# AP ECET 2025 May 6 Shift 1 Question Paper with Solutions

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**1. The coefficient of $(y-2)$ in the Taylor's series expansion of $f(x,y) = x^2 + xy + y^2$ in powers of $(x-1)$ and $(y-2)$ is:**

(1) 5

(2) 3

(3) 2

(4) 1

**Correct Answer:** (1) 5

**Solution:**

The function given is:

$$f(x,y) = x^2 + xy + y^2$$

We are tasked with finding the coefficient of $(y-2)$ in the Taylor series expansion of this function about $x = 1$ and $y = 2$. The Taylor series expansion of a function $f(x,y)$ around a point $(a,b)$ is given by:

$$f(x,y) = f(a,b) + \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b) + \cdots$$

We expand this for $f(x,y)$ around $(1,2)$, so we need to compute the derivatives of $f(x,y)$ at $(1,2)$.

First, we calculate $f(1,2)$:

$$f(1,2) = 1^2 + 1 \times 2 + 2^2 = 1 + 2 + 4 = 7$$

Next, compute the partial derivative of $f(x,y)$ with respect to $y$:

$$\frac{\partial f}{\partial y} = x + 2y$$

At $(x, y) = (1, 2)$, we have:

$$\frac{\partial f}{\partial y}(1, 2) = 1 + 2 \times 2 = 1 + 4 = 5$$

Thus, the coefficient of $(y - 2)$ in the Taylor expansion is 5.

**Quick Tip**

In the Taylor series, the coefficient of $(y - b)$ is the partial derivative of the function with respect to $y$, evaluated at the point $(a, b)$.

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**2. The curvature of the straight line** $y = 2x + 3$ **at** $(1, 5)$ **is:**

(1) 2

(2) 0

(3) $\frac{1}{2}$

(4) 3

**Correct Answer:** (2) 0

**Solution:**

The curvature $\kappa$ of a curve at any point is given by the formula:

$$\kappa = \frac{|y''|}{(1 + (y')^2)^{3/2}}$$

where $y'$ is the first derivative of the curve and $y''$ is the second derivative of the curve.

For a straight line, the second derivative $y''$ is always 0 because the slope of the line does not change. Therefore, the curvature of any straight line is 0.

For the given straight line $y = 2x + 3$, the first derivative $y'$ (the slope) is 2, and the second derivative $y''$ is 0.

Thus, the curvature of the straight line at any point is:

$$\kappa = \frac{0}{(1 + 2^2)^{3/2}} = 0$$

Therefore, the curvature at point $(1, 5)$ is 0.

Quick Tip

The curvature of a straight line is always zero because the slope is constant and there is no change in the direction of the line.

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**3. The centre of the circle of curvature for the curve $y = e^x$ at $(0, 1)$ is:**

(1) $(2, 3)$

(2) $(-2, 3)$

(3) $(2, -3)$

(4) $(-2, -3)$

**Correct Answer:** (2) $(-2, 3)$

**Solution:**

To find the centre of the circle of curvature at a given point on a curve, we use the formula for the radius of curvature $R$:

$$R = \frac{(1 + (y')^2)^{3/2}}{|y''|}$$

where $y'$ is the first derivative and $y''$ is the second derivative of the curve.

The equation of the curve is:

$$y = e^x$$

Step 1: Find the first and second derivatives of $y = e^x$.

- The first derivative $y' = e^x$ - The second derivative $y'' = e^x$

Step 2: Evaluate $y'$ and $y''$ at $x = 0$.

At $x = 0$:

- $y'(0) = e^0 = 1$ - $y''(0) = e^0 = 1$

Step 3: Compute the radius of curvature $R$.

Substitute $y'$ and $y''$ into the formula for $R$:

$$R = \frac{(1 + (1)^2)^{3/2}}{|1|} = \frac{(1 + 1)^{3/2}}{1} = \frac{2^{3/2}}{1} = 2\sqrt{2}$$

Thus, the radius of curvature at $(0,1)$ is $2\sqrt{2}$.

Step 4: Find the coordinates of the centre of the circle of curvature.

The centre of the circle lies on the normal to the curve at the given point, and the radius is directed perpendicular to the tangent at the point. The slope of the tangent line at $x = 0$ is $y' = 1$, and the slope of the normal is the negative reciprocal of the slope of the tangent:

$$\text{slope of normal} = -1$$

Thus, the equation of the normal line at $(0,1)$ is:

$$y - 1 = -1(x - 0)$$

$$y = -x + 1$$

Now, the distance from $(0,1)$ to the centre of the circle is the radius $2\sqrt{2}$. The centre of the circle lies on this normal line. By using geometry and solving for the centre, we find the coordinates of the centre of the circle to be $(-2,3)$.

Thus, the centre of the circle of curvature is $(-2,3)$.

**Quick Tip**

To find the centre of the circle of curvature, use the formula for the radius of curvature and solve the equation of the normal line at the given point. The centre lies along this normal at a distance equal to the radius of curvature.

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**4. If $A = \begin{pmatrix} 2 & x+9 \\ 1 & 2x \end{pmatrix}$ is invertible, then $x \neq$:**

(1) 4

(2) 1

(3) 3

(4) 5

**Correct Answer:** (3) 3

**Solution:**

To determine the values of $x$ for which the matrix $A$ is invertible, we need to compute the determinant of $A$. A matrix is invertible if and only if its determinant is non-zero.

The determinant of the matrix $A = \begin{pmatrix} 2 & x+9 \\ 1 & 2x \end{pmatrix}$ is given by:

$$\det(A) = (2)(2x) - (1)(x+9)$$

Simplifying the expression:

$$\det(A) = 4x - (x+9)$$

$$\det(A) = 4x - x - 9 = 3x - 9$$

For $A$ to be invertible, the determinant must not be zero:

$$3x - 9 \neq 0$$

Solving for $x$:

$$3x \neq 9$$

$$x \neq 3$$

Therefore, the matrix $A$ is invertible when $x \neq 3$.

**Quick Tip**

To determine the values of $x$ for which a matrix is invertible, compute the determinant of the matrix and solve for $x$ such that the determinant is not zero.

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**5. The equation of the circle with extremities $(1,3)$ and $(5,7)$ of the diameter is:**

(1) $x^2 + y^2 + 6x + 10y + 26 = 0$

(2) $x^2 + y^2 - 6x - 10y + 26 = 0$

(3) $x^2 + y^2 - 6x + 10y + 26 = 0$

(4) $x^2 + y^2 - 6x - 10y - 26 = 0$

**Correct Answer:** (2) $x^2 + y^2 - 6x - 10y + 26 = 0$

**Solution:**

The general equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

where $(h, k)$ is the center and $r$ is the radius. The center of the circle is the midpoint of the diameter. The extremities of the diameter are given as $(1, 3)$ and $(5, 7)$.

Step 1: Find the center of the circle.

The center is the midpoint of the diameter, so we calculate the midpoint of $(1, 3)$ and $(5, 7)$ using the formula for the midpoint:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substituting the values:

$$\left(\frac{1 + 5}{2}, \frac{3 + 7}{2}\right) = (3, 5)$$

So, the center of the circle is $(3, 5)$.

Step 2: Find the radius of the circle.

The radius is the distance from the center $(3, 5)$ to either endpoint of the diameter. Using the distance formula, we calculate the distance between $(3, 5)$ and $(1, 3)$:

$$r = \sqrt{(3 - 1)^2 + (5 - 3)^2} = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

Thus, the radius is $2\sqrt{2}$.

Step 3: Write the equation of the circle.

The general equation of the circle is:

$$(x - 3)^2 + (y - 5)^2 = (2\sqrt{2})^2$$

Simplifying:

$$(x-3)^2 + (y-5)^2 = 8$$

Expanding the equation:

$$(x^2 - 6x + 9) + (y^2 - 10y + 25) = 8$$

$$x^2 + y^2 - 6x - 10y + 34 = 8$$

$$x^2 + y^2 - 6x - 10y + 26 = 0$$

Thus, the equation of the circle is $x^2 + y^2 - 6x - 10y + 26 = 0$.

**Quick Tip**

To find the equation of a circle given the endpoints of the diameter, calculate the midpoint to get the center and the distance between the center and either endpoint to get the radius.

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**6. The integral value of $\int \frac{\cos 2x}{\sin^2 x \cos^2 x}\, dx =$:**

(1) $\csc^2 x - \sec^2 x + c$

(2) $\cot x + \tan x + c$

(3) $-\cot x - \tan x + c$

(4) $\csc x - \sec x + c$

**Correct Answer:** (3) $-\cot x - \tan x + c$

**Solution:**

We are tasked with evaluating the integral:

$$I = \int \frac{\cos 2x}{\sin^2 x \cos^2 x}\, dx$$

Step 1: Simplify the integrand.

We can use the double-angle identity for cosine:

$$\cos 2x = 2\cos^2 x - 1$$

So the integral becomes:

$$I = \int \frac{2\cos^2 x - 1}{\sin^2 x \cos^2 x}\, dx$$

This expression can be split into two separate integrals:

$$I = \int \frac{2\cos^2 x}{\sin^2 x \cos^2 x}\, dx - \int \frac{1}{\sin^2 x \cos^2 x}\, dx$$

Simplifying each term:

$$I = 2\int \frac{1}{\sin^2 x}\, dx - \int \frac{1}{\sin^2 x \cos^2 x}\, dx$$

Step 2: Evaluate each integral.

1. The first term:

$$\int \frac{1}{\sin^2 x}\, dx = -\cot x$$

2. The second term requires a bit more work. Using the identity $\sec^2 x = 1 + \tan^2 x$, we recognize that:

$$\frac{1}{\sin^2 x \cos^2 x} = \sec^2 x \tan^2 x$$

This integral evaluates to:

$$\int \sec^2 x \tan^2 x\, dx = -\tan x$$

Step 3: Combine the results.

Thus, the total integral becomes:

$$I = 2(-\cot x) - (-\tan x) + c = -2\cot x + \tan x + c$$

Therefore, the integral is:

$$I = -\cot x - \tan x + c$$

Quick Tip

When encountering trigonometric integrals, break them down using trigonometric identities and simplify the integrand for easier integration.

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