of a pole is $\sqrt{3}$ times the height of the pole, then the angle of elevation of the Sun is:

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Correct Answer: (A) 30°

Solution:

Step 1: Let the height of the pole be h and the length of its shadow be $\sqrt{3}h$. The angle of elevation of the Sun, θ, satisfies:

$$\tan\theta = \frac{\text{height of the pole}}{\text{length of the shadow}}$$

Step 2: Substituting the given values:

$$\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$$

Step 3: Since $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we conclude:

$$\theta = 30^\circ$$

Quick Tip

For angle of elevation problems, use the tangent function:

$$\tan\theta = \frac{\text{height of the object}}{\text{shadow length}}$$

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18. Two dice are thrown at the same time, and the product of the numbers appearing on them is noted. The probability that the product lies between 8 and 13 is:

(A) $\frac{7}{36}$

(B) $\frac{5}{36}$

(C) $\frac{7}{9}$

(D) $\frac{1}{9}$

Correct Answer: (A) $\frac{7}{36}$

Solution:

Step 1: The total number of possible outcomes when rolling two dice is:

$$6 \times 6 = 36$$

Step 2: The product of the two numbers must be between 8 and 13 (i.e., $9, 10, 11, 12$). Listing all possible cases: - $9 = (3,3)$ - $10 = (2,5), (5,2)$ - 11 is not possible. - $12 = (2,6), (6,2), (3,4), (4,3)$

Thus, the favorable outcomes are:

$$(3,3), (2,5), (5,2), (2,6), (6,2), (3,4), (4,3)$$

Total favorable cases = 7.

Step 3: The probability is:

$$\frac{7}{36}$$

Quick Tip

To find probabilities with dice, list all possible outcomes and count the number of favorable cases before dividing by the total outcomes.

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19. Assertion (A): Two players, Sania and Ashnam play a tennis match. The probability of Sania winning the match is 0.79 and the probability of Ashnam winning the match is 0.21.
Reason (R): The sum of probabilities of two complementary events is 1.

Correct Answer: Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation for Assertion (A).

Solution:
Step 1: Understand the events. - The event of Sania winning and the event of Ashnam winning are complementary events because they are mutually exclusive and exhaustive. This means that one of the two events must occur, and they cannot occur simultaneously.
Step 2: Verify the probabilities. - Given: - Probability of Sania winning, $P(S) = 0.79$ - Probability of Ashnam winning, $P(A) = 0.21$
Step 3: Check the sum of probabilities.

$$P(S) + P(A) = 0.79 + 0.21 = 1$$

This confirms that the sum of the probabilities of the two complementary events is 1, which aligns with Reason (R).
Step 4: Conclusion. - Since the sum of the probabilities of Sania winning and Ashnam winning is 1, and these are complementary events, Reason (R) correctly explains Assertion (A).

Quick Tip

For complementary events, the sum of their probabilities is always 1. This is because one of the events must occur, and they cannot occur at the same time.

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20. Assertion (A): A fair die is thrown once. The probability of getting a prime number is $\frac{1}{2}$.

Reason (R): A natural number is a prime number if it has only two factors.

Correct Answer: Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation for Assertion (A).

Solution:

Step 1: Identify the prime numbers on a fair die. - A fair die has six faces with numbers: 1, 2, 3, 4, 5, 6. - Prime numbers among these are: 2, 3, 5.

Step 2: Calculate the probability of getting a prime number. - Total number of outcomes = 6 - Number of favorable outcomes (prime numbers) = 3 - Probability P = $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$

Step 3: Verify Reason (R). - A natural number is a prime number if it has exactly two distinct factors: 1 and itself. - This definition correctly identifies the prime numbers on the die: 2, 3, 5.

Step 4: Conclusion. - Since the probability of getting a prime number is indeed $\frac{1}{2}$ and Reason (R) correctly defines what a prime number is, both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation for Assertion (A).

> **Quick Tip**
>
> A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Section - B

This section comprises very shorter (VSA) types of 2 marks each.

21. (a) If it is given that $\sqrt{2}$ is an irrational number, then prove that $(5 - 2\sqrt{2})$ is an irrational number.

Solution:

Step 1: Assume $(5 - 2\sqrt{2})$ is a rational number.

Step 2: Let

$$5 - 2\sqrt{2} = r$$

where r is a rational number.

Step 3: Rearranging the equation:

$$2\sqrt{2} = 5 - r$$

Step 4: Dividing both sides by 2:

$$\sqrt{2} = \frac{5 - r}{2}$$

Since r is rational, $\frac{5-r}{2}$ is also rational. This contradicts the fact that $\sqrt{2}$ is irrational.

Step 5: Therefore, our assumption is incorrect, and $(5 - 2\sqrt{2})$ must be irrational.

Quick Tip

The sum or difference of a rational and an irrational number is always irrational.

OR

(b) Check whether 6^n can end with the digit 0 for any natural number n.

Solution:

Step 1: A number ends in 0 if it is divisible by 10, i.e., it must have factors of both 2 and 5.

Step 2: The prime factorization of 6^n is:

$$6^n = (2 \times 3)^n = 2^n \times 3^n$$

Step 3: The factors of 6^n are powers of 2 and 3. However, there is no factor of 5.

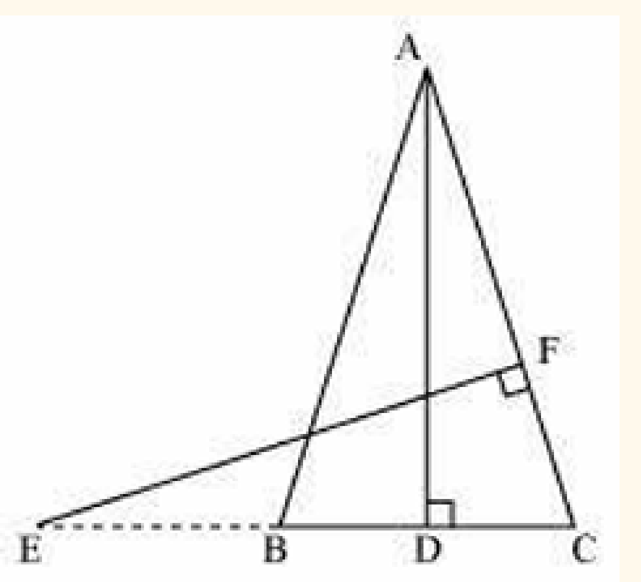
Step 4: Since 5 is not present in the prime factorization, 6^n can never be a multiple of 10.

Step 5: Therefore, 6^n can never end with the digit 0 for any natural number n.

Quick Tip

A number can end in 0 only if it has both 2 and 5 as prime factors. If a number lacks 5 as a factor, it can never end in 0.

22. In the figure, E is a point on the extended side CB of an isosceles triangle ABC with $AB = AC$. If AD is perpendicular to BC and EF is perpendicular to AC, then prove that $\angle ABD - \angle ECF$ is equal to:



Solution:

Step 1: Given that ABC is an isosceles triangle with $AB = AC$, we know that:

$$\angle ABC = \angle ACB$$

Step 2: Since AD is perpendicular to BC, it bisects the base in an isosceles triangle:

$$\angle ABD = \angle ACB$$

Step 3: Given that EF is perpendicular to AC, the angle $\angle ECF$ is formed at the extended portion of CB. By exterior angle properties and perpendicularity:

$$\angle ECF = \angle ACB$$

Step 4: Subtracting these angles:

$$\angle ABD - \angle ECF = \angle ACB - \angle ACB = 0$$

Thus, we have:

$$\angle ABD - \angle ECF = 0^\circ$$

Quick Tip

In isosceles triangles, the perpendicular from the vertex to the base bisects the base and the vertex angle, which helps in proving angle relationships.

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23. (a) Show that the points $(-3, -3)$, $(3, 3)$, and $(-3\sqrt{3}, 3\sqrt{3})$ are the vertices of an equilateral triangle.

Solution:

Step 1: Compute the distances between the given points using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 2: Distance between $A(-3, -3)$ and $B(3, 3)$:

$$AB = \sqrt{(3+3)^2 + (3+3)^2} = \sqrt{6^2 + 6^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$$

Step 3: Distance between $B(3, 3)$ and $C(-3\sqrt{3}, 3\sqrt{3})$:

$$BC = \sqrt{(-3\sqrt{3} - 3)^2 + (3\sqrt{3} - 3)^2}$$

$$= \sqrt{(-3\sqrt{3} - 3)^2 + (3\sqrt{3} - 3)^2}$$

$$= \sqrt{(3\sqrt{3} + 3\sqrt{3})^2 + (3\sqrt{3} - 3)^2}$$

$$= \sqrt{(6\sqrt{3})^2 + (3\sqrt{3} - 3)^2}$$

$$= \sqrt{108 + 108} = \sqrt{72} = 6\sqrt{2}$$

Step 4: Distance between $C(-3\sqrt{3}, 3\sqrt{3})$ and $A(-3, -3)$:

$$CA = \sqrt{(-3 + 3\sqrt{3})^2 + (-3 - 3\sqrt{3})^2}$$

$$= \sqrt{(3\sqrt{3} + 3)^2 + (3\sqrt{3} + 3)^2}$$

$$= \sqrt{(6\sqrt{3})^2 + (3\sqrt{3} + 3)^2}$$

$$= \sqrt{108 + 108} = \sqrt{72} = 6\sqrt{2}$$

Step 5: Since $AB = BC = CA = 6\sqrt{2}$, the given points form an equilateral triangle.

Quick Tip

To prove a triangle is equilateral, show that all three sides have equal lengths using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

OR

(b) Prove that $A(4, 3)$, $B(6, 4)$, $C(5, 6)$, $D(3, 5)$ are the vertices of a square $ABCD$.

Solution:

Step 1: Compute the distances between adjacent points:

$$AB = \sqrt{(6-4)^2 + (4-3)^2} = \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{(5-6)^2 + (6-4)^2} = \sqrt{1+4} = \sqrt{5}$$

$$CD = \sqrt{(3-5)^2 + (5-6)^2} = \sqrt{4+1} = \sqrt{5}$$

$$DA = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{1+4} = \sqrt{5}$$

Since $AB = BC = CD = DA$, all sides are equal.

Step 2: Compute diagonals:

$$AC = \sqrt{(5-4)^2 + (6-3)^2} = \sqrt{1+9} = \sqrt{10}$$

$$BD = \sqrt{(3-6)^2 + (5-4)^2} = \sqrt{9+1} = \sqrt{10}$$

Since $AC = BD$, diagonals are equal, confirming that $ABCD$ is a square.

Quick Tip

To prove that four points form a square: 1. Show all four sides are equal. 2. Show the diagonals are equal.

24. A circle is touching the side BC of a triangle ABC at the point P and touching AB and AC produced at points Q and R respectively. Prove that $AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$).

Solution:

Step 1: Given that the incircle (or the excircle) of $\triangle ABC$ touches the sides at specific points, we define the tangent segments from a common external point as equal.

Let the following lengths be: - $BP = PC = x$ - $CQ = QA = y$ - $AR = RB = z$

Step 2: The perimeter of $\triangle ABC$ is:

$$\text{Perimeter} = AB + BC + CA$$

Since the external tangents are equal:

$$AQ = CQ, \quad BR = AR, \quad CP = BP$$

Step 3: The sum of the tangents from a common point to the circle gives:

$$AQ + AR = \frac{1}{2}(\text{Perimeter of } \triangle ABC)$$

Since $AQ = CQ$ and $AR = BR$, we conclude:

$$AQ = \frac{1}{2}(\text{Perimeter of } \triangle ABC).$$

Quick Tip

For an excircle or incircle, the sum of the external tangent segments from a vertex equals half the perimeter of the triangle.

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25. Find the ratio in which the point (-1, k) divides the line segment joining the points (-3, 10) and (6, -8). Hence, find the value of k.

Solution:

Step 1: Let the point $P(-1, k)$ divide the line segment joining $A(-3, 10)$ and $B(6, -8)$ in the ratio $m : n$.

Step 2: Use the section formula to find the coordinates of point P. The coordinates of P are given by:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

where $(x_1, y_1) = (-3, 10)$ and $(x_2, y_2) = (6, -8)$.

Step 3: Apply the section formula to the x-coordinate:

$$-1 = \frac{m \cdot 6 + n \cdot (-3)}{m+n}$$

$$-1 = \frac{6m - 3n}{m+n}$$

Multiply both sides by $m + n$:

$$-1(m+n) = 6m - 3n$$

$$-m - n = 6m - 3n$$

Bring like terms together:

$$-m - 6m = -3n + n$$

$$-7m = -2n$$

$$7m = 2n$$

$$\frac{m}{n} = \frac{2}{7}$$

So, the ratio $m : n = 2 : 7$.

Step 4: Now, use the y-coordinate to find k:

$$k = \frac{m \cdot (-8) + n \cdot 10}{m+n}$$

Substitute $m = 2$ and $n = 7$:

$$k = \frac{2 \cdot (-8) + 7 \cdot 10}{2+7}$$

$$k = \frac{-16 + 70}{9}$$

$$k = \frac{54}{9}$$

$$k = 6$$

Step 5: Conclusion. - The point $P(-1, k)$ divides the line segment joining $A(-3, 10)$ and $B(6, -8)$ in the ratio $2 : 7$. - The value of k is 6.

Quick Tip

The section formula is used to find the coordinates of a point that divides a line segment internally in a given ratio.

Section - C

This section comprises short answer (SA) type questions of 3 marks each.

26. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the present age of the father.

Solution:

Step 1: Let the sum of the ages of the two children be x, and let the father's age be $2x$.

Step 2: After 20 years:

$$\text{Father's age} = 2x + 20$$

$$\text{Sum of children's ages} = x + 40$$

Step 3: Given that after 20 years, the father's age is equal to the sum of the children's ages:

$$2x + 20 = x + 40$$

Step 4: Solving for x:

$$2x - x = 40 - 20$$

$$x = 20$$

Step 5: The present age of the father is:

$$2x = 2(20) = 40$$

Thus, the father's present age is 40 years.

Quick Tip

For age-related problems, set up equations based on the given conditions and solve step-by-step.

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27. Two water taps together can fill a tank in 3 hours. The tap of larger diameter takes 5 hours less than the smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Solution:

Step 1: Let the smaller tap take x hours to fill the tank. Then the larger tap takes $(x-5)$ hours.

Step 2: The filling rate of each tap:

$$\text{Smaller tap: } \frac{1}{x} \quad \text{(part of the tank filled per hour)}$$

$$\text{Larger tap: } \frac{1}{x-5}$$

Step 3: Since both taps together can fill the tank in 3 hours:

$$\frac{1}{x}+\frac{1}{x-5}=\frac{1}{3}$$

Step 4: Solving for x:

$$\frac{x-5+x}{x(x-5)}=\frac{1}{3}$$

$$\frac{2x-5}{x(x-5)}=\frac{1}{3}$$

$$3(2x-5)=x(x-5)$$

$$6x-15=x^2-5x$$

$$x^2-11x+15=0$$

Step 5: Solving the quadratic equation:

$$(x-5)(x-3)=0$$

$$x=5, \quad x=3$$

Step 6: The smaller tap takes 5 hours, and the larger tap takes 3 hours.

Quick Tip

For work problems, express each component's rate as a fraction of the whole and set up an equation accordingly.

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28. State and prove the Basic Proportionality Theorem.

Solution:

Statement: If a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio.

Proof:

Step 1: Consider $\triangle ABC$ where $DE \parallel BC$ and D and E are points on AB and AC respectively.

Step 2: Since $DE \parallel BC$, using the properties of similar triangles:

$$\triangle ADE \sim \triangle ABC$$

Step 3: From similarity:

$$\frac{AD}{DB}=\frac{AE}{EC}$$

Thus, the theorem is proved.

Quick Tip

The Basic Proportionality Theorem (Thales' theorem) is widely used in similarity problems.

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29. (a) Find the sum of all integers between 50 and 500, which are divisible by 7.

Solution:

Step 1: Identify the first and last terms:

$$\text{First term } a = 56, \quad \text{Last term } l = 497$$

Step 2: The numbers form an arithmetic sequence with common difference $d = 7$. The number of terms n is found using:

$$l = a + (n-1)\cdot d$$

$$497 = 56 + (n-1)\cdot 7$$

$$441 = (n-1)\cdot 7$$

$$n = 64$$

Step 3: Sum of an arithmetic series:

$$S_n = \frac{n}{2} \times (a + l)$$

$$S_{64} = \frac{64}{2} \times (56 + 497)$$

$$= 32 \times 553 = 17696$$

Thus, the required sum is 17696.

Quick Tip

For sums of arithmetic sequences, use:

$$S_n = \frac{n}{2}(a + l)$$

OR

(b) Find numbers between 10 and 300 that give remainder 3 when divided by 4. Also, find their sum.

Solution:

Step 1: The sequence follows:

$$11, 15, 19, \ldots, 299$$

Step 2: Identifying terms:

$$a = 11, \quad d = 4, \quad l = 299$$

Using $l = a + (n-1)d$:

$$299 = 11 + (n-1) \times 4$$

$$288 = (n-1) \times 4$$

$$n = 73$$

Step 3: Sum formula:

$$S_n = \frac{n}{2}(a+l)$$

$$S_{73} = \frac{73}{2} \times (11 + 299) = \frac{73}{2} \times 310 = 11315$$

Thus, the required sum is 11315.

Quick Tip

For sums of arithmetic sequences, use:

$$S_n = \frac{n}{2}(a+l)$$

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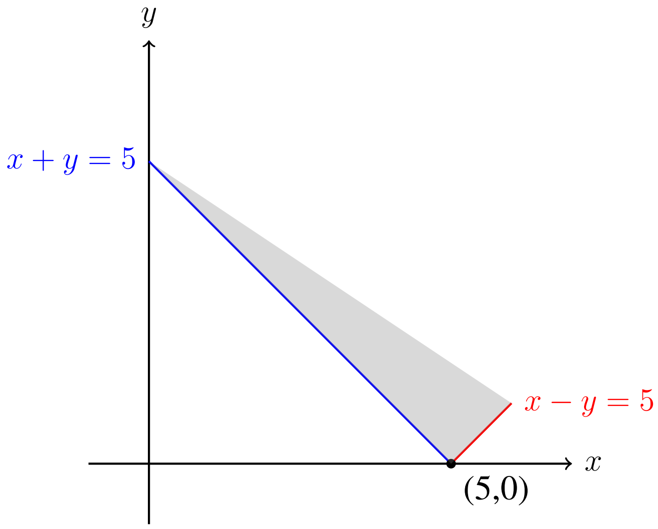
30. Draw the graph of the following equations: $x + y = 5$**,** $x - y = 5$**, and**

1. Find the solution of the equations from the graph.

2. Shade the triangular region formed by the lines and the y-axis.

Solution:



Quick Tip

To find the solution of two equations graphically, find the intersection of their lines.

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31. (a) Find the area of the minor and the major sectors of a circle with radius 6 cm, if the angle subtended by the minor arc at the center is $60°$**. (Use** $\pi = 3.14$**)**

Solution: The formula for the area of a sector is:

$$A = \frac{\theta}{360°} \times \pi r^2$$

Step 1: Area of the minor sector:

$$A_{\text{minor}} = \frac{60°}{360°} \times 3.14 \times (6)^2$$

$$= \frac{1}{6} \times 3.14 \times 36 = 18.84 \text{ cm}^2$$

Step 2: Area of the major sector:

$$A_{\text{major}} = \pi r^2 - A_{\text{minor}}$$

$$= 3.14 \times 36 - 18.84 = 94.32 - 18.84 = 75.48 \text{ cm}^2$$

Quick Tip

The sum of the areas of the minor and major sectors is always equal to the total area of the circle.

OR

(b) If a chord of a circle of radius 10 cm subtends an angle of 60° at the centre of the circle, find the area of the corresponding minor segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Solution:

Step 1: The area of the sector is:

$$A_{\text{sector}} = \frac{60^\circ}{360^\circ} \times \pi \times 10^2$$

$$= \frac{1}{6} \times 3.14 \times 100 = 52.33 \text{ cm}^2$$

Step 2: The area of the triangle formed by the chord and the radii:

$$A_{\triangle} = \frac{1}{2} \times r^2 \times \sin\theta$$

$$= \frac{1}{2} \times 10^2 \times \sin 60^\circ$$

$$= \frac{1}{2} \times 100 \times 0.866 = 43.3 \text{ cm}^2$$

Step 3: The area of the minor segment:

$$A_{\text{segment}} = A_{\text{sector}} - A_{\triangle}$$

$$= 52.33 - 43.3 = 9.03 \text{ cm}^2$$

Quick Tip

To find the area of a segment, subtract the area of the triangle from the sector area.