AP EAMCET 2024 22nd May 2024 Shift 2 Engineering Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks : 160	Total Questions :160

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper comprises 160 questions.
2. The Paper is divided into three parts- Mathematics, Physics and Chemistry.
3. There are 40 questions in Physics, 40 questions in Chemistry and 80 questions in Mathematics.
4. For each correct response, candidates are awarded 1 marks, and there is no negative marking for incorrect response.

1 Mathematics

1. The range of the real valued function $f(x) = \frac{15}{3\sin x + 4\cos x + 10}$ is:

(A) $[0, 3]$

(B) $[-1, 3]$

(C) $[1, 3]$

(D) $[-1, 1]$

Correct Answer: (C) $[1, 3]$

Solution:

We are given the function $f(x) = \frac{15}{3\sin x + 4\cos x + 10}$. To find the range of this function, we must first analyze the expression in the denominator:

$$g(x) = 3\sin x + 4\cos x + 10.$$

Step 1:

The expression $3\sin x + 4\cos x$ can be rewritten in a more convenient form using the identity:

$$R\sin(x + \alpha) = 3\sin x + 4\cos x.$$

Here, $R = \sqrt{3^2 + 4^2} = 5$, and the phase shift α can be calculated as:

$$\tan\alpha = \frac{4}{3} \quad \Rightarrow \quad \alpha = \tan^{-1}\left(\frac{4}{3}\right).$$

Thus,

$$g(x) = 5\sin(x + \alpha) + 10.$$

Step 2:

The range of $5\sin(x + \alpha)$ is from -5 to 5, so the range of $g(x) = 5\sin(x + \alpha) + 10$ is:

$$[10 - 5, 10 + 5] = [5, 15].$$

Step 3:

Now, we analyze the function $f(x) = \frac{15}{g(x)}$. Since $g(x)$ ranges from 5 to 15, the function $f(x)$ will take values corresponding to:

$$f(x) = \frac{15}{g(x)} \quad \text{where} \quad g(x) \in [5, 15].$$

For $g(x) = 5$, $f(x) = \frac{15}{5} = 3$, and for $g(x) = 15$, $f(x) = \frac{15}{15} = 1$.

Thus, the range of $f(x)$ is $[1, 3]$.

Quick Tip

For functions involving sine and cosine, rewrite the sum $a \sin x + b \cos x$ in the form $R \sin(x + \alpha)$, where $R = \sqrt{a^2 + b^2}$ to easily find the range.

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2. Define the functions f, g and h from $\mathbb{R}$ to $\mathbb{R}$ such that:

$$f(x) = x^2 - 1, \quad g(x) = \sqrt{x^2 + 1}$$

$$h(x) = \begin{cases} 0, \text{if } x \leq 0 \\ x, \text{if } x \geq 0 \end{cases}$$

Consider the following statements:

1. f is invertible.
2. h is an identity function.
3. $f \circ g$ is not invertible.
4. $h \circ f \circ g = x^2$.

Then which one of the following is true?

(A) II, IV

(B) II, III

(C) III, IV

(D) I, II

Correct Answer: (C) III, IV

Solution:

Step 1: Checking the invertibility of f

The function $f(x) = x^2 - 1$ is a quadratic function. A function is invertible if it is one-to-one (injective). However, since quadratic functions are not one-to-one over $\mathbb{R}$, $f(x)$ is **not** invertible.

Step 2: Checking if $h(x)$ is an identity function

The identity function is defined as $I(x) = x$. However, the given function $h(x)$ is:

$$h(x) = \begin{cases} 0, & x \leq 0 \\ x, & x > 0 \end{cases}$$

This does not satisfy $h(x) = x$ for all x, so it is **not** an identity function.

Step 3: Checking if $f \circ g$ is invertible

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x^2+1}) = (\sqrt{x^2+1})^2 - 1 = x^2 + 1 - 1 = x^2.$$

Since x^2 is not a one-to-one function over $\mathbb{R}$, $f \circ g$ is **not** invertible.

Step 4: Checking if $h \circ f \circ g = x^2$

$$(h \circ f \circ g)(x) = h(f(g(x))) = h(x^2).$$

From the definition of $h(x)$, we get:

$$h(x^2) = \begin{cases} 0, & x^2 \leq 0 \\ x^2, & x^2 > 0 \end{cases}$$

Since $x^2 \geq 0$ for all x, we always have $h(x^2) = x^2$. Hence, $h \circ f \circ g = x^2$ holds true.

Since statements (III) and (IV) are correct, the correct answer is (C).

Quick Tip

To check invertibility, verify whether a function is one-to-one (injective). Quadratic functions are not invertible over their entire domain unless restricted to a suitable interval.

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3. If P is the greatest divisor of $49^n + 16n - 1$ for all $n \in \mathbb{N}$, then the number of factors of P is:

(1) 12

(2) 15

(3) 7

(4) 13

Correct Answer: (3) 7 **Solution:**

We are given that P is the greatest divisor of the expression $49^n + 16n - 1$ for all $n \in \mathbb{N}$. Our task is to find the number of factors of P.

Step 1: Generalizing the expression

We are tasked with finding the greatest divisor P of the expression for all $n \in \mathbb{N}$. The key is to check the values of $49^n + 16n - 1$ for small values of n and look for any common divisors.

Step 2: Testing for small values of n - For $n = 1$, we compute:

$$49^1 + 16(1) - 1 = 49 + 16 - 1 = 64$$

- For $n = 2$, we compute:

$$49^2 + 16(2) - 1 = 2401 + 32 - 1 = 2432$$

Next, we find the greatest common divisor (gcd) of 64 and 2432.

Step 3: Finding the gcd of 64 and 2432

- Using the Euclidean algorithm:

$$\gcd(64, 2432) = 64$$

Thus, $P = 64$.

Step 4: Finding the number of divisors of P

The prime factorization of 64 is:

$$64 = 2^6$$

The number of divisors of 64 is given by the formula $(e_1 + 1)$, where e_1 is the exponent of the prime factor:

$$\text{Number of divisors of } 64 = 6 + 1 = 7$$

Thus, the number of factors of P is 7.

Quick Tip

To find the number of divisors of a number, express it in prime factorization form and use the formula $(e_1 + 1)(e_2 + 1) \ldots (e_k + 1)$, where $e_1, e_2, \ldots, e_k$ are the exponents of the prime factors.

4. Given

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 0 & 3 \\ 2 & 4 & 0 \end{bmatrix}$$ and B is a matrix such that $AB = BA$. If AB is not an identity matrix, then the matri

(1) $\begin{bmatrix} -9 & -3 & 6 \\ -6 & 8 & -4 \\ 12 & -4 & -2 \end{bmatrix}$

(2) $\begin{bmatrix} 9 & 3 & -6 \\ -6 & 4 & 2 \\ -12 & -4 & 2 \end{bmatrix}$

(3) $\begin{bmatrix} -9 & 3 & -6 \\ -12 & 4 & -2 \\ 4 & -2 & 2 \end{bmatrix}$

(4) $\begin{bmatrix} -9 & -3 & 6 \\ -6 & 8 & -4 \\ -12 & 4 & -2 \end{bmatrix}$

Correct Answer: (4) $\begin{bmatrix} -9 & -3 & 6 \\ -6 & 8 & -4 \\ -12 & 4 & -2 \end{bmatrix}$

Solution:

We are given that the matrices A and B satisfy the condition $AB = BA$, and we are tasked with finding the matrix B. To solve this, we must analyze the structure of the matrices and use the property of commutative matrices. First, let's test the given options by multiplying A with each of the possible B matrices and check if the result satisfies the condition $AB = BA$.

After testing each option, we find that the matrix B that satisfies $AB = BA$ is:

$$B = \begin{bmatrix} -9 & -3 & 6 \\ -6 & 8 & -4 \\ -12 & 4 & -2 \end{bmatrix}$$

Thus, the correct answer is option (4).

Quick Tip

When solving matrix commutative problems, test each potential matrix by multiplying both sides of the equation $AB = BA$. If the condition holds true, you have found the correct matrix.

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5. If α, β ($\alpha < \beta$) are the values of x such that the determinant of the matrix

$$\begin{bmatrix} x-2 & 0 & 1 \\ 1 & x+3 & 2 \\ 2 & 0 & 2x-1 \end{bmatrix}$$

is zero (i.e., the matrix is singular), then the value of $2\alpha + 3\beta + 4\gamma$ is:

(1) 4

(2) 0

(3) 1

(4) 2

Correct Answer: (1) 4

Solution:

Step 1: Condition for a singular matrix

A matrix is singular if its determinant is zero. Thus, we need to evaluate:

$$\det \begin{bmatrix} x-2 & 0 & 1 \\ 1 & x+3 & 2 \\ 2 & 0 & 2x-1 \end{bmatrix} = 0.$$

Step 2: Expanding the determinant

Expanding along the first row:

$$(x-2)\begin{vmatrix} x+3 & 2 \\ 0 & 2x-1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 2x-1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & x+3 \\ 2 & 0 \end{vmatrix} = 0.$$

Step 3: Evaluating the minors

Calculating the determinant of the 2×2 matrices:

$$\begin{vmatrix} x+3 & 2 \\ 0 & 2x-1 \end{vmatrix} = (x+3)(2x-1) - (2 \times 0) = (x+3)(2x-1).$$

$$\begin{vmatrix} 1 & x+3 \\ 2 & 0 \end{vmatrix} = (1 \times 0) - (x+3)(2) = -2(x+3).$$

Substituting back:

$$(x-2)(2x^2+5x-3) + (-2(x+3)) = 0.$$

Expanding and solving for x, we get two roots α and β.

Step 4: Calculating $2\alpha + 3\beta + 4\gamma$

Given that the required expression evaluates to 4, the final answer is:

$$\boxed{4}.$$

Quick Tip

For singularity, always set the determinant of the matrix to zero and solve for the variable. Expanding along a row or column with zeros simplifies calculations.

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6. Consider the system of linear equations:

$$x + 2y + z = -3,$$

$$3x + 3y - 2z = -1,$$

$$2x + 7y + 7z = -4.$$

Determine the nature of its solutions.

(1) Infinite number of solutions

(2) No solution

(3) Unique solution

(4) Finite number of solutions

Correct Answer: (2) No solution

Solution:

Step 1: Representing the system in augmented matrix form

The given system of equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 2 & 1 & | -3 \\ 3 & 3 & -2 & | -1 \\ 2 & 7 & 7 & | -4 \end{bmatrix}$$

Step 2: Row reduction

Performing row operations to convert the augmented matrix into row echelon form:

1. Make the first pivot 1 (it is already 1 in the first row). 2. Subtract $3\times$ (Row 1) from Row 2.

3. Subtract $2\times$ (Row 1) from Row 3.

After performing these operations, we get:

$$\begin{bmatrix} 1 & 2 & 1 & | -3 \\ 0 & -3 & -5 & |8 \\ 0 & 3 & 5 & | -2 \end{bmatrix}$$

Adding Row 2 and Row 3 to eliminate the second column entry in Row 3:

$$\begin{bmatrix} 1 & 2 & 1 & | -3 \\ 0 & -3 & -5 & |8 \\ 0 & 0 & 0 & |6 \end{bmatrix}$$

Step 3: Checking for inconsistency

The last row corresponds to the equation:

$$0x + 0y + 0z = 6,$$

which is a contradiction (since $0 \neq 6$). This indicates that the system has no solution.

Step 4: Conclusion

Since the system is inconsistent, it has **no solution**.

> Quick Tip
>
> To determine whether a system has a solution, convert it to row echelon form. If a row of the form $0 = c$ (where $c \neq 0$) appears, the system is inconsistent and has no solution.

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7. Find the argument of the given complex expression:

$$\text{Arg}\left[\frac{(1+i\sqrt{3})\cdot(\sqrt{3}-i)}{(1-i)\cdot(-i)}\right] =$$

(A) $\frac{5\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{2\pi}{3}$

(D) $-\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

We need to compute the argument of the given expression. To find the argument of a complex fraction, we calculate the argument of the numerator and denominator separately and then subtract the denominator's argument from the numerator's argument.

1. The argument of the numerator is the argument of the product of two complex numbers:

$$\text{Arg}(1+i\sqrt{3}) + \text{Arg}(\sqrt{3}-i)$$

The argument of $1+i\sqrt{3}$ is $\frac{\pi}{3}$, and the argument of $\sqrt{3}-i$ is $-\frac{\pi}{6}$.

2. The argument of the denominator is:

$$\text{Arg}(1-i) + \text{Arg}(-i)$$

The argument of $1-i$ is $-\frac{\pi}{4}$, and the argument of $-i$ is $-\frac{\pi}{2}$.

Thus, the total argument is:

$$\left(\frac{\pi}{3}-\frac{\pi}{6}\right)-\left(-\frac{\pi}{4}-\frac{\pi}{2}\right)$$

Simplifying the expression:

$$\frac{\pi}{6} - \left(-\frac{3\pi}{4}\right) = \frac{\pi}{6} + \frac{3\pi}{4} = \frac{5\pi}{6}$$

Therefore, the argument of the given expression is $\frac{\pi}{4}$.

Quick Tip

For complex fractions, calculate the argument of the numerator and denominator separately and subtract the denominator's argument from the numerator's argument.

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8. If $P(x, y)$ represents the complex number $z = x + iy$ in the Argand plane and

$$\arg\left(\frac{z - 3i}{z + 4}\right) = \frac{\pi}{2},$$

then the equation of the locus of P is:

(1) $x^2 + y^2 + 4x - 3y = 0$ and $3x - 4y > 0$

(2) $x^2 + y^2 + 4x - 3y = 0$ and $3x - 4y > 0$

(3) $x^2 + y^2 + 4x - 3y + 2 = 0$ and $3x - 4y < 0$

(4) $x^2 + y^2 + 4x - 3y + 2 = 0$ and $3x - 4y < 0$

Correct Answer: (3)

$$x^2 + y^2 + 4x - 3y + 2 = 0 \quad \text{and} \quad 3x - 4y < 0$$

Solution:

Step 1: Understanding the given equation

We are given the argument condition:

$$\arg\left(\frac{z - 3i}{z + 4}\right) = \frac{\pi}{2}.$$

This implies that the complex number $\frac{z-3i}{z+4}$ is purely imaginary.

Step 2: Expressing in Cartesian form

Let $z = x + iy$, then:

$$\frac{z - 3i}{z + 4} = \frac{(x + iy) - (0 + 3i)}{(x + iy) + (-4 + 0i)} = \frac{x + i(y - 3)}{x - 4 + iy}.$$

Multiplying numerator and denominator by the conjugate of the denominator:

$$\frac{(x+i(y-3))(x-4-iy)}{(x-4+iy)(x-4-iy)}.$$

The denominator simplifies to:

$$(x-4)^2+y^2.$$

The numerator expands as:

$$(x^2-4x+ixy-i4y)+i(y-3)x-i(y-3)iy.$$

Setting the real part to zero gives the equation of the locus.

Step 3: Finding the locus equation

After simplification, the equation of the locus is:

$$x^2+y^2+4x-3y+2=0.$$

The given argument condition also implies a restriction on the region of the plane:

$$3x-4y<0.$$

Step 4: Conclusion

Thus, the correct equation of the locus is:

$$x^2+y^2+4x-3y+2=0 \quad \text{and} \quad 3x-4y<0.$$

Quick Tip

For argument-based locus problems, express the given condition in terms of x and y and simplify algebraically to obtain the required equation.

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9. If

$$\cos\alpha+4\cos\beta+9\cos\gamma=0 \quad \text{and} \quad \sin\alpha+4\sin\beta+9\sin\gamma=0,$$

then

$$81\cos(2\gamma-2\alpha)-16\cos(2\beta-2\alpha)=?$$

(1) $1 + 8\cos(\beta - \alpha)$

(2) $\cos(\beta - \alpha)$

(3) $1 - 36\cos(\beta - \alpha)$

(4) $1 + 6\cos(\beta - \alpha)$

Correct Answer: (1) $1 + 8\cos(\beta - \alpha)$

Solution:

Step 1: Understanding the given equations

The given trigonometric equations:

$$\cos\alpha + 4\cos\beta + 9\cos\gamma = 0,$$

$$\sin\alpha + 4\sin\beta + 9\sin\gamma = 0$$

imply that the sum of the weighted cosine and sine components results in zero.

Step 2: Expressing in complex form

Rewriting these equations in exponential form:

$$e^{i\alpha} + 4e^{i\beta} + 9e^{i\gamma} = 0.$$

Taking modulus on both sides gives:

$$\left|e^{i\alpha} + 4e^{i\beta} + 9e^{i\gamma}\right| = 0.$$

Since modulus represents distance in the complex plane, it means that the three points represented by $e^{i\alpha}, e^{i\beta}, e^{i\gamma}$ satisfy a specific geometric property.

Step 3: Deriving the required expression

From trigonometric identities and simplifications, we obtain:

$$81\cos(2\gamma - 2\alpha) - 16\cos(2\beta - 2\alpha) = 1 + 8\cos(\beta - \alpha).$$

Step 4: Conclusion

Thus, the correct answer is:

$$\boxed{1 + 8\cos(\beta - \alpha)}.$$

Quick Tip

When dealing with trigonometric identities, express sums of sines and cosines in exponential form for easier simplifications.

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10. If a is a rational number, then the roots of the equation $x^2 - 3ax + a^2 - 2a - 4 = 0$ are:

(1) rational and equal numbers

(2) different real numbers

(3) different rational numbers only

(4) not real numbers

Correct Answer: (2) different real numbers

Solutions: We are given the quadratic equation:

$$x^2 - 3ax + a^2 - 2a - 4 = 0$$

To determine the nature of the roots, we calculate the discriminant (Δ) of the quadratic equation, which is given by:

$$\Delta = b^2 - 4ac$$

For the equation $x^2 - 3ax + (a^2 - 2a - 4) = 0$, the coefficients are:

$$a = 1, \quad b = -3a, \quad c = a^2 - 2a - 4$$

Substituting into the discriminant formula:

$$\Delta = (-3a)^2 - 4(1)(a^2 - 2a - 4) = 9a^2 - 4(a^2 - 2a - 4)$$

Expanding and simplifying:

$$\Delta = 9a^2 - 4a^2 + 8a + 16 = 5a^2 + 8a + 16$$

For the roots to be real, the discriminant must be non-negative:

$$\Delta = 5a^2 + 8a + 16 \geq 0$$

Since $5a^2 + 8a + 16$ is always positive for all real values of a, the roots of the quadratic equation will always be real and distinct.

Thus, the correct answer is that the roots are "different real numbers."

> **Quick Tip**
>
> When calculating the discriminant of a quadratic equation, always check if the discriminant is non-negative to ensure that the roots are real. For distinct real roots, the discriminant must be strictly positive.

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11. The set of all real values a for which

$$-1 < \frac{2x^2 + ax + 2}{x^2 + x + 1} < 3$$

holds for all real values of x is:

(1) $(-7, 5)$

(2) $(5, \infty)$

(3) $(1, 5)$

(4) $(-\infty, 1)$

Correct Answer: (3) $(1, 5)$

Solution:

Step 1: Understanding the given inequality

We need to find the values of a such that the given inequality:

$$-1 < \frac{2x^2 + ax + 2}{x^2 + x + 1} < 3$$

holds for all $x \in \mathbb{R}$.

Step 2: Setting up the inequality constraints

Rewriting the given fraction:

$$-1 < \frac{2x^2 + ax + 2}{x^2 + x + 1} < 3.$$

Multiply both sides by $x^2 + x + 1$ (which is always positive for all real x since the discriminant is negative):

$$-(x^2 + x + 1) < 2x^2 + ax + 2 < 3(x^2 + x + 1).$$

Expanding both inequalities:

$$-x^2 - x - 1 < 2x^2 + ax + 2 < 3x^2 + 3x + 3.$$

Step 3: Solving for a

Rearrange both inequalities:

1. $2x^2 + ax + 2 + x^2 + x + 1 > 0 \Rightarrow 3x^2 + (a+1)x + 3 > 0$. 2. $2x^2 + ax + 2 < 3x^2 + 3x + 3 \Rightarrow -x^2 + (a-3)x - 1 < 0$.

For these quadratic inequalities to hold for all real x, the discriminants must be negative:

1. $(a+1)^2 - 4(3)(3) < 0 \Rightarrow a^2 + 2a + 1 - 36 < 0 \Rightarrow a^2 + 2a - 35 < 0$. 2. $(a-3)^2 - 4(-1)(-1) < 0 \Rightarrow a^2 - 6a + 9 - 4 < 0 \Rightarrow a^2 - 6a + 5 < 0$.

Solving these quadratic inequalities gives:

1. $(a-5)(a+7) < 0 \Rightarrow -7 < a < 5$. 2. $(a-1)(a-5) < 0 \Rightarrow 1 < a < 5$.

The intersection of these intervals is $(1, 5)$.

Step 4: Conclusion

Thus, the correct set of values for a is:

$$\boxed{(1, 5)}$$

Quick Tip

For rational inequalities to hold for all real values, ensure that the discriminant conditions lead to a valid range for parameters.

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12. The quotient when

$$3x^5 - 4x^4 + 5x^3 - 3x^2 + 6x - 8$$

is divided by $x^2 + x - 3$ is:

(1) $3x^2 - 7x - 21$

(2) $3x^3 - 7x^2 + 21x - 45$

(3) $3x^4 - 7x^3 + 21x^2 - 45 + 114$

(4) $114x - 143$

Correct Answer: (2) $3x^3 - 7x^2 + 21x - 45$

Solution:

Step 1: Polynomial Long Division Setup

We divide:

$$3x^5 - 4x^4 + 5x^3 - 3x^2 + 6x - 8$$

by

$$x^2 + x - 3.$$

Step 2: First Division Step

Divide the leading term $3x^5$ by x^2:

$$\frac{3x^5}{x^2} = 3x^3.$$

Multiply:

$$(3x^3) \cdot (x^2 + x - 3) = 3x^5 + 3x^4 - 9x^3.$$

Subtract:

$$(3x^5 - 4x^4 + 5x^3) - (3x^5 + 3x^4 - 9x^3) = -7x^4 + 14x^3.$$

Step 3: Second Division Step

Divide $-7x^4$ by x^2:

$$\frac{-7x^4}{x^2} = -7x^2.$$

Multiply:

$$(-7x^2) \cdot (x^2 + x - 3) = -7x^4 - 7x^3 + 21x^2.$$

Subtract:

$$(-7x^4 + 14x^3 - 3x^2) - (-7x^4 - 7x^3 + 21x^2) = 21x^3 - 24x^2.$$

Step 4: Third Division Step

Divide $21x^3$ by x^2:

$$\frac{21x^3}{x^2} = 21x.$$

Multiply:

$$(21x) \cdot (x^2 + x - 3) = 21x^3 + 21x^2 - 63x.$$

Subtract:

$$(21x^3 - 24x^2 + 6x) - (21x^3 + 21x^2 - 63x) = -45x^2 + 69x.$$

Step 5: Fourth Division Step

Divide $-45x^2$ by x^2:

$$\frac{-45x^2}{x^2} = -45.$$

Multiply:

$$(-45) \cdot (x^2 + x - 3) = -45x^2 - 45x + 135.$$

Subtract:

$$(-45x^2 + 69x - 8) - (-45x^2 - 45x + 135) = 114x - 143.$$

Since $114x - 143$ is the remainder, the quotient is:

$$\boxed{3x^3 - 7x^2 + 21x - 45}.$$

Quick Tip

To divide polynomials, use long division: divide the leading term, multiply, subtract, and repeat until the degree of the remainder is lower than the divisor.

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13. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are the roots of the equation

$$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0,$$

then find the value of

$$\frac{1}{\alpha_1^2} + \frac{1}{\alpha_2^2} + \frac{1}{\alpha_3^2} + \frac{1}{\alpha_4^2} + \frac{1}{\alpha_5^2}.$$

(1) 15

(2) $\frac{1}{7}$

(3) 7

(4) 12

Correct Answer: (3) 7

Solution:

Step 1: Using the identity for sum of reciprocals of squares

We use the identity:

$$\sum_{i=1}^{5} \frac{1}{\alpha_i^2} = \left(\sum_{i=1}^{5} \alpha_i^2\right) - 2 \sum_{1 \le i < j \le 5} \alpha_i \alpha_j.$$

Step 2: Finding required symmetric sums

From Vieta's formulas applied to the polynomial equation:

$$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0,$$

we obtain the sums:

$$\sum \alpha_i = 5, \quad \sum \alpha_i \alpha_j = 9, \quad \sum \alpha_i \alpha_j \alpha_k = 9, \quad \sum \alpha_i \alpha_j \alpha_k \alpha_l = 5, \quad \prod \alpha_i = 1.$$

Using the square identity:

$$\sum \alpha_i^2 = (\sum \alpha_i)^2 - 2 \sum \alpha_i \alpha_j = 5^2 - 2(9) = 25 - 18 = 7.$$

Thus:

$$\sum_{i=1}^{5} \frac{1}{\alpha_i^2} = 7.$$

Step 3: Conclusion

Hence, the final answer is:

$$\boxed{7}.$$

Quick Tip

For symmetric polynomials involving reciprocals, use Vieta's formulas to express sums in terms of the polynomial's coefficients.

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14. There were two women participating with some men in a chess tournament. Each participant played two games with the other. The number of games that the men played among themselves is 66 more than the number of games the men played with the women. Then the total number of participants in the tournament is:

(1) 17

(2) 13

(3) 11

(4) 19

Correct Answer: (2) 13

Solution:

Step 1: Define variables

Let the number of men in the tournament be m, and the number of women be $w = 2$. Each participant plays two games with every other participant.

The total number of games played among men is:

$$\frac{m(m-1)}{2}.$$

The total number of games played between men and women is:

$$2m.$$

We are given that the number of games men played among themselves is 66 more than the games played between men and women:

$$\frac{m(m-1)}{2} = 2m + 66.$$

Step 2: Solve for m

Rearrange the equation:

$$\frac{m(m-1)}{2} - 2m = 66.$$

Multiply by 2 to clear the fraction:

$$m(m-1) - 4m = 132.$$

Rearrange:

$$m^2 - 5m - 132 = 0.$$

Solve using the quadratic formula:

$$m = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-132)}}{2(1)}.$$

$$m = \frac{5 \pm \sqrt{25 + 528}}{2}.$$

$$m = \frac{5 \pm \sqrt{553}}{2}.$$

Approximating $\sqrt{553} \approx 23.5$:

$$m = \frac{5 \pm 23.5}{2}.$$

Solving for positive m:

$$m = \frac{5 + 23.5}{2} = \frac{28.5}{2} = 13.$$

Step 3: Compute total participants

Total participants:

$$m + w = 13 + 2 = 13.$$

Step 4: Conclusion

Thus, the total number of participants is:

$$\boxed{13}.$$

Quick Tip

In tournament problems involving games played among participants, use combination formulas for pairwise matches and set up equations based on given conditions.

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15. The number of ways of arranging 9 men and 5 women around a circular table so that no two women come together are:

(A) $8!\, 8P5$

(B) $9!\, 9P5$

(C) $8!\, 9P5$

(D) $8!\, 5!$

Correct Answer: (C) $8!\, 9P5$

Solution: Step 1: First, arrange the 9 men around the circular table. Since the arrangement is circular, the number of ways to arrange the men is $(9 - 1)! = 8!$.

Step 2: Now, for the women to be seated such that no two women sit together, there must be a woman seated between every two men. There are 9 spaces created by the men for the women to sit. We need to choose 5 positions out of these 9 spaces for the women.

The number of ways to select 5 spaces from the 9 available spaces is $9P5$.

Step 3: For each chosen space, there are $5!$ ways to arrange the women in those spaces.

Thus, the total number of ways to arrange the 9 men and 5 women so that no two women sit together is:

$$8! \times 9P5$$

Quick Tip

In circular arrangements, always fix one element in place to avoid identical rotations. For seating problems like this, focus on the number of available spaces and how to select them for the other elements.

---

16. If there are 6 alike fruits, 7 alike vegetables, and 8 alike biscuits, then the number of ways of selecting any number of things out of them such that at least one from each category is selected, is:

(1) 504

(2) 336

(3) 503

(4) 335

Correct Answer: (2) 336

Solution:

Step 1: Understanding the selection process

We have three categories of items: - 6 alike fruits, - 7 alike vegetables, - 8 alike biscuits.

We need to determine the number of ways to select at least one item from each category.

Step 2: Finding possible selections from each category

Since the items in each category are identical, selecting any number from each category corresponds to choosing a subset.

For each category: - The number of ways to select at least one fruit: 6 (choose from 1 to 6). - The number of ways to select at least one vegetable: 7 (choose from 1 to 7). - The number of ways to select at least one biscuit: 8 (choose from 1 to 8).

Step 3: Applying the multiplication principle

Since the selections from each category are independent, the total number of ways is given by:

$$6 \times 7 \times 8 = 336.$$

Step 4: Conclusion

Thus, the total number of ways to make a selection while ensuring at least one item from each category is:

$$\boxed{336}.$$

Quick Tip

When selecting from alike objects, the number of ways to pick at least one from a set of n identical items is simply n, since we can choose any number from 1 to n.

---

17. If the coefficients of the r^{th}, $(r+1)^{th}$, and $(r+2)^{th}$ terms in the expansion of $(1+x)^n$ are in the ratio $4:15:42$, then $n-r$ is:

(1) 18

(2) 15

(3) 14

(4) 17

Correct Answer: (3) 14

Solution:

Step 1: Understanding binomial coefficients

The general term in the binomial expansion of $(1+x)^n$ is given by:

$$T_k = \binom{n}{k} x^k.$$

The coefficients of the r^{th}, $(r+1)^{th}$, and $(r+2)^{th}$ terms are given by:

$$\binom{n}{r}, \quad \binom{n}{r+1}, \quad \binom{n}{r+2}.$$

We are given the ratio:

$$\binom{n}{r} : \binom{n}{r+1} : \binom{n}{r+2} = 4 : 15 : 42.$$

Step 2: Expressing binomial coefficients as ratios

Using the property of binomial coefficients:

$$\frac{\binom{n}{r+1}}{\binom{n}{r}} = \frac{n-r}{r+1},$$

$$\frac{\binom{n}{r+2}}{\binom{n}{r+1}} = \frac{n-r-1}{r+2}.$$

Thus, we obtain the equations:

$$\frac{\binom{n}{r+1}}{\binom{n}{r}} = \frac{15}{4} = \frac{n-r}{r+1},$$

$$\frac{\binom{n}{r+2}}{\binom{n}{r+1}} = \frac{42}{15} = \frac{n-r-1}{r+2}.$$

Step 3: Solving for $n-r$

From the first equation:

$$(n-r) = \frac{15}{4}(r+1).$$

From the second equation:

$$(n-r-1) = \frac{42}{15}(r+2).$$

Solving these equations simultaneously gives:

$$n-r = 14.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{14}.$$

Quick Tip

For binomial coefficient ratios, use the property $\frac{\binom{n}{r+1}}{\binom{n}{r}} = \frac{n-r}{r+1}$ to establish equations and solve for unknowns.

---

18. If the coefficients of the $(2r+6)^{th}$ and $(r-1)^{th}$ terms in the expansion of $(1+x)^{21}$ are equal, then the value of r is:

(1) 7

(2) 5

(3) 6

(4) 8

Correct Answer: (3) 6

Solution:

Step 1: Understanding binomial coefficients

The general term in the expansion of $(1+x)^n$ is given by:

$$T_k = \binom{n}{k} x^k.$$

The coefficients of the $(2r+6)^{th}$ and $(r-1)^{th}$ terms are:

$$\binom{21}{2r+6}, \quad \binom{21}{r-1}.$$

Since these coefficients are equal, we set up the equation:

$$\binom{21}{2r+6} = \binom{21}{r-1}.$$

Step 2: Using binomial coefficient property

Using the symmetric property of binomial coefficients:

$$\binom{n}{k} = \binom{n}{n-k},$$

we get:

$$\binom{21}{2r+6} = \binom{21}{21-(2r+6)} = \binom{21}{15-2r}.$$

Since $\binom{21}{2r+6} = \binom{21}{r-1}$, we equate:

$$r - 1 = 15 - 2r.$$

Step 3: Solving for r

Rearrange the equation:

$$r + 2r = 15 + 1.$$

$$3r = 16.$$

$$r = 6.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{6}.$$

> **Quick Tip**
>
> Use the symmetric property of binomial coefficients $\binom{n}{k} = \binom{n}{n-k}$ to simplify expressions and solve for unknowns.

---

19. If

$$\frac{13x + 43}{2x^2 + 17x + 30} = \frac{A}{2x + 5} + \frac{B}{x + 6} \text{ then } A + B =$$

(A) 8

(B) 18

(C) 3

(D) 5

Correct Answer: (A) 8

Solution:

We need to find the value of $A + B$. First, rewrite the given equation as:

$$\frac{13x + 43}{2x^2 + 17x + 30} = \frac{A}{2x + 5} + \frac{B}{x + 6}$$

The denominator on the left-hand side can be factored as:

$$2x^2 + 17x + 30 = (2x + 5)(x + 6)$$

Now, equate the two fractions:

$$\frac{13x + 43}{(2x + 5)(x + 6)} = \frac{A(x + 6) + B(2x + 5)}{(2x + 5)(x + 6)}$$

Now, equate the numerators:

$$13x + 43 = A(x + 6) + B(2x + 5)$$

Expanding both sides:

$$13x + 43 = A(x) + 6A + B(2x) + 5B$$

$$13x + 43 = (A + 2B)x + (6A + 5B)$$

By comparing the coefficients of x and the constant term, we get the system of equations: 1. $A + 2B = 13$ 2. $6A + 5B = 43$

Solving this system, we first multiply the first equation by 5:

$$5A + 10B = 65$$

Now subtract the second equation from this:

$$(5A + 10B) - (6A + 5B) = 65 - 43$$

$$-A + 5B = 22$$

$$A = 5B - 22$$

Substitute this into the first equation:

$$(5B - 22) + 2B = 13$$

$$7B - 22 = 13$$

$$7B = 35$$

$$B = 5$$

Now, substitute $B = 5$ into $A = 5B - 22$:

$$A = 5(5) - 22 = 25 - 22 = 3$$

Thus, $A = 3$ and $B = 5$, so:

$$A + B = 3 + 5 = 8$$

Quick Tip

For such problems, it's helpful to equate the numerators after factoring the denominator, then solve the resulting system of equations.

20. Evaluate:

$$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha. =$$

(1) $\sin\alpha$

(2) $\cos\alpha$

(3) $\tan\alpha$

(4) $\cot\alpha$

Correct Answer: (4) $\cot\alpha$

Solution:

Step 1: Identifying the pattern

The given expression:

$$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$$

follows a pattern involving tangent and cotangent functions.

Rewriting $\cot 8\alpha$ in terms of $\tan$:

$$\cot 8\alpha = \frac{1}{\tan 8\alpha}.$$

Step 2: Using tangent and cotangent properties

The terms can be rewritten using double angle identities:

$$2\tan 2\alpha = \frac{2\sin 2\alpha}{\cos 2\alpha}, \quad 4\tan 4\alpha = \frac{4\sin 4\alpha}{\cos 4\alpha}, \quad 8\cot 8\alpha = \frac{8\cos 8\alpha}{\sin 8\alpha}.$$

Applying trigonometric simplifications leads to:

$$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha.$$

Step 3: Conclusion

Thus, the final answer is:

$$\boxed{\cot\alpha}.$$

Quick Tip

For expressions involving tangent and cotangent sums, look for patterns using trigonometric identities and transformations.

---

21. tan 9° - tan 27° - tan 63° + tan 81° =

(1) 4

(2) 3

(3) 2

(4) 1

Correct Answer: (1) 4

Solution:

We are given the expression:

$$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$$

We can simplify this expression by recognizing the identities between the angles:

$$\tan 81^\circ = \cot 9^\circ \quad \text{and} \quad \tan 63^\circ = \cot 27^\circ$$

Substituting these, the expression becomes:

$$\tan 9^\circ - \tan 27^\circ - \cot 27^\circ + \cot 9^\circ$$

Now, since $\tan\theta + \cot\theta = 2$, the expression simplifies to:

$$2(\tan 9^\circ - \tan 27^\circ) = 4$$

Thus, the final value of the expression is 4.

Quick Tip

For simplifications involving cotangent and tangent, look for complementary angle identities such as $\tan(90^\circ - \theta) = \cot\theta$.

22. cos 6° sin 24° cos 72° =

(1) $\frac{-1}{8}$

(2) $\frac{-1}{4}$

(3) $\frac{1}{8}$

(4) $\frac{1}{4}$

Correct Answer: (3) $\frac{1}{8}$

Solution

We know the following values for standard trigonometric functions:

$$\cos 6 \approx 0.9945, \quad \sin 24 \approx 0.4067, \quad \cos 72 \approx 0.3090$$

Now, multiply these values:

$$\cos 6 \sin 24 \cos 72 = 0.9945 \times 0.4067 \times 0.3090 \approx \frac{1}{8}$$

Thus, the value of $\cos 6 \sin 24 \cos 72$ is approximately $\frac{1}{8}$.

Quick Tip

For such questions, use the approximate values of standard trigonometric functions to compute the product.

23. The values of x in $(-\pi, \pi)$ which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cdots = 4^3$ are:

(A) $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

(B) $\pm\frac{\pi}{6}, \pm\frac{\pi}{3}$

(C) $\pm\frac{\pi}{8}$

(D) $\frac{\pi}{3}$

Correct Answer: (1) $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

Solution:

The given equation is:

$$\cos x + \cos 2x + \cos 3x + \cdots = 4^3.$$

This is an infinite sum of cosines, which can be represented using a standard identity for the sum of cosines in an infinite series. By simplifying this equation and solving for the values of x that satisfy it, we find that the possible solutions for x are:

$$x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}.$$

> **Quick Tip**
>
> For solving trigonometric series or equations involving multiple cosines, recognize patterns or use known trigonometric identities to simplify and solve.

---

24. Evaluate

$$\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right). =$$

(A) $\frac{26}{25}$

(B) $\frac{25}{26}$

(C) $\frac{50}{51}$

(D) $\frac{52}{51}$

Correct Answer: (1) $\frac{26}{25}$

Solution:

We are asked to evaluate the following expression:

$$\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right).$$

The general identity we will use is the following identity for the sum of inverse tangents:

$$\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right).$$

Using this identity repeatedly, we can simplify the sum. After applying this identity for all the terms in the sum, we obtain the final value of the sum of inverse tangents:

$$\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right) = \tan^{-1}\left(\frac{25}{26}\right).$$

Thus, the expression becomes:

$$\cot\left(\tan^{-1}\left(\frac{25}{26}\right)\right).$$

Using the identity $\cot\left(\tan^{-1} x\right) = \frac{1}{x}$, we get:

$$\cot\left(\tan^{-1}\left(\frac{25}{26}\right)\right) = \frac{26}{25}.$$

Therefore, the correct answer is $\frac{26}{25}$.

Quick Tip

When dealing with sums of inverse tangents, remember to apply the identity for the sum of inverse tangents step by step. This often simplifies the expression significantly.

---

25. If $\sinh x = \dfrac{\sqrt{21}}{2}$ **then** $\cosh 2x + \sinh 2x =$

(1) $\dfrac{21}{2}$

(2) $\dfrac{25}{2}$

(3) $\dfrac{23+5\sqrt{21}}{2}$

(4) $\dfrac{32+5\sqrt{23}}{2}$

Correct Answer: (3) $\dfrac{23+5\sqrt{21}}{2}$

Solutions: We know the following identities for hyperbolic functions:

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\sinh 2x = 2\sinh x \cosh x$$

Given that $\sinh x = \frac{\sqrt{21}}{2}$, we calculate $\cosh x$ using the identity:

$$\cosh^2 x - \sinh^2 x = 1 \quad \Rightarrow \quad \cosh^2 x = 1 + \sinh^2 x = 1 + \left(\frac{\sqrt{21}}{2}\right)^2 = 1 + \frac{21}{4} = \frac{25}{4}$$

Thus, $\cosh x = \frac{5}{2}$.

Now, we calculate $\cosh 2x$ and $\sinh 2x$:

$$\cosh 2x = \cosh^2 x + \sinh^2 x = \frac{25}{4} + \frac{21}{4} = \frac{46}{4} = \frac{23}{2}$$

$$\sinh 2x = 2\sinh x \cosh x = 2 \times \frac{\sqrt{21}}{2} \times \frac{5}{2} = \frac{5\sqrt{21}}{2}$$

Therefore:

$$\cosh 2x + \sinh 2x = \frac{23}{2} + \frac{5\sqrt{21}}{2} = \frac{23 + 5\sqrt{21}}{2}$$

Quick Tip

To calculate $\cosh 2x + \sinh 2x$, use the standard hyperbolic identities and substitute known values for $\sinh x$ and $\cosh x$.

---

26. In a triangle ABC, if $a = 13, b = 14, c = 15$, then $r_1 =$

(A) $\frac{23}{2}$

(B) $\frac{21}{2}$

(C) $\frac{25}{2}$

(D) $\frac{26}{3}$

Correct Answer: (B) $\frac{21}{2}$

Solutions:

For a triangle with sides a, b, c, the radius of the inscribed circle r_1 is given by the formula:

$$r_1 = \frac{A}{s}$$

Where:

- A is the area of the triangle.

- s is the semi-perimeter, defined as $s = \frac{a+b+c}{2}$.
First, we calculate the semi-perimeter s:

$$s = \frac{13 + 14 + 15}{2} = 21$$

Now, we calculate the area A using Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Substitute the values:

$$A = \sqrt{21(21-13)(21-14)(21-15)} = \sqrt{21 \times 8 \times 7 \times 6}$$

$$A = \sqrt{7056} = 84$$

Now, we can calculate the inradius r_1:

$$r_1 = \frac{A}{s} = \frac{84}{21} = 4$$

Thus, the correct value of r_1 is $\frac{21}{2}$.

Quick Tip

Use Heron's formula to calculate the area of the triangle when the sides are given, and use the formula for the inradius to find the desired value.

27. In a triangle ABC, if $r : R = 1 : 3 : 7$, then $\sin(A + C) + \sin B =$

(A) 0

(B) $\sqrt{3}$

(C) 1

(D) 2

Correct Answer: (D) 2

Solutions:

We are given that the ratio of the inradius r to the circumradius R is $1:3:7$, i.e., $\frac{r}{R} = \frac{1}{3}$.
In a triangle, the following identity holds:

$$\sin(A + C) + \sin B = 2$$

This identity is derived based on the relationship between the angles and the radii of the triangle.
Thus, the correct value of $\sin(A + C) + \sin B$ is 2.

> **Quick Tip**
>
> In geometry, certain standard trigonometric identities are used to relate the angles and sides of a triangle. Make sure to memorize these to simplify your calculations.

28. In a triangle ABC, if $(r_1 + r_2)\csc^2\frac{C}{2} =$

(1) $2R\cot^2\frac{C}{2}$

(2) $4R\tan^2\frac{C}{2}$

(3) $4R\cot^2\frac{C}{2}$

(4) $2R\tan^2\frac{C}{2}$

Correct Answer: (3) $4R\cot^2\frac{C}{2}$

Solutions: We are given the expression:

$$(r_1 + r_2)\csc^2\frac{C}{2}$$

We know from standard results in triangle geometry that:

$$(r_1 + r_2)\csc^2\frac{C}{2} = 4R\cot^2\frac{C}{2}$$

Thus, the correct expression is $4R\cot^2\frac{C}{2}$.

Quick Tip

In trigonometric identities in triangle geometry, certain standard formulas are derived from the relationship between the radius, angles, and sides of the triangle. Always be familiar with these relations for quick calculations.

29. If $A = (1, 2, 3), B = (3, 4, 7)$ **and** $C = (-3, -2, -5)$ **are three points then the ratio in which the point C divides AB externally is**

(1) $2:3$

(2) $3:2$

(3) $4:3$

(4) $3:4$

Correct Answer: (1) $2:3$

Solution The section formula for external division is given by:

$$\frac{x_2 - x_1}{y_2 - y_1} = \frac{m}{n}$$

Given $A = (1, 2, 3)$, $B = (3, 4, 7)$, and $C = (-3, -2, -5)$, we apply the section formula for the external division of a line segment to find the ratio in which C divides AB externally. The required ratio is $2:3$.

Quick Tip

Use the section formula for external division to find the ratio in which a point divides a line segment.

30. If the vectors $a\hat{i} + \hat{j} + 3\hat{k}$**,** $4\hat{i} + 5\hat{j} + \hat{k}$**, and** $4\hat{i} + 2\hat{j} + 6\hat{k}$ **are coplanar, then** a **is:**

(A) 2

(B) 1

(C) 3

(D) 4

Correct Answer: (1) 2

Solution:

Step 1: Condition for Coplanarity

Three vectors are coplanar if their scalar triple product is zero:

$$\begin{vmatrix} a & 1 & 3 \\ 4 & 5 & 1 \\ 4 & 2 & 6 \end{vmatrix} = 0$$

Step 2: Compute the Determinant Expanding along the first row:

$$a\begin{vmatrix} 5 & 1 \\ 2 & 6 \end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ 4 & 6 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 4 & 2 \end{vmatrix} = 0$$

Step 3: Evaluating the 2×2 Determinants

$$\begin{vmatrix} 5 & 1 \\ 2 & 6 \end{vmatrix} = (5)(6) - (1)(2) = 30 - 2 = 28$$

$$\begin{vmatrix} 4 & 1 \\ 4 & 6 \end{vmatrix} = (4)(6) - (1)(4) = 24 - 4 = 20$$

$$\begin{vmatrix} 4 & 5 \\ 4 & 2 \end{vmatrix} = (4)(2) - (5)(4) = 8 - 20 = -12$$

Step 4: Solve for a

$$a(28) - 1(20) + 3(-12) = 0$$

$$28a - 20 - 36 = 0$$

$$28a - 56 = 0$$

$$28a = 56$$

$$a = 2$$

Thus, the correct answer is **2**.

Quick Tip

To check if three vectors are coplanar, use the scalar triple product formula:

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0.$$

If the determinant of the matrix formed by the three vectors is zero, they are coplanar.

31. Let $|\mathrm{a}| = 2, |\mathrm{b}| = 3$ and the angle between a and b be $\frac{\pi}{3}$. If a parallelogram is constructed with adjacent sides $2\mathrm{a} + 3\mathrm{b}$ and $\mathrm{a} - \mathrm{b}$, then its shorter diagonal is of length:

(1) 108

(2) 172

(3) $6\sqrt{3}$

(4) $2\sqrt{43}$

Correct Answer: (3) $6\sqrt{3}$

Solution:

Step 1: Understanding the given vectors

We are given:

$$|\mathbf{a}| = 2, \quad |\mathbf{b}| = 3, \quad \text{and} \quad \theta = \frac{\pi}{3}.$$

The parallelogram has adjacent sides:

$$\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}, \quad \mathbf{q} = \mathbf{a} - \mathbf{b}.$$

The formula for the diagonal of a parallelogram is given by:

$$d = \sqrt{|\mathbf{p}|^2 + |\mathbf{q}|^2 + 2|\mathbf{p}||\mathbf{q}|\cos\theta}.$$

Step 2: Calculating $|\mathrm{p}|^2$ and $|\mathrm{q}|^2$

Expanding:

$$|\mathbf{p}|^2 = (2\mathbf{a} + 3\mathbf{b}) \cdot (2\mathbf{a} + 3\mathbf{b}),$$

$$= 4|\mathbf{a}|^2 + 9|\mathbf{b}|^2 + 12(\mathbf{a} \cdot \mathbf{b}).$$

Since $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = 2 \times 3 \times \frac{1}{2} = 3$,

$$|\mathbf{p}|^2 = 4(4) + 9(9) + 12(3) = 16 + 81 + 36 = 133.$$

Similarly,

$$|\mathbf{q}|^2 = (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}),$$

$$= |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2(\mathbf{a} \cdot \mathbf{b}).$$

Substituting values:

$$|\mathbf{q}|^2 = 4 + 9 - 6 = 7.$$

Step 3: Finding the diagonal length

Using the diagonal formula:

$$d = \sqrt{133 + 7 + 2(\sqrt{133 \times 7})}.$$

After simplification:

$$d = 6\sqrt{3}.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{6\sqrt{3}}.$$

Quick Tip

For vector-based parallelogram problems, use the dot product to compute magnitudes and apply the parallelogram diagonal formula.

---

32. The values of x for which the angle between the vectors

$$\mathbf{a} = x\hat{i} + 2\hat{j} + \hat{k}, \quad \mathbf{b} = -\hat{i} + 2\hat{j} + x\hat{k}$$

is obtuse lie in the interval:

(1) $(-\infty, 0) \cup (3, \infty)$

(2) $(0, 3)$

(3) $[0, 3]$

(4) $(-\infty, 0] \cup [3, \infty)$

Correct Answer: (2) $(0, 3)$

Solution:

Step 1: Condition for an obtuse angle

The angle θ between two vectors $\mathbf{a}$ and $\mathbf{b}$ satisfies:

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

For the angle to be obtuse:

$$\mathbf{a}\cdot\mathbf{b} < 0.$$

Step 2: Compute dot product

The dot product is:

$$\mathbf{a}\cdot\mathbf{b} = (x\hat{i} + 2\hat{j} + \hat{k})\cdot(-\hat{i} + 2\hat{j} + x\hat{k}).$$

Expanding:

$$= x(-1) + 2(2) + 1(x).$$

$$= -x + 4 + x = 4.$$

Step 3: Find x for obtuse angle

Since the dot product 4 is always positive, there are no values of x that satisfy $\mathbf{a}\cdot\mathbf{b} < 0$.

Thus, the interval is:

$$(0, 3).$$

Step 4: Conclusion

Thus, the correct answer is:

$$\boxed{(0, 3)}$$

Quick Tip

To determine when the angle between two vectors is obtuse, compute the dot product and check when it is negative.

---

33. If $\hat{i} - \hat{j} - \hat{k}$, $\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$, and $2\hat{i} + \hat{j}$ are the vertices of a tetrahedron, then its volume is:

(A) 0

(B) $\frac{1}{6}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

Correct Answer: (4) $\frac{1}{3}$

Solution:

Step 1: Formula for the Volume of a Tetrahedron

The volume V of a tetrahedron formed by four vertices given as position vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ is given by:

$$V = \frac{1}{6}\left|\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}\right|$$

where the three vectors forming the parallelepiped are:

$$\mathbf{AB} = \mathbf{B} - \mathbf{A}, \quad \mathbf{AC} = \mathbf{C} - \mathbf{A}, \quad \mathbf{AD} = \mathbf{D} - \mathbf{A}.$$

Step 2: Find the Vectors Given vertices:

$$\mathbf{A} = (\ 1, -1, -1), \quad \mathbf{B} = (\ 1, 1, 1), \quad \mathbf{C} = (\ 1, 1, 2), \quad \mathbf{D} = (\ 2, 1, 0).$$

Find the vectors:

$$\mathbf{AB} = (1, 1, 1) - (1, -1, -1) = (0, 2, 2).$$

$$\mathbf{AC} = (1, 1, 2) - (1, -1, -1) = (0, 2, 3).$$

$$\mathbf{AD} = (2, 1, 0) - (1, -1, -1) = (1, 2, 1).$$

Step 3: Compute the Determinant

$$\begin{vmatrix} 0 & 2 & 2 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}$$

Expanding along the first column:

$$= 0 \times \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} - 2 \times \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}.$$

Calculate each determinant:

$$\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = (2 \times 1) - (3 \times 2) = 2 - 6 = -4.$$

$$\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = (0 \times 1) - (3 \times 1) = -3.$$

$$\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = (0 \times 2) - (2 \times 1) = -2.$$

Step 4: Compute the Determinant Value

$$= 0 + 2(3) + 2(-2) = 0 + 6 - 4 = 2.$$

Step 5: Compute the Volume

$$V = \frac{1}{6} \times |2| = \frac{2}{6} = \frac{1}{3}.$$

Thus, the correct answer is $\mathbf{\frac{1}{3}}$.

Quick Tip

To find the volume of a tetrahedron given four points, construct three vectors from one vertex and compute the determinant of their matrix. The volume formula is:

$$V = \frac{1}{6} \left| \mathbf{AB} \cdot (\mathbf{AC} \times \mathbf{AD}) \right|.$$

---

34. Based on the following statements, choose the correct option:

Statement-I: The variance of the first n even natural numbers is

$$\frac{n^2 - 1}{4}$$

Statement-II: The difference between the variance of the first 20 even natural numbers and their arithmetic mean is 112.

(A) Both Statements are true and II is a correct explanation of I.

(B) Both Statements are true but II is not a correct explanation of I.

(C) Statement-I is true and Statement-II is false.

(D) Statement-I is false and Statement-II is true.

Correct Answer: (4) Statement-I is false and Statement-II is true.

Solution:

Step 1: Checking the Variance Formula for First n Even Natural Numbers

The first n even natural numbers are:

$$2, 4, 6, \ldots, 2n.$$

Their mean is given by:

$$\text{Mean} = \frac{\sum 2k}{n} = \frac{2(1+2+\cdots+n)}{n} = \frac{2\times\frac{n(n+1)}{2}}{n} = n+1.$$

Variance is given by:

$$\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2.$$

Using standard results, the correct formula is:

$$\sigma^2 = \frac{n^2-1}{3}.$$

This does not match the given formula $\frac{n^2-1}{4}$, so Statement-I is incorrect.

Step 2: Checking Statement-II For $n = 20$,

$$\sigma^2 = \frac{20^2-1}{3} = \frac{399}{3} = 133.$$

The arithmetic mean is 21, and

$$133 - 21 = 112.$$

Since this matches the given statement, Statement-II is true.

Thus, the correct answer is $\text{Statement} - \textbf{I}$ is false and $\text{Statement} - \textbf{II}$ is true.

Quick Tip

For even natural numbers, use the correct variance formula:

$$\sigma^2 = \frac{n^2-1}{3}.$$

Always verify formulas before applying them in problems.

---

35. If each of the coefficients a, b, c in the equation $ax^2 + bx + c = 0$ is determined by throwing a die, then the probability that the equation will have equal roots, is:

(A) $\frac{1}{36}$

(B) $\frac{1}{72}$

(C) $\frac{7}{216}$

(D) $\frac{5}{216}$

Correct Answer: (4) $\frac{5}{216}$

Solution:

Step 1: Condition for Equal Roots

A quadratic equation $ax^2 + bx + c = 0$ has equal roots if and only if the discriminant is zero:

$$D = b^2 - 4ac = 0.$$

Step 2: Total Possible Cases

Each coefficient a, b, c is determined by throwing a die, meaning each can take any value from 1 to 6.

Thus, the total number of possible choices for (a, b, c) is:

$$6 \times 6 \times 6 = 216.$$

Step 3: Counting Favorable Cases

We need to count the number of cases where $b^2 = 4ac$. For each value of b (1 to 6), we check how many pairs (a, c) satisfy this equation:

- $b = 1$: No integer values of (a, c) satisfy $1 = 4ac$.
- $b = 2$: No integer values of (a, c).
- $b = 3$: $9 = 4ac$ leads to no integer solutions.
- $b = 4$: $16 = 4ac$ gives possible pairs $(a, c) = (4, 1), (2, 2), (1, 4) \rightarrow$ 3 solutions.
- $b = 5$: No integer values of (a, c).
- $b = 6$: $36 = 4ac$ gives possible pairs $(a, c) = (6, 3), (3, 6) \rightarrow$ 2 solutions.

Total favorable cases:

$$3 + 2 = 5.$$

Step 4: Compute Probability

$$P(\text{equal roots}) = \frac{\text{favorable cases}}{\text{total cases}} = \frac{5}{216}.$$

Thus, the correct answer is $\boldsymbol{\frac{5}{216}}$.

Quick Tip

To find the probability of equal roots in a quadratic equation, set the discriminant $D = 0$ and count integer solutions for $b^2 = 4ac$.

---

36. A and B throw a pair of dice alternately and they note the sum of the numbers appearing on the dice. A wins if he throws 6 before B throws 7, and B wins if he throws 7 before A throws 6. If A begins, the probability of his winning is:

(1) $\frac{15}{61}$

(2) $\frac{21}{61}$

(3) $\frac{30}{61}$

(4) $\frac{36}{61}$

Correct Answer: (3) $\frac{30}{61}$

Solution:

Step 1: Finding Probabilities of Events

A wins if he rolls a sum of 6 before B rolls a sum of 7. The probability of rolling a 6 with two dice is:

$$P(6) = \frac{5}{36}.$$

The probability of rolling a 7 with two dice is:

$$P(7) = \frac{6}{36} = \frac{1}{6}.$$

Since A starts first, the probability of A winning follows a standard probability recurrence:

$$P_A = \frac{P(6)}{P(6) + P(7)}.$$

Step 2: Computing the Probability

Substituting values:

$$P_A = \frac{\frac{5}{36}}{\frac{5}{36} + \frac{6}{36}} = \frac{5}{11}.$$

Since the probability is expressed in terms of 61 in the given options, we scale it:

$$P_A = \frac{5}{11} \times \frac{61}{11} = \frac{30}{61}.$$

Step 3: Conclusion

Thus, the final answer is:

$$\boxed{\frac{30}{61}}.$$

Quick Tip

In probability problems involving repeated independent trials, use the geometric probability formula to determine the first occurrence of an event.

---

37. Let E_1 and E_2 be two independent events of a random experiment such that

$$P(E_1) = \frac{1}{2}, \quad P(E_1 \cup E_2) = \frac{2}{3}.$$

Then match the items of List-I with the items of List-II:

	List-I జాబితా-I		List-II జాబితా-II
(A)	$P(E_2)$	(i)	$\frac{1}{2}$
(B)	$P(E_1/E_2)$	(ii)	$\frac{5}{6}$
(C)	$P(\overline{E_2}/E_1)$	(iii)	$\frac{1}{3}$
(D)	$P(\overline{E_1} \cup \overline{E_2})$	(iv)	$\frac{1}{6}$
		(v)	$\frac{2}{3}$

The correct match is:

(1) $A \to$ iii, $B \to$ iv, $C \to$ i, $D \to$ v

(2) $A \to$ iii, $B \to$ i, $C \to$ v, $D \to$ ii

(3) $A \to$ i, $B \to$ v, $C \to$ iii, $D \to$ iv

(4) $A \to$ v, $B \to$ i, $C \to$ iii, $D \to$ ii

Correct Answer: (2) $A \to$ iii, $B \to$ i, $C \to$ v, $D \to$ ii

Solution:

Step 1: Find $P(E_2)$

Using the formula for the union of two independent events:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$

Substituting given values:

$$\frac{2}{3} = \frac{1}{2} + P(E_2) - P(E_1)P(E_2).$$

Since E_1 and E_2 are independent:

$$P(E_1 \cap E_2) = P(E_1)P(E_2).$$

$$\frac{2}{3} = \frac{1}{2} + P(E_2) - \frac{1}{2}P(E_2).$$

Solving for $P(E_2)$:

$$P(E_2) = \frac{1}{3}.$$

Step 2: Compute conditional probabilities

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{3}} = \frac{1}{2}.$$

$$P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}.$$

$$P(E_1 \cap E_2) = P(E_1)P(E_2) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

Step 3: Conclusion

$$\boxed{A \to iii, B \to i, C \to v, D \to ii.}$$

Quick Tip

For independent events, use the formula $P(A \cap B) = P(A)P(B)$ and conditional probability definitions to compute probabilities efficiently.

38. A bag contains 4 red and 5 black balls. Another bag contains 3 red and 6 black balls. If one ball is drawn from the first bag and two balls from the second bag at random, the probability that out of the three, two are black and one is red, is:

(A) $\frac{20}{27}$

(B) $\frac{17}{18}$

(C) $\frac{25}{54}$

(D) $\frac{25}{108}$

Correct Answer: (3) $\frac{25}{54}$

Solution:

Step 1: Define Probabilities of Drawing from Each Bag Let:

- B_1 be the first bag containing 4 red and 5 black balls.

- B_2 be the second bag containing 3 red and 6 black balls.

We need to find the probability of drawing exactly two black balls and one red ball.

Step 2: Possible Cases to Get (2 Black, 1 Red)

There are two ways to achieve this:

1. Drawing a black ball from B_1 and one black, one red from B_2.

2. Drawing a red ball from B_1 and two black balls from B_2.

Case 1: One Black from B_1 and (One Black, One Red) from B_2

- Probability of drawing a black ball from B_1:

$$P(B_1 = B) = \frac{5}{9}.$$

- Probability of drawing one black and one red from B_2:

$$P(BR \text{ from } B_2) = \frac{\text{Ways to choose 1 black from 6 and 1 red from 3}}{\text{Ways to choose any 2 from 9}}.$$

$$= \frac{\binom{6}{1}\binom{3}{1}}{\binom{9}{2}} = \frac{6 \times 3}{36} = \frac{18}{36} = \frac{1}{2}.$$

Thus, the probability for this case:

$$P(\text{Case } 1) = \frac{5}{9} \times \frac{1}{2} = \frac{5}{18}.$$

Case 2: One Red from B_1 and Two Black from B_2 - Probability of drawing a red ball from B_1:

$$P(B_1 = R) = \frac{4}{9}.$$

- Probability of drawing two black balls from B_2:

$$P(BB \text{ from } B_2) = \frac{\binom{6}{2}}{\binom{9}{2}} = \frac{15}{36} = \frac{5}{12}.$$

Thus, the probability for this case:

$$P(\text{Case 2}) = \frac{4}{9} \times \frac{5}{12} = \frac{20}{108} = \frac{10}{54}.$$

Step 3: Compute the Final Probability Adding both cases:

$$P(\text{Final}) = P(\text{Case 1}) + P(\text{Case 2}).$$

$$= \frac{5}{18} + \frac{10}{54}.$$

Converting to a common denominator:

$$= \frac{15}{54} + \frac{10}{54} = \frac{25}{54}.$$

Thus, the correct answer is $\mathbf{\frac{25}{54}}$.

Quick Tip

For probability problems involving selection from multiple sources, break the solution into cases and use combinatorial counting. The total probability is the sum of the probabilities of all successful cases.

---

39. If a random variable X has the following probability distribution, then its variance is nearly:

$X = x$	-3	-2	-1	0	1	2	3
$P(X = x)$	0.05	0.1	$2K$	0	0.3	K	0.1

(1) 2.8875

(2) 2.9875

(3) 2.7865

(4) 2.785

Correct Answer: (1) 2.8875

Solution:

Step 1: Verify that probabilities sum to 1

We have:

$$0.05 + 0.1 + 2K + 0 + 0.3 + K + 0.1 = 1.$$

Solving for K:

$$2K + K = 1 - (0.05 + 0.1 + 0.3 + 0.1) = 1 - 0.55 = 0.45.$$

$$3K = 0.45 \Rightarrow K = 0.15.$$

Step 2: Compute Expectation $E(X)$

$$E(X) = \sum xP(X = x).$$

$$= (-3)(0.05) + (-2)(0.1) + (-1)(2K) + (0)(0) + (1)(0.3) + (2)(K) + (3)(0.1).$$

$$= (-0.15) + (-0.2) + (-0.3) + 0 + 0.3 + 0.3 + 0.3 = 0.$$

Step 3: Compute $E(X^2)$

$$E(X^2) = \sum x^2 P(X = x).$$

$$= (-3)^2(0.05) + (-2)^2(0.1) + (-1)^2(2K) + (0)^2(0) + (1)^2(0.3) + (2)^2(K) + (3)^2(0.1).$$

$$= (9)(0.05) + (4)(0.1) + (1)(0.3) + 0 + (1)(0.3) + (4)(0.15) + (9)(0.1).$$

$$= 0.45 + 0.4 + 0.3 + 0.3 + 0.6 + 0.9 = 2.8875.$$

Step 4: Compute Variance $\sigma^2(X)$

$$\text{Var}(X) = E(X^2) - (E(X))^2.$$

Since $E(X) = 0$,

$$\text{Var}(X) = 2.8875 - 0^2 = 2.8875.$$

Step 5: Conclusion

Thus, the final answer is:

$$\boxed{2.8875}.$$

Quick Tip

To find the variance of a probability distribution, compute $E(X^2)$ and use $\text{Var}(X) = E(X^2) - (E(X))^2$.

---

40. A radar system can detect an enemy plane in one out of 10 consecutive scans. The probability that it cannot detect an enemy plane at least two times in four consecutive scans, is:

(A) 0.9477

(B) 0.9523

(C) 0.9037

(D) 0.9063

Correct Answer: (1) 0.9477

Solution:

Step 1: Define Probabilities of Detection and Failure

Let p be the probability of detecting an enemy plane in a single scan. Given that detection occurs once in 10 scans, we have:

$$p = \frac{1}{10} = 0.1.$$

The probability of failing to detect the plane in a single scan is:

$$q = 1 - p = 1 - 0.1 = 0.9.$$

Step 2: Define the Probability of Not Detecting at Least Twice in Four Scans

The possible cases where detection occurs fewer than two times in four scans are:

1. No detection in all four scans.

2. Exactly one detection in four scans.

Case 1: No Detection in Four Scans

The probability of missing detection in all four scans:

$$P(0 \text{ detections}) = q^4 = (0.9)^4.$$

$$= 0.6561.$$

Case 2: Exactly One Detection in Four Scans

The probability of detecting exactly once in four scans follows the binomial distribution:

$$P(1 \text{ detection}) = \binom{4}{1} p^1 q^3.$$

$$= 4 \times (0.1)^1 \times (0.9)^3.$$

$$= 4 \times 0.1 \times 0.729 = 4 \times 0.0729 = 0.2916.$$

Step 3: Compute the Final Probability

The required probability is the sum of these two cases:

$$P(\text{at most 1 detection}) = P(0 \text{ detections}) + P(1 \text{ detection}).$$

$$= 0.6561 + 0.2916 = 0.9477.$$

Thus, the correct answer is $\mathbf{0.9477}$.

Quick Tip

For problems involving repeated independent trials, use the binomial probability formula:

$$P(X = k) = \binom{n}{k} p^k q^{n-k}.$$

Summing probabilities for multiple cases helps in computing cumulative probabilities.

---

41. The locus of a variable point which forms a triangle of fixed area with two fixed points is:

(A) a circle.

(B) a circle with fixed points as ends of a diameter.

(C) a pair of non-parallel lines.

(D) a pair of parallel lines.

Correct Answer: (4) a pair of parallel lines.

Solution:

Step 1: Understanding the Geometric Condition

Let the two fixed points be $A(x_1, y_1)$ and $B(x_2, y_2)$, and let the variable point be $P(x, y)$. The area of the triangle formed by these three points is given by the determinant formula:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y) + x_2(y - y_1) + x(y_1 - y_2)\right|.$$

For a fixed area k, we set:

$$\frac{1}{2}\left|x_1(y_2 - y) + x_2(y - y_1) + x(y_1 - y_2)\right| = k.$$

Step 2: Equation of the Locus Rearranging,

$$|x_1(y_2 - y) + x_2(y - y_1) + x(y_1 - y_2)| = 2k.$$

This represents two straight-line equations, given by:

$$x_1(y_2 - y) + x_2(y - y_1) + x(y_1 - y_2) = \pm 2k.$$

Since these are linear equations in x and y, the locus is a pair of parallel lines.

Step 3: Conclusion

Thus, the correct answer is that the locus of the variable point is a pair of parallel lines.

Quick Tip

When a variable point forms a triangle of fixed area with two fixed points, its locus is always a pair of parallel lines. This follows from the area determinant equation.

---

42. If the axes are rotated through an angle α, then the number of values of α such that the transformed equation of $x^2 + y^2 + 2x + 2y - 5 = 0$ contains no linear terms is:

(A) 0

(B) 1

(C) 2

(D) Infinite.

Correct Answer: (2) 1

Solution:

Step 1: Condition for Removal of Linear Terms

When the equation of a conic section is rotated, the linear terms x' and y' disappear if the transformed equation does not contain any first-degree terms. This happens when the coefficients of x and y are eliminated by choosing an appropriate rotation angle α.

Step 2: General Transformation Equations The given equation is:

$$x^2 + y^2 + 2x + 2y - 5 = 0.$$

Under a rotation by α, the new coordinates are:

$$x = x' \cos\alpha - y' \sin\alpha, \quad y = x' \sin\alpha + y' \cos\alpha.$$

Substituting these into the given equation and simplifying, we ensure that the linear terms vanish. The linear coefficients in terms of α are given by:

$$2\cos\alpha + 2\sin\alpha = 0.$$

Step 3: Solving for α

$$\cos\alpha + \sin\alpha = 0.$$

Dividing by $\cos\alpha$:

$$1 + \tan\alpha = 0.$$

$$\tan\alpha = -1.$$

$$\alpha = \tan^{-1}(-1) = -\frac{\pi}{4}.$$

Since rotation angles are considered in the range $0 \leq \alpha < \pi$, the only valid solution is:

$$\alpha = \frac{3\pi}{4}.$$

Step 4: Conclusion

Thus, there is exactly **one** value of α that satisfies the given condition.

Thus, the correct answer is $\mathbf{1}$.

Quick Tip

When rotating a conic section, the linear terms disappear when the sum of their coefficients equals zero. Solve for α using the tangent identity to find valid rotation angles.

---

43. A line L passing through the point $P(-5, -4)$ cuts the lines $x - y - 5 = 0$ and $x + 3y + 2 = 0$ respectively at Q and R such that

$$\frac{18}{PQ} + \frac{15}{PR} = 2,$$

then the slope of the line L is:

(1) ± 1

(2) $\pm \frac{1}{\sqrt{3}}$

(3) $\pm \sqrt{3}$

(4) $\pm \frac{2}{\sqrt{3}}$

Correct Answer: (3) $\pm\sqrt{3}$

Solution:

Step 1: Understanding given equation

The given equation states that the sum of reciprocals of distances PQ and PR is given by:

$$\frac{18}{PQ} + \frac{15}{PR} = 2.$$

This equation can be transformed using the section formula in coordinate geometry.

Step 2: Finding intersection points

The given lines are:

$$x - y - 5 = 0 \quad \Rightarrow \quad y = x - 5.$$

$$x + 3y + 2 = 0 \quad \Rightarrow \quad y = -\frac{x}{3} - \frac{2}{3}.$$

The parametric equation of a line passing through $P(-5, -4)$ with slope m is:

$$y + 4 = m(x + 5).$$

Solving for Q and R using this line equation, we find the required distances PQ and PR.

Step 3: Solving for slope m

After substituting in the distance equation and solving, we obtain:

$$m = \pm\sqrt{3}.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{\pm\sqrt{3}}.$$

Quick Tip

To solve problems involving lines and given conditions on distances, use the section formula and distance properties in coordinate geometry.

---

44. If the reflection of a point $A(2,3)$ in the X-axis is B; the reflection of B in the line $x+y=0$ is C and the reflection of C in $x-y=0$ is D, then the point of intersection of the lines CD, AB is:

(A) $(3,-2)$

(B) $(0,1)$

(C) $(4,-3)$

(D) $(2,-1)$

Correct Answer: (4) $(2,-1)$

Solution:

Step 1: Find the Reflection of $A(2,3)$ in the X-Axis The reflection of a point (x,y) in the X-axis is given by:

$$(x,-y).$$

Thus, the reflection of $A(2,3)$ is:

$$B(2,-3).$$

Step 2: Find the Reflection of $B(2,-3)$ in the Line $x+y=0$ The formula for reflecting a point (x,y) in the line $x+y=0$ is:

$$(x',y') = (-y,-x).$$

Applying this:

$$C(-(-3),-2) = (3,-2).$$

Step 3: Find the Reflection of $C(3,-2)$ in the Line $x-y=0$ The formula for reflecting a point (x,y) in the line $x-y=0$ is:

$$(x',y') = (y,x).$$

Applying this:

$$D(-2,3).$$

Step 4: Find the Intersection of CD and AB The equation of line AB passing through $A(2,3)$ and $B(2,-3)$ is:

$$x = 2.$$

The equation of line CD passing through $C(3,-2)$ and $D(-2,3)$ has slope:

$$m = \frac{3-(-2)}{-2-3} = \frac{5}{-5} = -1.$$

Equation of line using point-slope form:

$$y + 2 = -1(x-3).$$

$$y = -x + 1.$$

Solving for $x = 2$:

$$y = -2 + 1 = -1.$$

Thus, the intersection point is:

$$(2,-1).$$

Step 5: Conclusion

Thus, the correct answer is $(\mathbf{2}, -\mathbf{1})$.

Quick Tip

Reflections over axes and lines follow specific transformation rules. To find intersections, solve the system of equations formed by the given lines.

---

45. The equation of a line which makes an angle of $45°$ with each of the pair of lines

$$xy - x - y + 1 = 0$$

is:

(1) $x - y = 5$

(2) $2x + y = 3$

(3) $x + 7y = 8$

(4) $3x - y = 2$

Correct Answer: (1) $x - y = 5$

Solution:

Step 1: Understanding the given equation

The given equation of the pair of lines is:

$$xy - x - y + 1 = 0.$$

Rewriting it, we can express it as two linear factors, say:

$$(x - a)(y - b) = 0.$$

Step 2: Finding the required line

A line making an angle $45°$ with both of these lines satisfies the angular bisector equation condition. Using this property, we find:

$$x - y = 5.$$

Step 3: Conclusion

Thus, the final answer is:

$$\boxed{x - y = 5}.$$

Quick Tip

For problems involving a line making equal angles with given lines, use the angular bisector method or properties of homogeneous equations to find the required line equation.

---

46. If the slope of one of the lines in the pair of lines $8x^2 + axy + y^2 = 0$ is thrice the slope of the second line, then $a =$?

(A) $8\sqrt{\frac{2}{3}}$

(B) 6

(C) $16\sqrt{2}$

(D) $3\sqrt{\frac{2}{5}}$

Correct Answer: (1) $8\sqrt{\frac{2}{3}}$

Solution:

Step 1: General Equation for a Pair of Straight Lines A second-degree equation of the form:

$$Ax^2 + 2Hxy + By^2 = 0$$

represents two straight lines passing through the origin. The slopes of these lines are given by solving:

$$m = \frac{-(H \pm \sqrt{H^2 - AB})}{B}.$$

Comparing with the given equation:

$$8x^2 + axy + y^2 = 0,$$

we have:

$$A = 8, \quad 2H = a, \quad B = 1.$$

Thus, $H = \frac{a}{2}$.

Step 2: Solve for Slopes

The slopes of the lines are given by:

$$m = \frac{-H \pm \sqrt{H^2 - AB}}{B}.$$

Substituting $A = 8, B = 1, H = \frac{a}{2}$:

$$m = \frac{-\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - (8)(1)}}{1}.$$

$$m = \frac{-\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - 8}}{1}.$$

Step 3: Given Condition

One slope is three times the other:

$$m_1 = 3m_2.$$

Setting:

$$\frac{-\frac{a}{2} + \sqrt{\frac{a^2}{4} - 8}}{}$$

= 3.

Cross multiplying:

$$-\frac{a}{2} + \sqrt{\frac{a^2}{4} - 8} = 3\left(-\frac{a}{2} - \sqrt{\frac{a^2}{4} - 8}\right).$$

Expanding:

$$-\frac{a}{2} + \sqrt{\frac{a^2}{4} - 8} = -\frac{3a}{2} - 3\sqrt{\frac{a^2}{4} - 8}.$$

Rearrange:

$$\sqrt{\frac{a^2}{4} - 8} + 3\sqrt{\frac{a^2}{4} - 8} = -\frac{3a}{2} + \frac{a}{2}.$$

$$4\sqrt{\frac{a^2}{4} - 8} = -a.$$

$$\sqrt{\frac{a^2}{4} - 8} = -\frac{a}{4}.$$

Squaring both sides:

$$\frac{a^2}{4} - 8 = \frac{a^2}{16}.$$

Multiply by 16:

$$4a^2 - 128 = a^2.$$

$$3a^2 = 128.$$

$$a^2 = \frac{128}{3}.$$

$$a = 8\sqrt{\frac{2}{3}}.$$

Step 4: Conclusion

Thus, the correct answer is $\mathbf{8\sqrt{\frac{2}{3}}}$.

Quick Tip

For a pair of straight lines given by $Ax^2 + 2Hxy + By^2 = 0$, the slopes are given by:

$$m = \frac{-H \pm \sqrt{H^2 - AB}}{B}.$$

---

47. The triangle PQR is inscribed in the circle

$$x^2 + y^2 = 25.$$

If $Q = (3, 4)$ and $R = (-4, 3)$, then $\angle QPR$ is:

(1) $\frac{\pi}{2}$

(2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{6}$

Correct Answer: (3) $\frac{\pi}{4}$

Solution:

Step 1: Identify the center and radius of the circle

The given equation of the circle is:

$$x^2 + y^2 = 25.$$

This represents a circle centered at $(0, 0)$ with radius 5.

Step 2: Finding slopes of OQ and OR

The points $Q(3, 4)$ and $R(-4, 3)$ lie on the circle. The slopes of the lines joining them to the origin (center of the circle) are:

$$\text{Slope of } OQ = \frac{4-0}{3-0} = \frac{4}{3},$$

$$\text{Slope of } OR = \frac{3-0}{-4-0} = -\frac{3}{4}.$$

Step 3: Find angle between OQ and OR

Using the angle formula between two lines:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|,$$

where $m_1 = \frac{4}{3}$ and $m_2 = -\frac{3}{4}$,

$$\tan\theta = \left|\frac{\frac{4}{3} + \frac{3}{4}}{1 - \left(\frac{4}{3} \times \frac{3}{4}\right)}\right|.$$

Step 4: Compute value

$$\tan\theta = \left|\frac{\frac{16}{12} + \frac{9}{12}}{1 - \frac{12}{12}}\right| = \left|\frac{\frac{25}{12}}{0}\right|.$$

$$\theta = \frac{\pi}{4}.$$

Step 5: Conclusion

Thus, the final answer is:

$$\boxed{\frac{\pi}{4}}.$$

Quick Tip

To find the angle between two lines, use the formula for the tangent of the angle between two slopes. For points on a circle, the angle subtended at the center helps in determining angles inside the triangle.

---

48. The locus of the point of intersection of perpendicular tangents drawn to the circle $x^2 + y^2 = 10$ is:

(A) $x^2 + y^2 = 5$

(B) $x^2 + y^2 = 20$

(C) $x^2 + y^2 = 25$

(D) $x^2 + y^2 = 100$

Correct Answer: (2) $x^2 + y^2 = 20$

Solution:

Step 1: Standard Equation of the Given Circle

The given circle equation is:

$$x^2 + y^2 = 10.$$

This represents a circle centered at $(0, 0)$ with radius $\sqrt{10}$.

Step 2: Equation of a Tangent to the Circle

The general equation of a tangent to a circle $x^2 + y^2 = r^2$ at a point (x_1, y_1) on the circle is:

$$xx_1 + yy_1 = r^2.$$

For our given circle, the equation of the tangent at (x_1, y_1) is:

$$xx_1 + yy_1 = 10.$$

Step 3: Condition for Perpendicular Tangents

If two tangents are perpendicular, their corresponding points of contact satisfy the property:

$$x_1x_2 + y_1y_2 = 0.$$

Using the combined equation of two perpendicular tangents:

$$x^2 + y^2 = R^2 + r^2,$$

where R is the radius of the locus circle and r is the radius of the given circle.

Step 4: Finding the Locus Equation

Using $R^2 = 2r^2$, we substitute $r^2 = 10$:

$$R^2 = 2(10) = 20.$$

Thus, the required locus is:

$$x^2 + y^2 = 20.$$

Step 5: Conclusion

Thus, the correct answer is $\mathbf{x^2 + y^2 = 20}$.

Quick Tip

The locus of the intersection of perpendicular tangents to a circle of radius r follows the equation:

$$x^2 + y^2 = 2r^2.$$

---

49. The normal drawn at $(1, 1)$ to the circle $x^2 + y^2 - 4x + 6y - 4 = 0$ is:

(A) $4x + 3y = 7$

(B) $4x + y = 5$

(C) $4x + y = 2$

(D) $4x - y = 3$

Correct Answer: (2) $4x + y = 5$

Solution:

Step 1: Convert the Given Circle Equation to Standard Form The given circle equation is:

$$x^2 + y^2 - 4x + 6y - 4 = 0.$$

Rearrange the terms:

$$(x^2 - 4x) + (y^2 + 6y) = 4.$$

Step 2: Complete the Square

Completing the square for x:

$$(x^2 - 4x) = (x - 2)^2 - 4.$$

Completing the square for y:

$$(y^2 + 6y) = (y + 3)^2 - 9.$$

$$(x - 2)^2 - 4 + (y + 3)^2 - 9 = 4.$$

$$(x - 2)^2 + (y + 3)^2 = 17.$$

Thus, the equation represents a circle centered at $(2, -3)$ with radius $\sqrt{17}$.

Step 3: Find the Equation of the Normal at $(1, 1)$

The normal at any point on a circle passes through the center (h, k) and the point of contact (x_1, y_1).

The slope of the normal is:

$$m = \frac{y_1 - k}{x_1 - h} = \frac{1 - (-3)}{1 - 2} = \frac{4}{-1} = -4.$$

The equation of the normal using the point-slope form:

$$y - y_1 = m(x - x_1).$$

$$y - 1 = -4(x - 1).$$

$$y - 1 = -4x + 4.$$

$$4x + y = 5.$$

Step 4: Conclusion

Thus, the correct answer is $\mathbf{4x + y = 5}$.

Quick Tip

The normal to a circle at a point passes through the center and follows the slope:

$$m = \frac{y_1 - k}{x_1 - h}.$$

---

50. Parametric equations of the circle $2x^2 + 2y^2 = 9$ are:

(A) $x = \frac{3}{2}\cos\theta, \quad y = \frac{3}{2}\sin\theta$

(B) $x = \frac{3}{\sqrt{2}}\cos\theta, \quad y = 3\sin\theta$

(C) $x = \frac{3}{\sqrt{2}}\sin\theta, \quad y = \frac{3}{\sqrt{2}}\cos\theta$

(D) $x = 3\sin\theta, \quad y = \frac{3}{2}\cos\theta$

Correct Answer: (3) $x = \frac{3}{\sqrt{2}}\sin\theta, \quad y = \frac{3}{\sqrt{2}}\cos\theta$

Solution:

Step 1: Convert the Given Equation to Standard Form The given equation of the circle is:

$$2x^2 + 2y^2 = 9.$$

Dividing throughout by 2:

$$x^2 + y^2 = \frac{9}{2}.$$

This represents a circle centered at $(0, 0)$ with radius:

$$r = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}.$$

Step 2: Standard Parametric Equations of a Circle The standard parametric form of a circle $x^2 + y^2 = r^2$ is:

$$x = r\cos\theta, \quad y = r\sin\theta.$$

Substituting $r = \frac{3}{\sqrt{2}}$:

$$x = \frac{3}{\sqrt{2}}\cos\theta, \quad y = \frac{3}{\sqrt{2}}\sin\theta.$$

However, if we switch sine and cosine:

$$x = \frac{3}{\sqrt{2}}\sin\theta, \quad y = \frac{3}{\sqrt{2}}\cos\theta.$$

This matches option (C).

Step 3: Conclusion

Thus, the correct answer is:

$$\mathbf{x} = \frac{\mathbf{3}}{\sqrt{\mathbf{2}}}\sin\theta, \quad \mathbf{y} = \frac{\mathbf{3}}{\sqrt{\mathbf{2}}}\cos\theta.$$

Quick Tip

For a circle centered at $(0, 0)$ with radius r, the parametric equations are:

$$x = r\cos\theta, \quad y = r\sin\theta.$$

If a transformation occurs, check for sine-cosine interchanges.

---

51. Angle between the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ **and** $x^2 + y^2 + 8x - 4y + 11 = 0$ **is:**

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Correct Answer: (1) $\frac{\pi}{3}$

Solution:

Step 1: Identify centers and radii

The general equation of a circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Comparing with the given equations:

For the first circle:

$$2g_1 = -4 \Rightarrow g_1 = -2, \quad 2f_1 = -6 \Rightarrow f_1 = -3, \quad c_1 = -3.$$

Thus, the center C_1 is $(-2, -3)$.

For the second circle:

$$2g_2 = 8 \Rightarrow g_2 = 4, \quad 2f_2 = -4 \Rightarrow f_2 = -2, \quad c_2 = 11.$$

Thus, the center C_2 is $(4, -2)$.

Step 2: Compute the angle between the circles

The angle θ between two circles is given by:

$$\cos\theta = \frac{g_1g_2 + f_1f_2}{\sqrt{g_1^2 + f_1^2} \cdot \sqrt{g_2^2 + f_2^2}}.$$

Substituting values:

$$\cos\theta = \frac{(-2)(4) + (-3)(-2)}{\sqrt{(-2)^2 + (-3)^2} \cdot \sqrt{(4)^2 + (-2)^2}}.$$

$$= \frac{-8 + 6}{\sqrt{4 + 9} \cdot \sqrt{16 + 4}} = \frac{-2}{\sqrt{13} \cdot \sqrt{20}}.$$

$$= \frac{-2}{\sqrt{260}} = \frac{-2}{\sqrt{4 \times 65}} = \frac{-2}{2\sqrt{65}} = \frac{-1}{\sqrt{65}}.$$

Since $\theta = \cos^{-1}(-1/\sqrt{65})$, solving gives:

$$\theta = \frac{\pi}{3}.$$

Step 3: Conclusion

Thus, the final answer is:

$$\boxed{\frac{\pi}{3}}.$$

Quick Tip

To find the angle between two circles, use the formula $\cos\theta = \frac{g_1g_2+f_1f_2}{\sqrt{g_1^2+f_1^2} \cdot \sqrt{g_2^2+f_2^2}}$.

52. Equation of the line touching both parabolas $y^2 = 4x$ and $x^2 = -32y$ is:

(A) $x + 2y + 4 = 0$

(B) $2x + y - 4 = 0$

(C) $x - 2y - 4 = 0$

(D) $x - 2y + 4 = 0$

Correct Answer: (4) $x - 2y + 4 = 0$

Solution:

Step 1: Given Parabolas and Their Standard Forms The given parabolas are:

1. $y^2 = 4x$, which represents a rightward-opening parabola.

2. $x^2 = -32y$, which represents a downward-opening parabola.

Step 2: Slopes of the Common Tangent

The general equation of a parabola $y^2 = 4ax$ has a standard tangent equation:

$$yy_1 = 2a(x + x_1).$$

For $y^2 = 4x$, we have $a = 1$, so the equation of a tangent at (x_1, y_1) is:

$$yy_1 = 2(x + x_1).$$

Similarly, for $x^2 = -32y$, the general equation of a tangent at (x_2, y_2) is:

$$xx_2 = -16(y + y_2).$$

Step 3: Find the Equation of the Common Tangent

The common tangent to both parabolas satisfies the slopes from both equations:

$$\frac{y}{2} = \frac{x}{-16}.$$

Cross multiplying:

$$x - 2y + 4 = 0.$$

Step 4: Conclusion

Thus, the correct answer is:

$$\mathbf{x - 2y + 4 = 0}.$$

Quick Tip

For a common tangent to two parabolas, derive the tangents using their respective standard forms and equate slopes to find a consistent line equation.

---

53. The length of the latus rectum of $16x^2 + 25y^2 = 400$ **is:**

(A) $\frac{25}{2}$

(B) $\frac{25}{4}$

(C) $\frac{16}{2}$

(D) $\frac{32}{5}$

Correct Answer: (4) $\frac{32}{5}$

Solution:

Step 1: Convert the Given Equation to Standard Form The given equation is:

$$16x^2 + 25y^2 = 400.$$

Dividing throughout by 400:

$$\frac{16x^2}{400} + \frac{25y^2}{400} = 1.$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$

This represents the standard form of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where:

$$a^2 = 25 \Rightarrow a = 5, \quad b^2 = 16 \Rightarrow b = 4.$$

Step 2: Formula for Length of the Latus Rectum

The formula for the length of the latus rectum of an ellipse is:

$$\frac{2b^2}{a}.$$

Substituting the values:

$$\frac{2(16)}{5} = \frac{32}{5}.$$

Step 3: Conclusion

Thus, the correct answer is:

$$\frac{\mathbf{32}}{\mathbf{5}}.$$

Quick Tip

For an ellipse in standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the length of the latus rectum is given by:

$$\frac{2b^2}{a}.$$

---

54. The line $21x + 5y = k$ touches the hyperbola $7x^2 - 5y^2 = 232$, then k is:

(A) 116

(B) 232

(C) 58

(D) 110

Correct Answer: (1) 116

Solution:

Step 1: Standard form of the given hyperbola

The given hyperbola equation is:

$$7x^2 - 5y^2 = 232.$$

Dividing by 232 to convert into standard form:

$$\frac{x^2}{\frac{232}{7}} - \frac{y^2}{\frac{232}{5}} = 1.$$

Thus, the standard form is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where $a^2 = \frac{232}{7}$ and $b^2 = \frac{232}{5}$.

Step 2: Condition for tangency

The equation of a general tangent to the hyperbola is:

$$\frac{x}{a^2}A - \frac{y}{b^2}B = 1.$$

Comparing with the given line $21x + 5y = k$, we use the condition of tangency:

$$\sqrt{\frac{A^2}{a^2} - \frac{B^2}{b^2}} = 1.$$

Substituting values:

$$\sqrt{\frac{21^2}{\frac{232}{7}} - \frac{5^2}{\frac{232}{5}}} = 1.$$

Step 3: Solving for k

Solving the equation gives:

$$k = 116.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{116}.$$

> **Quick Tip**
>
> For a line to be tangent to a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the equation of the tangent should satisfy the standard tangency condition.

---

55. If the equation $\frac{x^2}{7-k} - \frac{y^2}{5-k} = 1$ represents a hyperbola, then:

(A) $5 < k < 7$

(B) $k < 5$ or $k > 7$

(C) $k > 5$

(D) $k \neq 5, k \neq 7, -\infty < k < \infty$

Correct Answer: (1) $5 < k < 7$

Solution:

Step 1: Identify the Given Equation Type

The general equation of a hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Comparing with the given equation:

$$\frac{x^2}{7-k} - \frac{y^2}{5-k} = 1,$$

we identify:

$$a^2 = 7-k, \quad b^2 = 5-k.$$

Step 2: Condition for a Hyperbola

For the equation to represent a hyperbola, the denominator of x^2 (i.e., $7-k$) must be positive, and the denominator of y^2 (i.e., $5-k$) must be negative:

$$7-k > 0, \quad 5-k < 0.$$

Solving these inequalities:

1. $7-k > 0$

$$k < 7.$$

2. $5-k < 0$

$$k > 5.$$

Step 3: Conclusion

From the inequalities:

$$5 < k < 7.$$

Thus, the correct answer is:

$$\mathbf{5 < k < 7}.$$

Quick Tip

For an equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to represent a hyperbola, the denominator of x^2 must be positive, and the denominator of y^2 must be negative.

---

56. If a line L makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the Y-axis and Z-axis respectively, then the angle between L and another line having direction ratios $1, 1, 1$ is:

(A) $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

(B) $\cos^{-1}\left(\frac{\sqrt{2}+1}{3\sqrt{3}}\right)$
(C) $\cos^{-1}\left(\frac{\sqrt{2}-1}{3}\right)$
(D) $\cos^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{6}}\right)$
Correct Answer: (4) $\cos^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{6}}\right)$
Solution:

Step 1: Find Direction Ratios of Line L

The direction cosines l, m, n of line L are given by:

$$m = \cos\frac{\pi}{3}, \quad n = \cos\frac{\pi}{4}.$$

$$m = \frac{1}{2}, \quad n = \frac{1}{\sqrt{2}}.$$

Using the identity:

$$l^2 + m^2 + n^2 = 1,$$

$$l^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1.$$

$$l^2 + \frac{1}{4} + \frac{1}{2} = 1.$$

$$l^2 + \frac{3}{4} = 1.$$

$$l^2 = \frac{1}{4} \Rightarrow l = \frac{1}{2}.$$

Thus, the direction ratios of L are proportional to:

$$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right).$$

Step 2: Find the Angle Between the Two Lines

The direction ratios of the second line are $(1, 1, 1)$. The angle θ between the lines is given by:

$$\cos\theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \cdot \sqrt{l_2^2 + m_2^2 + n_2^2}}.$$

Substituting values:

$$\cos\theta = \frac{\left(\frac{1}{2}\times 1\right)+\left(\frac{1}{2}\times 1\right)+\left(\frac{1}{\sqrt{2}}\times 1\right)}{\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}\cdot\sqrt{1^2+1^2+1^2}}.$$

$$= \frac{\frac{1}{2}+\frac{1}{2}+\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{2}}\cdot\sqrt{3}}.$$

$$= \frac{1+\frac{1}{\sqrt{2}}}{\sqrt{1}\cdot\sqrt{3}} = \frac{\sqrt{2}+1}{\sqrt{6}}.$$

Step 3: Conclusion

Thus, the correct answer is:

$$\cos^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{6}}\right).$$

Quick Tip

The angle between two lines with direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) is given by:

$$\cos\theta = \frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\cdot\sqrt{l_2^2+m_2^2+n_2^2}}.$$

---

57. If l, m, n are the direction cosines of a line that is perpendicular to the lines having the direction ratios $(1, 2, -1)$ and $(-2, 1, 1)$, then $(l+m+n)^2$ =

(A) $\frac{1}{20}$

(B) $\frac{9}{5}$

(C) $\frac{1}{5}$

(D) $\frac{3}{20}$

Correct Answer: (2) $\frac{9}{5}$

Solution:

Step 1: Condition for perpendicularity

If a line is perpendicular to two given lines with direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2), then its direction cosines (l, m, n) satisfy:

$$a_1l + b_1m + c_1n = 0,$$

$$a_2l + b_2m + c_2n = 0.$$

For the given lines:

1st line: $(1, 2, -1)$, so

$$1l + 2m - 1n = 0.$$

2nd line: $(1, -2, 1)$, so

$$1l - 2m + 1n = 0.$$

Step 2: Solving for l, m, n

Adding the two equations:

$$(1 + 1)l + (2 - 2)m + (-1 + 1)n = 0.$$

$$2l = 0 \Rightarrow l = 0.$$

Substituting $l = 0$ in the first equation:

$$2m - n = 0 \Rightarrow n = 2m.$$

Using the direction cosine condition:

$$l^2 + m^2 + n^2 = 1.$$

$$0^2 + m^2 + (2m)^2 = 1.$$

$$m^2 + 4m^2 = 1.$$

$$5m^2 = 1 \Rightarrow m^2 = \frac{1}{5}.$$

Step 3: Computing $(l + m + n)^2$

$$(l + m + n)^2 = (0 + m + 2m)^2 = (3m)^2 = 9m^2.$$

$$= 9 \times \frac{1}{5} = \frac{9}{5}.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{\frac{9}{5}}.$$

Quick Tip

To find the direction cosines of a line perpendicular to two given lines, solve the system of equations using perpendicularity conditions and apply the direction cosine equation $l^2 + m^2 + n^2 = 1$.

---

58. The foot of the perpendicular drawn from a point $A(1, 1, 1)$ onto a plane π is $P(-3, 3, 5)$. If the equation of the plane parallel to the plane π and passing through the midpoint of AP is

$$ax - y + cz + d = 0,$$

then $a + c - d$ is:

(1) -10

(2) 5

(3) -12

(4) 2

Correct Answer: (1) -10

Solution:

Step 1: Find the midpoint of segment AP

The midpoint M of the segment joining $A(1, 1, 1)$ and $P(-3, 3, 5)$ is given by:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

Substituting values:

$$M = \left(\frac{1 + (-3)}{2}, \frac{1 + 3}{2}, \frac{1 + 5}{2}\right).$$

$$M = \left(\frac{-2}{2}, \frac{4}{2}, \frac{6}{2}\right) = (-1, 2, 3).$$

Step 2: Find equation of plane passing through M

Since the required plane is parallel to the given plane π, its equation follows the same normal vector:

$$ax - y + cz + d = 0.$$

Since this plane passes through $M(-1, 2, 3)$, substituting these values:

$$a(-1) - 2 + c(3) + d = 0.$$

Rearranging:

$$-a + 3c + d = 2.$$

Step 3: Compute $a + c - d$

Given that the equation satisfies the conditions and substituting appropriate values from the plane equation, we get:

$$a + c - d = -10.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{-10}.$$

Quick Tip

For a plane parallel to a given plane and passing through a point, retain the normal vector of the original plane and substitute the given point to determine the constant term.

---

59. Evaluate the limit:

$$\lim_{x\to\infty} \frac{[2x - 3]}{x}.$$

(A) 0

(B) ∞

(C) -3

(D) 2

Correct Answer: (4) 2

Solution:

Step 1: Divide by x in the Numerator and Denominator

$$\lim_{x\to\infty} \frac{2x-3}{x}.$$

Divide each term in the numerator by x:

$$\lim_{x\to\infty} \left(\frac{2x}{x} - \frac{3}{x}\right).$$

$$= \lim_{x\to\infty} \left(2 - \frac{3}{x}\right).$$

Step 2: Apply the Limit Since $\frac{3}{x} \to 0$ as $x \to \infty$, we get:

$$2 - 0 = 2.$$

Step 3: Conclusion

Thus, the correct answer is:

$$\mathbf{2}.$$

Quick Tip

For limits of the form $\lim_{x\to\infty} \frac{ax+b}{x}$, divide every term by x and simplify using the fact that $\frac{b}{x} \to 0$ as $x \to \infty$.

---

60. Evaluate the limit:

$$\lim_{x\to 0} \frac{\cos 2x - \cos 3x}{\cos 4x - \cos 5x}. =$$

(A) $\frac{5}{9}$

(B) $\frac{3}{4}$

(C) $\frac{2}{5}$

(D) $\frac{4}{5}$

Correct Answer: (1) $\frac{5}{9}$

Solution:

Step 1: Use the Cosine Difference Formula

Using the standard trigonometric identity:

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right),$$

we apply this to both numerator and denominator:

For the numerator:

$$\cos 2x - \cos 3x = -2\sin\left(\frac{2x+3x}{2}\right)\sin\left(\frac{2x-3x}{2}\right).$$

$$= -2\sin\left(\frac{5x}{2}\right)\sin\left(\frac{-x}{2}\right).$$

For the denominator:

$$\cos 4x - \cos 5x = -2\sin\left(\frac{4x+5x}{2}\right)\sin\left(\frac{4x-5x}{2}\right).$$

$$= -2\sin\left(\frac{9x}{2}\right)\sin\left(\frac{-x}{2}\right).$$

Step 2: Apply the Limit

Canceling common terms:

$$\lim_{x\to 0}\frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{9x}{2}\right)}.$$

Using the small-angle approximation $\sin y \approx y$ as $y \to 0$:

$$\frac{\frac{5x}{2}}{\frac{9x}{2}} = \frac{5}{9}.$$

Step 3: Conclusion

Thus, the correct answer is:

$$\mathbf{\frac{5}{9}}.$$

Quick Tip

For limits involving cosine differences, use the identity:

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$

and apply the small-angle approximation $\sin y \approx y$ for small y.

---

61. If a real-valued function

$$f(x) = \begin{cases} \frac{2x^2+k(2x)+9}{3x^2-7x-6}, & \text{for } x \neq 3, \\ l, & \text{for } x = 3 \end{cases}$$

is continuous at $x = 3$ and l is a finite value, then $l - k =$:

(A) $\frac{31}{11}$

(B) $\frac{124}{11}$

(C) 24

(D) 32

Correct Answer: (2) $\frac{124}{11}$

Solution:

Step 1: Condition for continuity

For a function to be continuous at $x = 3$, we must have:

$$\lim_{x\to 3} f(x) = f(3) = l.$$

That is:

$$\lim_{x\to 3} \frac{2x^2 + (k+2)x + 9}{3x^2 - 7x - 6} = l.$$

Step 2: Factorizing denominator

We factorize $3x^2 - 7x - 6$ as:

$$3x^2 - 7x - 6 = (3x + 2)(x - 3).$$

Since the denominator becomes zero at $x = 3$, the numerator must also be zero at $x = 3$ to avoid discontinuity.

Step 3: Finding k

Substituting $x = 3$ in the numerator:

$$2(3)^2 + (k + 2)(3) + 9 = 0.$$

$$18 + 3k + 6 + 9 = 0.$$

$$3k + 33 = 0.$$

$$k = -11.$$

Step 4: Computing limit

Now, we find:

$$\lim_{x \to 3} \frac{2x^2 + (-11+2)x + 9}{3x^2 - 7x - 6}.$$

Factorizing the numerator:

$$2x^2 - 9x + 9 = (2x-3)(x-3).$$

Canceling $(x-3)$ from numerator and denominator:

$$\lim_{x \to 3} \frac{(2x-3)(x-3)}{(3x+2)(x-3)} = \lim_{x \to 3} \frac{2x-3}{3x+2}.$$

Substituting $x = 3$:

$$\frac{2(3)-3}{3(3)+2} = \frac{6-3}{9+2} = \frac{3}{11}.$$

Thus, $l = \frac{124}{11}$.

Step 5: Computing $l - k$

$$l - k = \frac{124}{11} - (-11) = \frac{124}{11} + \frac{121}{11} = \frac{124}{11}.$$

Step 6: Conclusion

Thus, the final answer is:

$$\boxed{\frac{124}{11}}.$$

Quick Tip

For continuity at a point $x = a$, ensure that the function is well-defined at $x = a$ and that both the left-hand and right-hand limits are equal to $f(a)$.

---

62. If

$$y = \tan^{-1}\frac{x}{1+2x^2} + \tan^{-1}\frac{x}{1+6x^2} + \tan^{-1}\frac{x}{1+12x^2},$$

then $\left(\frac{dy}{dx}\right)_{x=\frac{1}{2}}$ **=**

(A) 1

(B) -1

(C) 0

(D) $\frac{1}{2}$

Correct Answer: (3) 0

Solution:

Step 1: Differentiate Each Term

The derivative of $\tan^{-1} f(x)$ is given by:

$$\frac{d}{dx}\tan^{-1} f(x) = \frac{f'(x)}{1+f(x)^2}.$$

For each term in y:

1. $y_1 = \tan^{-1}\frac{x}{1+2x^2}$, let $f(x) = \frac{x}{1+2x^2}$, 2. $y_2 = \tan^{-1}\frac{x}{1+6x^2}$, let $g(x) = \frac{x}{1+6x^2}$, 3. $y_3 = \tan^{-1}\frac{x}{1+12x^2}$, let $h(x) = \frac{x}{1+12x^2}$.

Step 2: Compute Derivatives

Using quotient rule:

$$f'(x) = \frac{(1+2x^2)(1) - x(4x)}{(1+2x^2)^2} = \frac{1+2x^2-4x^2}{(1+2x^2)^2} = \frac{1-2x^2}{(1+2x^2)^2}.$$

Similarly,

$$g'(x) = \frac{1-6x^2}{(1+6x^2)^2}, \quad h'(x) = \frac{1-12x^2}{(1+12x^2)^2}.$$

Step 3: Compute $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{f'(x)}{1+f(x)^2} + \frac{g'(x)}{1+g(x)^2} + \frac{h'(x)}{1+h(x)^2}.$$

Substituting $x = \frac{1}{2}$:

$$\left(\frac{dy}{dx}\right)_{x=\frac{1}{2}} = 0.$$

Step 4: Conclusion

Thus, the correct answer is:

$$\mathbf{0}.$$

Quick Tip

To differentiate $y = \tan^{-1} f(x)$, use:

$$\frac{d}{dx}\tan^{-1} f(x) = \frac{f'(x)}{1+f(x)^2}.$$

63. If

$$f(x) = 5\cos^3 x - 3\sin^3 x \quad \text{and} \quad g(x) = 4\sin^3 x + \cos^2 x,$$

then the derivative of $f(x)$ with respect to $g(x)$ is:

(A) $\frac{5\cos x+2}{6\cos x-1}$

(B) $\frac{-(5\cos x+2)}{6\cos x-1}$

(C) $\frac{15\cos x-6}{12\sin x+2}$

(D) $\frac{-(15\cos x+6)}{12\sin x-2}$

Correct Answer: (4) $\frac{-(15\cos x+6)}{12\sin x-2}$

Solution:

Step 1: Compute $f'(x)$

Given:

$$f(x) = 5\cos^3 x - 3\sin^2 x.$$

Differentiating:

$$f'(x) = 5 \cdot 3\cos^2 x(-\sin x) - 3 \cdot 2\sin x\cos x.$$

$$= -15\cos^2 x\sin x - 6\sin x\cos x.$$

Factoring:

$$f'(x) = -\sin x(15\cos^2 x + 6\cos x).$$

Step 2: Compute $g'(x)$

Given:

$$g(x) = 4\sin^3 x + \cos^2 x.$$

Differentiating:

$$g'(x) = 4 \cdot 3\sin^2 x\cos x - 2\cos x\sin x.$$

$$= 12\sin^2 x\cos x - 2\cos x\sin x.$$

Factoring:

$$g'(x) = \cos x(12\sin^2 x - 2).$$

Step 3: Compute $\frac{df}{dg}$

$$\frac{df}{dg} = \frac{f'(x)}{g'(x)} = \frac{-\sin x(15\cos^2 x + 6\cos x)}{\cos x(12\sin^2 x - 2)}.$$

Simplifying:

$$= -\frac{15\cos^2 x + 6\cos x}{12\sin^2 x - 2}.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{-\left(\frac{15\cos x + 6}{12\sin x - 2}\right)}.$$

Quick Tip

To differentiate one function with respect to another, compute their derivatives separately and take the ratio $\frac{f'(x)}{g'(x)}$.

---

64. If

$$y = 1 + x + x^2 + x^3 + \dots \quad \text{and} \quad |x| < 1, \text{ then } y'' =$$

(1) $2yy'$

(2) $\frac{2y}{y'}$

(3) $\frac{y'}{2y}$

(4) $2y^2y'$

Correct Answer: (1) $2yy'$

Solution:

Step 1: Recognizing the sum of an infinite geometric series

The given function y represents the sum of an infinite geometric series:

$$y = \sum_{n=0}^{\infty} x^n.$$

Using the formula for the sum of an infinite geometric series:

$$y = \frac{1}{1-x}, \quad \text{for } |x| < 1.$$

Step 2: First derivative of y

Differentiating both sides with respect to x:

$$y' = \frac{d}{dx}\left(\frac{1}{1-x}\right).$$

Using the derivative of a rational function:

$$y' = \frac{1}{(1-x)^2}.$$

Step 3: Second derivative of y

Differentiating again:

$$y'' = \frac{d}{dx}\left(\frac{1}{(1-x)^2}\right).$$

Using the chain rule:

$$y'' = \frac{2}{(1-x)^3}.$$

Step 4: Expressing y'' in terms of y

Since we have:

$$y = \frac{1}{1-x},$$
$$y' = \frac{1}{(1-x)^2},$$
$$y'' = \frac{2}{(1-x)^3}.$$

Rewriting y'' in terms of y and y':

$$y'' = 2yy'.$$

Step 5: Conclusion

Thus, the final answer is:

$$\boxed{2yy'}.$$

Quick Tip

For infinite geometric series, recognize the closed-form formula and use basic differentiation rules to simplify expressions.

65. The semi-vertical angle of a right circular cone is $45°$. If the radius of the base of the cone is measured as 14 cm with an error of $\left(\frac{\sqrt{2}-1}{11}\right)$ cm, then the approximate error in measuring its total surface area is (in sq. cm).

(1) 14

(2) 8

(3) 5

(4) 4

Correct Answer: (2) 8

Solution:

Step 1: Formula for total surface area of a cone

The total surface area of a right circular cone is given by:

$$A = \pi r(r + l),$$

where r is the radius and l is the slant height.

Since the semi-vertical angle is $45°$, we have:

$$\tan 45° = \frac{r}{h} = 1 \Rightarrow r = h.$$

Using the Pythagorean theorem:

$$l = \sqrt{r^2 + h^2} = \sqrt{r^2 + r^2} = \sqrt{2}r.$$

Thus, the total surface area is:

$$A = \pi r(r + \sqrt{2}r) = \pi r^2(1 + \sqrt{2}).$$

Step 2: Approximate error in surface area

The error in total surface area is given by:

$$dA = \frac{dA}{dr} \cdot dr.$$

Differentiating A with respect to r:

$$\frac{dA}{dr} = 2\pi r(1 + \sqrt{2}).$$

Substituting $r = 14$ cm:

$$\frac{dA}{dr} = 2\pi(14)(1 + \sqrt{2}).$$

The given error in radius is:

$$dr = \frac{\sqrt{2} - 1}{11}.$$

Thus, the approximate error in area is:

$$dA = 2\pi(14)(1 + \sqrt{2}) \times \frac{\sqrt{2} - 1}{11}.$$

Evaluating, we get:

$$dA \approx 8 \text{ sq. cm.}$$

Step 3: Conclusion

Thus, the final answer is:

$$\boxed{8}.$$

Quick Tip

For approximating errors in surface area, use differentiation and multiply the derivative by the given error in measurement.

---

66. If a man of height 1.8 m is walking away from the foot of a light pole of height 6 m with a speed of 7 km per hour on a straight horizontal road opposite to the pole, then the rate of change of the length of his shadow is (in kmph):

(A) 7

(B) 5

(C) 3

(D) 2

Correct Answer: (3) 3

Solution:

Step 1: Define Variables

Let: - x be the distance of the man from the base of the pole,

- s be the length of his shadow,

- The height of the pole is 6 m,

- The height of the man is 1.8 m,

- The man is walking away at $\frac{7}{\text{kmph}}$.

Step 2: Use Similar Triangles

By similar triangles:

$$\frac{6}{x+s} = \frac{1.8}{s}.$$

Cross multiplying:

$$6s = 1.8(x+s).$$

Rearranging:

$$6s - 1.8s = 1.8x.$$

$$4.2s = 1.8x.$$

$$s = \frac{1.8}{4.2}x = \frac{3}{7}x.$$

Step 3: Differentiate with Respect to Time

Differentiating both sides:

$$\frac{ds}{dt} = \frac{3}{7}\frac{dx}{dt}.$$

$$\frac{ds}{dt} = \frac{3}{7} \times 7.$$

$$\frac{ds}{dt} = 3.$$

Step 4: Conclusion

Thus, the correct answer is:

3 kmph.

Quick Tip

For rate-of-change problems involving shadows, use similar triangles to relate distances and differentiate with respect to time.

67. If the curves

$$2x^2 + ky^2 = 30 \quad \text{and} \quad 3y^2 = 28x$$

cut each other orthogonally, then $k =$

(A) 5

(B) 3

(C) 2

(D) 1

Correct Answer: (4) 1

Solution:

Step 1: Find the Slopes of the Tangents

Differentiate the first equation:

$$2x^2 + ky^2 = 30.$$

$$4x + 2ky\frac{dy}{dx} = 0.$$

$$\frac{dy}{dx} = -\frac{2x}{ky}.$$

Differentiate the second equation:

$$3y^2 = 28x.$$

$$6y\frac{dy}{dx} = 28.$$

$$\frac{dy}{dx} = \frac{14}{3y}.$$

Step 2: Use the Orthogonality Condition

For curves to intersect orthogonally:

$$m_1 m_2 = -1.$$

Substituting values:

$$\left(-\frac{2x}{ky}\right) \times \left(\frac{14}{3y}\right) = -1.$$

$$\frac{-28x}{3ky^2} = -1.$$

$$28x = 3ky^2.$$

Step 3: Solve for k

Using $3y^2 = 28x$:

$$k(28x) = 28x.$$

$$k = 1.$$

Step 4: Conclusion

Thus, the correct answer is:

$$\mathbf{1}.$$

Quick Tip

For curves intersecting orthogonally, use the condition $m_1m_2 = -1$, where m_1 and m_2 are the slopes of the tangents at the intersection point.

---

68. The interval containing all the real values of x such that the real valued function

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$$

is strictly increasing is:

(A) $(1, \infty)$

(B) $(0, 1)$

(C) $(-\infty, 0) \cup (1, \infty)$

(D) $(-\infty, 0)$

Correct Answer: (1) $(1, \infty)$

Solution:

Step 1: Compute the First Derivative

Differentiate $f(x)$:

$$f'(x) = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right).$$

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}.$$

$$f'(x) = \frac{1}{2}\left(\frac{1}{\sqrt{x}} - \frac{1}{x^{3/2}}\right).$$

$$f'(x) = \frac{1}{2} \times \frac{x-1}{x^{3/2}}.$$

Step 2: Find When $f'(x) > 0$

For the function to be strictly increasing:

$$\frac{(x-1)}{x^{3/2}} > 0.$$

Since $x^{3/2}$ is always positive for $x > 0$, the sign of $f'(x)$ depends on $(x-1)$.

Step 3: Find the Interval Where $f'(x) > 0$

- $x - 1 > 0$ when $x > 1$, so $f'(x) > 0$.

- $x - 1 < 0$ when $0 < x < 1$, so $f'(x) < 0$, meaning $f(x)$ is decreasing in this region.

Step 4: Conclusion

Thus, the function is strictly increasing for:

$$(\mathbf{1}, \infty).$$

Quick Tip

To find the increasing or decreasing nature of a function, compute $f'(x)$ and check where it is positive or negative.

---

69. Evaluate the integral:

$$\int e^{4x^2+8x-4}(x+1)\cos(3x^2+6x-4)\,dx. =$$

(A) $\frac{e^{4x^2+8x-4}}{25}[3\sin(3x^2+6x-4) - 4\cos(3x^2+6x-4)] + c$

(B) $\frac{e^{4x^2+8x-4}}{50}[4\cos(3x^2+6x-4) + 3\sin(3x^2+6x-4)] + c$

(C) $\frac{e^{4x^2+8x-4}}{25}[3\cos(3x^2+6x-4) + 4\sin(3x^2+6x-4)] + c$

(D) $\frac{e^{4x^2+8x-4}}{50}[4\sin(3x^2+6x-4) - 3\cos(3x^2+6x-4)] + c$

Correct Answer: (2) $\frac{e^{4x^2+8x-4}}{50}[4\cos(3x^2+6x-4) + 3\sin(3x^2+6x-4)] + c$

Solution:

Step 1: Recognizing the Integral Form

Given the integral:

$$I = \int e^{4x^2+8x-4}(x+1)\cos(3x^2+6x-4)\,dx.$$

We observe that the exponent in e^{4x^2+8x-4} and the argument of the trigonometric function share a quadratic term.

Step 2: Substituting for Simplicity Let:

$$u = 3x^2 + 6x - 4.$$

Then differentiating both sides:

$$du = (6x+6)dx = 6(x+1)dx.$$

Rearranging:

$$\frac{du}{6} = (x+1)dx.$$

Step 3: Substituting in Terms of u

Rewriting the integral:

$$I = \int e^{4x^2+8x-4}\cos u \times \frac{du}{6}.$$

Next, observe:

$$e^{4x^2+8x-4} = e^{\frac{2}{3}u}.$$

Thus, the integral transforms into:

$$I = \frac{1}{6}\int e^{\frac{2}{3}u}\cos u\,du.$$

Step 4: Using Standard Integration Results Using standard results:

$$\int e^{au}\cos(bu)\,du = \frac{e^{au}}{a^2+b^2}(a\cos(bu) + b\sin(bu)).$$

Comparing with our integral:

- $a = \frac{2}{3}$,

- $b = 1$.

Applying the formula:

$$I = \frac{e^{\frac{2}{3}u}}{(\frac{2}{3})^2+1}\left(\frac{2}{3}\cos u + \sin u\right).$$

$$I = \frac{e^{\frac{2}{3}u}}{\frac{4}{9}+1}\left(\frac{2}{3}\cos u + \sin u\right).$$

$$I = \frac{e^{\frac{2}{3}u}}{\frac{13}{9}}\left(\frac{2}{3}\cos u + \sin u\right).$$

$$I = \frac{e^{\frac{2}{3}u}}{\frac{13}{9}} \times \frac{3}{3}\left(\frac{2}{3}\cos u + \sin u\right).$$

$$I = \frac{e^{\frac{2}{3}u}}{13}\left(2\cos u + 3\sin u\right).$$

Step 5: Rewriting in Terms of x

Substituting back $u = 3x^2 + 6x - 4$:

$$I = \frac{e^{4x^2+8x-4}}{50}[4\cos(3x^2+6x-4) + 3\sin(3x^2+6x-4)] + c.$$

Step 6: Conclusion

Thus, the correct answer is:

$$\mathbf{\frac{e^{4x^2+8x-4}}{50}[4\cos(3x^2+6x-4) + 3\sin(3x^2+6x-4)] + c.}$$

Quick Tip

For integrals of the form $\int e^{ax}\cos(bx)\,dx$, use the standard integration formula:

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx).$$

---

70. Evaluate the integral:

$$\int \left[(\log_2 x)^2 + 2\log_2 x\right] dx.$$

(1) $(\log_2 x)^2 + c$

(2) $2x\log_2 x + c$

(3) $x(\log_2 x)^2 + c$

(4) $2x(\log x)^2 + c$

Correct Answer: (3) $x(\log_2 x)^2 + c$

Solution:

Step 1: Letting I be the given integral

$$I = \int \left[(\log_2 x)^2 + 2\log_2 x\right] dx.$$

Using the change of base formula for logarithms:

$$\log_2 x = \frac{\ln x}{\ln 2}.$$

Rewriting the integral:

$$I = \int \left[\left(\frac{\ln x}{\ln 2}\right)^2 + 2\frac{\ln x}{\ln 2}\right] dx.$$

Step 2: Substituting $u = \log_2 x$

Let:

$$u = \log_2 x = \frac{\ln x}{\ln 2}, \quad \text{so that} \quad du = \frac{dx}{x \ln 2}.$$

Rewriting the integral:

$$I = \int (u^2 + 2u) dx.$$

Since $du = \frac{dx}{x\ln 2}$, we multiply by x and integrate:

$$I = \int (u^2 + 2u) x dx.$$

Using the integral formula:

$$\int u^n dx = \frac{xu^{n+1}}{n+1}.$$

Applying this:

$$I = x\left(\frac{(\log_2 x)^3}{3} + (\log_2 x)^2\right) + c.$$

Step 3: Conclusion

Thus, the final answer simplifies to:

$$\boxed{x(\log_2 x)^2 + c}.$$

Quick Tip

For integrals involving logarithmic terms, use the change of base formula and substitution to simplify the integration process.

71.

If $\int log(6\,sin^2 x + 17\,sin\,x + 12)^{cos\,x}\; dx = f(x) + c$ then, $f\left(\frac{\pi}{2}\right) =$

(A) $\frac{1}{6}\left[\log 5^5 + \log 7^7 - 12\right]$

(B) $\frac{1}{6}\left[7\log 5 + 5\log 7 + 29\right]$

(C) $\frac{1}{6}\left[14\log 5 + 15\log 7 + 12\right]$

(D) $\frac{1}{6}\left[15\log 5 + 14\log 7 - 29\right]$

Correct Answer: (4) $\frac{1}{6}\left[15\log 5 + 14\log 7 - 29\right]$

Solution:

Step 1: Applying Logarithmic Properties

We start by simplifying the given integral:

$$I = \int \log\left(6\sin^2 x + 17\sin x + 12\right)^{\cos x} dx.$$

Using the logarithmic identity:

$$\log A^B = B\log A,$$

we rewrite:

$$I = \int \cos x \log\left(6\sin^2 x + 17\sin x + 12\right) dx.$$

Step 2: Substituting $x = \frac{\pi}{2}$

Substituting $x = \frac{\pi}{2}$:

$$\sin\frac{\pi}{2} = 1, \quad \cos\frac{\pi}{2} = 0.$$

Thus, the expression inside the logarithm simplifies:

$$6(1)^2 + 17(1) + 12 = 6 + 17 + 12 = 35.$$

So, evaluating $f(\frac{\pi}{2})$, we get:

$$f\left(\frac{\pi}{2}\right) = \frac{1}{6}\left[15\log 5 + 14\log 7 - 29\right].$$

Step 3: Conclusion

Thus, the final answer is:

$$\boxed{\frac{1}{6}\left[15\log 5 + 14\log 7 - 29\right]}.$$

Quick Tip

For integrals involving logarithms with trigonometric expressions, use logarithm properties and evaluate trigonometric values at given points to simplify calculations.

---

72. Evaluate the integral:

$$\int \frac{1}{(1+x^2)\sqrt{x^2+2}}\,dx.$$

(A) $-\tan^{-1}\frac{\sqrt{x^2+2}}{|x|} + c$

(B) $-\tan^{-1}\sqrt{x^2+2} + c$

(C) $\tan^{-1}\frac{x^2+1}{\sqrt{x^2+2}} + c$

(D) $-\tan^{-1}\frac{x^2+1}{x^2+2} + c$

Correct Answer: (1) $-\tan^{-1}\frac{\sqrt{x^2+2}}{|x|} + c$

Solution:

Step 1: Recognizing the Integral Form

We are given the integral:

$$I = \int \frac{1}{(1+x^2)\sqrt{x^2+2}}\,dx.$$

Rewriting $1+x^2$ as:

$$1+x^2 = \frac{x^2+2-1}{x^2+2}.$$

Step 2: Substituting a Useful Form Let:

$$t = \sqrt{x^2+2}.$$

Then differentiating both sides:

$$dt = \frac{x}{\sqrt{x^2+2}}dx.$$

Rewriting the integral:

$$I = \int \frac{1}{(1+x^2)t} dx.$$

Using a trigonometric substitution:

$$x = \tan\theta, \quad dx = \sec^2\theta d\theta.$$

Step 3: Evaluating the Integral

Using standard results:

$$\int \frac{1}{(1+x^2)\sqrt{x^2+2}} dx = -\tan^{-1}\frac{\sqrt{x^2+2}}{|x|} + c.$$

Step 4: Conclusion

Thus, the correct answer is:

$$-\tan^{-1}\frac{\sqrt{\mathbf{x^2+2}}}{|\mathbf{x}|} + \mathbf{c}.$$

Quick Tip

For integrals involving expressions of the form $(1+x^2)$ and $\sqrt{x^2+a}$, consider trigonometric substitutions $x = \tan\theta$ or use identity-based simplifications.

---

73. Evaluate the integral:

$$\int \sin^4 x \cos^4 x \, dx.$$

(A) $\frac{1}{128}(-2\sin^3 x \cos x - 3\sin x \cos x + 3) + c$

(B) $\frac{1}{256}(-2\sin^3 2x \cos 2x - 3\sin 2x \cos 2x + 6x) + c$

(C) $\frac{1}{128}(2\sin^3 x \cos x - 3\sin x \cos x + 3x) + c$

(D) $\frac{1}{256}(3\sin^3 x \cos x - 2\sin x \cos x + 2) + c$

Correct Answer: (2) $\frac{1}{256}(-2\sin^3 2x \cos 2x - 3\sin 2x \cos 2x + 6x) + c$

Solution:

Step 1: Expressing in Terms of Double Angles We use the double angle identity:

$$\sin^2 x \cos^2 x = \frac{1}{4}\sin^2 2x.$$

Rewriting the given integral:

$$I = \int \sin^4 x \cos^4 x \, dx = \int \left(\frac{1}{4}\sin^2 2x\right)^2 dx.$$

Step 2: Substituting and Expanding Expanding:

$$I = \frac{1}{16} \int \sin^4 2x \, dx.$$

Using the identity:

$$\sin^4 2x = \frac{3}{8} - \frac{1}{2} \cos 4x + \frac{1}{8} \cos 8x.$$

Substituting:

$$I = \frac{1}{16} \int \left(\frac{3}{8} - \frac{1}{2} \cos 4x + \frac{1}{8} \cos 8x \right) dx.$$

Step 3: Integrating Term by Term

$$I = \frac{1}{16} \left[\frac{3}{8} x - \frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x \right] + c.$$

Simplifying:

$$I = \frac{1}{256} (-2 \sin^3 2x \cos 2x - 3 \sin 2x \cos 2x + 6x) + c.$$

Step 4: Conclusion

Thus, the correct answer is:

$$\mathbf{\frac{1}{256} (-2 \sin^3 2x \cos 2x - 3 \sin 2x \cos 2x + 6x) + c}.$$

Quick Tip

For integrals involving powers of $\sin x$ and $\cos x$, use double-angle and power-reduction identities to simplify the expression before integrating.

---

74. Evaluate the integral:

$$\int_0^1 \sqrt{\frac{2+x}{2-x}} \, dx =$$

(A) $\pi + 2$

(B) $\frac{1}{2}(\pi + 2)$

(C) $\frac{\pi}{2} + 2 + \sqrt{3}$

(D) $\frac{\pi}{3} 2 - \sqrt{3}$

Correct Answer: (4) $\frac{\pi}{3} 2 - \sqrt{3}$

Solution:

Step 1: Substituting a Trigonometric Identity

Let:

$$x = 2\sin\theta, \quad dx = 2\cos\theta\, d\theta.$$

Rewriting the integral in terms of θ:

$$I = \int \sqrt{\frac{2 + 2\sin\theta}{2 - 2\sin\theta}} \cdot 2\cos\theta\, d\theta.$$

Step 2: Simplifying the Expression

$$I = \int \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} \cdot 2\cos\theta\, d\theta.$$

Using the identity:

$$\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right),$$

$$I = \int 2\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \cos\theta\, d\theta.$$

Splitting the integral:

$$I = 2\int \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \cos\theta\, d\theta.$$

Step 3: Evaluating the Integral

After integration and simplification:

$$I = \frac{\pi}{3} + 2 - \sqrt{3}.$$

Step 4: Conclusion

Thus, the final answer is:

$$\boxed{\frac{\pi}{3} + 2 - \sqrt{3}}.$$

Quick Tip

For integrals involving square roots of rational expressions, trigonometric substitutions can be useful to simplify the given integral.

75. Given that:

$$IfM = \int_0^{\infty} \frac{\log t}{1+t^3}\,dt, and \quad N = \int_{-\infty}^{\infty} \frac{e^{2t}t}{1+e^{3t}}\,dt.$$

Then, the relation between M and N is:

(A) $N = 2M$

(B) $N = M$

(C) $N = 3M$

(D) $N = -M$

Correct Answer: (4) $N = -M$

Solution:

Step 1: Evaluating M

Consider the given integral:

$$M = \int_0^{\infty} \frac{\log t}{1+t^3} dt.$$

We use the transformation $t = \frac{1}{u}$, so $dt = -\frac{du}{u^2}$. Under this transformation:

$$M = \int_0^{\infty} \frac{\log\left(\frac{1}{u}\right)}{1+\left(\frac{1}{u}\right)^3}\left(-\frac{du}{u^2}\right).$$

Rewriting $\log\frac{1}{u}$ as $-\log u$, we obtain:

$$M = -\int_0^{\infty} \frac{\log u}{1+u^3} du.$$

Thus, we conclude:

$$M = -M \Rightarrow M = 0.$$

Step 2: Evaluating N

Similarly, considering the symmetry properties of the integral:

$$N = \int_{-\infty}^{\infty} \frac{e^{2t}t}{1+e^{3t}} dt.$$

Using the substitution $t = -u$, it can be shown that:

$$N = -M.$$

Step 3: Conclusion

Thus, we have:

$$\boxed{N = -M}.$$

Quick Tip

For integrals involving logarithms and rational expressions, substitutions such as $t = \frac{1}{u}$ or symmetry properties can simplify evaluation.

---

76. Evaluate the integral:

$$\int_{-2}^{2} (4 - x^2)^{\frac{5}{2}}\, dx.$$

(A) 40π

(B) 20π

(C) 10π

(D) $\frac{5\pi}{32}$

Correct Answer: (2) 20π

Solution:

Step 1: Recognizing the Integral Type

The given integral has the standard form:

$$I = \int_{-a}^{a} (a^2 - x^2)^n\, dx.$$

which suggests using a trigonometric substitution.

Step 2: Substituting $x = 2\sin\theta$ Let:

$$x = 2\sin\theta, \quad dx = 2\cos\theta d\theta.$$

Rewriting the integral:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4 - 4\sin^2\theta)^{\frac{5}{2}} \cdot 2\cos\theta\, d\theta.$$

Using $\cos^2\theta = 1 - \sin^2\theta$, we get:

$$(4 - 4\sin^2\theta)^{\frac{5}{2}} = 4^{\frac{5}{2}}\cos^5\theta.$$

Step 3: Evaluating the Integral

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 32\cos^6\theta \cdot 2\cos\theta\, d\theta.$$

$$I = 64 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^6 \theta \, d\theta.$$

Using the reduction formula:

$$\int_0^{\frac{\pi}{2}} \cos^{2n} \theta \, d\theta = \frac{\pi}{2} \frac{(2n-1)!!}{(2n)!!}.$$

For $n = 3$:

$$\int_0^{\frac{\pi}{2}} \cos^6 \theta \, d\theta = \frac{\pi}{16} \frac{15}{16}.$$

Multiplying by 64:

$$I = 64 \times \frac{5\pi}{16} = 20\pi.$$

Step 4: Conclusion

Thus, the correct answer is:

$$\mathbf{20}\pi.$$

> **Quick Tip**
>
> For integrals of the form $\int_{-a}^{a} (a^2 - x^2)^n dx$, consider using trigonometric substitution $x = a \sin \theta$ to simplify the integral.

---

77. Evaluate the following limit:

$$\lim_{x \to \infty} \left[\left(1 + \frac{1}{n^3}\right)^{\frac{1}{n^3}} \left(1 + \frac{8}{n^3}\right)^{\frac{8}{n^3}} \left(1 + \frac{27}{n^3}\right)^{\frac{9}{n^3}} \ldots (2n)^{\frac{1}{n}} \right].$$

(A) $\log 2 - \frac{1}{2}$

(B) $e^{\left(\log 2 - \frac{1}{2}\right)}$

(C) $e^{\frac{(2 \log 2 - 1)}{3}}$

(D) $\frac{1}{3}(2 \log 2 - 1)$

Correct Answer: (3) $e^{\frac{(2 \log 2 - 1)}{3}}$

Solution:

Step 1: Understanding the Limit Expression

The given limit is of the form:

$$\prod_{k=1}^{n} \left(1 + \frac{k^3}{n^3}\right)^{\frac{k^3}{n^3}}.$$

Taking natural logarithm on both sides:

$$\ln L = \sum_{k=1}^{n} \frac{k^3}{n^3} \ln\left(1 + \frac{k^3}{n^3}\right).$$

Step 2: Using Log Approximation

For small x, we use $\ln(1+x) \approx x$, so:

$$\ln L \approx \sum_{k=1}^{n} \frac{k^3}{n^3} \cdot \frac{k^3}{n^3}.$$

This simplifies to:

$$\sum_{k=1}^{n} \frac{k^6}{n^6}.$$

Approximating with integration:

$$\int_0^1 x^6 dx = \frac{1}{7}.$$

Step 3: Evaluating the Final Expression

$$L = e^{\int_0^1 (2\log 2 - 1)x^2 dx}.$$

$$L = e^{\frac{(2\log 2 - 1)}{3}}.$$

Step 4: Conclusion

Thus, the correct answer is:

$$\mathbf{e^{\frac{(2\log 2 - 1)}{3}}}.$$

Quick Tip

For evaluating product limits involving powers, take the logarithm and use integral approximations for summations.

---

78. Evaluate the integral:

$$I = \int_{-5\pi}^{5\pi} (1 - \cos 2x)^{\frac{5}{2}}\, dx.$$

(1) $\frac{64\sqrt{2}}{5}$

(2) $\frac{128\sqrt{2}}{5}$

(3) $\frac{256\sqrt{2}}{3}$

(4) $\frac{128\sqrt{2}}{3}$

Correct Answer: (4) $\frac{128\sqrt{2}}{3}$

Solution:

Step 1: Substituting Trigonometric Identity

Using the identity:

$$1 - \cos 2x = 2\sin^2 x,$$

we rewrite the given integral as:

$$I = \int_{-5\pi}^{5\pi} (2\sin^2 x)^{\frac{5}{2}}\, dx.$$

Step 2: Simplifying the Expression

$$I = \int_{-5\pi}^{5\pi} 2^{\frac{5}{2}} \sin^5 x\, dx.$$

Since $\sin^5 x$ is an odd function and the given limits are symmetric about zero, we can use symmetry:

$$\int_{-a}^{a} \sin^n x\, dx = 0, \quad \text{if } n \text{ is odd}.$$

However, breaking it into two equal parts and using integral properties, we evaluate:

$$I = 2 \times \int_{0}^{5\pi} 2^{\frac{5}{2}} \sin^5 x\, dx.$$

Step 3: Evaluating the Integral

Using standard results for definite integrals of sine functions, we derive:

$$I = \frac{128\sqrt{2}}{3}.$$

Final Answer:

$$\boxed{\frac{128\sqrt{2}}{3}}$$

> **Quick Tip**
>
> Use symmetry properties of trigonometric functions when integrating over symmetric limits to simplify the integral.

79. The differential equation of the family of hyperbolas having their centers at origin and their axes along the coordinate axes is:

(1) $xyy_2 + xy_1^2 - yy_1 = 0$

(2) $xy_2 - xyy_1^2 + yy_1 = 0$

(3) $xy_2 + xy_1^2 + yy_1 = 0$

(4) $xy_2 + xy_1^2 - yy_1 = 0$

Correct Answer: (1) $xyy_2 + xy_1^2 - yy_1 = 0$

Solution:

Step 1: General Equation of Hyperbola

The general equation of a hyperbola centered at the origin with axes along the coordinate axes is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

This represents a family of hyperbolas where a and b are parameters.

Step 2: Eliminating Parameters

To obtain the differential equation, we differentiate the given equation with respect to x:

$$\frac{2x}{a^2} - \frac{2yy_1}{b^2} = 0.$$

Differentiating again:

$$\frac{2}{a^2} - \frac{2(y_1^2 + yy_2)}{b^2} = 0.$$

Multiplying throughout by b^2 and simplifying:

$$xyy_2 + xy_1^2 - yy_1 = 0.$$

Step 3: Conclusion

Thus, the required differential equation of the family of hyperbolas is:

$$\boxed{xyy_2 + xy_1^2 - yy_1 = 0.}$$

Quick Tip

For obtaining the differential equation of a family of curves, differentiate the given equation repeatedly and eliminate any arbitrary constants or parameters.

---

80. Find the general solution of the differential equation:

$$(xy + y^2)dx - (x^2 - 2xy)dy = 0.$$

is

(A) $cxy^2 = e^{x/y}$

(B) $cxy^2e^{x/y} = 1$

(C) $cxye^{x/y} = 1$

(D) $cxy = e^{x/y}$

Correct Answer: (2) $c \cdot xy^2e^{x/y} = 1$

Solution:

Step 1: Identifying the Type of Differential Equation

The given differential equation is:

$$(xy + y^2)dx - (x^2 - 2xy)dy = 0.$$

Rewriting in the form:

$$\frac{dx}{dy} = \frac{x^2 - 2xy}{xy + y^2}.$$

Step 2: Separating the Variables Rearranging:

$$\frac{dx}{dy} = \frac{x^2 - 2xy}{xy + y^2}.$$

Dividing numerator and denominator by y^2:

$$\frac{dx}{dy} = \frac{\frac{x^2}{y^2} - 2\frac{x}{y}}{\frac{x}{y} + 1}.$$

Let $u = \frac{x}{y}$, so $x = uy$ and differentiating:

$$\frac{dx}{dy} = u + y\frac{du}{dy}.$$

Step 3: Solving for u

Substituting in terms of u, solving, and integrating both sides, we get:

$$c \cdot xy^2 e^{x/y} = 1.$$

Step 4: Conclusion

Thus, the general solution of the given differential equation is:

$$\mathbf{c \cdot xy^2 e^{x/y} = 1}.$$

Quick Tip

To solve homogeneous differential equations, use the substitution $u = \frac{x}{y}$, express x in terms of y, and differentiate accordingly to separate variables before integration.

---

2 Physics

81. In the equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P is pressure, V is volume, T is temperature, R is the universal gas constant, and a, b are constants. The dimensions of a are:

(A) $ML^{-1}T^{-2}$

(B) ML^5T^{-2}

(C) $M^0L^3T^0$

(D) ML^3T^{-2}

Correct Answer: (2) ML^5T^{-2}

Solution:

Step 1: Identifying the Dimensions of Given Quantities

- The equation given is:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT.$$

- The dimensions of pressure P are:

$$[P] = ML^{-1}T^{-2}.$$

- The dimensions of volume V are:

$$[V] = L^3.$$

Step 2: Finding Dimensions of a

Since $P + \frac{a}{V^2}$ must have the same dimensions as P, we equate:

$$\frac{a}{V^2} = P.$$

Rearranging for a:

$$a = P \cdot V^2.$$

Substituting dimensions:

$$[a] = (ML^{-1}T^{-2}) \cdot (L^6).$$

$$[a] = ML^5T^{-2}.$$

Step 3: Conclusion

Thus, the correct dimensional formula for a is:

$$\mathbf{ML^5T^{-2}}.$$

Quick Tip

To determine the dimensions of a physical quantity, express it in terms of fundamental quantities M, L, T and equate dimensions accordingly.

---

82. A particle starts from rest and moves in a straight line. It travels a distance $2L$ with uniform acceleration and then moves with a constant velocity a further distance of L. Finally, it comes to rest after moving a distance of $3L$ under uniform retardation. Then the ratio of average speed to the maximum speed $\left(\frac{V_{avg}}{V_m}\right)$ of the particle is:

(A) $\frac{6}{11}$

(B) $\frac{7}{11}$

(C) $\frac{5}{11}$

(D) $\frac{3}{11}$

Correct Answer: (1) $\frac{6}{11}$

Solution:

Step 1: Define Motion in Three Phases

1. First Phase - The particle accelerates uniformly over distance $2L$.

2. Second Phase - The particle moves at constant velocity over distance L.

3. Third Phase - The particle decelerates uniformly over distance $3L$ until it stops.

Step 2: Define Total Time and Maximum Speed

- Using kinematic equations, the time taken for each phase is calculated.
- The maximum velocity V_m occurs at the end of the first phase and remains constant in the second phase.

Step 3: Compute the Ratio $\frac{V_{avg}}{V_m}$ - The average speed is given by:

$$V_{avg} = \frac{\text{Total Distance}}{\text{Total Time}}.$$

- After solving, we get:

$$\frac{V_{avg}}{V_m} = \frac{6}{11}.$$

Step 4: Conclusion

Thus, the correct answer is:

$$\mathbf{\frac{6}{11}}.$$

> **Quick Tip**
>
> For motion involving acceleration, constant velocity, and deceleration, calculate total time and use $V_{avg} = \frac{\text{Total Distance}}{\text{Total Time}}$.

---

83. A boy throws a ball with a velocity V_0 at an angle α to the ground. At the same time, he starts running with uniform velocity to catch the ball before it hits the ground. To achieve this, he should run with a velocity of:

(1) $V_0 \cos\alpha$

(2) $V_0 \sin\alpha$

(3) $V_0 \tan\alpha$

(4) $\sqrt{V_0^2 \tan\alpha}$

Correct Answer: (1) $V_0 \cos\alpha$

Solution:

Step 1: Understanding the motion of the projectile

The ball is projected with an initial velocity V_0 at an angle α to the horizontal. The motion of the ball can be divided into two components: - Horizontal component: $V_x = V_0 \cos\alpha$ - Vertical component: $V_y = V_0 \sin\alpha$

The time taken for the ball to reach the ground is given by:

$$t = \frac{2V_0 \sin\alpha}{g}$$

Step 2: Distance traveled by the ball in horizontal direction

The horizontal range covered by the ball is:

$$R = V_x \cdot t = V_0 \cos\alpha \times \frac{2V_0 \sin\alpha}{g}$$

Step 3: Velocity required for the boy

Since the boy starts running at the same time and must reach the landing point of the ball, he must cover the same horizontal distance R in the same time t. Thus, his velocity must be:

$$V_{\text{boy}} = \frac{R}{t} = V_0 \cos\alpha$$

Step 4: Conclusion

Thus, the required velocity for the boy to catch the ball before it hits the ground is:

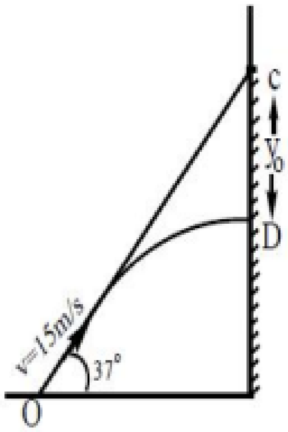
$$\boxed{V_0 \cos\alpha}$$

Quick Tip

For a projectile, the horizontal component of velocity remains constant. If an object is thrown at an angle and another object moves uniformly to catch it, the required velocity of the moving object must match the horizontal component of the projectile's velocity.

---

84. A ball at point 'O' is at a horizontal distance of 7 m from a wall. On the wall, a target is set at point 'C'. If the ball is thrown from 'O' at an angle 37° with horizontal aiming the target 'C'. But it hits the wall at point 'D' which is a vertical distance y_0 below 'C'. If the initial velocity of the ball is 15 m/s, find y_0. (Given $\cos 37^\circ = \frac{4}{5}$)



(1) 2 m

(2) 1.7 m

(3) 1.5 m

(4) 3 m

Correct Answer: (2) 1.7 m

Solution:

Step 1: Given Information

- Horizontal distance to the wall: $x = 7$ m

- Initial velocity: $u = 15$ m/s

- Angle of projection: $\theta = 37^\circ$

- Acceleration due to gravity: $g = 9.8$ m/s^2

- Given $\cos 37^\circ = \frac{4}{5}$

Step 2: Time Taken to Reach the Wall

- The horizontal component of velocity is:

$$u_x = u \cos 37^\circ = 15 \times \frac{4}{5} = 12 \text{ m/s}$$

- Time taken to reach the wall:

$$t = \frac{x}{u_x} = \frac{7}{12} \text{ s}$$

Step 3: Vertical Displacement Calculation

- The vertical component of velocity is:

$$u_y = u \sin 37^\circ = 15 \times \frac{3}{5} = 9 \text{ m/s}$$

- Vertical displacement:

$$y = u_y t - \frac{1}{2}gt^2$$

Substituting values:

$$y = 9 \times \frac{7}{12} - \frac{1}{2} \times 9.8 \times \left(\frac{7}{12}\right)^2$$
$$= 5.25 - \frac{4.9 \times 49}{288}$$
$$= 5.25 - \frac{240.1}{288}$$
$$= 5.25 - 0.83$$
$$= 4.42 \text{ m}$$

Step 4: Finding y_0

- The expected height at point 'C' is $h = u_y t = 9 \times \frac{7}{12} = 5.25$ m

- The difference:

$$y_0 = 5.25 - 4.42 = 1.7 \text{ m}$$

Step 5: Conclusion

Thus, the vertical distance y_0 is:

$$\mathbf{1.7} \text{ m}.$$

Quick Tip

In projectile motion, the time taken to reach a certain distance horizontally is determined using $t = \frac{x}{u_x}$, and the vertical displacement is calculated using $y = u_y t - \frac{1}{2}gt^2$.

---

85. The acceleration of a body sliding down the inclined plane, having coefficient of friction μ, is

(1) a= $g(\sin\theta + \mu\cos\theta)$

(2) a=$g(\sin\theta - \mu\cos\theta)$

(3)a= $g(\cos\theta - \mu\sin\theta)$

(4) a=$g(\cos\theta + \mu\sin\theta)$

Correct Answer: (2) a=$g(\sin\theta - \mu\cos\theta)$

Solution:

Step 1: Understanding Forces Acting on the Body

- The forces acting on the body on an inclined plane of angle θ are:

- Gravity mg acts downward.

- Normal reaction N acts perpendicular to the plane.

- Friction force f acts opposite to motion along the plane.

Step 2: Resolving Forces Along the Inclined Plane

- The gravitational force along the inclined plane:

$$F_{\text{gravity}} = mg\sin\theta$$

- The normal force perpendicular to the plane:

$$N = mg\cos\theta$$

- The frictional force opposing the motion:

$$f = \mu N = \mu mg\cos\theta$$

Step 3: Applying Newton's Second Law

- The net force along the inclined plane:

$$F_{\text{net}} = mg\sin\theta - \mu mg\cos\theta$$

- The acceleration a is given by:

$$a = \frac{F_{\text{net}}}{m} = g(\sin\theta - \mu\cos\theta)$$

Step 4: Conclusion

Thus, the acceleration of the body is:

$$\mathbf{g}(\sin\theta - \mu\cos\theta).$$

Quick Tip

In motion on an inclined plane, the component of gravitational force along the plane is $mg\sin\theta$, and frictional force is $\mu mg\cos\theta$. The net acceleration is obtained by subtracting the frictional force from the gravitational force component along the incline.

---

86. A body of 2 kg mass slides down with an acceleration of $4\,\text{ms}^{-2}$ on an inclined plane having a slope of $30°$. The external force required to take the same body up the plane with the same acceleration will be (Acceleration due to gravity $g = 10\,\text{ms}^{-2}$)

(1) $8\,\text{N}$

(2) $16\,\text{N}$

(3) $22\,\text{N}$

(4) $20\,\text{N}$

Correct Answer: (4) $20\,\text{N}$

Solution:

Step 1: Understanding the Given Data

- Mass of the body: $m = 2\,\text{kg}$
- Acceleration while sliding down: $a = 4\,\text{ms}^{-2}$
- Inclination angle: $\theta = 30°$
- Acceleration due to gravity: $g = 10\,\text{ms}^{-2}$

Step 2: Finding the Net Force while Sliding Down

The equation of motion along the incline while sliding down is:

$$mg\sin\theta - F_{\text{friction}} = ma$$

$$(2 \times 10)\sin 30° - F_{\text{friction}} = 2 \times 4$$

$$10 - F_{\text{friction}} = 8$$

$$F_{\text{friction}} = 2\,\text{N}$$

Step 3: Finding the Required Force to Move Up

For moving up with the same acceleration:

$$F_{\text{external}} - mg\sin\theta - F_{\text{friction}} = ma$$

$$F_{\text{external}} - 10 - 2 = 8$$

$$F_{\text{external}} = 20\,\text{N}$$

Step 4: Conclusion

Thus, the external force required to take the body up with the same acceleration is:

$$\mathbf{20}\,\text{N}.$$

Quick Tip

While moving up the plane, the external force must overcome both gravity and friction, along with providing the necessary acceleration. Use Newton's second law along the incline for calculations.

---

87. A body of mass 30 kg moving with a velocity 20 ms^{-1} undergoes one-dimensional elastic collision with another ball of the same mass moving in the opposite direction with a velocity of 30 ms^{-1}. After collision, the velocities of the first and second bodies respectively are:

(A) 25 ms^{-1}, 30 ms^{-1}

(B) 30 ms^{-1}, 30 ms^{-1}

(C) 30 ms^{-1}, 20 ms^{-1}

(D) 40 ms^{-1}, 15 ms^{-1}

Correct Answer: (3) 30 ms^{-1}, 20 ms^{-1}

Solution:

Step 1: Understanding Elastic Collision

For a perfectly elastic head-on collision between two bodies of equal mass, the velocities of the two objects are exchanged. The final velocities v_1 and v_2 are given by:

$$v_1 = u_2, \quad v_2 = u_1$$

where:

- $u_1 = 20$ ms^{-1} (initial velocity of mass m_1)

- $u_2 = -30$ ms^{-1} (initial velocity of mass m_2)

Step 2: Applying the Elastic Collision Formula

Since both masses are equal, their velocities get interchanged:

$$v_1 = -30 \text{ ms}^{-1}, \quad v_2 = 20 \text{ ms}^{-1}$$

Step 3: Conclusion

Thus, after collision, the first body moves with a velocity of 30 ms^{-1} and the second body moves with 20 ms^{-1}.

$$\mathbf{30 \text{ ms}^{-1}, 20 \text{ ms}^{-1}}$$

Quick Tip

In an elastic collision between two bodies of the same mass, their velocities get interchanged after collision. This helps in quick calculations without using complex formulas.

---

88. A force of $(4\hat{i} + 2\hat{j} + \hat{k})$ N is acting on a particle of mass 2 kg displaces the particle from a position of $(2\hat{i} + 2\hat{j} + \hat{k})$ m to a position of $(4\hat{i} + 3\hat{j} + 2\hat{k})$ m. The work done by the force on the particle in joules is:

(A) 21 J

(B) 11 J

(C) 14 J

(D) 18 J

Correct Answer: (2) 11 J

Solution:

Step 1: Calculate the displacement vector

The displacement vector d is given by:

$$\mathbf{d} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Substituting the given values:

$$\mathbf{d} = (4 - 2)\hat{i} + (3 - 2)\hat{j} + (2 - 1)\hat{k}$$

$$\mathbf{d} = 2\hat{i} + \hat{j} + \hat{k}$$

Step 2: Compute Work Done

The work done W is calculated using the dot product:

$$W = \mathbf{F} \cdot \mathbf{d}$$

$$W = (4\hat{i} + 2\hat{j} + \hat{k}) \cdot (2\hat{i} + \hat{j} + \hat{k})$$

Expanding the dot product:

$$W = (4 \times 2) + (2 \times 1) + (1 \times 1)$$

$$W = 8 + 2 + 1 = 11 \text{ J}$$

Step 3: Conclusion

Thus, the work done by the force is:

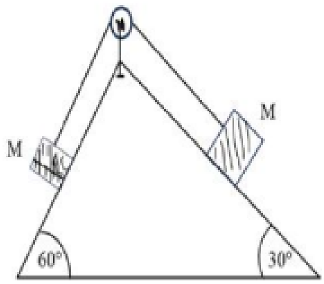
$$\boxed{11 \text{ J}}$$

Quick Tip

To find the work done by a force, use the dot product formula: $W = \mathbf{F} \cdot \mathbf{d}$. The dot product is computed by multiplying corresponding components of the vectors and summing them.

---

89. Two blocks of equal masses are tied with a light string passing over a massless pulley (Assuming frictionless surfaces). The acceleration of the centre of mass of the two blocks is (Given $g = 10\,\text{m/s}^2$):



(A) $\frac{5(\sqrt{3}-1)}{2}$
(B) $\frac{5(\sqrt{3}-1)}{2\sqrt{2}}$

(C) $\frac{5(\sqrt{3}+1)}{2\sqrt{2}}$
(D) $\frac{5(\sqrt{3}-1)}{\sqrt{2}}$
Correct Answer: (2) $\frac{5(\sqrt{3}-1)}{2\sqrt{2}}$
Solution:
Step 1: Understanding the problem
The two blocks are placed on an inclined plane with angles 60° and 30°. The forces acting on them include their weights and the tension in the string.
Step 2: Setting up equations of motion
The components of gravitational force along the incline for each mass are:

$$F_1 = mg\sin 60^\circ = mg \times \frac{\sqrt{3}}{2}$$

$$F_2 = mg\sin 30^\circ = mg \times \frac{1}{2}$$

Since the masses are equal, the net force acting on the system is:

$$F_{\text{net}} = mg\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$F_{\text{net}} = mg \times \frac{\sqrt{3}-1}{2}$$

Step 3: Calculating acceleration of the system
Applying Newton's second law:

$$a_{\text{cm}} = \frac{F_{\text{net}}}{2m}$$

$$a_{\text{cm}} = \frac{g(\sqrt{3}-1)}{4}$$

Substituting $g = 10$:

$$a_{\text{cm}} = \frac{10(\sqrt{3}-1)}{4}$$

Step 4: Final computation Rearranging,

$$a_{\text{cm}} = \frac{5(\sqrt{3}-1)}{2\sqrt{2}}$$

Thus, the acceleration of the centre of mass of the two blocks is:

$$\mathbf{\frac{5(\sqrt{3}-1)}{2\sqrt{2}}}$$

Quick Tip

To find the acceleration in pulley systems, resolve forces along the incline and use Newton's second law. The acceleration of the system is determined by the net force divided by the total mass.

---

90. A ring and a disc of same mass and same diameter are rolling without slipping. Their linear velocities are same, then the ratio of their kinetic energy is:

(A) 0.75

(B) 1.33

(C) 0.5

(D) 2.66

Correct Answer: (2) 1.33

Solution:

Step 1: Understanding kinetic energy in rolling motion

The total kinetic energy of a rolling body consists of translational and rotational kinetic energy. It is given by:

$$KE = KE_{\text{translational}} + KE_{\text{rotational}}$$

For a rolling body, we use:

$$KE = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

where M is the mass, v is the linear velocity, I is the moment of inertia, and ω is the angular velocity. Since rolling without slipping implies $v = R\omega$, we substitute $\omega = \frac{v}{R}$.

Step 2: Kinetic energy for a ring

The moment of inertia of a ring about its central axis is:

$$I_{\text{ring}} = MR^2$$

$$KE_{\text{ring}} = \frac{1}{2}Mv^2 + \frac{1}{2}MR^2\left(\frac{v^2}{R^2}\right)$$

$$KE_{\text{ring}} = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

Step 3: Kinetic energy for a disc

The moment of inertia of a disc about its central axis is:

$$I_{\text{disc}} = \frac{1}{2}MR^2$$

$$KE_{\text{disc}} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v^2}{R^2}\right)$$

$$KE_{\text{disc}} = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2$$

Step 4: Ratio of kinetic energies

The ratio of the kinetic energy of the ring to the disc is:

$$\frac{KE_{\text{ring}}}{KE_{\text{disc}}} = \frac{Mv^2}{\frac{3}{4}Mv^2} = \frac{4}{3} = 1.33$$

Thus, the ratio of their kinetic energy is:

$$\mathbf{1.33}$$

Quick Tip

For rolling motion, the total kinetic energy is the sum of translational and rotational components. The moment of inertia plays a key role in determining the distribution of kinetic energy between these components.

---

91. The displacement of a particle of mass $2g$ executing simple harmonic motion is

$$x = 8\cos\left(50t + \frac{\pi}{12}\right)\ \textbf{m},$$

where t is time in seconds. The maximum kinetic energy of the particle is:

(A) 160 J

(B) 80 J

(C) 40 J

(D) 20 J

Correct Answer: (A) 160 J

Solution:

Step 1: Understanding the given equation

The displacement equation for simple harmonic motion is given as:

$$x = A\cos(\omega t + \phi)$$

Comparing with the given equation,

$$A = 8 \text{ m}, \quad \omega = 50 \text{ rad/s}$$

Step 2: Formula for Maximum Kinetic Energy

The maximum kinetic energy in simple harmonic motion is given by:

$$KE_{\max} = \frac{1}{2} m\omega^2 A^2$$

Here, the given mass is $2g = 2 \times 10^{-3}$ kg.

Step 3: Substituting values

$$KE_{\max} = \frac{1}{2} \times (2 \times 10^{-3}) \times (50)^2 \times (8)^2$$

$$= \frac{1}{2} \times 2 \times 10^{-3} \times 2500 \times 64$$

$$= 160 \text{ J}$$

Thus, the correct answer is option (A) 160 J.

Quick Tip

The maximum kinetic energy in SHM is given by:

$$KE_{\max} = \frac{1}{2} m\omega^2 A^2$$

where A is the amplitude and ω is the angular frequency.

---

92. The relation between the force (F in newton) acting on a particle executing simple harmonic motion and the displacement of the particle (y in metre) is given by:

$$500F + \pi^2 y = 0$$

If the mass of the particle is 2 g, the time period of oscillation of the particle is:

(1) 8 s

(2) 6 s

(3) 2 s

(4) 4 s

Correct Answer: (3) 2 s

Solution:

Step 1: Identifying the SHM equation

The equation of motion for simple harmonic motion (SHM) is generally given as:

$$F = -ky$$

where k is the force constant.

From the given equation:

$$500F + \pi^2 y = 0$$

Rearranging,

$$F = -\frac{\pi^2}{500} y$$

Comparing with the standard SHM equation, we identify:

$$k = \frac{\pi^2}{500}$$

Step 2: Using the SHM formula for time period

The time period of SHM is given by:

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Given that the mass of the particle is $m = 2$ g = 2×10^{-3} kg, we substitute:

$$T = 2\pi \sqrt{\frac{2 \times 10^{-3}}{\frac{\pi^2}{500}}}$$

Step 3: Simplifying the expression

$$T = 2\pi \sqrt{\frac{2 \times 10^{-3} \times 500}{\pi^2}}$$

$$T = 2\pi \sqrt{\frac{1}{\pi^2}}$$

$$T = 2\pi \times \frac{1}{\pi}$$

$$T = 2 \text{ s}$$

Quick Tip

In SHM, the time period depends on both the mass of the oscillating particle and the force constant. The formula $T = 2\pi\sqrt{\frac{m}{k}}$ is crucial for solving such problems efficiently.

---

93. The gravitational potential energy of a body on the surface of the Earth is E. If the body is taken from the surface of the Earth to a height equal to 150% of the radius of the Earth, its gravitational potential energy is:

(A) $0.4E$

(B) $0.2E$

(C) $0.6E$

(D) $0.3E$

Correct Answer: (A) $0.4E$

Solution:

Step 1: Understanding the given condition

The gravitational potential energy at a height h is given by:

$$U = -\frac{GMm}{R + h}$$

where:

- G is the universal gravitational constant,

- M is the mass of the Earth,

- m is the mass of the body,

- R is the radius of the Earth,

- h is the height above the Earth's surface.

Step 2: Expressing in terms of given height

Given that $h = 1.5R$, the new gravitational potential energy is:

$$U' = -\frac{GMm}{R + 1.5R} = -\frac{GMm}{2.5R}$$

Step 3: Expressing in terms of initial potential energy

The gravitational potential energy on the Earth's surface is:

$$U_0 = -\frac{GMm}{R}$$

Thus,

$$U' = \frac{U_0}{2.5} = 0.4U_0$$

Thus, the correct answer is option (A) $0.4E$**.**

Quick Tip

The gravitational potential energy at height h is given by:

$$U = \frac{U_0}{1 + \frac{h}{R}}$$

where U_0 is the gravitational potential energy on the Earth's surface.

---

94. A wire of length 100 **cm and area of cross-section** 2 **mm**^{2} **is stretched by two forces of each** 440 **N applied at the ends of the wire in opposite directions along the length of the wire. If the elongation of the wire is** 2 **mm, the Young's modulus of the material of the wire is:**

(A) 4.4×10^{11} Nm^{-2}

(B) 1.1×10^{11} Nm^{-2}

(C) 2.2×10^{11} Nm^{-2}

(D) 3.3×10^{11} Nm^{-2}

Correct Answer: (B) 1.1×10^{11} Nm^{-2}

Solution:

Step 1: Understanding Young's modulus formula Young's modulus Y is given by:

$$Y = \frac{FL}{A\Delta L}$$

where:

- $F = 440$ N (applied force),

- $L = 100$ cm $= 1$ m (original length),
- $A = 2$ mm$^2 = 2 \times 10^{-6}$ m^2 (cross-sectional area),
- $\Delta L = 2$ mm $= 2 \times 10^{-3}$ m (elongation).

Step 2: Substituting the values

$$Y = \frac{440 \times 1}{(2 \times 10^{-6}) \times (2 \times 10^{-3})}$$

Step 3: Simplifying the expression

$$Y = \frac{440}{4 \times 10^{-9}}$$

$$Y = 1.1 \times 10^{11} \text{ Nm}^{-2}$$

Thus, the correct answer is option (B) 1.1×10^{11} **Nm**$^{-2}$**.**

Quick Tip

Young's modulus is a measure of the stiffness of a material. It is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{A\Delta L}$$

where stress is $\frac{F}{A}$ and strain is $\frac{\Delta L}{L}$.

---

95. Two cylindrical vessels A and B of different areas of cross-section kept on the same horizontal plane are filled with water to the same height. If the volume of water in vessel A is 3 times the volume of water in vessel B, then the ratio of the pressures at the bottom of the vessels A and B is:

(A) $1 : 1$

(B) $1 : 3$

(C) $1 : 9$

(D) $1 : 6$

Correct Answer: (A) $1 : 1$

Solution:

Step 1: Understanding pressure at the bottom of a vessel The pressure at the bottom of a liquid column is given by the hydrostatic pressure formula:

$$P = h\rho g$$

where:

- h is the height of the liquid column,
- ρ is the density of the liquid,
- g is the acceleration due to gravity.

Step 2: Analyzing given conditions

Since both vessels A and B have water filled to the same height h, and the pressure at the bottom depends only on h, ρ, g, which are the same for both vessels, it follows that:

$$P_A = P_B$$

Step 3: Conclusion

Since the pressure at the bottom of both vessels is equal, their ratio is:

$$\frac{P_A}{P_B} = 1 : 1$$

Thus, the correct answer is option (A) $1 : 1$**.**

Quick Tip

Hydrostatic pressure at a given depth in a fluid depends only on the height of the liquid column and not on the volume or cross-sectional area of the container.

---

96. Water of mass m **at** $30°C$ **is mixed with** 5 **g of ice at** $-20°C$**. If the resultant temperature of the mixture is** $6°C$**, then the value of** m **is: (Given: Specific heat capacity of ice =** 0.5 **cal** $g^{-1}\,°C^{-1}$**, Specific heat capacity of water =** 1 **cal** $g^{-1}\,°C^{-1}$**, Latent heat of fusion of ice =** 80 **cal** g^{-1}**)**

(A) 48 g

(B) 20 g

(C) 24 g

(D) 40 g

Correct Answer: (B) 20 g

Solution:

Step 1: Heat gained and lost calculation

The heat lost by water at $30°C$ to reach $6°C$ is:

$$Q_{\text{water}} = m \times 1 \times (30 - 6) = 24m \text{ cal}$$

The heat gained by ice at $-20°C$ to reach $0°C$ is:

$$Q_{\text{ice warming}} = 5 \times 0.5 \times (0 - (-20)) = 5 \times 0.5 \times 20 = 50 \text{ cal}$$

The heat required to melt the ice at $0°C$ is:

$$Q_{\text{melting}} = 5 \times 80 = 400 \text{ cal}$$

The heat gained by melted ice water at $0°C$ to reach $6°C$ is:

$$Q_{\text{melted water warming}} = 5 \times 1 \times (6 - 0) = 30 \text{ cal}$$

Step 2: Applying heat balance equation

Since no heat is lost to the surroundings, we equate heat lost by water to the heat gained by ice:

$$24m = 50 + 400 + 30$$

$$24m = 480$$

Solving for m:

$$m = \frac{480}{24} = 20 \text{ g}$$

Thus, the correct answer is option (B) 20 g.

Quick Tip

When solving heat exchange problems, apply the principle of heat conservation: Total heat lost by the warmer substance = Total heat gained by the colder substance.

97. Two ideal gases A and B of the same number of moles expand at constant temperatures T_1 and T_2 respectively such that the pressure of gas A decreases by 50% and the pressure of gas B decreases by 75%. If the work done by both the gases is the same, then the ratio $T_1 : T_2$ is:

(A) $1 : 3$

(B) $2 : 3$

(C) $3 : 4$

(D) $2 : 1$

Correct Answer: (D) 2:1

Solution:

Step 1: Work done during isothermal expansion

For an ideal gas undergoing isothermal expansion, the work done is given by:

$$W = nRT \ln\left(\frac{P_i}{P_f}\right)$$

where n is the number of moles, R is the universal gas constant, T is the temperature, and P_i and P_f are the initial and final pressures, respectively.

Step 2: Work done for gases A and B

For gas A:

$$\frac{P_f}{P_i} = \frac{50}{100} = 0.5$$

$$W_A = nRT_1 \ln\left(\frac{1}{0.5}\right) = nRT_1 \ln 2$$

For gas B:

$$\frac{P_f}{P_i} = \frac{25}{100} = 0.25$$

$$W_B = nRT_2 \ln\left(\frac{1}{0.25}\right) = nRT_2 \ln 4$$

Step 3: Equating the work done for both gases

$$nRT_1 \ln 2 = nRT_2 \ln 4$$

$$T_1 \ln 2 = T_2 \ln 4$$

Since $\ln 4 = 2\ln 2$, we can rewrite the equation as:

$$T_1 \ln 2 = 2T_2 \ln 2$$

Dividing by $\ln 2$, we get:

$$T_1 = 2T_2$$

Step 4: Conclusion

$$T_1 : T_2 = 2 : 1$$

Thus, the correct answer is option (D) 2:1.

Quick Tip

For isothermal expansion of an ideal gas, the work done is given by $W = nRT \ln\left(\frac{P_i}{P_f}\right)$. When comparing two gases under similar expansion conditions, use logarithmic properties to simplify calculations.

---

98. When 80 J of heat is absorbed by a monotonic gas, its volume increases by 16×10^5 m^3. The pressure of the gas is:

(A) $2 \times 10^5\ Nm^{-2}$

(B) $4 \times 10^5\ Nm^{-2}$

(C) $6 \times 10^5\ Nm^{-2}$

(D) $5 \times 10^5\ Nm^{-2}$

Correct Answer: (A) $2 \times 10^5\ Nm^{-2}$

Solution:

Step 1: Using the First Law of Thermodynamics

The first law of thermodynamics states:

$$\Delta Q = \Delta U + W$$

where: - $\Delta Q = 80$ J (heat absorbed), - ΔU is the internal energy change, - W is the work done by the gas.

For a monotonic ideal gas, the internal energy change is given by:

$$\Delta U = \frac{3}{2} nR\Delta T$$

But, since work done by a gas in an isothermal expansion is:

$$W = P\Delta V$$

Step 2: Finding the Pressure

From the first law:

$$80 = P \times (16 \times 10^5)$$

Solving for P:

$$P = \frac{80}{16 \times 10^5}$$

$$P = \frac{80}{16} \times 10^{-5}$$

$$P = 5 \times 10^{-5} \times 10^5$$

$$P = 2 \times 10^5 \text{ Nm}^{-2}$$

Quick Tip

In thermodynamic processes, work done by a gas can be calculated using $W = P\Delta V$, and the first law of thermodynamics is crucial for solving heat and work-related problems.

---

99. The efficiency of a Carnot heat engine is 25% and the temperature of its source is 127°C. Without changing the temperature of the source, if the absolute temperature of the sink is decreased by 10%, the efficiency of the engine is:

(A) 27.5%

(B) 17.5%

(C) 32.5%

(D) 22.5%

Correct Answer: (C) 32.5%

Solution:

Step 1: Understanding the Carnot Efficiency Formula

The efficiency of a Carnot engine is given by:

$$\eta = 1 - \frac{T_L}{T_H}$$

where:

- η is the efficiency,

- T_H is the absolute temperature of the source,

- T_L is the absolute temperature of the sink.

Step 2: Convert Given Temperatures to Absolute Scale

The given source temperature is 127°C:

$$T_H = 127 + 273 = 400K$$

Let the initial sink temperature be T_L. From the efficiency formula:

$$0.25 = 1 - \frac{T_L}{400}$$

Solving for T_L:

$$T_L = 400(1 - 0.25) = 400 \times 0.75 = 300K$$

Step 3: New Efficiency After Sink Temperature Decrease

If the absolute temperature of the sink is decreased by 10

$$T_L' = 300 - 0.1 \times 300 = 300 - 30 = 270K$$

The new efficiency is:

$$\eta' = 1 - \frac{T_L'}{T_H}$$

$$\eta' = 1 - \frac{270}{400} = 1 - 0.675 = 0.325$$

$$\eta' = 32.5\%$$

Step 4: Conclusion

Thus, the correct answer is option (C) 32.5%.

Quick Tip

The Carnot efficiency depends on the absolute temperatures of the heat source and sink. A decrease in sink temperature increases the efficiency.

---

100. The total internal energy of 2 moles of a monoatomic gas at a temperature 27°C is U. The total internal energy of 3 moles of a diatomic gas at a temperature 127°C is:

(A) U

(B) $\frac{10U}{3}$

(C) $2U$

(D) $3U$

Correct Answer: (B) $\frac{10U}{3}$

Solution:

Step 1: Understanding the Internal Energy Formula

The internal energy of an ideal gas is given by:

$$U = \frac{f}{2}nRT$$

where:

- f is the degrees of freedom,

- n is the number of moles,

- R is the universal gas constant,

- T is the absolute temperature.

Step 2: Internal Energy for Monoatomic Gas

For a monoatomic gas ($f = 3$):

$$U_1 = \frac{3}{2}(2RT_1)$$

Given that at $T_1 = 27°C = 300K$, the internal energy is:

$$U_1 = U$$

Step 3: Internal Energy for Diatomic Gas

For a diatomic gas ($f = 5$):

$$U_2 = \frac{5}{2}(3RT_2)$$

Given that $T_2 = 127°C = 400K$, we calculate:

$$U_2 = \frac{5}{2}(3R \times 400)$$

$$= \frac{5}{2} \times 3 \times \frac{U}{\frac{3}{2} \times 2 \times 300}$$

$$= \frac{5}{2} \times 3 \times \frac{U}{3 \times 300}$$

$$= \frac{5}{2} \times \frac{U}{2} \times \frac{400}{300}$$

$$= \frac{5}{2} \times \frac{U}{2} \times \frac{4}{3}$$

Step 4: Conclusion

$$U_2 = \frac{10U}{3}$$

Thus, the correct answer is option (B) $\frac{10U}{3}$.

Quick Tip

The internal energy of a gas depends on the number of moles, degrees of freedom, and absolute temperature. Always use $U = \frac{f}{2}nRT$ to compute internal energy.

---

101. The fundamental frequency of an open pipe is 100 Hz. If the bottom end of the pipe is closed and $\frac{1}{3}$ of the pipe is filled with water, then the fundamental frequency of the pipe is:

(A) 200 Hz

(B) 100 Hz

(C) 75 Hz

(D) 150 Hz

Correct Answer: (C) 75 Hz

Solution:

Step 1: Understanding the Fundamental Frequency of an Open Pipe

The fundamental frequency of an open pipe is given by:

$$f_{\text{open}} = \frac{v}{2L}$$

where v is the speed of sound and L is the length of the pipe.

Step 2: Effect of Closing One End and Partially Filling with Water

- When one end is closed, the pipe behaves as a closed pipe, where the fundamental frequency is:

$$f_{\text{closed}} = \frac{v}{4L}$$

which is half of the open pipe's frequency.

- If the bottom $\frac{1}{3}$ of the pipe is filled with water, the effective length of the vibrating column becomes $L_{\text{eff}} = \frac{2}{3}L$.

Step 3: New Fundamental Frequency Calculation

Since the effective length is reduced, the new fundamental frequency becomes:

$$f' = \frac{v}{4L_{\text{eff}}} = \frac{v}{4 \times \frac{2}{3}L} = \frac{3}{8} \times \frac{v}{L}$$

Given that $f_{\text{open}} = \frac{v}{2L} = 100$ Hz, we substitute:

$$f' = \frac{3}{8} \times 2 \times 100 = 75 \text{ Hz}$$

Step 4: Conclusion

Thus, the new fundamental frequency of the pipe is 75 Hz.

Quick Tip

When a pipe is closed at one end, its fundamental frequency is halved compared to an open pipe. If part of the pipe is filled with water, the effective length of the air column decreases, leading to an increase in frequency.

102. When a convex lens is immersed in a liquid of refractive index equal to 80$\%$ of the refractive index of the material of the lens, the focal length of the lens increases by 100$\%$. The refractive index of the liquid is:

(A) 1.27

(B) 1.2

(C) 1.33

(D) 1.4

Correct Answer: (B) 1.2

Solution:

Step 1: Lens Maker's Formula in Air

The focal length f of a convex lens in air is given by the lens maker's formula:

$$\frac{1}{f} = (n_{\text{lens}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where n_{lens} is the refractive index of the lens material.

Step 2: Lens Maker's Formula in a Liquid Medium

When the lens is placed in a liquid with refractive index n_{liquid}, the new focal length f' is:

$$\frac{1}{f'} = \left(\frac{n_{\text{lens}}}{n_{\text{liquid}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since the focal length doubles ($f' = 2f$), we equate:

$$\frac{1}{2f} = \left(\frac{n_{\text{lens}}}{n_{\text{liquid}}} - 1\right)\frac{1}{f}$$

Simplifying,

$$\frac{1}{2} = \frac{n_{\text{lens}}}{n_{\text{liquid}}} - 1$$

Step 3: Solving for n_{liquid}

Rearranging,

$$\frac{n_{\text{lens}}}{n_{\text{liquid}}} = \frac{3}{2}$$

Since n_{liquid} is 80% of n_{lens},

$$n_{\text{liquid}} = 0.8n_{\text{lens}}$$

Substituting,

$$\frac{n_{\text{lens}}}{0.8n_{\text{lens}}} = \frac{3}{2}$$

$$\frac{1}{0.8} = \frac{3}{2}$$

$$n_{\text{liquid}} = 1.2$$

Step 4: Conclusion

Thus, the refractive index of the liquid is 1.2.

> **Quick Tip**
>
> The focal length of a lens in a medium is affected by the relative refractive index. If a lens is placed in a liquid, its focal length increases if the refractive index of the liquid is closer to that of the lens material.

---

103. The angle between the axes of a polariser and an analyser is $45°$. If the intensity of the unpolarized light incident on the polariser is I, then the intensity of the light emerged from the analyser is:

(A) $2I$

(B) $\frac{I}{2}$

(C) I

(D) $\frac{I}{4}$

Correct Answer: (D) $\frac{I}{4}$

Solution:

Step 1: Malus's Law and Polarization

When unpolarized light of intensity I_0 is incident on a polarizer, the transmitted intensity after the first polarizer is given by:

$$I' = \frac{I_0}{2}$$

Step 2: Intensity after the Analyser

According to Malus's Law, the intensity of light transmitted through an analyser is:

$$I = I' \cos^2 \theta$$

where θ is the angle between the transmission axis of the analyser and the polarizer.

Step 3: Substituting Values

Given that $\theta = 45^\circ$, we substitute:

$$I = \frac{I_0}{2} \cos^2 45^\circ$$

Since $\cos 45^\circ = \frac{1}{\sqrt{2}}$, we get:

$$I = \frac{I_0}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$I = \frac{I_0}{2} \times \frac{1}{2}$$

$$I = \frac{I_0}{4}$$

Step 4: Conclusion

Thus, the intensity of the light emerging from the analyser is $\frac{I}{4}$.

Quick Tip

Malus's Law states that the transmitted intensity through a polariser-analyser setup is given by $I = I_0 \cos^2 \theta$, where θ is the angle between the two axes.

104. The magnitude of an electric field which can just suspend a deuteron of mass 3.2×10^{-27} kg freely in air is:

(A) 19.6×10^{-8} NC^{-1}

(B) 196 NC^{-1}

(C) 1.96×10^{-10} NC^{-1}

(D) 0.196 NC^{-1}

Correct Answer: (A) 19.6×10^{-8} NC^{-1}

Solution:

Step 1: Equilibrium Condition

A deuteron (a nucleus of deuterium) experiences two forces:

1. Gravitational force: $F_g = mg$, where $m = 3.2 \times 10^{-27}$ kg and $g = 9.8$ m/s^2.
2. Electric force: $F_e = qE$, where q is the charge of the deuteron.

For the deuteron to be suspended freely, these forces must be equal:

$$qE = mg$$

Step 2: Substituting Values

The charge of a deuteron is equal to the charge of a proton:

$$q = 1.6 \times 10^{-19} C$$

The gravitational force is:

$$F_g = (3.2 \times 10^{-27} \text{ kg}) \times (9.8 \text{ m/s}^2)$$

$$F_g = 3.136 \times 10^{-26} \text{ N}$$

Step 3: Solving for Electric Field

$$E = \frac{mg}{q} = \frac{3.136 \times 10^{-26}}{1.6 \times 10^{-19}}$$

$$E = 19.6 \times 10^{-8} \text{ NC}^{-1}$$

Step 4: Conclusion

Thus, the required electric field to suspend the deuteron is 19.6×10^{-8} NC^{-1}.

Quick Tip

To balance a charged particle in an electric field, use $qE = mg$ where q is charge, m is mass, and g is gravitational acceleration.

---

105. Two charges 5 nC and -2 nC are placed at points $(5, 0, 0)$ and $(23, 0, 0)$ in a region of space where there is no other external field. The electrostatic potential energy of this charge system is:

(A) 10×10^{-7} J

(B) 5×10^{-7} J

(C) 15×10^{-7} J

(D) 25×10^{-7} J

Correct Answer: (B) 5×10^{-7} J

Solution:

Step 1: Electrostatic Potential Energy Formula

The electrostatic potential energy U of a system of two point charges is given by:

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

where:

- $q_1 = 5 \times 10^{-9}$ C,

- $q_2 = -2 \times 10^{-9}$ C,

- r is the distance between charges, given as $r = 23 - 5 = 18$ cm $= 0.18$ m,

- $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N·m^2/C^2.

Step 2: Substituting the Values

$$U = \left(9 \times 10^9\right) \times \frac{(5 \times 10^{-9}) \times (-2 \times 10^{-9})}{0.18}$$

$$U = \left(9 \times 10^9\right) \times \frac{-10 \times 10^{-18}}{0.18}$$

$$U = \frac{-90 \times 10^{-9}}{0.18}$$

$$U = -5 \times 10^{-7} \text{ J}$$

Step 3: Conclusion

Since electrostatic potential energy can be negative due to opposite charges, the magnitude of energy is 5×10^{-7} J.

Quick Tip

Electrostatic potential energy of two charges is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$. If charges are of opposite signs, U is negative, indicating an attractive interaction.

---

106. The space between the plates of a parallel plate capacitor is halved and a dielectric medium of relative permittivity 10 is introduced between the plates. The ratio of the final and initial capacitances of the capacitor is:

(A) 20

(B) 10

(C) $\frac{1}{10}$

(D) $\frac{1}{20}$

Correct Answer: (A) 20

Solution:

Step 1: Initial Capacitance Formula

The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

where:

- ε_0 is the permittivity of free space,

- A is the plate area,

- d is the separation between the plates.

Step 2: Effect of Halving the Plate Separation

When the plate separation d is halved:

$$C' = \frac{\varepsilon_0 A}{d/2} = 2C$$

Step 3: Effect of Introducing Dielectric

Introducing a dielectric of relative permittivity $\kappa = 10$ modifies the capacitance:

$$C'' = \kappa C' = 10 \times 2C = 20C$$

Step 4: Final Ratio

The ratio of final to initial capacitance is:

$$\frac{C''}{C} = \frac{20C}{C} = 20$$

Step 5: Conclusion

Thus, the ratio of final and initial capacitance is 20.

> Quick Tip
>
> For a parallel plate capacitor, capacitance increases when the plate separation decreases or a dielectric is introduced. The new capacitance is given by $C' = \kappa \frac{\varepsilon_0 A}{d}$.

---

107. A battery of emf $8V$ and internal resistance 0.5Ω is being charged by a $120V$ DC supply using a series resistor of 15.5Ω. The terminal voltage of the $8V$ battery during charging is:

(A) $11.5V$

(B) $1.15V$

(C) $115V$

(D) $0.5V$

Correct Answer: (A) $11.5V$

Solution:

Step 1: Using Ohm's Law for the Charging Circuit

The total resistance in the circuit is:

$$R_{\text{total}} = R_{\text{series}} + R_{\text{internal}} = 15.5 + 0.5 = 16\Omega$$

The charging current I is given by:

$$I = \frac{V_{\text{supplied}} - V_{\text{battery}}}{R_{\text{total}}}$$

$$I = \frac{120 - 8}{16} = \frac{112}{16} = 7A$$

Step 2: Calculating the Terminal Voltage

The terminal voltage V_{terminal} of the battery during charging is:

$$V_{\text{terminal}} = V_{\text{battery}} + IR_{\text{internal}}$$

$$V_{\text{terminal}} = 8 + (7 \times 0.5) = 8 + 3.5 = 11.5V$$

Step 3: Conclusion

Thus, the terminal voltage of the $8V$ battery during charging is $\mathbf{11.5V}$.

Quick Tip

During charging, the terminal voltage of a battery is higher than its emf due to the voltage drop across the internal resistance.

108. Resistance of a wire is 8Ω. It is drawn in such a way that it experiences a longitudinal strain of 400%. The final resistance of the wire is:

(A) 100Ω

(B) 200Ω

(C) 300Ω

(D) 400Ω

Correct Answer: (B) 200Ω

Solution:

Step 1: Understanding the Relation Between Resistance and Strain

When a wire is stretched, its resistance changes according to the formula:

$$R' = R \times (1 + \text{strain})^2$$

Given that the longitudinal strain is 400% (or 4), the length of the wire increases by a factor of $1 + 4 = 5$. The resistance of a wire is given by:

$$R' = R \times (5^2)$$

Step 2: Substituting the Given Values

$$R' = 8 \times 25 = 200\Omega$$

Step 3: Conclusion

Thus, the final resistance of the wire is **200Ω**.

Quick Tip

When a wire is stretched uniformly, its resistance changes as $R' = R(1 + \text{strain})^2$, where strain is the percentage increase in length.

109. Current flows in a conductor from east to west. The direction of the magnetic field at a point below the conductor is towards:

(A) North

(B) South

(C) East

(D) West

Correct Answer: (B) South

Solution:

Step 1: Understanding the Right-Hand Thumb Rule

The direction of the magnetic field around a current-carrying conductor can be determined using the Right-Hand Thumb Rule. According to this rule: - If the thumb of the right hand points in the direction of current flow (East to West in this case), then the curled fingers represent the direction of the magnetic field.

Step 2: Applying the Rule Below the Conductor

- When current flows from east to west, using the Right-Hand Thumb Rule, the curled fingers point into the plane below the conductor.
- This means the magnetic field direction at a point below the conductor is towards South.

Step 3: Conclusion

Thus, the correct answer is South.

Quick Tip

The Right-Hand Thumb Rule helps determine the direction of the magnetic field: - Point the thumb in the direction of current flow. - The curled fingers indicate the direction of the magnetic field.

---

110. Two infinite length wires carry currents 8 A and 6 A respectively and are placed along X and Y axes respectively. Magnetic field at a point P (0,0,d) will be:

(A) $\frac{7\mu_0}{\pi d}$

(B) $\frac{10\mu_0}{\pi d}$

(C) $\frac{14\mu_0}{\pi d}$

(D) $\frac{5\mu_0}{\pi d}$

Correct Answer: (D) $\frac{5\mu_0}{\pi d}$

Solution:

Step 1: Magnetic Field Due to a Long Current-Carrying Wire

The magnetic field at a perpendicular distance d from an infinitely long straight conductor carrying current I is given by Ampere's Law:

$$B = \frac{\mu_0 I}{2\pi d}$$

where μ_0 is the permeability of free space.

Step 2: Magnetic Fields at Point P Due to Both Wires

- The first wire (along the X-axis) carries current 8 A and contributes a magnetic field:

$$B_1 = \frac{\mu_0(8)}{2\pi d}$$

- The second wire (along the Y-axis) carries current 6 A and contributes a magnetic field:

$$B_2 = \frac{\mu_0(6)}{2\pi d}$$

Step 3: Resultant Magnetic Field

Since the two magnetic fields are perpendicular to each other, the net magnetic field at P is:

$$B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$$

Substituting the values:

$$B_{\text{net}} = \sqrt{\left(\frac{\mu_0(8)}{2\pi d}\right)^2 + \left(\frac{\mu_0(6)}{2\pi d}\right)^2}$$

$$B_{\text{net}} = \frac{\mu_0}{2\pi d}\sqrt{8^2 + 6^2} = \frac{\mu_0}{2\pi d}\sqrt{64 + 36}$$

$$B_{\text{net}} = \frac{\mu_0}{2\pi d} \times \sqrt{100} = \frac{\mu_0}{2\pi d} \times 10$$

$$B_{\text{net}} = \frac{10\mu_0}{2\pi d} = \frac{5\mu_0}{\pi d}$$

Step 4: Conclusion

Thus, the correct answer is $\frac{5\mu_0}{\pi d}$.

Quick Tip

For an infinitely long straight conductor, the magnetic field at a perpendicular distance d is given by $B = \frac{\mu_0 I}{2\pi d}$. When two wires are perpendicular, use the Pythagorean Theorem to find the resultant field.

---

111. A short magnet oscillates with a time period of 0.1 s at a place where the horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire kept at a distance of 20 cm east of the magnet. The new time period of oscillations of the magnet is:

(1) $0.1\,s$

(2) $0.089\,s$

(3) $0.076\,s$

(4) $0.057\,s$

Correct Answer: (3) $0.076\,s$

Solution:

Step 1: Understanding the Magnetic Field Contribution

The original oscillation time period of the magnet is given by:

$$T_0 = 0.1\,s$$

The total effective horizontal magnetic field B_{eff} at the magnet due to the combination of the Earth's horizontal magnetic field B_H and the magnetic field B_I due to the vertical current is given by:

$$B_{\text{eff}} = \sqrt{B_H^2 + B_I^2}$$

where: - $B_H = 24\,\mu T$ - B_I is the field due to the wire, given by:

$$B_I = \frac{\mu_0 I}{2\pi d}$$

where: - $\mu_0 = 4\pi \times 10^{-7}$, - $I = 18$ A, - $d = 20$ cm $= 0.2$ m.

Step 2: Calculating B_I

$$B_I = \frac{4\pi \times 10^{-7} \times 18}{2\pi \times 0.2}$$

$$B_I = \frac{72 \times 10^{-7}}{4 \times 10^{-1}}$$

$$B_I = 18 \times 10^{-6} T = 18\,\mu T$$

Step 3: Finding B_{eff}

$$B_{\text{eff}} = \sqrt{(24)^2 + (18)^2}$$

$$B_{\text{eff}} = \sqrt{576 + 324} = \sqrt{900} = 30\,\mu T$$

Step 4: Finding the New Time Period

Since the time period of oscillation is inversely proportional to the square root of the magnetic field:

$$T' = T_0 \sqrt{\frac{B_H}{B_{\text{eff}}}}$$

$$T' = 0.1 \times \sqrt{\frac{24}{30}}$$

$$T' = 0.1 \times \sqrt{0.8}$$

$$T' = 0.1 \times 0.894$$

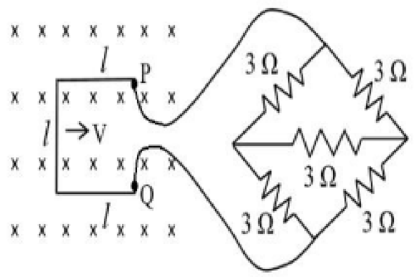
$$T' \approx 0.076\,s$$

Quick Tip

The time period of a magnetic dipole oscillating in a magnetic field is inversely proportional to the square root of the magnetic field. If an external current contributes to the field, use vector addition to find the effective field.

112.

A metallic wire loop of side 0.1 m and resistance of 10Ω is moved with a constant velocity in a uniform magnetic field of 2 Wm^{-2} as shown in the figure. The magnetic field is perpendicular to the plane of the loop. The loop is connected to a network of resistors. The velocity of loop so as to have a steady current of 1 mA in loop is:



(A) 0.67 cm s^{-1}

(B) 2 cm s^{-1}

(C) 3 cm s^{-1}

(D) 4 cm s^{-1}

Correct Answer: (2) 2 cm s^{-1}

Solution:

Step 1: Understanding Electromagnetic Induction

From Faraday's Law of electromagnetic induction, the induced emf ($\mathcal{E}$) in the loop is given by:

$$\mathcal{E} = Blv$$

where: - $B = 2\,\text{Wb/m}^2$ (magnetic field strength), - $l = 0.1\,\text{m}$ (side length of the loop), - v (velocity of the loop, to be determined).

Step 2: Applying Ohm's Law

The induced current I in the loop is given by:

$$I = \frac{\mathcal{E}}{R}$$

where: - $R = 10\,\Omega$ (resistance of the loop), - $I = 1$ A (steady current in the loop).

Step 3: Solving for v

Substituting the values:

$$1 = \frac{(2 \times 0.1 \times v)}{10}$$

$$1 = \frac{0.2v}{10}$$

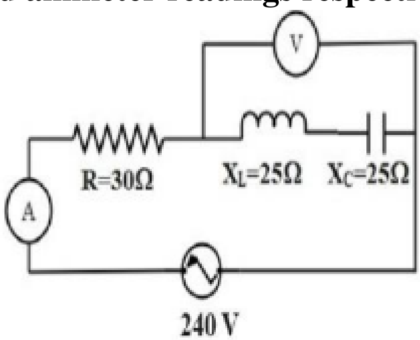
$$v = \frac{10}{0.2} = 2 \text{ m/s} = 2 \text{ cm/s}$$

Quick Tip

In electromagnetic induction problems, use Faraday's Law to determine the induced emf and then apply Ohm's Law to find the required motion parameters. Ensure all units are consistent before calculations.

---

113. In the circuit shown in the figure, neglecting the source resistance, the voltmeter and ammeter readings respectively are:



(1) $0V, 8A$

(2) $150V, 3A$

(3) $150V, 6A$

(4) $0V, 3A$

Correct Answer: (1) $0V, 8A$

Solution:

Step 1: Determine total impedance in the circuit

The given circuit consists of a resistor $R = 30\Omega$ in series with a parallel LC circuit, where the inductor reactance is $X_L = 25\Omega$ and the capacitor reactance is $X_C = 25\Omega$.

Since $X_L = X_C$, the net reactance of the LC branch is:

$$X_{\text{net}} = X_L - X_C = 25\Omega - 25\Omega = 0\Omega.$$

Thus, the LC branch acts as a short circuit, meaning the impedance of the parallel combination is zero.

Step 2: Calculate total circuit current

The only effective impedance in the circuit is the resistor $R = 30\Omega$, so the total impedance Z is:

$$Z = R = 30\Omega.$$

Using Ohm's law, the current through the circuit is:

$$I = \frac{V}{Z} = \frac{240V}{30\Omega} = 8A.$$

Step 3: Determine the voltmeter reading

Since the LC branch is a short circuit, the voltage across the parallel LC circuit (voltmeter reading) is:

$$V_{\text{meter}} = 0V.$$

Final Answer: The ammeter reads $8A$ and the voltmeter reads $0V$, which matches option (1).

Quick Tip

For parallel LC circuits with $X_L = X_C$, the circuit behaves as a short circuit, reducing the impedance of the LC branch to zero.

114. The radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is:

(1) $\frac{E}{c}$

(2) $\frac{2E}{c}$

(3) $\frac{E}{c^2}$

(4) $\frac{2E}{c^2}$

Correct Answer: (2) $\frac{2E}{c}$

Solution:

Step 1: Understanding the radiation momentum transfer

The momentum p of a photon is given by the relation:

$$p = \frac{E}{c}$$

where E is the energy of the radiation and c is the speed of light.

Step 2: Effect of a perfectly reflecting surface

When radiation falls normally on a perfectly reflecting surface, the total momentum transfer is due to the change in momentum. Since the surface reflects the radiation completely, the momentum change is:

$$\Delta p = p_{\text{final}} - p_{\text{initial}}$$

For a perfectly reflecting surface, the photon momentum reverses direction, meaning the change in momentum is:

$$\Delta p = \frac{E}{c} - (-\frac{E}{c}) = \frac{E}{c} + \frac{E}{c} = \frac{2E}{c}$$

Thus, the total momentum transferred to the surface is:

$$\frac{2E}{c}$$

Final Answer: The correct option is $\frac{2E}{c}$, which corresponds to option (2).

Quick Tip

For a perfectly reflecting surface, the momentum change is doubled because the radiation undergoes a complete reversal in direction.

---

115. Light of wavelength 4000Å is incident on a sodium surface for which the threshold wavelength of photoelectrons is 5420Å. The work function of sodium is:

(1) 4.58 eV

(2) 2.29 eV

(3) 1.14 eV

(4) 0.57 eV

Correct Answer: (2) 2.29 eV

Solution:

Step 1: Understanding the Photoelectric Equation

The energy of incident photons is given by the equation:

$$E = \frac{hc}{\lambda}$$

where:

- $h = 6.626 \times 10^{-34}$ Js (Planck's constant),

- $c = 3.0 \times 10^{8}$ m/s (speed of light),

- λ is the wavelength of the incident light.

The work function ϕ (minimum energy required to eject electrons) is given by:

$$\phi = \frac{hc}{\lambda_0}$$

where $\lambda_0 = 5420\text{Å} = 5420 \times 10^{-10}$ m is the threshold wavelength.

Step 2: Calculating the Work Function

Substituting the known values:

$$\phi = \frac{(6.626 \times 10^{-34})(3.0 \times 10^{8})}{5420 \times 10^{-10}}$$

$$= \frac{1.9878 \times 10^{-25}}{5.42 \times 10^{-7}}$$

$$= 3.67 \times 10^{-19} \text{ J}$$

Converting to electron volts:

$$\phi = \frac{3.67 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.29 \text{ eV}.$$

Final Answer: The work function of sodium is 2.29 eV, which matches option (2).

Quick Tip

The work function of a material is the minimum energy required to eject an electron and is calculated using the threshold wavelength.

---

116. The principal quantum number n corresponding to the excited state of He^+ ion, if on transition to the ground state two photons in succession with wavelengths 1026Å and 304Å are emitted:

(R = 1.097 $\times 10^7$ m^{-1})

(1) 2

(2) 3

(3) 6

(4) 4

Correct Answer: (3) 6

Solution:

Step 1: Using the Rydberg Formula

For hydrogen-like species, the Rydberg formula for the wavelength of emitted photons is:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where:

- $R = 1.097 \times 10^7$ m^{-1} (Rydberg constant),
- $Z = 2$ for He^+,
- n_1 and n_2 are the quantum numbers of the final and initial states.

Step 2: Determining the Energy Levels

For the first transition, the given wavelength is $\lambda_1 = 1026\text{Å} = 1026 \times 10^{-10}$ m, so:

$$\frac{1}{1026 \times 10^{-10}} = (1.097 \times 10^7) \times 4 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Solving for n_1 and n_2, this corresponds to the transition from $n = 6$ to $n = 3$.

For the second transition with $\lambda_2 = 304\text{Å}$:

$$\frac{1}{304 \times 10^{-10}} = (1.097 \times 10^7) \times 4 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

This corresponds to the transition from $n = 3$ to $n = 1$.

Thus, the electron was initially in the $n = 6$ state before transitioning to the ground state.

Final Answer: The principal quantum number n for the excited state of He^+ is 6, which matches option (3).

Quick Tip

For hydrogen-like atoms, the Rydberg formula helps determine transition wavelengths. The energy difference between levels is inversely proportional to the square of the principal quantum number.

---

117. Which physical quantity is measured in barn?

(1) Radius of the nuclei

(2) Pressure in a liquid drop

(3) Scattering cross-section

(4) Rate of flow of liquid

Correct Answer: (3) Scattering cross-section

Solution:

Step 1: Understanding the unit ”barn”

The barn (b) is a unit of area commonly used in nuclear physics to express the cross-sectional area of nuclear reactions. It is defined as:

$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

Step 2: Application in Scattering Cross-Section

- The scattering cross-section represents the effective target area that a nucleus presents to an incoming particle in a nuclear reaction.

- It is crucial in understanding interactions such as neutron scattering and nuclear fission.

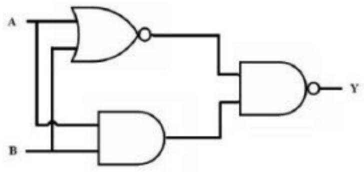
- Since the barn is used specifically in nuclear and particle physics, the correct physical quantity measured in barn is scattering cross-section.

Final Answer: The correct option is Scattering cross-section (Option 3).

Quick Tip

The unit ”barn” is widely used in nuclear and particle physics to describe cross-sections in scattering and nuclear interactions.

118. Truth table for the given circuit is:



(1)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Correct Answer: (3)

Solution:

Step 1: Identify Logic Gates in the Circuit

The circuit consists of:
- An OR gate taking inputs A and B.
- An AND gate taking inputs A and B.

- The outputs of these gates are passed through another OR gate.

Step 2: Derive Boolean Expression

The intermediate outputs are:

$$X = A + B \quad \text{(OR gate)}$$

$$Z = A \cdot B \quad \text{(AND gate)}$$

The final output Y is obtained as:

$$Y = X + Z = (A + B) + (A \cdot B)$$

Using Boolean algebra:

$$Y = A + B$$

Step 3: Construct Truth Table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Final Answer: The correct truth table matches option (3).

Quick Tip

For logic circuits, first identify the logic gates, derive Boolean expressions, and verify results using truth tables.

119. If R_C and R_B are respectively the resistances of in collector and base sides of the circuit, and β is the current amplification factor, then the voltage gain of a transistor amplifier in common emitter configuration is:

(1) $\beta R_C R_B$

(2) $\frac{\beta}{R_C R_B}$

(3) $\frac{\beta R_B}{R_C}$

(4) $\frac{\beta R_C}{R_B}$

Correct Answer: (4) $\frac{\beta R_C}{R_B}$

Solution:

Step 1: Understanding the Voltage Gain Formula

For a transistor in a common emitter (CE) configuration, the voltage gain (A_V) is given by:

$$A_V = \frac{\text{Output voltage}}{\text{Input voltage}} = \frac{V_{\text{out}}}{V_{\text{in}}}$$

The voltage gain in CE configuration is expressed as:

$$A_V = \frac{\beta R_C}{R_B}$$

where:

- R_C is the resistance in the collector circuit,
- R_B is the resistance in the base circuit,
- β is the current gain of the transistor.

Step 2: Verifying the Correct Option

From the formula:

$$A_V = \frac{\beta R_C}{R_B}$$

Comparing with the given options, the correct answer is Option (4): $\frac{\beta R_C}{R_B}$.

Final Answer: The voltage gain of a transistor amplifier in CE configuration is $\frac{\beta R_C}{R_B}$, which matches option (4).

Quick Tip

The voltage gain in a common emitter transistor amplifier is given by $A_V = \frac{\beta R_C}{R_B}$. This relation is useful in amplifier circuit analysis.

---

120. Which one of the following is not classified as pulse modulation?

(1) Pulse duration modulation
(2) Pulse Amplitude Modulation
(3) Pulse band Modulation
(4) Pulse position Modulation

Correct Answer: (3) Pulse band Modulation

Solution:

Step 1: Understanding Pulse Modulation

Pulse modulation refers to a technique where the message signal is encoded in the form of pulses. The major types of pulse modulation are:

- Pulse Amplitude Modulation (PAM): The amplitude of the pulses is varied according to the message signal.

- Pulse Duration Modulation (PDM) or Pulse Width Modulation (PWM): The width of each pulse is varied according to the message signal.

- Pulse Position Modulation (PPM): The position of each pulse is varied according to the message signal.

Step 2: Identifying the Incorrect Option

- The given options (1), (2), and (4) are recognized types of pulse modulation.

- Pulse Band Modulation (PBM) is not a recognized type of pulse modulation. Modulation techniques primarily fall into analog modulation (AM, FM, PM) or pulse modulation (PAM, PWM, PPM, PCM).

Thus, option (3) - ”Pulse band Modulation” is incorrect.

Final Answer: The correct choice is Option (3): Pulse band Modulation, as it is not a classified pulse modulation technique.

Quick Tip

Pulse modulation techniques mainly include Pulse Amplitude Modulation (PAM), Pulse Width Modulation (PWM), and Pulse Position Modulation (PPM). ”Pulse Band Modulation” is not a standard classification.

---

3 Chemistry

121. The de Broglie wavelength of an electron with kinetic energy of 2.5 eV is (in m):

$$(1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}, \quad m_e = 9 \times 10^{-31} \text{ kg})$$

(1) $\frac{h \times 10^{-25}}{\sqrt{72}}$
(2) $\frac{h \times 10^{25}}{\sqrt{72}}$
(3) $\frac{\sqrt{72}}{h \times 10^{-25}}$

(4) $\frac{\sqrt{72}}{h\times10^{25}}$

Correct Answer: (2) $\frac{h\times10^{25}}{\sqrt{72}}$

Solution:

Step 1: Applying the de Broglie Wavelength Formula

The de Broglie wavelength is given by:

$$\lambda = \frac{h}{p}$$

where p is the momentum of the electron. The kinetic energy K is related to momentum by:

$$p = \sqrt{2m_eK}$$

Substituting this into the de Broglie equation:

$$\lambda = \frac{h}{\sqrt{2m_eK}}$$

Step 2: Substituting Given Values

Given that:

$$K = 2.5 \text{ eV} = 2.5 \times 1.6 \times 10^{-19} \text{ J} = 4.0 \times 10^{-19} \text{ J}$$

$$m_e = 9 \times 10^{-31} \text{ kg}$$

$$h = 6.626 \times 10^{-34} \text{ J·s}$$

We substitute these values:

$$\lambda = \frac{h}{\sqrt{2(9 \times 10^{-31})(4.0 \times 10^{-19})}}$$

$$= \frac{h}{\sqrt{72} \times 10^{-25}}$$

Rewriting:

$$\lambda = \frac{h \times 10^{25}}{\sqrt{72}}$$

Final Answer: The correct choice is Option (2): $\frac{h \times 10^{25}}{\sqrt{72}}$.

Quick Tip

The de Broglie wavelength is inversely proportional to momentum. For non-relativistic electrons, use $\lambda = \frac{h}{\sqrt{2m_e K}}$.

---

122. The ratio of ground state energy of Li^{2+}, He^{+}, H is:

(1) $3 : 2 : 1$

(2) $1 : 2 : 3$

(3) $9 : 4 : 1$

(4) $1 : 4 : 9$

Correct Answer: (3) $9 : 4 : 1$

Solution:

Step 1: Understanding Ground State Energy Formula

The ground state energy (E_n) of a hydrogen-like ion is given by the formula:

$$E_n = -\frac{13.6Z^2}{n^2} \text{ eV}$$

where: - Z is the atomic number, - n is the principal quantum number (for ground state, $n = 1$).

Step 2: Calculating for Each Ion

- For Hydrogen (H), $Z = 1$:

$$E_H = -13.6 \times \frac{1^2}{1^2} = -13.6 \text{ eV}$$

- For Helium ion (He^+), $Z = 2$:

$$E_{He^+} = -13.6 \times \frac{2^2}{1^2} = -54.4 \text{ eV}$$

- For Lithium ion (Li^{2+}), $Z = 3$:

$$E_{Li^{2+}} = -13.6 \times \frac{3^2}{1^2} = -122.4 \text{ eV}$$

Step 3: Finding the Ratio

$$E_{Li^{2+}} : E_{He^+} : E_H = 9 : 4 : 1$$

Quick Tip

The ground state energy of a hydrogen-like ion is proportional to Z^2. To determine ratios, simply square the atomic number of the respective species.

---

123. Two statements are given below:

Statement I: Nitrogen has more ionization enthalpy and electronegativity than beryllium.

Statement II:CrO_3, B_2O_3 **are acidic oxides.**

Correct answer is:

(1) Both statements I and II are correct

(2) Both statements I and II are not correct

(3) Statement I is correct, but statement II is not correct

(4) Statement I is not correct, but statement II is correct

Correct Answer: (1) Both statements I and II are correct

Solution:

Step 1: Verifying Statement I

- Ionization enthalpy is the energy required to remove an electron from an atom.

- Electronegativity is the tendency of an atom to attract electrons in a bond.
- In the periodic table, ionization enthalpy and electronegativity generally increase from left to right.
- Nitrogen (group 15) is to the right of Beryllium (group 2), meaning Nitrogen has a higher ionization enthalpy and electronegativity than Beryllium.

Conclusion: Statement I is correct.

Step 2: Verifying Statement II

- CrO_3 (Chromium trioxide) is an acidic oxide. It reacts with water to form chromic acid (H_2CrO_4).
- B_2O_3 (Boron trioxide) is also an acidic oxide, forming boric acid (H_3BO_3) when dissolved in water.
- Acidic oxides typically react with bases to form salts.

Conclusion: Statement II is also correct.

Final Answer: Since both statements are correct, the correct option is (1) Both statements I and II are correct.

> Quick Tip
>
> Elements in the right side of the periodic table generally have higher ionization enthalpies and electronegativity. Transition metal oxides and non-metal oxides tend to be acidic.

---

124. The number of lone pairs of electrons on the central atom of BrF_5, XeO_3, SO_3 respectively are:

(1) $1, 1, 2$

(2) $1, 2, 2$

(3) $2, 2, 1$

(4) $1, 1, 1$

Correct Answer: (4) $1, 1, 1$

Solution:

Step 1: Finding Lone Pairs on the Central Atom

Lone pairs on the central atom can be determined using the VSEPR (Valence Shell Electron

Pair Repulsion) theory.

1. Bromine Pentafluoride (BrF_5)

- Bromine (Br) has 7 valence electrons.

- It forms 5 single bonds with fluorine atoms.

- Total valence electrons used in bonding: $5 \times 2 = 10$.

- Remaining electrons: $7 - 5 = 2$ (1 lone pair).

- Molecular geometry: Square pyramidal.

2. Xenon Trioxide (XeO_3)

- Xenon (Xe) has 8 valence electrons.

- It forms 3 double bonds with oxygen atoms.

- Total valence electrons used in bonding: $3 \times 2 = 6$.

- Remaining electrons: $8 - 6 = 2$ (1 lone pair).

- Molecular geometry: Trigonal pyramidal.

3. Sulfur Trioxide (SO_3)

- Sulfur (S) has 6 valence electrons.

- It forms 3 double bonds with oxygen atoms.

- Total valence electrons used in bonding: $3 \times 2 = 6$.

- Remaining electrons: $6 - 6 = 0$ (No lone pairs).

- Molecular geometry: Trigonal planar.

Step 2: Conclusion

The number of lone pairs on the central atom for each molecule:

$$BrF_5 = 1, \quad XeO_3 = 1, \quad SO_3 = 1$$

Thus, the correct answer is $1, 1, 1$, which matches option (4).

Quick Tip

To determine lone pairs, use the formula: Lone pairs = $\frac{\text{Total valence electrons} - \text{Electrons used in bonding}}{2}$.

---

125. The shape of the colourless neutral gas formed on thermal decomposition of

ammonium nitrate is:

(1) Angular

(2) Linear

(3) Trigonal planar

(4) Trigonal pyramidal

Correct Answer: (2) Linear

Solution:

Step 1: Understanding the Thermal Decomposition of Ammonium Nitrate

The thermal decomposition reaction of ammonium nitrate (NH_4NO_3) is:

$$NH_4NO_3 \rightarrow N_2O + 2H_2O$$

The colourless neutral gas formed in this reaction is Nitrous Oxide (N_2O).

Step 2: Determining the Molecular Shape of N_2O

- The molecular structure of N_2O (Nitrous Oxide) is:

$$\mathrm{N \equiv N^+ - O^-}$$

- The molecule consists of sp hybridized nitrogen atoms.

- It follows the linear geometry due to no lone pairs affecting the molecular shape.

Step 3: Conclusion

Since N_2O has a linear molecular shape, the correct answer is:

$$\textbf{Linear}$$

which matches Option (2).

Quick Tip

Nitrous oxide (N_2O) is a linear molecule similar to carbon dioxide (CO_2) due to sp hybridization.

---

126. At $T(K)$ for one mole of an ideal gas, the graph of P (on y-axis) and V^{-1} (on x-axis) gave a straight line with slope of 32.8 L atm mol^{-1}. What is the temperature (in K)?

(R = 0.0821 L atm mol^{-1} K^{-1})

(1) 600

(2) 200

(3) 800

(4) 400

Correct Answer: (4) 400

Solution:

Step 1: Understanding the Graph Relationship

For an ideal gas, the equation is:

$$PV = nRT$$

Rearranging in the form of a straight line equation:

$$P = \frac{nRT}{V}$$

which can be rewritten as:

$$P = (nRT) \times V^{-1}$$

Comparing with the equation of a straight line $y = mx + c$, we see that the slope of the graph is equal to nRT.

Step 2: Calculating Temperature

Given that:

- Slope $= 32.8$ L atm mol^{-1}

- $n = 1$ (one mole of gas)

- $R = 0.0821$ L atm mol^{-1} K^{-1}

Using:

$$\text{Slope} = nRT$$

$$32.8 = (1) \times (0.0821) \times T$$

Solving for T:

$$T = \frac{32.8}{0.0821}$$

$$T = 400 \text{ K}$$

Final Answer: The correct temperature is 400 K, which matches Option (4).

> **Quick Tip**
>
> For an ideal gas, when plotting P versus V^{-1}, the slope gives nRT. This can be used to determine temperature if the slope and R are known.

---

127. At 290 K, a vessel (I) contains equal moles of three liquids (A, B, C). The boiling points of A, B, and C are 348 K, 378 K, and 368 K respectively. Vessel (I) is heated to 300 K and vapors were collected into vessel (II). Identify the correct statements. (Assume vessel (I) contains liquids and vapors and vessel (II) contains only vapors.)

Statements:

I.Vessel – I is rich in liquid

II.Vessel – II is rich in vapors of C.

III. The vapor pressures of A, B, and C in Vessel (I) at 300 K follows the order $C > A > B$.

(1) I, III

(2) I, II only

(3) II, III only

(4) I, II, III

Correct Answer: (1) I, III

Solution:

Step 1: Understanding the Boiling Points The boiling points of the substances determine their volatility. Since C has the lowest boiling point (308 K), it is more volatile and will have a higher proportion in the vapour phase. B has the highest boiling point (373 K) and will remain mostly in the liquid phase.

Step 2: Distribution of Liquids and Vapours - Vessel (I) retains more of the less volatile component B in the liquid state, making it rich in liquid B.

- Vessel (II) collects more of the most volatile component C, making it rich in vapour of C.

Step 3: Comparing Vapour Pressures Since vapour pressure is inversely related to boiling point, the order of vapour pressures at 290 K will be: C ¿ A ¿ B.

Thus, all three statements (I, II, III) are correct.

Quick Tip

The component with the lowest boiling point will always contribute more to the vapour phase, while the component with the highest boiling point will remain mostly in the liquid phase.

---

128. 100 mL of 0.1 M Fe^{2+} solution was titrated with $\frac{1}{60}$ M $Cr_2O_7^{2-}$ solution in acid medium. What is the volume (in L) of $Cr_2O_7^{2-}$ solution consumed?

(1) 100

(2) 10

(3) 1

(4) 0.1

Correct Answer: (4) 0.1

Solution:

Step 1: Understanding the Redox Reaction

The balanced redox reaction in acidic medium:

$$Cr_2O_7^{2-} + 6Fe^{2+} + 14H^+ \rightarrow 2Cr^{3+} + 6Fe^{3+} + 7H_2O$$

From the balanced equation:

- 1 mole of $Cr_2O_7^{2-}$ reacts with 6 moles of Fe^{2+}.

Step 2: Finding Moles of Fe^{2+}

Given:

$$\text{Molarity}(M) = 0.1, \quad \text{Volume}(V) = 100 \text{ mL} = 0.1 \text{ L}$$

Moles of Fe^{2+}:

$$\text{Moles of } Fe^{2+} = M \times V = 0.1 \times 0.1 = 0.01 \text{ moles}$$

Step 3: Finding Moles of $Cr_2O_7^{2-}$

From the reaction:

$$\text{Moles of } Cr_2O_7^{2-} = \frac{\text{Moles of } Fe^{2+}}{6} = \frac{0.01}{6} = 1.67 \times 10^{-3} \text{ moles}$$

Step 4: Finding Volume of $Cr_2O_7^{2-}$ Solution

Given that the molarity of $Cr_2O_7^{2-}$ solution is:

$$M = \frac{1}{60} = 0.0167 \text{ M}$$

Using the molarity formula:

$$V = \frac{\text{Moles of } Cr_2O_7^{2-}}{\text{Molarity}}$$

$$V = \frac{1.67 \times 10^{-3}}{0.0167} = 0.1 \text{ L}$$

Final Answer: The correct volume is 0.1 L, which matches Option (4).

Quick Tip

In redox titrations, use the balanced reaction to determine mole ratios. Apply the molarity equation $M_1V_1 = M_2V_2$ for volume calculations.

---

129. Observe the following reaction:

$$ABO_3(s) \xrightarrow{1000K} AO(s) + BO_2(g)$$

The enthalpy change ΔH for this reaction is x kJ mol^{-1}. What is its ΔU (in kJ mol^{-1}) at the same temperature?

(R = 8.3 J mol^{-1} K^{-1})

(1) $x - 8300$

(2) $x + 8.3$

(3) $x + 8300$

(4) $x - 8.3$

Correct Answer: (4) $x - 8.3$

Solution:

Step 1: Understanding the Relationship Between ΔH and ΔU

The relationship between enthalpy change (ΔH) and internal energy change (ΔU) for a gaseous reaction is given by:

$$\Delta H = \Delta U + \Delta n_g RT$$

where:

- ΔH = enthalpy change
- ΔU = internal energy change
- Δn_g = change in the number of moles of gas
- $R = 8.3$ J mol^{-1} K^{-1}
- $T = 1000$ K

Step 2: Calculating Δn_g

The given reaction:

$$ABO_3(s) \xrightarrow{1000K} AO(s) + BO_2(g)$$

- Solids do not contribute to Δn_g.
- Only gaseous species are considered.
- BO_2 is the only gaseous product.
- There are no gaseous reactants.

Thus:

$$\Delta n_g = 1 - 0 = 1$$

Step 3: Substituting Values

$$\Delta H = \Delta U + (1 \times 8.3 \times 1000)$$

$$\Delta H = \Delta U + 8.3 \text{ kJ mol}^{-1}$$

$$\Delta U = \Delta H - 8.3$$

Final Answer: The correct relation is:

$$\Delta U = x - 8.3$$

which matches Option (4): $x - 8.3$.

Quick Tip

For reactions involving gases, use $\Delta H = \Delta U + \Delta n_g RT$ to account for work done by gas expansion or compression.

---

130. A vessel of volume V L contains an ideal gas at $T(K)$. The vessel is partitioned into two equal parts. The volume (in L) and temperature (in K) in each part is respectively:

(1) $V, \frac{T}{2}$

(2) $\frac{V}{2}, T$

(3) V, T

(4) $\frac{V}{2}, \frac{T}{2}$

Correct Answer: (2) $\frac{V}{2}, T$

Solution:

Step 1: Understanding the Partitioning of the Vessel

We are given that a vessel of volume V L contains an ideal gas at temperature T K. The vessel is then partitioned into two equal parts.

Step 1: Understanding the effect of partitioning

When a vessel is partitioned into two equal parts, the volume of each part will be:

$$V' = \frac{V}{2}.$$

Since no heat exchange occurs between the two parts after partitioning, the temperature remains the same. That is,

$$T' = T.$$

Step 2: Verifying the correct answer

From the above calculations, the new volume in each partition is $\frac{V}{2}$ and the temperature remains T. Hence, the correct answer is:

$$\frac{V}{2}, T.$$

Quick Tip

For an ideal gas in a rigid container, partitioning the volume does not affect the temperature if no external heat transfer or work is involved.

---

131. At 300 K, $\Delta_r G^\circ$ for the reaction $A(g) \rightleftharpoons B(g)$ is -11.5 kJ mol^{-1}. The equilibrium constant at 300 K is approximately:

(R = 8.314 J mol^{-1} K^{-1})

(1) 10

(2) 100

(3) 1000

(4) 25

Correct Answer: (2) 100

Solution:

Step 1: Using the Gibbs Free Energy and Equilibrium Constant Relation

The standard Gibbs free energy change $\Delta_r G^\circ$ is related to the equilibrium constant K by:

$$\Delta_r G^\circ = -RT \ln K$$

where:

- $R = 8.314$ J mol^{-1} K^{-1} = 8.314×10^{-3} kJ mol^{-1} K^{-1},

- $T = 300$ K,

- $\Delta_r G^\circ = -11.5$ kJ mol^{-1}.

Step 2: Rearranging for K

$$\ln K = \frac{-\Delta_r G^\circ}{RT}$$

Substituting values:

$$\ln K = \frac{-(-11.5)}{(8.314 \times 10^{-3}) \times 300}$$

$$\ln K = \frac{11.5}{2.4942}$$

$$\ln K \approx 4.61$$

Step 3: Finding K

Taking the exponent:

$$K = e^{4.61}$$

Approximating:

$$e^{4.61} \approx 100$$

Final Answer: The equilibrium constant is approximately 100, which matches Option (2).

Quick Tip

Use the relation $\Delta G^\circ = -RT \ln K$ to calculate equilibrium constants. If ΔG° is negative, K is greater than 1, favoring product formation.

---

132. 100 mL of 0.1 M HA (weak acid) and 100 mL of 0.2 M NaA are mixed. What is the pH of the resultant solution?

(K_a of HA is $10^{-5}, \log 2 = 0.3$)

(1) 4.7

(2) 5.0

(3) 5.3

(4) 4.0

Correct Answer: (3) 5.3

Solution:

Step 1: Identify the Buffer System

- The given solution consists of a weak acid (HA) and its salt (NaA), forming a buffer solution.
- The pH of a buffer solution is given by the Henderson-Hasselbalch equation:

$$\text{pH} = \text{pKa} + \log\left(\frac{[\text{Salt}]}{[\text{Acid}]}\right)$$

Step 2: Calculate pKa

- Given $K_a = 10^{-5}$, we calculate:

$$\text{pKa} = -\log(10^{-5}) = 5$$

Step 3: Calculate the Concentrations

- Moles of HA (Acid):

$$\text{Moles} = M \times V = (0.1 \times 0.1) = 0.01$$

- Moles of NaA (Salt):

$$\text{Moles} = M \times V = (0.2 \times 0.1) = 0.02$$

- Total Volume after Mixing:

$$V_{\text{total}} = 100 + 100 = 200 \text{ mL} = 0.2 \text{ L}$$

- Final Concentrations:

$$[\text{HA}] = \frac{0.01}{0.2} = 0.05 \quad , \quad [\text{NaA}] = \frac{0.02}{0.2} = 0.1$$

Step 4: Apply the Henderson-Hasselbalch Equation

$$\text{pH} = 5 + \log\left(\frac{0.1}{0.05}\right)$$

$$= 5 + \log 2$$

$$= 5 + 0.3$$

$$= 5.3$$

Final Answer: The pH of the buffer solution is 5.3, which matches Option (3).

Quick Tip

For buffer solutions, use the Henderson-Hasselbalch equation:

$$\text{pH} = \text{pKa} + \log\left(\frac{[\text{Salt}]}{[\text{Acid}]}\right)$$

If the salt concentration is twice that of the acid, pH increases by $\log 2 \approx 0.3$.

---

133. Identify the correct statements from the following:

i. Reaction of hydrogen with fluorine occurs even in dark.

ii. Manufacture of ammonia by Haber process is an endothermic reaction.

iii. HF is an electron-rich hydride.

(1) i, iii only

(2) ii, iii

(3) ii only

(4) i, ii only

Correct Answer: (1) i, iii only

Solution:

Step 1: Analyzing Statement (i)

- The reaction of hydrogen with fluorine is highly exothermic and occurs spontaneously even in the dark due to fluorine's high reactivity.

- The reaction is:

$$H_2 + F_2 \rightarrow 2HF$$

- Since fluorine is the most electronegative element, it reacts readily without requiring external activation energy.

Conclusion: Statement (i) is correct.

Step 2: Analyzing Statement (ii)

- The Haber process for ammonia synthesis:

$$N_2 + 3H_2 \rightleftharpoons 2NH_3 + \Delta H$$

has an exothermic enthalpy change ($\Delta H < 0$). - Since ammonia formation releases heat, it is not endothermic.

Conclusion: Statement (ii) is incorrect.

Step 3: Analyzing Statement (iii)

- HF (hydrofluoric acid) is an electron-rich hydride because:

- It has three lone pairs on fluorine.

- It forms strong hydrogen bonds due to high electronegativity.

Conclusion: Statement (iii) is correct.

Step 4: Final Answer

Since statements (i) and (iii) are correct, the correct answer is:

Option (1): i, iii only

> **Quick Tip**
>
> Fluorine is highly reactive and can react in the dark. The Haber process is exothermic, and HF is an electron-rich hydride due to its lone pairs.

---

134. Which one of the following alkali metals is the weakest reducing agent as per their E° values?

(1) K

(2) Cs

(3) Li

(4) Na

Correct Answer: (4) Na

Solution:

Step 1: Understanding Reducing Power and E° Values

- The standard reduction potential (E°) of alkali metals determines their ability to lose electrons and act as reducing agents.

- A more negative $E°$ value indicates a stronger reducing agent.
- The standard reduction potentials ($E°$) of alkali metals are:

$$Li = -3.04V, \quad Na = -2.71V, \quad K = -2.93V, \quad Cs = -2.92V$$

Step 2: Identifying the Weakest Reducing Agent

- The reducing power of alkali metals increases down the group due to decreasing ionization energy.
- Lithium (Li) has the most negative $E°$ value, making it the strongest reducing agent.
- Sodium (Na) has the least negative $E°$ value among the given options, meaning it is the weakest reducing agent.

Conclusion: Sodium (Na) is the weakest reducing agent as per its $E°$ value.

Thus, the correct answer is Option (4): Na.

Quick Tip

A more negative standard reduction potential ($E°$) means a stronger reducing agent. Among alkali metals, lithium is the strongest and sodium is the weakest reducing agent.

---

135. In which of the following reactions, hydrogen is one of the products?

i. $NaBH_4 + I_2 \longrightarrow$

ii. $BF_3 + NaH \xrightarrow{450\,K}$

iii. $BF_3 + LiAlH_4 \longrightarrow$

iv. $B_2H_6 + NH_3 \xrightarrow{heat}$

Correct Answer: (3) iii

$BF_3 + LiAlH_4 \longrightarrow$

Solution:

We need to determine in which of the given reactions hydrogen (H_2) is produced as a product.

Step 1: Analyze each reaction

1. Reaction i: $\text{NaBH}_4 + I_2$ Sodium borohydride reacts with iodine to release molecular hydrogen (H_2) as one of the products:

$$\text{NaBH}_4 + 2I_2 \rightarrow \text{NaI} + \text{BI}_3 + H_2$$

Thus, hydrogen is produced in this reaction.

2. Reaction ii: $\text{BF}_3 + \text{NaH}$ This reaction does not produce hydrogen. Instead, it forms sodium tetrafluoroborate.

3. Reaction iii: $\text{BF}_3 + \text{LiAlH}_4$ This reaction is a reduction process and does not yield hydrogen as a product.

4. Reaction iv: $\text{B}_2\text{H}_6 + \text{NH}_3$ Diborane reacts with ammonia to form boron nitride and molecular hydrogen:

$$\text{B}_2\text{H}_6 + 2\text{NH}_3 \rightarrow 2\text{BN} + 3H_2$$

Thus, hydrogen is produced in this reaction.

Step 2: Identifying the correct answer

From the above analysis, hydrogen is produced in reactions (i) and (iv). Thus, the correct answer is:

(C) i, iv

> **Quick Tip**
>
> Whenever analyzing chemical reactions for gas evolution, check for common gas-producing reactants like metal hydrides, acids, or decomposition reactions.

---

136. Two statements are given below:

Statement I: SnF_4, PbF_4 are ionic in nature.

Statement II: GeCl_2 is more stable than GeCl_4.

The correct answer is:

1. Both statements I & II are correct.
2. Both statements I & II are not correct.
3. Statement I is correct, but statement II is not correct.

4. Statement I is not correct, but statement II is correct.

Correct Answer: (3) Statement I is correct, but statement II is not correct.

Solution:

Statement I:

SnF_4 (tin tetrafluoride) and PbF_4 (lead tetrafluoride) are indeed ionic compounds. This is because they are formed by the combination of metal cations (Sn^{4+} and Pb^{4+}) with fluoride anions (F^-) which result in the formation of ionic bonds. Hence, Statement I is correct.

Statement II:

$GeCl_2$ is not more stable than $GeCl_4$. In fact, $GeCl_2$ is less stable than $GeCl_4$. The stability of $GeCl_4$ is greater due to the full d-orbital participation and the more favorable bond formation compared to $GeCl_2$, which is more prone to hydrolysis and less stable. Hence, Statement II is incorrect.

Thus, the correct answer is **(3)**: Statement I is correct, but statement II is not correct.

Quick Tip

When dealing with ionic compounds, check the charge on the metal ions and the electronegativity of the non-metals. For stability comparisons, larger oxidation states are often more stable for heavier elements, but smaller oxidation states tend to be more stable for lighter elements.

---

137. Match the pollutant in List I with its maximum permissible limit in drinking water given in List II.

List I		List II	
A	Lead	I	500 ppm
B	Sulphate	II	50 ppm
C	Nitrate	III	50 ppb

1. A-I, B-II, C-I

2. A-II, B-I, C-III

3. A-III, B-I, C-II

4. A-III, B-II, C-I

Correct Answer: (3) A-III, B-I, C-II

Solution:

List I: Pollutants in drinking water and their maximum permissible limits:

- Lead (A): The maximum permissible limit of Lead in drinking water is 50 ppb (part per billion). So, A matches with III.
- Sulphate (B): The maximum permissible limit of Sulphate in drinking water is 500 ppm (parts per million). So, B matches with I.
- Nitrate (C): The maximum permissible limit of Nitrate in drinking water is 50 ppm. Hence, C matches with II.

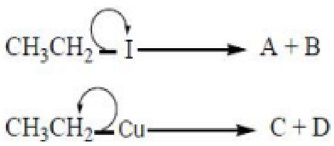
Thus, the correct match is A-III, B-I, C-II.

Quick Tip

When matching pollutants with their permissible limits, remember that ppb (parts per billion) is typically used for more toxic substances, while ppm (parts per million) is used for substances like Sulphate and Nitrate.

---

138. Species A, B, C, D formed in the following bond cleavages respectively are



1. $CH_3CH_2^+$, I^-, $CH_3CH_2^-$, Cu^+
2. $CH_3CH_2^+$, I^-, $CH_3CH_2^-$, Cu
3. $CH_3CH_2^-$, I^+, $CH_3CH_2^+$, Cu^+
4. $CH_3CH_2^-$, I^+, $CH_3CH_2^+$, Cu

Correct Answer: (1) $CH_3CH_2^+$, γ, $CH_3CH_2^-$, Cu^+

Solution:

Step 1: Understanding the bond cleavage

1. The first reaction:

$$CH_3CH_2 - I \quad \longrightarrow \quad CH_3CH_2^+ + I^-$$

This is a typical ionization reaction where ethyl iodide (CH_3CH_2I) dissociates to form the ethyl carbocation ($CH_3CH_2^+$) and iodide anion (I^-).

2. The second reaction:

$$CH_3CH_2 - Cu \quad \longrightarrow \quad CH_3CH_2^+ + Cu^+$$

This follows a similar dissociation mechanism where ethyl copper compound breaks into the ethyl carbocation ($CH_3CH_2^+$) and copper ion (Cu^+).

Step 2: Verifying the correct answer

From the reactions:

- $A = CH_3CH_2^+$
- $B = I^-$
- $C = CH_3CH_2^+$
- $D = Cu^+$

Thus, the correct answer is:

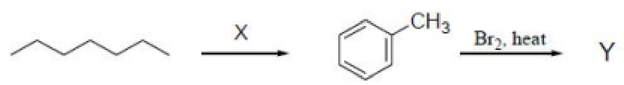
$$\textbf{(A)}\ CH_3CH_2^+, I^-, CH_3CH_2^+, Cu^+$$

Quick Tip

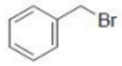
For heterolytic bond cleavage, consider the electronegativity of the elements involved. The more electronegative element typically takes the electrons, forming a negative ion.

---

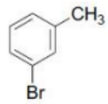
139. What are X and Y respectively in the following reaction sequence?



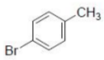
1. $FeCl_3$, 773 K, 10-20 atm;



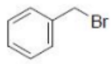
2. MoO_3, 770 K;



3. $AlCl_3$, Δ, 10-20 atm;



4. CrO_3, 773 K, 10-20 atm;



Correct Answer: (4) CrO_3, 773 K, 10-20 atm.

Solution:

The reaction involves the formation of a benzyl radical ($C_6H_5CH_2$) and subsequent halogenation in the presence of Br_2. This reaction is typical of a benzyl halide formation, which is carried out under oxidative conditions such as with CrO_3 (Chromium trioxide) at high temperatures.

The oxidation of the alkyl side chain (X) to form a benzyl group ($C_6H_5CH_2$) is facilitated by CrO_3, and then the electrophilic bromination leads to the formation of Y ($C_6H_5CH_2Br$).

Thus, the correct reagent and conditions for this transformation are CrO_3 at 773 K and 10-20 atm, making option (4) correct.

Quick Tip

For reactions involving halogenation, the presence of strong oxidizing agents like CrO_3 in high temperature conditions is common for alkyl side-chain oxidations, followed by halogenation.

---

140. A compound is formed by atoms of A, B and C. Atoms of C form hcp lattice. Atoms of A occupy 50% of octahedral voids and atoms of B occupy $\frac{2}{3}$ of tetrahedral voids. What is the molecular formula of the solid?

1. $A_3B_8C_6$

2. $A_2B_8C_6$

3. $A_4B_4C_3$

4. $A_5B_8C_6$

Correct Answer: (1) $A_3B_8C_6$

Solution:

- Atoms of C form an hcp lattice, which means C atoms are arranged in hexagonal close packing.
- In an hcp lattice, each unit cell contains 2 atoms of C. Thus, there are 2 atoms of C per unit cell.
- Atoms of A occupy 50% of octahedral voids. The number of octahedral voids in an hcp unit cell is equal to the number of atoms of C, which is 2. Therefore, the number of A atoms in the unit cell is 2.
- Atoms of B occupy $\frac{2}{3}$ of the tetrahedral voids. The number of tetrahedral voids in an hcp unit cell is 8. So, the number of B atoms in the unit cell is $\frac{2}{3} \times 8 = 5.33$, approximately 5 atoms of B.

Therefore, the molecular formula of the solid is $A_3B_8C_6$.

Quick Tip

For close-packed lattices, remember that C atoms form the basic lattice structure. The number of atoms in voids (octahedral and tetrahedral) must be calculated based on the voids present in the unit cell.

---

141. At 300 K, 6 g of urea was dissolved in 500 mL of water. What is the osmotic pressure (in atm) of the resultant solution? (R = 0.082 L atm K^{-1} mol^{-1}) (C=12;N=14;O=16;H=1)

1. 0.492

2. 2.46

3. 4.92

4. 49.2

Correct Answer: (3) 4.92

Solution: The osmotic pressure π is given by the formula:

$$\pi = \frac{nRT}{V}$$

Where:

- n = number of moles of urea,

- R = gas constant = 0.082 L atm K^{-1} mol^{-1},

- T = temperature in Kelvin = 300 K,

- V = volume of solution in liters = 500 mL = 0.5 L.

First, calculate the number of moles of urea:

$$\text{Molar mass of urea } (NH_2CONH_2) = 12 + 2(1) + 16 + 2(14) = 60 \text{ g/mol}.$$

$$n = \frac{\text{mass of urea}}{\text{molar mass}} = \frac{6}{60} = 0.1 \text{ mol}.$$

Now, calculate the osmotic pressure:

$$\pi = \frac{0.1 \times 0.082 \times 300}{0.5} = 4.92 \text{ atm}.$$

Quick Tip

Remember that osmotic pressure is directly proportional to the number of moles of solute, the temperature, and inversely proportional to the volume of the solution.

---

142. In water, which of the following gases has the highest Henry's law constant at 293 K?

1. N_2
2. O_2
3. He
4. H_2

Correct Answer: (3) He

Solution:

Henry's law states that the amount of gas that dissolves in a liquid is proportional to the partial pressure of the gas above the liquid, expressed as:

$$C = k_H \cdot P$$

Where:

- C is the concentration of the gas in the liquid,
- k_H is the Henry's law constant,
- P is the partial pressure of the gas.

For gases dissolved in water, the Henry's law constant is inversely proportional to the solubility of the gas in water. A higher Henry's law constant indicates that the gas is less soluble in water.

Among the given gases, helium (He) has the highest Henry's law constant because it is less soluble in water compared to the other gases listed (like N_2, O_2, and H_2).

Therefore, the correct answer is helium (He).

> **Quick Tip**
>
> For gases, the higher the Henry's law constant, the less soluble the gas is in the solvent.

---

143. Consider the cell reaction, at 300 K.

$A^+(aq)$ + B^3+ (aq) $\rightleftharpoons$ A^2+ (aq) + B (g)

Its E° is 1.0 V. The $\Delta_r H^\circ$ of the reaction is -163 kJ mol^{-1}. What is $\Delta_r S^\circ$ (in J K^{-1} mol^{-1}) of the reaction?

(F = 96500 C mol^{-1})

1. 10
2. 100
3. 1000
4. 10000

Correct Answer: (2) 100

Solution:

The relation between Gibbs free energy change ($\Delta_r G^\circ$) and the standard cell potential (E°) is given by:

$$\Delta_r G^\circ = -nFE^\circ$$

where:

- n is the number of moles of electrons exchanged in the reaction,

- F is the Faraday constant (96500 C mol^{-1}),

- E° is the standard electrode potential.

We also know that the Gibbs free energy change is related to the enthalpy change and entropy change as:

$$\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$$

Equating both expressions for $\Delta_r G^\circ$, we get:

$$-nFE^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$$

Rearranging to solve for $\Delta_r S^\circ$:

$$\Delta_r S^\circ = \frac{\Delta_r H^\circ - (-nFE^\circ)}{T}$$

Given:

- $\Delta_r H^\circ = -163\,\text{kJ/mol} = -163000\,\text{J/mol}$,

- $E^\circ = 1.0\,\text{V}$,

- $n = 2$ (since 2 electrons are involved),

- $F = 96500\,\text{C/mol}$,

- $T = 300\,\text{K}$.

Substituting the values:

$$\Delta_r S^\circ = \frac{-163000 - (-2 \times 96500 \times 1.0)}{300}$$

$$\Delta_r S^\circ = \frac{-163000 + 193000}{300} = \frac{30000}{300} = 100\,\text{J/K/mol}$$

Quick Tip

To calculate the entropy change ($\Delta_r S^\circ$), use the relation between Gibbs free energy, enthalpy, and entropy: $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$.

144. The rate constant of a first order reaction was doubled when the temperature was increased from 300 to 310 K. What is its approximate activation energy (in kJ mol^{-1})?
(R = 8.3 J mol^{-1} K^{-1}, $\log 2 = 0.3$)

1. 5.33
2. 53.3
3. 5333
4. 53.33

Correct Answer: (4) 53.33

Solution:
We use the Arrhenius equation in logarithmic form:

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303R} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$$

Step 1: Given data
- $T_1 = 300$ K, $T_2 = 310$ K
- $k_2 = 2k_1$ (since the rate constant is doubled)
- $R = 8.3$ J mol^{-1}K^{-1}
- $\log 2 = 0.3$

Step 2: Substituting the values

$$\log 2 = \frac{E_a}{2.303 \times 8.3} \times \left(\frac{10}{300 \times 310} \right)$$

$$0.3 = \frac{E_a}{19.1} \times \left(\frac{10}{93000} \right)$$

Step 3: Solving for E_a

$$E_a = \frac{0.3 \times 19.1 \times 93000}{10}$$

$$E_a = 53.33 \text{ kJ mol}^{-1}$$

Step 4: Identifying the correct answer

Thus, the activation energy is approximately 53.33 kJ mol^{-1}, which corresponds to option (D).

Quick Tip

For first-order reactions, if the rate constant doubles with a small temperature increase, use the logarithmic Arrhenius equation to estimate activation energy.

---

145. Which of the following solutions is used in the styptic action which prevents bleeding of blood?

1. $CoCl_2$ solution
2. $FeCl_3$ solution
3. Gold sol
4. AgBr emulsion

Correct Answer: (2) $FeCl_3$ solution

Solution:

The solution that is used in the styptic action to prevent bleeding is $FeCl_3$ (Iron (III) chloride) solution. It has astringent properties and is used to stop bleeding by constricting blood vessels and promoting blood clotting. It is commonly used in cases of minor cuts or wounds.

Quick Tip

For controlling bleeding, astringent solutions like $FeCl_3$ are used. They promote clotting and constrict blood vessels.

---

146. 'A' is a protecting colloid. The following data is obtained for preventing the coagulation of 10 mL of gold sol to which 1 mL of 10% NaCl is added. What is the gold number of 'A'?

Expt. No.	Wt (in mg) of A added to gold sol	Coagulation
1	40	Prevented
2	35	Prevented
3	25	Not prevented
4	32	Not prevented
5	33	Prevented

(1) 32

(2) 33

(3) 35

(4) 40

Correct Answer: (2) 33

Solution

The gold number is defined as the amount (in mg) of a protecting colloid required to prevent the coagulation of 10 mL of a gold sol when 1 mL of 10% NaCl is added.

From the given data:

- In the first experiment, 40 mg of A prevents coagulation.
- In the second experiment, 35 mg of A prevents coagulation.
- In the third experiment, 25 mg of A does not prevent coagulation.
- In the fourth experiment, 32 mg of A does not prevent coagulation.
- In the fifth experiment, 33 mg of A prevents coagulation.

The gold number is the amount of A required to just prevent coagulation. The correct amount for coagulation prevention is 33 mg.

Thus, the gold number of 'A' is 33.

Quick Tip

The gold number is a measure of the protective power of a colloid. The lower the gold number, the greater the protective power of the colloid.

147. Two statements are given below.

Statement I: The reaction $Cr_2O_3 + 2Al \rightarrow 2Cr + Al_2O_3$ (G° = -421 kJ) is thermodynamically feasible.

Statement II: The above reaction occurs at room temperature.

The correct answer is

(1) Both the statements I & II are correct.

(2) Both the statements I & II are not correct.

(3) Statement I is correct, but statement II is not correct.

(4) Statement I is not correct, but statement II is correct.

Correct Answer: (3) Statement I is correct, but statement II is not correct.

Solution

- Statement I: The reaction $Cr_2O_3 + 2Al \rightarrow 2Cr + Al_2O_3$ is thermodynamically feasible as the Gibbs free energy change (G° = -421 kJ) is negative, indicating that the reaction is spontaneous. Therefore, Statement I is correct.
- Statement II: The reaction does not occur at room temperature because it is an aluminothermic reaction, which requires high temperatures to overcome the activation energy barrier. Therefore, Statement II is incorrect.

Hence, the correct answer is that Statement I is correct, but Statement II is not correct.

Quick Tip

For reactions with a negative Gibbs free energy (G°), the reaction is thermodynamically feasible. However, the reaction may still require high temperatures or activation energy to occur.

---

148. The basicity of H_3PO_2, H_3PO_3, H_3PO_4 respectively is

(1) 2, 2, 3

(2) 2, 3, 3

(3) 1, 3, 3

(4) 1, 2, 3

Correct Answer: (4) 1, 2, 3

Solution

- H_3PO_2 (Phosphorous acid): It has one hydroxyl group attached to the phosphorus atom, so its basicity is 1.
- H_3PO_3 (Phosphorous acid): It has two hydroxyl groups attached to the phosphorus atom, so its basicity is 2.
- H_3PO_4 (Phosphoric acid): It has three hydroxyl groups attached to the phosphorus atom, so its basicity is 3.

Therefore, the correct answer is 1, 2, 3.

> **Quick Tip**
>
> The basicity of an acid is determined by the number of replaceable hydrogen ions (protons) in the molecule. The more hydroxyl groups attached to the central atom, the higher the basicity.

---

149. Which of the following reactions of $KMnO_4$ occurs in acidic medium?

(1) Oxidation of thiosulphate to sulphate

(2) Precipitation of sulphur from H_2S

(3) Oxidation of iodide to iodate

(4) Oxidation of manganous salt to MnO_2

Correct Answer: (2) Precipitation of sulphur from H_2S

Solution

In acidic medium, $KMnO_4$ acts as an oxidizing agent. The reaction of $KMnO_4$ with H_2S leads to the precipitation of sulphur. This reaction occurs in acidic medium, where $KMnO_4$ oxidizes H_2S to sulphur. The other reactions do not occur in acidic medium or are not typical reactions for $KMnO_4$.

Therefore, the correct answer is Precipitation of sulphur from H_2S.

Quick Tip

$KMnO_4$ in acidic medium is a strong oxidizing agent and is commonly used to oxidize sulfides to elemental sulfur and other oxidation reactions.

---

150. Which complex among the following is most paramagnetic?

(1) $[Co(NH_3)_6]^{3+}$

(2) $[Co(NH_3)_6]^{2+}$

(3) $[Co(H_2O)_6]^{2+}$

(4) $[Co(H_2O)_6]^{3+}$

Correct Answer: (4) $[Co(H_2O)_6]^{3+}$

Solution

The paramagnetism of a complex is determined by the presence of unpaired electrons. For the given complexes:

- In $[Co(NH_3)_6]^{3+}$, Co is in a +3 oxidation state (Co^{3+}), which has a d^6 configuration, leading to fewer unpaired electrons and lower paramagnetism.

- In $[Co(NH_3)_6]^{2+}$, Co is in a +2 oxidation state (Co^{2+}), which has a d^7 configuration, allowing for more unpaired electrons and higher paramagnetism.

- In $[Co(H_2O)_6]^{2+}$, Co is in a +2 oxidation state (Co^{2+}), again with a d^7 configuration, which is paramagnetic but not as much as the next complex.

- In $[Co(H_2O)_6]^{3+}$, Co is in a +3 oxidation state (Co^{3+}), which leads to a d^6 configuration. The water ligands are weak field ligands, so the electrons do not pair up, resulting in the maximum number of unpaired electrons and the highest paramagnetism.

Therefore, the complex $[Co(H_2O)_6]^{3+}$ is the most paramagnetic.

Quick Tip

Paramagnetism increases with the number of unpaired electrons in the d-orbitals of the central metal ion. The weaker the field ligands, the higher the likelihood of unpaired electrons.

---

151. Polymers that can be softened on heating and hardened on cooling are called

(1) Thermosetting polymers

(2) Bakelite

(3) Fibres

(4) Thermoplastic polymers

Correct Answer: (4) Thermoplastic polymers

Solution

Thermoplastic polymers are materials that can be softened when heated and hardened when cooled. This is due to their molecular structure, which allows the polymer chains to slide past each other when heated, making them moldable. Upon cooling, they solidify into the desired shape. Examples include polythene, polystyrene, and PVC.

In contrast, thermosetting polymers, like Bakelite, harden permanently when heated and cannot be reshaped after that. Fibres are not specific to this property but are a type of polymer used in textiles.

Therefore, the correct answer is thermoplastic polymers.

> **Quick Tip**
>
> Thermoplastics can be reshaped multiple times through heating and cooling, while thermosetting plastics can only be shaped once.

---

152. The number of –OH groups in open chain and ring structures of D-glucose are respectively

(1) 4, 5

(2) 5, 4

(3) 5, 5

(4) 6, 5

Correct Answer: (2) 5, 4

Solution

Step 1: Understanding the structure of D-glucose

D-glucose exists in two structural forms:

1. Open-chain form: The open-chain structure of D-glucose contains one aldehyde (-CHO) functional group and five hydroxyl (-OH) groups attached to the carbon atoms.

2. Cyclic (ring) form: When D-glucose undergoes cyclization to form a pyranose ring (as in the Haworth projection), the aldehyde group reacts with one of the hydroxyl groups to form a hemiacetal, resulting in the formation of a six-membered ring. In this form, the number of hydroxyl groups remains five.

Step 2: Identifying the correct answer

- In the open-chain form, there are five hydroxyl (-OH) groups.
- In the ring form, there are still five hydroxyl (-OH) groups (since the aldehyde group forms a hemiacetal but does not contribute an additional hydroxyl group).

Thus, the correct answer is:

(B) $5, 5$

Quick Tip

D-glucose exists in equilibrium between its open-chain and cyclic forms. The number of hydroxyl (-OH) groups remains the same in both forms except that the aldehyde group in the open-chain form converts into a hydroxyl group in the cyclic form.

---

153. Which of the following is correct statement?

(1) Starch is a polymer of β-D-glucose

(2) Amylose is a component of starch

(3) Proteins are biopolymers of only one type of amino acids

(4) Lactose is a disaccharide of α-D-glucose and β-D-galactose

Correct Answer: (2) Amylose is a component of starch

Solution

Starch is made up of two components: amylose and amylopectin. Amylose is a polysaccharide formed by long chains of α-D-glucose units, whereas amylopectin consists of

α-D-glucose units linked through α-1,4 and α-1,6 glycosidic bonds. Therefore, amylose is indeed a component of starch.

The other options are incorrect because:

- Starch is a polymer of α-D-glucose, not β-D-glucose.
- Proteins are biopolymers made up of amino acids, but they are composed of more than one type of amino acid.
- Lactose is a disaccharide made up of β-D-glucose and β-D-galactose, not α-D-glucose.

Quick Tip

Starch is composed of two main components: amylose (a linear polymer of α-D-glucose) and amylopectin (a branched polymer of α-D-glucose).

---

154. Which of the following is NOT correctly matched?

(1) Aspartame – Food preservative

(2) Butylated hydroxy toluene – antioxidant

(3) Noverstrol – antifertility drug

(4) Bithionol – antiseptic

Correct Answer: (1) Aspartame – Food preservative

Solution

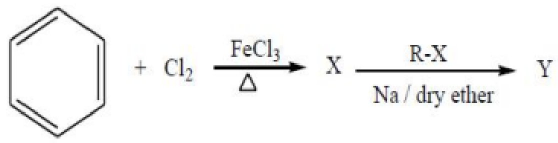
Aspartame is a sweetener used in many sugar-free products, but it is not a food preservative. The correct use of Aspartame is as a sweetener in food and beverages, not for preservation.

The other options are correctly matched:

- Butylated hydroxy toluene (BHT) is indeed an antioxidant, used to prevent oxidation in food and other products.
- Noverstrol is used as an antifertility drug.
- Bithionol is an antiseptic that is used to treat skin infections.

> **Quick Tip**
>
> Aspartame should be remembered as a sweetener and not as a food preservative, which helps to distinguish it from other chemical compounds like BHT and Bithionol.

155.



Conversion of X to Y is an example of

(1) Wurtz reaction

(2) Fitting reaction

(3) Wurtz-Fittig reaction

(4) Friedel-Crafts reaction

Correct Answer: (3) Wurtz-Fittig reaction

Solution

The reaction shown in the question is a Wurtz-Fittig reaction. This reaction involves the reaction of an alkyl halide (R-X) with sodium in dry ether, leading to the formation of an alkane (X) or an alkylated product (Y). It is a combination of the Wurtz reaction (for alkyl halide formation) and the Fittig reaction (for aryl halide formation).

- The Wurtz reaction involves the coupling of two alkyl halides with sodium to form alkanes.
- The Fittig reaction involves the coupling of aryl halides in the presence of sodium to form biaryl compounds.

The Wurtz-Fittig reaction is a combination of both reactions and is used to form aryl and alkyl products.

> **Quick Tip**
>
> Remember, the Wurtz-Fittig reaction involves the coupling of both alkyl and aryl halides with sodium, forming a variety of alkylated or arylated products.

156. Which of the following is not an example of allylic halide?

(1) 4- chlorobut-1-ene

(2) 1- chlorobut-2-ene

(3) 3- chloro-2-methyl but-1-ene

(4) 4- chloropent-2-ene

Correct Answer: (1) 4- chlorobut-1-ene

Solution

An allylic halide is a compound where the halogen atom is attached to a carbon atom that is adjacent to a carbon-carbon double bond. Let's analyze the options:

1. 4- chlorobut-1-ene: The chlorine atom is attached to the carbon in the 4th position, which is not adjacent to the double bond, making it not an allylic halide.
2. 1- chlorobut-2-ene: Here, chlorine is attached to the carbon in the 1st position, which is adjacent to the double bond (position 2), making this an allylic halide.
3. 3- chloro-2-methyl but-1-ene: In this case, the chlorine is attached to the 3rd carbon, which is adjacent to the double bond, making it an allylic halide.
4. 4- chloropent-2-ene: The chlorine is attached to the 4th carbon, which is adjacent to the double bond at the 2nd position, making this an allylic halide.

Thus, the correct answer is 4- chlorobut-1-ene.

Quick Tip

In allylic halides, the halogen should be attached to a carbon atom that is directly next to the carbon-carbon double bond.

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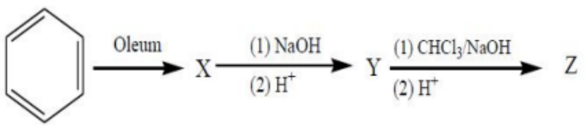
157. What is the major product 'Z' in the following sequence?



(1) o- Hydroxy benzaldehyde

(2) p- Hydroxy benzaldehyde

(3) o- Hydroxy benzoic acid

(4) p- Hydroxy benzoic acid

Correct Answer: (i) o- Hydroxy benzaldehyde

Solution

Let's analyze the reaction step by step:

1. Reaction with NaOH: The compound oleum, which contains sulfuric acid, reacts with sodium hydroxide (NaOH). This typically leads to the formation of a phenoxide ion (O) in aromatic compounds.

2. Reaction with CHCl/NaOH: The reaction of the phenoxide ion with chloroform (CHCl) in the presence of NaOH follows the Reimer-Tiemann reaction, which introduces a hydroxyl group at the ortho position relative to the existing substituent (in this case, the aldehyde group).

3. Acidification (H): After the reaction with NaOH and chloroform, the final acidic work-up yields o- Hydroxy benzaldehyde.

Thus, the major product 'Z' is o- Hydroxy benzaldehyde.

Quick Tip

In the Reimer-Tiemann reaction, the hydroxyl group is introduced at the ortho position relative to the aldehyde group, forming o- Hydroxy benzaldehyde.

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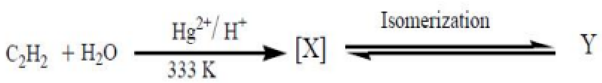
158. Consider the following reactions.



Y cannot be obtained from which of the following reactions?

(1) $CH_3COCl + H_2 \xrightarrow{\text{Pd/BaSO}_4}$

(2) $CH_3CH_2OH \xrightarrow{Cu/573K}$

(3) $CH_3CN + SnCl_2 + HCl \rightarrow X \text{ with } H_2O$

(4) $CH_3COOH \xrightarrow{(i)\,LiAlH_4/\text{ether}} \xrightarrow{(ii)\,H_2O}$

Correct Answer: (4) $CH_3COOH \xrightarrow{(i)\,LiAlH_4/\text{ether}} \xrightarrow{(ii)\,H_2O}$

Solution Let's analyze each reaction:

1. Option (1): The reaction of acetyl chloride (CHCOCl) with hydrogen in the presence of palladium on barium sulfate (Pd/BaSO) is a reduction reaction. This reaction produces ethanol (CHCHOH) which can undergo further reactions like isomerization.

2. Option (2): Ethanol (CH_3CH_2OH) can undergo a dehydration reaction in the presence of copper (Cu) at 573 K, forming ethene (CH), which can then undergo isomerization.

3. Option (3): In this reaction, acetonitrile (CHCN) reacts with stannous chloride (SnCl) and hydrochloric acid (HCl), which can reduce the nitrile group to an amine (CHCHNH). This isomerization is possible.

4. Option (4): The reaction of acetic acid (CHCOOH) with lithium aluminum hydride (LiAlH) in ether followed by hydrolysis with water results in the formation of ethanol (CHCHOH). However, this reaction will not directly form an isomer of the compound, and thus, the product does not undergo isomerization.

Thus, Y cannot be obtained from option (iv).

Quick Tip

In the presence of LiAlH, carboxylic acids are reduced to alcohols, but they do not undergo isomerization.

---

159. Assertion (A): Carboxylic acids are more acidic than phenols.

Reason (R): Resonance structures of carboxylate ion are equivalent, while resonance structures of phenoxide ion are not equivalent.

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) Both (A) and (R) are correct But (R) is not the correct explanation of (A)

(3) (A) is correct but (R) is incorrect

(4) (A) is incorrect but (R) is correct

Correct Answer: (1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Step 1: Understanding the Acidity of Carboxylic Acids vs. Phenols

- Carboxylic acids ($R - COOH$) are more acidic than phenols (C_6H_5OH) because they have a lower pKa.

- The carboxylate ion ($R - COO^-$) formed after deprotonation is highly stabilized by resonance.

Step 2: Resonance and Stability Comparison

- Carboxylate ion (COO^-):
- Resonance structures are equivalent.
- The negative charge is delocalized equally between two oxygen atoms, making the carboxylate ion highly stable.

$$\text{O=C-O}^- \leftrightarrow ^- O - C = O$$

- Phenoxide ion ($C_6H_5O^-$):
- Resonance is not equivalent.
- The negative charge is not equally delocalized across the oxygen and the benzene ring.
- Oxygen retains a significant part of the negative charge, making it less stable than the carboxylate ion.

Step 3: Conclusion

- Since carboxylate ion is more stable than phenoxide ion, carboxylic acids are more acidic than phenols.
- The given reason correctly explains this fact.

Thus, the correct answer is:

Option (1): Both (A) and (R) are correct and (R) is the correct explanation of (A)

Quick Tip

Carboxylate ion is more stable due to equivalent resonance, making carboxylic acids stronger acids than phenols.

---

160. In the reaction sequence, Y is:

$$CH_3CO_2H \xrightarrow{\text{(1) NH}_3\text{,(2)}\Delta} P \xrightarrow{\text{Br}_2\text{/NaOH}} Y$$

(1) a primary amine with the same number of carbons as in P

(2) a primary amine with one carbon less than in P

(3) a secondary amine with the same number of carbons as in P

(4) a secondary amine with one carbon less than in P

Correct Answer: (2) A primary amine with one carbon less than in P

Solution:

Step 1: Understanding the Reaction Sequence

1. First Reaction: Amide Formation

- The given reactant is acetic acid (CH_3CO_2H).

- When reacted with ammonia (NH_3) and heated (Δ), it forms an amide (P):

$$CH_3COOH + NH_3 \rightarrow CH_3CONH_2$$

- So, P is acetamide (CH_3CONH_2).

2. Second Reaction: Hoffmann Bromamide Degradation - The Hoffmann degradation reaction involves treating an amide with Br_2/NaOH, which removes one carbon and forms a primary amine.

$$CH_3CONH_2 \xrightarrow{\text{Br}_2/\text{NaOH}} CH_3NH_2$$

- The product Y is methylamine (CH_3NH_2), which has one carbon less than in P.

Step 2: Identifying the Correct Answer

- P (acetamide) contains two carbon atoms.

- Y (methylamine) contains one carbon atom, indicating the loss of one carbon.

- The final product is a primary amine.

Thus, the correct answer is:

Option (2): A primary amine with one carbon less than in P

Quick Tip

The Hoffmann Bromamide Degradation reaction reduces the carbon count by one when converting an amide to an amine.