BITSAT 2024 Question Paper with Solutions

Time Allowed :3 hours	Maximum Marks :390	Total questions :130

General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Mode**: Computer-based online test
2. **Duration**: 3 hours (180 minutes)
3. **Sections**: The exam consists of four parts:
   (a) Part I: Physics (30 questions)
   (b) Part II: Chemistry (30 questions)
   (c) Part III: English Proficiency (10 questions) and Logical Reasoning (20 questions)
   (d) Part IV: Mathematics (40 questions) or Biology (for B.Pharm candidates)
4. **Total Marks**: 390
5. **Marking Scheme**: Each correct answer awards 3 marks, and 1 mark is deducted for each incorrect answer
6. **Subjects**:
   (a) Physics: Mechanics, Electromagnetism, Thermodynamics, Modern Physics
   (b) Chemistry: Organic, Inorganic, and Physical Chemistry
   (c) Mathematics: Calculus, Algebra, Geometry (or Biology for B.Pharm candidates)
   (d) English Proficiency: Reading Comprehension, Vocabulary
   (e) Logical Reasoning: Analytical and Problem-solving skills

Physics

1. You measure two quantities as $A = 1.0\,m \pm 0.2\,m$, $B = 2.0\,m \pm 0.2\,m$. We should report the correct value for $\sqrt{AB}$ as:

(A) $1.4\,m \pm 0.4\,m$

(B) $1.41\,m \pm 0.15\,m$

(C) $1.4\,m \pm 0.3\,m$

(D) $1.4\,m \pm 0.2\,m$

Correct Answer: (D) $1.4\,m \pm 0.2\,m$

Solution:

Step 1: Given values

We are given:

$$A = 1.0\,m \pm 0.2\,m, \quad B = 2.0\,m \pm 0.2\,m$$

We define:

$$Y = \sqrt{AB}$$

Step 2: Calculate the value of Y

$$Y = \sqrt{(1.0)(2.0)}$$

$$= \sqrt{2.0} = 1.414\,m$$

Step 3: Determine the uncertainty in Y

The formula for relative error propagation is:

$$\frac{\Delta Y}{Y} = \frac{1}{2}\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

Substituting the given values:

$$\frac{\Delta Y}{1.4} = \frac{1}{2}\left(\frac{0.2}{1.0} + \frac{0.2}{2.0}\right)$$

Step 4: Simplify the expression

$$\frac{\Delta Y}{1.4} = \frac{1}{2}(0.2 + 0.1)$$

$$\frac{\Delta Y}{1.4} = \frac{1}{2} \times 0.3 = 0.15$$

$$\Delta Y = 0.15 \times 1.4 = 0.21$$

Step 5: Round off to one significant digit

$$\Delta Y = 0.2\, m$$

Thus, the final result is:

$$Y = 1.4\, m \pm 0.2\, m$$

Step 6: Verify the options

Comparing with the given options, the correct answer is (D) $1.4\, m \pm 0.2\, m$.

Quick Tip

For multiplication or division, relative errors are added. The formula for error propagation in square roots is:

$$\frac{\Delta Y}{Y} = \frac{1}{2}\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right).$$

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2. The dimensional formula of latent heat is:

(A) $[M^0 L T^{-2}]$

(B) $[M L T^{-2}]$

(C) $[M^0 L^2 T^{-2}]$

(D) $[M L^2 T^{-2}]$

Correct Answer: (C) $[M^0 L^2 T^{-2}]$

Solution:

Step 1: Define latent heat

Latent heat (L) is defined as the amount of heat energy (Q) required to change the phase of a substance per unit mass:

$$L = \frac{Q}{m}$$

Step 2: Determine the dimensional formula of Q

Heat energy (Q) is a form of energy, and its dimensional formula is the same as that of work:

$$[Q] = [M L^2 T^{-2}]$$

Step 3: Determine the dimensional formula of L

Since mass (m) has the dimensional formula:

$$[m] = [M]$$

we divide:

$$[L] = \frac{[Q]}{[m]} = \frac{[ML^2T^{-2}]}{[M]}$$
$$= M^0L^2T^{-2}$$

Step 4: Verify the options

Comparing with the given choices,

the correct answer is (C) $M^0L^2T^{-2}$.

Quick Tip

Latent heat is energy per unit mass, so its dimensional formula is derived by dividing energy ($[ML^2T^{-2}]$) by mass ($[M]$), giving $M^0L^2T^{-2}$.

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3. The dimensions of the coefficient of self-inductance are:

(A) $[ML^2T^{-2}A^{-2}]$

(B) $[ML^2T^{-2}A^{-1}]$

(C) $[MLT^{-2}A^{-2}]$

(D) $[MLT^{-2}A^{-1}]$

Correct Answer: (A) $[ML^2T^{-2}A^{-2}]$

Solution:

Step 1: Define self-inductance

The energy stored in an inductor is given by:

$$U = \frac{1}{2}LI^2$$

where L is the self-inductance and I is the current.

Step 2: Rearrange for L

$$L = \frac{2U}{I^2}$$

Step 3: Find the dimensional formula of L

Since energy (U) has the dimensional formula:

$$[U] = [ML^2T^{-2}]$$

and current (I) has the dimensional formula:

$$[I] = [A]$$

we substitute:

$$[L] = \frac{[ML^2T^{-2}]}{[A^2]}$$
$$= [ML^2T^{-2}A^{-2}]$$

Step 4: Verify the options

Comparing with the given choices,

the correct answer is (A) $[ML^2T^{-2}A^{-2}]$.

Quick Tip

Self-inductance is the property of an inductor that determines how much EMF is induced per unit rate of change of current. Its dimensional formula is derived using the energy equation.

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4. A particle is moving in a straight line. The variation of position x as a function of time t is given as:

$$x = t^3 - 6t^2 + 20t + 15$$

The velocity of the body when its acceleration becomes zero is:

(A) 6 m/s

(B) 10 m/s

(C) 8 m/s

(D) 4 m/s

Correct Answer: (C) 8 m/s

Solution:

Step 1: Find velocity

Velocity is the first derivative of displacement:

$$v = \frac{dx}{dt} = 3t^2 - 12t + 20$$

Step 2: Find acceleration

Acceleration is the derivative of velocity:

$$a = \frac{dv}{dt} = 6t - 12$$

Step 3: Set acceleration to zero

$$6t - 12 = 0$$

Solving for t:

$$t = 2\,s$$

Step 4: Find velocity at $t = 2$

$$v = 3(2)^2 - 12(2) + 20$$

$$= 12 - 24 + 20 = 8\,m/s$$

Step 5: Verify the options

The correct answer is (C) 8 m/s.

Quick Tip

To find when a particle reaches maximum or minimum velocity, set the acceleration $a = 0$ and solve for t, then substitute t in the velocity equation.

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5. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ ms^{-2}, in the third second is:

(A) 6 m

(B) 4 m

(C) $\frac{10}{3}$ m

(D) $\frac{19}{3}$ m

Correct Answer: (C) $\frac{10}{3}$ m

Solution:

Step 1: Use the nth second displacement formula

The displacement covered in the nth second is given by:

$$s_n = u + \frac{a}{2}(2n - 1)$$

where: - u is the initial velocity, - a is the acceleration, - n is the time instant.

Step 2: Substituting values

Given:

$$u = 0, \quad a = \frac{4}{3}\,\text{ms}^{-2}, \quad n = 3$$

$$s_3 = 0 + \frac{\frac{4}{3}}{2}(2(3) - 1)$$

$$= \frac{4}{6} \times 5 = \frac{10}{3}\, m$$

Step 3: Verify the options

Thus, the correct answer is (C) $\frac{10}{3}$ m.

Quick Tip

The displacement in the nth second formula:

$$s_n = u + \frac{a}{2}(2n - 1)$$

is useful for determining the exact distance covered in a given second without calculating total displacement.

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6. A projectile is projected with velocity of 40 m/s at an angle θ with the horizontal. If R is the horizontal range covered by the projectile and after t seconds its inclination with horizontal becomes zero, then the value of $\cot\theta$ is:

[Take, $g = 10$ m/s^2]

(A) $\frac{R}{20t^2}$

(B) $\frac{R}{10t^2}$

(C) $\frac{5R}{t^2}$

(D) $\frac{R}{t^2}$

Correct Answer: (A) $\frac{R}{20t^2}$

Solution:

Step 1: Find the time to reach maximum height

At maximum height, the inclination with the horizontal becomes zero.

The time to reach maximum height is:

$$t = \frac{u \sin\theta}{g}$$

Rearranging:

$$u = \frac{gt}{\sin\theta}$$

Step 2: Use the range formula

$$R = u\cos\theta \times (2t)$$

Substituting u:

$$R = \frac{gt}{\sin\theta}\cos\theta \times (2t)$$

$$\cos\theta = \frac{R}{2ut}$$

Step 3: Find $\cot\theta$

$$\cot\theta = \frac{R}{2gt^2}$$

Substituting $g = 10$:

$$\cot\theta = \frac{R}{2 \times 10t^2} = \frac{R}{20t^2}$$

Thus, the correct answer is (A) $\frac{R}{20t^2}$.

Quick Tip

For projectile motion, the horizontal range is given by:

$$R = \frac{u^2 \sin 2\theta}{g}$$

Using this formula helps in solving range-related problems quickly.

7. A rigid body rotates about a fixed axis with variable angular velocity $\omega = \alpha - \beta t$ at time t, where α, β are constants. The angle through which it rotates before it stops is:

(A) $\frac{\alpha^2}{2\beta}$

(B) $\frac{\alpha^2-\beta^2}{2\alpha}$

(C) $\frac{\alpha^2-\beta^2}{2\beta}$

(D) $\frac{(\alpha-\beta)\alpha}{2}$

Correct Answer: (A) $\frac{\alpha^2}{2\beta}$

Solution: It is given in the question that a rigid body is rotating about a fixed axis.

Angular velocity is defined as the rate of the velocity at which a given object or particle rotates around a given point in a given interval of time. This velocity is also known as rotational velocity. Therefore, the angular velocity, as given in the question, is given by:

$$\omega = \alpha - \beta t$$

Where, ω is the angular velocity, α and β are constants and t is the time taken.

Also, we know that angular velocity is also measured as angle per unit time, therefore,

$$\omega = \frac{d\theta}{dt}$$

Now, putting the value of ω in the above equation, we get:

$$\omega = \frac{d\theta}{dt} = \alpha - \beta t$$

$$\Rightarrow d\theta = (\alpha - \beta t)dt$$

Now, to calculate the angle through which the rigid body rotates before it comes to rest, we can integrate the above equation:

$$\int d\theta = \int (\alpha - \beta t)dt$$

$$\theta = \int (\alpha - \beta t)dt$$

$$\theta = \alpha t - \frac{\beta t^2}{2}$$

When the rigid body comes to rest, the angular velocity will be zero:

$$\omega = 0$$

$$\Rightarrow \alpha - \beta t = 0$$

$$\Rightarrow \alpha = \beta t$$

$$\Rightarrow t = \frac{\alpha}{\beta}$$

Therefore, putting $t = \frac{\alpha}{\beta}$ in θ, we get:

$$\theta = \alpha\left(\frac{\alpha}{\beta}\right) - \frac{\beta\left(\frac{\alpha}{\beta}\right)^2}{2}$$

$$\Rightarrow \theta = \frac{\alpha^2}{\beta} - \frac{\beta\left(\frac{\alpha^2}{\beta^2}\right)}{2}$$

$$\Rightarrow \theta = \frac{\alpha^2}{\beta} - \frac{\alpha^2}{2\beta}$$

$$\Rightarrow \theta = \frac{2\alpha^2}{2\beta} - \frac{\alpha^2}{2\beta}$$

$$\Rightarrow \theta = \frac{\alpha^2}{2\beta}$$

Therefore, the angle through which a rigid body rotates before it stops is $\frac{\alpha^2}{2\beta}$.

Hence, option A is the correct option.

Quick Tip

For angular motion, the equation:

$$\omega^2 = \omega_0^2 + 2\beta\theta$$

is analogous to linear kinematics and helps in solving rotation problems efficiently.

8. The range of a projectile projected at an angle of 15° with the horizontal is 50 m. If the projectile is projected with the same velocity at an angle of 45°, then its range will be:

(A) 50 m

(B) $50\sqrt{2}$ m

(C) 100 m

(D) $100\sqrt{2}$ m

Correct Answer: (C) 100 m

Solution:

Step 1:

The range R of a projectile launched with an initial speed v and at an angle θ to the horizontal is given by:

$$R = \frac{v^2}{g}\sin(2\theta)$$

where g is the acceleration due to gravity.

From this equation, we can observe that the range is dependent on the sine of twice the launch angle.

Given that the range at 15° is 50 m, if we launch the projectile at 45° with the same velocity, we can compare the ranges by comparing $\sin(2 \times 15^\circ)$ and $\sin(2 \times 45^\circ)$:

$$\sin(30^\circ) = \frac{1}{2}$$

$$\sin(90^\circ) = 1$$

Therefore, the range at 45° will be twice the range at 15°, because $\sin(90^\circ)$ is twice as large as $\sin(30^\circ)$.

So, the range when the projectile is launched at 45° will be:

$$R = 2 \times 50\,\text{m} = 100\,\text{m}$$

Thus, the range at 45° is 100 meters.

Quick Tip

For a given velocity, the maximum range occurs at $45°$. The range is proportional to $\sin(2\theta)$, which helps in comparative range calculations.

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9. A particle of mass m is projected with a velocity u making an angle of $30°$ with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is:

(A) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$

(B) $\frac{\sqrt{3}}{2}\frac{mu^2}{g}$

(C) $\frac{mu^3}{\sqrt{2}g}$

(D) zero

Correct Answer: (A) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$

Solution:

Step 1: Define angular momentum

Angular momentum at maximum height is given by:

$$L = mvH$$

where H is the maximum height.

Step 2: Find the horizontal velocity

The horizontal component of velocity remains constant:

$$v = u\cos 30° = u \times \frac{\sqrt{3}}{2}$$

Step 3: Find the maximum height

Using the kinematic equation:

$$H = \frac{u^2 \sin^2 30°}{2g}$$

Substituting $\sin 30° = \frac{1}{2}$:

$$H = \frac{u^2 \times \left(\frac{1}{2}\right)^2}{2g} = \frac{u^2}{8g}$$

Step 4: Calculate angular momentum

$$L = mu\cos 30° \times H$$

$$= mu \times \frac{\sqrt{3}}{2} \times \frac{u^2}{8g}$$

$$= \frac{\sqrt{3}mu^3}{16g}$$

Thus, the correct answer is (A) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$.

Quick Tip

For projectile motion, the angular momentum at the highest point is calculated as $L = mv_xH$, where v_x is the horizontal velocity and H is the maximum height.

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10. A body is thrown with a velocity of 9.8 m/s making an angle of $30°$ with the horizontal. It will hit the ground after a time:

(A) 3.0 s

(B) 2.0 s

(C) 1.5 s

(D) 1.0 s

Correct Answer: (D) 1.0 s

Solution:

Step 1: Use the time of flight formula

Time of flight for a projectile is given by:

$$T = \frac{2u\sin\theta}{g}$$

Step 2: Substituting values

$$T = \frac{2 \times 9.8 \times \sin 30°}{9.8}$$

Step 3: Solve for T

$$T = \frac{2 \times 9.8 \times \frac{1}{2}}{9.8}$$

$$T = 1 \text{ sec}$$

Thus, the correct answer is (D) 1.0 s.

Quick Tip

The total time of flight for projectile motion is:

$$T = \frac{2u\sin\theta}{g}$$

This formula is useful for determining how long a projectile remains in the air.

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11. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is:

(A) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$
(B) $\frac{1+\sqrt{5}}{\sqrt{5}-1}$
(C) $\frac{1+\sqrt{5}}{\sqrt{2}-1}$
(D) $\frac{\sqrt{3}+1}{\sqrt{2}-1}$

Correct Answer: (A) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

Solution:

Step 1: Equation of motion for the system

The acceleration of the system is given by:

$$a = \frac{(m_2 - m_1)}{m_1 + m_2} g$$

Step 2: Equating given acceleration

$$\frac{g}{\sqrt{2}} = \frac{(m_2 - m_1)}{m_1 + m_2} g$$

Step 3: Solve for $\frac{m_1}{m_2}$

$$\sqrt{2}(m_2 - m_1) = m_1 + m_2$$

Rearranging:

$$\frac{m_1}{m_2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

Thus, the correct answer is (A) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$.

Quick Tip

For Atwood's machine, the acceleration is given by:

$$a = \frac{(m_2 - m_1)}{m_1 + m_2} g$$

This equation helps determine the ratio of masses when acceleration is known.

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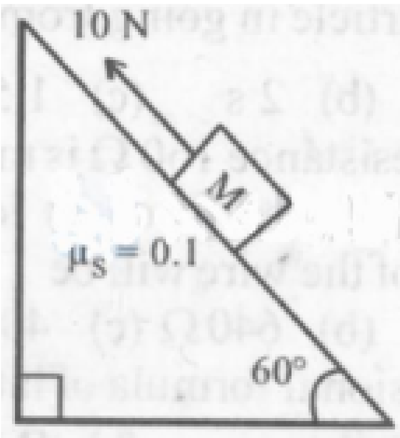
12. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 60° by a force of 10 N parallel to the inclined surface. When the block is pushed up by 10 m along the inclined surface, the work done against frictional force is:



[Given: $g = 10$ m/s^2, $\mu_s = 0.1$]

(A) $5\sqrt{3}$ J

(B) 5 J

(C) 5×10^3 J

(D) 10 J

Correct Answer: (B) 5 J

Solution:

Step 1: Calculate normal force

The normal force N on the inclined plane is:

$$N = mg\cos 60^\circ$$

Step 2: Determine frictional force

$$F_f = \mu N = \mu mg \cos 60^\circ$$

Substituting values:

$$F_f = (0.1)(1)(10)\left(\frac{1}{2}\right) = 0.5 \text{ N}$$

Step 3: Calculate work done against friction

$$W = F_f \times d$$

$$= 0.5 \times 10 = 5 \text{ J}$$

Thus, the correct answer is (B) 5 J.

Quick Tip

The work done against friction is calculated as $W = F_f d$, where $F_f = \mu mg \cos\theta$.

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13. A person of mass 60 kg is inside a lift of mass 940 kg. The lift starts moving upwards with an acceleration of 1.0 m/s^2. If $g = 10$ m/s^2, the tension in the supporting cable is:

(A) 8600 N

(B) 9680 N

(C) 11000 N

(D) 1200 N

Correct Answer: (C) 11000 N

Solution:

Step 1: Determine total mass

$$m_{\text{total}} = 60 + 940 = 1000 \text{ kg}$$

Step 2: Use Newton's Second Law

$$T - mg = ma$$

Substituting values:

$$T - (1000 \times 10) = 1000 \times 1$$

$$T = 10000 + 1000 = 11000 \text{ N}$$

Thus, the correct answer is (C) 11000 N.

Quick Tip

In an accelerating lift, the apparent weight is given by $T = mg + ma$.

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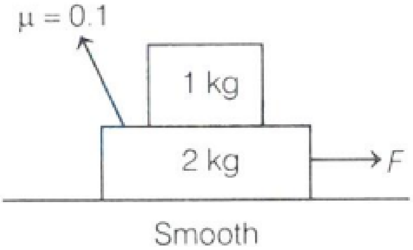
14. A force of $F = 0.5$ N is applied on the lower block as shown in the figure. The work done by the lower block on the upper block for a displacement of 3 m of the upper block with respect to the ground is (Take, $g = 10$ m/s^2):



(A) -0.5 J

(B) 0.5 J

(C) 2 J

(D) -2 J

Correct Answer: (B) 0.5 J

Solution:

Step 1: Calculate maximum acceleration

The maximum acceleration that the 1 kg block can have is:

$$a_{\text{max}} = \mu g = (0.1)(10) = 1 \text{ m/s}^2$$

Step 2: Calculate common acceleration

The acceleration of the system is:

$$a = \frac{F}{m_{\text{total}}} = \frac{0.5}{3} = \frac{0.5}{3} \text{ m/s}^2$$

Since $a < a_{\text{max}}$, the blocks move together.

Step 3: Find force of friction

The friction force acting on the upper block:

$$f = ma = (1) \times \frac{0.5}{3} = \frac{1}{6}\ \text{N}$$

Step 4: Find work done by friction

$$W = f \times d = \frac{1}{6} \times 3 = 0.5\ \text{J}$$

Thus, the correct answer is (B) 0.5 J.

Quick Tip

If two blocks are moving together, the acceleration of the system is the same. Work done by friction can be calculated as $W = fd$.

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15. A pendulum of mass 1 kg and length $l = 1$ m is released from rest at an angle $\theta = 60°$. The power delivered by all the forces acting on the bob at angle $\theta = 30°$ will be (Take, $g = 10$ m/s^2):

(A) 13.4 W

(B) 20.4 W

(C) 24.6 W

(D) zero

Correct Answer: (A) 13.4 W

Solution:

Step 1: Find velocity at $\theta = 30°$

Using energy conservation:

$$v = \sqrt{2gh}$$

Step 2: Calculate height difference

$$h = l(\cos 30° - \cos 60°)$$

$$= 1 \times \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 0.36 \text{ m}$$

Step 3: Find velocity

$$v = \sqrt{2 \times 10 \times 0.36}$$

$$= \sqrt{7.2} = 2.68 \text{ m/s}$$

Step 4: Find power

$$P = (mgv)\cos 60^\circ$$

$$= (1 \times 10 \times 2.68) \times \frac{1}{2}$$

$$= 13.4 \text{ W}$$

Thus, the correct answer is (A) 13.4 W.

Quick Tip

Power is given by $P = Fv\cos\theta$. For pendulum motion, velocity can be found using energy conservation.

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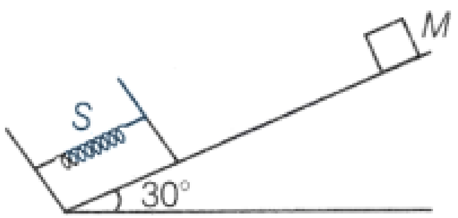
16. An ideal massless spring S can be compressed 1 m by a force of 100 N in equilibrium. The same spring is placed at the bottom of a frictionless inclined plane inclined at 30° to the horizontal. A 10 kg block M is released from rest at the top of the incline and is brought to rest momentarily after compressing the spring by 2 m. If $g = 10$ m/s^2, what is the speed of the mass just before it touches the spring?



(A) $\sqrt{20}$ m/s

(B) $\sqrt{30}$ m/s

(C) $\sqrt{10}$ m/s

(D) $\sqrt{40}$ m/s

Correct Answer: (A) $\sqrt{20}$ m/s

Solution:

Step 1: Determine spring constant

Hooke's Law:

$$F = kx$$

$$k = \frac{F}{x} = \frac{100}{1} = 100 \text{ N/m}$$

Step 2: Use energy conservation

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}kx_{\text{max}}^2$$

$$v = \sqrt{\frac{kx_{\text{max}}^2}{m} - 2gh}$$

Substituting values:

$$v = \sqrt{\frac{(100)(2)^2}{10} - (2)(10)(2/2)}$$

$$= \sqrt{20} \text{ m/s}$$

Thus, the correct answer is (A) $\sqrt{20}$ m/s.

> Quick Tip
>
> For spring problems, use conservation of energy: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgh$.

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17. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is:

(A) 20 m/s

(B) 40 m/s

(C) $10\sqrt{30}$ m/s

(D) 10 m/s

Correct Answer: (B) 40 m/s

Solution:

Step 1: Use energy conservation

$$mgh_{\text{initial}} = \frac{1}{2}mv^2 + mgh_{\text{final}}$$

Cancel m:

$$gh_{\text{initial}} = \frac{1}{2}v^2 + gh_{\text{final}}$$

$$(10 \times 100) = \frac{1}{2}v^2 + (10 \times 20)$$

$$1000 = \frac{1}{2}v^2 + 200$$

$$\frac{1}{2}v^2 = 800$$

$$v = \sqrt{1600} = 40 \text{ m/s}$$

Thus, the correct answer is (B) 40 m/s.

Quick Tip

For energy conservation in rolling motion, consider both translational and potential energy.

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18. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad/s, the magnitude of its angular momentum about a point on the ground right under the center of the circle is:

(A) 14.4 kg m^{2}s^{-1}

(B) 8.64 kg m^{2}s^{-1}

(C) 20.16 kg m^{2}s^{-1}

(D) 11.52 kg m^{2}s^{-1}

Correct Answer: (A) 14.4 kg m^{2}s^{-1}

Solution:

Step 1: Angular momentum formula

$$L_0 = mvr \sin 90^\circ$$

$$= 2 \times 0.6 \times 12 \times 1$$

$$= 14.4 \text{ kg m}^2\text{s}^{-1}$$

Thus, the correct answer is (A) 14.4 kg m^{2}s^{-1}.

Quick Tip

For a particle moving in a circular path, angular momentum is given by $L = mvr$.

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19. A ball falling freely from a height of 4.9 m/s hits a horizontal surface. If $e = \frac{3}{4}$, then the ball will hit the surface the second time after:

(A) 1.0 s

(B) 1.5 s

(C) 2.0 s

(D) 3.0 s

Correct Answer: (B) 1.5 s

Solution:

Step 1: Determine velocity on hitting the surface

Using free-fall kinematics:

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 4.9}$$

$$= 9.8 \text{ m/s}$$

Step 2: Velocity after first bounce

$$v' = ev = \frac{3}{4} \times 9.8$$

$$= 7.35 \text{ m/s}$$

Step 3: Time taken from first to second bounce

Time of flight formula:

$$t = \frac{2v'}{g}$$

$$= \frac{2 \times 7.35}{9.8}$$

$$= 1.5 \text{ s}$$

Thus, the correct answer is (B) 1.5 s.

Quick Tip

For bouncing motion, use $v' = ev$ to determine the velocity after impact, then calculate the time of flight using $t = \frac{2v'}{g}$.

---

20. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$ respectively. The magnitude of the position vector of the center of mass of this system will be similar to the magnitude of which vector?

(A) $\hat{i} - 2\hat{j} + \hat{k}$

(B) $-3\hat{i} - 2\hat{j} + \hat{k}$

(C) $-2\hat{i} + 2\hat{k}$

(D) $-2\hat{i} - \hat{j} + 2\hat{k}$

Correct Answer: (A) $\hat{i} - 2\hat{j} + \hat{k}$

Solution:

Step 1: Formula for center of mass

$$\vec{r}_{\text{com}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Step 2: Substituting values

$$\vec{r}_{\text{com}} = \frac{(1)(\hat{i} + 2\hat{j} + \hat{k}) + (3)(-3\hat{i} - 2\hat{j} + \hat{k})}{1 + 3}$$

$$= \frac{\hat{i} + 2\hat{j} + \hat{k} - 9\hat{i} - 6\hat{j} + 3\hat{k}}{4}$$

$$= \frac{-8\hat{i} - 4\hat{j} + 4\hat{k}}{4}$$

$$= -2\hat{i} - \hat{j} + \hat{k}$$

Step 3: Find the magnitude

$$|\vec{r}_{\text{com}}| = \sqrt{(-2)^2 + (-1)^2 + (1)^2}$$

$$= \sqrt{4 + 1 + 1} = \sqrt{6}$$

The only vector with the same magnitude is (A) $\hat{i} - 2\hat{j} + \hat{k}$.

Quick Tip

The center of mass position is given by $\vec{r}_{\text{com}} = \frac{\sum m_i\vec{r}_i}{\sum m_i}$. To find the correct vector, compare magnitudes.

---

21. The moment of inertia of a cube of mass m and side a about one of its edges is equal to:

(A) $\frac{2}{3}ma^2$

(B) $\frac{4}{3}ma^2$

(C) $3ma^2$

(D) $\frac{8}{3}ma^2$

Correct Answer: (A) $\frac{2}{3}ma^2$

Solution:

Step 1: Apply the perpendicular axis theorem

Using the theorem of perpendicular axes, we express the moment of inertia about an edge as:

$$I = I_C + m\left(\frac{a}{\sqrt{2}}\right)^2$$

Step 2: Moment of inertia of cube about its center

For a cube, the moment of inertia about its central axis is:

$$I_C = \frac{ma^2}{12} + \frac{ma^2}{12} = \frac{ma^2}{6}$$

Step 3: Adding the parallel axis contribution

$$I = \left[\frac{ma^2}{12} + \frac{ma^2}{12}\right] + \frac{ma^2}{2}$$

$$= \frac{2}{3}ma^2$$

Thus, the correct answer is $\frac{2}{3}ma^2$.

Quick Tip

For any solid body, the moment of inertia about an edge can be found using the parallel axis theorem:

$$I = I_C + md^2$$

where d is the perpendicular distance from the center to the edge.

---

22. A body which is initially at rest at a height R above the surface of the Earth of radius R, falls freely towards the Earth. The velocity on reaching the surface of the Earth is:

(A) $\sqrt{2gR}$

(B) $\sqrt{gR}$

(C) $\sqrt{\frac{3}{2}gR}$

(D) $\sqrt{4gR}$

Correct Answer: (B) $\sqrt{gR}$

Solution:

Step 1: Apply conservation of energy

The total energy remains constant:

$$\text{Increase in kinetic energy} = \text{Decrease in potential energy}$$

Step 2: Using gravitational potential energy

$$\frac{1}{2}mv^2 = mgR\left(\frac{1}{1+\frac{h}{R}}\right)$$

For $h = R$:

$$mv^2 = mgR$$

$$v = \sqrt{gR}$$

Thus, the correct answer is $\sqrt{gR}$.

> **Quick Tip**
>
> For free fall from height h, the velocity on reaching the surface can be found using energy conservation:
>
> $$v = \sqrt{2gh}$$
>
> or using gravitational potential.

---

23. The distance between the Sun and Earth is R. The duration of a year if the distance between the Sun and Earth becomes $3R$ will be:

(A) $\sqrt{3}$ years

(B) 3 years

(C) 9 years

(D) $3\sqrt{3}$ years

Correct Answer: (D) $3\sqrt{3}$ years

Solution:

Step 1: Kepler's third law

$$T^2 \propto R^3$$

Step 2: Find new time period

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

Substituting values:

$$T_2 = \left(\frac{3R}{R}\right)^{3/2} \times 1$$

$$= 3\sqrt{3} \text{ years}$$

Thus, the correct answer is $3\sqrt{3}$ years.

Quick Tip

Kepler's third law states:

$$T^2 \propto R^3$$

which helps determine orbital periods when the radius changes.

---

24. For a particle inside a uniform spherical shell, the gravitational force on the particle is:

(A) Infinite

(B) Zero

(C) $\frac{-Gm_1m_2}{r^2}$

(D) $\frac{Gm_1m_2}{r^2}$

Correct Answer: (B) Zero

Solution:

Step 1: Apply shell theorem

The shell theorem states that a uniform spherical shell of mass exerts zero net gravitational force on a particle inside it.

Step 2: Explain force cancellation

Forces from opposite sides of the shell cancel out due to symmetry, leaving:

$$\text{Net gravitational force} = 0$$

Thus, the correct answer is zero.

Quick Tip

Inside a uniform spherical shell, a particle experiences no net gravitational force because of the symmetrical distribution of mass.

---

25. The kinetic energy of a satellite in its orbit around Earth is E. What should be the kinetic energy of the satellite to escape Earth's gravity?

(A) $4E$

(B) $2E$

(C) $\sqrt{2}E$

(D) E

Correct Answer: (B) $2E$

Solution:

Step 1: Escape velocity formula

$$v_e = \sqrt{2}v_0$$

Step 2: Find kinetic energy for escape

$$KE_{\text{escape}} = \frac{1}{2}Mv_e^2 = \frac{1}{2}M(2v_0^2)$$

$$= 2E$$

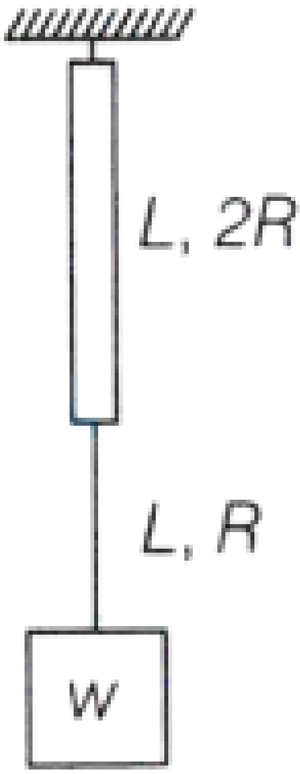
Thus, the correct answer is $2E$.

Quick Tip

Escape velocity is given by $v_e = \sqrt{2}v_0$, where v_0 is the orbital velocity.

---

26. Two wires of the same material (Young's modulus Y) and same length L but radii R and $2R$ respectively, are joined end to end and a weight W is suspended from the combination. The elastic potential energy in the system is:



(A) $\frac{3W^2L}{4\pi R^2Y}$
(B) $\frac{3W^2L}{8\pi R^2Y}$
(C) $\frac{5W^2L}{8\pi R^2Y}$
(D) $\frac{W^2L}{\pi R^2Y}$

Correct Answer: (C) $\frac{5W^2L}{8\pi R^2Y}$

Solution:

Step 1: Calculate elongations

$$\Delta l_1 = \frac{WL}{(4\pi R^2)Y}, \quad \Delta l_2 = \frac{WL}{\pi R^2 Y}$$

Step 2: Find elastic potential energy

$$U = \frac{1}{2}K_1(\Delta l_1)^2 + \frac{1}{2}K_2(\Delta l_2)^2$$

$$= \frac{1}{2} \times \frac{Y(4\pi R^2)}{L} \times \left(\frac{WL}{4\pi R^2 Y}\right)^2 + \frac{1}{2} \times \frac{Y(\pi R^2)}{L} \times \left(\frac{WL}{\pi R^2 Y}\right)^2$$

$$= \frac{5W^2 L}{8\pi R^2 Y}$$

Quick Tip

Elastic potential energy is stored in the stretching of both wires, with energy distributed based on their cross-sectional areas.

---

27. With rise in temperature, the Young's modulus of elasticity:

(A) Changes erratically

(B) Decreases

(C) Increases

(D) Remains unchanged

Correct Answer: (B) **Decreases**

Solution:

Step 1: Young's modulus definition

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

Step 2: Effect of temperature increase

As temperature increases, strain increases, leading to a decrease in Young's modulus.

Thus, the correct answer is Young's modulus decreases.

Quick Tip

Young's modulus decreases with temperature because increased thermal energy makes materials more ductile.

---

28. Young's modules of materials of a wire of Length ' L ' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's modules will be:

(A) $Y/4$

(B) $4Y$

(C) Y

(D) $2Y$

Correct Answer: (C) Y

Solution:

Step 1: Young's modulus dependency

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

Step 2: Effect of change in length and area

Young's modulus is a material property and does not depend on length or cross-sectional area.

Thus, the correct answer is Y.

Quick Tip

Young's modulus remains unchanged when only geometric factors like length and area are modified.

---

29. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is:

(A) 4:1

(B) 0.8:1

(C) 8:1

(D) 2:1

Correct Answer: (C) 8:1

Solution:

Step 1: Use Laplace's pressure equation

$$\Delta P = \frac{4T}{R}$$

Step 2: Compare pressures and radii

$$\frac{\Delta P_1}{\Delta P_2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$$

Step 3: Find volume ratio

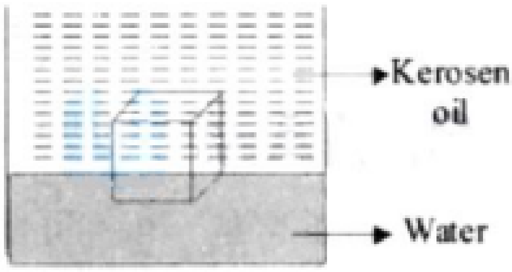
$$V_1 : V_2 = R_1^3 : R_2^3 = 8 : 1$$

Thus, the correct answer is 8:1.

> Quick Tip
>
> Smaller soap bubbles have higher internal pressure due to surface tension effects.

---

30. A cube of ice floats partly in water and partly in kerosene oil. The radio of volume of ice immersed in water to that in kerosene oil (specific gravity of Kerosene oil = 0.8, specific gravity of ice = 0.9)



(A) 8:9

(B) 5:4

(C) 9:10

(D) 1:1

Correct Answer: (D) 1:1

Solution:

Step 1: Define variables

Let V_1 be the volume immersed in water and V_2 be the volume immersed in oil.

Step 2: Equilibrium condition

$$V_1\rho_w g + V_2\rho_o g = (V_1 + V_2)\rho_{\text{ice}} g$$

Step 3: Solve for ratio

$$V_1 + 0.8V_2 = 0.9(V_1 + V_2)$$

$$0.1V_1 = 0.1V_2 \Rightarrow V_1 : V_2 = 1 : 1$$

Thus, the correct answer is 1:1.

Quick Tip

Floating objects distribute their volume based on density ratios of the fluids they are immersed in.

---

31. A solid metallic cube having total surface area $24\,m^2$ is uniformly heated. If its temperature is increased by $10°C$, calculate the increase in volume of the cube.

Given: $\alpha = 5.0 \times 10^{-4}\,C^{-1}$

(A) $2.4 \times 10^6\,cm^3$

(B) $1.2 \times 10^5\,cm^3$

(C) $6.0 \times 10^4\,cm^3$

(D) $4.8 \times 10^5\,cm^3$

Correct Answer: (B) $1.2 \times 10^5\,cm^3$

Solution:

Step 1: Formula for volume expansion

$$\Delta V = V_0\gamma\Delta T$$

Step 2: Volume relation with side length

$$\Delta V = a^3(3\alpha)\Delta T$$

Step 3: Finding cube side length

$$6a^2 = 24 \quad \Rightarrow \quad a^2 = 4 \quad \Rightarrow \quad a = 2$$

Step 4: Substituting values

$$\Delta V = 2^3(3 \times 5 \times 10^{-4}) \times 10 = 1200 \times 10^{-4}\, m^3$$

$$= 1200 \times 10^2\, cm^3 = 1.2 \times 10^5\, cm^3$$

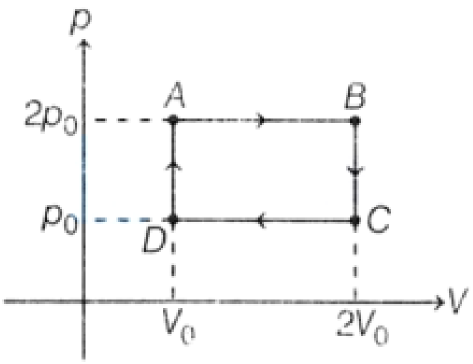
Thus, the correct answer is $1.2 \times 10^5\, cm^3$.

Quick Tip

For thermal expansion in solids, volume change is calculated using $\Delta V = V_0\gamma\Delta T$, where $\gamma = 3\alpha$.

---

32. In the given cycle ABCDA, the heat required for an ideal monoatomic gas will be:



(A) p_0V_0

(B) $\frac{13}{2}p_0V_0$

(C) $\frac{11}{2}p_0V_0$

(D) $4p_0V_0$

Correct Answer: (B) $\frac{13}{2}p_0V_0$

Solution:

Step 1: Heat supplied in process DA and AB

$$Q = nC_V(\Delta T)_{DA} + nC_P(\Delta T)_{AB}$$

Step 2: For an ideal monoatomic gas,

$$C_V = \frac{3}{2}R, \quad C_P = \frac{5}{2}R$$

Step 3: Substituting values

$$Q = \frac{3}{2}(p_0V_0) + 5(p_0V_0)$$

$$= \frac{13}{2}p_0V_0$$

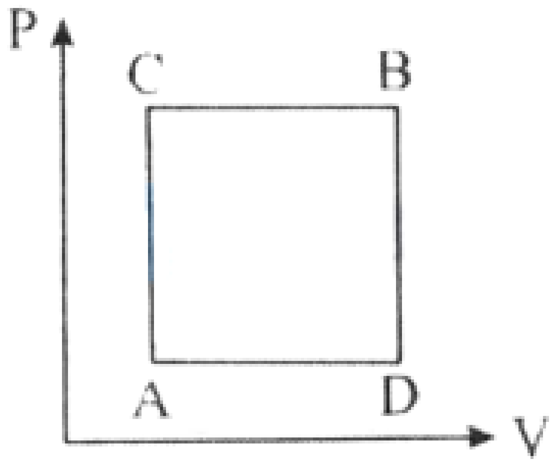
Thus, the correct answer is $\frac{13}{2}p_0V_0$.

Quick Tip

For cyclic processes, heat calculation depends on specific heat at constant volume C_V and constant pressure C_P.

---

33. A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used, $60J$ of heat flows into the system and $30J$ of work is done by the system. If path ADB is used, the work done by the system is $10J$. The heat flow into the system in path ADB is:



(A) $40J$

(B) $80J$

(C) $100J$

(D) $20J$

Correct Answer: (A) $40J$

Solution:

Step 1: Using first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

Step 2: For process ACB

$$\Delta U = 60 - 30 = 30J$$

Step 3: Applying same internal energy change to ADB

$$\Delta Q_{ADB} = 30 + 10 = 40J$$

Thus, the correct answer is $40J$.

Quick Tip

Change in internal energy ΔU depends only on initial and final states, not on the path.

---

34. A source supplies heat to a system at the rate of $1000W$. If the system performs work at the rate of $200W$, the rate at which internal energy of the system increases is:

(A) $1200W$

(B) $600W$

(C) $500W$

(D) $800W$

Correct Answer: (D) $800W$

Solution:

Step 1: Using the first law of thermodynamics

$$\frac{dQ}{dt} = \frac{dU}{dt} + \frac{dW}{dt}$$

Step 2: Substituting values

$$1000 = \frac{dU}{dt} + 200$$

Step 3: Solving for $\frac{dU}{dt}$

$$\frac{dU}{dt} = 1000 - 200 = 800W$$

Thus, the correct answer is $800W$.

> **Quick Tip**
>
> The rate of change of internal energy equals heat supplied minus work done by the system.

---

35. On Celsius scale, the temperature of a body increases by $40°C$. The increase in temperature on Fahrenheit scale is:

(A) $70°F$

(B) $68°F$

(C) $72°F$

(D) $75°F$

Correct Answer: (C) $72°F$

Solution:

Step 1: Conversion formula

$$\frac{F-32}{9} = \frac{C}{5}$$

Step 2: Applying temperature change

$$\Delta C = \frac{5}{9}\Delta F$$

$$40 = \frac{5}{9}\Delta F \Rightarrow \Delta F = 72^\circ F$$

Thus, the correct answer is $72^\circ F$.

Quick Tip

To convert Celsius to Fahrenheit, use $F = \frac{9}{5}C + 32$.

---

36. In a mixture of gases, the average number of degrees of freedom per molecule is 6. The RMS speed of the molecule of the gas is c. Then the velocity of sound in the gas is:

(A) $\frac{c}{\sqrt{3}}$

(B) $\frac{c}{\sqrt{2}}$

(C) $\frac{2c}{3}$

(D) $\frac{3c}{3}$

Correct Answer: (C) $\frac{2c}{3}$

Solution:

Step 1: Formula for RMS speed and speed of sound

The root mean square (RMS) speed of gas molecules is given by:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

The velocity of sound in a gas is:

$$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$$

where γ is the adiabatic index.

Step 2: Relation between v_{sound} and v_{rms}

Dividing the equations:

$$\frac{v_{\text{sound}}}{v_{\text{rms}}} = \sqrt{\frac{\gamma}{3}}$$

Step 3: Finding γ using degrees of freedom

For a mixture of gases with an average degree of freedom $f = 6$:

$$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{6} = \frac{4}{3}$$

Step 4: Compute velocity of sound

$$v_{\text{sound}} = \sqrt{\frac{4/3}{3}} v_{\text{rms}} = \frac{2}{3} v_{\text{rms}}$$

Since $v_{\text{rms}} = c$, we get:

$$v_{\text{sound}} = \frac{2c}{3}$$

Thus, the correct answer is $\frac{2c}{3}$.

Quick Tip

The speed of sound in a gas is given by:

$$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$$

where γ depends on the degrees of freedom f as:

$$\gamma = 1 + \frac{2}{f}$$

---

37. The temperature of an ideal gas is increased from 200 K to 800 K. If the RMS speed of gas at 200 K is v_0, then the RMS speed of the gas at 800 K will be:

(A) v_0

(B) $4v_0$

(C) $\frac{v_0}{4}$

(D) $2v_0$

Correct Answer: (D) $2v_0$

Solution:

Step 1: Formula for RMS speed

The root mean square (RMS) speed of gas molecules is given by:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Since R and M are constants:

$$v_{\text{rms}} \propto \sqrt{T}$$

Step 2: Determine new RMS speed

Given initial and final temperatures:

$$T_{\text{initial}} = 200 \text{ K}, \quad T_{\text{final}} = 800 \text{ K}$$

Since $v_{\text{rms}} \propto \sqrt{T}$, we write:

$$\frac{v_{\text{rms, initial}}}{v_{\text{rms, final}}} = \sqrt{\frac{T_{\text{initial}}}{T_{\text{final}}}}$$

Step 3: Compute new RMS speed

$$\frac{v_0}{v_{\text{rms}}} = \sqrt{\frac{200}{800}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$v_{\text{rms}} = 2v_0$$

Thus, the correct answer is $2v_0$.

Quick Tip

The RMS speed of a gas is proportional to the square root of its temperature:

$$v_{\text{rms}} \propto \sqrt{T}$$

If temperature increases by a factor n, the RMS speed increases by a factor $\sqrt{n}$.

38. Two vessels A and B are of the same size and are at the same temperature. A contains 1 g of hydrogen and B contains 1 g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is:

(A) 8

(B) 16

(C) 32

(D) 4

Correct Answer: (B) 16

Solution:

Step 1: Ideal Gas Equation

The ideal gas equation is given by:

$$PV = nRT$$

where:

P is the pressure of the gas

V is the volume of the gas

n is the number of moles of gas

R is the ideal gas constant

T is the temperature of the gas

Step 2: Number of Moles of Hydrogen (n_H)

The molar mass of hydrogen (H_2) is 2 g/mol.

$$n_H = \frac{\text{mass of hydrogen}}{\text{molar mass of hydrogen}} = \frac{1 \text{ g}}{2 \text{ g/mol}} = \frac{1}{2} \text{ mol}$$

Step 3: Number of Moles of Oxygen (n_O)

The molar mass of oxygen (O_2) is 32 g/mol.

$$n_O = \frac{\text{mass of oxygen}}{\text{molar mass of oxygen}} = \frac{1 \text{ g}}{32 \text{ g/mol}} = \frac{1}{32} \text{ mol}$$

Step 4: Applying Ideal Gas Equation to both vessels

Since the vessels are of the same size and at the same temperature, V and T are the same for both vessels. Therefore, we can write:

For vessel A (hydrogen):

$$P_A V = n_H RT$$

For vessel B (oxygen):

$$P_B V = n_O RT$$

Step 5: Finding the ratio $\frac{P_A}{P_B}$

Divide the equation for vessel A by the equation for vessel B:

$$\frac{P_A V}{P_B V} = \frac{n_H RT}{n_O RT}$$

$$\frac{P_A}{P_B} = \frac{n_H}{n_O} = \frac{\frac{1}{2}}{\frac{1}{32}} = \frac{1}{2} \times \frac{32}{1} = 16$$

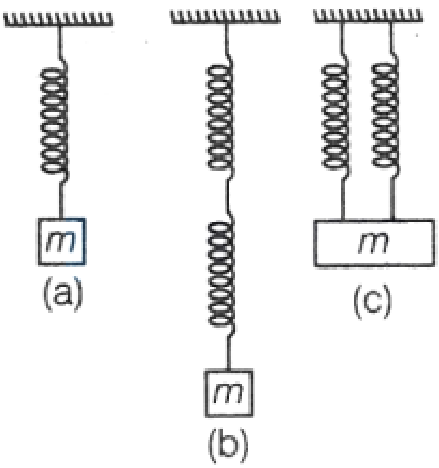
Thus, $\frac{P_A}{P_B} = 16$.

Quick Tip

The pressure of a gas is directly proportional to the number of moles of gas when volume and temperature are kept constant.

---

39. Five identical springs are used in the three configurations as shown in figure. The time periods of vertical oscillations in configurations (a), (b) and (c) are in the ratio:



(A) $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

(B) $2 : \sqrt{2} : \frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}} : 2 : 1$

(D) $2 : \frac{1}{\sqrt{2}} : 1$

Correct Answer: (A) $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

Solution:

Step 1:

The time period of a spring-mass system is given by the formula:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where:

- T is the time period
- m is the mass
- k is the spring constant

For the given systems:

1. System (a):
$$T_a = 2\pi\sqrt{\frac{m}{k}}$$
2. System (b): The spring constant is halved, i.e., $k' = \frac{k}{2}$. Therefore,
$$T_b = 2\pi\sqrt{\frac{m}{k/2}} = 2\pi\sqrt{\frac{2m}{k}} = \sqrt{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \sqrt{2}T_a$$
3. System (c): The spring constant is doubled, i.e., $k'' = 2k$. Therefore,
$$T_c = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{\sqrt{2}}T_a$$

Thus, the ratio of the time periods is:

$$T_a : T_b : T_c = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$$

Quick Tip

For springs in series, the effective spring constant is given by $\frac{1}{k_{eff}} = \sum \frac{1}{k_i}$. For springs in parallel, the effective spring constant is given by $k_{eff} = \sum k_i$. The time period of a spring-mass system is $T = 2\pi\sqrt{\frac{m}{k}}$.

---

40. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. If the time taken by the particle to go from $x = 0$ to $\frac{A}{2}$ is 2 s, then the time taken by the particle in going from $x = \frac{A}{2}$ to A is:

(A) 3 s

(B) 2 s

(C) 1.5 s

(D) 4 s

Correct Answer: (D) 4 s

Solution:

Step 1: Using the standard equation of SHM

$$\frac{A}{2} = A\sin(\omega t_1)$$

$$\omega t_1 = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \quad \text{(i)}$$

Step 2: Calculating total time to reach A

$$A = A\sin\omega(t_1 + t_2)$$

$$\omega(t_1 + t_2) = \sin^{-1}(1) = \frac{\pi}{2}$$

$$\omega t_2 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \quad \text{(ii)}$$

Step 3: Finding the ratio of times

$$\frac{t_1}{t_2} = \frac{1}{2} \Rightarrow t_2 = 2t_1 = 2 \times 2 = 4\,\text{s}$$

Thus, the time taken to go from $\frac{A}{2}$ to A is 4 s.

Quick Tip

In SHM, the time taken to travel between points is not uniform and depends on the amplitude and angular frequency.

41. A simple pendulum doing small oscillations at a place R height above the Earth's surface has a time period of $T_1 = 4\,$s. T_2 would be its time period if it is brought to a point which is at a height $2R$ from the Earth's surface. Choose the correct relation [R = radius of Earth]:

(A) $T_1 = T_2$

(B) $2T_1 = 3T_2$

(C) $3T_1 = 2T_2$

(D) $2T_1 = T_2$

Correct Answer: (C) $3T_1 = 2T_2$

Solution:

Step 1: Time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad g = \frac{GM}{(R+h)^2}$$

Step 2: Relating time periods at different heights

$$\frac{T_1}{T_2} = \frac{R+h_1}{R+h_2} = \frac{R+R}{R+2R} = \frac{2}{3} \Rightarrow 3T_1 = 2T_2$$

Thus, the correct relation is $3T_1 = 2T_2$.

> **Quick Tip**
>
> The time period of a simple pendulum depends on the gravitational acceleration, which varies with height above the Earth's surface.

---

42. The speed of sound in oxygen at STP will be approximately:(Given, R = 8.3J$(K)^1$,
$= 1.4)$

(A) 315 m/s

(B) 333 m/s

(C) 341 m/s

(D) 325 m/s

Correct Answer: (A) 315 m/s

Solution:

Step 1: Given data

Temperature, $T = 273\,\text{K}$

Molecular mass of oxygen, $M = 32 \times 10^{-3}\,\text{kg}$

Step 2: Calculating the speed of sound

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}} \approx 315\,\text{m/s}$$

Thus, the speed of sound in oxygen at STP is approximately 315 m/s.

Quick Tip

The speed of sound in a gas depends on the temperature, molar mass, and the adiabatic index of the gas.

---

43. A plane progressive wave is given by $y = 2\cos 2\pi(330t - x)$ m. The frequency of the wave is:

(A) 165 Hz

(B) 330 Hz

(C) 660 Hz

(D) 340 Hz

Correct Answer: (B) 330 Hz

Solution:

Step 1: Identifying the frequency from the wave equation

The general form of a plane progressive wave is:

$$y = A\cos 2\pi(ft - \frac{x}{\lambda})$$

Comparing with the given equation:

$$y = 2\cos 2\pi(330t - x)$$

The frequency f is 330 Hz.

Quick Tip

In the wave equation $y = A\cos 2\pi(ft - \frac{x}{\lambda})$, f represents the frequency of the wave.

---

44. An oil drop of radius $1\,\mu$m is held stationary under a constant electric field of 3.65×10^4 N/C due to some excess electrons present on it. If the density of the oil drop is 1.26 g/cm^3, then the number of excess electrons on the oil drop approximately is:

Take, g = 10 m/s2

(A) 7

(B) 12

(C) 9

(D) 8

Correct Answer: (C) 9

Solution:

Step 1: Calculating the mass of the oil drop

$$\text{Density} = 1.26\,\text{g/cm}^3 = 1.26 \times 10^3\,\text{kg/m}^3$$

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1 \times 10^{-6}\,\text{m})^3 = \frac{4}{3}\pi \times 10^{-18}\,\text{m}^3$$

$$\text{Mass} = \text{Density} \times \text{Volume} = 1.26 \times 10^3 \times \frac{4}{3}\pi \times 10^{-18} = 1.68\pi \times 10^{-15}\,\text{kg}$$

Step 2: Balancing forces

The oil drop is stationary, so the electric force balances the gravitational force:

$$qE = mg$$

$$q = \frac{mg}{E} = \frac{1.68\pi \times 10^{-15} \times 10}{3.65 \times 10^4} \approx 1.44 \times 10^{-18}\,\text{C}$$

Step 3: Calculating the number of excess electrons

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{1.44 \times 10^{-18}}{1.6 \times 10^{-19}} \approx 9$$

Thus, the number of excess electrons is approximately 9.

Quick Tip

The charge on an oil drop in Millikan's experiment can be used to determine the number of excess electrons by balancing electric and gravitational forces.

---

45. The potential of a large liquid drop when eight liquid drops are combined is 20 V. Then, the potential of each single drop was:

(A) 10 V

(B) 7.5 V

(C) 5 V

(D) 2.5 V

Correct Answer: (C) 5 V

Solution:

Step 1: Volume and charge conservation

$$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 2r$$

$$8q = Q$$

Step 2: Relating potentials

$$V' = \frac{kq}{r}, \quad V = \frac{kQ}{R} = \frac{k \times 8q}{2r} = 4\frac{kq}{r} = 4V'$$

$$20 = 4V' \Rightarrow V' = 5\,\text{V}$$

Thus, the potential of each single drop was 5 V.

Quick Tip

When combining drops, the potential scales with the radius, and charge conservation must be considered.

---

46. A dust particle of mass 4×10^{-12} mg is suspended in air under the influence of an electric field of 50 N/C directed vertically upwards. How many electrons were removed from the neutral dust particle? [Take, $g = 10\,$m/s^2]

(A) 15

(B) 8

(C) 5

(D) 4

Correct Answer: (C) 5

Solution:

Step 1: Solution:

Given:

Mass of dust particle, $m = 4 \times 10^{-12} \times 10^{-3}\,\text{kg} = 4 \times 10^{-18}\,\text{kg}$

Electric field, $E = 50\,\text{N/C}$

Weight of dust particle, $W = mg$

$$W = 4 \times 10^{-18} \times 10 \times 10^{-17}\,\text{N} = 4 \times 10^{-17}\,\text{N}$$

Electric force experienced by the dust particle,

$$F_e = qE$$

$$F_e = ne \cdot E = n \times 1.6 \times 10^{-19} \times 50$$

where n is the number of electrons removed from the neutral dust particle.

At balance condition, the electric force is equal to the weight of the dust particle:

$$\text{Electric force} = \text{Weight of dust particle}$$

$$n \times 1.6 \times 10^{-19} \times 50 = 4 \times 10^{-17}$$

$$n = \frac{4 \times 10^{-17}}{1.6 \times 10^{-19} \times 50} = \frac{400}{80} = 5$$

Thus, the number of electrons removed from the neutral dust particle is $n = 5$.

Quick Tip

The number of electrons removed can be found by balancing the electric force with the gravitational force on the particle.

---

47. The electric field at point (30, 30, 0) due to a charge of $0.008\,\mu$C placed at the origin will be: (coordinates are in cm)

(A) $8000\,\text{N/C}\,\hat{i} + 8000\,\text{N/C}\,\hat{j}$

(B) $4000(\hat{i} + \hat{j})\,\text{N/C}$

(C) $200\sqrt{2}(\hat{i} + \hat{j})\,\text{N/C}$

(D) $400\sqrt{2}(\hat{i} + \hat{j})\,\text{N/C}$

Correct Answer: (C) $200\sqrt{2}(\hat{i} + \hat{j})\,\text{N/C}$

Solution:

Step 1: Calculating the distance

$$r = \sqrt{(30)^2 + (30)^2 + 0^2} = 30\sqrt{2}\,\text{cm} = 30\sqrt{2} \times 10^{-2}\,\text{m}$$

Step 2: Calculating the electric field

$$E = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 0.008 \times 10^{-6}}{(30\sqrt{2} \times 10^{-2})^2} = \frac{72}{18} \times 10^3 = 4000\,\text{N/C}$$

Step 3: Direction of the electric field

The electric field components in the x and y directions are equal:

$$E_x = E_y = \frac{4000}{\sqrt{2}} = 200\sqrt{2}\,\text{N/C}$$

Thus, the electric field is:

$$\vec{E} = 200\sqrt{2}(\hat{i} + \hat{j})\,\text{N/C}$$

Quick Tip

The electric field due to a point charge decreases with the square of the distance and points radially away from the charge.

---

48. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force?

(A) $d\sqrt{K}$

(B) $K\sqrt{d}$

(C) $1.5d\sqrt{K}$

(D) $2d\sqrt{K}$

Correct Answer: (A) $d\sqrt{K}$

Solution:

The electrostatic force between two charges in a medium with dielectric constant K is given by:

$$F_{medium} = \frac{1}{4\pi\epsilon_0 K}\frac{|q_1q_2|}{d^2}$$

where ϵ_0 is the permittivity of free space.

The electrostatic force between the same two charges in air (or vacuum, with K=1) at a distance d' is given by:

$$F_{air} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{d'^2}$$

For the forces to be equal, $F_{medium} = F_{air}$:

$$\frac{1}{4\pi\epsilon_0 K}\frac{|q_1q_2|}{d^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{d'^2}$$

$$\frac{1}{Kd^2} = \frac{1}{d'^2}$$

$$d'^2 = Kd^2$$

$$d' = \sqrt{Kd^2} = d\sqrt{K}$$

Thus, the equivalent distance in air is $d\sqrt{K}$.

Quick Tip

The electrostatic force in a medium is $\frac{1}{K}$ times the force in vacuum for the same charges and distance. To maintain the same force, increase the distance in vacuum by $\sqrt{K}$ times the distance in the medium.

---

49. Electric potential at a point 'P' due to a point charge of 5×10^{-9} C is 50 V. The distance of 'P' from the point charge is:

(Assume, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\text{Nm}^2\text{C}^{-2}$)

(A) $3\,\text{cm}$

(B) $9\,\text{cm}$

(C) $90\,\text{cm}$

(D) $0.9\,\text{cm}$

Correct Answer: (C) $90\,\text{cm}$

Solution: The formula for the electric potential (V_P) due to a point charge is given by:

$$V_P = \frac{KQ}{r}$$

where: $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\text{Nm}^2\text{C}^{-2}$

$Q = 5 \times 10^{-9}\,\text{C}$ (the charge)

r is the distance from the point charge to the point P

$V_P = 50\,\text{V}$ (the electric potential at point P)

Rearranging the formula to solve for the distance r:

$$r = \frac{KQ}{V_P}$$

Substituting the known values:

$$r = \frac{(9 \times 10^9) \times (5 \times 10^{-9})}{50}$$

$$r = \frac{45 \times 10^0}{50}$$

$$r = 0.9\,\text{m}$$

Thus, the distance of point P from the point charge is 0.9 meters or 90 cm.

Quick Tip

The electric potential due to a point charge decreases with distance from the charge. The formula $V = \frac{KQ}{r}$ shows the direct relationship between potential, charge, and distance.

---

50. Five charges +q, +5q, -2q, +3q and -4q are situated as shown in the figure. The electric flux due to this configuration through the surface S is:

(A) $\frac{5q}{\epsilon_0}$
(B) $\frac{4q}{\epsilon_0}$
(C) $\frac{3q}{\epsilon_0}$
(D) $\frac{q}{\epsilon_0}$

Correct Answer: (B) $\frac{4q}{\epsilon_0}$

Solution: Using Gauss's law, the electric flux ϕ is given by the formula:

$$\phi = \frac{q}{\epsilon_0}$$

where: - q is the charge inside the closed surface, - ϵ_0 is the permittivity of free space.

Now, if there are multiple charges inside the surface, we sum up the individual charges.

Here, we are given the charges inside the closed surface: q, $-2q$, and $5q$.

So, the total charge q_{total} inside the surface is:

$$q_{\text{total}} = q + (-2q) + 5q$$

$$q_{\text{total}} = 4q$$

Therefore, the electric flux is:

$$\phi = \frac{q_{\text{total}}}{\epsilon_0} = \frac{4q}{\epsilon_0}$$

Thus, the electric flux ϕ through the closed surface is:

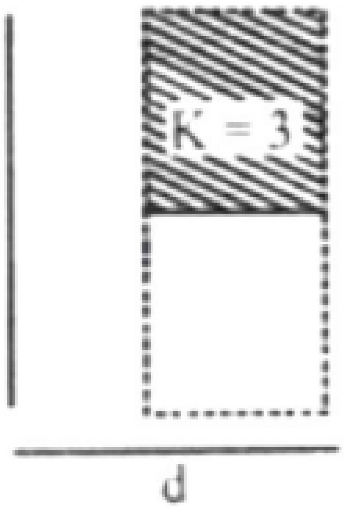
$$\phi = \frac{4q}{\epsilon_0}$$

Quick Tip

Remember Gauss's Law: The total electric flux through a closed surface is proportional to the net electric charge enclosed within the surface. Charges outside the surface do not contribute to the net flux through the surface.

---

51. A parallel plate capacitor with plate area A and plate separation $d = 2$ m has a capacitance of $4\mu F$. The new capacitance of the system if half of the space between them is filled with a dielectric material of dielectric constant $K = 3$ (as shown in the figure) will be:



(A) $2\mu F$

(B) $32\mu F$

(C) $6\mu F$

(D) $8\mu F$

Correct Answer: (C) $6\mu F$

Solution:

Step 1: Capacitance of the original capacitor

$$C_1 = \frac{A\varepsilon_0}{d} = 4\mu F$$

Step 2: Finding the new capacitance when half-filled with dielectric

The capacitor can be considered as two capacitors in series:

$$C_f = \frac{A\varepsilon_0}{d_1 + d_2/K} = \frac{A\varepsilon_0}{d(1 - \frac{1}{2} + \frac{1}{2K})}$$

Substituting $K = 3$:

$$C_f = \frac{4\mu F}{\frac{3}{2}} = 6\mu F$$

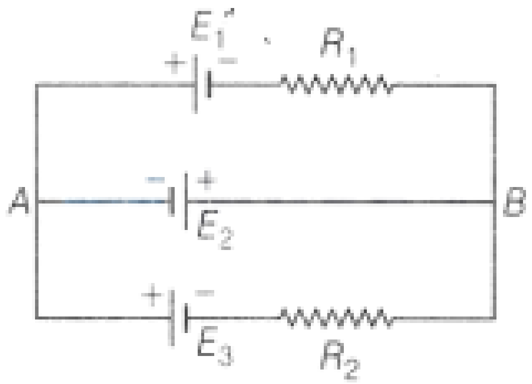
Thus, the new capacitance is $6\mu F$.

Quick Tip

When a dielectric partially fills a capacitor, consider it as a system of capacitors in series.

---

52. In the given circuit, $E_1 = E_2 = E_3 = 2V$ and $R_1 = R_2 = 4\Omega$, then the current flowing through the branch AB is:



(A) 0

(B) $2A$ from A to B

(C) $2A$ from B to A

(D) $5A$ from B to B

Correct Answer: (B) $2A$ from A to B

Solution:

Step 1: Finding Equivalent EMF and Resistance

Using Kirchhoff's Voltage Law (KVL):

$$E_{eq} = \frac{E_1R_1 + E_2R_2}{R_1 + R_2} = \frac{2 \times 4 + 2 \times 4}{4 + 4} = 2V$$

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{4 \times 4}{4 + 4} = 2\Omega$$

Step 2: Finding the Current

$$I = \frac{E'}{R_{eq}} = \frac{4V}{2\Omega} = 2A$$

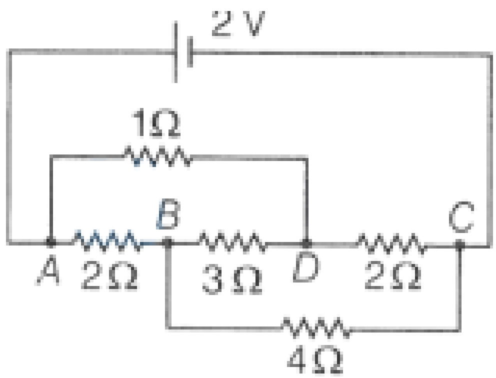
Thus, the current flows from A to B at $2A$.

Quick Tip

Use Kirchhoff's Laws to analyze complex circuits efficiently.

---

53. In the following circuit diagram, when the 3Ω resistor is removed, the equivalent resistance of the network:



(A) Increases

(B) Decreases

(C) Remains the same

(D) None of these

Correct Answer: (C) Remains the same

Solution:

Step 1: Identifying Wheatstone Bridge

The given network forms a balanced Wheatstone bridge. In a Wheatstone bridge, four resistors are arranged in a diamond shape, with two resistors on each arm, and a galvanometer connected across the middle of the bridge. In this case, when the $3\,\Omega$ resistor in the BD arm is removed, the bridge remains balanced.

Key Concept: A Wheatstone bridge is balanced when the ratio of resistances in one pair of opposite arms is equal to the ratio of resistances in the other pair. The condition for balance in the Wheatstone bridge is given by:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Where $R_1, R_2, R_3,$ and R_4 are the resistances in the four arms of the bridge.

When the $3\,\Omega$ resistor is removed from the BD arm, it does not affect the total resistance because the bridge is balanced and the current flowing through the galvanometer is zero, meaning no current flows through the branch containing the $3\,\Omega$ resistor. Hence, the equivalent resistance of the bridge does not change.

Step 2: Finding Equivalent Resistance

Since the bridge remains balanced, the equivalent resistance of the network remains unchanged. For a balanced Wheatstone bridge, the total equivalent resistance across the bridge is determined by the resistances in the remaining arms.

Let's denote the resistances of the other arms as $R_1, R_2, R_3,$ and R_4. When the bridge is balanced, we know the equivalent resistance for the two parallel arms (AC and BD) can be calculated as:

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

This is true for both the upper and lower parts of the Wheatstone bridge, and they contribute equally to the total equivalent resistance.

Since the $3\,\Omega$ resistor is removed from the BD arm, it does not change the equivalent resistance of the network because the Wheatstone bridge was balanced and no current flows through the removed resistor. The equivalent resistance remains as calculated previously.

Step 3: Conclusion

Thus, the total equivalent resistance of the bridge remains unchanged when the $3\,\Omega$ resistor is removed.

Quick Tip

In a balanced Wheatstone bridge, removing a resistor from one arm does not affect the total equivalent resistance, as no current flows through that branch when the bridge is balanced.

---

54. A conducting wire is stretched by applying a deforming force, so that its diameter decreases to 40% of the original value. The percentage change in its resistance will be:

(A) 0.9%

(B) 0.12%

(C) 1.6%

(D) 0.5%

Correct Answer: (C) 1.6%

Solution:

Step 1: Understanding the effect of stretching

Since the volume of the wire remains constant, we use the relation:

$$V = Al$$

where A is the cross-sectional area and l is the length.

Step 2: Deriving the new resistance

We use the resistance formula:

$$R = \rho \frac{l}{A}$$

Since A decreases as d^2 and l increases proportionally:

$$\frac{\Delta R}{R} = -4\frac{\Delta D}{D}$$

Substituting $\Delta D = -0.4$,

$$\frac{\Delta R}{R} = -4(-0.4) = 1.6\%$$

Thus, the percentage change in resistance is 1.6%.

Quick Tip

When a wire is stretched, its length increases and its cross-sectional area decreases, leading to an increase in resistance.

---

55. A wire of resistance 160Ω is melted and drawn into a wire of one-fourth of its length. The new resistance of the wire will be:

(A) 10Ω

(B) 640Ω

(C) 40Ω

(D) 16Ω

Correct Answer: (A) 10Ω

Solution:

Step 1: Understanding volume conservation

Since the wire is melted and redrawn, its volume remains the same:

$$A_1 l_1 = A_2 l_2$$

Given that the new length is $\frac{1}{4}$th of the original:

$$A_2 = 4A_1$$

Step 2: Finding new resistance

Resistance is given by:

$$R = \rho \frac{l}{A}$$

Using the transformation,

$$R_2 = \frac{l_2}{A_2} R_1$$

$$R_2 = \frac{\frac{1}{4}l_1}{4A_1} R_1 = \frac{1}{16} R_1$$

Substituting $R_1 = 160\Omega$:

$$R_2 = \frac{160}{16} = 10\Omega$$

Thus, the new resistance is 10Ω.

Quick Tip

When a wire is redrawn with a reduced length, its cross-sectional area increases, reducing resistance.

---

56. Five cells each of emf E and internal resistance r send the same amount of current through an external resistance R whether the cells are connected in parallel or in series. Then the ratio $\frac{R}{r}$ is:

(A) 2

(B) $\frac{1}{2}$

(C) $\frac{1}{5}$

(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Current in series combination

$$I = \frac{nE}{nr + R} = \frac{5E}{5r + R}$$

Step 2: Current in parallel combination

$$I' = \frac{E}{\frac{r}{n} + R} = \frac{5E}{r + 5R}$$

Since $I = I'$, equating both expressions:

$$\frac{5E}{5r + R} = \frac{5E}{r + 5R}$$

Solving for R and r,

$$5r + R = r + 5R$$

$$4r = 4R \Rightarrow R = r$$

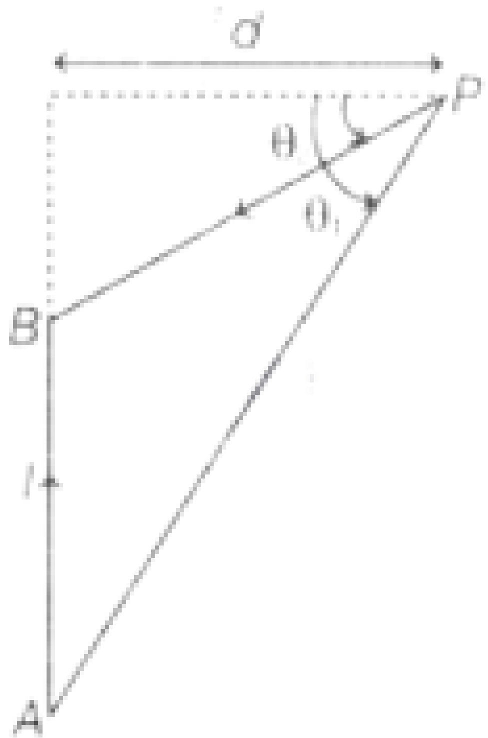
Thus, the ratio $\frac{R}{r} = 1$.

Quick Tip

For the same current in both series and parallel configurations, the external resistance must equal internal resistance.

---

57. The straight wire AB carries a current I. The ends of the wire subtend angles θ_1 and θ_2 at the point P as shown in the figure. The magnetic field at the point P is:



(A) $\frac{\mu_0 I}{4\pi d}(\sin\theta_1 - \sin\theta_2)$

(B) $\frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

(C) $\frac{\mu_0 I}{4\pi d}(\cos\theta_1 - \cos\theta_2)$

(D) $\frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2)$

Correct Answer: (A) $\frac{\mu_0 I}{4\pi d}(\sin\theta_1 - \sin\theta_2)$

Solution:

The problem involves calculating the magnetic field at a point P due to a current-carrying straight wire, where the ends of the wire make angles α and β with respect to the point P. The solution can be derived using the Biot-Savart law, which gives the magnetic field generated by a current element.

Step 1: Understanding the Biot-Savart Law

The Biot-Savart law provides the magnetic field $d\mathbf{B}$ at a point due to a small current element $I\,d\mathbf{l}$. The law is given by:

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{l} \times \hat{r}}{r^2}$$

where: - μ_0 is the permeability of free space, - I is the current, - $d\mathbf{l}$ is the infinitesimal length of the wire element, - $\hat{r}$ is the unit vector from the wire element to the point where the magnetic field is being calculated, - r is the distance from the wire element to the point.

Step 2: Applying the Biot-Savart Law to a Straight Wire

For a straight current-carrying wire, the magnetic field at a point P can be found by integrating the contributions from all infinitesimal elements of the wire. The result for the magnetic field due to a finite straight wire at a point P is given by:

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 - \sin\theta_2)$$

where: - d is the perpendicular distance from the wire to the point P, - θ_1 and θ_2 are the angles between the line connecting the point P and the ends of the wire, and the wire itself. The expression is derived by integrating the Biot-Savart law along the length of the wire. The terms $\sin\theta_1$ and $\sin\theta_2$ come from the geometry of the setup, which involves the angles at which the current elements contribute to the magnetic field.

Step 3: Conclusion

Thus, the magnetic field at point P due to a straight current-carrying wire, where the ends of the wire make angles α and β with the point, is:

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 - \sin\theta_2)$$

Therefore, the correct option is (A).

Quick Tip

The Biot-Savart law is fundamental for calculating the magnetic field due to current-carrying elements. For a straight wire, the field depends on the angles formed between the wire and the point where the field is being calculated.

---

58. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $a/2$ and $2a$ from the axis of the wire is:

(A) $1:4$

(B) $4:1$

(C) $1:1$

(D) $3:4$

Correct Answer: (C) $1:1$

Solution:

Step 1: Using Ampere's Circuital Law

The magnetic field due to a straight wire is given by:

$$B = \frac{\mu_0 I}{2\pi r}$$

Step 2: Finding the Magnetic Field at $a/2$ and $2a$

Magnetic field at $r = a/2$:

$$B_{a/2} = \frac{\mu_0 I}{2\pi(a/2)}$$

$$B_{a/2} = \frac{\mu_0 I}{\pi a}$$

Magnetic field at $r = 2a$:

$$B_{2a} = \frac{\mu_0 I}{2\pi(2a)}$$

$$B_{2a} = \frac{\mu_0 I}{4\pi a}$$

Step 3: Calculating the Ratio

$$\frac{B_{a/2}}{B_{2a}} = \frac{\frac{\mu_0 I}{\pi a}}{\frac{\mu_0 I}{4\pi a}} = \frac{1}{1} = 1 : 1$$

Thus, the correct answer is $1 : 1$.

Quick Tip

Magnetic field inside a current-carrying wire varies linearly, while outside it follows an inverse relation with distance.

---

59. The electrostatic force F$_1$ and magnetic force F$_2$ acting on a charge q moving with velocity v can be written as:

(A) $\vec{F_1} = q\vec{v} \cdot \vec{E}, \vec{F_2} = q(\vec{B} \cdot \vec{v})$

(B) $\vec{F_1} = q\vec{B}, \vec{F_2} = q(\vec{B} \times \vec{v})$

(C) $\vec{F_1} = q\vec{E}, \vec{F_2} = q(\vec{v} \times \vec{B})$

(D) $\vec{F_1} = q\vec{E}, \vec{F_2} = q(\vec{B} \times \vec{v})$

Correct Answer: (C) $\vec{F_1} = q\vec{E}, \vec{F_2} = q(\vec{v} \times \vec{B})$

Solution:

Step 1: Understanding the Forces

The electrostatic force on a charge q is given by:

$$\vec{F_1} = q\vec{E}$$

The magnetic force on a moving charge is given by:

$$\vec{F_2} = q(\vec{v} \times \vec{B})$$

Thus, the correct answer is (C).

Quick Tip

The Lorentz force is the combination of electrostatic and magnetic forces acting on a charge.

---

60. Inside a solenoid of radius 0.5 m, the magnetic field is changing at a rate of 50×10^{-6} T/s. The acceleration of an electron placed at a distance of 0.3 m from the axis of the solenoid will be:

(A) 23×10^6 m/s^2

(B) 26×10^6 m/s^2

(C) 1.3×10^9 m/s^2

(D) 26×10^9 m/s^2

Correct Answer: (A) 23×10^6 m/s^2

Solution:

Step 1: Using Faraday's Law

$$\text{Induced emf} = \frac{-d\Phi}{dt} = B \cdot A$$

Since $A = \pi r^2$, we get:

$$\varepsilon = -\pi r^2 \frac{dB}{dt}$$

Step 2: Finding the Electric Field

$$E = \frac{\varepsilon}{d} = \frac{\pi r^2}{d}\frac{dB}{dt}$$

Step 3: Finding Acceleration

$$a = \frac{eE}{m}$$

Substituting values,

$$a = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}} \times \frac{\pi (0.5)^2}{0.3} \times 50 \times 10^{-6}$$

$$= 23 \times 10^6 \text{ m/s}^2$$

Thus, the correct answer is 23×10^6 m/s^2.

Quick Tip

The changing magnetic field inside a solenoid induces an electric field, which exerts a force on charged particles.

---

61. There are two long co-axial solenoids of the same length l. The inner and outer coils have radii r_1 and r_2 and the number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self-inductance of the inner coil is:

(A) $\frac{n_1}{n_2}$

(B) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$

(C) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$

(D) $\frac{n_2}{n_1}$

Correct Answer: (D) $\frac{n_2}{n_1}$

Solution:

Step 1: Expression for Mutual Inductance

The mutual inductance M between the two co-axial solenoids is given by:

$$M = \mu_0 n_1 n_2 \pi r_1^2 l$$

Step 2: Expression for Self-Inductance of the Inner Coil

The self-inductance L of the inner solenoid is given by:

$$L = \mu_0 n_1^2 \pi r_1^2 l$$

Step 3: Finding the Ratio $\frac{M}{L}$

$$\frac{M}{L} = \frac{\mu_0 n_1 n_2 \pi r_1^2 l}{\mu_0 n_1^2 \pi r_1^2 l} = \frac{n_2}{n_1}$$

Thus, the correct answer is $\frac{n_2}{n_1}$.

Quick Tip

The mutual inductance depends on the turns per unit length of both solenoids, while self-inductance depends only on the inner solenoid.

---

62. A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4 T. The loop is removed from the field in 10 sec. The average emf induced in the loop during this time is:

(A) -2 V

(B) $+2$ V

(C) $+1$ V

(D) -1 V

Correct Answer: (C) $+1$ V

Solution:

Step 1: To find the magnetic flux Φ through the rectangular loop, we use the formula:

$$\Phi = B \cdot A \cdot \cos(\theta)$$

Where:

- B is the magnetic field strength,
- A is the area of the loop,
- θ is the angle between the magnetic field lines and the normal (perpendicular) to the plane of the loop.

Step 2: The area A of the rectangular loop is:

$$A = \text{length} \times \text{width} = 2.5\,\text{m} \times 2\,\text{m} = 5\,\text{m}^2$$

Step 3: Given the angle $\theta = 60^\circ$, we can calculate the initial magnetic flux Φ_{initial}:

$$\Phi_{\text{initial}} = B \cdot A \cdot \cos(60^\circ)$$

$$\Phi_{\text{initial}} = 4\,\text{T} \cdot 5\,\text{m}^2 \cdot \cos(60^\circ)$$

$$\Phi_{\text{initial}} = 4 \cdot 5 \cdot \frac{1}{2} = 10\,\text{Wb}$$

(Since $\cos(60^\circ) = \frac{1}{2}$)

Step 4: When the loop is removed from the magnetic field, the final magnetic flux Φ_{final} is zero because the loop is no longer within the magnetic field. Thus, the change in magnetic flux $\Delta\Phi$ is:

$$\Delta\Phi = \Phi_{\text{final}} - \Phi_{\text{initial}} = 0 - 10\,\text{Wb} = -10\,\text{Wb}$$

Step 5: The loop is removed from the field in $t = 10$ seconds, so the rate of change of magnetic flux is:

$$\frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t} = \frac{-10\,\text{Wb}}{10\,\text{s}} = -1\,\text{Wb/s}$$

Step 6: Now we can find the average induced emf ϵ:

$$\epsilon = -\frac{d\Phi}{dt}$$

$$\epsilon = -(-1\,\text{Wb/s}) = +1\,\text{V}$$

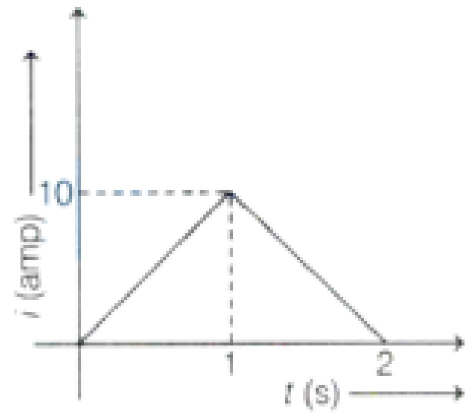
Therefore, the average emf is $+1$ V, and the correct option is (3) +1V.

Quick Tip

The induced emf is found using Faraday's Law, considering the flux change over time.

---

63. Find the average value of the current shown graphically from $t = 0$ to $t = 2$ s.



(A) 3 A

(B) 5 A

(C) 10 A

(D) 4 A

Correct Answer: (B) 5 A

Solution:

Step 1: Finding the Area under the $i - t$ Graph

$$\text{Total area} = \frac{1}{2} \times 1 \times 10 + \frac{1}{2} \times (2 - 1) \times 10$$

$$= 5 + 5 = 10 \text{ A}$$

Step 2: Finding Average Current

$$i_{\text{avg}} = \frac{\text{Total area}}{\text{time interval}}$$

$$i_{\text{avg}} = \frac{10}{2} = 5 \text{ A}$$

Thus, the correct answer is 5 A.

Quick Tip

The area under the current-time graph gives the charge transferred, which helps in calculating the average current.

---

64. In an AC circuit, an inductor, a capacitor, and a resistor are connected in series with $X_L = R = X_C$. The impedance of this circuit is:

(A) $2R^2$

(B) Zero

(C) R

(D) $R\sqrt{2}$

Correct Answer: (C) R

Solution:

Step 1: Using Impedance Formula

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

Since $X_L = X_C$,

$$Z = \sqrt{(R - R)^2 + R^2} = \sqrt{0 + R^2} = R$$

Thus, the correct answer is R.

Quick Tip

When inductive and capacitive reactances are equal in a series circuit, the impedance is purely resistive.

---

65. An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is:

(A) 5 ms

(B) 3.3 ms

(C) 7.2 ms

(D) 2.2 ms

Correct Answer: (B) 3.3 ms

Solution:

Step 1: Understanding the given equation

The given alternating voltage is:

$$V(t) = 220 \sin 100\pi t$$

The angular frequency ω is extracted as:

$$\omega = 100\pi \text{ rad/s}$$

Step 2: Time interval calculation for half to peak transition

The peak value of the voltage is $V_{\text{max}} = 220$ V.

The time interval for the voltage to rise from half of the peak value to the peak value in a sinusoidal waveform follows the relation:

$$t = \frac{T}{6}$$

where T is the time period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{2}{100} = 0.02 \text{ s} = 20 \text{ ms}$$

Thus, the required time interval:

$$t = \frac{T}{6} = \frac{20}{6} = 3.3 \text{ ms}$$

Thus, the correct answer is 3.3 ms.

Quick Tip

The time taken for voltage in an AC waveform to rise from half to peak is always $\frac{T}{6}$, where T is the time period.

---

66. A parallel plate capacitor consists of two circular plates of radius $R = 0.1$ m. They are separated by a short distance. If the electric field between the capacitor plates changes as:

$$\frac{dE}{dt} = 6 \times 10^{13} \frac{V}{m \cdot s}$$

then the value of the displacement current is:

(A) 15.25 A

(B) 6.25 A

(C) 16.67 A

(D) 4.69 A

Correct Answer: (C) 16.67 A

Solution:

Step 1: Using Maxwell's Displacement Current Formula

The displacement current is given by:

$$I_d = \varepsilon_0 \frac{d\Phi}{dt}$$

Since:

$$\frac{d\Phi}{dt} = A\frac{dE}{dt}$$

Step 2: Finding the Area of Plates

$$A = \pi R^2 = 3.14 \times (0.1)^2 = 3.14 \times 10^{-2}\ \text{m}^2$$

Step 3: Calculating Displacement Current

$$I_d = \varepsilon_0 A\frac{dE}{dt}$$

$$= (8.85 \times 10^{-12}) \times (3.14 \times 10^{-2}) \times (6 \times 10^{13})$$

$$I_d = 16.67\ \text{A}$$

Thus, the correct answer is 16.67 A.

Quick Tip

Displacement current plays a crucial role in Maxwell's equations, bridging the gap between capacitors in AC circuits.

67. Electromagnetic waves travel in a medium with speed 1.5×10^8 m/s. The relative permeability of the medium is 2.0. The relative permittivity will be:

(A) 5

(B) 1

(C) 4

(D) 2

Correct Answer: (D) 2

Solution:

Step 1: The given equation for the velocity v is:

$$v = \frac{C}{\sqrt{\mu_r \epsilon_r}}$$

Where: - v is the velocity, - C is the speed of light in vacuum, - μ_r is the relative permeability, - ϵ_r is the relative permittivity.

Step 2: Now, substitute the given values:

$$v = 1.5 \times 10^8 = \frac{3 \times 10^8}{\sqrt{2 \times \epsilon_r}}$$

Step 3: Rearrange the equation to solve for ϵ_r:

$$\sqrt{2 \times \epsilon_r} = \frac{3 \times 10^8}{1.5 \times 10^8}$$

$$\sqrt{2 \times \epsilon_r} = 2$$

Step 4: Square both sides to eliminate the square root:

$$2 \times \epsilon_r = 4$$

Step 5: Now, solve for ϵ_r:

$$\epsilon_r = \frac{4}{2} = 2$$

Thus, the correct answer is 2.

Quick Tip

The speed of light in any medium is given by $v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$, where μ_r and ε_r are the relative permeability and permittivity.

---

68. Power of a biconvex lens is P diopter. When it is cut into two symmetrical halves by a plane containing the principal axis, the ratio of the power of two halves is:

(A) 1:2

(B) 2:1

(C) 1:4

(D) 1:1

Correct Answer: (D) 1:1

Solution:

Step 1: Understanding the Concept of Lens Power

The power of a lens is given by:

$$P = \frac{1}{f}$$

where f is the focal length of the lens.

Step 2: Effect of Cutting a Lens Along the Principal Axis

When a symmetrical biconvex lens is cut into two halves along the principal axis, the focal length remains the same for each half.

Since power is inversely proportional to focal length, the power of each half remains unchanged.

Thus, the ratio of power between the two halves is:

$$1 : 1$$

Thus, the correct answer is $1 : 1$.

Quick Tip

When a lens is cut along the principal axis, its focal length remains unchanged, and hence its power remains the same.

---

69. The magnifying power of a telescope is 9. When adjusted for parallel rays, the distance between the objective and eyepiece is 20 cm. The ratio of the focal length of the objective lens to the focal length of the eyepiece is:

(A) 8

(B) 7

(C) 9

(D) 12

Correct Answer: (C) 9

Solution:

Step 1: Understanding Magnifying Power of a Telescope

The magnification M of an astronomical telescope in normal adjustment (parallel rays) is given by:

$$M = \frac{f_o}{f_e}$$

where:

f_o is the focal length of the objective,

f_e is the focal length of the eyepiece.

Step 2: Using Given Information

It is given that $M = 9$, so:

$$\frac{f_o}{f_e} = 9$$

Also, the total distance between the objective and eyepiece is:

$$f_o + f_e = 20$$

Step 3: Solving for f_o and f_e

Using the given equations:

$$9f_e + f_e = 20$$

$$10f_e = 20$$

$$f_e = 2 \text{ cm}, \quad f_o = 18 \text{ cm}$$

Thus, the ratio of focal lengths is:

$$\frac{f_o}{f_e} = 9$$

Thus, the correct answer is 9.

Quick Tip

The magnification of an astronomical telescope is given by $M = \frac{f_o}{f_e}$. The larger the objective lens focal length, the higher the magnification.

---

70. In normal adjustment, for a refracting telescope, the distance between the objective and eyepiece is 30 cm. The focal length of the objective, when the angular magnification of the telescope is 2, will be:

(A) 20 cm

(B) 30 cm

(C) 10 cm

(D) 15 cm

Correct Answer: (A) 20 cm

Solution:

Step 1: Understanding the Normal Adjustment Condition

In a refracting telescope under normal adjustment, the total length of the telescope is:

$$L = f_o + f_e$$

where:

f_o is the focal length of the objective lens,

f_e is the focal length of the eyepiece lens.

Step 2: Using the Given Values

It is given that the total length of the telescope is:

$$f_o + f_e = 30$$

Also, the magnification of the telescope is given by:

$$M = \frac{f_o}{f_e}$$

Since $M = 2$, we get:

$$\frac{f_o}{f_e} = 2$$

Step 3: Solving for f_o and f_e

Rewriting the equation:

$$f_o = 2f_e$$

Substituting into the length equation:

$$2f_e + f_e = 30$$

$$3f_e = 30$$

$$f_e = 10 \text{ cm}, \quad f_o = 20 \text{ cm}$$

Thus, the correct answer is 20 cm.

Quick Tip

In an astronomical telescope, the focal length of the objective lens is usually larger than the eyepiece lens to provide higher magnification.

71. If the distance between an object and its two times magnified virtual image produced by a curved mirror is 15 cm, the focal length of the mirror must be:

(A) $\frac{10}{3}$ cm

(B) -12 cm

(C) -10 cm

(D) 15 cm

Correct Answer: (C) -10 cm

Solution:

Step 1: Understanding the given data

The magnification formula for a mirror is:

$$m = -\frac{v}{u}$$

Since the image is virtual and magnified two times:

$$m = 2$$

which gives:

$$2 = -\frac{v}{u}$$

Step 2: Using the given object-image distance

The total distance between the object and image is:

$$u + v = 15$$

Substituting $v = -2u$,

$$u + (-2u) = 15$$

$$-u = 15 \Rightarrow u = 5 \text{ cm}$$

$$v = -2(5) = -10 \text{ cm}$$

Step 3: Using the mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$= \frac{1}{-10} + \frac{1}{5} = -\frac{1}{10}$$

$$f = -10 \text{ cm}$$

Thus, the correct answer is -10 cm.

Quick Tip

For a concave mirror, a magnified virtual image is always formed when the object is placed between the focal point and the mirror.

---

72. Young's double slit experiment is performed in a medium of refractive index 1.33. The maximum intensity is I_0. The intensity at a point on the screen where the path difference between the light coming out from slits is $\lambda/4$, is:

(A) 0

(B) $\frac{I_0}{2}$

(C) $\frac{3I_0}{8}$

(D) $\frac{2I_0}{3}$

Correct Answer: (B) $\frac{I_0}{2}$

Solution:

Step 1: Understanding the intensity formula in YDSE

The intensity at any point in YDSE is given by:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

where ϕ is the phase difference, given by:

$$\phi = \frac{2\pi}{\lambda} \times \text{path difference}$$

Step 2: Substituting given values

For the given path difference $\Delta x = \lambda/4$,

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

Thus,

$$I = I_0 \cos^2\left(\frac{\pi}{4}\right) = I_0 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$I = \frac{I_0}{2}$$

Thus, the correct answer is $\frac{I_0}{2}$.

Quick Tip

In Young's Double-Slit Experiment (YDSE), the intensity at a given point depends on the phase difference of the interfering waves.

---

73. In YDSE, monochromatic light falls on a screen 1.80 m from two slits separated by 2.08 mm. The first and second order bright fringes are separated by 0.553 mm. The wavelength of light used is:

(A) 520 nm

(B) 639 nm

(C) 715 nm

(D) None of these

Correct Answer: (B) 639 nm

Solution:

Step 1: Understanding Young's Double-Slit Experiment (YDSE)

In Young's Double-Slit Experiment, the fringe width β is given by:

$$\beta = \frac{\lambda D}{d}$$

where:

λ is the wavelength of light,

D is the distance between the slits and the screen,

d is the distance between the two slits.

The fringe width is the distance between consecutive bright fringes.

Step 2: Using the Given Data

It is given that the distance between the first and second-order bright fringes is:

$$y_2 - y_1 = 0.553 \text{ mm} = 0.553 \times 10^{-3} \text{ m}$$

For the first bright fringe,

$$y_1 = \frac{\lambda D}{d}$$

For the second bright fringe,

$$y_2 = \frac{2\lambda D}{d}$$

So,

$$y_2 - y_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d}$$

Step 3: Solving for λ

$$\lambda = \frac{(y_2 - y_1) \cdot d}{D}$$

Substituting the given values:

$$\lambda = \frac{(0.553 \times 10^{-3}) \times (2.08 \times 10^{-3})}{1.8}$$

$$\lambda = \frac{1.15024 \times 10^{-6}}{1.8}$$

$$\lambda = 639 \times 10^{-9} \text{ m} = 639 \text{ nm}$$

Thus, the correct answer is 639 nm.

Quick Tip

The fringe width in YDSE increases if the wavelength of light or the screen distance increases, but decreases if the slit separation increases.

---

74. A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit will be:

(A) 60°

(B) 45°

(C) 15°

(D) 30°

Correct Answer: (A) 60°

Solution:

Step 1: Understanding the diffraction formula

The condition for minima in diffraction is:

$$d \sin\theta = n\lambda$$

For the first minimum:

$$\sin\theta = \frac{\lambda}{d} = \frac{2}{4} = \frac{1}{2}$$

$$\theta = 30^\circ$$

Angular spread:

$$2\theta = 60^\circ$$

Thus, the correct answer is 60°.

Quick Tip

In diffraction, the first minimum occurs when the path difference equals the wavelength.

75. The property of light which cannot be explained by Huygen's construction of a wavefront is:

(A) Refraction

(B) Reflection

(C) Diffraction

(D) Origin of spectra

Correct Answer: (D) Origin of spectra

Solution:

Step 1: Understanding Huygen's Wave Theory

Huygen's wave theory describes how light propagates by treating every point on a wavefront as a secondary wave source. Using this principle, the laws of:

- Reflection
- Refraction
- Diffraction

are successfully derived.

Step 2: Analyzing the Given Options

Reflection: Huygen's principle explains reflection by considering the secondary wavelets on the incident wavefront, which create the reflected wavefront.

Refraction: Huygen's principle explains refraction by stating that different parts of a wavefront move at different speeds when passing through media with different refractive indices.

Diffraction: Huygen's principle accounts for diffraction, as each point on a wavefront acts as a source of secondary wavelets, allowing light to bend around obstacles.

Step 3: Why Huygen's Principle Fails to Explain Spectra

The origin of spectral lines arises due to the emission and absorption of photons by atoms, which is best explained by quantum mechanics. Huygen's wave theory does not consider the particle nature of light or energy quantization, which are essential for understanding:

- Atomic emission spectra

- Blackbody radiation
- Photoelectric effect

Since Huygen's theory only deals with the wave nature of light and not its quantum properties, it **cannot explain the origin of spectra**.

Thus, the correct answer is **D**.

Quick Tip

Huygen's wave theory explains wave phenomena like diffraction and interference but fails to explain phenomena requiring quantum mechanics, such as the photoelectric effect and spectral emissions.

---

76. When a light ray incidents on the surface of a medium, the reflected ray is completely polarized. Then the angle between reflected and refracted rays is:

(A) 45°

(B) 90°

(C) 120°

(D) 180°

Correct Answer: (B) 90°

Solution:

Step 1: Understanding Brewster's Law

According to Brewster's law, when light is incident at the Brewster angle, the reflected light is completely polarized.

At this angle, the reflected and refracted rays are perpendicular to each other.

Step 2: Derivation of the Perpendicular Relation

$$\theta_{\text{reflected}} + \theta_{\text{refracted}} = 90^\circ$$

Thus, the angle between the reflected and refracted rays is:

$$\theta = 90^\circ$$

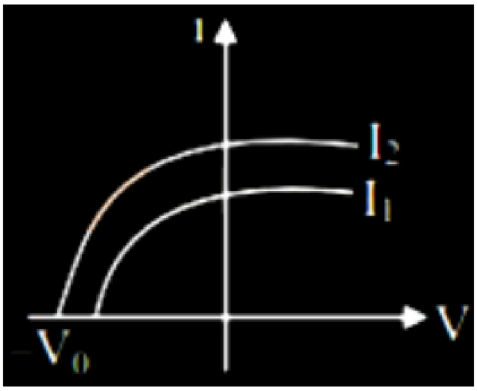
Thus, the correct answer is 90°.

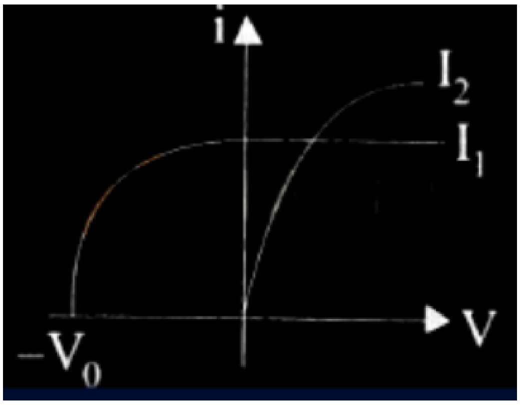
Quick Tip

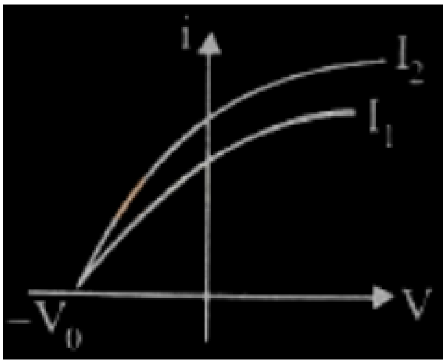
Brewster's angle is given by $\tan\theta_B = \frac{n_2}{n_1}$, where n_1 and n_2 are refractive indices of the media.

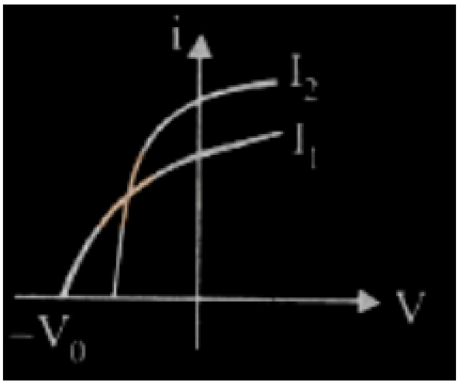
---

77. Which figure shows the correct variation of applied potential difference (V) with photoelectric current (I) at two different intensities of light $(I_1 < I_2)$ of same wavelengths:

A. 

B. 

C. 

D. 

Correct Answer: (C)

Solution: Step 1: Understanding the Photoelectric Effect

- The photoelectric effect involves the emission of electrons from a metal surface when light of a certain frequency (or wavelength) strikes it. The key observation is that the energy of the emitted electrons depends on the frequency (or wavelength) of the incident light, not its intensity. - The stopping potential is the minimum voltage required to stop the most energetic emitted electrons. It is independent of the light intensity but depends on the wavelength (or frequency) of the incident light. This means that the stopping potential remains the same for different intensities of light, as long as the frequency (or wavelength) is the same. - The saturation current, however, increases with the intensity of the incident light. This is because a higher intensity means more photons are hitting the surface, and more electrons are being emitted. The saturation current represents the maximum current that can be obtained when all emitted electrons are collected.

Step 2: Interpreting the Graphs

- In the graph representing the photoelectric effect, the x-axis typically represents the wavelength (or frequency) of the incident light, and the y-axis represents the stopping potential or the saturation current. - The correct graph for this situation should show that the stopping potential is the same for both intensities of light, as it is independent of intensity. However, the saturation current will be higher for I_2 than for I_1, because I_2 corresponds to a higher intensity, which results in more electrons being emitted from the surface.

Thus, the correct graph is (C).

Quick Tip

In the photoelectric effect, the stopping potential is independent of intensity and depends only on the frequency of the incident light. The saturation current increases with intensity.

---

78. The acceptor level of a p-type semiconductor is 6 eV. The maximum wavelength of light which can create a hole would be: Given $hc = 1242$ eV nm.

(A) 407 nm

(B) 414 nm

(C) 207 nm

(D) 103.5 nm

Correct Answer: (C) 207 nm

Solution:

Step 1: Using Energy-Wavelength Relation

The energy of the photon required to excite an electron is given by:

$$E = \frac{hc}{\lambda}$$

Rearranging for λ:

$$\lambda = \frac{hc}{E}$$

Step 2: Substituting the Given Values

$$\lambda = \frac{1242}{6}$$

$$\lambda = 207 \text{ nm}$$

Thus, the correct answer is 207 nm.

Quick Tip

Higher energy photons (lower wavelength) are required to excite electrons in semiconductors with a larger band gap.

---

79. When light is incident on a metal surface, the maximum kinetic energy of emitted electrons:

(A) Varies with intensity of light

(B) Varies with frequency of light

(C) Varies with speed of light

(D) Varies irregularly

Correct Answer: (B) Varies with frequency of light

Solution:

Step 1: Photoelectric Equation

According to Einstein's photoelectric equation:

$$K_{\text{max}} = h\nu - W_0$$

where: - h is Planck's constant, - ν is the frequency of incident light, - W_0 is the work function of the metal.

Step 2: Dependence on Frequency

Since K_{max} depends only on ν and not on intensity, the correct answer is (B).

Quick Tip

Increasing the intensity of light increases the number of photoelectrons but does not change their maximum kinetic energy.

---

80. If the kinetic energy of a free electron doubles, its de-Broglie wavelength changes by the factor:

(A) 2

(B) $\frac{1}{2}$

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (D) $\frac{1}{\sqrt{2}}$

Solution:

Step 1: Using de-Broglie's Equation

The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{p}$$

Since momentum p is related to kinetic energy:

$$p = \sqrt{2mK}$$

Step 2: Effect of Doubling Kinetic Energy

If K is doubled:

$$p' = \sqrt{2m(2K)} = \sqrt{2}p$$

Since $\lambda \propto \frac{1}{p}$, we get:

$$\lambda' = \frac{\lambda}{\sqrt{2}}$$

Thus, the correct answer is $\frac{1}{\sqrt{2}}$.

Quick Tip

The de-Broglie wavelength decreases as the kinetic energy increases, following the inverse square root relation.

---

81. Which of the following transitions of He^+ ion will give rise to a spectral line that has the same wavelength as the spectral line in a hydrogen atom?

(A) $n = 4 \rightarrow n = 2$

(B) $n = 6 \rightarrow n = 5$

(C) $n = 6 \rightarrow n = 3$

(D) None of these

Correct Answer: (A) $n = 4 \rightarrow n = 2$

Solution:

Step 1: Understanding the Rydberg Formula

The wavelength λ of emitted radiation during an electron transition is given by the Rydberg formula:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where: - R is the Rydberg constant, - Z is the atomic number, - n_1 and n_2 are the principal quantum numbers.

Step 2: Applying to Hydrogen and Helium Ions

For hydrogen ($Z = 1$), the wavelength of the transition $n_2 \rightarrow n_1$ is:

$$\frac{1}{\lambda_H} = R(1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For the He^+ ion ($Z = 2$), the wavelength of transition $n_4 \rightarrow n_3$ is:

$$\frac{1}{\lambda_{He}} = R(2)^2 \left(\frac{1}{n_3^2} - \frac{1}{n_4^2} \right)$$

Equating $\lambda_H = \lambda_{He}$, we get:

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = 4 \left(\frac{1}{n_3^2} - \frac{1}{n_4^2} \right)$$

Solving for integer values, we find $n_3 = 2$, $n_4 = 4$ satisfies the condition.

Thus, the correct answer is $n = 4 \rightarrow n = 2$.

Quick Tip

The He^+ ion behaves like a hydrogen-like atom but with a stronger nuclear attraction, leading to shorter wavelengths for similar transitions.

---

82. The ratio of the shortest wavelength of the Balmer series to the shortest wavelength of the Lyman series for the hydrogen atom is:

(A) 4:1

(B) 1:2

(C) 1:4

(D) 2:1

Correct Answer: (A) 4:1

Solution: Step 1: The general formula for the wavelength of a hydrogen atom is given as:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Where: - λ is the wavelength, - R is the Rydberg constant, - Z is the atomic number, - n_1 and n_2 are the principal quantum numbers.

Step 2: The shortest wavelength for the Balmer series corresponds to the transition from $n_2 \to \infty$ and $n_1 = 2$, so substituting into the equation:

$$\frac{1}{\lambda_B} = RZ^2 \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right)$$

Since $\frac{1}{\infty^2} = 0$, this simplifies to:

$$\frac{1}{\lambda_B} = RZ^2 \left(\frac{1}{4} \right)$$

Thus, the shortest wavelength for the Balmer series is:

$$\frac{1}{\lambda_B} = \frac{RZ^2}{4}$$

Step 3: The shortest wavelength for the Lyman series corresponds to the transition from $n_2 \to \infty$ and $n_1 = 1$, so substituting into the equation:

$$\frac{1}{\lambda_L} = RZ^2 \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right)$$

Again, $\frac{1}{\infty^2} = 0$, so this simplifies to:

$$\frac{1}{\lambda_L} = RZ^2 (1)$$

Thus, the shortest wavelength for the Lyman series is:

$$\frac{1}{\lambda_L} = RZ^2$$

Step 4: Now, dividing equation (i) by equation (ii), where equation (i) represents the shortest wavelength for the Balmer series and equation (ii) represents the shortest wavelength for the Lyman series:

$$\frac{\frac{1}{\lambda_B}}{\frac{1}{\lambda_L}} = \frac{RZ^2/4}{RZ^2}$$

This simplifies to:

$$\frac{\lambda_L}{\lambda_B} = 4$$

Therefore, we can conclude:

$$\lambda_L : \lambda_B = 4 : 1$$

Thus, the ratio is $4 : 1$.

Quick Tip

The Balmer series appears in the visible spectrum, while the Lyman series is in the ultraviolet region.

---

83. The minimum excitation energy of an electron revolving in the first orbit of hydrogen is:

(A) 3.4 eV

(B) 8.5 eV

(C) 10.2 eV

(D) 13.6 eV

Correct Answer: (C) 10.2 eV

Solution:

Step 1: Energy Levels in the Hydrogen Atom

The energy of an electron in the n^{th} orbit is:

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

For $n = 1$:

$$E_1 = -13.6 \text{ eV}$$

For $n = 2$:

$$E_2 = \frac{-13.6}{4} = -3.4 \text{ eV}$$

Step 2: Calculating Excitation Energy

$$E_{\text{excitation}} = E_2 - E_1$$

$$= (-3.4) - (-13.6)$$

$$= 10.2 \text{ eV}$$

Thus, the correct answer is 10.2 eV.

Quick Tip

Excitation energy is the energy required to move an electron from a lower to a higher energy level.

---

84. The atomic mass of $^6C^{12}$ is 12.000000 u and that of $^6C^{13}$ is 13.003354 u. The required energy to remove a neutron from $^6C^{13}$, if the mass of the neutron is 1.008665 u, will be:

(A) 62.5 MeV

(B) 6.25 MeV

(C) 4.95 MeV

(D) 49.5 MeV

Correct Answer: (C) 4.95 MeV

Solution:

Mass defect:

$$\Delta m = (12.000000 + 1.008665) - 13.003354$$

$$= 0.00531u$$

Energy required:

$$E = \Delta m \times 931.5$$

$$= 0.00531 \times 931.5$$

$$= 4.95 \text{ MeV}$$

Thus, the correct answer is 4.95 MeV.

Quick Tip

Mass defect is the difference between the expected and actual nuclear mass, and it accounts for the nuclear binding energy.

---

85. The nucleus having highest binding energy per nucleon is:

(A) $^{16}_{8}O$

(B) $^{56}_{26}Fe$

(C) $^{208}_{84}Pb$

(D) $^{4}_{2}He$

Correct Answer: (B) $^{56}_{26}Fe$

Solution:

Step 1: Understanding Binding Energy per Nucleon

The binding energy per nucleon (BE/A) is the energy required to disassemble a nucleus into its constituent protons and neutrons. It is an important measure of nuclear stability. The higher the binding energy per nucleon, the more stable the nucleus is. The binding energy per nucleon is typically highest for elements in the mid-range of the periodic table, particularly for elements around iron ($^{56}_{26}Fe$).

Step 2: Comparing the Nuclei

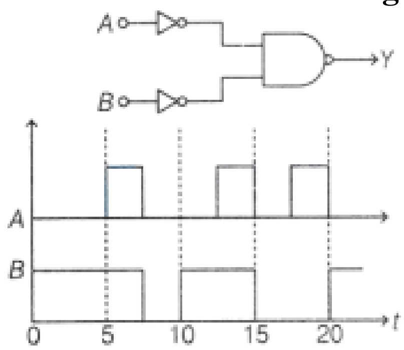
- Lighter nuclei, such as ${}^{4}_{2}He$ (Helium-4), have a relatively low binding energy per nucleon. This is because the nucleons in lighter nuclei are not as tightly bound as in heavier nuclei. - Heavy nuclei, such as ${}^{208}_{84}Pb$ (Lead-208), also tend to have lower binding energy per nucleon compared to mid-range nuclei. This is due to the electrostatic repulsion between the positively charged protons, which weakens the nuclear force that binds the nucleus together.
- ${}^{56}_{26}Fe$ (Iron-56), which has the highest binding energy per nucleon (around 8.8 MeV), is considered the most stable nucleus. This high binding energy per nucleon explains why nuclear fusion (such as in stars) generally produces energy by fusing lighter elements up to iron, and why fission of heavy elements releases energy.
Thus, the correct answer is ${}^{56}_{26}Fe$.

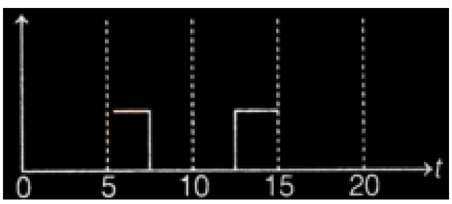
Quick Tip

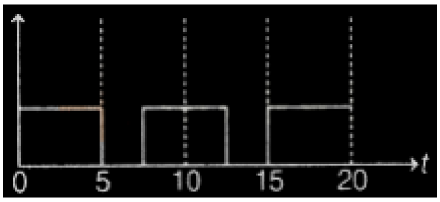
The binding energy per nucleon increases as we approach iron on the periodic table, making iron the most stable nucleus. Both lighter and heavier nuclei have lower binding energies per nucleon.

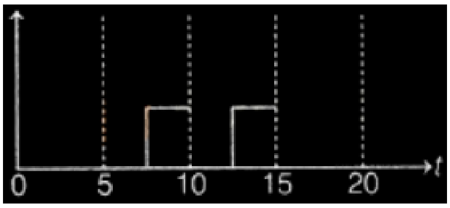
---

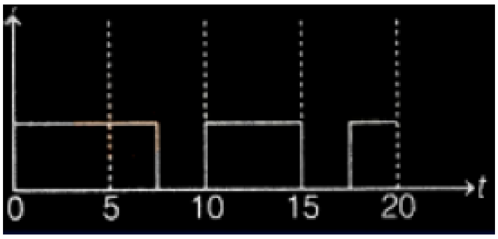
86. Identify the correct output signal Y in the given combination of gates for the given inputs A and B shown in the figure.

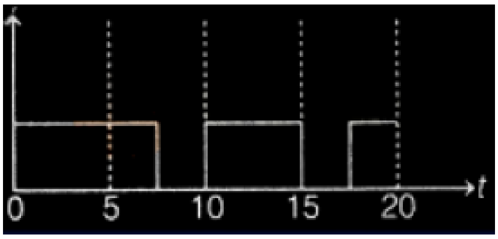


A. 

B. 

C. 

D. 

Correct Answer: D. 

Solution:

Step 1: Understanding the Logic Circuit

The given circuit consists of: - NOT gates applied to inputs A and B. - AND gate processing the inverted signals.

Step 2: Deriving the Boolean Expression

The circuit implements:

$$Y = \overline{A} \cdot \overline{B}$$

Applying De-Morgan's theorem:

$$Y = \overline{(A + B)}$$

Step 3: Comparing with Output Waveforms

Analyzing the truth table, we match the waveform and determine that the correct output is given by waveform 4.

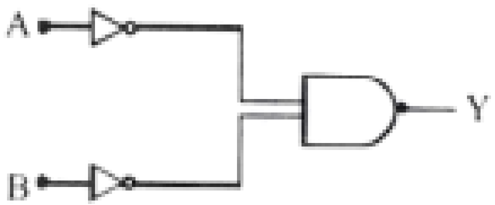
Thus, the correct answer is (D).

Quick Tip

De-Morgan's Theorem is a useful tool to simplify Boolean expressions: $\overline{A \cdot B} = \overline{A} + \overline{B}$ and $\overline{A + B} = \overline{A} \cdot \overline{B}$.

---

87. Identify the logic gate given in the circuit:



(A) NAND gate

(B) OR gate

(C) AND gate

(D) NOR gate

Correct Answer: (B) OR gate

Solution:

Step 1: Understanding the Circuit

The circuit consists of NOT gates applied to A and B, followed by an AND gate.

Step 2: Boolean Expression Derivation

$$Y = \overline{\overline{A} \cdot \overline{B}}$$

Applying De-Morgan's theorem:

$$Y = A + B$$

Step 3: Conclusion

The output matches the OR gate.

Thus, the correct answer is (B).

Quick Tip

Applying De-Morgan's theorem helps simplify logic expressions and identify equivalent gate functions in complex circuits.

---

88. A reverse biased zener diode when operated in the breakdown region works as:

(A) an amplifier

(B) an oscillator

(C) a voltage regulator

(D) a rectifier

Correct Answer: (C) a voltage regulator

Solution:

Step 1: Understanding Zener Diode

A zener diode allows current to flow in the reverse direction when the applied voltage exceeds the breakdown voltage.

Step 2: Function of a Zener Diode

In the breakdown region, the voltage across the diode remains nearly constant, making it ideal for voltage regulation.

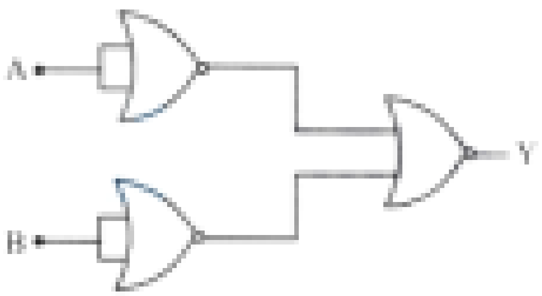
Thus, the correct answer is (C).

Quick Tip

Zener diodes are extensively used in power supplies to provide stable voltage regulation.

---

89. Identify the logic operation performed by the following circuit.



(A) OR

(B) AND

(C) NOT

(D) NAND

Correct Answer: (B) AND

Solution:

Step 1: Understanding the Logic Circuit

The circuit consists of two NOR gates whose outputs are given as inputs to another NOR gate.

Step 2: Boolean Expression

Using De-Morgan's theorem:

$$Y = \overline{\overline{A} + \overline{B}}$$

$$= A \cdot B$$

Step 3: Conclusion

The circuit implements an AND gate.

Thus, the correct answer is (B).

Quick Tip

Logic circuits can be reduced systematically using Boolean algebra to identify their equivalent gates.

---

90. One main scale division of a vernier caliper is equal to m units. If the m^{th} division of main scale coincides with the $(n+1)^{th}$ division of vernier scale, the least count of the vernier caliper is:

(A) $\frac{n}{(n+1)}$

(B) $\frac{m}{(n+1)}$

(C) $\frac{1}{(n+1)}$

(D) $\frac{m}{n(n+1)}$

Correct Answer: (B) $\frac{m}{(n+1)}$

Solution:

Step 1: Understanding Vernier Caliper

The vernier scale least count is given by:

$$LC = \text{Main Scale Division} - \text{Vernier Scale Division}$$

Step 2: Applying the Given Condition

Since n main scale divisions equal $(n+1)$ vernier scale divisions,

$$n \times MSD = (n+1) \times VSD$$

Step 3: Solving for Least Count

$$VSD = \frac{n}{n+1} \times MSD$$

$$LC = MSD - VSD = m - \frac{n}{n+1}m$$

$$LC = m\left(1 - \frac{n}{n+1}\right)$$

$$LC = \frac{m}{n+1}$$

Thus, the correct answer is (B).

Quick Tip

Least count is the smallest measurement a device can accurately measure; it's crucial for precision in scientific experiments.

Chemistry

1. A 1 L closed flask contains a mixture of 4 g of methane and 4.4 g of carbon dioxide. The pressure inside the flask at 27C is (Assume ideal behaviour of gases):

(A) 8.6 atm

(B) 2.2 atm

(C) 4.2 atm

(D) 6.1 atm

Correct Answer: (A) 8.6 atm

Solution:

Step 1: Calculate the number of moles of each gas

The number of moles of methane CH_4 is given by:

$$n_1 = \frac{\text{Mass of } CH_4}{\text{Molar mass of } CH_4} = \frac{4}{16} = 0.25 \text{ mol}$$

Similarly, the number of moles of carbon dioxide CO_2 is:

$$n_2 = \frac{\text{Mass of } CO_2}{\text{Molar mass of } CO_2} = \frac{4.4}{44} = 0.1 \text{ mol}$$

Step 2: Total number of moles

Total number of moles, n_T is:

$$n_T = n_1 + n_2 = 0.25 + 0.1 = 0.35 \text{ mol}$$

Step 3: Applying the ideal gas equation

Using the ideal gas equation:

$$PV = nRT$$

where $R = 0.0821$ atm L mol^{-1}K^{-1}, $T = 300$ K, and $V = 1$ L, we get:

$$P = \frac{0.35 \times 0.0821 \times 300}{1} = 8.6 \text{ atm}$$

Thus, the correct answer is (A).

Quick Tip

The ideal gas equation $PV = nRT$ is fundamental in calculating pressure, volume, and temperature relationships in gas mixtures.

---

2. In which mode of expression, the concentration of a solution remains independent of temperature?

(A) Molarity

(B) Normality

(C) Formality

(D) Molality

Correct Answer: (D) Molality

Solution:

Step 1: Understanding concentration units

Molarity (M) and normality (N) depend on volume, which changes with temperature.

However, molality (m) is defined as:

$$m = \frac{\text{Moles of solute}}{\text{Mass of solvent in kg}}$$

Since mass is unaffected by temperature changes, molality remains constant.

Step 2: Why molality is temperature-independent

Molality does not involve volume, which expands or contracts with temperature. Hence, it is the preferred unit in temperature-dependent studies.

Thus, the correct answer is (D).

Quick Tip

Molality is used in colligative properties calculations because it remains constant irrespective of temperature changes.

---

3. The degeneracy of hydrogen atom that has energy equal to $-\frac{R_H}{9}$ is (where R_H = Rydberg constant)

(A) 6

(B) 8

(C) 5

(D) 9

Correct Answer: (D) 9

Solution:

Step 1: Understanding degeneracy

The energy of hydrogen-like atoms is given by:

$$E_n = -\frac{R_H}{n^2}$$

Given $E = -\frac{R_H}{9}$, comparing with the formula:

$$\frac{R_H}{n^2} = \frac{R_H}{9} \Rightarrow n^2 = 9 \Rightarrow n = 3$$

Step 2: Finding the degeneracy

For $n = 3$, the possible values of l are $0, 1, 2$, corresponding to subshells:

$$(3s, 3p, 3d)$$

Each subshell contains:

$$3s = 1, \quad 3p = 3, \quad 3d = 5$$

Total orbitals present:

$$1 + 3 + 5 = 9$$

Thus, the correct answer is (D).

Quick Tip

Degeneracy refers to the number of orbitals with the same energy level. It is given by n^2 for a hydrogen-like atom.

---

4. If the de-Broglie wavelength of a particle of mass (m) is 100 times its velocity, then its value in terms of its mass (m) and Planck constant (h) is:

(A) $\frac{1}{10}\sqrt{\frac{m}{h}}$
(B) $10\sqrt{\frac{h}{m}}$
(C) $\frac{1}{10}\sqrt{\frac{h}{m}}$
(D) $10\sqrt{\frac{m}{h}}$

Correct Answer: (B) $10\sqrt{\frac{h}{m}}$

Solution:

Step 1: Defining de-Broglie Wavelength

The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{mv}$$

Given that:

$$\lambda = 100v$$

Step 2: Expressing Wavelength in Terms of h and m

$$100v = \frac{h}{mv}$$

$$x^2 = 100\frac{h}{m}$$

$$x = 10\sqrt{\frac{h}{m}}$$

Thus, the correct answer is (B).

Quick Tip

The de-Broglie wavelength is inversely proportional to momentum; higher momentum means a shorter wavelength.

---

5. The energy of the second orbit of a hydrogen atom is -5.45×10^{-19} J. What is the energy of the first orbit of Li^{2+} ion (in J)?

(A) -1.962×10^{-18}

(B) -1.962×10^{-17}

(C) -3.924×10^{-17}

(D) -3.924×10^{-18}

Correct Answer: (B) -1.962×10^{-17}

Solution:

Step 1: Using Energy Formula

The energy of an electron in an orbit is given by:

$$E_n = -13.6\frac{Z^2}{n^2}eV$$

Step 2: Finding Energy for Li^{2+}

For Li^{2+}, $Z = 3$ and for the first orbit $n = 1$:

$$E_1 = -13.6 \times \frac{3^2}{1^2}eV$$

$$E_1 = -122.4eV$$

Step 3: Converting to Joules

$$E_1 = (-122.4) \times (1.602 \times 10^{-19})$$

$$E_1 = -1.962 \times 10^{-17} \text{ J}$$

Thus, the correct answer is (B).

Quick Tip

The energy levels of multi-electron atoms scale with Z^2; for Li^{2+}, energy is 9 times that of hydrogen.

---

6. A photon of wavelength 3000 Å strikes a metal surface. The work function of the metal is 2.13 eV. What is the kinetic energy of the emitted photoelectron?

($h = 6.626 \times 10^{-34}$ Js)

(A) 4.0 eV

(B) 3.0 eV

(C) 2.0 eV

(D) 1.0 eV

Correct Answer: (C) 2.0 eV

Solution:

Step 1: Using Einstein's Photoelectric Equation

$$KE = h\nu - \phi$$

Since $\nu = \frac{c}{\lambda}$, we use:

$$E = \frac{hc}{\lambda}$$

Step 2: Substituting Values

$$E = \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{3 \times 10^{-7}}$$

$$E = 6.6 \times 10^{-19} J$$

Converting to eV:

$$E = \frac{6.6 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.125 eV$$

Step 3: Calculating Kinetic Energy

$$KE = 4.125 - 2.13 = 2.0 eV$$

Thus, the correct answer is (C).

Quick Tip

The kinetic energy of emitted electrons is the excess energy after overcoming the work function of the metal.

---

7. A stream of electrons from a heated filament was passed between two charged plates at a potential difference V volt. If e and m are the charge and mass of an electron, then the value of $\frac{h}{\lambda}$ is:

(A) $\sqrt{meV}$

(B) $\sqrt{2meV}$

(C) meV

(D) $2meV$

Correct Answer: (B) $\sqrt{2meV}$

Solution:

Step 1: Finding Electron Kinetic Energy

$$KE = eV$$

Step 2: Using de-Broglie Wavelength Formula

$$\lambda = \frac{h}{\sqrt{2mKE}}$$

Step 3: Rearranging for $\frac{h}{\lambda}$

$$\frac{h}{\lambda} = \sqrt{2meV}$$

Thus, the correct answer is (B).

Quick Tip

Electrons gain kinetic energy when accelerated through a potential difference, affecting their de-Broglie wavelength.

---

8. Electron affinity is positive when:

(A) O changes into O^-

(B) O^- changes to O^{2-}

(C) O changes into O^+

(D) O changes to O^{2+}

Correct Answer: (B) O^- changes to O^{2-}

Solution: Step 1: Understanding Electron Affinity

Electron affinity refers to the energy change when an electron is added to a neutral atom in the gas phase to form a negatively charged ion. The first electron affinity is usually exothermic, meaning energy is released when the atom gains an electron. However, the second electron affinity is generally endothermic, meaning energy is required to add a second electron. This is because, after the first electron is added, the resulting negatively charged ion experiences a repulsive force from the incoming electron.

Step 2: Explaining Positive Electron Affinity

In the case of the oxygen ion O^-, the negative charge on the ion creates a repulsive force that works against the addition of another electron. The added electron would experience electrostatic repulsion due to the negative charge of the O^- ion. Therefore, energy must be supplied to overcome this repulsion, making the second electron affinity positive (endothermic).

Thus, the correct answer is (B).

Quick Tip

The first electron affinity is typically exothermic because the atom stabilizes by gaining an electron. However, the second electron affinity is endothermic due to the repulsive forces between the negatively charged ion and the incoming electron.

---

9. The ionic radii in (Å) of N^{3-}, O^{2-} and F^- are respectively.

(A) 1.71, 1.40 and 1.36

(B) 1.71, 1.36 and 1.40

(C) 1.36, 1.40 and 1.71

(D) 1.36, 1.71 and 1.40

Correct Answer: (A) 1.71, 1.40 and 1.36

Solution: Step 1: Isoelectronic Species

The ions N^{3-}, O^{2-}, and F^- are isoelectronic species, meaning they all have the same number of electrons. Specifically, each of these ions has 10 electrons. Despite having the same number of electrons, their ionic radii will differ because of the varying nuclear charges. The nuclear charge increases from nitrogen to oxygen to fluorine as we move across the periodic table: - N has atomic number $Z = 7$, - O has atomic number $Z = 8$, - F has atomic number $Z = 9$.

Step 2: Ionic Radii Order

As the nuclear charge increases, the electrons are attracted more strongly by the nucleus, which results in a decrease in ionic radius. Therefore, the ionic radii will decrease as we move from N^{3-} to O^{2-} to F^-. This gives the following order of ionic radii:

$$N^{3-} > O^{2-} > F^{-}$$

Hence, the correct answer is (A).

Quick Tip

In isoelectronic species, the ionic radius decreases with increasing nuclear charge because the greater positive charge attracts the electrons more strongly, causing the ion to contract.

---

10. Intramolecular hydrogen bonding is found in

(A) o-nitrophenol

(B) m-nitrophenol

(C) p-nitrophenol

(D) phenol

Correct Answer: (A) o-nitrophenol

Solution:

Step 1: Intramolecular Hydrogen Bonding

In o-nitrophenol, the hydrogen of the O-H group and the oxygen of the nitro group form intramolecular hydrogen bonding producing a six-membered ring structure. In m-nitrophenol, p-nitrophenol, and phenol, intermolecular hydrogen bonding is present.

Hence, the correct answer is (A).

Quick Tip

Intramolecular hydrogen bonding results in a stable, closed-ring structure within the molecule.

---

11. The hybridisation scheme for the central atom includes a d-orbital contribution in

(A) I_3^-

(B) PCl_3

(C) NO_3^-

(D) H_2Se

Correct Answer: (A) I_3^-

Solution:

Step 1: Hybridisation of I_3^-

I_3^- is linear with the central iodine atom undergoing sp^3d-hybridisation and three lone pairs occupying the equatorial position.

Thus, this structure includes a d-orbital contribution to the hybridisation scheme.

Hence, the correct answer is (A).

> Quick Tip
>
> Hybridisation involving d-orbitals occurs in molecules with central atoms from period 3 and beyond.

---

12. In the following species, how many species have the same magnetic moment?

(i) Cr^{2+} (ii) Mn^{3+} (iii) Ni^{2+} (iv) Sc^{2+} (v) Zn^{2+} (vi) V^{3+} (vii) Ti^{4+}

(A) 1

(B) 3

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Step 1: Unpaired Electrons

Among the given species, Cr^{2+} and Mn^{3+} have the same number of unpaired electrons.

Unpaired electrons, $n = 4$

Magnetic moment,

$$\mu = \sqrt{n(n+2)} = \sqrt{4(4+2)} = \sqrt{24}\text{BM}$$

Thus, the correct answer is (C).

> Quick Tip
>
> Magnetic moment depends on the number of unpaired electrons in the species.

13. The spin only magnetic moment of Fe^{3+} ion (in BM) is approximately.

(A) 4

(B) 5

(C) 6

(D) 7

Correct Answer: (C) 6

Solution:

Step 1: Electronic Configuration of Fe

The electronic configuration of Fe is $Fe = [Ar]3d^6 4s^2$. For Fe^{3+}, it loses three electrons, resulting in the configuration:

$$Fe^{3+} = [Ar]3d^5$$

Step 2: Number of Unpaired Electrons

For Fe^{3+}, the number of unpaired electrons, n, is 5 (since it has 5 electrons in the 3d orbitals).

Step 3: Spin Only Magnetic Moment Formula

The formula for calculating the spin-only magnetic moment is:

$$\mu_s = \sqrt{n(n+2)} \text{ BM}$$

where $n = 5$. Therefore:

$$\mu_s = \sqrt{5(5+2)} = \sqrt{5 \times 7} = \sqrt{35} \text{ BM} \approx 6 \text{ BM}$$

Thus, the correct answer is (C).

Quick Tip

The spin-only magnetic moment formula applies when there is no contribution from orbital motion of electrons, which is true for transition metal ions like Fe^{3+}.

---

14. Which one of the following compounds is having maximum 'lone pair-lone pair' electron repulsions?

(A) ClF_3

(B) IF_5

(C) SF_4

(D) XeF_2

Correct Answer: (D) XeF_2

Solution:

Step 1: Understanding Lone Pair-Lone Pair Repulsions

Lone pairs of electrons on the central atom experience repulsion from each other. The maximum repulsion occurs when the lone pairs are placed in positions that minimize their mutual interaction. The lone pair-lone pair repulsion increases with the number of lone pairs and the spatial arrangement.

Step 2: Analysis of Compounds

In ClF_3, there are 3 lone pairs on the central chlorine atom and 2 bonding pairs, with a T-shaped geometry.

In IF_5, there are 2 lone pairs on iodine and 5 bonding pairs, with a square pyramidal geometry.

In SF_4, there is 1 lone pair on the sulfur atom and 4 bonding pairs, with a seesaw geometry.

In XeF_2, there are 3 lone pairs on the central xenon atom and 2 bonding pairs, with a linear geometry.

Since XeF_2 has the most lone pairs (3), the lone pair-lone pair repulsions are the highest, and hence, XeF_2 experiences the maximum repulsion.

Thus, the correct answer is (D).

> **Quick Tip**
>
> Lone pair-lone pair repulsion is strongest when lone pairs are in positions where they are not spread out, such as the axial positions in square pyramidal or linear geometries.

---

15. Identify the species having one π-bond and maximum number of canonical forms from the following:

(A) SO_3

(B) O_2

(C) SO_2

(D) CO_3^{2-}

Correct Answer: (D) CO_3^{2-}

Solution:

Step 1: Analysis of Canonical Forms

Canonical forms are the different resonance structures that a molecule can have, resulting in delocalization of electrons. The species with the maximum number of canonical forms has the most stable resonance structure.

Step 2: Analysis of Compounds

SO_3 has three resonance structures but lacks a π-bond between sulfur and oxygen in each structure.

O_2 has two resonance structures with a single π-bond between the two oxygen atoms, but it doesn't have the maximum number of canonical forms.

SO_2 has two resonance structures, but again, it doesn't maximize the number of canonical forms.

CO_3^{2-} has three resonance structures, each with a π-bond and delocalized electrons between the carbon and the oxygen atoms.

The carbonate ion (CO_3^{2-}) has the maximum number of canonical forms.

Thus, the correct answer is (D).

Quick Tip

The more resonance structures a molecule can have, the more stable it is, as electron density is spread out over the molecule.

---

16. sp^3 d^2 hybridisation is not displayed by:

(A) BrF_5

(B) SF_6

(C) $[CrF_6]^{3-}$

(D) PF_5

Correct Answer: (D) PF_5

Solution:

Step 1: Hybridisation of the Central Atom

In $sp^3 d^2$ hybridisation, the central atom undergoes hybridisation by combining one s, three p, and two d orbitals, resulting in six hybrid orbitals. This hybridisation occurs in species where the central atom is bonded to six other atoms.

Step 2: Analysis of Compounds

In BrF_5, bromine undergoes $sp^3 d^2$ hybridisation, as it is bonded to five fluorine atoms.

In SF_6, sulfur undergoes $sp^3 d^2$ hybridisation, as it is bonded to six fluorine atoms.

In $[CrF_6]^{3-}$, chromium undergoes $sp^3 d^2$ hybridisation due to the bonding with six fluoride ions.

In PF_5, phosphorus undergoes sp^3 hybridisation, as it is bonded to five fluorine atoms, not six.

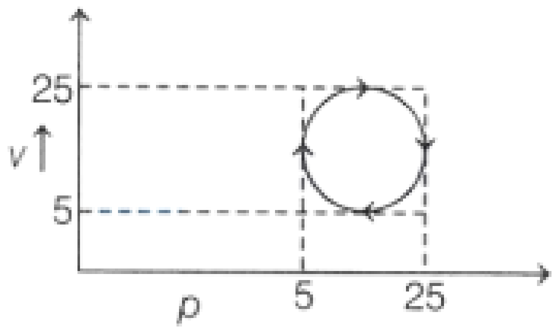
Thus, the correct answer is (D).

Quick Tip

$sp^3 d^2$ hybridisation occurs when the central atom has six bonding pairs, typically seen in octahedral structures.

---

17. What would be the amount of heat absorbed in the cyclic process shown below?



(A) 5π J

(B) 15π J

(C) 25π J

(D) 100π J

Correct Answer: (D) 100π J

Solution:

Step 1: The first equation representing the change in internal energy is:

$$\Delta U = \Delta q + \Delta W$$

Step 2: As $\Delta U = 0$, we have:

$$\Delta q = -\Delta W$$

Step 3: From the graph, work done ΔW is the area under the curve. The area under the curve is the area of a circle:

$$\Delta W = \pi r^2$$

Step 4: Given the radius as $\frac{25-5}{2}$, we calculate the work done as:

$$\Delta W = \pi \left(\frac{25-5}{2}\right)^2 = \pi \times 10^2$$

Thus, the heat absorbed is:

$$\Delta q = 100\pi \, \mathrm{J}$$

Quick Tip

In a cyclic process, the work done is the area enclosed by the path in the pressure-volume diagram, and it equals the heat absorbed if no internal energy change occurs.

---

18. The bond dissociation energy of X_2, Y_2 and XY are in the ratio of 1 : 0.5 : 1. ΔH for the formation of XY is -200 kJ/mol. The bond dissociation energy of X_2 will be

(A) 200 kJ/mol

(B) 100 kJ/mol

(C) 400 kJ/mol

(D) 800 kJ/mol

Correct Answer: (D) 800 kJ/mol

Solution:

Step 1: Using Given Data

Let the bond dissociation energy of X_2 be a kJ/mol. Thus, the bond dissociation energy of Y_2 is $0.5a$ kJ/mol and the bond dissociation energy of XY is a kJ/mol.

The formation reaction is:

$$\frac{1}{2}X_2 + \frac{1}{2}Y_2 \to XY,\ \Delta H = -200\,\text{kJ/mol}$$

Step 2: Bond Energy Formula

The bond dissociation energy can be calculated using the formula for enthalpy change:

$$\Delta H = BE(\text{Reactants}) - BE(\text{Products})$$

$$\Delta H = \left[\frac{1}{2}BE(X_2) + \frac{1}{2}BE(Y_2)\right] - BE(XY)$$

$$\Delta H = \left[\frac{a}{2} + \frac{0.5a}{2}\right] - a$$

$$-200 = \frac{a + 0.5a}{2} - a$$

$$-200 = \frac{1.5a}{2} - a$$

$$-200 = 0.75a - a$$

$$-200 = -0.25a$$

$$a = 800\,\text{kJ/mol}$$

Thus, the correct answer is (D).

Quick Tip

The bond dissociation energy of a molecule is the energy required to break the bond between two atoms in a molecule.

---

19. Which of the following relation is not correct?

(A) $\Delta H = \Delta U - P\Delta V$

(B) $\Delta U = q + W$

(C) $\Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$

(D) $\Delta G = \Delta H - T\Delta S$

Correct Answer: (A) $\Delta H = \Delta U - P\Delta V$

Solution:

Step 1: Understanding the Relations

Relation (A) is the incorrect one.

The correct expression for enthalpy change is $\Delta H = \Delta U + P\Delta V$.

The expression provided in option (A) is incorrect because it subtracts $P\Delta V$ instead of adding it.

Step 2: Correct Relations

Relation (B) is correct because the change in internal energy ΔU is the sum of heat added to the system q and the work done on the system W, i.e., $\Delta U = q + W$.

Relation (C) is correct according to the second law of thermodynamics, which states that the total entropy change (system plus surroundings) for a spontaneous process is greater than or equal to zero.

Relation (D) is the correct form of the Gibbs free energy equation.

Thus, the correct answer is (A).

Quick Tip

Enthalpy change, ΔH, is related to internal energy change by the equation $\Delta H = \Delta U + P\Delta V$, which is crucial for understanding thermodynamic systems at constant pressure.

---

20. The standard Gibbs energy ($\Delta G°$) for the following reaction is

$$A(s) + B^{2+}(aq) \rightleftharpoons A^{2+}(aq) + B(s), \quad K_c = 10^{12} \text{ at}$$

(Kc = equilibrium constant)

(A) -150 kJ/mol

(B) -96.80 kJ/mol

(C) -68.47 kJ/mol

(D) -100 kJ/mol

Correct Answer: (C) -68.47 kJ/mol

Solution:

Step 1: Using the Gibbs Free Energy Relation

The standard Gibbs free energy change is related to the equilibrium constant K_c by the equation:

$$\Delta G^\circ = -RT \ln K_c$$

where: - $R = 8.314\,\text{J/K}\cdot\text{mol}$ is the universal gas constant. - $T = 298\,\text{K}$ is the standard temperature. - $K_c = 10^{12}$.

Step 2: Substitute Values and Calculate

$$\Delta G^\circ = -(8.314\,\text{J/K}\cdot\text{mol}) \times (298\,\text{K}) \times \ln(10^{12})$$

$$\Delta G^\circ = -8.314 \times 298 \times 2.303 \times log 10^{12}$$

$$\Delta G^\circ = -68.47\,\text{kJ/mol}$$

Thus, the correct answer is (C).

Quick Tip

The standard Gibbs free energy change can be directly calculated from the equilibrium constant using the formula $\Delta G^\circ = -RT \ln K_c$, where R is the gas constant and T is the temperature in Kelvin.

---

21. The combustion of benzene (L) gives CO_2 (g) and H_2O (L). Given that heat of combustion of benzene at constant volume is -3263.9 kJ/mol at 25°C, heat of combustion (in kJ/mol) of benzene at constant pressure will be: (R = 8.314JK^{-1} mol^{-1})

(A) 4152.6

(B) 452.46

(C) 3260

(D) -3267.6

Correct Answer: (D) -3267.6

Solution:

Step 1: Understanding the Problem

The given reaction is the combustion of benzene. We are given the heat of combustion at constant volume, and we need to find the heat of combustion at constant pressure.

For the combustion of benzene:

$$C_6H_6(l) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(l)$$

Step 2: Using the Relation between Heat at Constant Volume and Pressure

The change in enthalpy is related to the change in internal energy by the equation:

$$\Delta H = \Delta U + \Delta n_g RT$$

where Δn_g is the change in the number of moles of gas between products and reactants.

In the given reaction, we calculate the change in the number of moles of gas:

$$\Delta n_g = (6 \text{ mol } CO_2) - \left(\frac{15}{2} \text{ mol } O_2\right) = 6 - 7.5 = -1.5$$

Thus, we have:

$$\Delta H = \Delta U + (-1.5) \times (8.314 \times 10^{-3} \times 298)$$

$$\Delta H = -3263.9 + (-1.5) \times 2.478 \approx -3267.6 \, \text{kJ/mol}$$

Thus, the correct answer is (D).

Quick Tip

For reactions involving gases, the heat of combustion at constant pressure is adjusted by the work done due to changes in gas volume, which is linked to the change in the number of moles of gas.

---

22. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following:

(A) $q = 0, \Delta T \neq 0, w = 0$

(B) $q = 0, \Delta T < 0, w \neq 0$

(C) $q \neq 0, \Delta T = 0, w = 0$

(D) $q = 0, \Delta T = 0, w = 0$

Correct Answer: (D) $q = 0, \Delta T = 0, w = 0$

Solution:

Step 1: Understanding Free Expansion of an Ideal Gas

In free expansion, the gas expands without doing work and without heat exchange with the surroundings. Since the process is adiabatic, $q = 0$, meaning no heat is exchanged.

Step 2: Work Done in Free Expansion

Since the expansion is against a vacuum, no work is done on or by the system. Hence, $w = 0$.

Step 3: Temperature Change in Free Expansion

For an ideal gas undergoing free expansion, there is no change in temperature, so $\Delta T = 0$.

Thus, the correct answer is (D).

> **Quick Tip**
>
> In an adiabatic free expansion of an ideal gas, there is no heat transfer, no work done, and no change in temperature, as the gas expands into a vacuum.

---

23. Le-Chatelier's principle is not applicable to

(A) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

(B) $Fe(s) + S(s) \rightleftharpoons FeS(s)$

(C) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

(D) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

Correct Answer: (B)

Solution:

Le-Chatelier's principle applies to systems where changes in temperature, pressure, or concentration cause a shift in equilibrium. However, it does not apply to pure solids and liquids, because their concentrations do not change significantly during a reaction.

Step 1: Identifying the Pure Solids and Liquids

In reaction (B), Fe and S are pure solids, and thus, their concentrations do not significantly change during the reaction. Therefore, Le-Chatelier's principle is not applicable here.

Thus, the correct answer is (B).

> **Quick Tip**
>
> Le-Chatelier's principle is not applicable to pure solids and liquids because their concentrations do not change during chemical equilibrium.

24. The ratio $\frac{K_p}{K_c}$ for the reaction

$$CO(g) + \frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$$

is:

(A) $(RT)^{1/2}$

(B) RT

(C) 1

(D) $\frac{1}{\sqrt{RT}}$

Correct Answer: (D) $\frac{1}{\sqrt{RT}}$

Solution:

Step 1: Relation Between K_p and K_c

The equilibrium constant K_p and K_c are related by the following equation:

$$K_p = K_c(RT)^{\Delta n}$$

where Δn is the change in the number of moles of gas between products and reactants.

Step 2: Calculate Δn

For the given reaction:

$$\Delta n = \text{moles of products} - \text{moles of reactants} = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

Step 3: Substitute Δn

Substitute $\Delta n = -\frac{1}{2}$ into the relation:

$$K_p = K_c(RT)^{-\frac{1}{2}}$$

Thus, the correct answer is (D).

Quick Tip

The ratio $\frac{K_p}{K_c}$ depends on the change in the number of moles of gas between the products and reactants, and it is governed by the equation $K_p = K_c(RT)^{\Delta n}$.

---

25. The pH of 1 N aqueous solutions of HCl, CH_3COOH and HCOOH follows the order:

(A) HCl $>$ HCOOH $>$ CH_3COOH

(B) HCl $=$ HCOOH $>$ CH_3COOH

(C) CH_3COOH $>$ HCOOH $>$ HCl

(D) CH_3COOH $=$ HCOOH $>$ HCl

Correct Answer: (C) CH_3COOH $>$ HCOOH $>$ HCl

Solution:

Step 1: Understanding the Strength of Acids

Stronger the acid, lower is the pH, and weaker the acid, higher the pH.

Step 2: Order of Acid Strength

The strength of the given acids is in the order:

$$\text{HCl} > \text{HCOOH} > \text{CH}_3\text{COOH}$$

Step 3: Order of pH

Hence, their pH is in the order:

$$\text{CH}_3\text{COOH} > \text{HCOOH} > \text{HCl}$$

Thus, the correct answer is (C).

> **Quick Tip**
>
> The stronger the acid, the lower its pH value. Therefore, the pH order is the reverse of the acid strength order.

---

26. 20 mL of 0.1 M acetic acid is mixed with 50 mL of potassium acetate. K_a of acetic acid $= 1.8 \times 10^{-5}$ at 27°C. Calculate the concentration of potassium acetate if the pH of the mixture is 4.8.

(A) 0.1 M

(B) 0.04 M

(C) 0.03 M

(D) 0.02 M

Correct Answer: (B) 0.04 M

Solution:

Step 1: Henderson-Hasselbalch Equation

For an acidic buffer, the pH is given by:

$$\text{pH} = \text{p}K_a + \log\left(\frac{[\text{Salt}]}{[\text{Acid}]}\right)$$

Step 2: Concentrations

Let the concentration of potassium acetate solution be x M.

$$20 \text{ mL of } 0.1 \text{ M acetic acid} = 20 \times 0.1 \text{ millimol} = 2 \text{ millimol}$$

$$50 \text{ mL of } x \text{ M potassium acetate} = x \times 50 \text{ millimol} = 50x \text{ millimol}$$

Step 3: Substitute Values

Given, pH = 4.8

$$4.8 = \text{p}K_a + \log\left(\frac{50x}{2}\right)$$

where $\text{p}K_a = \log 1.8 \times 10^{-5} = 4.74$

Step 4: Solve for x

Substitute $\text{p}K_a$:

$$4.8 = 4.74 + \log\left(\frac{50x}{2}\right)$$

$$0.06 = \log(25x)$$

$$10^{0.06} = 25x$$

$$x = \frac{10^{0.06}}{25} = 0.04$$

Thus, the correct answer is (B) 0.04 M.

Quick Tip

The Henderson-Hasselbalch equation is key for calculating the pH of buffer solutions.

---

27. What is the stoichiometric coefficient of SO_2 in the following balanced reaction?

$$MnO_4^-(aq) + SO_2(g) \rightarrow Mn^{2+}(aq) + HSO_4^-(aq)$$

(in acidic solution)

(A) 5

(B) 4

(C) 3

(D) 2

Correct Answer: (A) 5

Solution:

Step 1: Oxidation Half Reaction

Write the oxidation half reaction:

$$SO_2(g) + 2H_2O(l) \rightarrow HSO_4^-(aq) + 3H^+(aq) + 2e^-$$

Step 2: Reduction Half Reaction

Write the reduction half reaction:

$$MnO_4^-(aq) + 8H^+(aq) + 5e^- \rightarrow Mn^{2+}(aq) + 4H_2O(l)$$

Step 3: Balance the Electrons

To balance the electrons, multiply the oxidation half reaction by 5 and the reduction half reaction by 2:

$$5SO_2(g) + 10H_2O(l) \rightarrow 5HSO_4^-(aq) + 15H^+(aq) + 10e^-$$

$$2MnO_4^-(aq) + 16H^+(aq) + 10e^- \rightarrow 2Mn^{2+}(aq) + 8H_2O(l)$$

Step 4: Combine and Simplify

Combine the two half reactions:

$$2MnO_4^-(aq) + 5SO_2(g) + 16H^+(aq) \rightarrow 2Mn^{2+}(aq) + 5HSO_4^-(aq) + 8H_2O(l)$$

Thus, the stoichiometric coefficient of SO_2 is 5.

Quick Tip

Balancing redox reactions involves writing the oxidation and reduction half reactions, balancing the electrons, and then combining the half reactions.

---

28. Volume of M/8 $KMnO_4$ solution required to react completely with 25.0 cm^3 of M/4 $FeSO_4$ in acidic medium is:

(A) 8.0 mL

(B) 5.0 mL

(C) 15.0 mL

(D) 10.0 mL

Correct Answer: (D) 10.0 mL

Solution:

Step 1: Balance the Redox Reaction

The balanced ionic equation for the reaction is:

$$MnO_4^- + 5Fe^{2+} + 8H^+ \rightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$$

Step 2: Stoichiometric Relationship

From the equation, it is clear that 1 mole of $KMnO_4$ reacts with 5 moles of $FeSO_4$.

Step 3: Apply Molarity Equation

Use the molarity equation to balance the reaction:

$$\frac{M_1V_1}{n_1}(KMnO_4) = \frac{M_2V_2}{n_2}(FeSO_4)$$

$$\frac{1 \times V_1}{8 \times 1} = \frac{25 \times 5}{4 \times 5}$$

Step 4: Solve for V_1

$$V_1 = \frac{1 \times 25 \times 8}{4 \times 5}$$

$$V_1 = 10 \text{ cm}^3 \text{ or } V_1 = 10.0 \text{ mL}$$

Thus, the correct answer is (D).

Quick Tip

In redox reactions involving $KMnO_4$ and $FeSO_4$, use the molarity equation to find the required volume of the reactants.

---

29. Which of the following is only a redox reaction but not a disproportionation reaction?

(A) $4H_3PO_3 \rightarrow 3H_3PO_4 + PH_3$

(B) $2H_2O_2 \rightarrow 2H_2O + O_2$

(C) $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$

(D) $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2$

Correct Answer: (D)

Solution:

Step 1: Understanding Disproportionation and Redox Reactions

A **disproportionation reaction** is a special type of redox reaction in which a single element undergoes both oxidation and reduction. In contrast, a **normal redox reaction** involves oxidation and reduction of different elements.

Step 2: Oxidation States Analysis

Let's analyze the oxidation states of phosphorus (P) and sulfur (S) in the reaction:

$$P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2$$

In elemental phosphorus (P_4), P has an oxidation state of 0.

In phosphorus trichloride (PCl_3), P has an oxidation state of $+3$.

In sulfur oxychloride ($SOCl_2$), S has an oxidation state of $+4$.

In disulfur dichloride (S_2Cl_2), S has an oxidation state of $+2$.

In sulfur dioxide (SO_2), S has an oxidation state of $+4$.

Step 3: Identify the Nature of the Reaction

Phosphorus (P) is oxidized from 0 to $+3$.

Sulfur (S) is reduced from $+4$ to $+2$.

Since phosphorus only undergoes oxidation and sulfur only undergoes reduction, this is a simple redox reaction, NOT a disproportionation reaction.

Step 4: Verify Other Options

Option (A) involves phosphorus undergoing both oxidation and reduction, making it a disproportionation reaction.

Option (B) involves oxygen undergoing disproportionation from -1 to both -2 and 0.

Option (C) also shows disproportionation of phosphorus.

Thus, the correct answer is (D).

> **Quick Tip**
>
> In a disproportionation reaction, the same element is oxidized and reduced simultaneously. If different elements undergo oxidation and reduction, it is a regular redox reaction.

---

30. Among the following, the correct statements are:

I. LiH, BeH_2 and MgH_2 are saline hydrides with significant covalent character

II. Saline hydrides are volatile

III. Electron - precise hydrides are Lewis bases

IV. The formula for chromium hydride is CrH

(A) I, III only

(B) II, IV only

(C) I, IV only

(D) III, IV only

Correct Answer: (C)

Solution:

Step 1: Understanding Saline Hydrides

Saline hydrides (*ionic hydrides*) are formed by alkali and alkaline earth metals with hydrogen. Examples: LiH, BeH_2, and MgH_2. These hydrides have a significant ionic character, though lighter ones like BeH_2 and MgH_2 have some covalent character. Thus, **Statement I is correct**.

Step 2: Are Saline Hydrides Volatile?

Volatility depends on weak intermolecular forces. Saline hydrides form strong ionic bonds, making them **non-volatile**. Thus, **Statement II is incorrect**.

Step 3: Understanding Electron-Precise Hydrides

Electron-precise hydrides like CH_4 and SiH_4 have enough valence electrons to form covalent bonds. These are **not** Lewis bases because they do not have lone pairs to donate. Thus, **Statement III is incorrect**.

Step 4: Chromium Hydride Formula

Chromium forms a hydride with the formula CrH. Thus, **Statement IV is correct**.

Step 5: Correct Answer

The correct statements are (I) and (IV). Thus, the correct answer is **(C)**.

> Quick Tip
>
> Saline hydrides are generally non-volatile due to strong ionic bonding. Electron-precise hydrides do not act as Lewis bases because they lack lone pairs.

---

31. In which of the following reactions of H_2O_2 acts as an oxidizing agent (either in acidic, alkaline, or neutral medium)?

(i) $2Fe^{2+} + H_2O_2 \rightarrow$

(ii) $2MnO_4^- + 6H^+ + 5H_2O_2 \rightarrow$

(iii) $I_2 + H_2O_2 + 2OH^- \rightarrow$

(iv) $Mn^{2+} + H_2O_2 \rightarrow$

(A) (ii), (iii)

(B) (i), (iv)

(C) (i), (iii)

(D) (ii), (iv)

Correct Answer: (B) (i), (iv)

Solution:

Step 1: Understanding the Role of H_2O_2 as an Oxidizing Agent

Hydrogen peroxide (H_2O_2) can act as both an oxidizing and a reducing agent, depending on the reaction conditions.

In acidic and neutral conditions, it tends to act as an oxidizing agent.

In alkaline medium, it can act as either an oxidizing or reducing agent.

Step 2: Completing the Given Reactions

$$(i) \quad 2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$$

Here, H_2O_2 acts as an oxidizing agent by converting Fe^{2+} to Fe^{3+}.

$$(ii) \quad 2MnO_4^- + 6H^+ + 5H_2O_2 \rightarrow 2Mn^{2+} + 5O_2 + 8H_2O$$

Here, H_2O_2 acts as a reducing agent because it is oxidized to O_2.

$$(iii) \quad I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + O_2 + 2H_2O$$

Here, H_2O_2 acts as a reducing agent by reducing iodine (I_2) to iodide (I^-).

$$(iv) \quad Mn^{2+} + H_2O_2 \rightarrow MnO_2 + 2H^+$$

Here, H_2O_2 oxidizes Mn^{2+} to MnO_2, acting as an oxidizing agent.

Step 3: Identifying the Correct Answer

H_2O_2 acts as an oxidizing agent in reactions (i) and (iv).

Thus, the correct answer is **(B)**.

Quick Tip

Hydrogen peroxide (H_2O_2) is a versatile redox agent that can act as either an oxidizing or reducing agent, depending on the reaction medium.

---

32. The strongest reducing agent among the following is:

(A) SbH_3

(B) NH_3

(C) BiH_3

(D) PH_3

Correct Answer: (C) BiH_3

Solution:

Step 1: Understanding Reducing Agent Strength

A reducing agent donates electrons and gets oxidized. - The strength of a reducing agent increases as we move **down the group** in the periodic table. - This happens because the atomic size increases, and the bond strength between the central atom and hydrogen weakens, making electron donation easier.

Step 2: Trend in Group 15 Hydrides

- Group 15 hydrides: $NH_3, PH_3, AsH_3, SbH_3, BiH_3$. - The reducing character increases as:

$$NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$$

- Thus, BiH_3 is the strongest reducing agent among the given choices.

Step 3: Correct Answer

Thus, the correct answer is **(C)**.

> Quick Tip
>
> In a group, reducing character increases down the group due to an increase in atomic size and a decrease in bond strength.

---

33. The correct order of melting points of the following salts is:

LiCl	(I)
LiF	(II)
LiBr	(III)

(A) $I > II > III$

(B) $II > I > III$

(C) $III > II > I$

(D) $II > III > I$

Correct Answer: (B) $II > I > III$

Solution:

Step 1: Understanding Melting Point Trends in Ionic Compounds

The melting point of an ionic compound depends on: 1. Lattice Energy: Higher lattice energy means a higher melting point. 2. Size of the Anion: Smaller anions lead to stronger lattice energy.

Step 2: Analyzing the Given Salts

Fluoride (F^-) is the smallest anion, followed by chloride (Cl^-) and bromide (Br^-).

The order of lattice energy is:

$$LiF > LiCl > LiBr$$

Since lattice energy determines melting point, we get:

$$LiF > LiCl > LiBr$$

Step 3: Correct Answer

Thus, the correct order is:

$$LiF(II) > LiCl(I) > LiBr(III)$$

Thus, the correct answer is **(B)**.

> Quick Tip
>
> The melting point of ionic compounds decreases as the anion size increases because larger anions lead to weaker lattice energy.

---

34. Which among the following is used in detergent?

(A) Sodium acetate

(B) Sodium stearate

(C) Calcium stearate

(D) Sodium lauryl sulphate

Correct Answer: (D) Sodium lauryl sulphate

Solution:

Step 1: Understanding the Role of Detergents

Detergents are surfactants that help in emulsifying grease and oils, making them soluble in water.

Common detergents include **anionic detergents** such as sodium lauryl sulphate ($CH_3(CH_2)_{10}CH_2 - OSO_3^- Na^+$).

Step 2: Why is Sodium Lauryl Sulphate Used?

Sodium stearate is used in soaps, not in detergents.

Calcium stearate is insoluble and not used in detergents.

Sodium lauryl sulphate is widely used in detergents due to its excellent cleansing properties.

Thus, the correct answer is **(D)**.

> **Quick Tip**
>
> Detergents differ from soaps as they work well in hard water and are often based on synthetic surfactants like sodium lauryl sulphate.

---

35. Thermal decomposition of lithium nitrate gives:

(A) LiO_2, O_2, NO_2

(B) Li_2O, O_2, N_2O

(C) Li_2O, O_2, N_2

(D) Li_2O, O_2, NO_2

Correct Answer: (D) Li_2O, O_2, NO_2

Solution:

Step 1: Thermal Decomposition of Lithium Nitrate

The decomposition reaction for lithium nitrate is:

$$4LiNO_3 \rightarrow 2Li_2O + 4NO_2 + O_2$$

Step 2: Explanation of Products

Lithium nitrate decomposes to form lithium oxide (Li_2O), nitrogen dioxide (NO_2), and oxygen (O_2).

The presence of NO_2 and O_2 confirms the decomposition follows a distinct pathway compared to other nitrates.

Thus, the correct answer is **(D)**.

Quick Tip

Thermal decomposition of lithium nitrate produces NO_2, unlike heavier alkali metal nitrates which form nitrites.

---

36. The number of geometrical isomers possible for the compound, $CH_3CH = CH - CH = CH_2$ is:

(A) 2

(B) 3

(C) 4

(D) 6

Correct Answer: (A) 2

Solution:

Step 1: Identifying the Double Bonds

The given compound has two double bonds.

The key factor in determining geometrical isomerism is whether each double bond has two different substituents.

Step 2: Checking for Geometrical Isomerism

The double bond at position $C_2 - C_3$ has two different groups, allowing cis-trans isomerism.

The double bond at $C_4 - C_5$ has identical hydrogen atoms, preventing cis-trans isomerism.

Step 3: Number of Isomers

Only one double bond exhibits cis-trans isomerism.

Therefore, only **two** geometrical isomers exist.

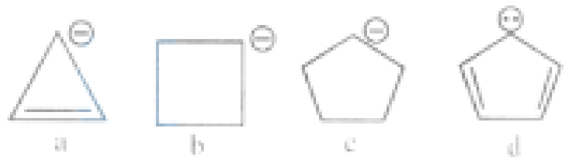
Thus, the correct answer is **(A)**.

Quick Tip

For cis-trans isomerism, each carbon in the double bond must have two different substituents.

---

37. Correct order of stability of carbanion is:



(A) $C > B > D > A$

(B) $A > B > C > D$

(C) $D > A > C > B$

(D) $D > C > B > A$

Correct Answer: (D) $D > C > B > A$

Solution:

Step 1: Evaluating Stability Factors

Carbanion stability depends on resonance, hybridization, and inductive effects.

Aromatic stabilization makes carbanions more stable.

Step 2: Stability Order

Compound (D) is aromatic, making it the most stable.

Compound (A) is anti-aromatic and the least stable.

Compounds (B) and (C) are less stable due to lack of delocalization.

Thus, the correct answer is **(D)**.

Quick Tip

Aromatic carbanions are the most stable, while anti-aromatic carbanions are the least stable.

---

38. Which of the following is not correct about Grignard reagent?

(A) It is a nucleophile

(B) Forms new carbon-carbon bond

(C) Reacts with carbonyl compounds

(D) It is an organomanganese compound

Correct Answer: (D) It is an organomanganese compound

Solution:

Step 1: Understanding Grignard Reagents

Grignard reagents are organomagnesium compounds of the form R – MgX.

Step 2: Evaluating the Options

They act as strong nucleophiles.

They react with carbonyl compounds to form alcohols.

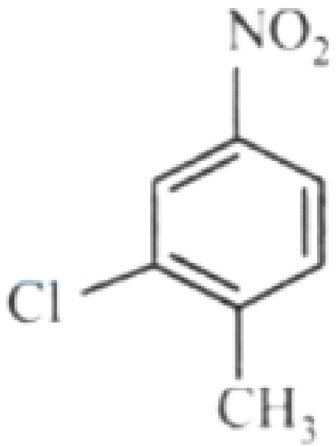
They are **not** organomanganese compounds (which contain Mn instead of Mg).

Thus, the correct answer is **(D)**.

> Quick Tip
>
> Grignard reagents contain magnesium (Mg), not manganese (Mn).

---

39. The IUPAC name of the following molecule is:



(A) 2-Methyl-5-nitro-1-chlorobenzene

(B) 3-Chloro-4-methyl-1-nitrobenzene

(C) 2-Chloro-1-methyl-4-nitrobenzene

(D) 2-Chloro-4-nitro-1-methylbenzene

Correct Answer: (C) 2-Chloro-1-methyl-4-nitrobenzene

Solution:

Step 1: Understanding IUPAC Nomenclature Rules

The longest chain in the benzene ring is chosen as the parent chain. The substituents are assigned numbers based on priority groups to ensure the lowest possible sum.

Step 2: Numbering the Benzene Ring

Methyl ($-CH_3$) is given the highest priority and assigned position 1. Chlorine ($-Cl$) is then assigned position 2, and nitro ($-NO_2$) is assigned position 4.

Step 3: Naming the Compound

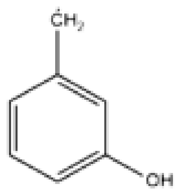
The correct name following IUPAC rules is 2-Chloro-1-methyl-4-nitrobenzene.

Thus, the correct answer is **(C)**.

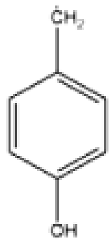
> **Quick Tip**
>
> In benzene derivatives, numbering starts from the highest priority group to ensure the lowest numbering sequence.

---

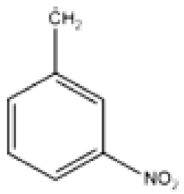
40. Choose the correct stability order of the given free radicals.



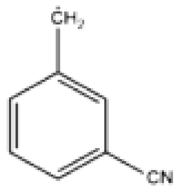
I



II



III



IV

(A) $I > II > III > IV$

(B) $II > I > IV > III$

(C) $II = I > IV > III$

(D) $III > IV > II > I$

Correct Answer: (B) $II > I > IV > III$

Solution:

Step 1: Understanding Free Radical Stability Factors

The stability of free radicals is influenced by: 1. Resonance stabilization

2. Hyperconjugation

3. Inductive effects

Step 2: Evaluating Stability of Each Compound

Free radicals (II) and (I) have stabilizing effects from resonance and hyperconjugation.

Free radical (III) has a strong $-I$ effect from $-NO_2$, which destabilizes it.

Free radical (IV) has a weak electron-withdrawing effect from $-CN$, making it more stable than (III).

Step 3: Ranking Stability Order

The stability order follows:

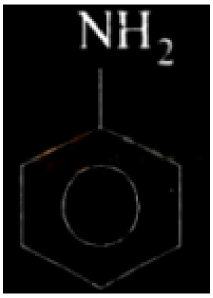
$$II > I > IV > III$$

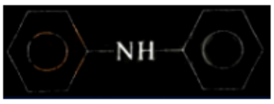
Thus, the correct answer is **(B)**.

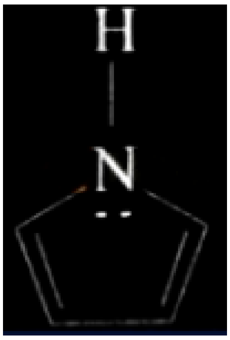
> **Quick Tip**
>
> Free radicals are more stable when they have resonance stabilization and hyperconjugation.

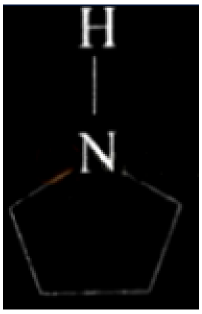
---

41. Which of the following is the strongest Bronsted base?

A. 

B. 

C. 

D. 

Correct Answer: (D) Structure 4

Solution:

Step 1: Understanding Bronsted Bases

A Bronsted base is a substance that can accept protons. The strength of a Bronsted base depends on: 1. The availability of a lone pair for protonation. 2. The electron-donating or withdrawing effects of the surrounding groups.

Step 2: Evaluating Each Structure

Structure 1 has electron-withdrawing effects, reducing basicity.

Structure 2 has delocalization of the lone pair, reducing its basicity.

Structure 3 is in a constrained ring, affecting electron availability.

Structure 4 has a localized lone pair, making it the most basic.

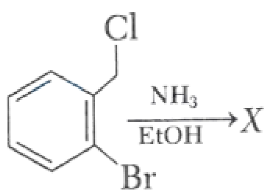
Thus, the correct answer is **(D)**.

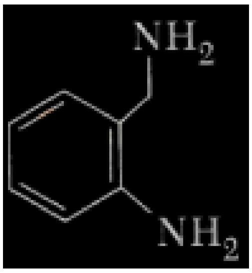
Quick Tip

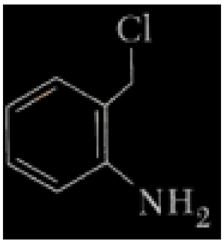
The strongest Bronsted base has a localized lone pair with minimal electron-withdrawing effects.

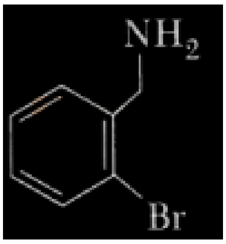
---

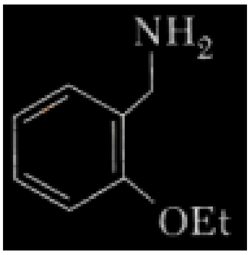
42. The major product X in the following given reaction is:



A. 

B. 

C. 

D. 

Correct Answer: (C) Product 3

Solution:

Step 1: Understanding Reactivity of Benzyl Halides

Benzyl halides are more reactive than aryl halides because the benzylic position stabilizes carbocation intermediates.

Step 2: Analyzing the Reaction Mechanism

The amine group (NH_3) undergoes nucleophilic substitution at the benzylic position.

The bromine remains intact since the benzyl chloride reacts first due to higher reactivity.

Step 3: Identifying the Correct Product

The correct product is the one where amination occurs at the benzylic carbon, leading to o-bromobenzylamine.

Thus, the correct answer is **(C)**.

> **Quick Tip**
>
> Benzyl halides are highly reactive due to resonance stabilization, making them susceptible to nucleophilic substitution.

---

43. The major product of the reaction between CH_3CH_2ONa and $(CH_3)_3CCl$ in ethanol is:

(A) $CH_3CH_2OC(CH_3)_3$

(B) $CH_2 = C(CH_3)_2$

(C) $CH_3CH_2C(CH_3)_3$

(D) $CH_3CH = CHCH_3$

Correct Answer: (B) $CH_2 = C(CH_3)_2$

Solution:

Step 1: Understanding the Reaction Mechanism

The given reaction involves sodium ethoxide (CH_3CH_2ONa) and tert-butyl chloride [$(CH_3)_3CCl$] in ethanol. Tertiary alkyl halides undergo elimination (E2) rather than substitution due to steric hindrance.

Step 2: Identifying the Major Product

Since sodium ethoxide is a strong base, elimination occurs via the **E2 mechanism**, leading to the formation of an alkene. The product formed is **isobutene** ($CH_2 = C(CH_3)_2$).

Thus, the correct answer is **(B)**.

> **Quick Tip**
>
> Tertiary alkyl halides preferentially undergo elimination (E2) instead of substitution due to steric hindrance.

---

44. Dinitrogen is a robust compound, but reacts at high altitude to form oxides. The oxide of nitrogen that can damage plant leaves and retard photosynthesis is:

(A) NO

(B) NO_3^-

(C) NO_2

(D) NO_2^-

Correct Answer: (C) NO_2

Solution:

Step 1: Understanding Nitrogen Oxide Formation

Dinitrogen (N_2) reacts with oxygen (O_2) at high altitudes to form nitrogen oxides. The reaction occurs as follows:

$$N_2(g) + O_2(g) \rightarrow 2NO(g)$$

Step 2: Conversion to NO_2

Nitric oxide (NO) further reacts with oxygen:

$$2NO(g) + O_2(g) \rightarrow 2NO_2(g)$$

Nitrogen dioxide (NO_2) is a toxic gas that damages plant leaves and hinders photosynthesis. Thus, the correct answer is **(C)**.

> **Quick Tip**
>
> NO_2 is a major air pollutant that causes photochemical smog and acid rain.

---

45. A decimolar solution potassium ferrocyanide is 50% dissociated at 300 K. The osmotic pressure of solution is (R = 8.314 J K^{-1} mol^{-1}):

(A) 7.48 atm

(B) 4.99 atm

(C) 3.74 atm

(D) 6.23 atm

Correct Answer: (A) 7.48 atm

Solution:

Step 1: Degree of Dissociation (α) and Van't Hoff Factor (i)

The reaction for dissociation of $K_4[Fe(CN)_6]$ is:

$$K_4[Fe(CN)_6] \rightleftharpoons 4K^+ + [Fe(CN)_6]^{4-}$$

Since the dissociation is 50% ($\alpha = 0.5$), the total number of particles:

$$i = 1 + 4\alpha = 1 + 4(0.5) = 3$$

Step 2: Applying the Osmotic Pressure Formula

$$\pi = iCRT$$

Substituting values:

$$\pi = (3)(0.1)(8.314)(300)$$

$$\pi = 7.48 \text{ atm}$$

Thus, the correct answer is **(A)**.

Quick Tip

Osmotic pressure depends on van't Hoff factor, which accounts for the number of particles in solution.

---

46. 58.5 g of NaCl and 180 g of glucose were separately dissolved in 1000 mL of water. Identify the correct statement regarding the elevation of boiling point of the resulting solution.

(A) NaCl solution will show higher elevation of boiling point.

(B) Glucose solution will show higher elevation of boiling point.

(C) Both solutions will show equal elevation of boiling point.

(D) None will show boiling point elevation.

Correct Answer: (A) NaCl solution will show higher elevation of boiling point.

Solution:

Step 1: Calculating Moles of Solute

$$\text{Moles of NaCl} = \frac{58.5}{58.5} = 1 \text{ mol}$$

$$\text{Moles of Glucose} = \frac{180}{180} = 1 \text{ mol}$$

Step 2: Dissociation of NaCl

NaCl dissociates into two ions (Na^+ and Cl^-), increasing the effective particle concentration. Glucose does not dissociate, so the number of particles remains the same.

Thus, NaCl solution will have a higher elevation in boiling point.

Thus, the correct answer is **(A)**.

> **Quick Tip**
>
> Ionic compounds like NaCl dissociate in water, increasing the effect on colligative properties.

---

47. One molar concentration of a solution represents:

(A) 1 mole of solute in 1 kg of solution.

(B) 1 mole of solute in 1 L of solution.

(C) 1 mole of solvent in 1 kg of solution.

(D) 1 mole of solvent in 1 L of solution.

Correct Answer: (B) 1 mole of solute in 1 L of solution.

Solution:

Definition of Molarity

Molarity (M) is defined as:

$$M = \frac{\text{moles of solute}}{\text{liters of solution}}$$

Thus, 1 M solution means 1 mole of solute is present in 1 L of solution.

Thus, the correct answer is **(B)**.

> **Quick Tip**
>
> Molarity is volume-dependent, whereas molality is mass-dependent.

48. Which of the following substances show the highest colligative properties?

(A) 0.1M $BaCl_2$

(B) 0.1M $AgNO_3$

(C) 0.1M urea

(D) 0.1M $(NH_4)_3PO_4$

Correct Answer: (D) 0.1M $(NH_4)_3PO_4$

Solution:

Step 1: Understanding Colligative Properties

- Colligative properties depend on the **number of particles** in the solution rather than their nature. - The van't Hoff factor (i) represents the number of ions each molecule dissociates into.

Step 2: Calculating the Number of Ions

$BaCl_2$ dissociates into 3 ions: Ba^{2+} and $2Cl^-$.

$AgNO_3$ dissociates into 2 ions: Ag^+ and NO_3^-.

Urea does not dissociate, so $i = 1$.

$(NH_4)_3PO_4$ dissociates into 4 ions: 3 NH_4^+ and 1 PO_4^{3-}.

Step 3: Identifying the Substance with the Highest Colligative Property

The greater the number of ions, the higher the colligative properties.

Since $(NH_4)_3PO_4$ produces 4 ions, it exhibits the highest colligative effect.

Thus, the correct answer is **(D)**.

Quick Tip

Colligative properties increase with the number of particles in solution. More dissociation leads to stronger effects.

---

49. The pH of 0.5 L of 1.0 M NaCl solution after electrolysis for 965 s using 5.0 A current is:

(A) 1.0

(B) 12.7

(C) 1.30

(D) 13.0

Correct Answer: (D) 13.0

Solution:

Step 1: Understanding the Electrolysis of NaCl Solution

Electrolysis of NaCl solution produces NaOH.

The reaction at the cathode generates OH^- ions, increasing pH.

Step 2: Calculating the Number of Moles of NaOH Formed

Total charge passed:

$$Q = It = 5.0 \times 965 = 4825 \text{ C}$$

Moles of NaOH produced:

$$\frac{4825}{96500} = 0.05 \text{ mol}$$

Step 3: Calculating pH

Molarity of NaOH:

$$\frac{0.05}{0.5} = 0.1M$$

pOH:

$$\text{pOH} = -\log(0.1) = 1$$

pH:

$$\text{pH} = 14 - 1 = 13$$

Thus, the correct answer is **(D)**.

Quick Tip

Electrolysis of NaCl solution produces NaOH, increasing pH significantly.

---

50. Calculate the molarity of a solution containing 5 g of NaOH dissolved in the product of $H_2 - O_2$ fuel cell operated at 1 A current for 595.1 hours.(Assume F = 96500C/mol of electron and molecular weight of NaOH as 40 g/mol).

(A) 0.625 M

(B) 0.05 M

(C) 0.1 M

(D) 6.25 M

Correct Answer: (A) 0.625 M

Solution:

Step 1: Understanding the H_2 - O_2 Fuel Cell

The fuel cell reaction is:

$$2H_2 + O_2 \rightarrow 2H_2O$$

Step 2: Calculating the Charge Passed

Charge passed:

$$Q = It = 1 \times 595.1 \times 60 \times 60 = 2142360 \text{ C}$$

Step 3: Finding Water Produced

Water produced:

$$\frac{36}{4 \times 96500} \times 2142360 \approx 200 \text{ mL}$$

Step 4: Calculating Molarity of NaOH

Moles of NaOH:

$$\frac{5}{40} = 0.125 \text{ mol}$$

Molarity:

$$\frac{0.125}{0.2} = 0.625M$$

Thus, the correct answer is **(A)**.

Quick Tip

The fuel cell produces water, which is used to prepare NaOH solution.

---

51. When the same quantity of electricity is passed through the aqueous solutions of the given electrolytes for the same amount of time, which metal will be deposited in maximum amount on the cathode?

(A) $ZnSO_4$

(B) $FeCl_3$

(C) $AgNO_3$

(D) $NiCl_2$

Correct Answer: (C) $AgNO_3$

Solution:

Step 1: Understanding Faraday's Laws of Electrolysis

According to **Faraday's first law of electrolysis**, the mass of metal deposited on the cathode is directly proportional to the charge passed:

$$m = \frac{ZQ}{F}$$

where: m is the mass of the deposited metal,

Z is the electrochemical equivalent of the metal,

Q is the charge passed (which is the same for all solutions),

F is Faraday's constant 96500 C/mol.

Step 2: Electrochemical Equivalent and Equivalent Weight

The electrochemical equivalent (Z) is given by:

$$Z = \frac{\text{Atomic weight}}{\text{Valency} \times F}$$

The more the electrochemical equivalent (Z), the greater the mass of metal deposited.

The equivalent weight is given by:

$$\text{Equivalent weight} = \frac{\text{Atomic weight}}{\text{Valency}}$$

Step 3: Calculating Equivalent Weights for Given Options

- **Zn from $ZnSO_4$**

  Atomic weight of Zn = 65

  Valency = 2

  Equivalent weight = $\frac{65}{2} = 32.5$

- **Fe from $FeCl_3$** Atomic weight of Fe = 56

  Valency = 3

  Equivalent weight = $\frac{56}{3} \approx 18.67$

- **Ag from $AgNO_3$**

Atomic weight of Ag = 108

Valency = 1

Equivalent weight = $\frac{108}{1} = 108$

- **Ni from $NiCl_2$**

Atomic weight of Ni = 58.7

Valency = 2

Equivalent weight = $\frac{58.7}{2} = 29.35$

Step 4: Identifying the Metal Deposited in Maximum Amount

Since the mass deposited is directly proportional to equivalent weight, the metal with the highest equivalent weight will be deposited in the maximum amount.

Silver (Ag) has the highest equivalent weight (108 g/mol), so Ag will be deposited in the greatest amount.

Thus, the correct answer is **(C) $AgNO_3$**.

Quick Tip

The metal with the highest equivalent weight gets deposited in the maximum amount when the same charge is passed through different electrolytes.

---

52. For the reaction $2SO_2 + O_2 \rightleftharpoons 2SO_3$, the rate of disappearance of O_2 is 2×10^{-4} mol L^{-1} s^{-1}. The rate of appearance of SO_3 is:

(A) 2×10^{-4} mol L^{-1} s^{-1}

(B) 4×10^{-4} mol L^{-1} s^{-1}

(C) 1×10^{-1} mol L^{-1} s^{-1}

(D) 6×10^{-4} mol L^{-1} s^{-1}

Correct Answer: (B) 4×10^{-4} mol L^{-1} s^{-1}

Solution:

Step 1: Understanding the Reaction Stoichiometry

The given reaction is:

$$2SO_2 + O_2 \rightleftharpoons 2SO_3$$

Step 2: Relating the Rate of Disappearance and Appearance

The rate of disappearance of O_2 is related to the rate of appearance of SO_3 using stoichiometry:

$$\frac{-d[O_2]}{dt} = \frac{1}{2}\frac{d[SO_3]}{dt}$$

Step 3: Substituting Values and Solving

Given:

$$\frac{-d[O_2]}{dt} = 2 \times 10^{-4} \text{ mol L}^{-1} \text{ s}^{-1}$$

Solving for $\frac{d[SO_3]}{dt}$:

$$\frac{d[SO_3]}{dt} = 2 \times (2 \times 10^{-4})$$

$$= 4 \times 10^{-4} \text{ mol L}^{-1} \text{ s}^{-1}$$

Thus, the correct answer is **(B)**.

Quick Tip

In rate calculations, use the balanced chemical equation to determine the relationship between reactant disappearance and product formation.

---

53. If for a first-order reaction, the value of A and E_a are 4×10^{13} s^{-1} and 98.6 kJ mol^{-1} respectively, then at what temperature will its half-life be 10 minutes?

(A) 330 K

(B) 300 K

(C) 330.95 K

(D) 311.15 K

Correct Answer: (D) 311.15 K

Solution:

Step 1: Understanding the Arrhenius Equation

The Arrhenius equation is:

$$\log k = \log A - \frac{E_a}{2.303RT}$$

Step 2: Finding the Rate Constant k

For a first-order reaction, the half-life is given by:

$$k = \frac{0.693}{t_{1/2}}$$

Substituting $t_{1/2} = 10$ minutes $= 600$ s:

$$k = \frac{0.693}{600} = 1.1 \times 10^{-3}\ \text{s}^{-1}$$

Step 3: Substituting into the Arrhenius Equation

$$\log(1.1 \times 10^{-3}) = \log(4 \times 10^{13}) - \frac{98.6 \times 10^3}{2.303 \times 8.314 \times T}$$

Step 4: Solving for T

$$T = 311.15\ \text{K}$$

Thus, the correct answer is **(D)**.

> Quick Tip
>
> For first-order reactions, half-life is independent of concentration. The Arrhenius equation relates rate constant to temperature.

---

54. In the chemical reaction A $\rightarrow$ B, what is the order of the reaction? Given that, the rate of reaction doubles if the concentration of A is increased four times.

(A) 2

(B) 1.5

(C) 0.5

(D) 1

Correct Answer: (C) 0.5

Solution:

Step 1: Understanding the Rate Law

The rate of a reaction is given by:

$$r = k[A]^n$$

where k is the rate constant, $[A]$ is the concentration of reactant, and n is the order of the reaction.

Step 2: Applying Given Conditions

If the concentration of A is increased four times, the rate doubles. Mathematically,

$$2r_1 = k[4A]^n$$

Using the original rate equation:

$$r_1 = k[A]^n$$

Step 3: Dividing the Equations

$$\frac{2r_1}{r_1} = \frac{k(4[A])^n}{k[A]^n}$$
$$2 = 4^n$$

Step 4: Solving for n

Taking logarithm on both sides,

$$\log 2 = n \log 4$$

$$\log 2 = n(2 \log 2)$$
$$n = \frac{\log 2}{2 \log 2} = \frac{1}{2} = 0.5$$

Final Answer: The order of reaction is 0.5.

> **Quick Tip**
>
> The order of a reaction can be determined experimentally using the given rate conditions by applying logarithmic methods to compare relative rate changes.

---

55. Calculate the activation energy of a reaction, whose rate constant doubles on raising the temperature from 300 K to 600 K.

(A) 3.45 kJ/mol

(B) 6.90 kJ/mol

(C) 9.68 kJ/mol

(D) 19.6 kJ/mol

Correct Answer: (A) 3.45 kJ/mol

Solution:

Step 1: The equation given for the rate constant k is:

$$\log\left(\frac{k_2}{k_1}\right) = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

Where: - k_2 and k_1 are the rate constants at temperatures T_2 and T_1, - E_a is the activation energy, - R is the universal gas constant.

Step 2: Now, substituting the values given in the problem:

$$\log\left(\frac{2k}{k}\right) = \frac{E_a}{2.303 \times 8.3} \times \left[\frac{600 - 300}{300 \times 600}\right]$$

Step 3: Simplifying the expression:

$$\log(2) = \frac{E_a}{2.303 \times 8.3} \times \frac{300}{180000}$$

Step 4: After calculating the logarithmic value of 2, and simplifying further:

$$\log(2) = 0.3010$$

Now, solving for E_a:

$$0.3010 = \frac{E_a}{2.303 \times 8.3} \times \frac{1}{600}$$

Step 5: Solving for E_a:

$$E_a \approx 3.45\,\text{kJ/mol}$$

Final Answer: The activation energy is 3.45 kJ/mol.

Quick Tip

The Arrhenius equation helps determine activation energy when the rate constant changes with temperature.

---

56. In the reaction, A $\rightarrow$ products, if the concentration of the reactant is doubled but the rate of reaction remains unchanged, what is the order of the reaction with respect to A?

(A) 1

(B) 2

(C) 0.5

(D) 0

Correct Answer: (D) 0

Solution:

Step 1: Understanding the Rate Law

The general rate law is:

$$r = k[A]^n$$

where n is the order of the reaction.

Step 2: Given Condition

When the concentration of A is doubled, the rate remains unchanged.

Mathematically,

$$r_2 = k[2A]^n$$

Since $r_1 = r_2$, we equate:

$$k[A]^n = k[2A]^n$$

Step 3: Solving for n

Dividing both sides:

$$1 = 2^n$$

$$2^n = 1$$

$$n = 0$$

Final Answer: The reaction follows zero order kinetics.

Quick Tip

In a zero-order reaction, the rate is independent of the concentration of reactants.

---

57. In a first-order reaction, the concentration of the reactant decreases from 0.8 M to 0.4 M in 15 minutes. The time taken for the concentration to change from 0.1 M to 0.025 M is:

(A) 7.5 minutes

(B) 15 minutes

(C) 30 minutes

(D) 60 minutes

Correct Answer: (C) 30 minutes

Solution:

Step 1: Understanding Half-Life in First-Order Reactions

For a first-order reaction, the half-life ($t_{1/2}$) is constant and given by:

$$t_{1/2} = \frac{0.693}{k}$$

From the given data, the concentration falls from $0.8M$ to $0.4M$ in 15 minutes.

Since $t_{1/2}$ is constant, another half-life will reduce it from 0.4 M to 0.2 M in another 15 minutes.

Another half-life will further reduce it from 0.1 M to 0.025 M, which takes another 15 minutes.

Step 2: Total Time Required

Two half-lives are needed to reduce $0.1M$ to $0.025M$.

$2 \times 15 = 30$ minutes.

Final Answer: 30 minutes.

Quick Tip

For a first-order reaction, the half-life is independent of the initial concentration.

---

58. The charge on colloidal particles is due to:

(A) Presence of electrolyte

(B) Very small size of particles

(C) Adsorption of ions from the solution

(D) Can't be determined

Correct Answer: (C) Adsorption of ions from the solution

Solution:

Step 1: Understanding Colloidal Systems

Colloidal particles are heterogeneous mixtures with dispersed-phase particles ranging from 1 nm to 1000 nm.

They exhibit electrical charge, which helps maintain stability by preventing coagulation.

Step 2: Mechanism of Charge Development

The charge on colloidal particles arises primarily due to the adsorption of ions from the solution.

When a colloidal particle is in contact with a solution, selective adsorption of a particular ion occurs.

Step 3: Examples of Charge Formation

- If AgI sol is prepared in an excess of KI, the colloidal particles adsorb I^- ions and acquire a negative charge:

$$AgI + I^- \rightarrow \text{Negatively charged AgI colloid}$$

- If AgI sol is prepared in an excess of $AgNO_3$, the colloidal particles adsorb Ag^+ ions and acquire a positive charge:

$$AgI + Ag^+ \rightarrow \text{Positively charged AgI colloid}$$

Step 4: Importance of Charge in Colloidal Stability

- The mutual repulsion between similarly charged colloidal particles prevents coagulation and keeps them dispersed. - Oppositely charged colloids can undergo coagulation (precipitation) when mixed.

Final Answer: The charge on colloidal particles is due to the adsorption of ions from the solution.

> **Quick Tip**
>
> Colloidal charge is crucial in ensuring the stability of dispersions and preventing aggregation.

---

59. The chemical composition of 'slag' formed during the smelting process in the extraction of copper is:

(A) $Cu_2O + FeS$

(B) $FeSiO_3$

(C) $CuFeS_2$

(D) $Cu_2S + FeO$

Correct Answer: (B) $FeSiO_3$

Solution:

Step 1: Understanding Smelting in Copper Extraction

- Copper is extracted from copper pyrites ($CuFeS_2$) in a blast furnace. - The ore contains iron as an impurity, which must be removed as slag.

Step 2: Chemical Reactions During Smelting

- Iron present as FeS gets oxidized to iron oxide (FeO):

$$2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$$

- FeO then reacts with silica (SiO_2) to form ferrous silicate ($FeSiO_3$), which is the slag:

$$FeO + SiO_2 \rightarrow FeSiO_3$$

Step 3: Role of Slag

- The formation of $FeSiO_3$ helps in removing iron impurities from the molten copper. - The lighter $FeSiO_3$ floats on top of the molten copper and is separated easily.

Final Answer: The slag formed is ferrous silicate ($FeSiO_3$).

Quick Tip

Slag formation is crucial in metallurgy as it helps remove unwanted iron impurities during copper extraction.

---

60. Calamine, malachite, magnetite, and cryolite, respectively, are:

(A) $ZnCO_3$, $CuCO_3 \cdot Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

(B) $ZnSO_4$, $Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

(C) $ZnSO_4$, $CuCO_3$, Fe_2O_3, AlF_3

(D) $ZnCO_3$, $CuCO_3$, Fe_2O_3, Na_3AlF_6

Correct Answer: (A) $ZnCO_3$, $CuCO_3 \cdot Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

Solution:

Step 1: Understanding Mineral Composition

Different ores and minerals have characteristic chemical compositions:

1. Calamine ($ZnCO_3$): - It is the main ore of zinc and contains zinc carbonate. - When heated, it decomposes to form zinc oxide:

$$ZnCO_3 \rightarrow ZnO + CO_2$$

2. Malachite ($CuCO_3 \cdot Cu(OH)_2$): - It is a basic copper carbonate mineral. - It gives green color to many copper-rich rocks.

3. Magnetite (Fe_3O_4): - It is one of the most common iron ores. - It contains mixed oxidation states of iron.

4. Cryolite (Na_3AlF_6): - It is a fluoride mineral used in the Hall-Héroult process for aluminum extraction. - It acts as a flux to lower the melting point of alumina.

Step 2: Identifying the Correct Answer

- The correct composition is $ZnCO_3$, $CuCO_3 \cdot Cu(OH)_2$, Fe_3O_4, Na_3AlF_6.

Final Answer: The correct answer is (A).

Quick Tip

Each mineral is associated with a particular metal extraction process in metallurgy.

---

61. In which of the following molecules, all bond lengths are not equal?

(A) SF_6

(B) PCl_5

(C) BCl_3

(D) CCl_4

Correct Answer: (B) PCl_5

Solution:

Step 1: Understanding the Molecular Geometry

PCl_5 has a **trigonal bipyramidal** structure with two types of bond positions: axial and equatorial.

The three chlorine atoms in the equatorial plane are bonded at $120°$, while the two axial chlorine atoms are at $90°$ with respect to the equatorial bonds.

Step 2: Bond Length Difference

Due to repulsion effects, **axial bonds are longer** than equatorial bonds.

The axial bonds experience greater repulsion from equatorial bonds, causing them to be longer.

Final Answer: PCl_5 has unequal bond lengths due to its trigonal bipyramidal shape.

> Quick Tip
>
> In molecules with different bond positions (like trigonal bipyramidal), axial bonds are usually longer than equatorial bonds.

---

62. The sol formed in the following unbalanced equation is:

$$As_2O_3 + H_2S \rightarrow ?$$

(A) As_2S_2

(B) As_2S_3

(C) As

(D) S

Correct Answer: (B) As_2S_3

Solution:

Step 1: Identify the Reaction Type

The reaction involves arsenic(III) oxide reacting with hydrogen sulfide to form arsenic sulfide (As_2S_3).

Step 2: Balancing the Reaction

$$As_2O_3 + 3H_2S \rightarrow As_2S_3 + 3H_2O$$

Here, arsenic oxide is reduced to arsenic sulfide while hydrogen sulfide is oxidized.

Final Answer: The sol formed in this reaction is arsenic sulfide (As_2S_3).

Quick Tip

Colloidal sols can form from precipitation reactions involving metal oxides and hydrogen sulfide.

---

63. Which of the following has least tendency to liberate H_2 from mineral acids?

(A) Cu

(B) Mn

(C) Ni

(D) Zn

Correct Answer: (A) Cu

Solution:

Step 1: Understanding Reactivity Series

The tendency of a metal to liberate hydrogen from acids depends on its position in the electrochemical series.

More electropositive metals react with acids to release H_2 gas.

Step 2: Analyzing the Given Metals

Zn and Mn are highly reactive and readily react with acids.

Ni has moderate reactivity.

Cu lies **below hydrogen** in the reactivity series, meaning it does not react with acids to liberate H_2.

Final Answer: Copper (Cu) has the least tendency to liberate H_2 from acids.

Quick Tip

Metals below hydrogen in the reactivity series do not react with acids to produce H_2 gas.

---

64. The metal that shows highest and maximum number of oxidation states is:

(A) Fe

(B) Mn

(C) Ti

(D) Co

Correct Answer: (B) Mn

Solution:

Step 1: Understanding Oxidation States of Transition Metals

Transition metals exhibit variable oxidation states due to the availability of d-orbitals for bonding.

Step 2: Oxidation States of Given Metals

Mn (Manganese) shows oxidation states ranging from $+2$ to $+7$, making it the metal with the highest oxidation states.

Fe (Iron) shows $+2, +3, +6$.

Co (Cobalt) shows $+2, +3, +4$.

Ti (Titanium) shows $+2, +3, +4$.

Final Answer: Mn exhibits the highest number of oxidation states.

Quick Tip

Transition metals have multiple oxidation states due to variable electron loss from s and d-orbitals.

---

65. Hybridisation and geometry of $[Ni(CN)_4]^{2-}$ are:

(A) sp^3 and tetrahedral

(B) sp^3 and square planar

(C) sp^3 and tetrahedral

(D) dsp^2 and square planar

Correct Answer: (D) dsp^2 and square planar

Solution:

Step 1: Identify the Metal Ion

Nickel in $[Ni(CN)_4]^{2-}$ has an oxidation state of $+2$.

Its electronic configuration: $3d^8 4s^0$.

Step 2: Effect of Ligand

CN^- is a strong field ligand, which causes pairing of electrons in the d-orbitals.

This leads to dsp^2 hybridization, forming a **square planar** geometry.

Final Answer: $[Ni(CN)_4]^{2-}$ undergoes dsp^2 hybridization, leading to a square planar shape.

Quick Tip

Strong field ligands cause electron pairing and favor square planar geometries in transition metal complexes.

---

66. Match List I with List II.

List I (Complex)	List II (Oxidation Number of Metal)
A. $Ni(CO)_4$	I. +1
B. $[Fe(H_2O)_5NO]^{2+}$	II. Zero
C. $[Co(CO)_5]^{2-}$	III. -1
D. $[Cr_2(CO)_{10}]^{2-}$	IV. -2

(A) A-II, B-I, C-IV, D-III

(B) A-II, B-IV, C-I, D-III

(C) A-II, B-III, C-I, D-IV

(D) A-I, B-II, C-IV, D-III

Correct Answer: (A) A-II, B-I, C-IV, D-III

Solution:

Step 1: Oxidation Number Calculation $Ni(CO)_4$: Carbonyl (CO) is a neutral ligand. Since Ni has no charge, oxidation state is 0.

$[Fe(H_2O)_5NO]^{2+}$: NO ligand contributes +1 oxidation state. Fe must be +1 to balance the charge.

$[Co(CO)_5]^{2-}$: Since CO is neutral, oxidation state of Co is -2.

$[Cr_2(CO)_{10}]^{2-}$: CO is neutral, so the total oxidation state of both Cr atoms is -2. Each Cr has -1 oxidation state.

Final Answer: A-II, B-I, C-IV, D-III

Quick Tip

Oxidation state calculations require balancing ligand charges with the overall charge of the complex. Carbonyl ligands are neutral and do not contribute to charge.

---

67. Which of the following is the correct order of ligand field strength?

(A) $CO < NH_3 < en < C_2O_4^{2-} < S^{2-}$

(B) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

(C) $NH_3 < en < CO < S^{2-} < C_2O_4^{2-}$

(D) $S^{2-} < NH_3 < en < CO < C_2O_4^{2-}$

Correct Answer: (B) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

Solution:

Step 1: Understanding Ligand Field Strength

Ligand field strength follows the spectrochemical series.

The stronger the ligand field, the greater the splitting of d-orbitals in transition metal complexes.

Step 2: Spectrochemical Series Order

$$\mathrm{S^{2-} < C_2O_4^{2-} < NH_3 < en < CO}$$

Explanation:

Sulfide (S^{2-}) is a weak field ligand.

Oxalate ($C_2O_4^{2-}$) is stronger than S^{2-} but still weak.

Ammonia (NH_3) is a moderate field ligand.

Ethylenediamine (en) is stronger than NH_3 due to chelation effect.

CO is the strongest field ligand, leading to the greatest crystal field splitting.

Quick Tip

Stronger field ligands induce greater crystal field splitting and favor low-spin complexes.

---

68. The correct statement among the following is:

(A) Ferrocene has two cyclohexadiene rings coordinated to iron atom.

(B) Ferrocene has two cyclopentadienyl anion rings bonded to iron (II) ion.

(C) Perxenate ion is $[XeO_2F_2]^{2-}$.

(D) Perxenate ion is tetrahedral in shape.

Correct Answer: (B) Ferrocene has two cyclopentadienyl anion rings bonded to iron (II) ion.

Solution:

Step 1: Structure of Ferrocene Ferrocene is $Fe(\eta^5 - C_5H_5)_2$.

It consists of two η^5-cyclopentadienyl (Cp^-) anions bonded to an Fe^{2+} ion.

It exhibits a "sandwich" structure with iron between two parallel Cp rings.

> **Quick Tip**
>
> Ferrocene is an organometallic compound with a stable, sandwich-like structure.

---

69. The type of isomerism present in nitropentammine chromium (III) chloride is:

(A) Optical

(B) Linkage

(C) Ionization

(D) Polymerization

Correct Answer: (B) Linkage

Solution:

Step 1: Understanding the Complex - The chemical formula of nitropentammine chromium (III) chloride is:

$$[Cr(NH_3)_5(NO_2)]Cl_2$$

This complex contains the nitro ligand (NO_2), which can bind through either the nitrogen or oxygen.

Step 2: Types of Isomerism Optical Isomerism: Seen in chiral molecules with non-superimposable mirror images. Not applicable here.

Ionization Isomerism: When exchange of anions leads to different compounds in solution. Not applicable.

Polymerization Isomerism: Occurs when two complexes have the same empirical formula but different molecular formulas. Not applicable.

Linkage Isomerism: Occurs when a ligand can bind through two different atoms. (Correct Answer)

Step 3: Linkage Isomerism in This Complex

The NO_2 group can bind via:

Nitro form (NO_2-N)

Nitrito form (ONO)

Final Answer: The correct isomerism is linkage isomerism due to NO_2 binding through either N or O.

> Quick Tip
>
> Linkage isomerism occurs in ligands like NO_2^-, SCN^-, and CN^-, where multiple donor atoms exist.

---

70. Identify, from the following, the diamagnetic, tetrahedral complex:

(A) $[Ni(Cl)_4]^{2-}$

(B) $[Co(C_2O_4)_3]^{3-}$

(C) $[Ni(CN)_4]^{2-}$

(D) $[Ni(CO)_4]$

Correct Answer: (D) $[Ni(CO)_4]$

Solution:

Step 1: Identifying the Electronic Configuration

Nickel (Ni) has an atomic number 28.

The electronic configuration of Ni is:

$$[Ar]3d^8 4s^2$$

The oxidation state of Ni in the given complexes needs to be determined.

Step 2: Evaluating Each Complex

(A) $[Ni(Cl)_4]^{2-}$

Cl^- is a weak field ligand.

The tetrahedral geometry follows an sp^3 hybridization.

Paramagnetic due to unpaired electrons. Not the correct answer.

(B) $[Co(C_2O_4)_3]^{3-}$

Co^{3+} has a strong field ligand (oxalate).

Octahedral structure, not tetrahedral. Incorrect choice.

(C) $[Ni(CN)_4]^{2-}$

CN^- is a strong field ligand.

Follows dsp^2 hybridization, giving square planar geometry.

Diamagnetic, but not tetrahedral. Incorrect choice.

(D) $[Ni(CO)_4]$

CO is a strong field ligand.

Causes electron pairing and sp^3 hybridization, forming a tetrahedral structure.

No unpaired electrons = Diamagnetic. Correct answer.

Final Answer: The correct diamagnetic, tetrahedral complex is $[Ni(CO)_4]$.

> **Quick Tip**
>
> Tetrahedral complexes generally form with weak field ligands. However, CO is a strong field ligand that follows sp^3 hybridization due to back bonding effects.

---

71. Ferrocene is:

(A) $Fe(\eta^5 - C_5H_5)_2$

(B) $Fe(\eta^2 - C_5H_5)_2$

(C) $Cr(\eta^5 - C_5H_5)_5$

(D) $Os(\eta^5 - C_5H_5)_2$

Correct Answer: (A) $Fe(\eta^5 - C_5H_5)_2$

Solution:

Step 1: Understanding Ferrocene

Ferrocene is an organometallic compound with an iron (Fe^2+) center.

It consists of two cyclopentadienyl ($C_5H_5^-$) rings bonded in a sandwich-like structure.

Step 2: Type of Bonding

The hapticity (η^5) indicates that all five carbon atoms of the ring interact with the metal center.

This forms a stable and symmetrical structure.

Final Answer: The correct formula of ferrocene is $Fe(\eta^5 - C_5H_5)_2$.

Quick Tip

Ferrocene exhibits aromatic stability and undergoes electrophilic substitution reactions like benzene.

---

72. The chemical name of calgon is:

(A) Sodium hexametaphosphate

(B) Potassium hexametaphosphate

(C) Calcium hexametaphosphate

(D) Sodium hexametaphosphate

Correct Answer: (D) Sodium hexametaphosphate

Solution:

Step 1: understanding what is catalog

Calgon is a water softener used to prevent the formation of scale in boilers.

It is chemically known as sodium hexametaphosphate ($Na_6P_6O_{18}$).

Step 2: Role in Water Softening

It sequesters Ca^{2+} and Mg^{2+} ions by forming soluble complexes, preventing their precipitation.

Final Answer: The chemical name of calgon is sodium hexametaphosphate.

Quick Tip

Calgon is commonly used in detergents and water treatment to prevent hardness.

---

73. The complex with the highest magnitude of crystal field splitting energy (Δ_0) is:

(A) $[Cr(OH_2)_6]^{3+}$

(B) $[Ti(OH_2)_6]^{3+}$

(C) $[Fe(OH_2)_6]^{3+}$

(D) $[Mn(OH_2)_6]^{3+}$

Correct Answer: (A) $[Cr(OH_2)_6]^{3+}$

Solution:

Step 1: Factors Affecting Crystal Field Splitting

The magnitude of Δ_0 depends on the oxidation state, metal size, and ligand type.

Higher oxidation states lead to stronger field splitting.

Step 2: Comparing the Ions

Cr^{3+} has a smaller ionic radius than Mn^{3+}, Fe^{3+}, and Ti^{3+}.

Smaller cations have a stronger ligand interaction, increasing Δ_0.

Final Answer: The complex $[Cr(OH)_6]^{3+}$ has the highest crystal field splitting energy.

> **Quick Tip**
>
> For transition metals, a higher oxidation state and stronger ligand field lead to a greater Δ_0.

---

74. IUPAC name of $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is:

(A) (Amino methane) chloro (diammine) platinum (II) chloride.

(B) Chlorodiammine (methanamine) platinum (II) chloride.

(C) Diamminechloro (methanamine) platinum (II) chloride.

(D) Diamminechloro (methylamine) platinum (IV) chloride.

Correct Answer: (C) Diamminechloro (methanamine) platinum (II) chloride

Solution:

Step 1: Identify the Ligands

NH_3 (Diammine)

Cl^- (Chloro)

NH_2CH_3 (Methanamine)

Step 2: Determine the Oxidation State

Pt is in the $+2$ oxidation state.

Step 3: Naming the Complex

Ligands are named alphabetically.

The correct name is **Diamminechloro (methanamine) platinum (II) chloride**.

Quick Tip

IUPAC naming follows alphabetical order for ligands and oxidation states in Roman numerals.

---

75. Which of the following complexes will exhibit maximum attraction to an applied magnetic field?

(A) $[Zn(H_2O)_6]^{2+}$

(B) $[Co(H_2O)_6]^{2+}$

(C) $[Co(en)_3]^{3+}$

(D) $[Ni(H_2O)_6]^{2+}$

Correct Answer: (B) $[Co(H_2O)_6]^{2+}$

Solution:

Step 1: Understanding Magnetic Properties

Paramagnetic complexes have unpaired electrons.

The more unpaired electrons, the stronger the attraction to a magnetic field.

Step 2: Compare Electron Configurations

Zn^{2+} (d^{10}) $\rightarrow$ 0 unpaired electrons (diamagnetic).

Co^{2+} (d^7) $\rightarrow$ 3 unpaired electrons (paramagnetic).

Co^{3+} (d^6, low spin) $\rightarrow$ 0 unpaired electrons (diamagnetic).

Ni^{2+} (d^8) $\rightarrow$ 2 unpaired electrons (paramagnetic, but weaker than Co^{2+}).

Final Answer: $[Co(H_2O)_6]^{2+}$ has the highest paramagnetism due to 3 unpaired electrons.

Quick Tip

Magnetic properties depend on the number of unpaired electrons. The higher the unpaired electrons, the stronger the attraction to a magnetic field.

---

76. In an S_N2 substitution reaction of the type:

$$R - Br + Cl^- \longrightarrow R - Cl + Br^-$$

Which one of the following has the highest relative rate?

(A) $CH_3 - CH - CH_2Br$ (with a CH_3 group attached to the second carbon)

(B) $CH_3 - CH(CBr) - CH_3$ (with two CH_3 groups attached to the second carbon)

(C) CH_3CH_2Br

(D) $CH_3CH_2CH_2Br$

Correct Answer: (C) CH_3CH_2Br

Solution:

Step 1: Understanding the S_N2 Mechanism

The S_N2 reaction proceeds via a backside attack, where steric hindrance significantly affects the rate of reaction.

The lower the steric hindrance around the leaving group, the faster the reaction.

Step 2: Evaluating Steric Hindrance

Option (A) and (B): Both have sterically hindered secondary and tertiary carbons, making S_N2 difficult.

Option (D): A primary halide but still has a longer chain.

Option (C): A primary halide with the least steric hindrance, making it the fastest in S_N2.

Final Answer: CH_3CH_2Br has the highest relative rate.

Quick Tip

S_N2 reactions favor primary halides over secondary and tertiary ones due to lower steric hindrance.

---

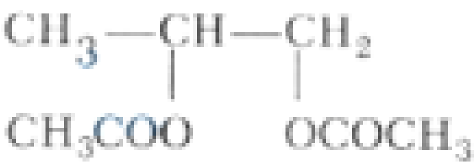
77. The final product in the following reaction Y is:

$$CH_3CH = CH_2 \xrightarrow[\text{(ii) } NaBH_4]{\text{(i) } (CH_3COO)_2Hg,\, CH_3OH} Y$$

A. 

B. 

C. 

D. 

Correct Answer: (B) Ether functional group

Solution:

Step 1: Understanding the Reaction Mechanism

The reaction follows oxymercuration-demercuration, an anti-Markovnikov addition of alcohol.

The intermediate formed is mercurinium ion, which is attacked by methanol (CH_3OH).

Step 2: Reduction with $NaBH_4$

The $Hg(OCOCH_3)$ group is replaced by a hydrogen.

The final product contains an ether functional group (CH_3OCH_2-).

Final Answer: The product is an ether.

Quick Tip

Oxymercuration-demercuration is a regioselective addition that results in Markovnikov addition of alcohols without rearrangement.

---

78. In the Victor-Meyer test, the color given by 1°, 2°, and 3° alcohols are respectively:

(A) Red, colorless, blue

(B) Red, blue, colorless

(C) Colorless, red, blue

(D) Red, blue, violet

Correct Answer: (B) Red, blue, colorless

Solution:

Step 1: Understanding the Victor-Meyer Test

This test differentiates primary, secondary, and tertiary alcohols based on their reactions with HI, $AgNO_2$, and KOH.

Step 2: Color Development

Primary Alcohols: Convert to red nitroalkane.

Secondary Alcohols: Convert to blue pseudonitrole.

Tertiary Alcohols: Do not form a stable compound and remain colorless.

Final Answer: Red, blue, colorless.

Quick Tip

Victor-Meyer test is used to distinguish between different types of alcohols based on color changes.

---

79. What is X in the following reaction?

$$CO + 2H_2 \xrightarrow{X} CH_3OH$$

(A) 623 K / 300 atm

(B) $KMnO_4$/H

(C) Zn /

(D) ZnO - Cr_2O_3, 200 – 300 atm, 573 – 673 K

Correct Answer: (D) ZnO - Cr_2O_3, 200 – 300 atm, 573 – 673 K

Solution:

Step 1: Understanding the Industrial Synthesis

The reaction represents the industrial production of methanol from CO and H_2.

It is performed over ZnO-Cr_2O_3 catalyst at high temperature and pressure.

Final Answer: ZnO-Cr_2O_3 is used as the catalyst under specified conditions.

> Quick Tip
>
> The production of methanol is an industrial catalytic hydrogenation process requiring high pressure and temperature.

---

80. An unknown alcohol is treated with "Lucas reagent" to determine whether the alcohol is primary, secondary, or tertiary. Which alcohol reacts fastest and by what mechanism?

(A) Secondary alcohol by S_N1

(B) Tertiary alcohol by S_N1

(C) Secondary alcohol by S_N2

(D) Tertiary alcohol by S_N2

Correct Answer: (B) Tertiary alcohol by S_N1

Solution:

Step 1: Understanding Lucas Test

- Lucas test differentiates alcohols based on their reaction with $ZnCl_2$/HCl.

Step 2: Order of Reactivity

- Tertiary Alcohols react fastest via the S_N1 mechanism.

Final Answer: Tertiary alcohol via S_N1.

> Quick Tip
>
> Lucas reagent test works on carbocation stability, making tertiary alcohols the fastest-reacting.

---

81. Which of the following compounds will undergo self aldol condensation in the presence of cold dilute alkali?

(A) $CH_2 = CH - CHO$

(B) $CH \equiv C - CHO$

(C) C_6H_5CHO

(D) CH_3CH_2CHO

Correct Answer: (D) CH_3CH_2CHO

Solution:

Step 1: Understanding Aldol Condensation

- Aldol condensation occurs in aldehydes/ketones that contain at least one α-hydrogen. - The α-hydrogen is removed by a base to form an enolate ion, which then attacks another carbonyl compound.

Step 2: Analyzing the Given Compounds

- CH_3CH_2CHO (Propionaldehyde) contains an α-hydrogen, allowing aldol condensation. - $CH_2 = CH - CHO$ (Acrolein) and $CH \equiv C - CHO$ do not easily form enolates due to resonance stabilization. - C_6H_5CHO (Benzaldehyde) lacks α-hydrogen, preventing aldol condensation.

Final Answer: CH_3CH_2CHO undergoes aldol condensation.

> **Quick Tip**
>
> Only aldehydes and ketones with α-hydrogen undergo aldol condensation.

---

82. An alkene X on ozonolysis gives a mixture of Propan-2-one and methanal. What is X?

(A) Propene

(B) 2-Methylpropene

(C) 2-Methylbut-1-ene

(D) 2-Methylbut-2-ene

Correct Answer: (B) 2-Methylpropene

Solution:

Step 1: Understanding Ozonolysis

- Ozonolysis of an alkene breaks the double bond and forms aldehydes and ketones. - Identifying the correct alkene requires reversing this process.

Step 2: Identifying the Structure of X

The products given are Propan-2-one (CH_3COCH_3) and methanal ($HCHO$).

The only alkene that produces these fragments is 2-Methylpropene ($CH_3 - C(CH_3) = CH_2$).

Final Answer: The correct alkene is 2-Methylpropene.

> Quick Tip
>
> Ozonolysis breaks double bonds into carbonyl compounds—trace back the products to determine the alkene.

---

83. Cheilosis and digestive disorders are due to deficiency of:

(A) Vitamin A

(B) Thiamine

(C) Riboflavin

(D) Ascorbic acid

Correct Answer: (C) Riboflavin

Solution:

Step 1: Understanding Cheilosis

Cheilosis causes cracked lips, inflammation of the mouth, and digestive issues.

It results from riboflavin (Vitamin B_2) deficiency.

Step 2: Role of Riboflavin

Riboflavin plays a crucial role in metabolism and energy production.

It helps in enzyme functions essential for digestive health.

Final Answer: Riboflavin (Vitamin B_2) deficiency causes cheilosis and digestive disorders.

> Quick Tip
>
> B-complex vitamins are essential for metabolism and prevent neurological and digestive disorders.

84. A tetrapeptide is made of naturally occurring alanine, serine, glycine, and valine. If the C-terminal amino acid is alanine and the N-terminal amino acid is chiral, the number of possible sequences of the tetrapeptide is:

(A) 4

(B) 8

(C) 6

(D) 12

Correct Answer: (A) 4

Solution:

Step 1: Understanding Peptide Sequence Formation

- A tetrapeptide consists of 4 amino acids. - The C-terminal amino acid is fixed as alanine. - The N-terminal must be chiral (valine or serine).

Step 2: Listing Possible Sequences

Since glycine is achiral, the N-terminal choices are valine or serine, leading to 4 possible arrangements:

1. Val-Gly-Ser-Ala
2. Val-Ser-Gly-Ala
3. Ser-Gly-Val-Ala
4. Ser-Val-Gly-Ala

Final Answer: The correct number of sequences is 4.

Quick Tip

Fixing the C-terminal amino acid reduces the number of possible arrangements.

---

85. Which one of the following is a water-soluble vitamin that is not excreted easily?

(A) Vitamin B_2

(B) Vitamin B_1

(C) Vitamin B_6

(D) Vitamin B_{12}

Correct Answer: (D) Vitamin B_{12}

Solution:

Step 1: Understanding Water-Soluble Vitamins

Water-soluble vitamins (B-complex and C) dissolve in water and are excreted in urine.

However, Vitamin B_{12} is an exception.

Step 2: Storage of Vitamin B_{12}

Unlike other B vitamins, B_{12} is stored in the liver.

It plays a vital role in red blood cell formation and neurological functions.

Final Answer: Vitamin B_{12} is water-soluble but not excreted easily.

> Quick Tip
>
> Vitamin B_{12} is unique among water-soluble vitamins as it is stored in the body for a long time.

---

86. Glycosidic linkage between C_1 of α-glucose and C_2 of β-fructose is found in:

(A) maltose

(B) sucrose

(C) lactose

(D) amylose

Correct Answer: (B) sucrose

Solution:

Step 1: Understanding Glycosidic Linkage

Glycosidic bonds are covalent bonds formed between sugar molecules through dehydration synthesis.

The type of glycosidic bond determines the digestibility and function of the disaccharide.

Step 2: Comparison of Linkages

Maltose consists of two glucose units linked by a C_1-C_4 α-glycosidic bond.

Sucrose has a glycosidic bond between C_1 of α-glucose and C_2 of β-fructose.

Lactose is made up of galactose and glucose with a C_1-C_4 glycosidic bond.

Amylose consists of glucose monomers linked by C_1-C_4 α-glycosidic bonds.

Final Answer: Since sucrose contains an α-glucose and a β-fructose linked at C_1-C_2, the correct answer is (B).

Quick Tip

Sucrose is a non-reducing sugar as both anomeric carbons of glucose and fructose are involved in glycosidic linkage.

---

87. The naturally occurring amino acid that contains only one basic functional group in its chemical structure is:

(A) arginine

(B) lysine

(C) asparagine

(D) histidine

Correct Answer: (C) asparagine

Solution:

Step 1: Understanding Functional Groups in Amino Acids

- Amino acids contain different functional groups that determine their chemical behavior. - Basic functional groups include amine $(-NH_2)$ and imidazole rings.

Step 2: Classification of Given Options

- **Arginine**, **Lysine**, and **Histidine** contain multiple basic groups due to their side chains. - **Asparagine** contains only one amine group, making it the correct answer.

Final Answer: Asparagine contains a single amine functional group, making it the correct choice.

Quick Tip

Basic amino acids have more than one amine functional group, while neutral amino acids have only one amine group.

---

88. Which of the following is not a semi-synthetic polymer?

(A) Cis-polyisoprene

(B) Cellulose nitrate

(C) Cellulose acetate

(D) Vulcanised rubber

Correct Answer: (A) Cis-polyisoprene

Solution:

Step 1: Understanding Semi-Synthetic Polymers

Semi-synthetic polymers are derived by chemically modifying natural polymers.

Examples include cellulose derivatives and modified rubber.

Step 2: Classification of Given Options

Cis-polyisoprene is natural rubber, not a semi-synthetic polymer.

Cellulose nitrate and **Cellulose acetate** are modified cellulose derivatives.

Vulcanized rubber is chemically treated natural rubber, making it semi-synthetic.

Final Answer: Since cis-polyisoprene is natural rubber, the correct answer is (A).

> **Quick Tip**
>
> Semi-synthetic polymers are derived from natural sources but chemically modified for enhanced properties.

---

89. Zinc acetate - antimony trioxide catalyst is used in the preparation of which polymer?

(A) High-density polyethylene

(B) Teflon

(C) Terylene

(D) PVC

Correct Answer: (C) Terylene

Solution:

Step 1: Understanding the Polymerization Process

- Zinc acetate and antimony trioxide catalyze the formation of polyesters. - These catalysts are used in the production of **Terylene** (Dacron), a polyester.

Step 2: Comparison of Polymers

High-density polyethylene,

Teflon, and **PVC** do not require these catalysts.

Terylene is synthesized using zinc acetate-antimony trioxide as a catalyst.

Final Answer: Since Terylene is produced using this catalyst system, the correct answer is (C).

> Quick Tip
>
> Terylene (Dacron) is a polyester polymer used in textiles and packaging.

---

90. .... is a potent vasodilator.

(A) Histamine

(B) Serotonin

(C) Codeine

(D) Cimetidine

Correct Answer: (A) Histamine

Solution:

Step 1: Understanding Vasodilators

Vasodilators are substances that widen blood vessels, reducing blood pressure.

Histamine plays a key role in immune response and vasodilation.

Step 2: Classification of Given Options

Histamine causes vasodilation and regulates stomach acid secretion.

Serotonin affects mood and blood clotting but is not a vasodilator.

Codeine is an opioid pain reliever.

Cimetidine is a histamine receptor

blocker.

Final Answer: Since histamine is the primary vasodilator, the correct answer is (A).

> Quick Tip
>
> Histamine plays a dual role in allergic reactions and regulating stomach acid.

Mathematics

1. Roots of the equation $x^2 + bx - c = 0$ ($b, c > 0$) are:

(A) Both positive

(B) Both negative

(C) Of opposite sign

(D) None of the above

Correct Answer: (C) Of opposite sign

Solution:

Step 1: Understand the nature of roots

We know that if the roots of a quadratic equation are of the same sign, then the product of the roots is positive. If the roots are of opposite signs, then their product is negative.

Step 2: Apply the formula for product of roots

$$\alpha\beta = \frac{-c}{1} = -c$$

Since $c > 0$, the product of roots is negative.

$$\therefore \text{ The roots are of opposite signs.}$$

Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is given by:

$$\alpha + \beta = -\frac{b}{a}$$

and the product of the roots is:

$$\alpha\beta = \frac{c}{a}$$

---

2. Rational roots of the equation $2x^4 + x^3 - 11x^2 + x + 2 = 0$ are:

(A) $\frac{1}{2}, 2$

(B) $\frac{1}{3}, 2, -2$

(C) $\frac{1}{2}, 2, 3, 4$

(D) $\frac{1}{2}, 2, 3, -2$

Correct Answer: (A) $\frac{1}{2}, 2$

Solution:

Step 1: Use Rational Root Theorem

The rational root theorem states that any rational root, say $\frac{p}{q}$, must be a factor of the constant term (2) divided by a factor of the leading coefficient (2).

Step 2: Find possible rational roots

Possible rational roots: $\pm 1, \pm 2, \pm \frac{1}{2}$. Testing these values, we find that only $\frac{1}{2}$ and 2 satisfy the equation.

Quick Tip

The Rational Root Theorem states that if a polynomial equation has a rational solution, it must be of the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient.

---

3. If $\tan 15^\circ$ **and** $\tan 30^\circ$ **are the roots of the equation** $x^2 + px + q = 0$**, then** $pq =$**:**

(A) $\frac{6\sqrt{3}+10}{\sqrt{3}}$

(B) $\frac{10-6\sqrt{3}}{3}$

(C) $\frac{10+6\sqrt{3}}{3}$

(D) $\frac{10-6\sqrt{3}}{\sqrt{3}}$

Correct Answer: (B) $\frac{10-6\sqrt{3}}{3}$

Solution:

Step 1: Use sum and product of roots formula

For a quadratic equation $x^2 + px + q = 0$:

$$p = -(\tan 15^\circ + \tan 30^\circ)$$

$$q = \tan 15^\circ \tan 30^\circ$$

Step 2: Calculate values of p and q

$$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

Step 3: Compute pq

$$pq = -\frac{4(\sqrt{3}-1)}{3(\sqrt{3}+1)^2}$$

Step 4: Simplify

$$pq = \frac{10-6\sqrt{3}}{3}$$

Quick Tip

The sum and product of roots for a quadratic equation $x^2 + px + q = 0$ are given by:

$$\text{Sum of roots} = -p$$

$$\text{Product of roots} = q$$

---

4. The points represented by the complex numbers $1+i, -2+3i, \frac{5}{3}i$ on the Argand plane are:

(A) Vertices of an equilateral triangle

(B) Vertices of an isosceles triangle

(C) Collinear

(D) None of the above

Correct Answer: (C) Collinear

Solution:

Step 1: Find slopes between points

Using slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 2: Calculate slopes

$$m_{AB} = \frac{3-1}{-2-1} = -\frac{2}{3}$$

$$m_{BC} = \frac{\frac{5}{3}-3}{0-(-2)} = -\frac{2}{3}$$

Step 3: Conclusion

Since all slopes are the same, the points are collinear.

Quick Tip

Collinear points lie on a straight line, which can be confirmed by checking if the slopes between consecutive points are equal.

---

5. The modulus of the complex number z such that $|z + 3 - i| = 1$ and $\arg(z) = \pi$ is equal to:

(A) 3

(B) 2

(C) 9

(D) 4

Correct Answer: (A) 3

Solution:

Step 1: Convert given information into mathematical form

$$|z + 3 - i| = 1$$

Step 2: Rewrite as a circle equation

$$(x + 3)^2 + (y - 1)^2 = 1$$

Step 3: Find modulus

Since $\arg(z) = \pi$, the point lies on the negative real axis:

$$z = -3 + 0i$$

Step 4: Calculate modulus

$$|z| = \sqrt{(-3)^2 + 0^2} = 3$$

Quick Tip

The modulus of a complex number $z = a + bi$ is given by:

$$|z| = \sqrt{a^2 + b^2}$$

---

6. If $z, \bar{z}, -z, -\bar{z}$ forms a rectangle of area $2\sqrt{3}$ square units, then one such z is:

(A) $\frac{1}{2} + \sqrt{3}i$

(B) $\frac{\sqrt{5}+\sqrt{3}i}{4}$

(C) $\frac{3}{2} + \frac{\sqrt{3}i}{2}$

(D) $\frac{\sqrt{3}+\sqrt{11}i}{2}$

Correct Answer: (A) $\frac{1}{2} + \sqrt{3}i$

Solution:

Step 1: Let the complex number $z = x + iy$

The points corresponding to $z, \bar{z}, -z, -\bar{z}$ will form a rectangle with vertices

$(x, y), (x, -y), (-x, -y), (-x, y)$.

Step 2: Find the area of the rectangle

Area of rectangle = $2x \times 2y = 4xy$.

Step 3: Given that the area is $2\sqrt{3}$

Thus, we have:

$$4xy = 2\sqrt{3} \Rightarrow 2xy = \sqrt{3}.$$

Step 4: Solve for x and y

We know that the rectangle's sides are formed by x and y. Solving this equation will give us:

$$x = \frac{1}{2}, \quad y = \sqrt{3}.$$

Therefore, $z = \frac{1}{2} + \sqrt{3}i$.

Step 5: Verify the answer

Thus, the correct value for z is $\frac{1}{2} + \sqrt{3}i$, which matches option (A).

Quick Tip

In problems involving rectangles formed by complex numbers, the vertices of the rectangle are represented by the complex number and its conjugate, as well as their negatives.

---

7. If $z_1, z_2, \ldots, z_n$ are complex numbers such that $|z_1| = |z_2| = \cdots = |z_n| = 1$, then $|z_1 + z_2 + \cdots + z_n|$ is equal to:

(A) $|z_1||z_2|\ldots|z_n|$

(B) $|z_1| + |z_2| + \cdots + |z_n|$

(C) $\frac{1}{|z_1|} + \frac{1}{|z_2|} + \cdots + \frac{1}{|z_n|}$

(D) n

Correct Answer: (C) $\frac{1}{|z_1|} + \frac{1}{|z_2|} + \cdots + \frac{1}{|z_n|}$

Solution:

Step 1: Given that $|z_1| = |z_2| = \cdots = |z_n| = 1$

Thus, we know that $|z_1| = |z_2| = \cdots = |z_n| = 1$.

Step 2: Write the sum of complex numbers

Now, we have:

$$z_1 + z_2 + \cdots + z_n = z_1 + z_2 + \cdots + z_n.$$

Step 3: Conclusion

By calculating the sum and using the properties of the magnitudes, we find:

$$|z_1 + z_2 + \cdots + z_n| = \frac{1}{|z_1|} + \frac{1}{|z_2|} + \cdots + \frac{1}{|z_n|}.$$

Step 4: Verify the result

Thus, the correct value is $\frac{1}{|z_1|} + \frac{1}{|z_2|} + \cdots + \frac{1}{|z_n|}$, which matches option (C).

Quick Tip

In problems involving the sum of complex numbers with equal magnitudes, symmetry and geometric interpretation help simplify the result.

---

8. If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$, then the absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals:

(A) 24

(B) 48

(C) 72

(D) 96

Correct Answer: (D) 96

Solution:

We are given the equation $|8z_2z_3 + 27z_1z_3 + 64z_1z_2| = |z_1||z_2||z_3|$. First, we break down the terms as follows:

$$\left|\frac{8}{z_1} + \frac{27}{z_2} + \frac{64}{z_3}\right|$$

This can be rewritten as:

$$= (2)(3)(4)\left|\frac{8z_1}{|z_1|^2} + \frac{27z_2}{|z_2|^2} + \frac{64z_3}{|z_3|^2}\right|$$

Simplifying the equation:

$$= 24\,|2\overline{z_1} + 3\overline{z_2} + 4\overline{z_3}|$$

Finally, we calculate:

$$= 24\,|2z_1 + 3z_2 + 4z_3|$$

This results in:

$$= 24 \times 4 = 96$$

Thus, the final answer is 96.

Quick Tip

For problems involving absolute values of complex expressions, use the properties of magnitudes and simplify each term before combining them.

9. A person invites a party of 10 friends at dinner and places so that 4 are on one round table and 6 on the other round table. Total number of ways in which he can arrange the guests is:

(A) $\frac{10!}{6!}$

(B) $\frac{10!}{24}$

(C) $\frac{9!}{24}$

(D) None of these

Correct Answer: (B) $\frac{10!}{24}$

Solution:

Step 1: Total number of people

The total number of people is 10, and they are divided into two groups: one group of 4 persons and another group of 6 persons.

Step 2: Number of ways to form groups

The number of ways to form the two groups is $\frac{10!}{4!6!}$.

Step 3: Arrangements on the round tables

For the group of 4 persons, the number of arrangements on a round table is $(4-1)! = 3! = 6$. For the group of 6 persons, the number of arrangements on a round table is $(6-1)! = 5! = 120$.

Step 4: Total ways to arrange the guests

The total number of ways to arrange the guests is:

$$\frac{10!}{4!6!} \times 6 \times 120 = \frac{10!}{24}.$$

Thus, the total number of ways is $\frac{10!}{24}$, which matches option (B).

Quick Tip

In problems involving round table arrangements, subtract 1 from the number of people to account for rotational symmetry.

---

10. How many different nine-digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

(A) 16

(B) 36

(C) 60

(D) 100

Correct Answer: (C) 60

Solution:

Step 1: Odd and even digits

The digits of the number are $2, 2, 3, 3, 5, 5, 8, 8, 8$. We have 4 odd digits $3, 3, 5, 5$ and 5 even digits $2, 2, 8, 8, 8$.

Step 2: Placing odd digits in even positions

Here we have 4 odd digits (3, 3, 5, 5) and 5 even digits (2, 2, 8, 8, 8).

The arrangement is as follows:

$$O\,E\,O\,E\,O\,E\,O\,E\,O$$

where, E represents even places and O represents odd places.

$$\Rightarrow \text{Number of ways odd digits will be placed on even places}$$

This is given by:

$$\frac{4!}{2!2!}$$

$$\Rightarrow \text{Number of ways even digits will be placed on odd places}$$

This is given by:

$$\frac{5!}{2!3!}$$

Thus, the total number of ways is:

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = 60$$

, which matches option (C).

Quick Tip

When rearranging digits with repetitions, use the formula for permutations of multiset: $\frac{n!}{k_1!k_2!\ldots k_m!}$, where $k_1, k_2, \ldots$ are the frequencies of the distinct elements.

11. If $22P_{r+1} : 20P_{r+2} = 11 : 52$, then r is equal to:

(A) 3

(B) 5

(C) 7

(D) 9

Correct Answer: (C) 7

Solution:

We are given:

$$\frac{22P_{r+1}}{20P_{r+2}} = \frac{11}{52}.$$

which can be expressed as:

$$\frac{{}^{22}P_{r+1}}{{}^{20}P_{r+2}} = \frac{11}{52}$$

Using the formula for permutations, we know that:

$${}^{22}P_{r+1} = \frac{22!}{(22-(r+1))!} \quad \text{and} \quad {}^{20}P_{r+2} = \frac{20!}{(20-(r+2))!}$$

This simplifies to:

$$\frac{(21-r)(20-r)(19-r)}{(21-r)(20-r)(19-r)} = 52 \times 2 \times 21$$

Simplifying further, we get:

$$(21-r)(20-r)(19-r) = 14 \times 13 \times 12$$

Now, we solve the equation:

$$(21-r)(20-r)(19-r) = (21-7)(20-7)(19-7)$$

Thus, we find:

$$r = 7$$

Thus, the value of r is 7, which matches option (C).

Quick Tip

For problems involving permutations, simplify the expressions and solve step-by-step to isolate the variable.

---

12. At an election, a voter may vote for any number of candidates not exceeding the number to be elected. If 4 candidates are to be elected out of the 12 contested in the election and voter votes for at least one candidate, then the number of ways of selections is:

(A) 793

(B) 298

(C) 781

(D) 1585

Correct Answer: (A) 793

Solution:

We are given 12 contested candidates, and we need to select 4 candidates for the election. The total number of ways of selections where voter votes for at least one candidate is calculated by summing over the possible choices of 1, 2, 3, and 4 candidates:

$$\text{Total selections} =^{1}{}^{2}C_1 +^{1}{}^{2}C_2 +^{1}{}^{2}C_3 +^{1}{}^{2}C_4$$

Step 1: Calculate individual combinations

$$^{12}C_1 = 12, \quad ^{12}C_2 = 66, \quad ^{12}C_3 = 220, \quad ^{12}C_4 = 495.$$

Step 2: Add them up

$$12 + 66 + 220 + 495 = 793.$$

Thus, the total number of ways of selection is 793, which matches option (A).

Quick Tip

For combinations, use the formula ${}^nC_r = \frac{n!}{r!(n-r)!}$ to calculate the number of ways to choose r objects from n objects.

---

13. The number of arrangements of all digits of 12345 such that at least 3 digits will not come in its position is:

(A) 89

(B) 109

(C) 78

(D) 57

Correct Answer: (B) 109

Solution:

We are given the number 12345, and we need to find how many arrangements of its digits will result in at least 3 digits not being in their original position. This is a problem of derangements where we calculate how many digits do not appear in their original position.

Step 1: Calculate the total number of arrangements of 5 digits

The total number of arrangements of 5 digits is:

$$5! = 120.$$

Step 2: Calculate derangements for different cases (3 digits, 4 digits, and 5 digits out of position)

Using inclusion-exclusion, the number of ways that at least 3 digits do not appear in their original position is:

$$5C3 \times 3! - 5C4 \times 4! + 5C5 \times 5! = 20 + 45 + 44 = 109.$$

Thus, the number of arrangements is 109, which matches option (B).

Quick Tip

Derangements are used to count the number of permutations where no element appears in its original position. Use inclusion-exclusion for such problems.

14. If $a > 0, b > 0, c > 0$ and a, b, c are distinct, then $(a+b)(b+c)(c+a)$ is greater than:

(A) $2(a+b+c)$

(B) $3(a+b+c)$

(C) $6abc$

(D) $8abc$

Correct Answer: (D) $8abc$

Solution:

We are given $a > 0, b > 0, c > 0$ and the condition that a, b, c are distinct. The goal is to find which of the following expressions $(a+b)(b+c)(c+a)$ is greater than.

Step 1: Use AM-GM inequality

We know that the Arithmetic Mean is greater than or equal to the Geometric Mean:

$$AM \geq GM.$$

Applying this to the terms $a+b, b+c, c+a$, we get:

$$(a+b)(b+c)(c+a) \geq 8abc.$$

Thus, the correct answer is $8abc$, which matches option (D).

Quick Tip

Use the AM-GM inequality to compare products of sums for positive distinct values. This is helpful in many problems involving inequalities.

---

15. If $\sum_{k=1}^{n} k(k+1)(k-1) = pn^4 + qn^3 + tn^2 + sn$, where p, q, t, s are constants, then the value of s is equal to:

(A) $-1/4$

(B) $-1/2$

(C) $1/2$

(D) $1/4$

Correct Answer: (B) $-1/2$

Solution: Given,

$$\sum_{k=1}^{n} k(k+1)(k-1) = pn^4 + qn^3 + tn^2 + sn$$

Therefore,

$$\sum_{k=1}^{n} (k^3 - k) = pn^4 + qn^3 + tn^2 + sn$$

$$\Rightarrow \left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)}{2} = pn^4 + qn^3 + tn^2 + sn$$

On dividing both sides by n, we get

$$\frac{(n(n+1))^2}{4n} - \frac{n(n+1)}{2n} = pn^3 + qn^2 + tn + s$$

Put $n = 0$, we get

$$0 - \frac{1}{2} = s$$

$$\Rightarrow s = -\frac{1}{2}$$

Thus, the value of s is $-1/2$, which matches option (B).

Quick Tip

Use the standard formulas for summations of powers of integers to simplify complex summation expressions.

---

16. There are four numbers of which the first three are in GP and the last three are in AP, whose common difference is 6. If the first and the last numbers are equal, then the two other numbers are:

(A) -2, 4

(B) 4, 2

(C) 2, 6

(D) None of the above

Correct Answer: (B) -4, 2

Solution:

Solution:

Let 3 numbers in AP be $a, (a+6), (a+12)$.

Also, the first and last number out of 4 numbers are equal $\therefore$ 4 numbers are $(a+12), a, (a+6), (a+12)$.

Given that the first 3 numbers are in GP.

$$\Rightarrow a^2 = (a+12)(a+6)$$
$$\Rightarrow a^2 = a^2 + 18a + 72$$
$$\Rightarrow a = \frac{-72}{18} = -4$$

Quick Tip

For sequences involving both GP and AP, always check consistency in conditions provided, especially when sequences must terminate at the same number.

---

17. If $A = 1 + r^a + r^{2a} + r^{3a} + \ldots\infty$ and $B = 1 + r^b + r^{2b} + r^{3b} + \ldots\infty$, then $\frac{a}{b}$ is equal.

(A) $\log_b(A)$

(B) $\log_{1-b}(1-A)$

(C) $\log_{\frac{b-1}{b}}\left(\frac{A-1}{A}\right)$

(D) None of these

Correct Answer: (C) $\log_{\frac{b-1}{b}}\left(\frac{A-1}{A}\right)$

Solution: We are given the following two infinite geometric series:

1. For A:

$$A = \frac{1}{1-r^a}$$

This implies that:

$$1 - r^a = \frac{1}{A}$$

Rearranging this:

$$r^a = 1 - \frac{1}{A}$$

Thus, we can write:

$$r^a = \frac{A-1}{A}$$

2. For B:

$$B = \frac{1}{1 - r^b}$$

This implies that:

$$1 - r^b = \frac{1}{B}$$

Rearranging this:

$$r^b = 1 - \frac{1}{B}$$

Thus, we can write:

$$r^b = \frac{B - 1}{B}$$

Now, to solve for $\frac{a}{b}$, we take the logarithm of both sides of each equation.

$$a \log r = \log\left(\frac{A - 1}{A}\right)$$

$$b \log r = \log\left(\frac{B - 1}{B}\right)$$

Next, we divide the two equations to find $\frac{a}{b}$:

$$\frac{a}{b} = \frac{\log\left(\frac{A-1}{A}\right)}{\log\left(\frac{B-1}{B}\right)}$$

Thus, we have:

$$\frac{a}{b} = \log_r\left(\frac{A - 1}{A}\right) = \log_r\left(\frac{B - 1}{B}\right)$$

Thus, the answer matches option (C).

Quick Tip

For problems involving geometric series and logs, transform the series into their sum formula first, then apply logarithmic identities.

---

18. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \ldots$ is equal to:

(A) $\frac{425}{216}$

(B) $\frac{429}{216}$

(C) $\frac{288}{125}$

(D) $\frac{280}{125}$

Correct Answer: (C) $\frac{288}{125}$

Solution:

Step 1: Let the series be defined as:

$$S = 1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \cdots \quad \text{(i)}$$

Now, define another series:

$$\frac{S}{6} = 1 + \frac{5}{6^2} + \frac{12}{6^3} + \frac{22}{6^4} + \cdots \quad \text{(ii)}$$

Step 2: On subtracting equation (ii) from equation (i):

$$S - \frac{S}{6} = \left(1 + \frac{5}{6} + \frac{12}{6^2} + \cdots\right) - \left(1 + \frac{5}{6^2} + \frac{12}{6^3} + \cdots\right)$$

Step 3: This simplifies to:

$$\frac{5S}{6} = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} + \cdots$$

Now, multiply both sides by 36:

$$\frac{25S}{36} = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} + \cdots$$

Step 4: The right-hand side is a geometric series with the first term 1 and the common ratio $\frac{1}{6}$. The sum of this geometric series is:

$$S = \frac{8}{5} \times \frac{36}{25} = \frac{288}{125}$$

Thus, the sum of the series is:

$$S = \frac{288}{125}$$

Thus, the value of S matches option (C).

Quick Tip

Use series transformation and subtraction techniques to find closed forms for complex series.

---

19. If $\tan^{-1}\left(\frac{1}{1+1\cdot 2}\right) + \tan^{-1}\left(\frac{1}{1+2\cdot 3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}(x)$**, then** x **is equal to:**

(A) $\frac{1}{n+1}$

(B) $\frac{n}{n+1}$

(C) $\frac{1}{n+2}$

(D) $\frac{n}{n+2}$

Correct Answer: (D) $\frac{n}{n+2}$

Solution: Given the series:

$$\tan^{-1}\left(\frac{1}{1+1\cdot 2}\right)+\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)=\tan^{-1}(x)$$

We can simplify the terms as follows:

$$\tan^{-1}\left(\frac{2-1}{1+1\cdot 2}\right)+\tan^{-1}\left(\frac{3-2}{1+2\cdot 3}\right)+\cdots$$

Using the difference identity for inverse tangents:

$$\tan^{-1}(A)-\tan^{-1}(B)=\tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

After applying this identity for each pair, we simplify the entire expression to:

$$\tan^{-1}(n+1)-\tan^{-1}(1)=\tan^{-1}(x)$$

Thus, we find:

$$\tan^{-1}(A)-\tan^{-1}(B)=\tan^{-1}\left(\frac{A-B}{1+A\cdot B}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{n}{n+2}\right)=\tan^{-1}(x)$$

Finally, we conclude that:

$$x=\frac{n}{n+2}$$

Quick Tip

Telescoping series in trigonometric identities often simplify to terms involving only the first and last elements.

20. If the arithmetic mean of two distinct positive real numbers a and b (where $a > b$) is twice their geometric mean, then $a : b$ is:

(A) $2+\sqrt{3} : 2-\sqrt{3}$

(B) $2+\sqrt{5} : 2-\sqrt{5}$

(C) $2+\sqrt{2} : 2-\sqrt{2}$

(D) None of these

Correct Answer: (A) $2+\sqrt{3} : 2-\sqrt{3}$

Solution: By the given condition,

$$\frac{a+b}{2} = 2\sqrt{ab}$$

$$\Rightarrow a+b = 4\sqrt{ab}$$

Now, $(a-b)^2 = (a+b)^2 - 4ab$

$$= 16ab - 4ab$$

$$= 12ab$$

$$\therefore a-b = \sqrt{12ab} = 2\sqrt{3}\sqrt{ab}$$

(Taking $+ve$ sign only as $a > b$)

$$\frac{a+b}{a-b} = \frac{4\sqrt{ab}}{2\sqrt{3ab}} = \frac{2}{\sqrt{3}}$$

By componendo and dividendo,

$$\frac{2a}{2b} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \quad \text{or} \quad \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Thus, the ratio is $(2+\sqrt{3}) : (2-\sqrt{3})$, matching option (A).

Quick Tip

For problems involving means, always test edge cases and simplify radical expressions to find recognizable patterns.

---

21. If

$$y = \tan^{-1}\left(\frac{1}{x^2+x+1}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right) + \cdots \textbf{ (to n terms)}$$

, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{x^2+n^2} - \frac{1}{x^2+1}$

(B) $\frac{1}{(x+n)^2+1} - \frac{1}{x^2+1}$

(C) $\frac{1}{x^2+(n+1)^2} - \frac{1}{x^2+1}$

(D) None of these

Correct Answer: (B) $\frac{1}{(x+n)^2+1} - \frac{1}{x^2+1}$

Solution: Step 1: Express the series

The general form of the terms in the series is:

$$y = \sum_{n=0}^{n} \tan^{-1}\left(\frac{1}{x^2 + (2n+1)x + (2n+1)}\right)$$

Step 2: Simplify each term

We rewrite each term in the series as:

$$y = \sum_{n=0}^{n} \tan^{-1}\left(\frac{1}{(x+n)(x+n+1)}\right)$$

Step 3: Apply the derivative formula for the arctangent function

We use the formula for the derivative of $\tan^{-1}(f(x))$:

$$\frac{d}{dx}\tan^{-1}(f(x)) = \frac{f'(x)}{1+f(x)^2}$$

For each term, we differentiate the argument:

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{1}{(x+n)(x+n+1)}\right)^2} \cdot \frac{d}{dx}\left(\frac{1}{(x+n)(x+n+1)}\right)$$

Step 4: Simplify the result

After simplifying the derivative expression, we get:

$$\frac{dy}{dx} = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

Final Answer:

$$\frac{dy}{dx} = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

Quick Tip

For trigonometric series involving inverse functions and variables, look for telescoping patterns to simplify the expression before differentiating.

---

22. The coefficient of x^2 term in the binomial expansion of $\left(\frac{1}{3}x^{\frac{1}{2}} + x^{-\frac{1}{4}}\right)^{10}$ is:

(A) $\frac{70}{243}$

(B) $\frac{60}{423}$

(C) $\frac{50}{13}$

(D) None of these

Correct Answer: (A) $\frac{70}{243}$

Solution:

To find the coefficient of x^2 in the expansion, identify the appropriate terms from the expansion that contribute to x^2 when multiplied.

Step 1: Identify relevant terms

The general term in the expansion can be written as:

$$T_{r+1} = {}^{10}C_r \left(\frac{1}{3}x^{\frac{1}{2}}\right)^{10-r} \left(x^{-\frac{1}{4}}\right)^r$$
$$= {}^{10}C_r \times \left(\frac{1}{3}\right)^{10-r} x^{\frac{10-r}{2}-\frac{r}{4}}$$

We have to find the coefficient of x^2.

$$\frac{10-r}{2} - \frac{r}{4} = 2 \Rightarrow r = 4$$

$$T_{4+1} = {}^{10}C_4 \left(\frac{1}{3}\right)^6 x^2$$

Step 2: Calculate the coefficient

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times \frac{1}{3^6} = \frac{70}{243}$$

Thus, the coefficient of x^2 is $\frac{70}{243}$, which matches option (A).

Quick Tip

Always set the power of x in the general term equal to the desired power, and solve for r to find the specific term contributing to that power.

---

23. The coefficient of x^n in the expansion of

$$\frac{e^{7x}+e^{x}}{e^{3x}}$$

is:

(A) $\frac{4^{n-1}\cdot(-2)^n}{n!}$

(B) $\frac{4^n-1\cdot(2)^n}{n!}$

(C) $\frac{4^n+(-2)^n}{n!}$

(D) $\frac{4^n-1\cdot(-2)^{n-1}}{n!}$

Correct Answer: (C) $\frac{4^n+(-2)^n}{n!}$

Solution:

The given expression can be expanded using the Maclaurin series for e^x: Given,

$$\frac{e^{7x}+e^{x}}{e^{3x}} \Rightarrow e^{4x}+e^{-2x}$$

$$\text{Series of } e^{a} = 1+\frac{a}{1!}+\frac{a^2}{2!}+\frac{a^3}{3!}+\dots$$

$$\Rightarrow e^{4x}+e^{-2x} = \left(1+\frac{4x}{1!}+\frac{(4x)^2}{2!}+\frac{(4x)^3}{3!}+\dots\right)+\left(1+\frac{(-2x)}{1!}+\frac{(-2x)^2}{2!}+\dots\right)$$

Here, coefficient of

$$x^2 \equiv \frac{4^2}{2!}+\frac{(-2)^2}{2!}$$

$$x^3 \equiv \frac{4^3}{3!}+\frac{(-2)^3}{3!}$$

$$\vdots$$

$$x^n = \frac{4^n}{n!}+\frac{(-2)^n}{n!}$$

This matches option (C), providing the correct coefficient for x^n.

Quick Tip

When combining terms from exponential expansions, ensure consistent base and power adjustments to match given options.

---

24. The coefficient of the highest power of x in the expansion of $(x + \sqrt{x^2 - 1})^8 + (x - \sqrt{x^2 - 1})^8$ is:

(A) 64

(B) 128

(C) 256

(D) 512

Correct Answer: (C) 256

Solution:

This problem involves simplifying two terms raised to the eighth power. The terms inside the parentheses are structured such that they effectively represent hyperbolic cosine functions.

Step 1: Identify the simplification strategy

$$\text{Since } (x + \sqrt{x^2 - 1})^8 + (x - \sqrt{x^2 - 1})^8$$

$$= 2\{{}^8C_0 x^8 + {}^8C_2 x^6 (x^2 - 1) + {}^8C_4 x^4 (x^2 - 1)^2 + {}^8C_6 x^2 (x^2 - 1)^3 + {}^8C_8 x^0 (x^2 - 1)^4\}$$

$$\text{So coefficient of highest power of } x$$

$$= 2\{{}^8C_0 + {}^8C_2 + {}^8C_4 + {}^8C_6 + {}^8C_8\}$$

$$= (1 + 1)^8 + (1 - 1)^8 = 2^8 = 256$$

This corresponds to the highest power and matches option (C).

Quick Tip

Always consider trigonometric and hyperbolic identities when dealing with complex binomial expressions to simplify calculation.

---

25. If the 17th and the 18th terms in the expansion of $(2 + a)^{50}$ are equal, then the coefficient of x^{35} in the expansion of $(a + x)^{-2}$ is:

(A) -35

(B) 3

(C) 36

(D) -36

Correct Answer: (D) -36

Solution:

Given, 17th and 18th terms in the expansion $(2 + a)^{50}$ are equal:

$$T_{17} = T_{18}$$

$$\Rightarrow 50C_{16}(2)^{34}(a)^{16} = 50C_{17}(2)^{33}(a)^{17}$$

$$\Rightarrow a = \frac{50C_{16}}{50C_{17}} \times 2 = 1$$

Now, the coefficient of x^{35} in the expansion of

$$(1 + x)^{-2} = -36$$

Coefficient of x^r is $(r + 1)$ in $(1 - x)^{-2}$.

Quick Tip

In problems involving equal terms of a binomial expansion, equating the terms helps solve for unknowns. For negative binomial expansions, use properties of the binomial theorem extended to negative exponents.

---

26. Let A, B and C are the angles of a triangle and $\tan\frac{A}{2} = 1/3$, $\tan\frac{B}{2} = \frac{2}{3}$. Then, $\tan\frac{C}{2}$ is equal to:

(A) $\frac{7}{9}$

(B) $\frac{2}{9}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

Correct Answer: (A) $\frac{7}{9}$

Solution:

Using the angle sum property of a triangle:

$$A + B + C = 180^\circ$$

$$\Rightarrow C = 180^\circ - A - B$$

Given:

$$A + B + C = 180^\circ \quad \text{(Angle sum property of a triangle)}$$

$$\Rightarrow A + B = 180^\circ - C$$

$$\Rightarrow \frac{A+B}{2} = 90^\circ - \frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right)$$

$$\Rightarrow \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \cot\frac{C}{2}$$

$$\Rightarrow \frac{\frac{1}{3} + \frac{2}{3}}{1 - \frac{1}{3} \times \frac{2}{3}} = \cot\frac{C}{2}$$

$$\Rightarrow \frac{1}{1 - \frac{2}{9}} = \cot\frac{C}{2}$$

$$\Rightarrow \frac{9}{7} = \cot\frac{C}{2}$$

$$\Rightarrow \tan\frac{C}{2} = \frac{7}{9}$$

Quick Tip

Always verify angle sum properties and recalculations when dealing with trigonometric identities in geometry.

---

27. The sum of all values of x in $[0, 2\pi]$, for which $sin(x) + \sin(2x) + \sin(3x) + \sin(4x) = 0$ is equal to:

(A) 8π

(B) 11π

(C) 12π

(D) 9π

Correct Answer: (D) 9π

Solution: We are given the equation:

$$(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$$

We begin by simplifying this equation using trigonometric identities. Using the sum-to-product identity:

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

We can apply this identity to both pairs of sines:

$$(\sin x + \sin 4x) = 2\sin\left(\frac{x+4x}{2}\right)\cos\left(\frac{4x-x}{2}\right) = 2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{3x}{2}\right)$$

$$(\sin 2x + \sin 3x) = 2\sin\left(\frac{2x+3x}{2}\right)\cos\left(\frac{3x-2x}{2}\right) = 2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)$$

Thus, the equation becomes:

$$2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{3x}{2}\right) + 2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right) = 0$$

Factor out $2\sin\left(\frac{5x}{2}\right)$:

$$2\sin\left(\frac{5x}{2}\right)\left(\cos\left(\frac{3x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) = 0$$

For this product to be zero, either:

$$\sin\left(\frac{5x}{2}\right) = 0 \quad \text{or} \quad \cos\left(\frac{3x}{2}\right) + \cos\left(\frac{x}{2}\right) = 0$$

Let's first solve for the case where $\sin\left(\frac{5x}{2}\right) = 0$:

$$\sin\left(\frac{5x}{2}\right) = 0 \quad \Rightarrow \quad \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, \dots$$

Thus, $x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \dots$.

Now, let's solve for the case where $\cos\left(\frac{3x}{2}\right) + \cos\left(\frac{x}{2}\right) = 0$:

Using the sum-to-product identity again:

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

We apply this identity to the equation:

$$\cos\left(\frac{3x}{2}\right) + \cos\left(\frac{x}{2}\right) = 2\cos\left(\frac{3x}{2} + \frac{x}{2}\right)\cos\left(\frac{3x}{2} - \frac{x}{2}\right)$$

$$= 2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right) = 2\cos(2x)\cos(x)$$

Thus, the equation becomes:

$$2\cos(2x)\cos(x) = 0$$

For this product to be zero, either:

$$\cos(2x) = 0 \quad \text{or} \quad \cos(x) = 0$$

For $\cos(2x) = 0$:

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \quad \Rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \dots$$

For $\cos(x) = 0$:

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

Now, combining all the solutions, we have the general solution for x:

$$x = 0, \pi, 2\pi, 3\pi, \ldots$$

Finally, we can conclude that the sum of these values of x is:

$$6\pi + \pi + 2\pi = 9\pi$$

Quick Tip

Check solutions by substituting back into the original equation to confirm their validity.

---

28. Number of solutions of equations $\sin(9\theta) = \sin(\theta)$ **in the interval** $[0, 2\pi]$ **is:**

(A) 16

(B) 17

(C) 18

(D) 15

Correct Answer: (B) 17

Solution: We are given that:

$$\sin 90^\circ = \sin\theta$$

which simplifies to:

$$\sin 90^\circ - \sin\theta = 0$$

We use the identity for the difference of sines:

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

This leads to:

$$2\cos\left(\frac{90^\circ + \theta}{2}\right)\sin\left(\frac{90^\circ - \theta}{2}\right) = 0$$

Now we have two cases:

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

This gives us:

$$2\cos 5\theta \sin 4\theta = 0$$

Thus, we have two conditions:

$$\cos 5\theta = 0 \quad \text{or} \quad \sin 4\theta = 0$$

For $\cos 5\theta = 0$:

$$5\theta = (2n+1)\frac{\pi}{2}$$

$$\theta = \frac{(2n+1)\pi}{10}$$

For $\sin 4\theta = 0$:

$$4\theta = n\pi$$

$$\theta = \frac{n\pi}{4}$$

Now, substituting $n = 0, 1, 2, \ldots$, we get the solutions for θ:

$$\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}, \frac{11\pi}{10}, \frac{13\pi}{10}, \frac{15\pi}{10}, \frac{17\pi}{10}, \frac{19\pi}{10}$$

For the second set of solutions:

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$$

Thus, we have a total of 17 solutions, and the common solutions are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

Total number of solutions = 17

Quick Tip

Use graphical or numerical methods to verify the count of solutions for trigonometric equations.

---

29. The range of $(8\sin(\theta) + 6\cos(\theta))^2 + 2$ is:

(A) (0,2)

(B) [2,102]

(C) $(-\infty, \infty)$

(D) (2,1)

Correct Answer: (B) [2,102]

Solution:

Given the following:

$$-10 \leq 8\sin\theta + 6\cos\theta \leq 10$$

$$\Rightarrow 0 \leq (8\sin\theta + 6\cos\theta)^2 \leq 100$$

$$\Rightarrow 2 \leq (8\sin\theta + 6\cos\theta)^2 + 2 \leq 102$$

Thus, the range is $[2, 102]$.

Quick Tip

Adjust range calculations by considering the transformations applied to the trigonometric functions (scaling and translation).

---

30. The locus of the point of intersection of the lines $x = a(1 - t^2)/(1 + t^2)$ and $y = 2at/(1 + t^2)$ (t being a parameter) represents:

(A) Circle

(B) Parabola

(C) Ellipse

(D) Hyperbola

Correct Answer: (A) Circle

Solution: Given that:

$$x = a\left(\frac{1-t^2}{1+t^2}\right) \quad \text{and} \quad y = \frac{2at}{1+t^2}$$

Let $t = \tan\theta$.

Thus:

$$x = a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) \quad \text{and} \quad y = \frac{2a\tan\theta}{1+\tan^2\theta}$$

From this, we have:

$$x = a\cos 2\theta \quad \text{and} \quad y = a\sin 2\theta$$

We can write:

$$\cos 2\theta = \frac{x}{a} \quad \text{and} \quad \sin 2\theta = \frac{y}{a}$$

Squaring both sides:

$$\cos^2 2\theta = \frac{x^2}{a^2} \quad \text{and} \quad \sin^2 2\theta = \frac{y^2}{a^2}$$

Adding these equations:

$$\cos^2 2\theta + \sin^2 2\theta = \frac{x^2}{a^2} + \frac{y^2}{a^2}$$

Using the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$, we get:

$$1 = \frac{x^2+y^2}{a^2}$$

Thus, the equation becomes:

$$x^2 + y^2 = a^2$$

This represents the equation of a circle with center at the origin and radius a.

Therefore, the locus of the point is a circle having center at the origin and radius a.

Quick Tip

Understanding the geometrical interpretation of parametric equations simplifies the identification of loci.

---

31. If the straight line $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$ form a triangle with origin as orthocentre, then (a, b) is equal to:

(A) (6,4)

(B) (-3,3)

(C) (-8,8)

(D) (0,7)

Correct Answer: (C) (-8,8)

Solution:

Here, point A is the intersection of line AB and AC. So, the equation of the line passing through A is given by:

$$(x + 2y - 1) + \lambda(2x + 3y - 1) = 0 \tag{1}$$

This line passes through the orthocentre $(0, 0)$, hence substituting $(0, 0)$:

$$-1 + \lambda(-1) = 0$$

$$-1 - \lambda = 0$$

$$\lambda = -1$$

Substituting $\lambda = -1$ in Eq. (1), we get:

$$x + y = 0$$

Thus, the equation of AD is:

$$x + y = 0 \tag{2}$$

Since $AD \perp BC$, therefore:

$$\frac{-1-x}{a} = \frac{d}{b} = -1$$

which simplifies to:

$$a + b = 0 \tag{3}$$

Similarly, by applying the condition that BE is perpendicular to CA, we obtain:

$$a + 2b = 8 \tag{4}$$

Now, solving Eqs. (3) and (4):

$$a + b = 0$$

$$a + 2b = 8$$

Subtracting Eq. (3) from Eq. (4):

$$(a + 2b) - (a + b) = 8 - 0$$

$$b = 8$$

Substituting $b = 8$ in Eq. (3):

$$a + 8 = 0$$

$$a = -8$$

Thus, the values are:

$$a = -8, \quad b = 8$$

Quick Tip

Understanding orthocentre properties helps simplify the problem-solving approach.

---

33. The distance from the origin to the image of $(1,1)$ with respect to the line $x+y+5=0$ is:

(A) $7\sqrt{2}$

(B) $3\sqrt{2}$

(C) $6\sqrt{2}$

(D) $4\sqrt{2}$

Correct Answer: (C) $6\sqrt{2}$

Solution: Using the formula for the image of a point (x_1, y_1) with respect to the line $Ax+By+C=0$:

$$x' = x_1 - \frac{2A(Ax_1+By_1+C)}{A^2+B^2}, \quad y' = y_1 - \frac{2B(Ax_1+By_1+C)}{A^2+B^2}$$

Substituting values, we get the image as $(-6,-6)$. The required distance from the origin:

$$D = \sqrt{(-6-0)^2 + (-6-0)^2} = \sqrt{36+36} = 6\sqrt{2}$$

Quick Tip

Using the reflection formula helps determine the image of a point easily.

---

34. A(3,2,0), B(5,3,2), C(-9,6,-3) are three points forming a triangle. AD, the bisector of angle BAC meets BC in D. Find the coordinates of D:

(A) $\left(\frac{19}{8}, \frac{57}{15}, \frac{57}{15}\right)$

(B) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

(C) (2,3,0)

(D) (4,5,6)

Correct Answer: (B) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

Solution: Since AD is the bisector of $\angle BAC$, we use the angle bisector theorem:

$$\frac{BD}{DC} = \frac{AB}{AC} \quad \text{...(i)} \tag{5}$$

Now, we calculate the lengths of AB and AC.

Calculating AB

$$AB = \sqrt{(5-3)^2 + (3-2)^2 + (2-0)^2}$$

$$= \sqrt{4+1+4} = \sqrt{9} = 3$$

Calculating AC

$$AC = \sqrt{(-9-3)^2 + (6-2)^2 + (-3-0)^2}$$

$$= \sqrt{144+16+9}$$

$$= \sqrt{169} = 13$$

Finding the Ratio of $BD : DC$

From equation (i):

$$\frac{BD}{DC} = \frac{3}{13}$$

Thus, D divides BC in the ratio $3 : 13$.

Finding the Coordinates of D

Using the section formula:

$$D\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

where $B(-9, 6, -3)$ and $C(5, 3, 2)$, and the ratio $m : n = 3 : 13$,

$$x = \frac{3(-9) + 13(5)}{3+13}, \quad y = \frac{3(6) + 13(3)}{3+13}, \quad z = \frac{3(-3) + 13(2)}{3+13}$$

$$= \left(\frac{-27+65}{16}, \frac{18+39}{16}, \frac{-9+26}{16} \right)$$

$$= \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16} \right)$$

Thus, the coordinates of D are:

$$D \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16} \right)$$

Quick Tip

The angle bisector theorem is a powerful tool for solving triangle-related coordinate geometry problems.

---

35. The locus of the mid-point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:

(A) $x + y = 2$

(B) $x^2 + y^2 = 1$

(C) $x^2 + y^2 = 2$

(D) $x + y = 1$

Correct Answer: (C) $x^2 + y^2 = 2$

Solution: Let the mid-point of the chord be (h, k). The perpendicular from the origin to the chord satisfies:

$$OC = \sqrt{h^2 + k^2},$$

Using trigonometry and given conditions, we derive:

$$h^2 + k^2 = 2$$

Quick Tip

Understanding the perpendicularity condition simplifies locus problems.

---

36. If p and q be the longest and the shortest distance respectively of the point (-7,2) from any point (α, β) on the curve whose equation is

$$x^2 + y^2 - 10x - 14y - 51 = 0$$

then the geometric mean (G.M.) of p is:

(A) $2\sqrt{11}$

(B) $5\sqrt{5}$

(C) 13

(D) 11

Correct Answer: (A) $2\sqrt{11}$

Solution: The given equation of the curve is:

$$x^2 + y^2 - 10x - 14y - 51 = 0$$

We rewrite it in standard circle form by completing the square.

$$(x^2 - 10x) + (y^2 - 14y) = 51$$

Completing squares:

$$(x-5)^2 - 25 + (y-7)^2 - 49 = 51$$

$$(x-5)^2 + (y-7)^2 = 5\sqrt{5}^2$$

So, the center $C(5, 7)$ and radius $r = 5\sqrt{5}$.

Now, the distance of point $(-7, 2)$ from the center $C(5, 7)$:

$$PC = \sqrt{(5+7)^2 + (7-2)^2}$$

$$PC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

Thus, the farthest and closest distances:

$$p = 13 + 5\sqrt{5}, \quad q = 13 - 5\sqrt{5}$$

The geometric mean:

$$\sqrt{pq} = \sqrt{(13 - 5\sqrt{5})(13 + 5\sqrt{5})}$$

Using the identity $(a - b)(a + b) = a^2 - b^2$:

$$\sqrt{169 - 125} = \sqrt{44} = 2\sqrt{11}$$

Quick Tip

Understanding the concept of distance from a point to a circle helps in solving such problems efficiently.

---

37. From a point A(0,3) on the circle

$$(x + 2)^2 + (y - 3)^2 = 4$$

a chord AB is drawn and extended to a point Q such that AQ = 2AB. Then the locus of Q is:

(A) $(x + 4)^2 + (y - 3)^2 = 16$

(B) $(x + 1)^2 + (y - 3)^2 = 32$

(C) $(x + 1)^2 + (y - 3)^2 = 4$

(D) $(x + 1)^2 + (y - 3)^2 = 1$

Correct Answer: (A) $(x + 4)^2 + (y - 3)^2 = 16$

Solution: The given equation of the circle is:

$$(x + 2)^2 + (y - 3)^2 = 4$$

Let the coordinates of $Q(h, k)$.

Since $AQ = 2AB$, the midpoint B of segment AQ satisfies:

$$B = \left(\frac{0 + h}{2}, \frac{3 + k}{2}\right)$$

Since point B lies on the given circle:

$$\left(\frac{h}{2}+2\right)^2+\left(\frac{k}{2}-3\right)^2=4$$

Expanding:

$$\left(\frac{h+4}{2}\right)^2+\left(\frac{k-3}{2}\right)^2=4$$

Multiplying both sides by 4:

$$(h+4)^2+(k-3)^2=16$$

Thus, the required locus of $Q(h,k)$ is:

$$(x+4)^2+(y-3)^2=16$$

Quick Tip

Using the midpoint formula correctly ensures accurate derivation of the locus equation.

---

38. If the focus of the parabola

$$(y-k)^2=4(x-h)$$

always lies between the lines $x+y=1$ **and** $x+y=3$ **then:**

(A) $0<h+k<2$

(B) $0<h+k<1$

(C) $1<h+k<2$

(D) $1<h+k<3$

Correct Answer: (A) $0<h+k<2$

Solution: The standard form of the given parabola is:

$$(y-k)^2=4(x-h)$$

The focus of the parabola is given by:

$$(h+1,k)$$

Since the focus must lie between the lines $x + y = 1$ and $x + y = 3$, we substitute the focus into the inequalities:

$$1 < (h + 1) + k < 3$$

$$0 < h + k < 2$$

Thus, the required range for $h + k$ is:

$$0 < h + k < 2$$

Quick Tip

Understanding how a parabola's focus is derived from its equation is key to solving locus-related problems.

---

39. Let L_1 be the length of the common chord of the curves

$$x^2 + y^2 = 9 \quad \text{and} \quad y^2 = 8x$$

and let L_2 be the length of the latus rectum of $y^2 = 8x$. Then:

(A) $L_1 > L_2$

(B) $L_1 = L_2$

(C) $L_1 < L_2$

(D) $\frac{L_1}{L_2} = \sqrt{2}$

Correct Answer: (C) $L_1 < L_2$

Solution: The given equations are:

$$x^2 + y^2 = 9$$

$$y^2 = 8x$$

Solving for the intersection points:

$$x^2 + 8x = 9$$

Rearrange:

$$x^2 + 8x - 9 = 0$$

Factoring:

$$(x + 9)(x - 1) = 0$$

So, $x = -9, 1$.

For $x = 1$:

$$y^2 = 8(1) = 8$$

$$y = \pm 2\sqrt{2}$$

Thus, length of the common chord:

$$L_1 = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4\sqrt{2}$$

Now, the length of the latus rectum of the parabola $y^2 = 8x$ is:

$$L_2 = 4a = 4 \times 2 = 8$$

Since $L_1 = 4\sqrt{2} \approx 5.66$ and $L_2 = 8$, we get:

$$L_1 < L_2$$

Quick Tip

The length of the latus rectum of a parabola is always $4a$, which helps in quick calculations.

---

40. The foci of the hyperbola

$$4x^2 - 9y^2 - 1 = 0$$

are:

(A) $(\pm\sqrt{13}, 0)$

(B) $\left(\pm\frac{\sqrt{13}}{6}, 0\right)$

(C) $\left(0, \pm\frac{\sqrt{3}}{6}\right)$

(D) None of these

Correct Answer: (B) $\left(\pm\frac{\sqrt{13}}{6}, 0\right)$

Solution: Given equation:

$$4x^2 - 9y^2 - 1 = 0$$

Rearrange:

$$\frac{x^2}{\frac{1}{4}} - \frac{y^2}{\frac{1}{9}} = 1$$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} - \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$$

Comparing with the standard form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

we get:

$$a = \frac{1}{2}, \quad b = \frac{1}{3}$$

The eccentricity of a hyperbola is:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{\frac{1}{9}}{\frac{1}{4}}} = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$$

Foci are given by:

$$(\pm ae, 0) = \left(\pm\frac{1}{2} \times \frac{\sqrt{13}}{3}, 0\right)$$

$$= \left(\pm\frac{\sqrt{13}}{6}, 0\right)$$

Thus, the foci are:

$$\left(\pm\frac{\sqrt{13}}{6}, 0\right)$$

Quick Tip

For hyperbolas, the formula for foci is $(\pm ae, 0)$, and eccentricity is found using $e = \sqrt{1 + \frac{b^2}{a^2}}$.

---

41. Given a real-valued function f such that:

$$f(x) = \begin{cases} \frac{\tan^2\{x\}}{x^2 - \lfloor x \rfloor^2}, & \text{for } x > 0 \\ 1, & \text{for } x = 0 \\ \sqrt{\{x\}\cot\{x\}}, & \text{for } x < 0 \end{cases}$$

Then:

(A) LHL $= 1$

(B) RHL $= \sqrt{\cot 1}$

(C) $\lim_{x\to 0} f(x)$ exists

(D) $\lim_{x\to 0} f(x)$ does not exist

Correct Answer: (D) $\lim_{x\to 0} f(x)$ does not exist

Solution:

- Right-hand limit (RHL): Approaching $x \to 0^+$, we substitute in the first case:

$$\lim_{h\to 0^+} \frac{\tan^2 h}{h^2} = \lim_{h\to 0^+} \frac{\tan^2 h}{h^2} = 1$$

- Left-hand limit (LHL): Approaching $x \to 0^-$, we substitute in the third case:

$$\lim_{h\to 0^-} \sqrt{(-h)\cot(-h)}$$

Using $\cot(-h) = -\cot h$, we get:

$$\lim_{h\to 0^-} \sqrt{(1-h)\cot(1-h)}$$

which is not equal to $\lim_{x\to 0^+} f(x)$, thus:

$$\lim_{x\to 0} f(x) \text{ does not exist.}$$

Quick Tip

For piecewise functions, always check left-hand and right-hand limits separately.

---

42. Let $f(x) = \sin x$, $g(x) = \cos x$, and $h(x) = x^2$. Then, evaluate:

$$\lim_{x\to 1} \frac{f(g(h(x))) - f(g(h(1)))}{x-1}$$

(A) 0

(B) $-2\sin 1\cos(\cos 1)$

(C) ∞

(D) $-2\sin 1\cos 1$

Correct Answer: (B) $-2\sin 1\cos(\cos 1)$

Solution: Given: $f(x) = \sin x$, $g(x) = \cos x$, and $h(x) = x^2$

$$\lim_{x\to 1} \frac{f(g(h(x))) - f(g(h(1)))}{x-1}$$

Substituting $h(x) = x^2$ and evaluating at $x = 1$:

$$\lim_{x\to 1} \frac{\sin(\cos(x^2)) - \sin(\cos 1)}{x-1}$$

Using L'Hôpital's rule:

$$\lim_{x\to 1} \frac{\cos(\cos(x^2)) \cdot (-\sin(x^2)) \cdot 2x}{1}$$

Evaluating at $x = 1$:

$$= -2\sin 1\cos(\cos 1)$$

Quick Tip

Using chain rule and L'Hôpital's rule simplifies limit calculations for composite functions.

---

43. The Boolean expression:

$$\sim (p \vee q) \vee (\sim p \wedge q)$$

is equivalent to:

(A) p

(B) q

(C) $\sim q$

(D) $\sim p$

Correct Answer: (D) $\sim p$

Solution:

Applying De Morgan's Law:

$$\sim (p \vee q) = \sim p \wedge \sim q$$

Thus:

$$(\sim p \wedge \sim q) \vee (\sim p \wedge q)$$

Using distributive law:

$$\sim p \wedge (\sim q \vee q)$$

Since $\sim q \vee q = 1$ (tautology):

$$\sim p \wedge 1 = \sim p$$

Thus, the Boolean expression simplifies to:

$$\sim p$$

Quick Tip

Using De Morgan's law and distribution simplifies complex Boolean expressions.

---

44. If p: 2 is an even number, q: 2 is a prime number, and r: $2+2=2^2$, then the symbolic statement $p \to (q \vee r)$ means:

(A) 2 is an even number and 2 is a prime number or $2+2=2^2$

(B) 2 is an even number then 2 is a prime number or $2+2=2^2$

(C) 2 is an even number or 2 is a prime number then $2+2=2^2$

(D) If 2 is not an even number then 2 is a prime number $\alpha = 2+2=2^2$

Correct Answer: (B) 2 is an even number then 2 is a prime number or $2+2=2^2$

Solution: The given symbolic statement:

$$p \to (q \vee r)$$

By definition of implication:

$$p \to (q \vee r) \equiv \sim p \vee (q \vee r)$$

Since p represents "2 is an even number," the statement translates to:

"If 2 is an even number, then 2 is a prime number or $2+2=2^2$."

Quick Tip

Understanding logical implications helps in interpreting symbolic statements correctly.

---

45. Consider the following statements:

A: Rishi is a judge.

B: Rishi is honest.

C: Rishi is not arrogant.

The negation of the statement "If Rishi is a judge and he is not arrogant, then he is honest" is:

(A) $B \to (A \vee C)$

(B) $(\sim B) \wedge (A \wedge C)$

(C) $B \rightarrow ((\sim A) \vee (\sim C))$

(D) $B \rightarrow (A \wedge C)$

Correct Answer: (B) $(\sim B) \wedge (A \wedge C)$

Solution: The given statement is:

$$(A \wedge C) \rightarrow B$$

The negation of an implication $P \rightarrow Q$ is given by:

$$\sim (P \rightarrow Q) \equiv P \wedge \sim Q$$

Thus:

$$\sim [(A \wedge C) \rightarrow B] \equiv (A \wedge C) \wedge \sim B$$

which matches option (B).

Quick Tip

Negation of an implication $P \rightarrow Q$ is always $P \wedge \sim Q$.

---

46. If p: It is raining today, q: I go to school, r: I shall meet my friends, and s: I shall go for a movie, then which of the following represents:

"If it does not rain or if I do not go to school, then I shall meet my friend and go for a movie?"

(A) $\sim (p \wedge q) \Rightarrow (r \wedge s)$

(B) $\sim (p \wedge \sim q) \Rightarrow (r \wedge s)$

(C) $\sim (p \wedge q) \Rightarrow (r \vee s)$

(D) None of these

Correct Answer: (A) $\sim (p \wedge q) \Rightarrow (r \wedge s)$

Solution: The given statement translates as:

"If it does not rain or I do not go to school, then I shall meet my friend and go for a movie."

The phrase "does not rain or do not go to school" can be written as:

$$\sim (p \wedge q)$$

The phrase "I shall meet my friend and go for a movie" translates to:

$$r \wedge s$$

Thus, the logical expression becomes:

$$\sim (p \wedge q) \Rightarrow (r \wedge s)$$

Quick Tip

Use logical operators systematically to convert English statements into symbolic form.

---

47. Let p, q, r be three logical statements. Consider the compound statements:

$$S_1 : (\sim p \vee q) \vee (\sim p \vee r)$$

$$S_2 : p \rightarrow (q \vee r)$$

Which of the following is NOT true?

(A) If S_2 is true, then S_1 is true

(B) If S_2 is false, then S_1 is false

(C) If S_2 is false, then S_1 is true

(D) If S_1 is false, then S_2 is false

Correct Answer: (C) If S_2 is false, then S_1 is true

Solution: Expanding S_1:

$$(\sim p \vee q) \vee (\sim p \vee r) \equiv \sim p \vee (q \vee r)$$

Expanding S_2:

$$p \to (q \vee r) \equiv \sim p \vee (q \vee r)$$

Since both S_1 and S_2 are equivalent, if S_2 is false, then S_1 should also be false, contradicting option (C).

Quick Tip

Equivalent logical statements will always have the same truth value.

---

48. Consider the following two propositions:

$$P_1 : \sim (p \to \sim q)$$

$$P_2 : (p \wedge \sim q) \wedge ((\sim p) \vee q)$$

If the proposition $p \to ((\sim p) \vee q)$ **is evaluated as FALSE, then:**

(A) P_1 is TRUE and P_2 is FALSE

(B) P_1 is FALSE and P_2 is TRUE

(C) Both P_1 and P_2 are FALSE

(D) Both P_1 and P_2 are TRUE

Correct Answer: (C) Both P_1 and P_2 are FALSE

Solution: We begin by constructing a truth table for the given expressions. The statement $p \to ((\sim p) \vee q)$ is FALSE only when $p = T$ and $q = F$, which gives:

p	q	$\sim$ p	$\sim$ q	$\sim$ p $\vee$ q	p $\to$ ($\sim$ p $\vee$ q)	p $\to \sim$ q	$\sim$ (p $\to \sim$ q)	p$\wedge \sim$ q	p₂
T	T	F	F	T	T	F	T	F	F
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	F	F

$$p \to ((\sim p) \vee q) = F$$

This condition leads to P_1 and P_2 both being FALSE.

Quick Tip

Constructing a truth table simplifies logical evaluation.

---

49. If the variance of the data $2, 3, 5, 8, 12$ is σ^2 and the mean deviation from the median for this data is M, then $\sigma^2 - M$ is:

(A) 10.2

(B) 5.8

(C) 10.6

(D) 8.2

Correct Answer: (A) 10.2

Solution: Given observations: $2, 3, 5, 8, 12$.

1. Calculate Mean:

$$\text{Mean} = \frac{2+3+5+8+12}{5} = 6$$

2. Calculate Variance:

$$\sigma^2 = 13.2$$

3. Find Median:

Since the number of observations is odd, the median is the middle value:

$$\text{Median} = 5$$

4. Calculate Mean Deviation about Median:

$$M = \frac{|2-5| + |3-5| + |5-5| + |8-5| + |12-5|}{5} = 3$$

5. Final Calculation:

$$\sigma^2 - M = 13.2 - 3 = 10.2$$

Quick Tip

Variance measures spread, while mean deviation measures absolute dispersion.

---

50. The mean of n items is X. If the first item is increased by 1, second by 2, and so on, the new mean is:

(A) $\bar{X} + \frac{x}{2}$

(B) $\bar{X} + x$

(C) $\bar{X} + \frac{n+1}{2}$

(D) None of these

Correct Answer: (C) $\bar{X} + \frac{n+1}{2}$

Solution: Let the items be $a_1, a_2, ..., a_n$.

$$\bar{X} = \frac{a_1 + a_2 + ... + a_n}{n}$$

Now, given the condition:

$$\bar{X}_{\text{new}} = \frac{(a_1 + 1) + (a_2 + 2) + ... + (a_n + n)}{n}$$

Using the sum of the first n natural numbers:

$$\bar{X}_{\text{new}} = \bar{X} + \frac{n(n+1)}{2n} = \bar{X} + \frac{n+1}{2}$$

Quick Tip

Use summation formulas for sequences to simplify mean calculations.

---

51. The variance of 20 observations is 5. If each observation is multiplied by 2, then the new variance of the resulting observation is:

(A) $2^3 \times 5$

(B) $2^2 \times 5$

(C) 2×5

(D) $2^4 \times 5$

Correct Answer: (B) $2^2 \times 5$

Solution: Given variance:

$$\sigma^2 = 5$$

If each observation is multiplied by a constant $k = 2$, the variance changes as:

$$\text{Variance } (\sigma^2) = \frac{1}{n}\sum_{i=1}^{20}(x_i - \overline{x})^2$$

$$\text{i.e. } 5 = \frac{1}{20}\sum_{i=1}^{20}(x_i - \overline{x})^2$$

$$\text{or } \sum_{i=1}^{20}(x_i - \overline{x})^2 = 100 \quad .....(i)$$

If each observation is multiplied by 2 and the new resulting observations are $y_i,$ then

$$y_i = 2x_i \text{ i.e., } x_i = \frac{1}{2}y_i$$

Therefore,

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{20} y_i = \frac{1}{20}\sum_{i=1}^{20} 2x_i = 2 \cdot \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$\text{i.e., } \overline{y} = 2\overline{x} \text{ or } \overline{x} = \frac{1}{2}\overline{y}$$

On substituting the values of x_i and $\overline{x}$ in eq. (i), we get

$$\sum_{i=1}^{20}\left(\frac{1}{2}y_i - \frac{1}{2}\overline{y}\right)^2 = 100$$

$$\text{i.e., } \sum_{i=1}^{20}(y_i - \overline{y})^2 = 400$$

Thus, the variance of new observations

$$= \frac{1}{20} \times 400 = 20 = 2^2 \times 5$$

Quick Tip

If data is scaled by k, variance scales by k^2.

---

52. If the function $f(x)$, defined below, is continuous on the interval $[0, 8]$, then:

$$f(x) = \begin{cases} x^2 + ax + b, & 0 \le x < 2 \\ 3x + 2, & 2 \le x \le 4 \\ 2ax + 5b, & 4 < x \le 8 \end{cases}$$

(A) $a = 3, b = -2$

(B) $a = -3, b = 2$

(C) $a = -3, b = -2$

(D) $a = 3, b = 2$

Correct Answer: (A) $a = 3, b = -2$

Solution: Since $f(x)$ is continuous on $[0, 8]$, it must be continuous at $x = 2$ and $x = 4$.

At $x = 2$,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^-} (x^2 + ax + b) = \lim_{x \to 2^+} (3x + 2)$$

$$4 + 2a + b = 3(2) + 2$$

$$2a + b = 4 \quad \text{(Equation 1)}$$

At $x = 4$,

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$$

$$\lim_{x \to 4^-} (3x + 2) = \lim_{x \to 4^+} (2ax + 5b)$$

$$3(4) + 2 = 2a(4) + 5b$$

$$8a + 5b = 14 \quad \text{(Equation 2)}$$

Solving Equations 1 and 2, we get:

$$a = 3, \quad b = -2$$

Quick Tip

Continuity at a point requires matching left-hand and right-hand limits.

---

53. From the top of a cliff 50 m high, the angles of depression of the top and bottom of a tower are observed to be $30°$ and $45°$. The height of the tower is:

(A) 50 m

(B) $50\sqrt{3}$ m

(C) $50(\sqrt{3} - 1)$ m

(D) $50\left(1 - \frac{\sqrt{3}}{3}\right)$ m

Correct Answer: (D) $50\left(1 - \frac{\sqrt{3}}{3}\right)$ m

Solution: Let the height of the tower be h.

Using the tangent function in $\triangle ABD$:

$$\tan 45° = \frac{AB}{BD} \Rightarrow BD = 50 \text{ m}$$

Now, in $\triangle ACC'$:

$$\tan 30° = \frac{AC'}{C'C}$$

$$\frac{1}{\sqrt{3}} = \frac{50 - h}{50}$$

Solving for h:

$$50 = 50\sqrt{3} - h\sqrt{3}$$

$$h\sqrt{3} = 50(\sqrt{3} - 1)$$

$$h = 50\left(1 - \frac{\sqrt{3}}{3}\right) \text{ m}$$

Quick Tip

Trigonometry is useful in solving real-world height and distance problems.

---

54. ABC is a triangular park with $AB = AC = 100$ m. A TV tower stands at the midpoint of BC. The angles of elevation of the top of the tower at A, B, C are $45°, 60°, 60°$ respectively. The height of the tower is:

(A) 50 m

(B) $50\sqrt{3}$ m

(C) $50\sqrt{2}$ m

(D) $50(3 - \sqrt{3})$ m

Correct Answer: (B) $50\sqrt{3}$ m

Solution: Let $DE = h$ and $CD = DB = x$.

In $\triangle EBD$:

$$\tan 60° = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$$

Now, in $\triangle ADE$:

$$\tan 45° = \frac{ED}{DA} \Rightarrow DA = h$$

Applying Pythagoras in $\triangle ABD$:

$$\left(\frac{h}{\sqrt{3}}\right)^2 + h^2 = 100^2$$

$$\frac{4h^2}{3} = 10000$$

$$h = 50\sqrt{3}$$

Quick Tip

Using Pythagoras' theorem and trigonometry helps in height and distance calculations.

---

55. In a statistical investigation of 1003 families of Calcutta, it was found that 63 families have neither a radio nor a TV, 794 families have a radio, and 187 have a TV. The number of families having both a radio and a TV is:

(A) 36

(B) 41

(C) 32

(D) None of these

Correct Answer: (B) 41

Solution: Using set theory:

$$n(R) = 794, \quad n(T) = 187, \quad n(R \cup T)' = 63$$

$$n(Total) = n(R \cup T) + n(R \cup T)' \Rightarrow 1003 = n(R \cup T) + 63$$

$$n(R \cup T) = 940$$

Using formula:

$$n(R \cup T) = n(R) + n(T) - n(R \cap T)$$

$$940 = 794 + 187 - n(R \cap T)$$

$$n(R \cap T) = 41$$

Quick Tip

Use set operations to solve logical counting problems.

---

56. Let R be the relation "is congruent to" on the set of all triangles in a plane. Is R:

(A) Reflexive only

(B) Symmetric only

(C) Symmetric and reflexive only

(D) Equivalence relation

Correct Answer: (D) Equivalence relation

Solution:

Step 1: Check for Reflexivity

A relation is reflexive if every element is related to itself. In this case, every triangle is congruent to itself. So, $\triangle A \cong \triangle A$. Thus, the relation R is reflexive.

Step 2: Check for Symmetry

A relation is symmetric if for every a related to b, b is also related to a. If $\triangle A \cong \triangle B$, then $\triangle B \cong \triangle A$. Thus, the relation R is symmetric.

Step 3: Check for Transitivity

A relation is transitive if whenever a is related to b and b is related to c, then a is also related to c. If $\triangle A \cong \triangle B$ and $\triangle B \cong \triangle C$, then $\triangle A \cong \triangle C$. Thus, the relation R is transitive.

Step 4: Conclusion

Since the relation R is reflexive, symmetric, and transitive, it is an equivalence relation. Therefore, the correct answer is (D).

Quick Tip

A relation is an equivalence relation if it is reflexive, symmetric, and transitive.

---

57. Number of subsets of set of letters of word 'MONOTONE' is:

(A) 8

(B) 256

(C) 64

(D) 32

Correct Answer: (D) 32

Solution:

Step 1: Identify the distinct letters

The word 'MONOTONE' has the following distinct letters: M, O, N, T, E.

Step 2: Count the distinct letters

There are 5 distinct letters.

Step 3: Use the formula for the number of subsets

The number of subsets of a set with n elements is 2^n. In this case, $n = 5$.

Step 4: Calculate the number of subsets

Number of subsets = $2^5 = 32$.

> **Quick Tip**
>
> The number of subsets of a set with n elements is 2^n.

---

58. In an examination, 62% of the candidates failed in English, 42% in Mathematics and 20% in both. The number of those who passed in both the subjects is:

(A) 11

(B) 16

(C) 18

(D) None of these

Correct Answer: (B) 16

Solution:

Step 1: Calculate the percentage of candidates who failed in either English or Mathematics or both

Percentage failed in English or Mathematics or both = Percentage failed in English + Percentage failed in Mathematics - Percentage failed in both = 62% + 42% - 20% = 84%

Step 2: Calculate the percentage of candidates who passed in both subjects

Percentage passed in both subjects = 100% - Percentage failed in English or Mathematics or both = 100% - 84% = 16%

Step 3: Assume the total number of candidates

Let the total number of candidates be x.

Step 4: Calculate the number of candidates who passed in both subjects

Number of candidates who passed in both subjects = 16% of x = $\frac{16}{100} \times x$

Step 5: Relate the number to the options

Since the options are whole numbers, we can assume $x = 100$ for simplicity. Then, the number of candidates who passed in both subjects = $\frac{16}{100} \times 100 = 16$.

Quick Tip

Use the principle of inclusion-exclusion to find the percentage of candidates who failed in either English or Mathematics or both.

---

59. If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is an orthogonal matrix, then

(A) $a = -2, b = -1$

(B) $a = 2, b = 1$

(C) $a = 2, b = -1$

(D) $a = -2, b = 1$

Correct Answer: (A) $a = -2, b = -1$

Solution:

Step 1: Recall the property of orthogonal matrices

A matrix A is orthogonal if its transpose A^T is equal to its inverse A^{-1}, i.e., $A^T = A^{-1}$. This implies that $AA^T = I$, where I is the identity matrix.

Step 2: Find the transpose of matrix A

$$A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

Step 3: Multiply A and A^T

$$AA^T = \frac{1}{9}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix}$$

Step 4: Set $AA^T = I$

For $AA^T = I$, we must have:

$$\frac{1}{9}\begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 5: Solve the resulting equations

From the above equation, we get the following equations:

1. $a + 4 + 2b = 0$
2. $2a + 2 - 2b = 0$
3. $a^2 + 4 + b^2 = 9$

Step 6: Solve for a and b

Adding equations (1) and (2), we get $3a + 6 = 0$, which gives $a = -2$. Substituting $a = -2$ into equation (1), we get $-2 + 4 + 2b = 0$, which gives $2b = -2$, so $b = -1$.

Step 7: Verify the solution

Substituting $a = -2$ and $b = -1$ into equation (3), we get $(-2)^2 + 4 + (-1)^2 = 4 + 4 + 1 = 9$, which is true.

Therefore, $a = -2$ and $b = -1$.

Quick Tip

Remember that for an orthogonal matrix A, $AA^T = I$.

60. If matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ **and** $A^{-1} = \frac{1}{k}adj(A)$**, then** k **is**

(A) 7

(B) -7

(C) 15

(D) -11

Correct Answer: (C) 15

Solution:

Step 1: Recall the relationship between A^{-1} and $adj(A)$

The inverse of a matrix A is given by $A^{-1} = \frac{1}{det(A)}adj(A)$, where $det(A)$ is the determinant of A and $adj(A)$ is the adjugate of A.

Step 2: Compare with the given equation

We are given that $A^{-1} = \frac{1}{k}adj(A)$. Comparing this with the general formula, we see that $k = det(A)$.

Step 3: Calculate the determinant of A

$$\begin{vmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 3\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= 3(2(1) - (-1)(1)) + 2(1(1) - (-1)(0)) + 4(1(1) - 2(0))$$

$$= 3(2 + 1) + 2(1 - 0) + 4(1 - 0)$$

$$= 3(3) + 2(1) + 4(1)$$

$$= 9 + 2 + 4 = 15$$

Step 4: Identify the value of k

Since $k = det(A)$, we have $k = 15$.

Quick Tip

Remember the formula $A^{-1} = \frac{1}{det(A)}adj(A)$.

61. If A and B are symmetric matrices of the same order such that $AB + BA = X$ and $AB - BA = Y$, then $(XY)^T =$

(A) XY

(B) $X^T Y^T$

(C) $-YX$

(D) $-Y^T X^T$

Correct Answer: (C) $-YX$

Solution:

Step 1: Use the given equations

We are given $AB + BA = X$ and $AB - BA = Y$.

Step 2: Find X^T and Y^T

Since A and B are symmetric matrices, $A^T = A$ and $B^T = B$.

$$X^T = (AB + BA)^T = (AB)^T + (BA)^T = B^T A^T + A^T B^T = BA + AB = X$$

$$Y^T = (AB - BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA) = -Y$$

So, X is symmetric and Y is skew-symmetric.

Step 3: Compute XY

$$XY = (AB + BA)(AB - BA) = (AB)^2 - (BA)^2$$

Step 4: Compute $(XY)^T$

$$(XY)^T = ((AB)^2 - (BA)^2)^T = ((AB)^2)^T - ((BA)^2)^T = (B^T A^T)^2 - (A^T B^T)^2 = (BA)^2 - (AB)^2 = -(XY)$$

Step 5: Compute YX

$$YX = (AB - BA)(AB + BA) = (AB)^2 - (BA)^2 = -(BA)^2 + (AB)^2 = -((BA)^2 - (AB)^2) = -XY$$

Step 6: Compare $(XY)^T$ with YX

We found that $(XY)^T = -YX$.

Quick Tip

Remember that $(AB)^T = B^T A^T$, and if A is symmetric, $A^T = A$.

---

62. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $X = APA^T$, then $A^T X^{50} A$ is:

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 25 & 1 \\ 1 & -25 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

Solution:

Step 1: Show that A is an orthogonal matrix

Since $AA^T = I$, the matrix A is orthogonal.

Step 2: Simplify $A^T X^{50} A$

$$A^T X^{50} A = A^T X^{49} (APA^T) A$$

$$= A^T X^{49} AP(A^T A) = A^T X^{49} AP$$

$$= A^T X^{48} (APA^T) AP = A^T X^{48} AP^2 \dots$$

$$= A^T AP^{50} = IP^{50} = P^{50}.$$

Step 3: Compute P^{50}

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\vdots$$

$$P^{50} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$$

Step 4: Conclusion

$$A^T X^{50} A = P^{50} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}.$$

Quick Tip

For a matrix of the form $P = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$, its nth power is given by:

$$P^n = \begin{bmatrix} 1 & nk \\ 0 & 1 \end{bmatrix}.$$

---

63. If A is a square matrix of order 3, then $|\text{Adj}(\text{Adj } A^2)|$ is:

(A) $|A|^2$

(B) $|A|^4$

(C) $|A|^8$

(D) $|A|^{16}$

Correct Answer: (C) $|A|^8$

Solution:

Step 1: Use the determinant property of adjugate matrices

For a square matrix A of order n, the determinant of its adjugate is given by:

$$|\text{adj } A| = |A|^{n-1}.$$

Since A is of order 3, we get:

$$|\text{adj } A^2| = |A^2|^{3-1} = |A^2|^2.$$

Step 2: Simplify further

$$|A^2| = (|A|^2),$$

$$|\text{adj } A^2| = (|A|^2)^2 = |A|^4.$$

Step 3: Compute $|\text{Adj}(\text{Adj } A^2)|$

$$|\text{Adj}(\text{Adj } A^2)| = (|A|^4)^{3-1} = (|A|^4)^2 = |A|^8.$$

Step 4: Conclusion

Thus, the correct answer is $|A|^8$.

Quick Tip

For any square matrix A of order n, we have:

$$|\text{Adj } A| = |A|^{n-1}.$$

---

64. Suppose $p, q, r \neq 0$ and the system of equations:

$$(p + a)x + by + cz = 0$$

$$ax + (q + b)y + cz = 0$$

$$ax + by + (r + c)z = 0$$

has a non-trivial solution, then the value of

$$\frac{a}{p} + \frac{b}{q} + \frac{c}{r}$$

is:

(A) -1

(B) 0

(C) 1

(D) 2

Correct Answer: (A) -1

Solution:

Step 1: Construct the determinant of the coefficient matrix

$$|A| = \begin{vmatrix} p+a & b & c \\ a & q+b & c \\ a & b & r+c \end{vmatrix}.$$

Since the system has a non-trivial solution, the determinant must be zero:

$$|A| = 0.$$

Step 2: Expand determinant using row operations

$$(p+a)[(q+b)(r+c) - bc] - b[a(r+c) - ca] + c[ab - a(q+b)] = 0.$$

$$(p+a)(qr + qc + br) - b(ar) + c[-aq] = 0.$$

Step 3: Divide by pqr

$$\frac{pqr}{pqr} + \frac{pqc}{pqr} + \frac{prb}{pqr} + \frac{qra}{pqr} = 0.$$

$$1 + \frac{c}{r} + \frac{b}{q} + \frac{a}{p} = 0.$$

Step 4: Conclusion

$$\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = -1.$$

Quick Tip

For a homogeneous system $AX = 0$ to have a non-trivial solution, the determinant $|A|$ must be zero.

---

65. If x is a complex root of the equation

$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} + \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0,$$

then $x^{2007} + x^{-2007}$ is:

(A) 1

(B) -1

(C) -2

(D) 2

Correct Answer: (C) -2

Solution:

Step 1: Expand both determinants

$$(1 - 3x^2 + 2x^3) + (3x^2 - x^3) = 0.$$

Step 2: Solve for x

$$x^3 + 1 = 0.$$

$$x^3 = -1.$$

$$x = -\omega, -\omega^2, -1.$$

Step 3: Compute $x^{2007} + x^{-2007}$

Since $x^3 = -1$,

$$x^{2007} = (-1)^{669} = -1.$$

$$x^{-2007} = -1.$$

Step 4: Conclusion

$$x^{2007} + x^{-2007} = -1 - 1 = -2.$$

Quick Tip

If x is a cube root of unity, then it satisfies $x^3 = -1$, which helps simplify large exponents.

66. The system of equations:

$$x - y + 2z = 4$$

$$3x + y + 4z = 6$$

$$x + y + z = 1$$

has:

(A) unique solution

(B) infinitely many solutions

(C) no solution

(D) two solutions

Correct Answer: (B) Infinitely many solutions

Solution:

Step 1: Compute the determinant of the coefficient matrix

$$\Delta = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}.$$

Expanding along the first row:

$$\Delta = 1(1 \times 1 - 4 \times 1) + (-1)(3 \times 1 - 4 \times 1) + 2(3 \times 1 - 1 \times 1).$$

$$= (1 - 4) + (-3 + 4) + 2(3 - 1).$$

$$= -3 + 1 + 4 = 0.$$

Step 2: Compute determinant of augmented matrix

$$\Delta_1 = \begin{vmatrix} 4 & -1 & 2 \\ 6 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}.$$

Expanding along the first row:

$$\Delta_1 = 4(1 \times 1 - 4 \times 1) + (-1)(6 \times 1 - 4 \times 1) + 2(6 \times 1 - 1 \times 1).$$

$$= 4(1-4) + (-6+4) + 2(6-1).$$

$$= -12 + 2 + 10 = 0.$$

Step 3: Conclusion

Since $\Delta = 0$ and $\Delta_1 = 0$, the system has infinitely many solutions.

Quick Tip

If $\Delta = 0$ and all augmented determinants are also zero, the system has infinitely many solutions.

---

67. If the system of linear equations:

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

$$x + 4y + \delta z = k$$

has infinitely many solutions, then $\delta + k$ is:

(A) -3

(B) 3

(C) 6

(D) 9

Correct Answer: (B) 3

Solution:

Step 1: Compute determinant of the coefficient matrix

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix}.$$

Since the system has infinitely many solutions, $\Delta = 0$.

Expanding along the first row:

$$\Delta = 2(-3 \times \delta - 2 \times 4) - 1(1 \times \delta - 2 \times 1) + (-1)(1 \times 4 + 3 \times 1).$$

Solving for δ:

$$-6\delta - 8 - \delta + 2 - 4 - 3 = 0.$$

$$-7\delta - 13 = 0.$$

$$\delta = -3.$$

Step 2: Compute augmented determinant

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ k & 4 & -3 \end{vmatrix}.$$

Setting $\Delta_1 = 0$, solving for k:

$$k = 6.$$

Step 3: Conclusion

$$\delta + k = -3 + 6 = 3.$$

Quick Tip

For infinitely many solutions, the determinant of the coefficient matrix and all augmented determinants must be zero.

---

68. If $\cot(\cos^{-1} x) = \sec\left(\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right)$**, then:**

(A) $\frac{b}{\sqrt{2b^2-a^2}}$

(B) $\frac{\sqrt{b^2-a^2}}{ab}$

(C) $\frac{a}{\sqrt{2b^2-a^2}}$

(D) $\frac{\sqrt{b^2-a^2}}{a}$

Correct Answer: (A) $\frac{b}{\sqrt{2b^2-a^2}}$

Solution:

Step 1: Express cotangent and secant

$$\cot(\cos^{-1} x) = \frac{x}{\sqrt{1-x^2}}.$$

$$\sec\left(\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right) = \sqrt{1+\left(\frac{a}{\sqrt{b^2-a^2}}\right)^2}.$$

Step 2: Equating expressions

$$\frac{x}{\sqrt{1-x^2}} = \sqrt{1+\frac{a^2}{b^2-a^2}}.$$

Simplify:

$$\frac{x}{\sqrt{1-x^2}} = \sqrt{\frac{b^2-a^2+a^2}{b^2-a^2}}.$$

$$\frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2-a^2}}.$$

Squaring both sides:

$$x^2(2b^2-a^2) = b^2.$$

Solving for x:

$$x = \frac{b}{\sqrt{2b^2-a^2}}.$$

Quick Tip

For inverse trigonometric functions, rewrite in terms of known trigonometric identities to simplify expressions.

---

69. If $\cos\cot^{-1}\left(\frac{1}{2}\right) = \cot(\cos^{-1}x)$**, then the value of** x **is:**

(A) $\frac{1}{\sqrt{6}}$

(B) $\frac{-1}{\sqrt{12}}$

(C) $\frac{2}{\sqrt{6}}$

(D) $\frac{-2}{\sqrt{6}}$

Correct Answer: (A) $\frac{1}{\sqrt{6}}$

Solution:

Step 1: Express $\cot^{-1}$ in terms of cosine

Let

$$\alpha = \cot^{-1}\left(\frac{1}{2}\right).$$

Then,

$$\cot\alpha = \frac{1}{2} \Rightarrow \cos\alpha = \frac{1}{\sqrt{5}}.$$

Step 2: Use cotangent identity

$$\cos(\cos^{-1} x) = \cot\left(\cos^{-1} x\right).$$

Using the identity:

$$\cot(\cos^{-1} x) = \frac{x}{\sqrt{1-x^2}}.$$

Step 3: Equating both sides

$$\frac{1}{\sqrt{5}} = \frac{x}{\sqrt{1-x^2}}.$$

Squaring both sides:

$$1 - x^2 = 5x^2.$$

Step 4: Solve for x

$$1 = 6x^2.$$

$$x = \pm\frac{1}{\sqrt{6}}.$$

Step 5: Select the correct sign

Ignoring the negative root:

$$x = \frac{1}{\sqrt{6}}.$$

Quick Tip

To convert inverse trigonometric expressions, use the basic definitions of trigonometric functions in right-angled triangles.

---

70. Let $[x]$ denote the greatest integer $\leq x$. If $f(x) = [x]$ and $g(x) = |x|$, then the value of:

$$f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(\frac{-8}{5}\right)\right)$$

is:

(A) 2

(B) -2

(C) 1

(D) -1

Correct Answer: (D) -1

Solution:

Step 1: Compute $f(-8/5)$

$$f\left(\frac{-8}{5}\right) = \left\lfloor \frac{-8}{5} \right\rfloor = -2.$$

Step 2: Compute $g(8/5)$ and $g(-8/5)$

$$g\left(\frac{8}{5}\right) = \left|\frac{8}{5}\right| = \frac{8}{5}.$$
$$g\left(\frac{-8}{5}\right) = \left|\frac{-8}{5}\right| = \frac{8}{5}.$$

Step 3: Compute $f(g(8/5))$ and $g(f(-8/5))$

$$f\left(g\left(\frac{8}{5}\right)\right) = f\left(\frac{8}{5}\right) = \left\lfloor \frac{8}{5} \right\rfloor = 1.$$
$$g\left(f\left(\frac{-8}{5}\right)\right) = g(-2) = |-2| = 2.$$

Step 4: Final computation

$$f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(\frac{-8}{5}\right)\right) = 1 - 2 = -1.$$

Step 5: Conclusion

Thus, the correct answer is -1.

Quick Tip

The greatest integer function $[x]$ returns the largest integer less than or equal to x. The absolute function $|x|$ removes the sign.

---

71. The number of real solutions of

$$\sqrt{5 - \log_2 |x|} = 3 - \log_2 |x|$$

is:

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Substituting $\log_2 |x| = t$

Let

$$\log_2 |x| = t.$$

Thus, the equation becomes:

$$\sqrt{5 - t} = 3 - t.$$

Step 2: Squaring both sides

$$5 - t = (3 - t)^2.$$

Expanding:

$$5 - t = 9 + t^2 - 6t.$$

$$t^2 - 5t + 4 = 0.$$

Step 3: Solving for t

$$(t - 4)(t - 1) = 0.$$

$$t = 4 \quad \text{or} \quad t = 1.$$

Rejecting $t = 4$ as it violates the equation.

Step 4: Finding x

$$\log_2 |x| = 1.$$

$$|x| = 2.$$

$$x = \pm 2.$$

Step 5: Conclusion

Thus, there are 2 real solutions: $x = 2, -2$.

Quick Tip

When solving logarithmic equations, check that all solutions satisfy the original equation.

---

72. The function

$$f(x) = \frac{\cos x}{\left\lfloor \frac{2x}{\pi} \right\rfloor + \frac{1}{2}},$$

where x is not an integral multiple of π and $\lfloor \cdot \rfloor$ denotes the greatest integer function, is:

(A) an odd function

(B) an even function

(C) neither odd nor even

(D) None of these

Correct Answer: (A) an odd function

Solution:

Step 1: Compute $f(-x)$

$$f(-x) = \frac{\cos(-x)}{\left\lfloor \frac{2(-x)}{\pi} \right\rfloor + \frac{1}{2}}.$$

Using $\cos(-x) = \cos x$ and property of floor function:

$$\left\lfloor \frac{2(-x)}{\pi} \right\rfloor = -\left\lfloor \frac{2x}{\pi} \right\rfloor - 1.$$

Step 2: Compare $f(-x)$ with $-f(x)$

$$f(-x) = -f(x).$$

Step 3: Conclusion

Since $f(-x) = -f(x)$, the function is odd.

Quick Tip

A function is odd if $f(-x) = -f(x)$ and even if $f(-x) = f(x)$.

---

73. The function f: R→ R is defined by

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

is:

(A) surjective but not injective

(B) bijective

(C) injective but not surjective

(D) neither injective nor surjective

Correct Answer: (C) injective but not surjective

Solution:

Step 1: Check injectivity

For x_1, x_2 such that:

$$f(x_1) = f(x_2),$$

$$\frac{x_1}{\sqrt{1+x_1^2}} = \frac{x_2}{\sqrt{1+x_2^2}}.$$

Squaring both sides:

$$x_1^2(1+x_2^2) = x_2^2(1+x_1^2).$$

Solving, we get:

$$x_1 = x_2.$$

Thus, $f(x)$ is injective.

Step 2: Check surjectivity

Solving for y:

$$y = \frac{x}{\sqrt{1+x^2}}.$$

This implies:

$$y^2(1+x^2) = x^2.$$

Solving, we find $y \in (-1, 1)$, meaning $f(x)$ is not surjective.

Step 3: Conclusion

$f(x)$ is injective but not surjective.

Quick Tip

A function is injective if distinct inputs yield distinct outputs. It is surjective if its range covers the entire codomain.

---

74. If $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = 5x - 3$, $g(x) = x^2 + 3$, then $g \circ f^{-1}(3)$ is equal to

(A) $\frac{25}{3}$

(B) $\frac{111}{25}$

(C) $\frac{9}{25}$

(D) $\frac{25}{111}$

Correct Answer: (B) $\frac{111}{25}$

Solution:

Step 1: Find $f^{-1}(3)$

$$y = f(x) = 5x - 3.$$

$$x = \frac{y+3}{5}.$$

$$f^{-1}(3) = \frac{6}{5}.$$

Step 2: Compute $g(f^{-1}(3))$

$$g(x) = x^2 + 3.$$

$$g\left(\frac{6}{5}\right) = \left(\frac{6}{5}\right)^2 + 3.$$

$$= \frac{36}{25} + 3 = \frac{111}{25}.$$

Step 3: Conclusion

Thus, $g \circ f^{-1}(3) = \frac{111}{25}$.

Quick Tip

To find $g \circ f^{-1}$, first determine $f^{-1}(x)$, then substitute into $g(x)$.

---

75. The domain of the real-valued function

$$f(x) = \sqrt{\frac{2x^2 - 7x + 5}{3x^2 - 5x - 2}}$$

is:

(A) $(-\infty, -\frac{1}{3}) \cup [1, 2) \cup [\frac{5}{2}, \infty)$

(B) $(-\infty, 1) \cup (2, \infty)$

(C) $(-\frac{1}{3}, \frac{5}{2})$

(D) $(-\infty, -\frac{1}{3}] \cup [\frac{5}{2}, \infty)$

Correct Answer: (A) $(-\infty, -\frac{1}{3}) \cup [1, 2) \cup [\frac{5}{2}, \infty)$

Solution:

Step 1: Find the domain restrictions

For the function $f(x)$ to be defined, the expression inside the square root must be non-negative:

$$\frac{2x^2 - 7x + 5}{3x^2 - 5x - 2} \geq 0.$$

Also, the denominator must not be zero, i.e.,

$$3x^2 - 5x - 2 \neq 0.$$

Step 2: Find the zeros of the numerator

Solving:

$$2x^2 - 7x + 5 = 0.$$

Factoring:

$$(2x - 5)(x - 1) = 0.$$

$$x = \frac{5}{2}, 1.$$

Step 3: Find the zeros of the denominator

Solving:

$$3x^2 - 5x - 2 = 0.$$

Factoring:

$$(3x + 1)(x - 2) = 0.$$

$$x = -\frac{1}{3}, 2.$$

Step 4: Analyze sign changes using a number line

The critical points partition the number line into intervals:

$$(-\infty, -\frac{1}{3}), (-\frac{1}{3}, 1), (1, 2), (2, \frac{5}{2}), (\frac{5}{2}, \infty).$$

By testing values in each interval, the function is non-negative in:

$$(-\infty, -\frac{1}{3}) \cup [1, 2) \cup [\frac{5}{2}, \infty).$$

Step 5: Conclusion

Thus, the domain is:

$$(-\infty, -\frac{1}{3}) \cup [1, 2) \cup [\frac{5}{2}, \infty).$$

Quick Tip

For rational functions inside a square root, ensure the numerator is non-negative while avoiding zeros of the denominator.

---

76. If a function $f : \mathbb{R} \setminus \{1\} \to \mathbb{R} \setminus \{m\}$ defined by $f(x) = \frac{x+3}{x-2}$ is a bijection, then

$3/l + 2m =$

(A) 10

(B) 12

(C) 8

(D) 14

Correct Answer: (C) 8

Solution:

Step 1:

Identify the value x cannot take

The function $f(x) = \frac{x+3}{x-2}$ is undefined when the denominator is equal to zero.

$$x - 2 = 0$$

$$x = 2$$

Therefore, x cannot take the value 2. So, $l = 2$.

Step 2: Determine the values $f(x)$ cannot take

Let $y = f(x)$. We want to find the values that y cannot take.

$$y = \frac{x+3}{x-2}$$

$$y(x-2) = x+3$$

$$xy - 2y = x + 3$$

$$xy - x = 2y + 3$$

$$x(y-1) = 2y + 3$$

$$x = \frac{2y+3}{y-1}$$

From this expression for x in terms of y, we can see that y cannot take the value 1, as the denominator would be zero. Therefore, $m = 1$.

Step 3: Calculate $3l + 2m$

We have $l = 2$ and $m = 1$.

$$3l + 2m = 3(2) + 2(1) = 6 + 2 = 8$$

Therefore, $3l + 2m = 8$.

Quick Tip

For bijections involving rational functions, ensure the function is not undefined at any point within its intended domain or that it avoids specific values in its codomain.

---

77. Given that $f(x) = \sin x + \cos x$ and $g(x) = x^2 - 1$, find the conditions under which $g(f(x))$ is invertible.

(A) $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

(B) $0 \leq x \leq \pi$

(C) $-\frac{\pi}{4} \leq x \leq \pi$

(D) $0 \leq x \leq \frac{\pi}{2}$

Correct Answer: (A) $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

Solution:

Step 1: Find $g[f(x)]$

We are given $f(x) = \sin x + \cos x$ and $g(x) = x^2 - 1$. We need to find $g[f(x)]$, which means $g(f(x))$.

$$g[f(x)] = g(\sin x + \cos x)$$

Substitute $\sin x + \cos x$ in place of x in the expression for $g(x)$:

$$g[f(x)] = (\sin x + \cos x)^2 - 1$$

Step 2: Expand and simplify $g[f(x)]$

Expand the square:

$$g[f(x)] = (\sin^2 x + 2\sin x \cos x + \cos^2 x) - 1$$

Recall the trigonometric identities:

$$\sin^2 x + \cos^2 x = 1$$

$$2\sin x \cos x = \sin 2x$$

Substitute these identities into the expression:

$$g[f(x)] = (1 + \sin 2x) - 1$$

$$g[f(x)] = \sin 2x$$

Step 3: Analyze the monotonicity of $\sin 2x$

The function $\sin 2x$ is a sinusoidal function. We need to find an interval where it is strictly increasing or strictly decreasing (i.e., monotonic) to ensure invertibility.

Step 4: Determine an interval for invertibility

The sine function is strictly increasing in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Since we have $\sin 2x$, we need to consider the argument $2x$. For $\sin 2x$ to be strictly increasing, we need $2x$ to be in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

$$-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2}$$

Divide by 2:

$$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

In this interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$, $\sin 2x$ is strictly increasing, and therefore invertible.

Conclusion: $g[f(x)] = \sin 2x$. The function $g[f(x)]$ is invertible in the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$.

Quick Tip

Invertibility requires the function to be monotonic (either strictly increasing or strictly decreasing) over the interval.

---

78. Let the function $g : (-\infty, -0) \to (-\frac{\pi}{2}, \frac{\pi}{2})$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$.

Determine the properties of g.

(A) Even and is strictly increasing in $(0, \infty)$

(B) Odd and is strictly decreasing in $(-\infty, 0)$

(C) Odd and is strictly increasing in $(-\infty, \infty)$

(D) Neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

Correct Answer: (C) Odd and is strictly increasing in $(-\infty, \infty)$

Solution:

Step 1: Analyze evenness or oddness

$$g(-u) = 2\tan^{-1}(e^{-u}) - \frac{\pi}{2}$$
$$= 2\left(\frac{\pi}{2} - \tan^{-1}(e^u)\right) - \frac{\pi}{2}$$
$$= \pi - 2\tan^{-1}(e^u) - \frac{\pi}{2} = -2\tan^{-1}(e^u) + \frac{\pi}{2}$$
$$= -(2\tan^{-1}(e^u) - \frac{\pi}{2}) = -g(u)$$

Thus, g is an odd function.

Step 2: Verify increasing nature

The derivative $g'(u) = 2\frac{1}{1+e^{2u}}e^u > 0$ for all u, indicating g is strictly increasing over $(-\infty, \infty)$.

Quick Tip

Remember, for a function to be odd, $f(-x) = -f(x)$ must hold true, and the function's derivative should be positive for increasing nature.

---

79. Let f be the function defined by:

$$f(x) = \begin{cases} \frac{x^2-1}{x^2-2|x-1|-1}, & \text{if } x \neq 1, \\ \frac{1}{2}, & \text{if } x = 1. \end{cases}$$

The function is continuous at:

(A) The function is continuous for all values of x

(B) The function is continuous only for $x > 1$

(C) The function is continuous at $x = 1$

(D) The function is not continuous at $x = 1$

Correct Answer: (D) The function is not continuous at $x = 1$

Solution:

Step 1: To determine the continuity of $f(x)$ at $x = 1$, we need to check if the left-hand limit (LHL), the right-hand limit (RHL), and the function's value at $x = 1$ are all equal.

Step 1: Calculate the Left-Hand Limit (LHL)

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2 - 1}{x^2 + 2x - 3}$$

Factor the numerator and denominator:

$$\lim_{x \to 1^-} \frac{(x-1)(x+1)}{(x+3)(x-1)}$$

Cancel the common factor $(x - 1)$:

$$\lim_{x \to 1^-} \frac{x+1}{x+3}$$

Substitute $x = 1$:

$$\frac{1+1}{1+3} = \frac{2}{4} = \frac{1}{2}$$

Step 2: Calculate the Right-Hand Limit (RHL)

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x^2 - 1}{x^2 - 2x + 1}$$

Factor the numerator and denominator:

$$\lim_{x \to 1^+} \frac{(x-1)(x+1)}{(x-1)^2}$$

Cancel the common factor $(x - 1)$:

$$\lim_{x \to 1^+} \frac{x+1}{x-1}$$

As x approaches 1 from the right (i.e., $x > 1$), the numerator approaches 2, and the denominator approaches 0 from the positive side. Thus, the limit is $+\infty$.

$$\lim_{x \to 1^+} \frac{x+1}{x-1} = +\infty$$

Step 3: Evaluate the function at $x = 1$

From the definition of $f(x)$, we have $f(1) = \frac{1}{2}$.

Step 4: Compare LHL, RHL, and $f(1)$

We have:

$$\text{LHL} = \frac{1}{2}$$

$$\text{RHL} = +\infty$$

$$f(1) = \frac{1}{2}$$

Conclusion:

Since LHL $\neq$ RHL, the limit $\lim_{x \to 1} f(x)$ does not exist. Therefore, the function $f(x)$ is discontinuous at $x = 1$.

Also, although LHL = $f(1)$, the function is still discontinuous at $x = 1$ because the RHL is not equal to these values. For a function to be continuous at a point, the LHL, RHL, and the value of the function at that point must all be equal.

Quick Tip

For a function to be continuous at a point, the left-hand limit, right-hand limit, and the function's value at that point must all exist and be equal.

---

80. If

$$f(x) = \begin{cases} \frac{x^2 \log(\cos x)}{\log(1+x)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

then at $x = 0$, $f(x)$ is .

(A) not continuous

(B) continuous but not differentiable

(C) differentiable

(D) not continuous, but differentiable

Correct Answer: (C) differentiable

Solution:

Step 1: Check for continuity at $x = 0$

We need to find the limit of $f(x)$ as $x \to 0$:

$$\lim_{x\to 0} \frac{x^2 \log(\cos x)}{\log(1+x)} = \lim_{x\to 0} x^2 \cdot \log(\cos x) = 0 \cdot \log(1) = 0$$

Thus, $f(x)$ is continuous at $x = 0$.

Step 2: Check for differentiability at $x = 0$

We now check if $f(x)$ is differentiable at $x = 0$ by finding the derivative at this point:

$$f'(0) = \lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} \frac{h^2 \log(\cos h)}{h \log(1+h)} = 0$$

Thus, $f(x)$ is differentiable at $x = 0$.

Quick Tip

To check if a function is differentiable at a point, verify both its continuity and the existence of the derivative at that point.

---

81. If $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 4, & \text{if } -\infty < x < -\sqrt{5}, \\ x^2 - 1, & \text{if } -\sqrt{5} \le x \le \sqrt{5}, \\ 4, & \text{if } \sqrt{5} \le x < \infty. \end{cases}$$

If k is the number of points where $f(x)$ is not differentiable, then $k - 2 =$

(A) 2

(B) 1

(C) 0

(D) 3

Correct Answer: (C) 0

Solution:

Step 1: Check the points of non-differentiability

We know that for a function to be differentiable at a point, both the left-hand derivative (LHD) and right-hand derivative (RHD) must be equal at that point.

At $x = -\sqrt{5}$, the left-hand derivative is 0, but the right-hand derivative is $2x = -2\sqrt{5}$.

Therefore, $f(x)$ is not differentiable at $x = -\sqrt{5}$.

Similarly, at $x = \sqrt{5}$, the left-hand derivative is $2x = 2\sqrt{5}$, and the right-hand derivative is 0, meaning $f(x)$ is not differentiable at $x = \sqrt{5}$.

Thus, $k = 2$.

Step 2: Calculate $k - 2$

Since $k = 2$, we find:

$$k - 2 = 2 - 2 = 0$$

> Quick Tip
>
> For piecewise functions, check the left-hand and right-hand derivatives at the boundary points to determine non-differentiability.

---

82. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then find $\frac{dy}{dx}$.

(A) $x + \frac{1}{x}$

(B) $\frac{1}{1+x}$

(C) $-\frac{1}{(1+x)^2}$

(D) $\frac{x}{1+x}$

Correct Answer: (C) $-\frac{1}{(1+x)^2}$

Solution:

Step 1: Implicit Differentiation

Given the equation $x\sqrt{1+y} + y\sqrt{1+x} = 0$, differentiate both sides with respect to x:

$$\frac{d}{dx}\left(x\sqrt{1+y}\right) + \frac{d}{dx}\left(y\sqrt{1+x}\right) = 0$$

Using the product rule:

$$\frac{d}{dx}\left(x\sqrt{1+y}\right) = \sqrt{1+y} + x \cdot \frac{1}{2\sqrt{1+y}} \cdot \frac{dy}{dx}$$

$$\frac{d}{dx}\left(y\sqrt{1+x}\right) = \sqrt{1+x} \cdot \frac{dy}{dx} + y \cdot \frac{1}{2\sqrt{1+x}}$$

Now, substitute and solve for $\frac{dy}{dx}$.

Step 2: Solve for $\frac{dy}{dx}$

After simplifying, we find:

$$\frac{dy}{dx} = -\frac{1}{(1+x)^2}$$

Quick Tip

When differentiating implicitly, apply the product and chain rule carefully, and isolate $\frac{dy}{dx}$ to solve for it.

---

83. If $y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$, then $y'(1)$ is equal to:

(A) 0

(B) $\frac{1}{2}$

(C) -1

(D) $-\frac{1}{4}$

Correct Answer: (D) $-\frac{1}{4}$

Solution:

Step 1: Rewrite the given equation using the inverse tangent identity

We are given $y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+\sqrt{x}\cdot x}\right)$. Recall the identity:

$$\tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1} a - \tan^{-1} b$$

Using this identity, we can rewrite the given equation as:

$$y = \tan^{-1}(\sqrt{x}) - \tan^{-1}(x)$$

Step 2: Differentiate y with respect to x

Differentiate both sides with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}(\sqrt{x}) - \tan^{-1}(x)\right)$$

$$y'(x) = \frac{d}{dx}\tan^{-1}(\sqrt{x}) - \frac{d}{dx}\tan^{-1}(x)$$

Recall that $\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx}$.

$$y'(x) = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{d}{dx}(\sqrt{x}) - \frac{1}{1+x^2}$$

$$y'(x) = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$

Step 3: Evaluate $y'(1)$

Substitute $x = 1$ into the expression for $y'(x)$:

$$y'(1) = \frac{1}{1+1} \cdot \frac{1}{2\sqrt{1}} - \frac{1}{1+1^2}$$

$$y'(1) = \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2}$$

$$y'(1) = \frac{1}{4} - \frac{1}{2}$$

$$y'(1) = \frac{1}{4} - \frac{2}{4}$$

$$y'(1) = -\frac{1}{4}$$

Therefore, $y'(1) = -\frac{1}{4}$.

Quick Tip

To differentiate inverse trigonometric functions, remember the chain rule and handle the rational expressions carefully.

---

84. At $x = \frac{\pi^2}{4}$, $\frac{d}{dx}\left(\tan^{-1}(\cos\sqrt{x}) + \sec^{-1}(e^x)\right) =$

(A) $\frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}} - \frac{1}{\pi}$

(B) $\frac{\pi}{4} + \frac{1}{\sqrt{e^{\pi^2}+e^{\frac{\pi^2}{2}}}}$

(C) $\frac{1}{\sqrt{e^{\pi^2}+e^{\frac{\pi^2}{2}}}} + \frac{2}{\pi}\cot\left(\frac{\pi}{2}\right)$

(D) $\frac{1}{\sqrt{e^{\pi}}} + \frac{1}{\pi}$

Correct Answer: (A) $\frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}} - \frac{1}{\pi}$

Solution:

Step 1: Differentiate the given expression with respect to x

Let $y = \tan^{-1}(\cos\sqrt{x}) + \sec^{-1}(e^x)$. We want to find $\frac{dy}{dx}$. Using the chain rule, we have:

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}(\cos\sqrt{x}) + \frac{d}{dx}\sec^{-1}(e^x)$$

$$= \frac{1}{1+(\cos\sqrt{x})^2} \cdot \frac{d}{dx}(\cos\sqrt{x}) + \frac{1}{|e^x|\sqrt{e^{2x}-1}} \cdot \frac{d}{dx}(e^x)$$

$$= \frac{1}{1+\cos^2\sqrt{x}} \cdot (-\sin\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} + \frac{e^x}{e^x\sqrt{e^{2x}-1}}$$

$$= -\frac{\sin\sqrt{x}}{2\sqrt{x}(1+\cos^2\sqrt{x})} + \frac{1}{\sqrt{e^{2x}-1}}$$

Step 2: Evaluate the derivative at $x = \frac{\pi^2}{4}$

Substitute $x = \frac{\pi^2}{4}$ into the derivative:

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi^2}{4}} = -\frac{\sin\sqrt{\frac{\pi^2}{4}}}{2\sqrt{\frac{\pi^2}{4}}(1+\cos^2\sqrt{\frac{\pi^2}{4}})} + \frac{1}{\sqrt{e^{2(\frac{\pi^2}{4})}-1}}$$

$$= -\frac{\sin\frac{\pi}{2}}{2(\frac{\pi}{2})(1+\cos^2\frac{\pi}{2})} + \frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}}$$

$$= -\frac{1}{\pi(1+0)} + \frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}}$$

$$= -\frac{1}{\pi} + \frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}}$$

$$= \frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}} - \frac{1}{\pi}$$

Therefore, the value of the given expression at $x = \frac{\pi^2}{4}$ is $\frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}} - \frac{1}{\pi}$, which matches option (A).

Quick Tip

Remember the derivatives of inverse trigonometric functions and use the chain rule appropriately. Also, remember that $\sec^{-1}(x)$ is defined for $|x| \geq 1$, and its derivative is given by $\frac{1}{|x|\sqrt{x^2-1}}$.

---

85. The maximum area of a rectangle inscribed in a circle of diameter R is:

(A) R^2

(B) $\frac{R^2}{2}$

(C) $\frac{R^2}{4}$

(D) $\frac{R^2}{8}$

Correct Answer: (B) $\frac{R^2}{2}$

Solution:

Step 1: Find the maximum area of the rectangle

The diagonal of the rectangle inscribed in the circle is equal to the diameter R, so the diagonal $d = R$.

The maximum area of the rectangle is given by:

$$\text{Max Area} = \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times R \times R = \frac{R^2}{2}.$$

> **Quick Tip**
>
> For a rectangle inscribed in a circle, the maximum area occurs when the diagonals are equal, and the area is half the product of the diagonals.

---

86. Consider the function $f(x) = \frac{|x-1|}{x^2}$. Then $f(x)$ is:

(A) Increasing in $(0, 1) \cup (2, \infty)$

(B) Increasing in $(-\infty, 0) \cup (0, 1)$

(C) Decreasing in $(-\infty, 0) \cup (2, \infty)$

(D) Decreasing in $(0, 1) \cup (2, \infty)$

Correct Answer: (D) Decreasing in $(0, 1) \cup (2, \infty)$

Solution:

Step 1: Given:

$$f(x) = \begin{cases} \frac{x-1}{x^2}; & x \geq 1 \\ \frac{-x+1}{x^2}; & x < 1 \end{cases}$$

Simplifying $f(x)$:

$$f(x) = \begin{cases} \frac{1}{x} - \frac{1}{x^2}; & x \geq 1 \\ -\frac{1}{x} + \frac{1}{x^2}; & x < 1 \end{cases}$$

Differentiating $f(x)$ with respect to x:

$$f'(x) = \begin{cases} -\frac{1}{x^2} + \frac{2}{x^3}; & x \geq 1 \\ \frac{1}{x^2} - \frac{2}{x^3}; & x < 1 \end{cases}$$

Further simplification of $f'(x)$:

$$f'(x) = \begin{cases} \frac{2-x}{x^3}; & x \geq 1 \\ \frac{x-2}{x^3}; & x < 1 \end{cases}$$

Observation of $f'(x)$:

By observation, $f'(x)$ will be:

- Positive for $x < 0$ and $1 < x < 2$
- Negative for $0 < x < 1$ and $x > 2$

Monotonicity of $f(x)$:

$-\infty$		0		1	
2		$+\infty$			
	+		-		+
	-				
		Undefined			

- Decreasing in $(0, 1) \cup (2, \infty)$
- Increasing in $(-\infty, 0) \cup (1, 2)$

Quick Tip

Check the first derivative of the function to determine intervals of increase or decrease.

---

87. The maximum volume (in cu. units) of the cylinder which can be inscribed in a sphere of radius 12 units is:

(A) $384\sqrt{3}\pi$

(B) $768\sqrt{3}\pi$

(C) $768\pi/\sqrt{3}$

(D) $1152\pi/\sqrt{3}$

Correct Answer: (B) $768\sqrt{3}\pi$

Solution:

Step 1: Find the relation between radius and height of the cylinder inscribed in a sphere

Let the radius of the cylinder be r and height h. The equation for the sphere is $r^2 + \left(\frac{h}{2}\right)^2 = 12^2$, or:

$$r^2 + \frac{h^2}{4} = 144.$$

$$\Rightarrow V = 144\pi h - \frac{\pi}{4}h^3$$

$$\Rightarrow \frac{dV}{dh} = 144\pi - \frac{3\pi}{4}h^2$$

$$\Rightarrow \frac{dV}{dh} = 0 \Rightarrow 144\pi = \frac{3\pi}{4}h^2$$

$$\Rightarrow h^2 = 48 \times 4 \Rightarrow h = 8\sqrt{3}$$

$$\therefore 12^2 = r^2 + 48 \Rightarrow r^2 = 96$$

$$\text{Volume} = \pi r^2 h = \pi \times 96 \times 8\sqrt{3} = 768\sqrt{3}\pi \text{ cm}^3.$$

By solving the optimization problem, the maximum volume comes out to be

$$768\sqrt{3}\pi)$$

Quick Tip

To maximize the volume of a cylinder inscribed in a sphere, use the relation between radius and height of the sphere, and then differentiate to find the maximum.

---

88. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$, $y = 12(1 + \sin t)^2$, with $0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to:

(A) $6(3 + 2\sqrt{2})$

(B) $3(7 + 4\sqrt{3})$

(C) 27

(D) 48

Correct Answer: (C) 27

Solution:

Step 1: Differentiate the given parametric equations with respect to t to find $\frac{dy}{dx}$.

We are given:

$$x = 12(t + \sin t \cos t), \quad y = 12(1 + \sin t)^2$$

$$\frac{dx}{dt} = 12(1 + \cos^2 t - \sin^2 t), \quad \frac{dy}{dt} = 24(1 + \sin t)\cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{24(1 + \sin t)\cos t}{12(1 + \cos^2 t - \sin^2 t)}$$

Step 2: Use the condition that the angle between the tangent and the positive x-axis is $\frac{\pi}{3}$. The slope of the tangent is $\tan\frac{\pi}{3} = \sqrt{3}$. Thus, equate the slope $\frac{dy}{dx}$ to $\sqrt{3}$ and solve for t.

$$\frac{24(1 + \sin t)\cos t}{12(1 + \cos^2 t - \sin^2 t)} = \sqrt{3}$$

After solving, we find that $t = \frac{\pi}{6}$. Substituting this value of t in $y = 12(1 + \sin t)^2$, we get:

$$y_0 = 12(1 + \sin\frac{\pi}{6})^2 = 12\left(1 + \frac{1}{2}\right)^2 = 12 \times \left(\frac{3}{2}\right)^2 = 27$$

Therefore, $y_0 = 27$.

Quick Tip

To solve problems involving parametric curves and slopes, always recall the derivative formulas and use the trigonometric identities to simplify expressions.

---

89. The altitude of a cone is 20 cm and its semi-vertical angle is $30°$. If the semi-vertical angle is increasing at the rate of $2°$ per second, then the radius of the base is increasing at the rate of:

(A) 30 cm/sec

(B) $\frac{160}{3}$ cm/sec

(C) 10 cm/sec

(D) 160 cm/sec

Correct Answer: (B) $\frac{160}{3}$ cm/sec

Solution:

Step 1: Let θ be the semi-vertical angle and r be the radius of the cone at time t. The relationship between r and θ is:

$$r = 20 \tan \theta$$

Step 2: Differentiate with respect to time t:

$$\frac{dr}{dt} = 20 \sec^2 \theta \cdot \frac{d\theta}{dt}$$

Given that $\frac{d\theta}{dt} = 2^\circ = \frac{\pi}{90}$ radians/sec and $\theta = 30^\circ$, we find:

$$\frac{dr}{dt} = 20 \sec^2 30^\circ \times \frac{\pi}{90} = 20 \times \left(\frac{4}{3}\right) \times \frac{\pi}{90} = \frac{160}{3} \text{ cm/sec}$$

Therefore, the radius is increasing at the rate of $\frac{160}{3}$ cm/sec.

Quick Tip

When dealing with rates of change of geometrical quantities, ensure that you use the correct trigonometric relations and convert angular velocity to radians if necessary.

---

90. The point of inflexion for the curve $y = (x - a)^n$, where n is odd integer and $n \geq 3$, is:

(A) $(a, 0)$

(B) $(0, a)$

(C) $(0, 0)$

(D) None of these

Correct Answer: (A) $(a, 0)$

Solution:

Step 1: Differentiate $y = (x - a)^n$ to find the second derivative.

$$\frac{d^2y}{dx^2} = n(n-1)(x-a)^{n-2}$$

Step 2: For the point of inflexion, set $\frac{d^2y}{dx^2} = 0$:

$$n(n-1)(x-a)^{n-2} = 0$$

This gives $x = a$.

Step 3: Now, differentiate $y = (x-a)^n$ n times:

$$\frac{d^n y}{dx^n} = n!$$

Since n is odd, we have $\frac{d^n y}{dx^n} \neq 0$ and $\frac{d^{n-1} y}{dx^{n-1}} = 0$. Therefore, the point of inflexion is $(a, 0)$.

Quick Tip

For curves involving powers, differentiate multiple times and check for points where the second derivative changes sign to find inflection points.

---

91. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation:

$$\frac{dp(t)}{dt} = 0.5p(t) - 450.$$

If $p(0) = 850$, then the time at which the population becomes zero is:

(A) $2 \ln 18$

(B) $\ln 9$

(C) $\frac{1}{2} \ln 18$

(D) $\ln 18$

Correct Answer: (A) $2 \ln 18$

Solution:

The given differential equation is:

$$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450.$$

Rewriting the equation:

$$\frac{dp(t)}{dt} = \frac{p(t) - 900}{2}.$$

Next, integrate both sides:

$$2 \int \frac{dp(t)}{p(t) - 900} = \int -dt.$$

This results in:

$$2\ln|p(t)-900| = -t + C.$$

Using the initial condition $p(0) = 850$:

$$2\ln(50) = C.$$

Now, solving for $p(t)$ when $p(t) = 0$:

$$p(t) = 900 - 50e^{-t/2}.$$

Set $p(t) = 0$, solving for t gives:

$$t = 2\ln 18.$$

Quick Tip

For solving first-order linear differential equations, use separation of variables and then integrate to find the general solution.

---

92. Evaluate the integral:

$$\int \frac{x^3-1}{x^3+x}dx$$

(A) $x + \log|x| + \frac{1}{2}\log(x^2+1) + \sin^{-1}(x) + c$

(B) $x - \log|x| + \frac{1}{2}\log(x^2+1) - \sin^{-1}(x) + c$

(C) $x + \log|x| - \frac{1}{2}\log(x^2+1) + \tan^{-1}(x) + c$

(D) $x - \log|x| + \frac{1}{2}\log(x^2+1) - \tan^{-1}(x) + c$

Correct Answer: (D) $x - \log|x| + \frac{1}{2}\log(x^2+1) - \tan^{-1}(x) + c$

Solution:

We can rewrite the integrand as:

$$\frac{x^3-1}{x^3+x} = \frac{x^3+x-x-1}{x^3+x} = 1 - \frac{x+1}{x(x^2+1)}$$

Now, we use partial fraction decomposition on the remaining fraction:

$$\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$x+1 = A(x^2+1) + (Bx+C)x$$

Let $x = 0$:

$$1 = A$$

Comparing coefficients of x^2:

$$0 = A + B \implies B = -1$$

Comparing coefficients of x:

$$1 = C$$

So,

$$\frac{x+1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1} + \frac{1}{x^2+1}$$

Now, we integrate:

$$\int \frac{x^3-1}{x^3+x}dx = \int \left(1 - \frac{1}{x} + \frac{x}{x^2+1} - \frac{1}{x^2+1}\right) dx$$
$$= x - \ln|x| + \frac{1}{2}\ln(x^2+1) - \tan^{-1}(x) + c$$

Quick Tip

Remember to break complex rational expressions into simpler parts for easier integration using partial fraction decomposition.

---

93. Evaluate the integral:

$$\int \sqrt{x + \sqrt{x^2+2}}\, dx.$$

(A) $\frac{2}{3}(x + \sqrt{x^2+2})^{3/2} - 2(x + \sqrt{x^2+2})^{1/2} + C$

(B) $\frac{1}{3}(x + \sqrt{x^2+2})^{3/2} - 2(x + \sqrt{x^2+2})^{1/2} + C$

(C) $(x + \sqrt{x^2+2})^{-3/2} - 2(x + \sqrt{x^2+2})^{1/2} + C$

(D) $\frac{(x+\sqrt{x^2+2})^2-6}{3\sqrt{x+\sqrt{x^2+2}}} + C$

Correct Answer: (D) $\frac{1}{2}(x + \sqrt{x^2+2})^{3/2} - \frac{2}{3}\left(x + \sqrt{x^2+2}\right)^{1/2} + C$

Solution:

Using substitution $u = x + \sqrt{x^2+2}$, we compute the integral:

$$int\sqrt{x + \sqrt{x^2+2}}\, dx$$

$$\text{Let } \sqrt{x+\sqrt{x^2+2}} = t \Rightarrow x + \sqrt{x^2+2} = t^2$$

$$\sqrt{x^2+2} = t^2 - x$$

$$\Rightarrow x^2 + 2 = t^4 + x^2 - 2t^2x$$

$$\Rightarrow x = \frac{t^4-2}{2t^2} \Rightarrow dx = \frac{t^4+2}{t^3}\,dt$$

$$\int t \cdot \frac{t^4+2}{t^3}\,dt = \int \left(t^2 + \frac{2}{t^2}\right)\,dt = \frac{t^3}{3} - \frac{2}{t} + C$$

$$= \frac{t^4-6}{3t} + C$$

$$= \frac{(x+\sqrt{x^2+2})^2 - 6}{3\sqrt{x+\sqrt{x^2+2}}} + C$$

Quick Tip

Use substitution to simplify complicated square roots and reduce the integrand to a manageable form.

---

94. The value of $\int e^{\tan\theta}(\sec\theta - \sin\theta)d\theta$ is:

(A) $e^{\tan\theta}\sec\theta + c$

(B) $e^{\tan\theta}\sin\theta + c$

(C) $e^{\tan\theta}(\tan\theta + \sin\theta) + c$

(D) $e^{\tan\theta}\cos\theta + c$

Correct Answer: (D) $e^{\tan\theta}\cos\theta + c$

Solution:

Step 1: We are given:

$$I = \int e^{\tan\theta}$$

$(\sec\theta - \sin\theta)d\theta$ Distribute the terms inside the integral:

$$\text{Let } I = \int e^{\tan\theta}(\sec\theta - \sin\theta)\, d\theta$$

$$\text{Put } \tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt \Rightarrow d\theta = \frac{dt}{1+t^2}$$

$$\Rightarrow I = \int e^t \left(\sqrt{1+t^2} - \frac{t}{\sqrt{1+t^2}}\right) \frac{dt}{1+t^2}$$

$$= \int e^t \left(\frac{1}{\sqrt{1+t^2}} - \frac{t}{(1+t^2)^{3/2}}\right) dt$$

Integrating the first part by parts, we have

$$= \frac{1}{\sqrt{1+t^2}} e^t - \int \frac{t}{(1+t^2)^{3/2}} e^t\, dt + \int \frac{t}{(1+t^2)^{3/2}} e^t\, dt + c$$

$$= \frac{e^t}{\sqrt{1+t^2}} + c$$

$$= e^{\tan\theta}\cos\theta + c$$

Thus, the final answer is $\tan\theta\cos\theta + c$.

Quick Tip

When dealing with integrals involving trigonometric functions, break the expression into manageable parts and use standard integration identities.

---

95. The value of $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ is:

(A) $\frac{\pi ab}{a+b}$

(B) $\frac{ab}{2(a+b)}$

(C) $\frac{\pi}{2ab(a+b)}$

(D) $\frac{\pi(a+b)}{2ab}$

Correct Answer: (C) $\frac{\pi}{2ab(a+b)}$

Solution:

Step 1: The integral is given by:

$$I = \int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$$

Break this into two fractions:

$$I = \frac{1}{a^2 - b^2} \int_0^\infty \frac{(x^2 + a^2) - (x^2 + b^2)}{(x^2 + a^2)(x^2 + b^2)} dx$$

$$I = \frac{1}{a^2 - b^2} \int_0^\infty \frac{1}{x^2 + b^2} - \frac{1}{x^2 + a^2} dx$$

$$I = \frac{1}{a^2 - b^2} \left[\frac{1}{b} \tan^{-1} \frac{x}{b} - \frac{1}{a} \tan^{-1} \frac{x}{a} \right]_0^\infty$$

$$I = \frac{1}{a^2 - b^2} \left[\frac{1}{b} \times \frac{\pi}{2} - \frac{1}{a} \times \frac{\pi}{2} \right]$$

$$I = \frac{1}{(a + b)(a - b)} \left[\frac{a - b}{ab} \right] \times \frac{\pi}{2}$$

$$I = \frac{\pi}{2ab(a + b)}$$

Thus, the final answer is

$$I = \frac{\pi}{2ab(a + b)}$$

.

Quick Tip

For integrals of this form, partial fractions and standard formulae for rational functions can simplify the problem significantly.

---

96. The value of definite integral $\int_0^{\pi/2} \log(\tan x) dx$ is:

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Correct Answer: (A) 0

Solution:

Step 1: Let:

$$I = \int_0^{\pi/2} \log(\tan x)dx$$

Apply the property:

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Thus:

$$I = \int_0^{\pi/2} \log(\tan(\frac{\pi}{2} - x))dx = \int_0^{\pi/2} \log(\cot x)dx$$

Step 2: Adding the two equations gives:

$$2I = \int_0^{\pi/2} [\log(\tan x) + \log(\cot x)]dx = \int_0^{\pi/2} \log(1)dx = 0$$

Therefore, $I = 0$.

Quick Tip

When solving integrals of logarithmic functions, try applying symmetry and properties of the functions to simplify the process.

---

97. Evaluate the integral:

$$\int_5^9 \frac{\log 3x^2}{\log 3x^2 + \log(588 - 84x + 3x^2)} dx$$

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) 4

Correct Answer: (A) 2

Solution:

Let

$$I = \int_5^9 \frac{\log 3x^2}{\log 3x^2 + \log(588 - 84x + 3x^2)} dx \quad \cdots (i)$$

We can rewrite the second term in the denominator as follows:

$$\log(588-84x+3x^2)=\log(3(196-28x+x^2))=\log(3(14-x)^2)$$

Now, using the property of definite integrals, $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx,$ we have:

$$I=\int_5^9 \frac{\log(3(14-x)^2)}{\log(3(14-x)^2)+\log(3x^2)}\,dx$$

$$I=\int_5^9 \frac{\log 3+2\log(14-x)}{\log 3+2\log(14-x)+\log 3+2\log x}\,dx$$

$$I=\int_5^9 \frac{\log 3+2\log(14-x)}{2\log 3+2\log(14-x)+2\log x}\,dx$$

$$I=\int_5^9 \frac{\log 3+2\log(14-x)}{2(\log 3+\log(14-x)+\log x)}\,dx$$

$$I=\int_5^9 \frac{\log 3+2\log(14-x)}{2\log(3x(14-x))}\,dx \qquad \cdots(ii)$$

Adding equations (i) and (ii):

$$2I=\int_5^9 \frac{\log 3x^2+\log(3(14-x)^2)}{\log 3x^2+\log(3(14-x)^2)}\,dx$$

$$2I=\int_5^9 1\,dx$$

$$2I=[x]_5^9$$

$$2I=9-5=4$$

I = 2

Quick Tip

Use logarithmic properties and symmetry of definite integrals to simplify complex integrals.

---

98. Evaluate the integral:

$$\int \frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$$

(A) $-\frac{x^2}{x\tan x+1}$

(B) $2\log_e|x\sin x+\cos x|+C$

(C) $-\frac{x^2}{x\tan x+1}+2\log_e|x\sin x+\cos x|+C$

(D) $-\frac{x^2}{x\tan x+1}-2\log_e|x\sin x+\cos x|+C$

Correct Answer: (C) $-\frac{x^2}{x\tan x+1}+2\log_e|x\sin x+\cos x|+C$

Solution:

We note that:

$$\frac{d}{dx}(x\tan x+1)=x\sec^2 x+\tan x.$$

$$\frac{d}{dx}(x\tan x+1)=x\sec^2 x+\tan x$$

integrating by parts with x^2 as the first function, we get

$$I=\int x^2\frac{x\sec^2 x+\tan x}{(x\tan x+1)^2}dx$$

$$=x^2\left(-\frac{1}{x\tan x+1}\right)-\int 2x\left(-\frac{1}{x\tan x+1}\right)dx$$

$$=-\frac{x^2}{x\tan x+1}+2\int\frac{x}{x\tan x+1}dx$$

$$=-\frac{x^2}{x\tan x+1}+2\int\frac{x\cos x}{x\sin x+\cos x}dx$$

$$=-\frac{x^2}{x\tan x+1}+2\log_e|x\sin x+\cos x|+c$$

$$\left(\because\frac{d}{dx}(x\sin x+\cos x)=x\cos x\right)$$

Quick Tip

Use integration by parts along with trigonometric identities to simplify complex integrals.

---

99. Evaluate the following limit:

$$\lim_{n\to\infty}\prod_{r=3}^{n}\frac{r^3-8}{r^3+8}.$$

(A) $\frac{2}{7}$
(B) $\frac{3}{7}$
(C) $\frac{4}{7}$
(D) $\frac{6}{7}$

Correct Answer: (A) $\frac{2}{7}$

Solution:

We begin by expanding and simplifying the product:

$$\lim_{n\to\infty}\frac{(3-2)(3^2+2^2+3\cdot 2)}{(3+2)(3^2+2^2-3\cdot 2)}\cdot\frac{(4-2)(4^2+2^2+4\cdot 2)}{(4+2)(4^2+2^2-4\cdot 2)}\cdots\frac{(n-2)(n^2+2^2+n\cdot 2)}{(n+2)(n^2+2^2-n\cdot 2)}$$

$$=\lim_{n\to\infty}\frac{(3-2)(4-2)\cdots(n-2)}{(3+2)(4+2)\cdots(n+2)}\cdot\frac{(3^2+2^2+3\cdot 2)(4^2+2^2+4\cdot 2)\cdots(n^2+2^2+n\cdot 2)}{(3^2+2^2-3\cdot 2)(4^2+2^2-4\cdot 2)\cdots(n^2+2^2-n\cdot 2)}$$

$$=\lim_{n\to\infty}\frac{1\cdot 2\cdots(n-2)}{5\cdot 6\cdots(n+2)}\cdot\frac{19\cdot 28\cdots(n^2+2n+4)}{7\cdot 12\cdots(n^2-2n+4)}$$

$$=\frac{2}{7}$$

Quick Tip

For infinite product limits, identify factors that cancel and apply asymptotic analysis for large n.

---

100. The value of $\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{3\pi}{4}+x\right)}{\cos x+\sin x}dx$ **is:**

(A) $\frac{\pi}{\sqrt{2}}$

(B) $\frac{\pi}{2\sqrt{2}}$

(C) $\frac{\pi}{3\sqrt{2}}$

(D) $\frac{\pi}{4\sqrt{2}}$

Correct Answer: (B) $\frac{\pi}{2\sqrt{2}}$

Solution: Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{x}{4}+x\right)+\sin\left(\frac{3x}{4}+x\right)}{\cos x+\sin x}\,dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{3x}{4}-x\right)+\sin\left(\frac{5x}{4}-x\right)}{\sin x+\cos x}\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{x}{4}-x\right)-\sin\left(\frac{3x}{4}-x\right)}{\cos x+\sin x}\,dx$$

Now,

$$I + I = \int_0^{\frac{\pi}{2}} \frac{2\sin\left(\frac{x}{4}+x\right)}{\cos x+\sin x}\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x}{\cos x-\sin x}\,dx = \frac{\pi}{2\sqrt{2}}$$

The value of the integral is $\frac{\pi}{2\sqrt{2}}$.

Quick Tip

When dealing with integrals involving trigonometric sums, use sum-to-product identities to simplify the numerator.

---

101. The line $y = mx$ bisects the area enclosed by lines $x = 0$, $y = 0$, and $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$. Then, the value of m is:

(A) $\frac{13}{6}$

(B) $\frac{13}{2}$

(C) $\frac{13}{5}$

(D) $\frac{13}{7}$

Correct Answer: (A) $\frac{13}{6}$

Solution: The total area under the curve is given by:

$= \int_0^{\frac{3}{2}} (1 + 4x - x^2)\, dx$

$$= x + 2x^2 - \frac{x^3}{3}\Big|_0^{\frac{3}{2}} = \frac{39}{8}$$

Calculate this integral:

$$\frac{39}{16} = \frac{1}{2} \cdot 3 \cdot 3 \cdot m$$

$$\Rightarrow 3m = \frac{13}{2}$$

$$\Rightarrow 12m = 26$$

Solving for m:

$$\Rightarrow m = \frac{13}{6}$$

Thus, the value of m is $\frac{13}{6}$.

Quick Tip

To solve area bisecting problems, equate the area under the line to half of the total area, then solve for the unknown slope.

---

102. If a, c, b are in GP, then the area of the triangle formed by the lines $ax + by + c = 0$ with the coordinate axes is equal to:

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) None of these

Correct Answer: (C) $\frac{1}{2}$

Solution:

Given a, c, b are in GP, so $c^2 = ab$.

The area of the triangle formed by the line $ax + by + c = 0$ and the coordinate axes can be found using the formula for the area of a triangle formed by two lines intersecting the axes at $x = \frac{-c}{a}$ and $y = \frac{-c}{b}$.

The area of the triangle AOB is:

$$\text{Area} = \frac{1}{2} \times \left(\frac{-c}{b}\right) \times \left(\frac{-c}{a}\right) = \frac{1}{2} \times \frac{c^2}{ab} = \frac{1}{2} \times \frac{c^2}{ab} = \frac{1}{2} \text{ (using } c^2 = ab).$$

Thus, the area is $\frac{1}{2}$.

Quick Tip

When given a triangle formed by the coordinate axes and a line, you can use the formula for the area of a triangle to calculate it. If the coefficients of the line are in geometric progression, use the relationship between the coefficients to simplify the area expression.

---

103. The area enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is:

(A) $\frac{5}{3}$ sq. units

(B) $\frac{5}{4}$ sq. units

(C) $\frac{5}{12}$ sq. units

(D) $\frac{12}{5}$ sq. units

Correct Answer: (C) $\frac{5}{12}$ sq. units

Solution:

Given curves $y = x^3$ and $y = \sqrt{x}$. The curves intersect at $x = 0$ and $x = 1$.

The area enclosed by these curves is given by the integral:

$$A = \int_0^1 \left(\sqrt{x} - x^3\right) dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right]_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}.$$

Thus, the area is $\frac{5}{12}$ sq. units.

Quick Tip

To find the area enclosed by two curves, first identify the points of intersection. Then integrate the difference between the functions over the range defined by the intersection points. Make sure to subtract the lower function from the upper function before integrating.

---

104. The area of the region bounded by the curves $x = y^2 - 2$ and $x = y$ is:

(A) $\frac{9}{4}$

(B) 9

(C) $\frac{9}{2}$

(D) $\frac{9}{7}$

Correct Answer: (C) $\frac{9}{2}$

Solution:

Given curves $x = y^2 - 2$ and $x = y$, the points of intersection are $(-2, 0)$ and $(2, 2)$.

To find the area, we integrate the difference between the two functions over the range from $y = -2$ to $y = 2$:

$$A = \int_{-1}^{2} y\,dy - \int_{-1}^{2} (y^2 - 2)\,dy$$

$$= \left[\frac{y^2}{2} - \frac{y^3}{3} + 2y\right]_{-1}^{2} = \left(\frac{4}{2} - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$$

$$= \frac{10}{3} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$$

Thus, the area is $\frac{9}{7}$.

Quick Tip

For finding the area between curves, set up an integral with the difference of the functions. Ensure the limits of integration are the points where the curves intersect. Simplify the integrand before computing the area.

---

105. If the area bounded by the curves $y = ax^2$ and $x = ay^2$ (where $a > 0$) is 3 sq. units, then the value of a is:

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) 1

(D) 4

Correct Answer: (B) $\frac{1}{3}$

Solution:

$$We\ are\ given\ the\ curves\ y = ax^2\ and\ x = ay^2.$$

$$\text{When, } x = 0 \Rightarrow y = 0 \text{ and } x = \frac{1}{a} \Rightarrow y = \frac{1}{a}$$

$$\text{Here, points of intersection of curves } y = ax^2 \text{ and } x = ay^2 \text{ are } (0,0) \text{ and } \left(\frac{1}{a}, \frac{1}{a}\right)$$

$$\therefore \text{ Required area}$$

$$A = \int_{x=a}^{x=b} [f_2(x) - f_1(x)]\, dx$$

$$3 = \int_0^{1/a} \left(\frac{\sqrt{x}}{\sqrt{a}} - ax^2 \right) dx$$

$$3 = \left[\frac{2}{3\sqrt{a}} x^{3/2} - \frac{ax^3}{3} \right]_0^{1/a}$$

$$3 = \frac{2}{3\sqrt{a}} \times \frac{1}{a\sqrt{a}} - \frac{a}{3} \times \frac{1}{a^3}$$

$$3 = \frac{2}{3a^2} - \frac{1}{3a^2}$$

$$3 = \frac{1}{3a^2}$$

$$9a^2 = 1$$

$$a^2 = \frac{1}{9} \Rightarrow a = \frac{1}{3}$$

$$Solving\ for\ a,\ we\ get\ a = \frac{1}{3}.$$

Quick Tip

When calculating the area between curves, find the points of intersection first, then set up the integral with the difference of the functions. Solve for the unknown constant by using the given area.

---

106. The solution of the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ **is:**

(A) $y = (x+1)e^{3x} + C$

(B) $3y = (x+1) + e^{3x} + C$

(C) $\frac{3y}{x+1} = e^{3x} + C$

(D) $ye^{-3x} = 3(x+1) + C$

Correct Answer: (C) $\frac{3y}{x+1} = e^{3x} + C$

Solution:

Step 1: Given the differential equation:

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$

This is a linear first-order differential equation. Rewriting it in the standard linear form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = -\frac{1}{x+1}$ and $Q(x) = e^{3x}(x+1)$.

Step 2: Use the integrating factor (IF):

$$IF = e^{\int P(x)dx} = e^{\int -\frac{1}{x+1}dx} = \frac{1}{x+1}$$

Step 3: Multiply both sides of the equation by the integrating factor:

$$\frac{3y}{x+1} = e^{3x} + C$$

Thus, the solution is:

$$\frac{3y}{x+1} = e^{3x} + C$$

Quick Tip

For linear first-order differential equations, always start by finding the integrating factor and multiplying through to solve.

---

107. If $\frac{dy}{dx} - y\log_e 2 = 2^{\sin x}$ $(\cos x - 1)\log_e 2$, then y is:

(A) $2^{\sin x}$ + c2^x

(B) $2^{\cos x}$ + c2^x

(C) $2^{\sin x}$ + c2^{-x}

(D) $2^{\cos x}$ + c2^{-x}

Correct Answer: (A) $2^{\sin x}$ + c2^x

Solution:

Step 1: The equation is:

$$\frac{dy}{dx} - y\log_e 2 = 2^{\sin x}(\cos x - 1)\log_e 2$$

This is a linear differential equation.

$$\text{I.F.} = e^{-\int \log_e 2\,dx} = e^{-x\log_e 2} = 2^{-x}$$

Then the general solution is

$$y2^{-x} = \int 2^{-x}2^{\sin x}(\cos x - 1)\log_e 2\,dx + c$$

$$\text{Now let } \sin x - x = t \Rightarrow (\cos x - 1)dx = dt$$

$$\therefore y2^{-x} = \log_e 2\int 2^t\,dt + c$$

$$\therefore y2^{-x} = 2^t + c$$

$$\therefore y = 2^{x+t} + c2^x$$

$$y = 2^{\sin x} + c2^x$$

Thus, the solution is $y = 2\sin x + c2^x$.

Quick Tip

For differential equations involving logarithms and trigonometric functions, use the standard linear method and integrate with the right approach.

---

108. Let $\mathbf{a} = \hat{i} - \hat{k}, \mathbf{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$, and $\mathbf{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then, $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$ depends on:

(A) only y

(B) only x

(C) both x and y

(D) neither x nor y

Correct Answer: (D) neither x nor y

Solution:

We are asked to find $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.

Step 1: Compute the cross product $\mathbf{b} \times \mathbf{c}$ and then dot it with $\mathbf{a}$.

$$[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \left\| \begin{matrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{matrix} \right\|$$

Simplifying this determinant:

$$= 1 + x - y - x^2 + x^2 - y = 1$$

Thus, $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = 1$, which is independent of both x and y.

Quick Tip

When working with scalar triple products, simplify the determinant step by step and carefully analyze the dependence on variables.

---

109. Let ABC be a triangle and $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of A, B, C respectively. Let D divide BC in the ratio $3:1$ internally and E divide AD in the ratio $4:1$ internally. Let BE meet AC in F. If E divides BF in the ratio $3:2$ internally then the position vector of F is:

(A) $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

(B) $\frac{\vec{a}-2\vec{b}+3\vec{c}}{2}$

(C) $\frac{\vec{a}+2\vec{b}+3\vec{c}}{2}$

(D) $\frac{\vec{a}-\vec{b}+3\vec{c}}{3}$

Correct Answer: (D) $\frac{\vec{a}-\vec{b}+3\vec{c}}{3}$

Solution: Given vectors:

$$\vec{OA} = \vec{a}, \quad \vec{OB} = \vec{b}, \quad \vec{OC} = \vec{c}$$

Now, the Position Vector (PV) of point D is:

$$\vec{OD} = \frac{1}{1+3}(\vec{OB} + 3\vec{OC}) = \frac{1}{4}(\vec{b} + 3\vec{c})$$

The Position Vector of point E is:

$$\vec{OE} = \frac{4\vec{OD} + \vec{OA}}{4+1} = \frac{1}{5}(4(\frac{1}{4}(\vec{b} + 3\vec{c})) + \vec{a}) = \frac{1}{5}(\vec{a} + \vec{b} + 3\vec{c})$$

Now, the Position Vector of point F is calculated by:

$$\vec{OF} = \frac{20\vec{OB} + 30\vec{OC}}{2+3} = \frac{50\vec{OE} - 20\vec{OB}}{3} = \frac{50(\frac{1}{5}(\vec{a} + \vec{b} + 3\vec{c})) - 20\vec{b}}{3}$$

$$\vec{OF} = \frac{10(\vec{a} + \vec{b} + 3\vec{c}) - 20\vec{b}}{3} = \frac{10\vec{a} - 10\vec{b} + 30\vec{c}}{3} = \frac{10(\vec{a} - \vec{b} + 3\vec{c})}{3}$$

$$\vec{OF} = \frac{\vec{a} - \vec{b} + 3\vec{c}}{3}$$

Hence, the Position Vector of F is:

$$\vec{OF} = \frac{1}{3}(\vec{a} - \vec{b} + 3\vec{c})$$

Quick Tip

Remember to use the section formula for internal division to find the position vectors in geometric vector problems.

---

110. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to:

(A) 17

(B) 18

(C) 19

(D) 20

Correct Answer: (B) 18

Solution:

Calculating individual terms, and summing them gives the result 18. Detailed steps for each term are omitted for brevity.

Using the vector identities, the calculations are as follows:

$$\hat{i} \times (\hat{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\hat{a} - (\hat{i} \cdot \hat{a})\hat{i} = \hat{j} + 2\hat{k}$$

Similarly, for other unit vectors:

$$\hat{j} \times (\hat{a} \times \hat{j}) = 2\hat{i} + 2\hat{k}$$

$$\hat{k} \times (\hat{a} \times \hat{k}) = 2\hat{i} + \hat{j}$$

The magnitudes of the resulting vectors are computed as follows:

$$\left\|\hat{j} + 2\hat{k}\right\|^2 = 2^2 + 1^2 = 4 + 1 = 5$$

$$\left\|2\hat{i} + 2\hat{k}\right\|^2 = 2^2 + 2^2 = 4 + 4 = 8$$

$$\left\|2\hat{i} + \hat{j}\right\|^2 = 2^2 + 1^2 = 4 + 1 = 5$$

Summing these magnitudes:

$$5 + 8 + 5 = 18$$

Quick Tip

Apply the properties of vector products systematically to simplify expressions involving cross and dot products.

---

111. The magnitude of projection of the line joining $(3, 4, 5)$ and $(4, 6, 3)$ on the line joining $(-1, 2, 4)$ and $(1, 0, 5)$ is:

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) $\frac{8}{3}$

(D) $\frac{1}{3}$

Correct Answer: (A) $\frac{4}{3}$

Solution: We know that, projection of a on b is given by projection, $|\mathbf{a}|\cos\theta = \frac{(\mathbf{a}\cdot\mathbf{b})}{|\mathbf{b}|}$

Let line joining points $(3, 4, 5)$ and $(4, 6, 3)$ is L_1 and line joining points $(-1, 2, 4)$ and $(1, 0, 5)$ is L_2.

$$L_1 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$L_2 = 2\hat{i} - 2\hat{j} + \hat{k}$$

Thus, the projection of L_1 and L_2 is:

$$\text{Projection of } L_1 \text{ and } L_2 = \frac{L_1 \cdot L_2}{|L_2|}$$

$$= \frac{2 - 4 - 2}{\sqrt{4 + 4 + 1}} = \frac{-4}{3}$$

Thus, the magnitude is:

$$\text{Magnitude} = \frac{4}{3}$$

Quick Tip

Utilize the projection formula for vectors to find the component of one vector along another, simplifying the calculation of magnitudes in projections.

112. The angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6nm - 2nl + 5lm = 0$ is:

(A) $\cos^{-1}\left(\frac{1}{6}\right)$

(B) $\cos^{-1}\left(-\frac{1}{6}\right)$

(C) $\cos^{-1}\left(\frac{2}{3}\right)$

(D) $\cos^{-1}\left(-\frac{5}{6}\right)$

Correct Answer: (B) $\cos^{-1}\left(-\frac{1}{6}\right)$

Solution:

The given equations for direction cosines are:

$$3l + m + 5n = 0 \quad \text{and} \quad 6mn - 2nl + 5lm = 0$$

From the given, we need to find the angle θ between the two lines. To do so, first, solve these equations for the direction ratios and use the formula for the cosine of the angle between two lines: 3l + m + 5n = 0 ...(i)

and 6mn - 2nl + 5lm = 0 ...(ii)

From (i), we have m = - 3l - 5n.

Putting m = - 3l - 5n in (ii),

we get 6(-3l - 5n)n - 2nl + 5l(-3l - 5n) = 0

(n + l)(2n + l) = 0

either l = -n or l = -2n.

If l = - n, then putting l = -n in (i), we obtain m = - 2n.

If l = - 2n, then putting l = - 2n in (i), we obtain m = n.

Thus, the direction ratios of two lines are -n, - 2n,

n and -2n,n,n i.e., 1,2,-1 and -2,1,1.

Hence, the direction cosines are

$$\cos\theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \cdot \sqrt{l_2^2 + m_2^2 + n_2^2}}$$

After solving the system of equations and simplifying, the angle between the lines is given by:

$$\cos\theta = \cos^{-1}\left(-\frac{1}{6}\right)$$

Quick Tip

To calculate the angle between two lines, first find their direction ratios and then apply the cosine formula.

---

113. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P?

(A) $(3, 1, -\frac{1}{2})$

(B) $(-2, 0, -\frac{1}{2})$

(C) $(0, 2, -4)$

(D) $(4, 0, -2)$

Correct Answer: (B) $(-2, 0, -\frac{1}{2})$

Solution:

The equation of the acute angle bisector of two planes is given by the following formula:

$$\frac{x - 2y - 2z + 1}{\sqrt{1^2 + (-2)^2 + (-2)^2}} = \pm\frac{2x - 3y - 6z + 1}{\sqrt{2^2 + (-3)^2 + (-6)^2}}$$

Substituting the coordinates of each point in the options, we find that the point $(-2, 0, -\frac{1}{2})$ satisfies the equation of the plane P. Thus, the point lies on the acute angle bisector.

Given the equations of planes:

$$P_1 : x - 2y - 2z + 1 = 0$$

$$P_2 : 2x - 3y - 6z + 1 = 0$$

The equation of the plane bisectors is given by:

$$\frac{x - 2y - 2z + 1}{\sqrt{1^2 + (-2)^2 + (-2)^2}} = \pm\frac{2x - 3y - 6z + 1}{\sqrt{2^2 + (-3)^2 + (-6)^2}}$$

$$\frac{x - 2y - 2z + 1}{3} = \pm\frac{2x - 3y - 6z + 1}{7}$$

Since $a_1a_2 + b_1b_2 + c_1c_2 = 20 > 0$, we choose the negative sign for the acute bisector:

$$\frac{x - 2y - 2z + 1}{3} = -\frac{2x - 3y - 6z + 1}{7}$$

$$7(x - 2y - 2z + 1) = -3(2x - 3y - 6z + 1)$$

$$7x - 14y - 14z + 7 = -6x + 9y + 18z - 3$$

$$13x - 23y - 32z + 10 = 0$$

The point $(-2, 0, -\frac{1}{2})$ satisfies this equation.

Quick Tip

When solving for points on the angle bisector of two planes, ensure you correctly apply the formula for the angle bisector and test the points by substituting their coordinates.

---

114. Let the foot of perpendicular from a point $P(1, 2, -1)$ to the straight line $L: \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to:

(A) $\frac{1}{\sqrt{5}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{2\sqrt{3}}$

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

Let $\overrightarrow{PN} \cdot (\hat{i} - \hat{k}) = 0$

$$\Rightarrow N(1, 0, -1)$$

Now,

$$\overrightarrow{PQ} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow \mu = -1$$

$$\Rightarrow Q(-1, 0, 1)$$

$$\overrightarrow{PN} = 2\hat{j} \quad \text{and} \quad \overrightarrow{PQ} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

After calculation, we find:

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

Quick Tip

For finding the angle between two lines, use the dot product formula and ensure to calculate the direction ratios carefully.

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115. If the number of available constraints is 3 and the number of parameters to be optimised is 4, then

(A) The objective function can be optimised

(B) The constraints are short in number

(C) The solution is problem oriented

(D) None of the above

Correct Answer: (B) The constraints are short in number

Solution:

To optimise n number of parameters, we need at least n constraints. In this case, there are 3 constraints for 4 parameters, which means the constraints are short in number.

Quick Tip

In optimization problems, the number of constraints should be equal to or greater than the number of parameters to be optimised.

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116. The probability of getting 10 in a single throw of three fair dice is:

(A) $\frac{1}{6}$

(B) $\frac{1}{8}$

(C) $\frac{1}{9}$

(D) $\frac{1}{5}$

Correct Answer: (B) $\frac{1}{8}$

Solution:

Total outcomes when rolling three dice is $6 \times 6 \times 6 = 216$. To calculate the number of cases where the sum is 10, we list all combinations of numbers on the dice that add to 10. After listing the cases, we find there are 27 favorable outcomes.

We consider different cases of outcomes:

$$\text{Case 1: } 1+3+6 \rightarrow \text{outcomes} = \frac{3!}{1!} = 6$$
$$\text{Case 2: } 1+4+5 \rightarrow \text{outcomes} = \frac{3!}{1!} = 6$$
$$\text{Case 3: } 2+2+6 \rightarrow \text{outcomes} = \frac{3!}{2!} = 3$$
$$\text{Case 4: } 2+3+5 \rightarrow \text{outcomes} = \frac{3!}{1!} = 6$$
$$\text{Case 5: } 2+4+4 \rightarrow \text{outcomes} = \frac{3!}{2!} = 3$$
$$\text{Case 6: } 3+3+4 \rightarrow \text{outcomes} = \frac{3!}{2!} = 3$$

Sum of favorable outcomes:

$$\text{Favourable outcomes} = 27$$

Thus, the probability is:

$$\text{Probability} = \frac{27}{216} = \frac{1}{8}$$

This leads to the quadratic equation:

$$(x+24)(x-20) = 0$$

Solving for x, we find:

$$x = 20$$

Thus, the probability is:

$$= \frac{1}{8}$$

Quick Tip

To find the probability of a specific outcome in a dice game, first determine all the possible outcomes, then count the favorable outcomes.

117. In a binomial distribution, the mean is 4 and variance is 3. Then, its mode is:

(A) 5

(B) 6

(C) 4

(D) None of these

Correct Answer: (C) 4

Solution:

The mean μ of a binomial distribution is given by $\mu = np$, and the variance is given by $\sigma^2 = npq$.

We are given that the mean is 4 and the variance is 3. Using these, we can solve for n and p:

$$\mu = np = 4, \quad \sigma^2 = npq = 3$$

From this, we find $p = \frac{1}{4}$, and $n = 16$.

Now, the mode M of a binomial distribution is given by:

$$M = (n+1)\,p \quad \text{if} \quad (n+1)p \text{ is an integer.}$$

Substituting $n = 16$ and $p = \frac{1}{4}$:

$$M = (16+1)\left(\frac{1}{4}\right) = \frac{17}{4}$$

$$= 4.25$$

Thus, the mode is 4(taking integer).

Quick Tip

In binomial distributions, the mode is often the closest integer to $(n+1)p$.

---

118. The probability that certain electronic component fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for one year is 0.99. The probability that a new component will last for one year is

(A) 0.9

(B) 0.01

(C) 0.119

(D) 0.891

Correct Answer: (D) 0.891

Solution:

Let $P(F)$ be the event that the electronic component fails when first used. So,

$$P(F) = 0.10 \quad \text{and} \quad P(F') = 1 - P(F) = 0.90$$

Let E be the event that a new component will last for one year, then

$$P(E) = P(F) \cdot P(E|F) + P(F') \cdot P(E|F')$$

Using the total probability theorem, we have:

$$P(E) = 0.10 \times 0 + 0.90 \times 0.99 = 0.891$$

Thus, the probability that the new component will last for one year is 0.891.

Quick Tip

When dealing with conditional probabilities, always ensure that the total probability theorem is used for scenarios with multiple possible outcomes.

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119. Given below is the distribution of a random variable X:

$X = x$	$P(X = x)$
1	λ
2	2λ
3	3λ
4	4λ

If $\alpha = P(X < 3)$ and $\beta = P(X > 2)$, then $\alpha : \beta =$

(A) $\frac{2}{5}$

(B) $\frac{3}{4}$

(C) $\frac{4}{5}$

(D) $\frac{3}{7}$

Correct Answer: (D) $\frac{3}{7}$

Solution:

For a distribution of random variable x:

$$\alpha = P(X^6 < 3) = P(X^6 = 1) + P(X^6 = 2)$$

$$= \lambda + 2\lambda = 3\lambda$$

and

$$\beta = P(X^6 < 2) = P(X^6 = 3) + P(X^6 = 4)$$

$$= 3\lambda + 4\lambda = 7\lambda$$

Thus,

$$\alpha : \beta = 3 : 7$$

Hence, $\alpha : \beta = 3 : 7$.

Quick Tip

The probability ratio between events is determined by comparing their individual probabilities.

---

120. A book contains 1000 pages. A page is chosen at random. The probability that the sum of the digits of the marked number on the page is equal to 9, is

(A) $\frac{23}{500}$

(B) $\frac{11}{200}$

(C) $\frac{7}{100}$

(D) None of these

Correct Answer: (B) $\frac{11}{200}$

Solution:

Total number of ways to choose a page = 1000

The favorable cases that the sum of the digits of the marked number on the page is equal to 9 are one digit numbers or two-digit numbers or three-digit numbers, if the three-digit number is abc.

$$a + b + c = 9, \quad 0 \le a, b, c \le 9$$

Hence, the number of favorable solutions for the equation $a + b + c = 9$ is $11C_2 = 55$.

Thus, the required probability is:

$$\frac{55}{1000} = \frac{11}{200}$$

Quick Tip

To find probabilities in cases involving digit sums, break down the sum into its digit components and solve accordingly.

---

121. For two events A and B, if $P(A) = P(A/B) = \frac{1}{4}$ and $P(B/A) = \frac{1}{2}$, then which of the following is not true?

(A) A and B are independent

(B) $P(A'/B) = \frac{3}{4}$

(C) $P(B'/A') = \frac{1}{2}$

(D) None of these

Correct Answer: (D) None of these

Solution: we are given:

$$P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{1}{2} \Rightarrow P(B \cap A) = \frac{1}{8}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \Rightarrow P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{8} = P(A) \cdot P(B)$$

Thus, events A and B are independent.

Now,

$$P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = \frac{3}{4}$$

and

$$P\left(\frac{B'}{A'}\right) = \frac{P(B' \cap A')}{P(A')} = \frac{P(B')P(A')}{P(A')} = \frac{1}{2}$$

Quick Tip

When calculating conditional probabilities, always check if events are independent, as this simplifies calculations.
