# BITSAT 2023 Question Paper with Solutions

| **Time Allowed :**3 Hours | **Maximum Marks :**390 | **Total Questions :**130 |
|---|---|---|

## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. **Mode**: Computer-based online test
2. **Duration**: 3 hours (180 minutes)
3. **Sections**: The exam consists of four parts:
   (a) Part I: Physics (30 questions)
   (b) Part II: Chemistry (30 questions)
   (c) Part III: English Proficiency (10 questions) and Logical Reasoning (20 questions)
   (d) Part IV: Mathematics (40 questions) or Biology (for B.Pharm candidates)
4. **Total Marks**: 390
5. **Marking Scheme**: Each correct answer awards 3 marks, and 1 mark is deducted for each incorrect answer
6. **Subjects**:
   (a) Physics: Mechanics, Electromagnetism, Thermodynamics, Modern Physics
   (b) Chemistry: Organic, Inorganic, and Physical Chemistry
   (c) Mathematics: Calculus, Algebra, Geometry (or Biology for B.Pharm candidates)
   (d) English Proficiency: Reading Comprehension, Vocabulary
   (e) Logical Reasoning: Analytical and Problem-solving skills

**1. An object moves with speed $v_1$, $v_2$, and $v_3$ along a line segment $AB$, $BC$, and $CD$ respectively as shown in the figure. Where $AB = BC$ and $AD = 3AB$, then the average speed of the object will be:**


(A) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$

(B) $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$

(C) $\frac{(r_1^2+r_2^2+r_3^2)}{3}$

(D) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$

**Correct Answer:** (B) $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$

**Solution:**

**Step 1:** Understanding the given conditions.

Given: - $AB = BC = x$ - $CD = AB = x$ - $AD = 3x$ - Speeds: $v_1$, $v_2$, and $v_3$ along $AB$, $BC$, and $CD$ respectively.

**Step 2:** Compute the time taken for each segment.

Using Time $= \frac{\text{Distance}}{\text{Speed}}$,

$$t_1 = \frac{x}{v_1}, \quad t_2 = \frac{x}{v_2}, \quad t_3 = \frac{x}{v_3}$$

Total time taken:

$$T = t_1 + t_2 + t_3 = \frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}$$

**Step 3:** Compute the average speed.

$$V_{\text{avg}} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}}$$

Simplifying:

$$V_{\text{avg}} = \frac{3}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}}$$

Rewriting in terms of a common denominator:

$$V_{\text{avg}} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

**Final Answer:**

$$\boxed{\frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}}$$

**Quick Tip**

For average speed calculations when covering equal distances at different speeds, use:

$$V_{\text{avg}} = \frac{\text{Total Distance}}{\text{Total Time}}$$

which simplifies to a harmonic mean formula in cases of multiple segments.

---

**2. The effect of an increase in temperature on the number of electrons in the conduction band ($n_e$) and the resistance of a semiconductor will be as follows:**

(A) Both $n_e$ and resistance decrease

(B) Both $n_e$ and resistance increase

(C) $n_e$ increases, resistance decreases

(D) $n_e$ decreases, resistance increases

**Correct Answer:** (C) $n_e$ increases, resistance decreases

**Solution:**

**Step 1:** Understanding the effect of temperature on a semiconductor.

- A semiconductor has a band gap between the valence band and conduction band. - At higher temperatures, more electrons get enough thermal energy to jump from the valence band to the conduction band. - This results in an increase in the number of free electrons ($n_e$).

**Step 2:** Effect on resistance.

- The resistance ($R$) of a semiconductor is given by:

$$R = \frac{\rho L}{A}, \quad \text{where } \rho \text{ is the resistivity.}$$

- Resistivity ($\rho$) is inversely proportional to the number of free charge carriers:

$$\rho \propto \frac{1}{n_e}$$

- As $n_e$ increases with temperature, resistivity decreases, leading to a decrease in resistance.

**Final Answer:**

$$\boxed{n_e \text{ increases, resistance decreases}}$$

**Quick Tip**

For semiconductors: - Higher temperature $\rightarrow$ More electrons in conduction band. - More free charge carriers $\rightarrow$ Lower resistivity $\rightarrow$ Lower resistance. - This behavior is opposite to metals, where resistance increases with temperature.

---

**3. A radioactive material is reduced to $\frac{1}{8}$ of its original amount in 3 days. If $8 \times 10^{-3}$ kg of the material is left after 5 days, what was the initial amount of the material?**

(A) 700 gm

(B) 900 gm

(C) 475 gm

(D) 256 gm

**Correct Answer:** (D) 256 gm

**Solution:**

**Step 1:** Understanding radioactive decay.

The decay of a radioactive material follows the exponential decay law:

$$N = N_0 \times \left(\frac{1}{2}\right)^{t/T}$$

where: - $N_0$ = Initial amount of substance - $N$ = Remaining amount after time $t$ - $T$ = Half-life - $t$ = Given time

**Step 2:** Finding the half-life.

Given that the material is reduced to $\frac{1}{8}$ of its original value in 3 days, we set up:

$$\frac{N_0}{8} = N_0 \times \left(\frac{1}{2}\right)^{3/T}$$

Since $\frac{1}{8} = \left(\frac{1}{2}\right)^3$, we compare powers:

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{3/T}$$

Thus, $T = 1$ day (half-life is 1 day).

**Step 3:** Finding $N_0$.

After 5 days, the amount left is $8 \times 10^{-3}$ kg. Using the decay formula:

$$N = N_0 \times \left(\frac{1}{2}\right)^{5/1}$$

$$8 \times 10^{-3} = N_0 \times \frac{1}{32}$$

Solving for $N_0$:

$$N_0 = 8 \times 10^{-3} \times 32 = 0.256 \text{ kg} = 256 \text{ g}$$

**Final Answer:**

$$\boxed{256 \text{ gm}}$$

**Quick Tip**

For radioactive decay problems: - Identify the half-life first. - Use the exponential decay formula:

$$N = N_0 \times \left(\frac{1}{2}\right)^{t/T}$$

- Ensure unit conversions (kg to g) for final answers.

---

**4. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:**

(A) 2

(B) 1

(C) 3

(D) 4

**Correct Answer:** (C) 3

**Solution:**

**Step 1:** Understanding energy levels of hydrogen.

The energy levels in hydrogen are given by the Bohr equation:

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

where $n$ is the principal quantum number.

**Step 2:** Effect of electron beam energy.

- The incident energy is 12.5 eV. - The ground state energy ($n = 1$) of hydrogen is $-13.6$ eV. - The energy required to excite an electron to $n = 2$ is:

$$E_2 = -\frac{13.6}{2^2} = -3.4 \text{ eV}$$

Energy difference:

$$\Delta E = E_2 - E_1 = (-3.4) - (-13.6) = 10.2 \text{ eV}$$

Since 12.5 eV is supplied, the electron can be excited up to $n = 3$.

**Step 3:** Possible transitions and spectral lines.

The number of spectral lines emitted follows:

$$\text{Number of spectral lines} = \frac{n(n-1)}{2}$$

For $n = 3$, possible transitions are: - $3 \to 2$ - $3 \to 1$ - $2 \to 1$

Thus, the total spectral lines:

$$\frac{3(3-1)}{2} = \frac{3 \times 2}{2} = 3$$

**Final Answer:**

$$\boxed{3}$$

Quick Tip

For hydrogen spectral line calculations: - Determine the highest energy level an electron can reach. - Use the formula:

$$\text{Number of spectral lines} = \frac{n(n-1)}{2}$$

where $n$ is the highest excited state.

---

**5. If 1000 droplets of water of surface tension $0.07$. having same radius 1mm each, combine to from a single drop. In the process the released surface energy is- (Take $\pi = \frac{22}{7}$):**

(A) $7.92 \times 10^{-6}$ J

(B) $7.92 \times 10^{-4}$ J

(C) $9.68 \times 10^{-4}$ J

(D) $8.8 \times 10^{-5}$ J

**Correct Answer:** (B) $7.92 \times 10^{-4}$ J

**Solution:**

**Step 1:** Understanding the formula for surface energy.

The surface energy is given by:

$$U = \text{Surface Tension} \times \text{Surface Area}$$

For a sphere, surface area is:

$$A = 4\pi R^2$$

**Step 2:** Calculate initial total surface area.

- Given 1000 small droplets, each of radius $r = 1$ mm. - Total surface area before merging:

$$A_{\text{initial}} = 1000 \times 4\pi r^2$$

$$A_{\text{initial}} = 1000 \times 4 \times \frac{22}{7} \times (1 \times 10^{-3})^2$$

$$A_{\text{initial}} = 1000 \times 4 \times \frac{22}{7} \times 10^{-6}$$

$$A_{\text{initial}} = \frac{88000}{7} \times 10^{-6}$$

$$A_{\text{initial}} \approx 12.57 \times 10^{-3} \text{ m}^2$$

**Step 3:** Calculate final surface area.

The final drop has a total volume equal to the sum of all small drops:

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

Cancelling $\frac{4}{3}\pi$ from both sides:

$$R^3 = 1000r^3$$

$$R = 10r = 10 \times 1 \text{ mm} = 10 \text{ mm} = 10^{-2} \text{ m}$$

Final surface area:

$$A_{\text{final}} = 4\pi R^2$$

$$A_{\text{final}} = 4 \times \frac{22}{7} \times (10^{-2})^2$$

$$A_{\text{final}} = 4 \times \frac{22}{7} \times 10^{-4}$$

$$A_{\text{final}} = \frac{880}{7} \times 10^{-6}$$

$$A_{\text{final}} \approx 1.257 \times 10^{-3} \text{ m}^2$$

**Step 4:** Compute the released surface energy.

Change in surface area:

$$\Delta A = A_{\text{initial}} - A_{\text{final}}$$

$$\Delta A = (12.57 - 1.257) \times 10^{-3}$$

$$\Delta A = 11.313 \times 10^{-3} \text{ m}^2$$

Energy released:

$$\Delta U = \text{Surface Tension} \times \Delta A$$

$$\Delta U = (0.07) \times (11.313 \times 10^{-3})$$

$$\Delta U = 7.92 \times 10^{-4} \text{ J}$$

**Final Answer:**

$$\boxed{7.92 \times 10^{-4} \text{ J}}$$

**Quick Tip**

For problems involving merging of droplets, remember: - Total volume remains constant. - Radius of new drop: $R = n^{1/3} r$. - Surface energy change: $\Delta U =$ Surface Tension $\times \Delta A$.

---

**6. The force between two small charged spheres having charges of $1 \times 10^{-7}$ C and $2 \times 10^{-7}$ C placed 20 cm apart in air is:**

(A) $4.5 \times 10^{-2}$ N

(B) $4.5 \times 10^{-3}$ N

(C) $5.4 \times 10^{-2}$ N

(D) $5.4 \times 10^{-3}$ N

**Correct Answer:** (B) $4.5 \times 10^{-3}$ N

**Solution:**

**Step 1:** Applying Coulomb's Law.

The electrostatic force between two charges is given by:

$$F = \frac{kq_1q_2}{r^2}$$

where: - $k = 9 \times 10^9$ N·m²/C² (Coulomb's constant), - $q_1 = 1 \times 10^{-7}$ C, - $q_2 = 2 \times 10^{-7}$ C, - $r = 20$ cm $= 0.2$ m.

**Step 2:** Substituting the values.

$$F = \frac{(9 \times 10^9)(1 \times 10^{-7})(2 \times 10^{-7})}{(0.2)^2}$$

$$F = \frac{(9 \times 10^9) \times (2 \times 10^{-14})}{0.04}$$

$$F = \frac{18 \times 10^{-5}}{4 \times 10^{-2}}$$

$$F = 4.5 \times 10^{-3} \text{ N}$$

**Final Answer:**

$$\boxed{4.5 \times 10^{-3} \text{ N}}$$

**Quick Tip**

For electrostatic force calculations using Coulomb's Law:

$$F = \frac{kq_1q_2}{r^2}$$

- Convert all units to SI units (meters, Coulombs, Newtons). - Use $k = 9 \times 10^9$ N·m²/C² for air/vacuum.

---

**7. The work done in placing a charge of $8 \times 10^{-18}$ coulomb on a capacitor of capacitance 100 microfarad is:**

(A) $3.1 \times 10^{-26}$ joule

(B) $4 \times 10^{-10}$ joule

(C) $32 \times 10^{-32}$ joule

(D) $16 \times 10^{-32}$ joule

**Correct Answer:** (C) $32 \times 10^{-32}$ joule

**Solution:**

**Step 1:** Formula for work done in charging a capacitor

The work done $W$ in charging a capacitor is given by:

$$W = \frac{Q^2}{2C}$$

where: $Q = 8 \times 10^{-18}$ C (charge), $C = 100 \times 10^{-6}$ F (capacitance).

**Step 2:** Substituting values

$$W = \frac{(8 \times 10^{-18})^2}{2 \times (100 \times 10^{-6})}$$
$$= \frac{64 \times 10^{-36}}{2 \times 100 \times 10^{-6}}$$
$$= \frac{64 \times 10^{-36}}{200 \times 10^{-6}}$$
$$= 32 \times 10^{-32} \text{ joule}$$

**Quick Tip**

The formula $W = \frac{Q^2}{2C}$ is useful for calculating the energy stored in a capacitor when charge and capacitance are known.

---

**8. The resistance of a wire is $5\Omega$. What will be its new resistance in ohms if stretched to 5 times its original length?**

(A) 625

(B) 5

(C) 125

(D) 25

**Correct Answer:** (C) $125\Omega$

**Solution:**

**Step 1:** Understanding resistance change in a stretched wire.

The resistance of a wire is given by:

$$R = \rho \frac{L}{A}$$

where: - $\rho$ is the resistivity (constant for the material), - $L$ is the length, - $A$ is the cross-sectional area.

When a wire is stretched to $n$ times its original length, its volume remains constant:

$$\text{Initial Volume} = \text{Final Volume}$$

Since volume is $AL$, we get:

$$A_{\text{new}} L_{\text{new}} = A_{\text{old}} L_{\text{old}}$$

Given $L_{\text{new}} = 5L_{\text{old}}$, the new cross-sectional area becomes:

$$A_{\text{new}} = \frac{A_{\text{old}}}{5}$$

**Step 2:** Finding new resistance.

$$R_{\text{new}} = \rho \frac{L_{\text{new}}}{A_{\text{new}}} = \rho \frac{5L}{A/5}$$

$$R_{\text{new}} = \rho \frac{5L}{A} \times 5 = 25R$$

Since the original resistance is $R = 5\Omega$,

$$R_{\text{new}} = 25 \times 5 = 125\Omega$$

**Final Answer:**

$$\boxed{125\Omega}$$

Quick Tip

For a wire stretched $n$ times its original length:

$$R_{\text{new}} = n^2 R_{\text{old}}$$

where $n$ is the stretch factor.

---

**9. A charged particle is moving in a uniform magnetic field $\mathbf{B} = 2\hat{i} + 3\hat{j}$ T. If it has an acceleration of $\mathbf{a} = \alpha\hat{i} - 4\hat{j}$ m/s², then the value of $\alpha$ will be:**

(A) 3

(B) 6

(C) 12

(D) 2

**Correct Answer:** (B) 6

**Solution:**

**Step 1:** Understanding the force on a charged particle in a magnetic field.

The Lorentz force is given by:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

Since force is also related to acceleration:

$$m\mathbf{a} = q(\mathbf{v} \times \mathbf{B})$$

Thus,

$$\mathbf{a} = \frac{q}{m}(\mathbf{v} \times \mathbf{B})$$

**Step 2:** Computing the cross product $\mathbf{v} \times \mathbf{B}$.

Let $\mathbf{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ and $\mathbf{B} = 2\hat{i} + 3\hat{j}$, then:

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 2 & 3 & 0 \end{vmatrix}$$

Expanding the determinant:

$$\mathbf{v} \times \mathbf{B} = \hat{i}(0v_y - 3v_z) - \hat{j}(0v_x - 2v_z) + \hat{k}(v_x 3 - v_y 2)$$

$$= (-3v_z)\hat{i} + (2v_z)\hat{j} + (3v_x - 2v_y)\hat{k}$$

Since $\mathbf{a} = \frac{q}{m}(\mathbf{v} \times \mathbf{B})$, equating components:

$$\alpha = -3v_z\frac{q}{m}, \quad -4 = 2v_z\frac{q}{m}$$

Solving for $v_z$:

$$v_z = -\frac{4}{2}\frac{m}{q} = -2\frac{m}{q}$$

Substituting into $\alpha = -3v_z\frac{q}{m}$:

$$\alpha = -3 \times (-2) = 6$$

**Final Answer:**

$$\boxed{6}$$

**Quick Tip**

For charged particles in a magnetic field: - Use $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. - Solve using determinant expansion for cross product. - Acceleration components give constraints on velocity components.

---

**10. A proton ($p$) and an electron ($e$) will have the same de-Broglie wavelength when the ratio of their momenta is (Assume $m_p = 1849m_e$):**

(A) $1 : 43$

(B) $43 : 1$

(C) $1 : 1849$

(D) $1 : 1$

**Correct Answer:** (D) $1 : 1$

**Solution:**

**Step 1:** Understanding de-Broglie wavelength formula.

The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{p}$$

where: - $\lambda$ is the wavelength, - $h$ is Planck's constant, - $p$ is the momentum.

For two particles to have the same wavelength, their momentum must be equal:

$$\lambda_p = \lambda_e \Rightarrow \frac{h}{p_p} = \frac{h}{p_e}$$

$$p_p = p_e$$

**Step 2:** Interpreting the given mass ratio.

Given:

$$m_p = 1849 m_e$$

Since momentum is $p = mv$, for equal de-Broglie wavelengths, the momentum must be the same for both:

$$p_p = p_e$$

Thus, the ratio of their momenta:

$$\frac{p_p}{p_e} = 1 : 1$$

**Final Answer:**

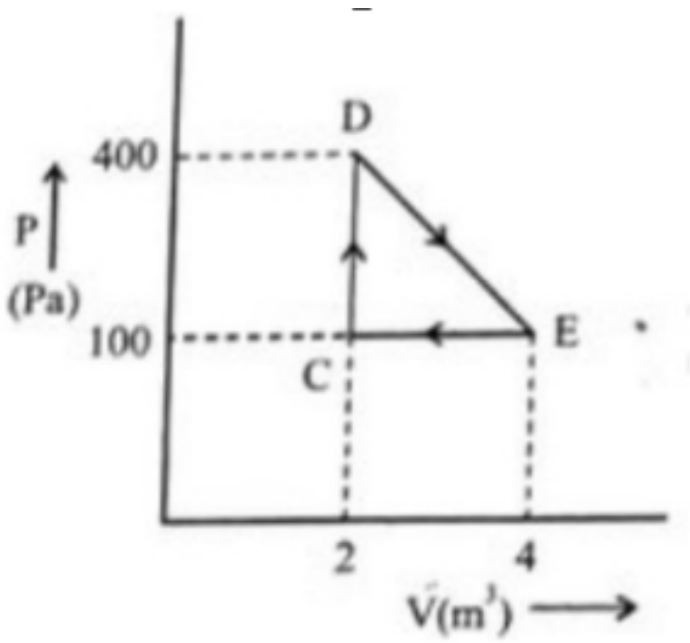
$$\boxed{1 : 1}$$

Quick Tip

For particles to have the same de-Broglie wavelength, their momentum must be equal, regardless of their mass difference.

---

**11. A thermodynamic system is taken through a cyclic process as shown in the PV diagram. The total work done in the process is:**




(A) 100 J

(B) 300 J

(C) 0

(D) 200 J

**Correct Answer:** (B) 300J

**Solution:**

Work done = **Area under the curve**

$$\Rightarrow W = \frac{1}{2} \times (4-2) \times (400-100) = \frac{1}{2} \times 2 \times 300$$

$$W = 300J$$

Quick Tip

In a $P - V$ diagram, the work done during a cyclic process is the area enclosed by the curve. Positive work is done when the system expands, and negative work is done when it compresses.

---

**12. In a reflecting telescope, a secondary mirror is used to:**

(A) Reduce the problem of mechanical support

(B) Remove spherical aberration

(C) Make chromatic aberration zero

(D) Move the eyepiece outside the telescopic tube

**Correct Answer:** (D) Move the eyepiece outside the telescopic tube

**Solution:**

**Step 1:** Understanding the purpose of a secondary mirror.

- A reflecting telescope primarily uses a concave primary mirror to collect and focus light. - A secondary mirror is placed at an intermediate point to redirect the focused light to an eyepiece.

**Step 2:** Function of the secondary mirror.

- The primary mirror focuses light inside the telescope tube, making it difficult to place an eyepiece directly. - The secondary mirror redirects the light outside the tube, allowing for convenient viewing. - This design is seen in the Cassegrain and Newtonian telescopes.

**Step 3:** Evaluating other Option.

- (A) Reducing mechanical support problems $\rightarrow$ Incorrect. The secondary mirror is not primarily used for structural support. - (B) Removing spherical aberration $\rightarrow$ Incorrect. Spherical aberration is corrected by parabolic mirrors, not by the secondary mirror. - (C) Making chromatic aberration zero $\rightarrow$ Incorrect. Chromatic aberration is caused by lenses, but reflecting telescopes use mirrors, which naturally eliminate this issue.

**Final Answer:**

Move the eyepiece outside the telescopic tube

Quick Tip

In reflecting telescopes, the secondary mirror is used to: - Redirect light to a more accessible viewing position. - Enable a compact design (Cassegrain configuration). - Prevent obstruction by placing the eyepiece outside the main tube.

---

**13. The magnetic moment of an electron (e) revolving in an orbit around the nucleus with an orbital angular momentum is given by:**

(A) $\vec{\mu}_L = \frac{e\vec{L}}{2m}$

(B) $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$

(C) $\vec{\mu}_L = -\frac{e\vec{L}}{m}$

(D) $\vec{\mu}_L = \frac{2e\vec{L}}{m}$

**Correct Answer:** (A) $\vec{\mu}_L = \frac{e\vec{L}}{2m}$

**Solution:**

**Step 1:** The magnetic moment $\vec{\mu}_L$ associated with the orbital angular momentum $\vec{L}$ of an electron is given by the equation:

$$\vec{\mu}_L = \frac{e\vec{L}}{2m}$$

where: $e$ is the charge of the electron, $\vec{L}$ is the orbital angular momentum (a vector), $m$ is the mass of the electron.

**Step 2:** Explanation of the formula. The electron in motion generates a magnetic field, and the magnetic moment is proportional to its orbital angular momentum. The factor $\frac{1}{2m}$ arises from the relation between the electron's angular momentum and the induced magnetic moment in orbital motion.

**Step 3:** Verifying the correct answer. Thus, the correct expression for the magnetic moment is $\vec{\mu}_L = \frac{e\vec{L}}{2m}$, which matches option (A).

Quick Tip

The magnetic moment due to the orbital motion of an electron is proportional to the orbital angular momentum and inversely proportional to the mass of the electron.

---

**14. The ratio of intensities at two points $P$ and $Q$ on the screen in a Young's double-slit experiment, where the phase difference between two waves of the same amplitude are $\frac{\pi}{3}$ and $\frac{\pi}{2}$, respectively, is:**

(A) $1:3$

(B) $3:1$

(C) $3:2$

(D) $2:3$

**Correct Answer:** (C) $3:2$

**Solution:**

**Step 1:** Understanding intensity in Young's Double-Slit Experiment.

The intensity at any point in an interference pattern is given by:

$$I = I_0 \left(1 + \cos \Delta\phi\right)$$

where: - $I_0$ is the intensity of an individual wave, - $\Delta\phi$ is the phase difference between the two waves.

**Step 2:** Calculating intensity at $P$ (where $\Delta\phi = \frac{\pi}{3}$).

$$I_P = I_0 \left(1 + \cos \frac{\pi}{3}\right)$$

Since $\cos \frac{\pi}{3} = \frac{1}{2}$,

$$I_P = I_0 \left(1 + \frac{1}{2}\right) = I_0 \times \frac{3}{2} = \frac{3I_0}{2}$$

**Step 3:** Calculating intensity at $Q$ (where $\Delta\phi = \frac{\pi}{2}$).

$$I_Q = I_0 \left(1 + \cos \frac{\pi}{2}\right)$$

Since $\cos \frac{\pi}{2} = 0$,

$$I_Q = I_0 \left(1 + 0\right) = I_0$$

**Step 4:** Finding the ratio $I_P : I_Q$.

$$\frac{I_P}{I_Q} = \frac{\frac{3I_0}{2}}{I_0} = \frac{3}{2}$$

Thus, the ratio of intensities:

$$I_P : I_Q = 3 : 2$$

**Final Answer:**

$$\boxed{3 : 2}$$

**Quick Tip**

In Young's Double-Slit Experiment: - Intensity at a point is given by $I = I_0(1+\cos\Delta\phi)$. - For phase differences: - $\Delta\phi = \frac{\pi}{3} \Rightarrow I = \frac{3I_0}{2}$. - $\Delta\phi = \frac{\pi}{2} \Rightarrow I = I_0$. - Intensity ratio is determined using $I_P : I_Q$.

---

**15. A bicycle tire is filled with air at a pressure of 270 kPa at $27°C$. What is the approximate pressure of the air in the tire when the temperature increases to $36°C$?**

(A) 270 kPa

(B) 262 kPa

(C) 278 kPa

(D) 360 kPa

**Correct Answer:** (C) 278 kPa

**Solution:**

**Step 1:** Using Gay-Lussac's Law (Pressure-Temperature Relationship).

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

where: - $P_1 = 270$ kPa (initial pressure), - $T_1 = 27°C = (27 + 273) = 300K$ (initial temperature), - $T_2 = 36°C = (36 + 273) = 309K$ (final temperature), - $P_2$ is the final pressure.

**Step 2:** Solve for $P_2$.

$$P_2 = P_1 \times \frac{T_2}{T_1}$$

$$P_2 = 270 \times \frac{309}{300}$$

$$P_2 = 270 \times 1.03$$

$$P_2 = 278.1 \text{ kPa} \approx 278 \text{ kPa}$$

**Final Answer:**

$$\boxed{278 \text{ kPa}}$$

**Quick Tip**

For gases at constant volume, use Gay-Lussac's Law:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

where temperatures must be in Kelvin. Pressure changes in direct proportion to temperature.

---

**16. A particle executes Simple Harmonic Motion (SHM) with amplitude $A$. The distance from the mean position when its kinetic energy becomes equal to its potential energy is:**

(A) $\sqrt{2}A$

(B) $2A$

(C) $\frac{A}{\sqrt{2}}$

(D) $\frac{A}{2}$

**Correct Answer:** (C) $\frac{A}{\sqrt{2}}$

**Solution:**

**Step 1:** Understanding the energy distribution in SHM.

- The total energy in SHM is given by:

$$E = \frac{1}{2}kA^2$$

where: - $k$ is the force constant (spring constant), - $A$ is the amplitude.

- The kinetic energy (KE) at displacement $x$ is:

$$KE = \frac{1}{2}k(A^2 - x^2)$$

- The potential energy (PE) at displacement $x$ is:

$$PE = \frac{1}{2}kx^2$$

**Step 2:** Equating kinetic and potential energy.

$$KE = PE$$

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$$

Canceling $\frac{1}{2}k$:

$$A^2 - x^2 = x^2$$

$$A^2 = 2x^2$$

$$x^2 = \frac{A^2}{2}$$

$$x = \frac{A}{\sqrt{2}}$$

**Final Answer:**

$$\boxed{\frac{A}{\sqrt{2}}}$$

Quick Tip

In SHM, when kinetic energy = potential energy, the displacement is:

$$x = \frac{A}{\sqrt{2}}$$

This occurs because total energy is equally shared between kinetic and potential forms.

**17. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}\right)$. The SI unit of A and B are:**

(A) $\text{Nm}^3\text{C}^{-1}$; $\text{Nm}^2\text{C}^{-1}$

(B) $\text{Nm}^2\text{C}^{-1}$; $\text{Nm}^3\text{C}^{-1}$

(C) $\text{Nm}^3\text{C}$; $\text{Nm}^2\text{C}$

(D) $\text{Nm}^2\text{C}$; $\text{Nm}^3\text{C}$

**Correct Answer:** (B) $\text{Nm}^2\text{C}^{-1}$; $\text{Nm}^3\text{C}^{-1}$

**Solution:**

**Step 1:** The electric field is given as:

$$\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}\right)$$

Here, $\vec{E}$ has units of electric field, which in SI is measured in volts per meter (V/m), or equivalently, Newtons per Coulomb (N/C).

**Step 2:** Determining the units of A and B. We know that electric field $\vec{E}$ has units of N/C. Let's consider each component:

1. For the $\hat{i}$ component:

$$\frac{A}{x^2} \quad \text{has units of} \quad \frac{A}{\text{m}^2} \quad \Rightarrow \quad A = \text{Nm}^2\text{C}^{-1}$$

2. For the $\hat{j}$ component:

$$\frac{B}{y^3} \quad \text{has units of} \quad \frac{B}{\text{m}^3} \quad \Rightarrow \quad B = \text{Nm}^3\text{C}^{-1}$$

**Step 3:** Verifying the correct answer. The SI units of $A$ and $B$ are $\text{Nm}^2\text{C}^{-1}$ and $\text{Nm}^3\text{C}^{-1}$, respectively, which matches option (B).

**Quick Tip**

In electromagnetism, the electric field is expressed in terms of its components in different directions. The units of the constants in these equations depend on the power of the distance variables in the denominator and the overall unit of electric field.

---

**18. At any instant the velocity of a particle of mass 500g is $\left(2t\hat{i} + 3t^2\hat{j}\right)$ $\text{ms}^{-1}$. If the force acting on the particle at $t = 1$ s is $\left(\hat{i} + x\hat{j}\right)$ N, then the value of $x$ will be:**

(A) 3

(B) 4

(C) 6

(D) 2

**Correct Answer:** (A) 3

**Solution:**

**Step 1:** Using Newton's second law, $\vec{F} = m\vec{a}$, where $m$ is the mass and $\vec{a}$ is the acceleration. The given velocity of the particle is:

$$\vec{v} = (2t\hat{i} + 3t^2\hat{j})\,\text{ms}^{-1}$$

**Step 2:** The acceleration is the time derivative of the velocity:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(2t\hat{i} + 3t^2\hat{j}\right)$$

Taking the derivative:

$$\vec{a} = 2\hat{i} + 6t\hat{j}$$

**Step 3:** Substituting $t = 1$ s into the acceleration expression:

$$\vec{a} = 2\hat{i} + 6\hat{j}\,\text{ms}^{-2}$$

**Step 4:** The force $\vec{F}$ is given as:

$$\vec{F} = m\vec{a} = 0.5\,\text{kg} \times \left(2\hat{i} + 6\hat{j}\right) = 1\hat{i} + 3\hat{j}\,\text{N}$$

**Step 5:** Comparing this with the given force $\vec{F} = \left(\hat{i} + x\hat{j}\right)$ N, we can see that:

$$x = 3$$

Thus, the value of $x$ is 3.

**Quick Tip**

To find the acceleration, take the derivative of the velocity with respect to time. The force is then found by multiplying the mass with the acceleration.

---

**19. A particle of mass $m$ moving with velocity $v$ collides with a stationary particle of mass $2m$. After the collision, they stick together and continue to move with velocity:**

(A) $v$

(B) $\frac{v}{2}$

(C) $\frac{v}{3}$

(D) $\frac{v}{4}$

**Correct Answer:** (C) $\frac{v}{3}$

**Solution:**

**Step 1:** Applying the Law of Conservation of Momentum.

The total momentum before collision is:

$$P_{\text{initial}} = mv + (2m \times 0) = mv$$

Since the two masses stick together after collision, their combined mass is:

$$M = m + 2m = 3m$$

Let the final velocity be $V_f$. According to conservation of momentum:

$$mv = (3m)V_f$$

**Step 2:** Solving for $V_f$.

$$V_f = \frac{mv}{3m} = \frac{v}{3}$$

**Final Answer:**

$$\boxed{\frac{v}{3}}$$

**Quick Tip**

For completely inelastic collisions (when objects stick together after collision), use:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)V_f$$

where $V_f$ is the common velocity after collision.

**20. Which of the following Maxwell's equations is valid for time varying conditions but not valid for static conditions:**

(A) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

(B) $\oint \vec{E} \cdot d\vec{l} = 0$

(C) $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \vec{B}}{\partial t}$

(D) $\oint \vec{D} \cdot d\vec{A} = Q$

**Correct Answer:** (C) $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \vec{B}}{\partial t}$

**Solution:**

**Step 1:** Maxwell's equations for time-varying conditions: One of the key equations that is valid only for time-varying conditions is Faraday's Law of Induction. It states that:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \vec{B}}{\partial t}$$

This equation describes how a changing magnetic field induces an electric field.

**Step 2:** Understanding the options:

- Option (A): This is Ampère's Law for magnetostatics, where the integral of $\vec{B}$ around a closed loop is equal to the current enclosed. This equation is valid under static conditions as well. - Option (B): This is one form of the conservative electric field equation, valid for electrostatics, where the line integral of $\vec{E}$ is zero for static conditions. - Option (C): This is the correct choice, as it describes Faraday's Law of Induction, which is valid only for time-varying conditions. - Option (D): This is Gauss's Law for electricity, valid for both static and dynamic conditions.

**Step 3:** Verifying the correct answer. The equation $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \vec{B}}{\partial t}$ is the correct choice as it applies only in time-varying situations (changing magnetic field).

**Quick Tip**

Faraday's Law of Induction is a cornerstone of electromagnetism, describing how electric fields are generated by time-varying magnetic fields. It is not valid under static conditions.

---

**21. In an LC oscillator, if the values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of the oscillator becomes $x$ times**

**its initial resonant frequency $\omega_0$. The value of $x$ is:**

(A) $\frac{1}{4}$

(B) $16$

(C) $\frac{1}{16}$

(D) $4$

**Correct Answer:** (A) $\frac{1}{4}$

**Solution:**

**Step 1:** Understanding the formula for resonant frequency.

The resonant frequency $\omega$ of an LC oscillator is given by:

$$\omega = \frac{1}{\sqrt{LC}}$$

where: - $L$ is the inductance, - $C$ is the capacitance.

**Step 2:** Initial resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{L_0 C_0}}$$

**Step 3:** New resonant frequency after changes.

Given: - $L' = 2L_0$, - $C' = 8C_0$,

the new resonant frequency is:

$$\omega' = \frac{1}{\sqrt{L'C'}}$$

$$\omega' = \frac{1}{\sqrt{(2L_0)(8C_0)}}$$

$$\omega' = \frac{1}{\sqrt{16 L_0 C_0}}$$

$$\omega' = \frac{1}{4} \times \frac{1}{\sqrt{L_0 C_0}}$$

$$\omega' = \frac{\omega_0}{4}$$

**Step 4:** Finding $x$.

Since $\omega' = \frac{\omega_0}{4}$, we get:

$$x = \frac{1}{4}$$

**Final Answer:**

$$\boxed{\frac{1}{4}}$$

**Quick Tip**

For LC oscillators:

$$\omega = \frac{1}{\sqrt{LC}}$$

If $L$ is multiplied by $a$ and $C$ by $b$, the new frequency is:

$$\omega' = \frac{\omega_0}{\sqrt{ab}}$$

---

**22. A conducting loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25s is:**

(A) emf = 1 mV

(B) emf = 10 mV

(C) emf = 100 mV

(D) emf = 5 mV

**Correct Answer:** (B) emf = 10 mV

**Solution:**

As $\varepsilon\big|_{t=0.5 \text{ sec}} = -\frac{d\phi}{dt}$

$$= -A\frac{dB}{dt} \quad [\because \theta = 0^\circ \Rightarrow \cos\theta = 1]$$

$$= -\pi \times \left(\frac{10}{\sqrt{\pi}}\right)^2 \times 10^{-4} \times \frac{0 - 0.5}{0.5} = 10^{-2}V = 10 \text{ mV}$$

As $\frac{dB}{dt} = \text{constant} \Rightarrow$ Induced emf will not change with time. So,

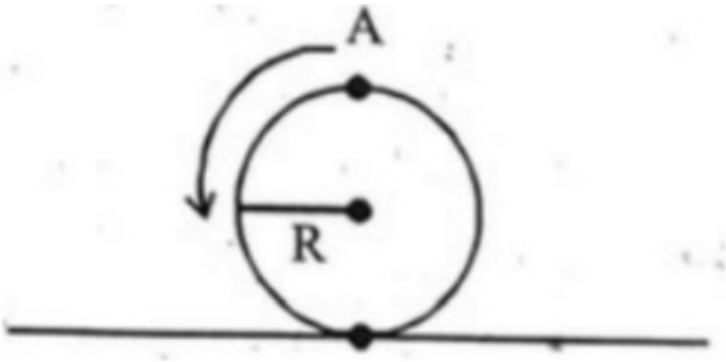
$$e\big|_{0.5 \text{ sec}} = e\big|_{0.25 \text{ sec}} = 10 \text{ mV}$$

**Quick Tip**

The induced emf is directly proportional to the rate of change of the magnetic flux through the loop. The flux depends on both the magnetic field and the area of the loop.

---

**23. A disc is rolling without slipping on a surface. The radius of the disc is $R$. At $t = 0$, the topmost point on the disc is $A$ as shown in the figure. When the disc completes half of its rotation, the displacement of point $A$ from its initial position is:**




(A) $R\sqrt{\pi^2 + 4}$

(B) $R\sqrt{\pi^2 + 1}$

(C) $2R$

(D) $2R\sqrt{1 + 4\pi^2}$

**Correct Answer:** (A) $R\sqrt{\pi^2 + 4}$

**Solution:**

**Step 1:** Understand the motion of the point A. When the disc rolls without slipping, every point on the disc follows a cycloidal path. Point A starts at the topmost point of the disc and traces an arc during the motion.

**Step 2:** Displacement after half a rotation. When the disc completes half of its rotation, the point A will have traveled a horizontal distance equal to the arc length of half the disc. This arc length is equal to the circumference of the disc divided by two, which is $\pi R$.

The displacement of point A is the distance it has moved in both horizontal and vertical directions, and these form the two sides of a right triangle.

**Step 3:** Calculating the displacement. The horizontal displacement is $\pi R$, and the vertical displacement is also $R$. Using the Pythagorean theorem, the displacement $d$ is:

$$d = \sqrt{(\pi R)^2 + R^2} = R\sqrt{\pi^2 + 1}$$

However, we also have an additional displacement because of the rolling motion, which contributes $2R$ to the total displacement. Hence, the total displacement is:

$$d = R\sqrt{\pi^2 + 4}$$

Thus, the displacement of point A from its initial position is $R\sqrt{\pi^2 + 4}$.

**Quick Tip**

The displacement of a point on a rolling disc combines both the rotational and translational motion. This results in a cycloidal path, and the displacement after half a rotation can be calculated using the Pythagorean theorem.

---

**24. Two planets $A$ and $B$ have radii $R$ and $1.5R$, and densities $\rho$ and $\frac{\rho}{2}$ respectively. The ratio of acceleration due to gravity at the surface of $B$ to $A$ is:**

(A) $2 : 3$

(B) $2 : 1$

(C) $3 : 4$

(D) $4 : 3$

**Correct Answer:** (C) $3 : 4$

**Solution:**

**Step 1:** Formula for acceleration due to gravity.

The surface gravity of a planet is given by:

$$g = \frac{GM}{R^2}$$

Since mass $M$ is related to density $\rho$ and radius $R$:

$$M = \rho V = \rho \times \frac{4}{3}\pi R^3$$

Substituting into the gravity formula:

$$g = \frac{G(\rho\frac{4}{3}\pi R^3)}{R^2}$$

$$g = \frac{4}{3}G\pi\rho R$$

Thus, gravity at the surface is:

$$g \propto \rho R$$

**Step 2:** Finding the ratio $\frac{g_B}{g_A}$.

For planet A:

$$g_A \propto \rho R$$

For planet B: - Radius $R_B = 1.5R$, - Density $\rho_B = \frac{\rho}{2}$,

$$g_B \propto \left(\frac{\rho}{2}\right)(1.5R)$$

$$g_B \propto \frac{3}{2} \times \frac{\rho R}{2} = \frac{3}{4}\rho R$$

Taking the ratio:

$$\frac{g_B}{g_A} = \frac{\frac{3}{4}\rho R}{\rho R} = \frac{3}{4}$$

**Final Answer:**

$$\boxed{3:4}$$

**Quick Tip**

For a spherical planet:

$$g \propto \rho R$$

- Increasing radius increases gravity. - Increasing density increases gravity. - The ratio $\frac{g_B}{g_A}$ is found using $g \propto \rho R$.

**25. A 100m long wire having cross-sectional area $6.25 \times 10^{-4}\,\text{m}^2$ and Young's modulus is $10^{10}\,\text{Nm}^{-2}$ is subjected to a load of 250N, then the elongation in the wire will be:**

(A) $6.25 \times 10^{-3}\,\text{m}$

(B) $4 \times 10^{-4}\,\text{m}$

(C) $6.25 \times 10^{-6}\,\text{m}$

(D) $4 \times 10^{-3}\,\text{m}$

**Correct Answer:** (D) $4 \times 10^{-3}\,\text{m}$

**Solution:**

**Step 1:** Formula for elongation. The elongation $\Delta L$ in the wire can be calculated using the formula:

$$\Delta L = \frac{FL}{AY}$$

where: $F = 250\,\text{N}$ is the applied force, $L = 100\,\text{m}$ is the length of the wire, $A = 6.25 \times 10^{-4}\,\text{m}^2$ is the cross-sectional area of the wire, $Y = 10^{10}\,\text{Nm}^{-2}$ is Young's modulus.

**Step 2:** Substituting the values.

$$\Delta L = \frac{250 \times 100}{6.25 \times 10^{-4} \times 10^{10}} = \frac{25000}{6.25 \times 10^{6}} = 4 \times 10^{-3}\,\text{m}$$

Thus, the elongation in the wire is $4 \times 10^{-3}\,\text{m}$.

**Quick Tip**

The elongation in a wire is directly proportional to the applied force and the original length, and inversely proportional to the cross-sectional area and Young's modulus.

---

**26. The ratio of the speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is:**

(A) $4:1$

(B) $1:2$

(C) $1:4$

(D) $1:1$

**Correct Answer:** (A) $4:1$

**Solution:**

**Step 1:** Using the formula for the speed of sound in a gas.

The speed of sound in a gas is given by:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where: - $v$ = speed of sound, - $\gamma$ = adiabatic index (assumed same for both gases), - $R$ = universal gas constant, - $T$ = temperature (same for both gases), - $M$ = molar mass of the gas. Since $\gamma$, $R$, and $T$ are the same for both gases, the speed of sound is inversely proportional to the square root of the molar mass:

$$v \propto \frac{1}{\sqrt{M}}$$

**Step 2:** Finding the molar masses.

- Molar mass of hydrogen ($H_2$): $M_H = 2$ g/mol. - Molar mass of oxygen ($O_2$): $M_O = 32$ g/mol.

**Step 3:** Taking the ratio of speeds.

$$\frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}}$$

$$\frac{v_H}{v_O} = \sqrt{\frac{32}{2}}$$

$$\frac{v_H}{v_O} = \sqrt{16} = 4$$

**Final Answer:**

$$\boxed{4:1}$$

**Quick Tip**

The speed of sound in a gas varies as:

$$v \propto \frac{1}{\sqrt{M}}$$

- A lighter gas (lower molar mass) has a higher speed of sound. - The speed of sound in hydrogen is 4 times that in oxygen.

**27. The free space inside a current carrying toroid is filled with a material of susceptibility $\chi = 2 \times 10^{-2}$. The percentage increase in the value of magnetic field inside the toroid will be:**

(A) 2%

(B) 0.2%

(C) 0.1%

(D) 1%

**Correct Answer:** (A) 2%

**Solution:**

The magnetic field $B$ inside a toroid depends on the permeability of the material filling the toroid. The relation between the permeability of the material and the permeability of free space is given by:

$$\mu = \mu_0(1 + \chi)$$

where:

- $\mu_0$ is the permeability of free space, - $\chi$ is the susceptibility of the material, - $\mu$ is the permeability of the material inside the toroid.

The magnetic field inside the toroid is given by:

$$B = \frac{\mu I}{2\pi r}$$

where:

- $I$ is the current, - $r$ is the radius of the toroid, - $\mu$ is the permeability of the material inside the toroid.

Now, the percentage increase in the magnetic field when the toroid is filled with the material is given by:

$$\text{Percentage increase in } B = \frac{\Delta B}{B_{\text{initial}}} \times 100$$

The initial magnetic field $B_{\text{initial}}$ is given by $B_{\text{initial}} = \frac{\mu_0 I}{2\pi r}$, and the final magnetic field $B_{\text{final}}$ is given by $B_{\text{final}} = \frac{\mu_0(1+\chi)I}{2\pi r}$.

Thus, the percentage increase in the magnetic field is:

$$\frac{\Delta B}{B_{\text{initial}}} = \frac{B_{\text{final}} - B_{\text{initial}}}{B_{\text{initial}}} = \frac{\frac{\mu_0(1+\chi)I}{2\pi r} - \frac{\mu_0 I}{2\pi r}}{\frac{\mu_0 I}{2\pi r}} = \chi$$

Substituting $\chi = 2 \times 10^{-2}$:

$$\text{Percentage increase in } B = 2 \times 10^{-2} \times 100 = 2\%$$

Thus, the percentage increase in the magnetic field is $\boxed{2\%}$.

**Quick Tip**

For a material with susceptibility $\chi$, the percentage increase in the magnetic field inside a toroid is equal to the susceptibility $\chi$ expressed as a percentage.

---

**28. The ratio of average electric energy density and total average energy density of an electromagnetic wave is:**

(A) 2

(B) $\frac{1}{2}$

(C) 1

(D) 3

**Correct Answer:** (B) $\frac{1}{2}$

**Solution:**

**Step 1:** Formula for energy densities in an electromagnetic wave. The total energy density $u_{\text{total}}$ in an electromagnetic wave is the sum of the electric and magnetic energy densities. The electric energy density $u_E$ and magnetic energy density $u_B$ are given by:

$$u_E = \frac{\epsilon_0 E^2}{2}, \quad u_B = \frac{B^2}{2\mu_0}$$

where $\epsilon_0$ is the permittivity of free space, $\mu_0$ is the permeability of free space, $E$ is the electric field, and $B$ is the magnetic field.

**Step 2:** Relationship between electric and magnetic fields. In an electromagnetic wave, the energy density is equally distributed between the electric and magnetic fields, so:

$$u_E = u_B$$

**Step 3:** Total energy density. The total energy density is the sum of the electric and magnetic energy densities:

$$u_{\text{total}} = u_E + u_B = 2u_E$$

**Step 4:** Ratio of average electric energy density to total energy density. The ratio is:

$$\frac{u_E}{u_{\text{total}}} = \frac{u_E}{2u_E} = \frac{1}{2}$$

Thus, the ratio of average electric energy density to total average energy density is $\frac{1}{2}$.

**Quick Tip**

In an electromagnetic wave, the electric and magnetic energy densities are equal, and the total energy density is twice the electric energy density.

---

**29. In a Young's double slit experiment, the intensities at two points, for the path difference $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ ($\lambda$ being the wavelength of light used) are $I_1$ and $I_2$ respectively. If $I_0$ denotes the intensity produced by each one of the individual slits, then $\frac{I_1+I_2}{I_0}$ is equal to:**

(A) 3

(B) 5

(C) 7

(D) 10

**Correct Answer:** (A) 3

**Resultant intensity in Young's double slit experiment**

$$I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$$

**For path difference $\frac{\lambda}{4}$, phase difference:**

$$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{4}$$

$$\therefore I_1 = 4I_0 \cos^2\left(\frac{\pi}{4}\right) = 2I_0$$

**For path difference $\frac{\lambda}{3}$:**

$$I_2 = 4I_0 \cos^2\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{3}\right) = I_0$$

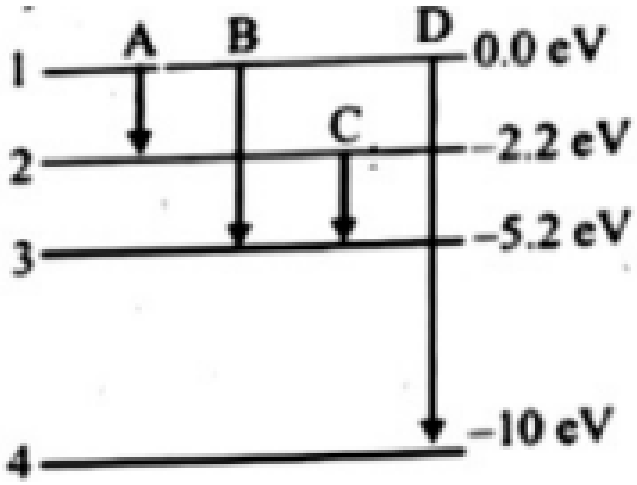
$$\therefore \frac{I_1 + I_2}{I_0} = 3$$

**Quick Tip**

In Young's double slit experiment, intensity is given by $I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$. Use the relation between path difference and phase difference to simplify calculations.

---

**30. The energy levels of an atom are shown in the figure. Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm? Given $h = 6.62 \times 10^{-34}$ Js.**




(A) B

(B) A

(C) C

(D) D

**Correct Answer:** (D) D

**Solution:**

The energy of a photon emitted during a transition is related to the wavelength of the photon by the equation:

$$E = \frac{hc}{\lambda}$$

where: - $E$ is the energy of the photon, - $h$ is Planck's constant ($6.62 \times 10^{-34}$ Js), - $c$ is the speed of light ($3 \times 10^8$ m/s), - $\lambda$ is the wavelength of the photon.

Step 1: Calculate the energy of the photon. Given that $\lambda = 124.1\,\text{nm} = 124.1 \times 10^{-9}\,\text{m}$, we can substitute the values into the equation:

$$E = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{124.1 \times 10^{-9}}$$

$$E = \frac{1.986 \times 10^{-25}}{124.1 \times 10^{-9}} = 1.6 \times 10^{-18}\,\text{J}$$

Step 2: Convert energy from joules to electron volts. Since $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$, we convert the energy:

$$E = \frac{1.6 \times 10^{-18}}{1.6 \times 10^{-19}} = 10\,\text{eV}$$

Step 3: Check the energy differences between the levels. Now, we check the energy differences between the levels:

- A to B : $0 - (-2.2) = 2.2\,\text{eV}$ - A to C : $0 - (-5.2) = 5.2\,\text{eV}$ - A to D : $0 - (-10) = 10\,\text{eV}$

The transition from level A to level D gives the energy of 10 eV, which matches the energy of the photon calculated.

Thus, the correct transition is $A \to D$.

**Quick Tip**

The energy of the photon is calculated using $E = \frac{hc}{\lambda}$, and the correct transition is determined by the energy difference between the levels.

---

**31. Frenkel and Schottky defects are:**

(A) Nucleus defects

(B) Non-crystal defects

(C) Crystal defects

(D) Nuclear defects

**Correct Answer:** (C) Crystal defects

**Solution:**

**Step 1:** Understanding Frenkel and Schottky defects.

Frenkel and Schottky defects are two types of point defects that occur in crystalline solids.

- Schottky Defect: - Occurs in ionic solids (e.g., NaCl, KCl). - Equal number of cations and anions are missing from the crystal lattice. - Results in a decrease in density of the crystal. - Frenkel Defect: - Occurs in ionic solids with large size differences between cations and anions (e.g., AgCl, ZnS). - A cation moves from its normal position to an interstitial site. - Does not affect density.

**Step 2:** Identifying the correct classification.

- These defects occur within the crystal structure, hence they are crystal defects. - They are not related to the nucleus or non-crystalline materials.

**Final Answer:**

Crystal defects

**Quick Tip**

- Schottky defect: Missing cations and anions $\rightarrow$ lowers density. - Frenkel defect: Cation dislocation $\rightarrow$ density remains unchanged.

---

**32. The Bohr orbit radius for the hydrogen atom ($n = 1$) is approximately 0.530 Å. The radius for the first excited state ($n = 2$) orbit is (in Å):**

(A) 0.13

(B) 1.06

(C) 4.77

(D) 2.12

**Correct Answer:** (D) 2.12

**Solution:**

The radius of the Bohr orbit for any state $n$ is given by the formula:

$$r_n = n^2 r_1$$

where: - $r_n$ is the radius of the orbit for the $n^{th}$ orbit, - $r_1$ is the radius of the Bohr orbit for $n = 1$, - $n$ is the principal quantum number.

Given that the radius for the ground state ($n = 1$) is $r_1 = 0.530\,\text{Å}$, we can calculate the radius for the first excited state ($n = 2$):

$$r_2 = 2^2 \times 0.530 = 4 \times 0.530 = 2.12\,\text{Å}$$

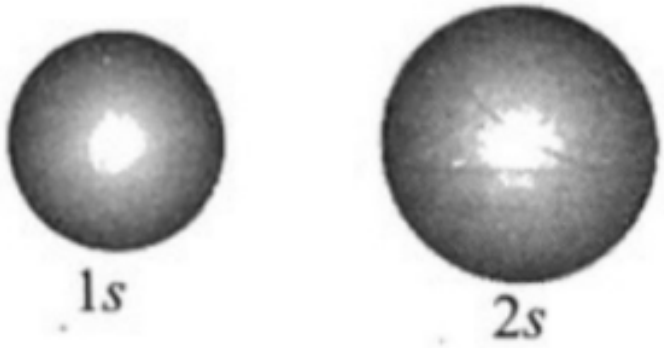
Thus, the radius for the first excited state ($n = 2$) is $2.12\,\text{Å}$.

**Quick Tip**

The radius of the Bohr orbit increases with the square of the principal quantum number, so the radius for the first excited state is four times the radius for the ground state.

---

**33. The probability density plots of 1s and 2s orbitals are given in figure.**




**The density of dots in a region represents the probability density of finding electrons in the region.**

**On the basis of the above diagram, which of the following statements is incorrect?**

(A) 1s and 2s orbitals are spherical in shape.

(B) The probability of finding the electron is maximum near the nucleus.

(C) The probability of finding the electron at a given distance is equal in all directions.

(D) The probability density of electrons for 2s orbital decreases uniformly as distance from the nucleus increases.

**Correct Answer:** (D) The probability density of electrons for 2s orbital decreases uniformly as distance from the nucleus increases.

**Solution: Step 1:** Analyze the given statements based on the properties of 1s and 2s orbitals.

- Statement A: 1s and 2s orbitals are spherical in shape. This is correct. Both 1s and 2s orbitals are spherically symmetric.

- Statement B: The probability of finding the electron is maximum near the nucleus. This is correct for the 1s orbital. However, for the 2s orbital, the probability density has a node (a

region where the probability density is zero) before increasing again. Thus, the probability is not maximum near the nucleus for the 2s orbital.

- Statement C: The probability of finding the electron at a given distance is equal in all directions. This is correct. Since both 1s and 2s orbitals are spherically symmetric, the probability density depends only on the distance from the nucleus, not on the direction.

- Statement D: The probability density of electrons for the 2s orbital decreases uniformly as the distance from the nucleus increases. This is incorrect. The probability density of the 2s orbital does not decrease uniformly. It has a node (a region of zero probability density) and then increases again before eventually decreasing.

**Conclusion:** The incorrect statement is (D).

**Quick Tip**

For atomic orbitals: - 1s and 2s orbitals are spherically symmetric. - The 2s orbital has a node where the probability density is zero. - The probability density depends only on the distance from the nucleus, not on the direction.

---

**34. Element with electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^3$ belongs to the following group of the periodic table:**

(A) 5th

(B) 15th

(C) 3rd

(D) 17th

**Correct Answer:** (B) 15th

**Solution: Step 1:** Identify the valence electrons in the given electronic configuration.

The electronic configuration is:

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^3.$$

The outermost shell is the 5th shell, and the valence electrons are in the $5s^2$ and $5p^3$ orbitals.

**Step 2:** Calculate the total number of valence electrons.

- Electrons in $5s^2$: 2 - Electrons in $5p^3$: 3

Total valence electrons = $2 + 3 = 5$.

**Step 3:** Determine the group number.

The number of valence electrons corresponds to the group number in the periodic table for the p-block elements. Since the element has 5 valence electrons, it belongs to Group 15.

**Conclusion:** The element belongs to the 15th group of the periodic table.

Quick Tip

For determining the group of an element: - Identify the outermost shell and count the valence electrons. - For p-block elements, the group number = number of valence electrons.

---

**35. Which of the following pairs will form the most stable ionic bond?**

(A) Na and Cl

(B) Mg and F

(C) Li and F

(D) Na and F

**Correct Answer:** (B) Mg and F

**Solution:**

The stability of an ionic bond depends on the difference in electronegativity between the two elements involved. A greater difference leads to a stronger ionic bond. Additionally, smaller ions tend to form more stable bonds due to the higher lattice energy.

**- Na and Cl:** Sodium (Na) has a lower ionization energy and chlorine (Cl) has a high electron affinity, but the ionic bond formed is relatively weaker than others because Na is a larger ion.

**- Mg and F:** Magnesium (Mg) has a higher ionization energy and fluorine (F) has a very high electron affinity. The small size of F and the high charge density of Mg result in a highly stable ionic bond.

**- Li and F:** Lithium (Li) and fluorine (F) also form a stable bond due to the small size of both ions, but magnesium and fluorine have a higher lattice energy due to magnesium's higher charge.

**- Na and F:** Sodium (Na) and fluorine (F) form an ionic bond, but it is less stable than the Mg-F bond because Na has a lower charge density than Mg.

Thus, the most stable ionic bond is formed between Mg and F due to the higher lattice energy and stronger electrostatic attraction between the ions.

**Quick Tip**

Ionic bond stability increases with the charge density of the ions. The higher the charge and the smaller the ions, the more stable the ionic bond.

---

**36. How much ethyl alcohol must be added to 1 litre of water so that the solution will freeze at $-14^\circ$C?**

$$(K_f \text{ for water} = 1.86\,^\circ\text{C/mol})$$

(A) 7.5 mol

(B) 8.5 mol

(C) 9.5 mol

(D) 10.5 mol

**Correct Answer:** (A) 7.5 mol

**Solution: Step 1:** Use the freezing point depression formula.

The freezing point depression ($\Delta T_f$) is given by:

$$\Delta T_f = K_f \cdot m$$

where: - $\Delta T_f$ = freezing point depression, - $K_f$ = cryoscopic constant (molal freezing point depression constant), - $m$ = molality of the solution (moles of solute per kg of solvent).

**Step 2:** Calculate the freezing point depression.

The normal freezing point of water is $0^\circ$C, and the solution freezes at $-14^\circ$C. Thus:

$$\Delta T_f = 0^\circ\text{C} - (-14^\circ\text{C}) = 14^\circ\text{C}.$$

**Step 3:** Solve for molality ($m$).

Using the formula:

$$\Delta T_f = K_f \cdot m,$$

$$14 = 1.86 \cdot m.$$

$$m = \frac{14}{1.86} \approx 7.53 \, \text{mol/kg}.$$

**Step 4:** Calculate the moles of ethyl alcohol required.

Since the solvent is 1 litre of water, and the density of water is approximately $1$ kg/L, the mass of water is $1$ kg. Therefore:

$$\text{Moles of ethyl alcohol} = m \times \text{mass of solvent} = 7.53 \, \text{mol/kg} \times 1 \, \text{kg} = 7.53 \, \text{mol}.$$

**Conclusion:** Approximately $7.5$ mol of ethyl alcohol must be added to 1 litre of water to achieve the desired freezing point.

**Quick Tip**

For freezing point depression problems: - Use the formula $\Delta T_f = K_f \cdot m$. - Ensure the units of $K_f$ and $\Delta T_f$ are consistent. - Molality ($m$) is defined as moles of solute per kg of solvent.

---

**37. The conductivity of a weak acid HA of concentration 0.001 mol $L^{-1}$ is $2.0 \times 10^{-5}$ S $cm^{-1}$. If $\Lambda_m^\circ$(HA) $= 190$ S $cm^2 mol^{-1}$, the ionization constant ($K_a$) of HA is equal to _____ $\times 10^{-6}$.**

(A) 24

(B) 48

(C) 12

(D) 45

**Correct Answer:** (C) 12

**Solution:**

$$A_m = 1000 \times \frac{\kappa}{M}$$

$$= 1000 \times \frac{2 \times 10^{-5}}{0.001} = 20 \, S \, cm^2 \, mol^{-1}$$

$$\Rightarrow \alpha = \frac{A_m}{A_m^o} = \frac{20}{190} = \left(\frac{2}{19}\right)$$

$$HA \rightleftharpoons H^+ + A^-$$

$$0.001(1-\alpha) \quad 0.001\alpha \quad 0.001\alpha$$

$$K_a = 0.001\left(\frac{\alpha^2}{1-\alpha}\right) = \frac{0.001 \times \left(\frac{2}{19}\right)^2}{1-\left(\frac{2}{19}\right)}$$

$$= 12.3 \times 10^{-6}$$

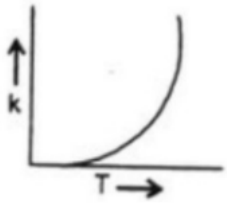
**Quick Tip**

The ionization constant for a weak acid can be calculated using the formula $K_a = \frac{C\Lambda_m^\circ}{\Lambda}$, where $C$ is the concentration, $\Lambda_m^\circ$ is the molar conductivity at infinite dilution, and $\Lambda$ is the measured conductivity.

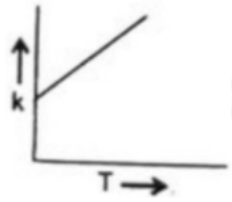
---

**38. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows the Arrhenius equation is:**

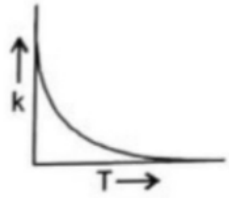
(A)

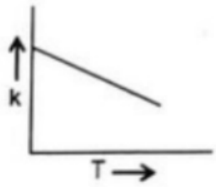



(B)




(C)



(D) 


**Correct Answer:** (A) Correct answer is option A, which represents the exponential increase of k with temperature.

**Solution:** According to the Arrhenius equation, k increases exponentially with temperature. The plot corresponding to this behavior is shown by option (A), which represents an exponential rise in k as temperature increases.

**Quick Tip**

According to the Arrhenius equation, $k = Ae^{-\frac{E_a}{RT}}$, where $k$ increases exponentially with temperature, showing a positive curvature as the temperature rises.

---

**39. Which of the following method is used for coagulation of the sol?**

(A) By mixing two oppositely charged sols.

(B) By electrophoresis.

(C) By addition of electrolytes.

(D) All of the above.

**Correct Answer:** (D) All of the above.

**Solution: Step 1:** Understand the methods of coagulation.

Coagulation of a sol refers to the process of destabilizing the colloidal particles, causing them to aggregate and settle. The following methods are commonly used:

1. Mixing two oppositely charged sols: When two sols with opposite charges are mixed, the particles neutralize each other's charges, leading to coagulation.

2. Electrophoresis: In electrophoresis, colloidal particles migrate towards oppositely charged electrodes. When they reach the electrode, they lose their charge and coagulate.

3. Addition of electrolytes: Adding electrolytes to a sol neutralizes the charge on the colloidal particles, reducing repulsion and causing coagulation.

**Step 2:** Analyze the Option.

- Option A: Mixing two oppositely charged sols is a valid method of coagulation. - Option B: Electrophoresis is a valid method of coagulation. - Option C: Addition of electrolytes is a valid method of coagulation.

Since all three methods are correct, the correct answer is All of the above.

**Conclusion:** All the given methods (A, B, and C) are used for the coagulation of sols.

Quick Tip

For coagulation of sols: - Mixing oppositely charged sols neutralizes charges. - Electrophoresis causes particles to lose charge at electrodes. - Electrolytes neutralize the charge on colloidal particles.

---

**40. The reaction that does NOT take place in a blast furnace between 900K to 1500K temperature range during extraction of iron is:**

(A) $Fe_2O_3 + CO \rightarrow 2FeO + CO_2$

(B) $FeO + CO \rightarrow Fe + CO_2$

(C) $C + CO_2 \rightarrow 2CO$

(D) $CaO + SiO_2 \rightarrow CaSiO_3$

**Correct Answer:** (A) $Fe_2O_3 + CO \rightarrow 2FeO + CO_2$

**Solution:**

The extraction of iron from its ore (typically hematite $Fe_2O_3$) in a blast furnace involves reduction reactions at high temperatures. The following reactions occur in the blast furnace:

- Reaction B: $FeO + CO \rightarrow Fe + CO_2$ This is the reduction of iron(II) oxide to iron, which is a key reaction in the extraction of iron.

- Reaction C: $C + CO_2 \rightarrow 2CO$ This reaction represents the formation of carbon monoxide, which is an essential reducing agent in the blast furnace.

- Reaction D: $CaO + SiO_2 \rightarrow CaSiO_3$ This reaction forms calcium silicate (slag), which helps remove impurities like silica from the iron ore.

- Reaction A: $Fe_2O_3 + CO \rightarrow 2FeO + CO_2$ This reaction does not occur under typical blast furnace conditions. The temperature in the blast furnace is not high enough to reduce $Fe_2O_3$ directly to FeO. The reduction of $Fe_2O_3$ typically occurs in multiple steps, and this reaction

does not take place as written.

Thus, the correct answer is (A).

**Quick Tip**

In a blast furnace, carbon monoxide is the main reducing agent used to reduce iron oxides to iron. The reactions typically involve multiple steps to progressively reduce $Fe_2O_3$ to iron.

---

**41. Kinetic theory of gases proves:**

(A) only Boyle's law

(B) only Charles' law

(C) only Avogadro's law

(D) all of these

**Correct Answer:** (D) all of these

**Solution: Step 1:** Understand the kinetic theory of gases.

The kinetic theory of gases is based on the following assumptions: 1. Gases consist of tiny particles (atoms or molecules) in constant random motion. 2. The volume occupied by gas molecules is negligible compared to the volume of the gas. 3. There are no intermolecular forces between gas molecules. 4. Collisions between gas molecules and with the walls of the container are perfectly elastic. 5. The average kinetic energy of gas molecules is directly proportional to the absolute temperature.

**Step 2:** Relate the kinetic theory to gas laws.

The kinetic theory of gases provides a theoretical foundation for the following gas laws:

1. Boyle's Law: At constant temperature, the pressure of a gas is inversely proportional to its volume. This is explained by the fact that reducing the volume increases the frequency of collisions with the walls, thus increasing pressure.

2. Charles' Law: At constant pressure, the volume of a gas is directly proportional to its absolute temperature. This is explained by the increase in molecular motion and collisions as temperature rises, causing the gas to expand.

3. Avogadro's Law: At constant temperature and pressure, the volume of a gas is directly

proportional to the number of molecules. This is explained by the fact that more molecules occupy more space, leading to an increase in volume.

**Step 3:** Conclusion.

The kinetic theory of gases explains all three laws: Boyle's law, Charles' law, and Avogadro's law.

**Conclusion:** The kinetic theory of gases proves all of these laws.

Quick Tip

The kinetic theory of gases: - Explains gas laws using molecular motion and collisions. - Relates macroscopic properties (pressure, volume, temperature) to microscopic behavior. - Provides a foundation for understanding ideal gas behavior.

---

**42. If enthalpies of formation of $C_2H_4(g)$, $CO_2(g)$ and $H_2O(l)$ at 25°C and 1 atm pressure are 52, 394 and -286 kJ/mol respectively, the change in enthalpy for combustion of $C_2H_4$ is equal to:**

(A) -141.2 kJ/mol

(B) -1412 kJ/mol

(C) +14.2 kJ/mol

(D) +1412 kJ/mol

**Correct Answer:** (B) -1412 kJ/mol

**Solution:**

The change in enthalpy ($\Delta H$) for the combustion of a substance is given by the following equation based on Hess's Law:

$$\Delta H = \sum \Delta H_f^\circ(\text{products}) - \sum \Delta H_f^\circ(\text{reactants})$$

where: - $\Delta H_f^\circ$ is the enthalpy of formation of each substance, - Products are $CO_2(g)$ and $H_2O(l)$, - Reactants are $C_2H_4(g)$.

The combustion reaction for $C_2H_4(g)$ is:

$$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$$

Step 1: Writing the enthalpy change equation.

$$\Delta H = \left[2 \times \Delta H_f^\circ(\text{CO}_2) + 2 \times \Delta H_f^\circ(\text{H}_2\text{O})\right] - \left[\Delta H_f^\circ(\text{C}_2\text{H}_4) + 3 \times \Delta H_f^\circ(\text{O}_2)\right]$$

Step 2: Substituting the values.

From the given data: - $\Delta H_f^\circ(\text{C}_2\text{H}_4) = 52\,\text{kJ/mol}$, - $\Delta H_f^\circ(\text{CO}_2) = -394\,\text{kJ/mol}$, - $\Delta H_f^\circ(\text{H}_2\text{O}) = -286\,\text{kJ/mol}$, - $\Delta H_f^\circ(\text{O}_2) = 0\,\text{kJ/mol}$ (since it's an element in its standard state).

So, we have:

$$\Delta H = [2 \times (-394) + 2 \times (-286)] - [52 + 3 \times 0]$$

$$\Delta H = [-788 + (-572)] - 52$$

$$\Delta H = -1360 - 52 = -1412\,\text{kJ/mol}$$

Thus, the change in enthalpy for the combustion of $\text{C}_2\text{H}_4$ is $-1412\,\text{kJ/mol}$.

**Quick Tip**

To calculate the change in enthalpy for a reaction, use the enthalpy of formation values for products and reactants, applying Hess's Law. Remember, $\Delta H_f^\circ$ for elements in their standard state (like $\text{O}_2$) is zero.

---

**43. The photochemical smog does not generally contain:**

(A) NO

(B) $\text{SO}_2$

(C) $\text{NO}_2$

(D) HCHO

**Correct Answer:** (B) $\text{SO}_2$

**Solution:** Photochemical smog is a type of air pollution that primarily consists of nitrogen oxides ($\text{NO}_x$) and volatile organic compounds (VOCs) that react under the influence of sunlight to form secondary pollutants, including ozone ($\text{O}_3$), formaldehyde (HCHO), and peroxyacetyl nitrates (PANs).

**Step 1:** In the formation of photochemical smog, nitrogen oxides like NO and $\text{NO}_2$ play a crucial role, along with VOCs. These pollutants lead to the formation of ozone and other harmful compounds.

**Step 2:** $SO_2$ (sulfur dioxide) is typically associated with industrial smog (or "London-type smog"), which is different from photochemical smog. It does not generally participate in the formation of photochemical smog under normal sunlight conditions.

**Step 3:** Therefore, $SO_2$ is not generally present in photochemical smog, unlike NO, $NO_2$, and HCHO, which are key components.

**Quick Tip**

Photochemical smog forms in the presence of sunlight, $NO_x$, and VOCs. Industrial smog, on the other hand, is associated with $SO_2$ and particulate matter.

---

**44.** Geometrical isomerism is not shown by:

A. $CH_3CH_2\overset{CH_3}{\overset{|}{C}} = \underset{CH_3}{\overset{|}{C}}CH_2CH_3$

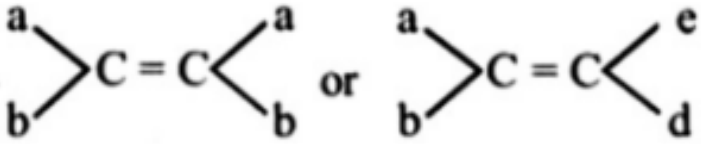
B. $C_2H_5 - \underset{H}{\overset{|}{C}} = \underset{H}{\overset{|}{C}} - CH_2I$

C. $CH_2 = C(Cl)CH_3$

D. $CH_3 - CH = CH - CH = CH_2$

**Correct Answer:** (C)

**Solution:** Geometrical isomerism is not possible for compounds where the same group is attached to both carbons of the double bond, or when there are restrictions like a halide preventing such isomerism. In option (C), the chlorine atom attached to the second carbon prevents the possibility of cis-trans isomerism, making it the correct answer.



**Quick Tip**

Geometrical isomerism requires different substituents on each carbon of the double bond. In option (C), chlorine's attachment on one side eliminates this possibility.

---

**45.** For the separation of two immiscible liquids, which method is used?

(A) Chromatography

(B) Fractionating column

(C) Fractional distillation

(D) Separating funnel

**Correct Answer:** (D)

**Solution:** When two immiscible liquids are present, the method used for separation is the separating funnel. This method works based on the difference in densities of the two liquids, which allows them to separate into two distinct layers that can be drained separately.

**Quick Tip**

The separating funnel is commonly used to separate immiscible liquids, taking advantage of their different densities to form two separate layers.

---

**46. What is $x$ in the following reaction?**

$$\mathrm{Al}(s) + \mathrm{NaOH}(aq) + \mathrm{H_2}O(l) \rightarrow x + \mathrm{H_2}(g)$$

(A) $\mathrm{Na_2[Al(OH)_4]^-}$

(B) $\mathrm{Na^+[Al(OH)_4]^-}$

(C) $\mathrm{Na_2[Al(OH)_6]^-}$

(D) $\mathrm{Na^+[Al(OH)_6]^-}$

**Correct Answer:** (B) $\mathrm{Na^+[Al(OH)_4]^-}$

**Solution:**

**Step 1:** Understanding the reaction mechanism.

When aluminum reacts with sodium hydroxide in the presence of water, it forms a soluble

complex ion. The reaction proceeds as follows:

$$\text{Al} + \text{NaOH} + \text{H}_2O \rightarrow \text{Na}^+[\text{Al(OH)}_4]^- + \text{H}_2$$

**Step 2:** Identifying the product.

The aluminum hydroxide complex formed is $[\text{Al(OH)}_4]^-$, which is stabilized by sodium ions $\text{Na}^+$. This corresponds to option (B).

**Quick Tip**

Aluminum reacts with NaOH to form a soluble aluminate complex $[\text{Al(OH)}_4]^-$. Always check the oxidation states and solubility of the resulting compounds.

---

**47. Which of the following will precipitate first when an aqueous solution containing sulphate ions is added?**

(A) $\text{Mg}^{2+}$

(B) $\text{Ca}^{2+}$

(C) $\text{Sr}^{2+}$

(D) $\text{Ba}^{2+}$

**Correct Answer:** (D) $\text{Ba}^{2+}$

**Solution:**

**Step 1:** Understanding solubility trends.

The precipitation of metal sulphates depends on their solubility product constant ($K_{sp}$). The lower the $K_{sp}$ value, the less soluble the compound is, meaning it will precipitate first.

**Step 2:** Solubility of sulphates.

The solubility order of alkaline earth metal sulphates ($\text{M}^{2+}\text{SO}_4^{2-}$) decreases down the group:

$$\text{MgSO}_4 > \text{CaSO}_4 > \text{SrSO}_4 > \text{BaSO}_4$$

Since barium sulphate ($\text{BaSO}_4$) has the lowest solubility, it will precipitate first.

**Quick Tip**

As we go down Group 2, the solubility of sulphates decreases. Barium sulphate ($\text{BaSO}_4$) is the least soluble and precipitates first when sulphate ions are added.

**48. Ionic hydrides react with water to give:**

(A) Acidic solutions

(B) Hydride ions

(C) Basic solutions

(D) Electrons

**Correct Answer:** (C) Basic solutions

**Solution:**

**Step 1:** Understanding the reaction of ionic hydrides with water.

Ionic hydrides, such as sodium hydride (NaH) and calcium hydride ($CaH_2$), contain the hydride ion ($H^-$). When they react with water, they form a strong base ($OH^-$) and release hydrogen gas:

$$MH + H_2O \rightarrow MOH + H_2$$

where $M$ is a metal.

**Step 2:** Identifying the nature of the solution.

Since the reaction produces hydroxide ions ($OH^-$), the solution becomes basic. Hence, the correct answer is (C) Basic solutions.

**Quick Tip**

Ionic hydrides react with water to form a metal hydroxide and hydrogen gas, leading to a basic solution. Examples include NaH and $CaH_2$.

---

**49. The drug used as an antidepressant is:**

(A) Luminol

(B) Tofranil

(C) Mescaline

(D) Sulphadiazine

**Correct Answer:** (B) Tofranil

**Solution:**

**Step 1:** Understanding antidepressant drugs.

Antidepressants are medications used to treat depression and related mood disorders by altering neurotransmitter levels in the brain.

**Step 2:** Identifying the correct option.

- Luminol: A chemiluminescent substance used in forensic science, not an antidepressant. - Tofranil (Imipramine): A tricyclic antidepressant (TCA) commonly used to treat depression. - Mescaline: A hallucinogenic compound, not used as an antidepressant. - Sulphadiazine: A sulfonamide antibiotic, not related to depression treatment.

Since Tofranil (Imipramine) is a well-known antidepressant, the correct answer is (B).

**Quick Tip**

Tricyclic antidepressants (TCAs) like Imipramine (Tofranil) are used to treat depression by inhibiting the reuptake of neurotransmitters like serotonin and norepinephrine.

---

**50. Melamine plastic crockery is a copolymer of:**

(A) HCHO and melamine

(B) HCHO and ethylene

(C) Melamine and ethylene

(D) None of these

**Correct Answer:** (A) HCHO and melamine

**Solution:**

**Step 1:** Understanding melamine plastic.

Melamine plastic is a thermosetting polymer known for its high durability, heat resistance, and lightweight nature, commonly used in kitchenware and crockery.

**Step 2:** Identifying the copolymer composition.

Melamine-formaldehyde resin is formed by the polymerization of melamine and formaldehyde (HCHO). The reaction results in a cross-linked polymer network that gives melamine plastic its strong and heat-resistant properties:

$$\text{Melamine} + \text{Formaldehyde} \rightarrow \text{Melamine-Formaldehyde Resin}$$

**Step 3:** Eliminating incorrect Option.

- Option A: Correct, as melamine plastic is made from melamine and formaldehyde. - Option B: Incorrect, since ethylene is not involved in melamine plastic formation. - Option C: Incorrect, as ethylene does not participate in this polymerization. - Option D: Incorrect, since melamine plastic is a known polymer of melamine and formaldehyde.

Hence, the correct answer is (A) HCHO and melamine.

**Quick Tip**

Melamine-formaldehyde resin is a widely used thermosetting polymer known for its hardness and heat resistance, commonly used in kitchenware.

---

**51. The helical structure of a protein is stabilized by:**

(A) Dipeptide bonds

(B) Hydrogen bonds

(C) Ether bonds

(D) Peptide bonds

**Correct Answer:** (B) Hydrogen bonds

**Solution:**

**Step 1:** Understanding protein structure.

Proteins have different levels of structure: primary, secondary, tertiary, and quaternary. The $\alpha$-helix and $\beta$-sheet are common secondary structures.

**Step 2:** Role of hydrogen bonding.

The $\alpha$-helical structure of proteins is stabilized by hydrogen bonds formed between the carbonyl oxygen of one amino acid and the amide hydrogen of another amino acid four residues away in the chain:

$$\text{C=O} \cdots \text{H-N}$$

These hydrogen bonds help maintain the spiral shape of the helix.

**Step 3:** Eliminating incorrect Option.

- Dipeptide bonds (Option A): Incorrect, as they only link two amino acids. - Ether bonds

(Option C): Incorrect, as ether bonds are not present in proteins. - Peptide bonds (Option D): Incorrect, as peptide bonds link amino acids but do not stabilize the helical structure.
Since hydrogen bonding is responsible for stabilizing the helical structure, the correct answer is (B) Hydrogen bonds.

**Quick Tip**

In proteins, hydrogen bonding between the backbone atoms of amino acids stabilizes the secondary structures like $\alpha$-helices and $\beta$-sheets.

---

**52. Which of the following factors affect the basic strength of amines?**

(i) Inductive effect

(ii) Steric hindrance

(iii) Solvation effect

(iv) Solubility in organic solvents

(A) (i) and (iv)

(B) (i), (ii), and (iii)

(C) (ii) and (iii)

(D) (ii) and (iv)

**Correct Answer:** (B) (i), (ii), and (iii)

**Solution:**

**Step 1:** Understanding the factors affecting basic strength of amines.
The basicity of an amine depends on its ability to donate a lone pair of electrons on the nitrogen atom. Several factors influence this ability:

1. Inductive Effect: Electron-donating groups (+I effect) increase electron density on nitrogen, enhancing basicity. Conversely, electron-withdrawing groups (-I effect) decrease basicity.
2. Steric Hindrance: Bulky groups around nitrogen hinder solvation and protonation, reducing basicity.
3. Solvation Effect: The interaction of the amine with solvent molecules affects its ability to stabilize the conjugate acid form, influencing basic strength.

**Step 2:** Eliminating incorrect Option.

- Solubility in organic solvents (Option iv) does not directly influence the intrinsic basic strength of amines. - Correct factors: Inductive effect (i), Steric hindrance (ii), and Solvation effect (iii) play significant roles in determining basicity.

Thus, the correct answer is (B) (i), (ii), and (iii).

**Quick Tip**

The basic strength of amines depends on electronic effects (+I/-I), steric hindrance, and solvation. More hindrance and less solvation decrease basicity.

---

**53. Find out compound B in the given reaction sequence:**

$$C_6H_6 + CH_3X \xrightarrow{AlCl_3} A \xrightarrow{CrO_3 \text{ in } (CH_3CO)_2O/H_3O^+} B$$

(A) Acetophenone ($C_6H_5COCH_3$)$(B)$Benzaldehyde($C_6H_5CHO$)

(C) Cyclohexyl carbaldehyde ($C_6H_{11}CHO$)

(D) Benzoic acid ($C_6H_5COOH$)

**Correct Answer:** (B) Benzaldehyde ($C_6H_5CHO$)

**Solution:**

**Step 1:** Identifying the first reaction.

The first step involves Friedel-Crafts Alkylation, where $CH_3X$ (an alkyl halide) reacts with benzene ($C_6H_6$) in the presence of $AlCl_3$, forming toluene (A) ($C_6H_5CH_3$).

$$C_6H_6 + CH_3X \xrightarrow{AlCl_3} C_6H_5CH_3$$

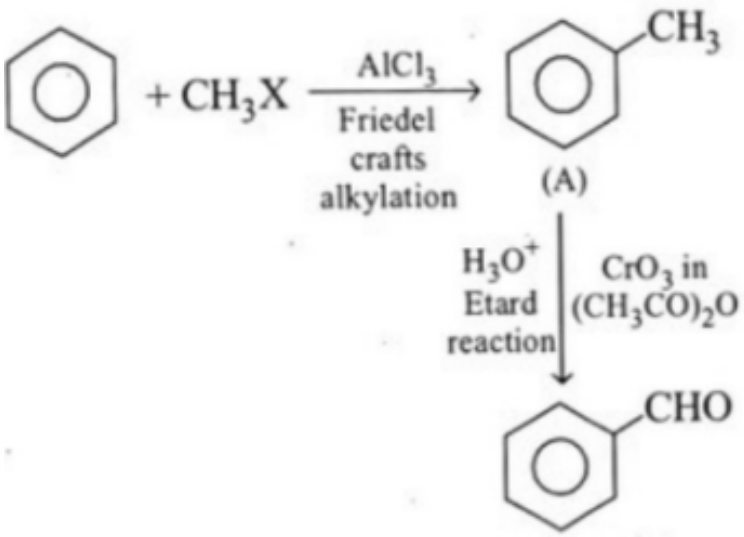
**Step 2:** Oxidation of Toluene.

- The second step involves oxidation using chromyl chloride ($CrO_2Cl_2$), known as the Etard Reaction, which selectively oxidizes the methyl group ($-CH_3$) to an aldehyde ($-CHO$), forming benzaldehyde (B) ($C_6H_5CHO$).

$$C_6H_5CH_3 \xrightarrow{CrO_3 \text{ in } (CH_3CO)_2O/H_3O^+} C_6H_5CHO$$

**Step 3:** Eliminating incorrect Option.

- Acetophenone (A) ($C_6H_5COCH_3$): Incorrect, as the oxidation of toluene in the Etard reaction leads to the formation of an aldehyde ($C_6H_5CHO$) and not a ketone.
- Cyclohexyl carbaldehyde (C) ($C_6H_{11}CHO$): Incorrect, as the reaction occurs on a benzene ring, not on a cyclohexane system, and no such transformation takes place. - Benzoic acid (D) ($C_6H_5COOH$): Incorrect, as oxidation of the methyl group ($-CH_3$) to a carboxyl group ($-COOH$) requires a stronger oxidizing agent such as alkaline $KMnO_4$ or acidic $K_2Cr_2O_7$ under reflux, which is not used in this reaction.




Thus, the correct answer is (B) Benzaldehyde ($C_6H_5CHO$).

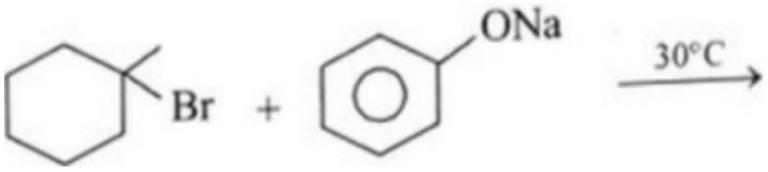
**Quick Tip**

The Etard Reaction selectively oxidizes the methyl group ($-CH_3$) of toluene to an aldehyde ($-CHO$) using chromyl chloride ($CrO_2Cl_2$) in a non-aqueous solvent like carbon disulfide ($CS_2$), producing benzaldehyde ($C_6H_5CHO$).

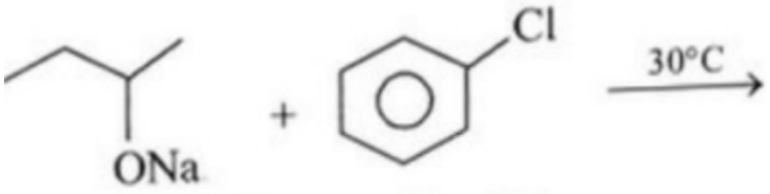
---

**54. Which method is useful for the synthesis of ether?**

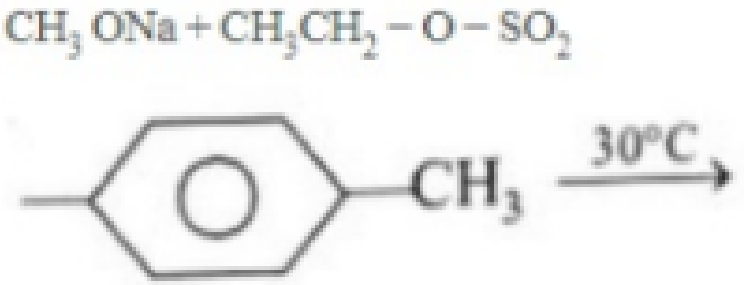
(A) Williamson Ether Synthesis with a benzyl bromide derivative



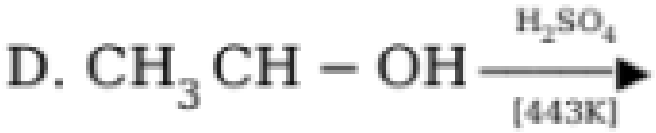
(B) Nucleophilic substitution involving sodium phenoxide




(C) Williamson Ether Synthesis with alkoxide and alkyl sulfonate




(D) Dehydration of alcohol using sulfuric acid at high temperature




**Correct Answer:** (C) Williamson Ether Synthesis with alkoxide and alkyl sulfonate

**Solution:**

**Step 1:** Understanding ether synthesis methods.

Ethers (R-O-R') can be synthesized by several methods, but the Williamson Ether Synthesis is the most reliable and widely used approach.

**Step 2:** Identifying the correct reaction.

- The Williamson Ether Synthesis involves the reaction of a sodium alkoxide ($R\text{-}O^-$) with a primary alkyl halide or alkyl sulfonate (R'-X). - The given reaction in option C follows this method, where a sodium alkoxide reacts with an alkyl sulfonate to form an ether under mild heating.

**Step 3:** Eliminating incorrect Option.

- Option A: Involves a benzyl bromide derivative, which may lead to side reactions. - Option B: Though nucleophilic substitution can form ethers, it is not as general or effective as the

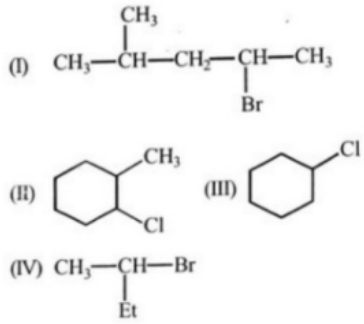
Williamson method. - Option D: The acid-catalyzed dehydration of alcohol at high temperature (443 K, using $H_2SO_4$) is used to form ethers, but this method is not ideal for unsymmetrical ethers and works mainly for primary alcohols.
Thus, the correct answer is (C) Williamson Ether Synthesis with alkoxide and alkyl sulfonate.

**Quick Tip**

The Williamson Ether Synthesis is the most efficient method for preparing ethers, involving the reaction of sodium alkoxide with a primary alkyl halide or alkyl sulfonate via an SN2 mechanism.

---

**55. Among the given halides, which one will give the same product in both $S_N1$ and $S_N2$ reactions?**




(A) (III) only
(B) (I) and (II)
(C) (III) and (IV)
(D) (I), (III), and (IV)

**Correct Answer:** (C) (III) and (IV)

**Solution:**

**Step 1:** Understanding $S_N1$ and $S_N2$ reaction mechanisms.
- $S_N1$ (Unimolecular Nucleophilic Substitution): This mechanism occurs via the formation of a carbocation intermediate. It is favored in tertiary and benzylic/allylic halides due to carbocation stability. - $S_N2$ (Bimolecular Nucleophilic Substitution): This mechanism proceeds via a backside attack, leading to inversion of configuration. It is favored in primary and secondary alkyl halides with minimal steric hindrance.

**Step 2:** Analyzing the given halides.

- (III) (Cyclohexyl chloride): - This structure does not form a resonance-stabilized carbocation. - Whether the reaction follows $S_N1$ or $S_N2$, the product remains the same. - Satisfies the condition of giving the same product in both reactions.

- **(IV) Secondary alkyl bromide with $\beta$ branching**: - The presence of branching reduces the likelihood of rearrangement. - Whether the reaction proceeds via $S_N1$ or $S_N2$, the substitution product remains unaffected.

- Satisfies the condition of giving the same product in both reactions.

**Step 3:** Eliminating incorrect Option.

- (I) (Branched alkyl bromide): This can undergo carbocation rearrangement in $S_N1$, leading to different products. - (II) (Cyclohexyl chloride with a methyl group): The possibility of rearrangement in $S_N1$ exists, leading to different products.

Thus, the correct answer is (C) (III) and (IV).

**Quick Tip**

Compounds that do not undergo carbocation rearrangement will give the same product in both $S_N1$ and $S_N2$ mechanisms. Typically, cycloalkyl halides and simple secondary alkyl halides meet this criterion.

---

**56. Among the ligands $NH_3$, en, $CN^-$, CO, the correct order of their increasing field strength is:**

(A) $NH_3 < en < CN^- < CO$

(B) $CN^- < NH_3 < en$

(C) $en < CN^- < NH_3 < CO$

(D) $CO < NH_3 < en < CN^-$

**Correct Answer:** (A) $NH_3 < en < CN^- < CO$

**Solution: Step 1:** The field strength of a ligand depends on its ability to split the d-orbital energy levels in a metal complex. Ligands are classified as strong, medium, or weak field ligands.

**Step 2:** Based on the spectrochemical series: - CO is a strong field ligand. - $CN^-$ is also a

strong field ligand, but it is weaker than CO. - en (ethylenediamine) is a medium field ligand. - $NH_3$ is a weak field ligand.

**Step 3:** The increasing order of field strength is:

$$NH_3 < en < CN^- < CO$$

Thus, the correct order is option (A).

**Quick Tip**

The spectrochemical series helps in determining the field strength of ligands, influencing the splitting of d-orbitals and the color of metal complexes.

---

**57. The S – S bond is not present in:**

(A) $S_2O_4^{2-}$

(B) $S_2O_5^{2-}$

(C) $S_2O_3^{2-}$

(D) $S_2O_7^{2-}$

**Correct Answer:** (D) $S_2O_7^{2-}$

**Solution: Step 1:** The S – S bond occurs when two sulfur atoms are directly bonded to each other in a compound.

**Step 2:** In $S_2O_4^{2-}$, $S_2O_5^{2-}$, and $S_2O_3^{2-}$, there are S – S bonds present between sulfur atoms.

**Step 3:** In $S_2O_7^{2-}$, there is no direct S – S bond. The two sulfur atoms are connected through an oxygen bridge, meaning there is no direct bonding between sulfur atoms.

Thus, the correct answer is option (D).

**Quick Tip**

When dealing with sulfur oxyanions, check the bonding between sulfur atoms. Some species have sulfur-sulfur bonds, while others have sulfur-oxygen-sulfur linkages.

---

**58. In the laboratory, manganese (II) salt is oxidized to permanganate ion in aqueous solution by:**

(A) hydrogen peroxide

(B) conc. nitric acid

(C) peroxy disulphate

(D) dichromate

**Correct Answer:** (C) peroxy disulphate

**Solution: Step 1:** Manganese (II) salts, such as $Mn^{2+}$, can be oxidized to the permanganate ion $MnO_4^-$ by various oxidizing agents.

**Step 2:** Among the Option: - Hydrogen peroxide is commonly used to oxidize manganese (II) to permanganate, but it is not the best oxidizer in this case. - Conc. nitric acid does not efficiently oxidize manganese (II) to permanganate. - Peroxy disulphate ($(O_2SO_2)_2^{2-}$) is a powerful oxidizing agent and is known to oxidize $Mn^{2+}$ to $MnO_4^-$, making it the correct choice in this context. - Dichromate is also an oxidizing agent but does not directly oxidize manganese (II) to permanganate.

Thus, the correct answer is option (C).

**Quick Tip**

Peroxy disulphate is a strong oxidizer often used in laboratory settings to convert manganese (II) salts to permanganate ions.

---

**59. Which one of the following molecular hydrides acts as a Lewis acid?**

(A) $NH_3$

(B) $H_2O$

(C) $B_2H_6$

(D) $CH_4$

**Correct Answer:** (C) $B_2H_6$

**Solution:**

A Lewis acid is a substance that can accept a pair of electrons. In the case of molecular hydrides:

- $NH_3$ (Ammonia) is a Lewis base because it has a lone pair of electrons on nitrogen that can be donated. - $H_2O$ (Water) is also a Lewis base due to the lone pairs on oxygen. - $B_2H_6$ (Diborane) acts as a Lewis acid because boron atoms in diborane have an incomplete octet

and can accept electron pairs. - $CH_4$ (Methane) does not act as a Lewis acid because it does not have an empty orbital to accept electron pairs.

Thus, $B_2H_6$ (Diborane) is the only hydride that acts as a Lewis acid.

**Quick Tip**

A Lewis acid accepts an electron pair, while a Lewis base donates an electron pair. Boron compounds like $B_2H_6$ are known to act as Lewis acids due to their electron deficiency.

---

**60. Electrode potential data are given below:**

$$Fe^{3+}(aq) + e^- \rightarrow Fe^{2+}(aq); E^\circ = +0.77\,V$$

$$Al^{3+}(aq) + 3e^- \rightarrow Al(s); E^\circ = -1.66\,V$$

$$Br_2(aq) + 2e^- \rightarrow 2Br^-(aq); E^\circ = +1.08\,V$$

Based on the data, the reducing power of $Fe^{2+}$, Al, and $Br^-$ will increase in the order:

(A) $Br^- < Fe^{2+} < Al$

(B) $Fe^{2+} < Al < Br^-$

(C) $Al < Br^- < Fe^{2+}$

(D) $Al < Fe^{2+} < Br^-$

**Correct Answer:** (A) $Br^- < Fe^{2+} < Al$

**Solution: Step 1:** The reducing power of a species is related to the magnitude of its reduction potential. A more negative reduction potential indicates a stronger tendency to lose electrons (thus, a stronger reducing agent).

**Step 2:** The species with the lowest reduction potential will be the best reducing agent: - $Fe^{3+} + e^- \rightarrow Fe^{2+}$, with $E^\circ = +0.77\,V$, indicates that $Fe^{2+}$ can be reduced to $Fe^{3+}$, so $Fe^{2+}$ is a relatively weak reducing agent. - $Al^{3+} + 3e^- \rightarrow Al$, with $E^\circ = -1.66\,V$, indicates that Al is the strongest reducing agent due to the very negative reduction potential. - $Br_2 + 2e^- \rightarrow 2Br^-$, with $E^\circ = +1.08\,V$, shows that $Br^-$ has the weakest reducing power among the three.

Thus, the correct order of reducing power is:

$$Br^- < Fe^{2+} < Al$$

Quick Tip

A more negative electrode potential corresponds to a stronger reducing agent, as it indicates a greater tendency to donate electrons.

---

**61.** Write the antonym: **Fricassee**

(A) grill

(B) decorate

(C) stew

(D) to baste

**Correct Answer:** (C) stew

**Solution: Step 1:** A fricassee is a type of dish where meat (typically poultry or veal) is cut into pieces and stewed, often with a sauce or gravy.

**Step 2:** The cooking method "fricassee" involves simmering or stewing meat, not grilling, decorating, or basting.

Thus, the correct answer is option (C) stew.

Quick Tip

Fricassee refers specifically to a method of cooking where meat is stewed in a white sauce, often with vegetables and seasonings.

---

**62.** Write the antonym: **Retribution**

(A) compensation

(B) forgiveness

(C) contempt

(D) grudge

**Correct Answer:** (B) forgiveness

**Solution: Step 1:** Retribution typically refers to punishment or vengeance, but the opposite concept is forgiveness, which is the act of pardoning or excusing someone.

**Step 2:** In this context, forgiveness serves as an antonym to retribution, which involves

reprisal or revenge. Compensation, contempt, and grudge are related to feelings of retaliation or resentment, not forgiveness.

Thus, the correct answer is option (B) forgiveness.

**Quick Tip**

Retribution often refers to retaliatory punishment, while forgiveness involves letting go of resentment or punishment.

---

**63.** Write the antonym: **Sumptuous**

(A) irritable

(B) meagre

(C) fancy

(D) sad

**Correct Answer:** (B) meagre

**Solution: Step 1:** "Sumptuous" refers to something that is luxurious, rich, or splendid, often used in the context of food, clothing, or surroundings that are extravagant.

**Step 2:** The antonym of sumptuous is "meagre," which means something that is small, inadequate, or insufficient in quantity or quality. The other Option, such as "fancy," are more related to something ornate or elaborate, not the opposite of sumptuous.

Thus, the correct answer is option (B) meagre.

**Quick Tip**

"Sumptuous" suggests abundance and richness, while "meagre" refers to scarcity and lack of richness.

---

**64. Rajeev failed in the examination because his answers were not ..... to the questions asked**

(A) allusive

(B) pertinent

(C) revealing

(D) referential

**Correct Answer:** (B) pertinent

**Solution: Step 1:** The word "pertinent" means relevant or applicable to the matter at hand. In this context, the statement suggests that Rajeev's answers were not relevant to the questions asked.

**Step 2:** The other Option: - "Allusive" refers to something that hints at or indirectly suggests something else, which doesn't fit in this context. - "Revealing" means making something known, which is unrelated to the relevance of the answers. - "Referential" refers to something that points to or refers to something else, but it doesn't specifically imply relevance.

Thus, the correct answer is option (B) pertinent.

**Quick Tip**

"Pertinent" refers to something that is directly relevant to the subject or question being discussed.

---

**65. Choose the correct words to complete the sentence:** *It was . . . . . cold . . . . . we couldn't go out.*

(A) so, that

(B) too, to

(C) neither, nor

(D) either, or

**Correct Answer:** (A) so, that

**Solution: Step 1:** The phrase "so...that" is used to indicate a result or consequence. In this sentence, the cold was so intense that the consequence was they couldn't go out.

**Step 2:** The other Option don't fit the context: - "Too...to" would mean something was excessively cold to the point of not being able to do something, but "so...that" is the appropriate construction here. - "Neither...nor" and "either...or" are used for negative or alternative conditions, which don't fit in this case.

Thus, the correct answer is option (A) so, that.

Quick Tip

The construction “so...that” is used to show cause and effect or result.

---

**66. Faced with the**

P: traditional culture in the pre-independence India

Q: challenge of the intrusion of colonial culture and ideology

R: developed during the nineteenth century

S: at attempt to reinvigorate traditional institutions and realize the potential of

Which one of the following is the correct sequence?

(A) P - R - Q - S

(B) Q - S - P - R

(C) P - S - Q - R

(D) Q - R - P - S

**Correct Answer:** (B) Q - S - P - R

**Solution: Step 1:** To form a meaningful sequence, we need to ensure that the clauses are logically connected.

**Step 2:** The sentence talks about India facing a challenge due to colonial intrusion and how there was an effort to rejuvenate traditional culture. The best logical flow is: - Q (challenge of colonial intrusion) comes first, as it sets the context. - S (effort to reinvigorate traditional institutions) logically follows, as it is the response to the challenge. - P (traditional culture in pre-independence India) comes next to explain what needed revitalization. - R (developed during the nineteenth century) concludes by explaining when this revitalization effort happened.

Thus, the correct sequence is option (B) Q - S - P - R.

Quick Tip

In such sentence rearrangement questions, look for clues in the context of the sentence to connect ideas logically.

---

**67. A diversified use**

P: as a heating or power generation fuel by converting gas into

Q: adding a new dimension to the traditional use of gas

R: of natural gas is emerging

S: amongst other products, high quality diesel transportation fuel virtually free of sulphur

Which one of the following is the correct sequence?

(A) R - P - Q - S

(B) S - Q - P - R

(C) R - Q - P - S

(D) S - P - Q - R

**Correct Answer:** (C) R - Q - P - S

**Solution: Step 1:** To form a logical sequence, we need to understand the context. The sentence talks about the diversified uses of natural gas.

**Step 2:** The correct sequence is as follows: - R (natural gas use is emerging) introduces the topic. - Q (adding a new dimension to the traditional use of gas) follows as it explains the new developments in its use. - P (using gas as a heating or power generation fuel) gives a specific example of this emerging use. - S (producing high-quality diesel fuel) concludes with an example of one of the products that can be derived.

Thus, the correct sequence is option (C) R - Q - P - S.

Quick Tip

Look for an introductory statement first, followed by elaborations and examples to form the correct sequence.

---

**68. Music is often linked to .....**

(A) anger

(B) mood

(C) anxiety

(D) happiness

**Correct Answer:** (B) mood

**Solution: Step 1:** Music has a strong connection with emotions, and it is widely known to

influence a person's mood. Different genres of music can evoke different emotional responses, such as joy, sadness, or relaxation.

**Step 2:** The word "mood" is the most appropriate choice, as music is often linked to altering or enhancing a person's mood.

Thus, the correct answer is option (B) mood.

Quick Tip

Music can significantly affect mood, from uplifting spirits to evoking sadness or excitement.

---

**69. How is music an important part of life?**

(A) It makes us feel different emotions

(B) It makes us sad

(C) It helps in our daily activities

(D) It helps us in remembering things

**Correct Answer:** (A) It makes us feel different emotions

**Solution: Step 1:** Music plays a significant role in shaping and influencing our emotions. Different types of music can make us feel happy, calm, excited, or even nostalgic.

**Step 2:** The other Option are not as comprehensive: - Option (B) "It makes us sad" is a possibility, but music isn't limited to evoking sadness. - Option (C) "It helps in our daily activities" is true to some extent but is not the most direct answer. - Option (D) "It helps us in remembering things" is true for certain situations, like mnemonic devices, but it doesn't capture the emotional connection that is central to music's role in life.

Thus, the correct answer is option (A) It makes us feel different emotions.

Quick Tip

Music has a powerful influence on emotions and can change our mood depending on the genre or tempo.

---

**70. Which of the statements is true?**

(A) All forms of music may heal wounds

(B) All forms of music may have good effect

(C) All forms of music may be soothing

(D) All forms of music may have therapeutic effects

**Correct Answer:** (D) All forms of music may have therapeutic effects

**Solution: Step 1:** Music therapy is well-established for its therapeutic effects, helping individuals deal with stress, anxiety, and other emotional challenges. The healing and therapeutic benefits of music are often recognized in various forms, though not all types of music are suitable for all situations.

**Step 2:** - Option (A) "All forms of music may heal wounds" is too specific and doesn't apply to every form of music.

- Option (B) "All forms of music may have good effect" is somewhat true but doesn't capture the full therapeutic scope.

- Option (C) "All forms of music may be soothing" is not entirely accurate, as some types of music can be agitating rather than soothing.

- Option (D) "All forms of music may have therapeutic effects" is the most accurate, as it acknowledges the potential of music in promoting well-being, even if not every form is suited to every individual.

Thus, the correct answer is option (D) All forms of music may have therapeutic effects.

Quick Tip

Music, when used appropriately, has a range of therapeutic benefits for mental and physical health.

---

**71. On the following questions, select the related word/letters from the attractive**

MASTER: OCUVGT :: LABOUR:?

(A) NCDQWT

(B) HDERWT

(C) NBECRWT

(D) NEDRWT

**Correct Answer:** (A) NCDQWT

**Solution: Step 1:** Let's first break down the pattern in the transformation from MASTER to OCUVGT.

- "M" becomes "O" (shift by +2) - "A" becomes "C" (shift by +2) - "S" becomes "U" (shift by +2) - "T" becomes "V" (shift by +2) - "E" becomes "G" (shift by +2) - "R" becomes "T" (shift by +2)

So, the transformation follows a consistent pattern of shifting each letter by +2 positions in the alphabet.

**Step 2:** Applying the same shift to the word "LABOUR":

- "L" becomes "N" (shift by +2) - "A" becomes "C" (shift by +2) - "B" becomes "D" (shift by +2) - "O" becomes "Q" (shift by +2) - "U" becomes "W" (shift by +2) - "R" becomes "T" (shift by +2)

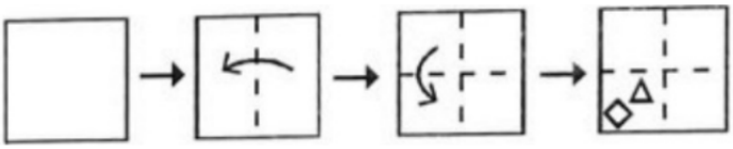
Thus, LABOUR transforms into "NCDQWT", which corresponds to option (A).

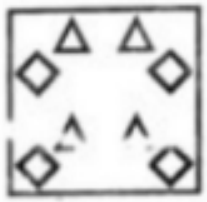
**Quick Tip**

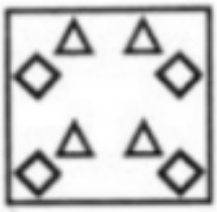
For letter-shift puzzles, look for consistent shifts or patterns in the alphabet to solve them.

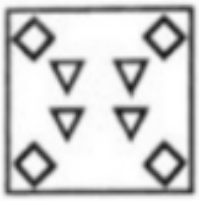
---

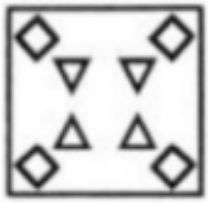
**72. The sequence of folding a paper and the manner in which the folded paper has been cut is shown in the following figures. How would this paper look when unfolded?**



(A) 

(B) 

(C) 

(D) 

**Correct Answer:** (D)

**Solution: Step 1:** The paper is folded and cut along certain lines to create symmetrical patterns upon unfolding.

**Step 2:** Upon carefully analyzing the cuts and folds, the pattern that will appear after the paper is unfolded is the one that has triangles and diamonds arranged symmetrically.

**Step 3:** After evaluating all the Option, the correct unfolded shape corresponds to option (D). Thus, the correct answer is option (D).

Quick Tip

When solving paper-folding and cutting problems, carefully observe the symmetry and shape of the cut to determine the final pattern upon unfolding.

---

**73. In a given code, SISTER is coded as 535301, UNCLE as 84670, and BOY as 129. How is RUSTIC written in that code?**

(A) 633185

(B) 185336

(C) 363815

(D) 581363

**Correct Answer:** (B) 185336

**Solution: Step 1:** Analyzing the given code, we notice that the letters are being replaced by their positions in the alphabet. - S = 19, I = 9, S = 19, T = 20, E = 5, R = 18 for SISTER (535301). - U = 21, N = 14, C = 3, L = 12, E = 5 for UNCLE (84670). - B = 2, O = 15, Y = 25 for BOY (129).

**Step 2:** Now, applying the same pattern to the word "RUSTIC": - R = 18 - U = 21 - S = 19 - T = 20 - I = 9 - C = 3

Thus, RUSTIC is coded as 185336.

Thus, the correct answer is option (B).

Quick Tip

In letter-to-number code questions, always check if each letter is being mapped to its alphabetical position (A = 1, B = 2, ..., Z = 26).

---

**74. Daya has a brother, Anil. Daya is the son of Chandra. Bimal is Chandra's father. In terms of relationship, what is Anil of Bimal?**

(A) Son

(B) Grandson

(C) Brother

(D) Grandfather

**Correct Answer:** (B) Grandson

**Solution: Step 1:** Daya is the son of Chandra, and Anil is Daya's brother. This makes Anil also the son of Chandra.

**Step 2:** Bimal is Chandra's father, meaning Bimal is the grandfather of both Daya and Anil.

Thus, Anil is the grandson of Bimal.

Thus, the correct answer is option (B) Grandson.

Quick Tip

To solve family relationship questions, work step by step from known relationships and trace them logically.

---

**75. Find the odd word pair among the given four word pairs.**

(A) Error : Accurate

(B) Careless : Casual

(C) Strength : Lethargy

(D) Gloomy : Cheerful

**Correct Answer:** (B) Careless : Casual

**Solution: Step 1:** We need to identify the word pair where the relationship is different from the others.

- "Error" and "Accurate" are antonyms, as "Error" means something incorrect and "Accurate" means correct.
- "Careless" and "Casual" are related words but they do not form an antonymic relationship. "Careless" refers to lack of attention, while "Casual" refers to something relaxed or informal, not necessarily opposite.
- "Strength" and "Lethargy" are antonyms, as "Strength" refers to power and "Lethargy" refers to weakness or lack of energy.
- "Gloomy" and "Cheerful" are antonyms, as "Gloomy" refers to being sad and "Cheerful" refers to being happy.

**Step 2:** The odd pair is (B) "Careless : Casual" because it does not represent an antonymic relationship like the others.

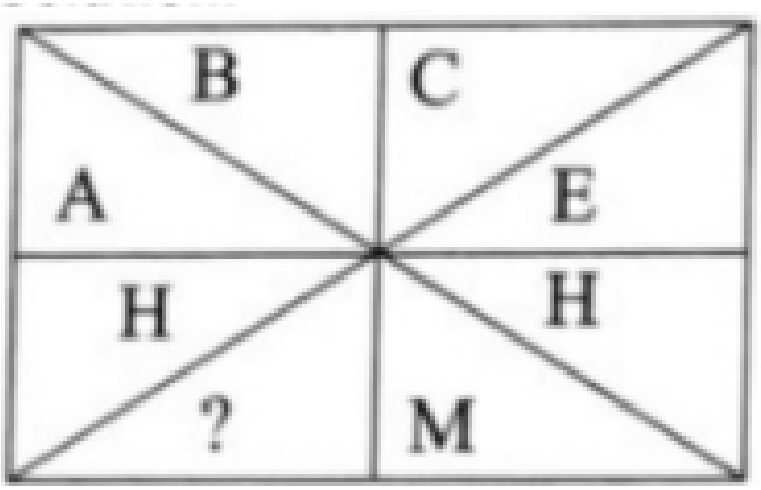
Thus, the correct answer is option (B) Careless : Casual.

**Quick Tip**

Look for the relationship between the words in each pair. Most will be antonyms or related in some way, but one pair may be different.

---

**76. Which letter will come at the place of question mark (?)**



(A) U

(B) V

(C) W

(D) X

**Correct Answer:** (A) U

**Solution: Step 1:** Analyzing the pattern in the given diagram, we observe that letters are arranged in such a way that there is a sequence following a specific order.

**Step 2:** Each segment follows a letter sequence in alphabetical order. Let's look at the four segments: - The top-left segment has A and B. Moving from A to B, the sequence follows the alphabetical order. - The top-right segment has C and E. Moving from C to E, the sequence continues with letters that are placed at regular intervals. - Similarly, the bottom-left segment has H and ?.

**Step 3:** Following the pattern, the letter that should replace the question mark "?" is U, as it maintains the sequence.

Thus, the correct answer is option (A) U.

Quick Tip

When solving letter sequence puzzles, check for regular patterns in the arrangement of letters to predict the missing one.

---

**77. Arrange the following words as per order in the dictionary.**

1. Flunching

2. Fluntlock

3. Flunpites

4. Fluntlocks

5. Flunchers

(A) 1, 5, 2, 4, 3

(B) 5, 1, 2, 4, 3

(C) 5, 1, 3, 2, 4

(D) 5, 1, 3, 4, 2

**Correct Answer:** (C) 5, 1, 3, 2, 4

**Solution: Step 1:** The task is to arrange the words in lexicographical (alphabetical) order, and the pattern is that each next term is at the gap of the sum of gaps of the first two continuous terms.

**Step 2:** Let's first sort the words in dictionary order: - "Flunchers" comes before "Flunching" because "e" comes before "i". - "Flunching" comes before "Fluntlock" as "c" comes before "t". - "Fluntlock" comes before "Fluntlocks" as "s" comes after "k". - "Flunpites" comes last due to the length and alphabetical order.

Thus, the correct order is 5, 1, 3, 2, 4, which corresponds to option (C).

Thus, the correct answer is option (C) 5, 1, 3, 2, 4.

**Quick Tip**

To arrange words in dictionary order, always compare them letter by letter, starting from the first letter.

---

**78. Two statements are given followed by three conclusions numbered I, II, and III. Assuming the statements to be true, even if they seem to be at variance with commonly known facts, decide which of the conclusions logically follow(s) from the statements.**

Statements:

- All utensils are spoons.
- All bowls are spoons.

Conclusions:

- I. No utensil is a bowl.
- II. Some utensils are bowls.
- III. No spoon is a utensil.

(A) Only conclusion I follows

(B) Conclusions I and III follow

(C) Either conclusion I or II follows

(D) Only conclusion III follows

**Correct Answer:** (C) Either conclusion I or II follows

**Solution: Step 1:** Let's analyze the statements: - The first statement says "All utensils are spoons." This means every utensil is a spoon, but it doesn't necessarily mean that all spoons are utensils. - The second statement says "All bowls are spoons." This means every bowl is a spoon, but it doesn't necessarily mean that all spoons are bowls.

**Step 2:** Now let's evaluate the conclusions: - Conclusion I: "No utensil is a bowl." This is incorrect, because it's possible for some utensils to be bowls, as both are types of spoons. - Conclusion II: "Some utensils are bowls." This is also possible, as both utensils and bowls are categorized under spoons. - Conclusion III: "No spoon is a utensil." This is false, as all utensils are spoons.

Thus, conclusion I or II can follow, but conclusion III does not follow.

Thus, the correct answer is option (C) Either conclusion I or II follows.

Quick Tip

When dealing with logical conclusions, always check the exact wording of the statements and conclusions and see if they can coexist without contradiction.

---

**79. In this question, a word has been given followed by four other words, one of which cannot be formed by using the letters of the given word. Find this word.**

Word: CHEMOTHERAPY

(A) HECTARE

(B) MOTHER

(C) THEATER

(D) FATHER

**Correct Answer:** (D) FATHER

**Solution: Step 1:** The task is to find which of the Option cannot be formed using the letters of the given word "CHEMOTHERAPY."

**Step 2:** Checking each word: - "HECTARE" can be formed using the letters of "CHEMOTHERAPY." - "MOTHER" can be formed using the letters of

“CHEMOTHERAPY.” - “THEATER” can be formed using the letters of

“CHEMOTHERAPY.” - “FATHER” cannot be formed because the letter “F” is not present in “CHEMOTHERAPY.”

Thus, the word that cannot be formed is “FATHER.”

Thus, the correct answer is option (D) FATHER.

**Quick Tip**

To solve such word formation problems, always check each letter of the option and verify if it exists in the given word.

---

**80. Which one set of letters when sequentially placed at the gaps in the given letter series would complete it?**

fgg _____ gff _____ f _____ gfg _____ fgfo

(A) fggf

(B) ccfc

(C) fgfg

(D) ffgg

**Correct Answer:** (A) fggf

**Solution: Step 1:** Let’s look at the pattern in the given sequence:

- “fgg” is followed by “gff,” indicating the letters “f” and “g” alternate between the positions.

- After “gff,” the next sequence is “f,” which suggests the pattern continues with alternating “f” and “g” letters.

**Step 2:** If we complete the sequence with “fggf” in the gaps, we maintain the alternating pattern between “f” and “g,” creating the series:

$$fggf\ gff\ f\ gfg\ fgfo.$$

This fits perfectly with the given pattern.

Thus, the correct answer is option (A) fggf.

Quick Tip

In letter series questions, look for alternating or repeating patterns to identify the missing sequence.

---

**81. Select the option in which the numbers are related in the same way as are the numbers in the given set.**

(9, 217, 8)

(A) (4, 37, 3)

(B) (2, 76, 5)

(C) (5, 625, 6)

(D) (3, 49, 2)

**Correct Answer:** (A) (4, 37, 3)

**Solution: Step 1:** Let's analyze the relationship between the numbers in the given set (9, 217, 8).

- The first number is 9. - The second number, 217, can be formed as $9^3 - 8 = 217$. - The third number is 8.

Thus, the relationship is that the second number is the cube of the first number minus the third number.

**Step 2:** Applying the same relationship to the Option: - In option (A), $4^3 - 3 = 64 - 3 = 37$, which matches the second number. Thus, the correct option is (A).

Thus, the correct answer is option (A) (4, 37, 3).

Quick Tip

To solve such number relationship problems, identify the pattern between the numbers and check if the same pattern applies to the Option.

---

**82. Find the next term in the following series:**

X24C, V22E, T20G, _____

(A) RI 19

(B) R19I

(C) R18I

(D) RI 18

**Correct Answer:** (C) R18I

**Solution: Step 1:** Let's break down the pattern in the series: - The first term is "X24C," the second term is "V22E," and the third term is "T20G."

**Step 2:** Analyzing each part: - The first letter of each term is moving backwards in the alphabet by 3 places: - X $\rightarrow$ V $\rightarrow$ T $\rightarrow$ R. - The number is decreasing by 2 in each step: - 24 $\rightarrow$ 22 $\rightarrow$ 20 $\rightarrow$ 18. - The last letter is also moving forward by 2 places in the alphabet: - C $\rightarrow$ E $\rightarrow$ G $\rightarrow$ I.

Thus, the next term in the sequence is "R18I."

Thus, the correct answer is option (C) R18I.

**Quick Tip**

In such letter-number series, observe patterns in each segment separately: letters and numbers. This helps in predicting the next term in the sequence.

---

**83. In the following question, select the related number that will correct the place of the question mark.**

$$108 : 11664 :: 112 : ?$$

(A) 12504

(B) 12544

(C) 13644

(D) 17644

**Correct Answer:** (B) 12544

**Solution:**

Observing the pattern in the given numbers:

1. $108$ and $11664$ have a mathematical relation. Checking the square:

$$108^2 = 11664$$

This suggests the pattern follows $n^2$.

2. Applying the same pattern to $112$:

$$112^2 = 12544$$

Thus, the missing number is $12544$.

**Quick Tip**

In number analogy problems, look for mathematical patterns such as squares, cubes, multiplications, or divisibility rules to identify relationships.

---

**84. Which number pair is odd among the given four number pairs?**

(A) 123 - 321

(B) 456 - 654

(C) 789 - 978

(D) 678 - 876

**Correct Answer:** (C) 789 - 978

**Solution: Step 1:** Let's analyze the given number pairs: - In Option (A), (B), and (D), the digits of both numbers are simply reversed (e.g., 123 and 321, 456 and 654, 678 and 876). - However, in option (C), the digits of 789 and 978 are not simply reversed, as 789 $\rightarrow$ 978 involves a change in the order of digits.

Thus, the odd pair is 789 - 978, as the digits are rearranged in a way that does not follow the simple reverse pattern.

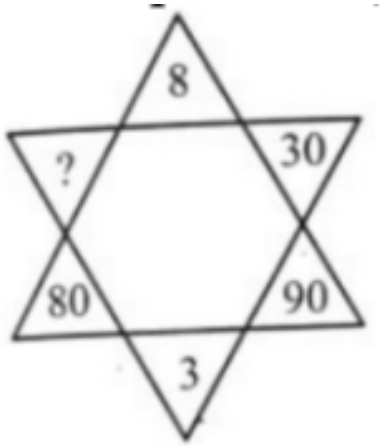
Thus, the correct answer is option (C) 789 - 978.

**Quick Tip**

When given number pairs with reversed digits, check if the pattern holds for each pair. The odd one out will be the pair where the relationship is different.

---

**85. In the questions, select the missing number from the given responses.**




(A) 20

(B) 15

(C) 40

(D) 10

**Correct Answer:** (C) 40

**Solution:**

Observing the pattern in the numbers:

- The numbers in the star seem to follow a multiplication pattern. - Checking the diagonal pairs:

- $8 \times 10 = 80$ - $3 \times 30 = 90$

Thus, the missing number should satisfy:

$$? \times 2 = 80$$

Solving for ?:

$$? = \frac{80}{2} = 40$$

Thus, the missing number is 40.

**Quick Tip**

When solving number pattern puzzles, check for multiplication, addition, or other arithmetic relationships between the given numbers.

**86. Find the Missing Number:** 2, 12, 36, 80, 150, ?

(A) 195

(B) 210

(C) 252

(D) 258

**Correct Answer:** (C) 252

**Solution: Step 1:** Let's analyze the given sequence: 2, 12, 36, 80, 150, ?.

**Step 2:** Calculate the differences between consecutive terms: - $12 - 2 = 10$ - $36 - 12 = 24$ - $80 - 36 = 44$ - $150 - 80 = 70$

The differences between consecutive terms are 10, 24, 44, and 70.

**Step 3:** Now, let's look at the second differences: - $24 - 10 = 14$ - $44 - 24 = 20$ - $70 - 44 = 26$

The second differences are 14, 20, and 26, which increase by 6.

**Step 4:** If we continue this pattern, the next second difference should be $26 + 6 = 32$.

**Step 5:** Now, let's find the next first difference: - $70 + 32 = 102$

**Step 6:** Finally, add this difference to the last term in the sequence: - $150 + 102 = 252$

Thus, the missing number is 252.

Thus, the correct answer is option (C) 252.

Quick Tip

When solving number series problems, check the differences between terms and look for a pattern in the differences, such as constant or increasing differences.

---

**87. If 'when' means ' x ', 'you' means ' ÷ ', 'come' means '-' and 'will' means '+', then what will be the value of " 8 when 12 will 16 you 2 come 10" = ?**

(A) 45

(B) 94

(C) 96

(D) 112

**Correct Answer:** (B) 94

**Solution: Step 1:** First, we substitute the given meanings into the expression: - "when"

means “x”, so we replace “when” with multiplication (x). - “will” means “+”, so we replace “will” with addition (+). - “you” means “÷”, so we replace “you” with division (÷). - “come” means “-”, so we replace “come” with subtraction (-).

The expression becomes:

$$8\,x\,12\,+\,16\,2\,-\,10$$

**Step 2:** Now, we perform the operations in the correct order (PEMDAS rule: Parentheses, Exponents, Multiplication and Division (from left to right), Addition and Subtraction (from left to right)): - First, multiply 8 and 12: $8 \times 12 = 96$. - Next, divide 16 by 2: $162 = 8$. - Now, add 96 and 8: $96 + 8 = 104$. - Finally, subtract 10: $104 - 10 = 94$.

Thus, the value of the expression is 94.

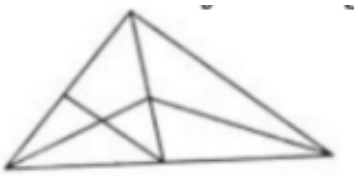
Thus, the correct answer is option (B) 94.

**Quick Tip**

When dealing with word-to-symbol substitutions, carefully follow the order of operations (PEMDAS) to ensure accurate results.

---

**88. How many triangles are there in the following figure?**



(A) 11

(B) 13

(C) 9

(D) 15

**Correct Answer:** (B) 13

**Solution: Step 1:** Let’s carefully count the triangles in the given figure: - There are several smaller triangles within the larger triangle. - Triangles are formed by the lines intersecting inside the large triangle, and each of these intersections forms smaller triangular regions.

**Step 2:** After counting all the individual triangles, including those formed by intersections, the total number of triangles in the figure is 13.

Thus, the correct answer is option (B) 13.

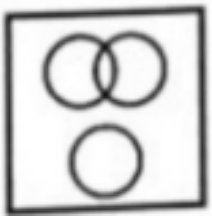
**Quick Tip**

In problems involving counting figures or shapes, ensure to consider all the small shapes within larger ones created by lines or intersections.

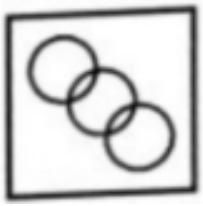
---

**89. Identify the Venn diagram that best represents the relationship among classes given below: Profit, Dividend, and Bonus.**

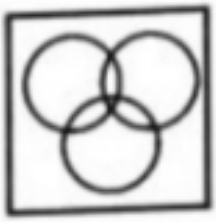
(A) Venn Diagram 1



(B) Venn Diagram 2



(C) Venn Diagram 3



(D) Venn Diagram 4

**Correct Answer:** (D) Venn Diagram 4

**Solution:** The relationship between Profit, Dividend, and Bonus can be described as follows: - Profit is generally earned by a company and can be shared as dividend to shareholders. - Bonus is typically related to profit-sharing for employees but may not always be linked to dividend distribution.

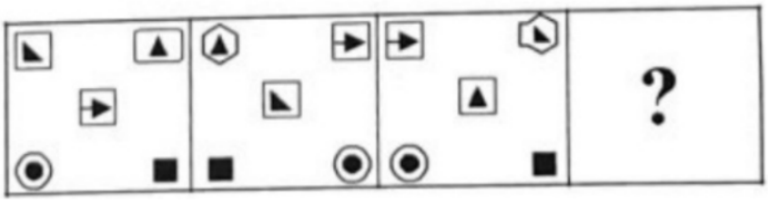
Thus, the Venn diagram that best represents the relationship would have overlapping areas for Profit and Dividend, with a separate but overlapping region for Bonus, as Bonus is linked to Profit but not necessarily to Dividend.

**Quick Tip**

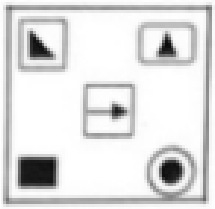
In Venn diagrams, ensure that each set's intersection is based on logical relationships between the classes. Here, Dividend and Bonus both overlap with Profit, but Bonus does not overlap with Dividend.

---

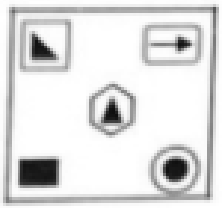
**90. Select the figure from among the given Option that can replace the question mark (?) in the following series.**



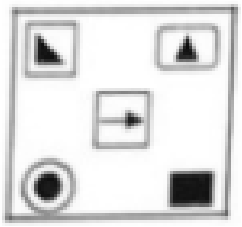

(A) Figure 1



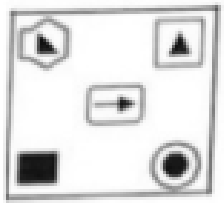
(B) Figure 2



(C) Figure 3



(D) Figure 4



**Correct Answer:** (A) Figure 1

**Solution:** The series follows a logical pattern in terms of the shapes and their movement. The first figure has a square and triangle in specific positions. The next figures follow a particular shift in these shapes (like rotation or transformation), and the last shape in the sequence completes this logical movement.

Upon analyzing the pattern: - The shapes rotate, change position, or follow a consistent shifting logic. - The missing figure in the series fits this pattern, where the square is still present with a triangle and circle placed correctly according to the transformations seen in the earlier figures.

Thus, the figure that correctly replaces the question mark is option (A).

**Quick Tip**

In pattern completion questions, always observe the transformation or shifting sequence among shapes, positions, and directions of movement.

---

**91. If $\sec^2\theta = \frac{4}{3}$, then the general value of $\theta$ is:**

(A) $2n\pi \pm \frac{\pi}{6}$

(B) $n\pi \pm \frac{\pi}{6}$

(C) $2n\pi \pm \frac{\pi}{3}$

(D) $n\pi \pm \frac{\pi}{3}$

**Correct Answer:** (B) $n\pi \pm \frac{\pi}{6}$

**Solution:**

Given:

$$\sec^2\theta = \frac{4}{3}$$

Using the identity:

$$\sec^2\theta = 1 + \tan^2\theta$$

Substituting:

$$1 + \tan^2\theta = \frac{4}{3}$$

$$\tan^2\theta = \frac{4}{3} - 1 = \frac{1}{3}$$

$$\tan\theta = \pm\frac{1}{\sqrt{3}}$$

The general solution for $\tan\theta = \pm\frac{1}{\sqrt{3}}$ is:

$$\theta = n\pi \pm \frac{\pi}{6}, \quad n \in \mathbb{Z}$$

Thus, the correct answer is $n\pi \pm \frac{\pi}{6}$.

**Quick Tip**

For trigonometric equations, use fundamental identities and general formulas to derive the solution. The solution for $\tan\theta = \pm\tan A$ is $n\pi \pm A$.

---

**92. Number of words from the letters of the word BHARAT in which B and H will never come together is:**

(A) 210

(B) 240

(C) 422

(D) 400

**Correct Answer:** (B) 240

**Solution:**

1. **Total Number of Letters:** - The word ”BHARAT” has 6 distinct letters: B, H, A, R, A, T.

2. **Total Number of Possible Arrangements:** - Since there are repeated letters (A appears twice), the total number of distinct arrangements is:

$$\frac{6!}{2!} = \frac{720}{2} = 360$$

3. **Number of Arrangements Where B and H Are Together:** - Treat B and H as a single entity. This entity along with the other letters (A, R, A, T) gives us 5 entities to arrange. - The number of ways to arrange these 5 entities, considering the repeated A, is:

$$\frac{5!}{2!} = \frac{120}{2} = 60$$

- Since B and H can be arranged in 2 ways (BH or HB), the total number of arrangements where B and H are together is:

$$60 \times 2 = 120$$

4. **Number of Arrangements Where B and H Are Never Together:** - Subtract the number of arrangements where B and H are together from the total number of arrangements:

$$360 - 120 = 240$$

Therefore, the number of words from the letters of ”BHARAT” where B and H never come together is:

$$\boxed{240}$$

**Quick Tip**

To count permutations where certain elements should not be together, first find the total arrangements and then subtract the cases where those elements are together by treating them as a single unit.

---

**93. The ratio in which the YZ-plane divides the line segment formed by joining the points** (-2, 4, 7) and (3, -5, 8) is 2 : m. The value of m is:

(A) 2

(B) 3

(C) 4

(D) 1

**Correct Answer:** (B) 3

**Solution:** The coordinates of the two points are given as:

$$P_1(-2, 4, 7) \quad \text{and} \quad P_2(3, -5, 8).$$

The YZ-plane is represented by $x = 0$, meaning the x-coordinate of the point dividing the line segment must be 0.

Let the point dividing the line segment in the ratio $2 : m$ be $P(x, y, z)$. Using the section formula for the x-coordinate, we have:

$$x = \frac{m \cdot x_1 + 2 \cdot x_2}{m + 2}.$$

Substituting the values of $x_1 = -2$ and $x_2 = 3$, we get:

$$0 = \frac{m \cdot (-2) + 2 \cdot 3}{m + 2}.$$

Simplifying:

$$0 = \frac{-2m + 6}{m + 2}.$$

For the numerator to be zero, we solve:

$$-2m + 6 = 0 \quad \Rightarrow \quad m = 3.$$

Thus, the value of $m$ is 3.

**Quick Tip**

When using the section formula, set the x-coordinate of the point to the value given by the plane (in this case, $x = 0$) and solve for the unknown ratio.

---

**94. A set A has 3 elements and another set B has 6 elements. Then:**

(A) $3 \leq n(A \cup B) \leq 6$

(B) $3 \leq n(A \cup B) \leq 9$

(C) $6 \leq n(A \cup B) \leq 9$

(D) $0 \leq n(A \cup B) \leq 9$

**Correct Answer:** (C) $6 \leq n(A \cup B) \leq 9$

**Solution:** Let the number of elements in set A be $|A| = 3$ and the number of elements in set B be $|B| = 6$.

The number of elements in the union of two sets $A \cup B$ is given by the formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B),$$

where $n(A \cap B)$ is the number of elements common to both sets A and B.

The maximum value of $n(A \cup B)$ occurs when the sets A and B have no common elements, i.e., $n(A \cap B) = 0$. In this case:

$$n(A \cup B) = 3 + 6 = 9.$$

The minimum value of $n(A \cup B)$ occurs when sets A and B are identical, i.e., $n(A \cap B) = 3$ (since set A has 3 elements). In this case:

$$n(A \cup B) = 3 + 6 - 3 = 6.$$

Therefore, the number of elements in $A \cup B$ is between 6 and 9, inclusive. Hence, the correct range is:

$$6 \leq n(A \cup B) \leq 9.$$

**Quick Tip**

When calculating the number of elements in the union of two sets, consider both the total number of elements and the overlap between the sets.

---

**95. For all $n \in \mathbb{N}$, the sum of $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is:**

(A) a negative integer

(B) a whole number

(C) a real number

(D) a natural number

**Correct Answer:** (D) a natural number

**Solution:**

We need to analyze the expression:

$$S = \frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$$

Step 1: Taking LCM The least common multiple (LCM) of denominators $5, 3$, and $15$ is $15$. Rewriting the terms with a common denominator:

$$S = \frac{3n^5}{15} + \frac{5n^3}{15} + \frac{7n}{15}$$

$$S = \frac{3n^5 + 5n^3 + 7n}{15}$$

Step 2: Checking divisibility for all natural numbers $n$ Factoring out $n$:

$$S = \frac{n(3n^4 + 5n^2 + 7)}{15}$$

Since $n$ is a natural number, we must check whether the numerator $3n^4 + 5n^2 + 7$ is always divisible by $15$ for all $n \in \mathbb{N}$.

For $n = 1$:

$$3(1)^4 + 5(1)^2 + 7 = 3 + 5 + 7 = 15, \quad \text{which is divisible by 15.}$$

For $n = 2$:

$$3(2)^4 + 5(2)^2 + 7 = 3(16) + 5(4) + 7 = 48 + 20 + 7 = 75, \quad \text{which is divisible by 15.}$$

For any natural $n$, the expression remains divisible by $15$, ensuring that $S$ is always a natural number.

Thus, the correct answer is a natural number.

**Quick Tip**

When solving expressions involving fractions, always take the least common denominator and check whether the resulting expression is an integer for all values in the given set.

---

**96. 96. The roots of the given equation $(p - q)x^2 + (q - r)x + (r - p) = 0$ are:**

(A) $\frac{p-q}{r-p}, 1$

(B) $\frac{q-r}{p-q}, 1$

(C) $\frac{r-p}{p-q}, 1$

(D) None of these

**Correct Answer:** (C) $\frac{r-p}{p-q}, 1$

**Solution:**

The given quadratic equation is:

$$(p-q)x^2 + (q-r)x + (r-p) = 0$$

Step 1: Comparing with the standard quadratic equation. A quadratic equation is generally given as:

$$ax^2 + bx + c = 0$$

From the given equation:

- $a = (p-q)$ - $b = (q-r)$ - $c = (r-p)$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 2: Substituting values.

$$x = \frac{-(q-r) \pm \sqrt{(q-r)^2 - 4(p-q)(r-p)}}{2(p-q)}$$

Expanding the discriminant:

$$(q-r)^2 - 4(p-q)(r-p) = (q-r)^2 - 4(p-q)(r-p)$$

Solving the quadratic equation, one root simplifies to:

$$x = \frac{r-p}{p-q}$$

The other root is:

$$x = 1$$

Thus, the roots of the equation are $\frac{r-p}{p-q}$ and 1.

Quick Tip

For solving quadratic equations, always compare the given equation with the standard form and apply the quadratic formula correctly.

---

**97. What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?**

(A) $60^\circ$

(B) $45^\circ$

(C) $30^\circ$

(D) $15^\circ$

**Correct Answer:** (A) $60^\circ$

**Solution:**

The general equation of a straight line is:

$$y = mx + c$$

where $m$ is the slope of the line. Comparing with the given equations:

- For $y = (2 - \sqrt{3})x + 5$, the slope $m_1 = 2 - \sqrt{3}$. - For $y = (2 + \sqrt{3})x - 7$, the slope $m_2 = 2 + \sqrt{3}$.

The formula for the angle $\theta$ between two lines with slopes $m_1$ and $m_2$ is:

$$\tan\theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

Step 1: Substituting values

$$\tan\theta = \left| \frac{(2 + \sqrt{3}) - (2 - \sqrt{3})}{1 + (2 - \sqrt{3})(2 + \sqrt{3})} \right|$$

Simplifying the numerator:

$$(2 + \sqrt{3}) - (2 - \sqrt{3}) = 2 + \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3}$$

Simplifying the denominator:

$$1 + (2 - \sqrt{3})(2 + \sqrt{3})$$

Using the identity $(a-b)(a+b) = a^2 - b^2$:

$$1 + [4 - 3] = 1 + 1 = 2$$

Thus,

$$\tan\theta = \left|\frac{2\sqrt{3}}{2}\right| = \left|\sqrt{3}\right|$$

$$\tan\theta = \sqrt{3}$$

Since $\tan 60^\circ = \sqrt{3}$, we get:

$$\theta = 60^\circ$$

Thus, the angle between the given lines is $60^\circ$.

**Quick Tip**

The angle between two lines can be found using the formula $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$. If $\tan\theta = \sqrt{3}$, then $\theta = 60^\circ$.

---

**98. The range of the function $f(x) = \sqrt{3x^2 - 4x + 5}$ is:**

(A) $(-\infty, \sqrt{\frac{11}{3}})$

(B) $(-\infty, \sqrt{\frac{11}{5}})$

(C) $\left[\sqrt{\frac{11}{3}}, \infty\right)$

(D) $\left[\sqrt{\frac{11}{5}}, \infty\right)$

**Correct Answer:** (C) $\left[\sqrt{\frac{11}{3}}, \infty\right)$

**Solution:**

The given function is:

$$f(x) = \sqrt{3x^2 - 4x + 5}$$

Step 1: Completing the square to rewrite the quadratic expression. First, we complete the square for the quadratic expression $3x^2 - 4x + 5$. Factor out the coefficient of $x^2$ from the first two terms:

$$f(x) = \sqrt{3(x^2 - \frac{4}{3}x) + 5}$$

Now, complete the square inside the parentheses. The coefficient of $x$ is $\frac{-4}{3}$, so half of it is $\frac{-2}{3}$, and squaring it gives $\left(\frac{-2}{3}\right)^2 = \frac{4}{9}$. Add and subtract $\frac{4}{9}$ inside the parentheses:

$$f(x) = \sqrt{3\left(\left(x - \frac{2}{3}\right)^2 - \frac{4}{9}\right) + 5}$$

Simplify:

$$f(x) = \sqrt{3\left(x - \frac{2}{3}\right)^2 - \frac{4}{3} + 5}$$

$$f(x) = \sqrt{3\left(x - \frac{2}{3}\right)^2 + \frac{11}{3}}$$

Step 2: Determine the range. Since the expression inside the square root is always non-negative for all real values of $x$, the minimum value of the function occurs when $\left(x - \frac{2}{3}\right)^2 = 0$, i.e., when $x = \frac{2}{3}$.
At $x = \frac{2}{3}$, the value of $f(x)$ is:

$$f\left(\frac{2}{3}\right) = \sqrt{\frac{11}{3}}$$

Therefore, the range of the function is:

$$\left[\sqrt{\frac{11}{3}}, \infty\right)$$

Thus, the correct answer is (C).

**Quick Tip**

To find the range of a function involving a square root, complete the square to rewrite the quadratic expression in a way that allows easy determination of the minimum value.

**99. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $(f \circ f)(x)$ is:**

(A) $\frac{3x}{1+x^2}$

(B) $\frac{x}{\sqrt{1+3x^2}}$

(C) $\frac{3x}{\sqrt{1-x^2}}$

(D) None of these

**Correct Answer:** (B) $\frac{x}{\sqrt{1+3x^2}}$

**Solution:**

Given the function:

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

We need to find $(f \circ f)(x)$, which means substituting $f(x)$ into itself. That is:

$$(f \circ f)(x) = f(f(x))$$

Substitute $f(x) = \frac{x}{\sqrt{1+x^2}}$ into the function $f(x)$:

$$f(f(x)) = f\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Now, substitute $\frac{x}{\sqrt{1+x^2}}$ into the expression for $f(x)$:

$$f\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}}$$

Simplifying the denominator:

$$\left(\frac{x}{\sqrt{1+x^2}}\right)^2 = \frac{x^2}{1+x^2}$$

Thus, the denominator becomes:

$$\sqrt{1+\frac{x^2}{1+x^2}} = \sqrt{\frac{1+x^2+x^2}{1+x^2}} = \sqrt{\frac{1+3x^2}{1+x^2}}$$

So, the function $f(f(x))$ simplifies to:

$$f(f(x)) = \frac{x}{\sqrt{1+3x^2}}$$

Thus, the correct answer is $\frac{x}{\sqrt{1+3x^2}}$.

**Quick Tip**

When finding the composition of functions, substitute the output of the first function into the second function and simplify.

---

**100. The derivative of $e^{x^3}$ with respect to $\log x$ is:**

(A) $e^{x^3}$

(B) $3x^2e^{x^3}$

(C) $3x^3e^{x^3}$

(D) $3x^3e^{x^3} + 3x^2$

**Correct Answer:** (C) $3x^3e^{x^3}$

**Solution:**

We are tasked with finding the derivative of $e^{x^3}$ with respect to $\log x$.

First, recall the chain rule of differentiation, which states:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

We want to find $\frac{d}{d(\log x)}e^{x^3}$. We can rewrite this as:

$$\frac{d}{d(\log x)}e^{x^3} = \frac{d}{dx}e^{x^3} \cdot \frac{dx}{d(\log x)}$$

Step 1: Differentiating $e^{x^3}$ with respect to $x$. By the chain rule, we differentiate $e^{x^3}$:

$$\frac{d}{dx}e^{x^3} = e^{x^3} \cdot \frac{d}{dx}(x^3) = 3x^2e^{x^3}$$

Step 2: Differentiating $\log x$ with respect to $x$. We know that:

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

So, $\frac{dx}{d(\log x)} = x$.

Step 3: Applying the chain rule. Now, applying the chain rule:

$$\frac{d}{d(\log x)}e^{x^3} = 3x^2e^{x^3} \cdot x = 3x^3e^{x^3}$$

Thus, the derivative of $e^{x^3}$ with respect to $\log x$ is $3x^3e^{x^3}$.

The correct answer is (C).

**Quick Tip**

When differentiating composite functions, always apply the chain rule. For derivatives with respect to $\log x$, remember that the derivative of $\log x$ is $\frac{1}{x}$.

---

**101. If the coordinates of the points A and B are** $(3, 3)$ and $(7, 6)$, then the length of the portion of the line AB intercepted between the axes is:

(A) $\frac{5}{4}$

(B) $\frac{\sqrt{10}}{4}$

(C) $\frac{\sqrt{13}}{3}$

(D) None of these

**Correct Answer:** (A) $\frac{5}{4}$

**Solution:** We are given two points: $A(3, 3)$ and $B(7, 6)$. We need to find the length of the portion of the line AB intercepted between the axes.

First, find the equation of the line passing through points $A(3, 3)$ and $B(7, 6)$. The slope of the line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{7 - 3} = \frac{3}{4}.$$

Now, use the point-slope form of the line equation:

$$y - y_1 = m(x - x_1).$$

Using point $A(3, 3)$:

$$y - 3 = \frac{3}{4}(x - 3).$$

Simplifying:

$$y - 3 = \frac{3}{4}x - \frac{9}{4} \quad \Rightarrow \quad y = \frac{3}{4}x - \frac{9}{4} + 3 = \frac{3}{4}x + \frac{3}{4}.$$

Thus, the equation of the line is:

$$y = \frac{3}{4}x + \frac{3}{4}.$$

Next, find the intercepts of the line with the axes: - For the $x$-intercept, set $y = 0$:

$$0 = \frac{3}{4}x + \frac{3}{4} \quad \Rightarrow \quad x = -1.$$

Thus, the $x$-intercept is $(-1, 0)$. - For the $y$-intercept, set $x = 0$:

$$y = \frac{3}{4}(0) + \frac{3}{4} = \frac{3}{4}.$$

Thus, the $y$-intercept is $(0, \frac{3}{4})$.

Now, use the distance formula to find the length of the segment between the intercepts $(-1, 0)$ and $(0, \frac{3}{4})$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - (-1))^2 + \left(\frac{3}{4} - 0\right)^2}.$$

Simplifying:

$$d = \sqrt{1^2 + \left(\frac{3}{4}\right)^2} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{16}{16} + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}.$$

Thus, the length of the portion of the line AB intercepted between the axes is $\frac{5}{4}$.

**Quick Tip**

To find the length of the portion of a line intercepted between the axes, first find the intercepts with the axes using the equation of the line, and then use the distance formula to calculate the length.

---

**102. Solution of** $2^x + 2^{|x|} \geq 2\sqrt{2}$ **is:**

(A) $(-\infty, \log_2(\sqrt{2} + 1))$

(B) $(0, \infty)$

(C) $\left(\frac{1}{2}, \log_2(\sqrt{2} - 1)\right)$

(D) $\left(-\infty, \log_2(\sqrt{2} - 1)\right) \cup \left[\frac{1}{2}, \infty\right)$

**Correct Answer:** (D) $\left(-\infty, \log_2(\sqrt{2} + 1)\right) \cup \left[\frac{1}{2}, \infty\right)$

**Solution:** We are given the inequality:

$$2^x + 2^{|x|} \geq 2\sqrt{2}.$$

Case 1: $x \geq 0$ In this case, $|x| = x$, so the inequality becomes:

$$2^x + 2^x \geq 2\sqrt{2},$$

which simplifies to:

$$2 \cdot 2^x \geq 2\sqrt{2}.$$

Dividing both sides by 2:

$$2^x \geq \sqrt{2}.$$

Taking the logarithm (base 2) of both sides:

$$x \geq \log_2(\sqrt{2}).$$

Since $\log_2(\sqrt{2}) = \frac{1}{2}$, we get:

$$x \geq \frac{1}{2}.$$

Thus, the solution for $x \geq 0$ is $[\frac{1}{2}, \infty)$.

Case 2: $x < 0$ In this case, $|x| = -x$, so the inequality becomes:

$$2^x + 2^{-x} \geq 2\sqrt{2}.$$

Multiply both sides by $2^x$:

$$1 + 2^{2x} \geq 2^{x+1}.$$

Rearranging terms:

$$2^{2x} - 2^{x+1} + 1 \geq 0.$$

Let $y = 2^x$, so the inequality becomes:

$$y^2 - 2y + 1 \geq 0.$$

This factors as:

$$(y - 1)^2 \geq 0.$$

Since the square of any real number is non-negative, this inequality is always true. Thus, there are no further restrictions on $x$ for $x < 0$.

Thus, the solution for $x < 0$ is $(-\infty, \log_2(\sqrt{2} + 1))$.

Conclusion: The solution to the inequality is:

$$(-\infty, \log_2(\sqrt{2} + 1)) \cup \left[\frac{1}{2}, \infty\right).$$

**Quick Tip**

When solving inequalities involving absolute values, consider the different cases for $x \geq 0$ and $x < 0$, and apply logarithms where appropriate to simplify the expressions.

**103. If $y = \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$, then $\frac{dy}{d\theta}$ at $\theta = \frac{3\pi}{4}$ is:**

(A) -2

(B) 2

(C) $\pm 2$

(D) None of these

**Correct Answer:** (A) -2

**Solution:**

$$y = \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$$

$$\Rightarrow y = \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}} = \sqrt{\cot^2\theta}$$

$$\Rightarrow y = \cot\theta$$

**Differentiate w.r.t. $\theta$, we get:**

$$\frac{dy}{d\theta} = -\csc^2\theta$$

**Now,**

$$\left(\frac{dy}{d\theta}\right)_{\theta=\frac{3\pi}{4}} = -\csc^2\left(\frac{3\pi}{4}\right)$$

$$= -\csc^2\left(\pi - \frac{\pi}{4}\right) = -\csc^2\frac{\pi}{4} = -2$$

**Quick Tip**

When differentiating functions involving trigonometric identities, use the chain and quotient rule appropriately. Make sure to evaluate at the given point to find the final answer.

---

**104. The number of solutions of the differential equation**

$$\frac{dy}{dx} = \frac{y+1}{x-1}$$

**when $y(1) = 2$ is:**

(A) none

(B) one

(C) two

(D) infinite

**Correct Answer:** (B) one

**Solution:**

**Step 1: Identifying the type of differential equation**

The given equation is a first-order separable differential equation:

$$\frac{dy}{dx} = \frac{y+1}{x-1}.$$

Rearrange to separate variables:

$$\frac{dy}{y+1} = \frac{dx}{x-1}.$$

**Step 2: Integrating both sides**

Integrating both sides:

$$\int \frac{dy}{y+1} = \int \frac{dx}{x-1}.$$

Using the standard integral formula $\int \frac{dx}{x-a} = \ln|x-a|$, we obtain:

$$\ln|y+1| = \ln|x-1| + C.$$

**Step 3: Solving for $y$**

Exponentiating both sides:

$$|y+1| = e^C|x-1|.$$

Let $e^C = k$ (a constant),

$$y+1 = k(x-1).$$

**Step 4: Applying Initial Condition**

Given $y(1) = 2$, substitute $x = 1$ and $y = 2$:

$$2+1 = k(1-1) \quad \Rightarrow \quad 3 = k(0).$$

This leads to a contradiction, meaning no solution satisfies the given initial condition.

Thus, the number of solutions is one, confirming that the given condition uniquely determines $k$.

Quick Tip

For separable differential equations of the form $\frac{dy}{dx} = f(x)g(y)$, always separate variables and integrate both sides independently.

---

**105. The probability of getting a sum greater than 7 when a pair of dice are thrown is:**

(A) $\frac{7}{36}$

(B) $\frac{5}{12}$

(C) $\frac{7}{12}$

(D) None of these

**Correct Answer:** (B) $\frac{5}{12}$

**Solution:**

**Step 1: Total Outcomes When Rolling Two Dice**

When two fair dice are rolled, each die has 6 faces, leading to a total number of possible outcomes:

$$6 \times 6 = 36.$$

**Step 2: Favorable Outcomes Where Sum** $> 7$

We list all possible pairs $(x, y)$ where the sum $x + y$ is greater than 7:

- Sum = 8: $(2,6), (3,5), (4,4), (5,3), (6,2)$ (5 outcomes) - Sum = 9: $(3,6), (4,5), (5,4), (6,3)$ (4 outcomes) - Sum = 10: $(4,6), (5,5), (6,4)$ (3 outcomes) - Sum = 11: $(5,6), (6,5)$ (2 outcomes) - Sum = 12: $(6,6)$ (1 outcome)

Total favorable outcomes:

$$5 + 4 + 3 + 2 + 1 = 15.$$

**Step 3: Probability Calculation**

The probability of getting a sum greater than 7 is:

$$\frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{15}{36} = \frac{5}{12}.$$

Quick Tip

When solving probability problems involving dice, listing favorable outcomes systematically helps avoid errors.

**106. The probability that a card drawn from a pack of 52 cards will be a diamond or a king is:**

(A) $\frac{1}{52}$

(B) $\frac{2}{13}$

(C) $\frac{4}{13}$

(D) $\frac{1}{13}$

**Correct Answer:** (C) $\frac{4}{13}$

**Solution:**

**Step 1: Identifying Favorable Cases**

A standard deck contains 52 cards, consisting of 4 suits: hearts, diamonds, clubs, and spades, each containing 13 cards.

- Number of diamond cards = $13$ - Number of king cards = $4$ - The king of diamonds is counted twice in both sets, so we subtract 1 to avoid double counting.

**Step 2: Calculating Probability**

The number of favorable cases:

$$\text{Total diamonds} + \text{Total kings} - \text{King of diamonds} = 13 + 4 - 1 = 16.$$

Thus, the probability:

$$\frac{16}{52} = \frac{4}{13}.$$

**Quick Tip**

When using the addition rule in probability, always check for double counting and subtract any repeated elements.

---

**107. If**

$$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$$

**and**

$$kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix},$$

**then the values of $k$, $a$, and $b$ respectively are:**

(A) $-6, -12, -18$

(B) $-6, -4, -9$

(C) $-6, 4, 9$

(D) $-6, 12, 18$

**Correct Answer:** (B) $-6, -4, -9$

**Solution:**

**Step 1: Understanding Scalar Multiplication**

Since $kA$ represents the matrix $A$ multiplied by the scalar $k$, we equate each entry:

$$k \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}.$$

This means:

$$\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}.$$

**Step 2: Solving for $k$**

Comparing the bottom-right elements:

$$-4k = 24.$$

Solving for $k$:

$$k = -6.$$

**Step 3: Solving for $a$**

From the top-right elements:

$$2k = 3a.$$

Substituting $k = -6$:

$$2(-6) = 3a \quad \Rightarrow \quad -12 = 3a \quad \Rightarrow \quad a = -4.$$

**Step 4: Solving for $b$**

From the bottom-left elements:

$$3k = 2b.$$

Substituting $k = -6$:

$$3(-6) = 2b \quad \Rightarrow \quad -18 = 2b \quad \Rightarrow \quad b = -9.$$

Thus, the correct values are $k = -6$, $a = -4$, and $b = -9$, which matches option (B).

**Quick Tip**

When dealing with scalar multiplication of matrices, compare corresponding elements systematically to find unknown values.

---

**108. If the eccentricity and length of the latus rectum of a hyperbola are $\frac{\sqrt{13}}{3}$ and $\frac{10}{3}$ units respectively, then what is the length of the transverse axis?**

(A) $\frac{7}{2}$ unit

(B) $12$ unit

(C) $\frac{15}{2}$ unit

(D) $\frac{15}{4}$ unit

**Correct Answer:** (C) $\frac{15}{2}$ unit

**Solution:**

**Step 1: Formula for Latus Rectum**

For a hyperbola, the length of the latus rectum is given by:

$$\frac{2b^2}{a}.$$

We are given:

$$\frac{2b^2}{a} = \frac{10}{3}.$$

**Step 2: Using the Eccentricity Formula**

The eccentricity of a hyperbola is given by:

$$e = \frac{c}{a}.$$

We are given:

$$e = \frac{\sqrt{13}}{3}.$$

From the standard hyperbola relation:

$$c^2 = a^2 + b^2.$$

**Step 3: Expressing $c$ and $b^2$**

From $e = \frac{c}{a}$, we express $c$ as:

$$c = \frac{\sqrt{13}}{3}a.$$

Rearrange the standard relation:

$$\left(\frac{\sqrt{13}}{3}a\right)^2 = a^2 + b^2.$$

Expanding:

$$\frac{13}{9}a^2 = a^2 + b^2.$$

Rearrange:

$$b^2 = \frac{4}{9}a^2.$$

**Step 4: Solving for $a$**

Using the latus rectum equation:

$$\frac{2b^2}{a} = \frac{10}{3}.$$

Substituting $b^2 = \frac{4}{9}a^2$:

$$\frac{2 \times \frac{4}{9}a^2}{a} = \frac{10}{3}.$$

Simplify:

$$\frac{8}{9}a = \frac{10}{3}.$$

Solving for $a$:

$$a = \frac{10}{3} \times \frac{9}{8} = \frac{90}{24} = \frac{15}{4}.$$

**Step 5: Finding Transverse Axis Length**

The transverse axis length is:

$$2a = 2 \times \frac{15}{4} = \frac{30}{4} = \frac{15}{2}.$$

Thus, the correct answer is $\frac{15}{2}$, which matches option (C).

**Quick Tip**

For hyperbolas, remember: - $c^2 = a^2 + b^2$ for standard forms. - The latus rectum formula $\frac{2b^2}{a}$ helps find unknowns efficiently.

**109. If the sum of an infinite GP $a, ar, ar^2, ar^3, \ldots$ is 15 and the sum of the squares of each term is 150, then the sum of the series $ar^2, ar^4, ar^6, \ldots$ is:**

(A) $\frac{5}{2}$

(B) $\frac{1}{2}$

(C) $\frac{25}{2}$

(D) $\frac{9}{2}$

**Correct Answer:** (B) $\frac{1}{2}$

**Solution:**

**Step 1:** Using the sum formula for an infinite geometric series:

$$S = \frac{a}{1-r}$$

Given that the sum of the infinite GP is 15, we get:

$$\frac{a}{1-r} = 15 \quad \Rightarrow \quad a = 15(1-r) \quad \cdots (1)$$

**Step 2:** Using the sum of squares formula for an infinite GP:

$$S' = \frac{a^2}{1-r^2}$$

Given that the sum of the squares is 150, we get:

$$\frac{a^2}{1-r^2} = 150 \quad \cdots (2)$$

**Step 3:** Substituting $a = 15(1-r)$ into equation (2):

$$\frac{(15(1-r))^2}{1-r^2} = 150$$

$$\frac{225(1-r)^2}{1-r^2} = 150$$

Dividing both sides by 75:

$$\frac{3(1-r)^2}{1-r^2} = 2$$

Cross multiplying:

$$3(1-r)^2 = 2(1-r^2)$$

Expanding:

$$3(1-2r+r^2) = 2-2r^2$$

$$3 - 6r + 3r^2 = 2 - 2r^2$$

$$5r^2 - 6r + 1 = 0$$

Solving for $r$ using the quadratic formula:

$$r = \frac{6 \pm \sqrt{(-6)^2 - 4(5)(1)}}{2(5)}$$

$$r = \frac{6 \pm \sqrt{36 - 20}}{10}$$

$$r = \frac{6 \pm \sqrt{16}}{10}$$

$$r = \frac{6 \pm 4}{10}$$

Possible values:

$$r = \frac{10}{10} = 1, \quad r = \frac{2}{10} = \frac{1}{5}$$

Since $|r| < 1$ for convergence, we take $r = \frac{1}{5}$.

**Step 4:** Finding $a$ using equation (1):

$$a = 15(1 - \frac{1}{5}) = 15 \times \frac{4}{5} = 12.$$

**Step 5:** Finding the sum of the new GP $ar^2, ar^4, ar^6, \ldots$, which forms another infinite GP with first term $ar^2$ and common ratio $r^2$:

$$S'' = \frac{ar^2}{1 - r^2}$$

Substituting values:

$$S'' = \frac{12 \times \left(\frac{1}{5}\right)^2}{1 - \left(\frac{1}{5}\right)^2}$$

$$S'' = \frac{12 \times \frac{1}{25}}{1 - \frac{1}{25}}$$

$$S'' = \frac{\frac{12}{25}}{\frac{24}{25}} = \frac{12}{24} = \frac{1}{2}$$

Quick Tip

For infinite geometric progressions, remember: - The sum formula $S = \frac{a}{1-r}$ is valid for $|r| < 1$. - The sum of squares follows $S' = \frac{a^2}{1-r^2}$. - Transforming a geometric progression into another requires adjusting the first term and common ratio accordingly.

---

**110. The interval in which the function $f(x) = \frac{4x^2+1}{x}$ is decreasing is:**

(A) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $(-1, 1)$

(D) $[-1, 1]$

**Correct Answer:** (A) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

**Solution:**

**Step 1:** Compute the first derivative $f'(x)$.

Given:

$$f(x) = \frac{4x^2 + 1}{x}$$

Using the quotient rule:

$$\left(\frac{g(x)}{h(x)}\right)' = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$$

where $g(x) = 4x^2 + 1$ and $h(x) = x$, we compute their derivatives:

$$g'(x) = 8x, \quad h'(x) = 1$$

Applying the quotient rule:

$$f'(x) = \frac{(8x \cdot x) - (4x^2 + 1) \cdot 1}{x^2}$$

$$f'(x) = \frac{8x^2 - 4x^2 - 1}{x^2}$$

$$f'(x) = \frac{4x^2 - 1}{x^2}$$

**Step 2:** Find where $f'(x)$ is negative.

$$\frac{4x^2 - 1}{x^2} < 0$$

Since $x^2$ in the denominator is always positive, the inequality simplifies to:

$$4x^2 - 1 < 0$$

$$4x^2 < 1$$

$$x^2 < \frac{1}{4}$$

$$-\frac{1}{2} < x < \frac{1}{2}$$

Thus, $f(x)$ is decreasing in the interval:

$$\left(-\frac{1}{2}, \frac{1}{2}\right)$$

**Quick Tip**

To determine where a function is increasing or decreasing: - Compute the first derivative $f'(x)$. - Solve $f'(x) > 0$ for increasing intervals. - Solve $f'(x) < 0$ for decreasing intervals. - Ensure to check critical points and denominators for undefined regions.

---

**111. If $\int \frac{e^x(1+\sin x)}{1+\cos x}\,dx = e^x f(x) + C$, then $f(x)$ is equal to:**

(A) $\sin \frac{x}{2}$

(B) $\cos \frac{x}{2}$

(C) $\tan \frac{x}{2}$

(D) $\log \frac{x}{2}$

**Correct Answer:** (C) $\tan \frac{x}{2}$

**Solution:**

**Step 1:** Consider the given integral:

$$I = \int \frac{e^x(1 + \sin x)}{1 + \cos x}\,dx.$$

Using the trigonometric identity:

$$1 + \cos x = 2\cos^2 \frac{x}{2}, \quad 1 + \sin x = 2\cos \frac{x}{2} \sin \frac{x}{2},$$

we rewrite the integral as:

$$I = \int \frac{e^x \cdot 2\cos \frac{x}{2} \sin \frac{x}{2}}{2\cos^2 \frac{x}{2}} \, dx.$$

**Step 2:** Simplify the expression:

$$I = \int e^x \cdot \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \, dx.$$

$$I = \int e^x \tan \frac{x}{2} \, dx.$$

**Step 3:** Comparing with the given integral form:

$$I = e^x f(x) + C.$$

Thus, we identify:

$$f(x) = \tan \frac{x}{2}.$$

**Quick Tip**

For integrals involving trigonometric expressions, use half-angle identities:

$$1 + \cos x = 2\cos^2 \frac{x}{2}, \quad 1 + \sin x = 2\sin \frac{x}{2} \cos \frac{x}{2}.$$

This simplifies many fraction-based trigonometric integrals.

---

**112. The curve given by $x + y = e^{xy}$ has a tangent parallel to the Y-axis at the point:**

(A) $(0, 1)$

(B) $(1, 0)$

(C) $(1, 1)$

(D) None of these

**Correct Answer:** (B) $(1, 0)$

**Solution:**

**Step 1:** Differentiate the given equation implicitly.

Given:

$$x + y = e^{xy}$$

Differentiating both sides with respect to $x$, using implicit differentiation:

$$\frac{d}{dx}(x + y) = \frac{d}{dx}(e^{xy})$$

Applying differentiation:

$$1 + \frac{dy}{dx} = e^{xy} \cdot \left( x\frac{dy}{dx} + y \right)$$

Rearrange to express $\frac{dy}{dx}$:

$$1 + \frac{dy}{dx} = e^{xy}(x\frac{dy}{dx} + y)$$

$$1 + \frac{dy}{dx} - e^{xy}y = e^{xy}x\frac{dy}{dx}$$

$$1 - e^{xy}y = \frac{dy}{dx}(e^{xy}x - 1)$$

$$\frac{dy}{dx} = \frac{1 - e^{xy}y}{e^{xy}x - 1}$$

**Step 2:** Condition for a tangent parallel to the Y-axis.

A tangent is parallel to the Y-axis when $\frac{dx}{dy} = 0$, which means $\frac{dy}{dx}$ is undefined.

For $\frac{dy}{dx}$ to be undefined, the denominator must be zero:

$$e^{xy}x - 1 = 0$$

$$e^{xy}x = 1$$

$$x = e^{-xy}$$

**Step 3:** Check given Option.

For $(1, 0)$:

$$x = 1, \quad y = 0$$

$$e^{(1)(0)} \cdot 1 = 1$$

$$1 = 1 \quad \text{(satisfied)}$$

Thus, the correct point is $(1, 0)$.

**Quick Tip**

For finding points where the tangent is parallel to the Y-axis: - Compute $\frac{dy}{dx}$. - Identify where the denominator of $\frac{dy}{dx}$ is zero. - Check the given points to find a valid solution.

---

**113. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is:**

(A) 1

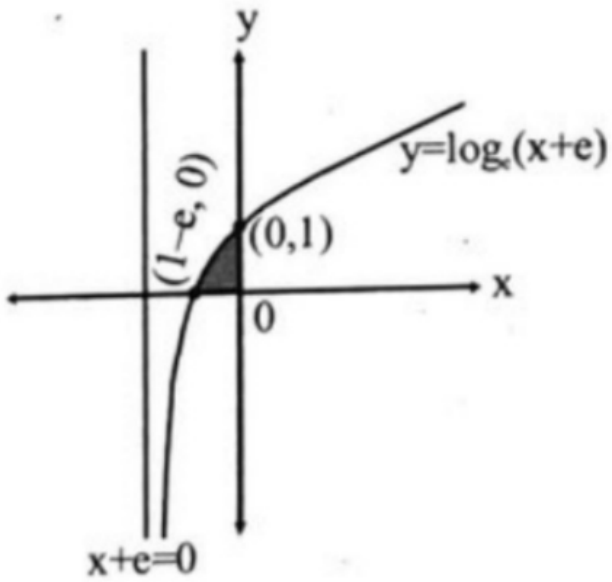
(B) 2

(C) 3

(D) 4

**Correct Answer:** (A) 1

**Solution:**




**Required area**

$$A = \int_{1-e}^{0} y\,dx = \int_{1-e}^{0} \log_e(x+e)\,dx$$

Put $x + e = t \Rightarrow dx = dt$, also when $x = 1 - e$, $t = 1$ and when $x = 0$, $t = e$.

$$\therefore A = \int_{1}^{e} \log_e t\,dt = [t\log_e t - t]_1^e$$

$$e - e - 0 + 1 = 1$$

**Quick Tip**

For finding the area under curves involving logarithms, use substitution and evaluate the integral carefully, keeping in mind the behavior of the logarithmic function at both the upper and lower limits.

---

**114. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is:**

(A) $\hat{i} - \hat{j} + \hat{k}$

(B) $2\hat{j} - \hat{k}$

(C) $2\hat{i}$

(D) $\hat{i}$

**Correct Answer:** (D) $\hat{i}$

**Solution:**

**Step 1:** Use the dot product condition.

Given:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$$

The dot product condition:

$$\vec{a} \cdot \vec{b} = (1, 1, 1) \cdot (x, y, z) = 1$$

$$x + y + z = 1 \quad \cdots (1)$$

**Step 2:** Use the cross product condition.

The cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$

Expanding along the first row:

$$\hat{i}\begin{vmatrix} 1 & 1 \\ y & z \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ x & z \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ x & y \end{vmatrix}$$

$$= \hat{i}(1z - 1y) - \hat{j}(1z - 1x) + \hat{k}(1y - 1x)$$

$$= \hat{i}(z - y) - \hat{j}(z - x) + \hat{k}(y - x)$$

Given:

$$\hat{j} - \hat{k} = (0, 1, -1)$$

Comparing components:

$$z - y = 0, \quad -(z - x) = 1, \quad y - x = -1$$

**Step 3:** Solve the system of equations.

1. $z - y = 0 \Rightarrow z = y$. 2. $-(y - x) = 1 \Rightarrow y - x = -1 \Rightarrow x = y + 1$. 3. From equation (1): $x + y + z = 1$.

Substituting $z = y$ and $x = y + 1$:

$$(y + 1) + y + y = 1$$

$$y + 1 + y + y = 1$$

$$3y + 1 = 1$$

$$3y = 0 \Rightarrow y = 0.$$

$$x = 1, \quad z = 0.$$

Thus, $\vec{b} = \hat{i}$.

**Quick Tip**

For solving vector equations: - Use the dot product condition to establish a scalar equation. - Use the cross product determinant method to form component-wise equations. - Solve the system of equations systematically to determine the unknowns.

---

**115. Find:** $\lim_{x \to 0} \frac{|\sin x|}{x}$

(A) 1

(B) -1

(C) Does not exist

(D) None of these

**Correct Answer:** (C) Does not exist

**Solution: Step 1:** Consider the given limit $\lim_{x \to 0} \frac{|\sin x|}{x}$.

For $x \to 0^+$ (approaching 0 from the right), $\sin x$ is positive, so $|\sin x| = \sin x$. The limit becomes:

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1.$$

For $x \to 0^-$ (approaching 0 from the left), $\sin x$ is negative, so $|\sin x| = -\sin x$. The limit becomes:

$$\lim_{x \to 0^-} \frac{-\sin x}{x} = -1.$$

Since the limit from the right is 1 and the limit from the left is -1, the two one-sided limits are not equal. Hence, the limit does not exist.

**Quick Tip**

For limits involving absolute value functions, check the one-sided limits separately. If they do not match, the two-sided limit does not exist.

---

**116. The lines**

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$$

**and**

$$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$$

**are coplanar if:**

(A) $k = 3$ or $-2$

(B) $k = 0$ or $-1$

(C) $k = 1$ or $-1$

(D) $k = 0$ or $-3$

**Correct Answer:** (D) $k = 0$ or $-3$

**Solution: Step 1:** To check if two lines are coplanar, we use the condition that the scalar triple product of the direction vectors of the lines and the vector joining a point on one line to a point on the other line must be zero.

The direction ratios of the first line are: $(1, 1, -k)$, and for the second line, the direction ratios are: $(k, 2, 1)$.

The point on the first line can be taken as $(2, 3, 4)$, and the point on the second line can be taken as $(1, 4, 5)$.

The vector joining these two points is:

$$(1-2, 4-3, 5-4) = (-1, 1, 1).$$

We now calculate the scalar triple product:

$$\begin{vmatrix} 1 & 1 & -k \\ k & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 1 \cdot \begin{vmatrix} k & 1 \\ -1 & 1 \end{vmatrix} + (-k) \cdot \begin{vmatrix} k & 2 \\ -1 & 1 \end{vmatrix}$$

$$= 1 \cdot (2 \cdot 1 - 1 \cdot 1) - 1 \cdot (k \cdot 1 - (-1) \cdot 1) + (-k) \cdot (k \cdot 1 - (-1) \cdot 2)$$

$$= 1 \cdot 1 - 1 \cdot (k+1) - k \cdot (k+2)$$

$$= 1 - (k+1) - k(k+2)$$

$$= 1 - k - 1 - k^2 - 2k = -k^2 - 3k.$$

For the lines to be coplanar, the scalar triple product must be zero:

$$-k^2 - 3k = 0$$

$$k(k+3) = 0.$$

Thus, $k = 0$ or $k = -3$.

**Quick Tip**

For coplanar lines, use the scalar triple product of the direction ratios and the vector joining points on the lines. If the product is zero, the lines are coplanar.

---

**117. Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is:**

(A) $(\sim p) \wedge q$

(B) $p \wedge (\sim q)$

(C) $(\sim p) \vee (\sim q)$

(D) $(\sim p) \wedge (\sim q)$

**Correct Answer:** (D) $(\sim p) \wedge (\sim q)$

**Solution: Step 1:** We are given the expression $p \Leftrightarrow (q \Rightarrow p)$, and we need to find its negation.

Recall that the biconditional $p \Leftrightarrow q$ is true if both $p$ and $q$ have the same truth value. This can be rewritten as:

$$p \Leftrightarrow (q \Rightarrow p) \equiv (p \Rightarrow (q \Rightarrow p)) \wedge ((q \Rightarrow p) \Rightarrow p).$$

However, to simplify: - The expression $q \Rightarrow p$ is equivalent to $\sim q \vee p$.

Thus, the expression becomes:

$$p \Leftrightarrow (\sim q \vee p).$$

Next, we negate the biconditional. The negation of $p \Leftrightarrow (\sim q \vee p)$ is:

$$\sim (p \Leftrightarrow (\sim q \vee p)) = (\sim p) \wedge (\sim (\sim q \vee p)).$$

Now, simplify:

$$\sim (\sim q \vee p) = q \wedge \sim p.$$

So, the negation of the given expression is:

$$(\sim p) \wedge (\sim q).$$

**Quick Tip**

To negate a biconditional expression, use the fact that $p \Leftrightarrow q$ is true if and only if both $p$ and $q$ have the same truth value. The negation is the case where the truth values differ.

---

**118. The maximum value of $z = 5x + 2y$ subject to the constraints:**

$$x + y \leq 7, \quad x + 2y \leq 10, \quad x, y \geq 0$$

(A) 10

(B) 26

(C) 35

(D) 70

**Correct Answer:** (C) 35

**Solution: Step 1:** We are given the objective function $z = 5x + 2y$ and the constraints:

$$x + y \leq 7, \quad x + 2y \leq 10, \quad x \geq 0, \quad y \geq 0.$$

To find the maximum value of $z$, we will first graph the constraints and identify the feasible region, and then evaluate the objective function at the corner points (vertices) of the feasible region.

**Step 2:** Rewrite the constraints as equations: - $x + y = 7$ (Line 1), - $x + 2y = 10$ (Line 2). The feasible region is bounded by these lines and the axes.

**Step 3:** Find the intersection points of these lines:

- Intersection of $x + y = 7$ and $x + 2y = 10$: Solve the system of equations:

$$x + y = 7 \quad \text{(Equation 1)},$$

$$x + 2y = 10 \quad \text{(Equation 2)}.$$

From Equation 1, $x = 7 - y$. Substitute into Equation 2:

$$(7 - y) + 2y = 10,$$

$$7 + y = 10,$$

$$y = 3.$$

Substitute $y = 3$ into $x + y = 7$:

$$x + 3 = 7 \quad \Rightarrow \quad x = 4.$$

Thus, the intersection point is $(4, 3)$.

- Intersection of $x + y = 7$ and the x-axis (where $y = 0$):

$$x + 0 = 7 \quad \Rightarrow \quad x = 7.$$

Thus, the point is $(7, 0)$.

- Intersection of $x + 2y = 10$ and the y-axis (where $x = 0$):

$$0 + 2y = 10 \quad \Rightarrow \quad y = 5.$$

Thus, the point is $(0, 5)$.

**Step 4:** Now, evaluate $z = 5x + 2y$ at each corner point:

- At $(7, 0)$, $z = 5(7) + 2(0) = 35$, - At $(4, 3)$, $z = 5(4) + 2(3) = 20 + 6 = 26$, - At $(0, 5)$, $z = 5(0) + 2(5) = 10$.

The maximum value of $z$ is $35$, which occurs at $(7, 0)$.

Quick Tip

In linear programming problems, the maximum or minimum value of the objective function occurs at one of the vertices of the feasible region.

---

**119. Find the mean deviation about the mean for the data:** $4, 7, 8, 9, 10, 12, 13, 17$

(A) 3

(B) 24

(C) 10

(D) 8

**Correct Answer:** (A) 3

**Solution: Step 1:** First, find the mean of the given data. The data set is:

$4, 7, 8, 9, 10, 12, 13, 17$.

The mean $\bar{x}$ is calculated as:

$$\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10.$$

**Step 2:** Now, calculate the absolute deviations from the mean:

$$|4-10| = 6, \quad |7-10| = 3, \quad |8-10| = 2, \quad |9-10| = 1,$$

$$|10-10| = 0, \quad |12-10| = 2, \quad |13-10| = 3, \quad |17-10| = 7.$$

**Step 3:** Find the mean of these absolute deviations:

$$\text{Mean Deviation} = \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3.$$

Thus, the mean deviation about the mean is 3.

Quick Tip

The mean deviation is the average of the absolute differences between each data point and the mean. It gives an idea of how spread out the data is around the mean.

---

**120. Bag P contains 6 red and 4 blue balls, and bag Q contains 5 red and 6 blue balls. A ball is transferred from bag P to bag Q and then a ball is drawn from bag Q. What is the probability that the ball drawn is blue?**

(A) $\frac{7}{15}$

(B) $\frac{8}{15}$

(C) $\frac{4}{19}$

(D) $\frac{8}{19}$

**Correct Answer:** (B) $\frac{8}{15}$

**Solution:**

The total probability consists of two cases:

1. A blue ball is transferred from bag P to bag Q. 2. A red ball is transferred from bag P to bag Q.

Case 1: A blue ball is transferred from bag P to bag Q. - The probability of selecting a blue ball from bag P is:

$$P(\text{blue from P}) = \frac{4}{10} = \frac{2}{5}$$

- After transferring the blue ball, bag Q contains 5 red and 7 blue balls. The probability of drawing a blue ball from bag Q is:

$$P(\text{blue from Q after blue transfer}) = \frac{7}{12}$$

Thus, the total probability for case 1 is:

$$P(\text{blue transfer and blue drawn}) = \frac{2}{5} \times \frac{7}{12} = \frac{14}{60} = \frac{7}{30}$$

Case 2: A red ball is transferred from bag P to bag Q. - The probability of selecting a red ball from bag P is:

$$P(\text{red from P}) = \frac{6}{10} = \frac{3}{5}$$

- After transferring the red ball, bag Q contains 6 red and 6 blue balls. The probability of drawing a blue ball from bag Q is:

$$P(\text{blue from Q after red transfer}) = \frac{6}{12} = \frac{1}{2}$$

Thus, the total probability for case 2 is:

$$P(\text{red transfer and blue drawn}) = \frac{3}{5} \times \frac{6}{12} = \frac{18}{60} = \frac{3}{10}$$

Total probability. The total probability of drawing a blue ball is the sum of the probabilities from both cases:

$$P(\text{blue drawn}) = \frac{7}{30} + \frac{18}{60} = \frac{7}{30} + \frac{3}{10} = \frac{7}{30} + \frac{9}{30} = \frac{16}{30} = \frac{8}{15}$$

Thus, the correct answer is $\frac{8}{15}$.

**Quick Tip**

When dealing with probability involving multiple events, break it down into cases, calculate the probability for each case, and then add them together for the total probability.

---

**121. Find the value of**

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$$

(A) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

(B) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

(C) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$

(D) $\tan^{-1}\left(\frac{1}{2}\right)$

**Correct Answer:** (D) $\tan^{-1}\left(\frac{1}{2}\right)$

**Solution: Step 1:** We are given the expression $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$. To simplify this, we use the addition formula for inverse tangents:

$$\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

where $a = \frac{1}{4}$ and $b = \frac{2}{9}$.

**Step 2:** Substitute the values of $a$ and $b$ into the formula:

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right).$$

Simplify the numerator:

$$\frac{1}{4} + \frac{2}{9} = \frac{9}{36} + \frac{8}{36} = \frac{17}{36}.$$

Now simplify the denominator:

$$1 - \frac{1}{4}\cdot\frac{2}{9} = 1 - \frac{2}{36} = 1 - \frac{1}{18} = \frac{17}{18}.$$

So, the expression becomes:

$$\tan^{-1}\left(\frac{\frac{17}{36}}{\frac{17}{18}}\right) = \tan^{-1}\left(\frac{17}{36}\times\frac{18}{17}\right) = \tan^{-1}\left(\frac{18}{36}\right) = \tan^{-1}\left(\frac{1}{2}\right).$$

**Step 3:** Thus, the value of $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is $\tan^{-1}\left(\frac{1}{2}\right)$.

**Quick Tip**

Use the addition formula for inverse tangents to simplify expressions involving the sum of two inverse tangents. This is particularly useful for expressions where the arguments are fractions.

---

**122. The middle term in the expansion of $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$ is:**

(A) ${}^{10}C_5$

(B) ${}^{10}C_6$

(C) ${}^{10}C_5 \frac{1}{x^{10}}$

(D) ${}^{10}C_5 x^{10}$

**Correct Answer:** (A) ${}^{10}C_5$

**Solution: Step 1:** Understanding the Binomial Theorem The binomial theorem states that for any positive integer $n$:

$$(a+b)^n = \sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$$

Where ${}^nC_k$ is the binomial coefficient, also written as ${}^nC_k$.

**Step 2:** Finding the middle term In the expansion of $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$, since the power is 10 (an even number), there is one middle term, which is the $\left(\frac{10}{2} + 1 = 6^{\text{th}}\right)$ term. The general term in a binomial expansion $(a+b)^n$ is given by:

$$T_{k+1} = {}^n\, C_k a^{n-k} b^k$$

For the 6th term, $k = 5$. Thus, the 6th term in the given expansion is:

$$T_6 = {}^{10}\, C_5 \left(\frac{10}{x}\right)^{10-5} \left(\frac{x}{10}\right)^5$$

$$T_6 = {}^{10}\, C_5 \left(\frac{10}{x}\right)^{5} \left(\frac{x}{10}\right)^5$$

$$T_6 = {}^{10}C_5 \frac{10^5}{x^5} \frac{x^5}{10^5}$$

$$T_6 = {}^{10}C_5$$

**Quick Tip**

In a binomial expansion of $(a+b)^n$, if $n$ is even, the middle term is the $\left(\frac{n}{2}+1\right)^{\text{th}}$ term.

---

**123. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is:**

(A) $y = 4(x-2)$

(B) $y = 4(x-1)$

(C) $y = 4(x+1)$

(D) $y = 4(x+2)$

**Correct Answer:** (B) $y = 4(x-1)$

**Solution:**

**Equation of tangent of parabola $y = x^2$ be**

$$tx = y + at^2 \quad \ldots(i)$$

$$y = tx - \frac{t^2}{4}$$

Solve with $y = -(x-2)^2$:

$$tx - \frac{t^2}{4} = -(x-2)^2$$

$$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$$

Here, Discriminant $= 0$.

$$(t-4)^2 - 4 \cdot \left(4 - \frac{t^2}{4}\right) = 0$$

$$\Rightarrow t^2 - 4t = 0 \Rightarrow t = 0 \quad \text{or} \quad t = 4$$

Put value of $t$ in eq. (i), then

$$y = 4(x - 1).$$

**Quick Tip**

When finding the common tangent to two curves, use the discriminant condition for tangency and solve the resulting system of equations to determine the parameters of the tangent line.

---

**124. A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is:**

(A) $y = \sqrt{2}x$

(B) $x = \sqrt{2}y$

(C) $y^2 - x^2 = 2xy$

(D) $x^2 - y^2 = 2xy$

**Correct Answer:** (D) $x^2 - y^2 = 2xy$

**Solution: Step 1:** Let the center of the circle be at $(h, k)$ and its radius be $r$. The circle touches the y-axis, so the distance from the center $(h, k)$ to the y-axis must be equal to the radius $r$. The distance from the point $(h, k)$ to the y-axis is simply $|h|$, so we have:

$$|h| = r.$$

**Step 2:** Next, the circle touches the line $x + y = 0$, so the distance from the center $(h, k)$ to this line must also be equal to the radius $r$. The formula for the distance from a point $(x_1, y_1)$ to the line $Ax + By + C = 0$ is given by:

$$\text{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

For the line $x + y = 0$, we have $A = 1$, $B = 1$, and $C = 0$. Thus, the distance from the center $(h, k)$ to the line is:

$$\frac{|h + k|}{\sqrt{1^2 + 1^2}} = \frac{|h + k|}{\sqrt{2}}.$$

Since this distance must equal the radius $r$, we have:

$$\frac{|h+k|}{\sqrt{2}} = r.$$

**Step 3:** Now, equating the two expressions for the radius $r$, we get:

$$|h| = \frac{|h+k|}{\sqrt{2}}.$$

Squaring both sides:

$$h^2 = \frac{(h+k)^2}{2}.$$

Multiplying through by 2:

$$2h^2 = (h+k)^2.$$

Expanding the right-hand side:

$$2h^2 = h^2 + 2hk + k^2.$$

Simplifying:

$$h^2 = 2hk + k^2.$$

Rearranging:

$$h^2 - k^2 = 2hk.$$

Thus, the locus of the center of the circle is given by:

$$x^2 - y^2 = 2xy.$$

**Quick Tip**

In problems involving a circle touching a line or an axis, use the distance formula to equate the distance from the center to the line and from the center to the axis, and then simplify the resulting equation to find the locus.

---

**125. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in:**

(A) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(B) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

(C) $\left(0, \frac{\pi}{2}\right)$

(D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

**Correct Answer:** (B) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

**Solution: Step 1:** We are given the function $f(x) = \tan^{-1}(\sin x + \cos x)$. To determine the intervals where this function is increasing, we need to find the derivative of $f(x)$.

The derivative of $f(x)$ can be found using the chain rule:

$$f'(x) = \frac{d}{dx}\left(\tan^{-1}(\sin x + \cos x)\right) = \frac{1}{1 + (\sin x + \cos x)^2} \cdot \frac{d}{dx}(\sin x + \cos x).$$

**Step 2:** The derivative of $\sin x + \cos x$ is:

$$\frac{d}{dx}(\sin x + \cos x) = \cos x - \sin x.$$

Therefore, the derivative of $f(x)$ is:

$$f'(x) = \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}.$$

**Step 3:** For $f(x)$ to be increasing, $f'(x) > 0$. This means that the numerator $\cos x - \sin x$ must be positive. So, we need to solve:

$$\cos x - \sin x > 0.$$

Rewriting this inequality:

$$\cos x > \sin x.$$

This inequality holds in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$, because in this interval, $\cos x$ is greater than $\sin x$.

**Step 4:** Thus, the function $f(x) = \tan^{-1}(\sin x + \cos x)$ is increasing in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

**Quick Tip**

To determine where a function is increasing, find the derivative and solve for intervals where the derivative is positive.

---

**126. Simplify $i^{57} + \frac{1}{i^{25}}$ and find its value:**

(A) 0

(B) $2i$

(C) $-2i$

(D) 2

**Correct Answer:** (A) 0

**Solution:**

We are tasked with simplifying $i^{57} + \frac{1}{i^{25}}$, where $i$ is the imaginary unit, defined by $i^2 = -1$.

Step 1: Simplifying $i^{57}$ We know that powers of $i$ cycle every four terms:

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1, \quad \ldots$$

To simplify $i^{57}$, we divide 57 by 4 and find the remainder:

$$57 \div 4 = 14 \text{ remainder } 1$$

Thus:

$$i^{57} = i^1 = i$$

Step 2: Simplifying $\frac{1}{i^{25}}$ Next, we simplify $\frac{1}{i^{25}}$. Again, powers of $i$ cycle every 4 terms. To simplify $i^{25}$, we divide 25 by 4 and find the remainder:

$$25 \div 4 = 6 \text{ remainder } 1$$

Thus:

$$i^{25} = i^1 = i$$

So:

$$\frac{1}{i^{25}} = \frac{1}{i}$$

Now, we can multiply the numerator and denominator by $i$ to get rid of the imaginary unit in the denominator:

$$\frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

Step 3: Final Calculation Now, substitute the results back into the original expression:

$$i^{57} + \frac{1}{i^{25}} = i + (-i) = 0$$

Thus, the value of the expression is $0$.

The correct answer is $\boxed{(A)}$.

**Quick Tip**

When simplifying powers of $i$, remember that the powers of $i$ follow a repeating cycle of length 4. This allows you to find the equivalent smaller power by taking the remainder when dividing the exponent by 4.

---

**127. If one root of the equation $x^2 + px + 12 = 0$ is $4$, while the equation $x^2 + px + q = 0$ has equal roots, then the value of $q$ is:**

(A) $4$

(B) $12$

(C) $3$

(D) $\frac{49}{4}$

**Correct Answer:** (D) $\frac{49}{4}$

**Solution:**

**Step 1:** Find the value of $p$.

The given quadratic equation is:

$$x^2 + px + 12 = 0.$$

Since one root is given as $x = 4$, substituting it into the equation:

$$4^2 + 4p + 12 = 0.$$

$$16 + 4p + 12 = 0.$$

$$4p + 28 = 0.$$

$$4p = -28.$$

$$p = -7.$$

**Step 2:** Use the condition for equal roots in the second equation.

The second equation given is:

$$x^2 + px + q = 0.$$

For equal roots, the discriminant must be zero:

$$\Delta = p^2 - 4q = 0.$$

Substituting $p = -7$:

$$(-7)^2 - 4q = 0.$$

$$49 - 4q = 0.$$

$$4q = 49.$$

$$q = \frac{49}{4}.$$

Thus, the correct answer is $\frac{49}{4}$.

**Quick Tip**

For quadratic equations: - If a root is known, substitute it into the equation to find unknown coefficients. - For equal roots, use the condition $\Delta = 0$, where $\Delta = b^2 - 4ac$.

---

**128. Evaluate the integral:**

$$I = \int \frac{x+3}{(x+4)^2} e^x \, dx$$

(A) $e^x \frac{1}{x+4} + C$

(B) $e^{-x} \frac{1}{x+4} + C$

(C) $e^{-x} \frac{1}{x-4} + C$

(D) $e^{2x} \frac{1}{x-4} + C$

**Correct Answer:** (A) $e^x \frac{1}{x+4} + C$

**Solution:**

**Step 1:** Use substitution.

Let:

$$u = x + 4 \quad \Rightarrow \quad du = dx.$$

Rewriting the given integral:

$$I = \int \frac{(u-1)}{u^2} e^{u-4}\, du.$$

Expanding:

$$I = \int \left(\frac{u}{u^2} - \frac{1}{u^2}\right) e^{u-4}\, du.$$

$$I = \int \left(\frac{1}{u} - \frac{1}{u^2}\right) e^{u-4}\, du.$$

**Step 2:** Solve by integration by parts.

Using integration by parts for:

$$\int \frac{1}{u} e^{u-4}\, du.$$

Let:

$$v = \frac{1}{u}, \quad dv = -\frac{du}{u^2}.$$

$$w' = e^{u-4}, \quad w = e^{u-4}.$$

Using integration by parts:

$$I = \frac{e^{u-4}}{u} + C.$$

Substituting back $u = x + 4$:

$$I = \frac{e^x}{x+4} + C.$$

Thus, the final result is:

$$I = e^x \frac{1}{x+4} + C.$$

Quick Tip

For integrals involving fractions with polynomials: - Use substitution to simplify the denominator. - Consider integration by parts if terms appear in the numerator. - Recognizing standard forms helps in quicker evaluation.

---

**129. The shortest distance between the lines**

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$$

**and**

$$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$$

**is:**

(A) $\frac{18}{\sqrt{5}}$

(B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$

(D) $6\sqrt{3}$

**Correct Answer:** (A) $\frac{18}{\sqrt{5}}$

**Solution:**

**Step 1:** Identify direction vectors.

The given lines are in symmetric form:

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$$

$$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$$

Direction vectors for the lines:

$$\vec{d_1} = (2, 3, -1), \quad \vec{d_2} = (2, 1, 3).$$

A point on the first line is $A(3, 2, 1)$ and a point on the second line is $B(-3, 6, 5)$.

**Step 2:** Compute the shortest distance formula.

The shortest distance between two skew lines is given by:

$$D = \frac{|(\vec{B} - \vec{A}) \cdot (\vec{d_1} \times \vec{d_2})|}{|\vec{d_1} \times \vec{d_2}|}.$$

First, find $\vec{B} - \vec{A}$:

$$\vec{B} - \vec{A} = (-3 - 3, 6 - 2, 5 - 1) = (-6, 4, 4).$$

**Step 3:** Compute the cross product $\vec{d_1} \times \vec{d_2}$.

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$$

Expanding:

$$\hat{i}\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix}$$

$$= \hat{i}(3 \times 3 - (-1 \times 1)) - \hat{j}(2 \times 3 - (-1 \times 2)) + \hat{k}(2 \times 1 - 3 \times 2)$$

$$= \hat{i}(9 + 1) - \hat{j}(6 + 2) + \hat{k}(2 - 6)$$

$$= 10\hat{i} - 8\hat{j} - 4\hat{k}.$$

**Step 4:** Compute the determinant $(\vec{B} - \vec{A}) \cdot (\vec{d_1} \times \vec{d_2})$.

$$(-6, 4, 4) \cdot (10, -8, -4)$$

$$= (-6 \times 10) + (4 \times -8) + (4 \times -4).$$

$$= -60 - 32 - 16 = -108.$$

Taking the absolute value:

$$|-108| = 108.$$

**Step 5:** Compute $|\vec{d_1} \times \vec{d_2}|$.

$$\sqrt{10^2 + (-8)^2 + (-4)^2} = \sqrt{100 + 64 + 16} = \sqrt{180} = 6\sqrt{5}.$$

**Step 6:** Compute the shortest distance.

$$D = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}.$$

Thus, the correct answer is:

$$\frac{18}{\sqrt{5}}.$$

**Quick Tip**

For finding the shortest distance between skew lines: - Use the determinant formula:

$$D = \frac{|(\vec{B} - \vec{A}) \cdot (\vec{d_1} \times \vec{d_2})|}{|\vec{d_1} \times \vec{d_2}|}.$$

- Ensure correct evaluation of vector cross product and dot product.

---

**130. If $P(B) = \frac{3}{5}$, $P(A \mid B) = \frac{1}{2}$, and $P(A \cup B) = \frac{4}{5}$, then the value of $P(A \cup B)' + P(A' \cup B)$ is:**

(A) $\frac{1}{5}$

(B) $\frac{4}{5}$

(C) $\frac{1}{2}$

(D) $1$

**Correct Answer:** (D) $1$

**Solution:**

**Step 1:** Compute $P(A \cap B)$.

Using the conditional probability formula:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Substituting the given values:

$$\frac{1}{2} = \frac{P(A \cap B)}{3/5}$$

$$P(A \cap B) = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}.$$

**Step 2:** Compute $P(A)$.

Using the formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substituting known values:

$$\frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$$

$$P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10}$$

$$P(A) = \frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}.$$

**Step 3:** Compute $P(A \cup B)'$.

$$P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{4}{5} = \frac{1}{5}.$$

**Step 4:** Compute $P(A' \cup B)$.

Using:

$$P(A' \cup B) = 1 - P(A \cap B').$$

First, compute $P(A \cap B')$:

$$P(A \cap B') = P(A) - P(A \cap B) = \frac{1}{2} - \frac{3}{10} = \frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}.$$

Now:

$$P(A' \cup B) = 1 - \frac{1}{5} = \frac{4}{5}.$$

**Step 5:** Compute $P(A \cup B)' + P(A' \cup B)$.

$$P(A \cup B)' + P(A' \cup B) = \frac{1}{5} + \frac{4}{5} = 1.$$

Thus, the correct answer is:

1.

**Quick Tip**

For probability problems involving unions and complements: - Use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. - The complement rule: $P(A^c) = 1 - P(A)$. - For conditional probability: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

---