BSEAP Mathematics Question Paper 2023 with solutions

Time Allowed :3 hours 15 minutes	Maximum Marks :100	Total questions :33

General Instructions

(i) In the duration of **3 hours 15 minutes**, **15 minutes** of time is allotted to read the question paper.

(ii) **All** answers shall be written in the answer booklet **only**.

(iii) The question paper consists of **4 Sections** and **33 questions**.

(iv) Internal choice is available in **Section IV only**.

(v) Answers shall be written **neatly and legibly**.

SECTION -1

Note : 1) Answer all the questions in one word or a phrase.

2) Each question carries 1 mark.

1. What is the HCF of 37 and 49?

Solution:

To find the Highest Common Factor (HCF) of 37 and 49, we first determine the prime factors of each number: - 37 is a prime number, so its only factors are 1 and 37. - 49 is 7^2, so its factors are 1, 7, and 49.

The common factor between 37 and 49 is 7. Therefore, the HCF of 37 and 49 is 7.

Thus, the correct answer is 7.

> **Quick Tip**
>
> The HCF of two numbers is the largest number that divides both without leaving a remainder. For prime numbers like 37, the HCF with any other number is 1 unless that number is a multiple of the prime number.

---

2. $H = \{1, 2, 3, \ldots\}$**. This set is an** ___ **set (Finite or Infinite)**

Solution:

To determine whether the set $H = \{1, 2, 3, \ldots\}$ is finite or infinite, follow these steps:

1. The set $H = \{1, 2, 3, \ldots\}$ includes all natural numbers, starting from 1 and continuing indefinitely.
2. A set is classified as *finite* if it contains a specific number of elements, i.e., the number of elements can be counted and has an end.
3. On the other hand, a set is classified as *infinite* if it contains an uncountable number of elements and has no end.
4. Since H contains all natural numbers and continues indefinitely, it does not have a finite number of elements.
5. Therefore, the set $H = \{1, 2, 3, \ldots\}$ is an infinite set.

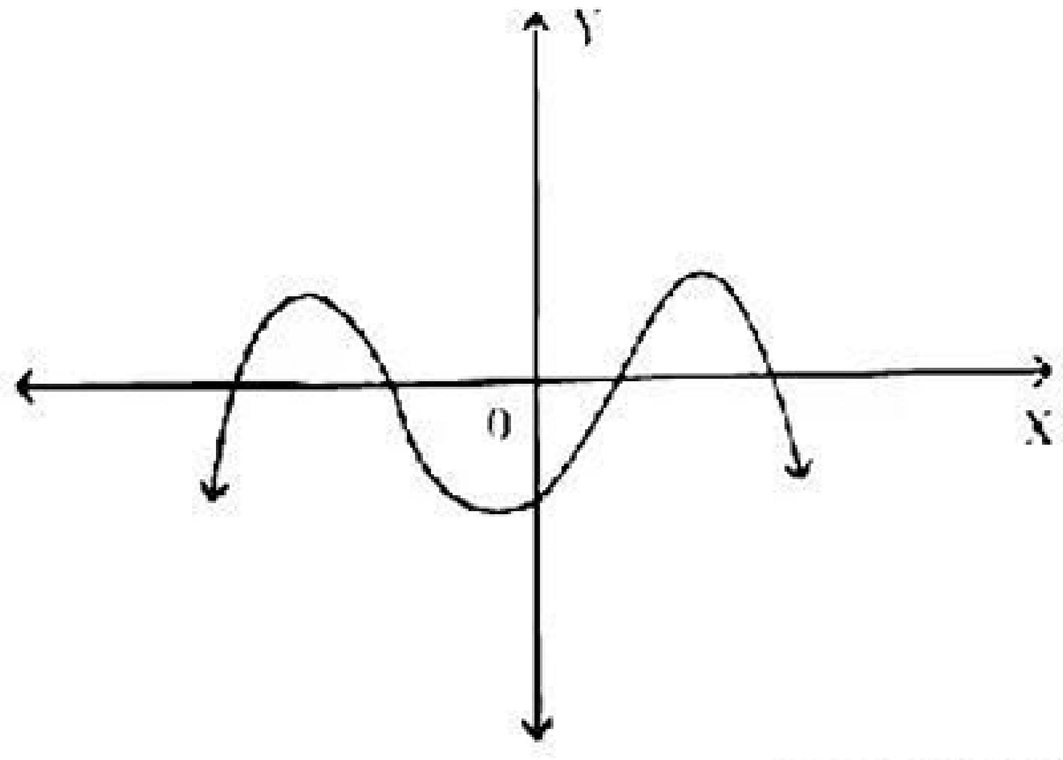
Thus, the correct answer is Infinite.

Quick Tip

A set is finite if it contains a specific, countable number of elements. A set is infinite if it continues without end, as in the case of the natural numbers.

---

3. Find the number of zeroes of the polynomial, whose graph is given.



Solution:

The zeroes (or roots) of a polynomial are the points where its graph intersects the x-axis. From the given graph, we can count the number of points where the curve crosses the x-axis. - The graph crosses the x-axis at three distinct points. - Therefore, the polynomial has three zeroes.

Thus, the number of zeroes of the polynomial is 3.

Quick Tip

The number of zeroes of a polynomial is equal to the number of times its graph intersects the x-axis. Each intersection represents a root of the polynomial.

---

4. Which of the following equations is not a linear equation?

(A) $5 + 4x = y - 3$

(B) $x + 2y = y - x$

(C) $3 - x = y^2 - 4$

(D) $x + y = 0$

Correct Answer: (C) $3 - x = y^2 - 4$

Solution:

A linear equation is an equation in which the highest power of the variables is 1, and there are no products of variables. Let's check each option:

1. In option (A), $5 + 4x = y - 3$, the highest power of the variables x and y is 1. Therefore, it is a linear equation.

2. In option (B), $x + 2y = y - x$, the highest power of the variables x and y is 1. Therefore, it is a linear equation.

3. In option (C), $3 - x = y^2 - 4$, the highest power of y is 2 (since there is y^2 on the right side of the equation). This is not a linear equation, as it involves a quadratic term in y.

4. In option (D), $x + y = 0$, the highest power of the variables x and y is 1. Therefore, it is a linear equation.

Thus, the correct answer is option (C) $3 - x = y^2 - 4$, as it is not a linear equation due to the presence of y^2.

Quick Tip

A linear equation involves only terms where the variables are raised to the power of 1, and there are no products of variables. Any equation involving a variable squared or higher powers is not linear.

5. Statement I: $2x^2 - x - 5 = 0$ **is a quadratic equation.**

Statement II: The general form of the quadratic equation is $ax^2 + bx + c = 0$**.**

Now, choose the correct answer:

(A) Both statements are true

(B) Statement I is true and Statement II is false

(C) Statement I is false and Statement II is true

(D) Both statements are false

Correct Answer: (A) Both statements are true

Solution:

- **Statement I:** $2x^2 - x - 5 = 0$ is indeed a quadratic equation because it has the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -1$, and $c = -5$. Thus, statement I is true. - **Statement II:** The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, and c are constants, and $a \neq 0$. This is a standard definition of a quadratic equation, so statement II is also true. Since both statements are true, the correct answer is option (A) Both statements are true.

Quick Tip

A quadratic equation always has the form $ax^2+bx+c = 0$, where a, b, and c are constants, and $a \neq 0$.

---

6. Match the following:

- $i. \tan\theta$ p. $\frac{1}{\cos\theta}$
- $ii. \sec\theta$ q. $\frac{1}{\sin\theta}$
- $iii. \csc\theta$ r. $\frac{\sin\theta}{\cos\theta}$

(A) $i \to r, ii \to p, iii \to q$

(B) $i \to p, ii \to q, iii \to r$

(C) $i \to p, ii \to q, iii \to r$

(D) $i \to q, ii \to p, iii \to r$

Correct Answer: (A) $i \to r, ii \to p, iii \to q$

Solution:

We match each trigonometric identity with its corresponding expression:

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$, so $i \to r$.
- $\sec\theta = \frac{1}{\cos\theta}$, so $ii \to p$.
- $\csc\theta = \frac{1}{\sin\theta}$, so $iii \to q$.

Thus, the correct matching is $i \to r, ii \to p, iii \to q$, which corresponds to option (A).

Quick Tip

Remember the basic trigonometric identities: - $\tan\theta = \frac{\sin\theta}{\cos\theta}$ - $\sec\theta = \frac{1}{\cos\theta}$ - $\csc\theta = \frac{1}{\sin\theta}$

---

7. Distance between the two points $(2, 0)$ and $(6, 0)$ is __ units.

Solution:

We use the distance formula to find the distance between the two points $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (6, 0)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting the coordinates:

$$d = \sqrt{(6 - 2)^2 + (0 - 0)^2} = \sqrt{4^2} = \sqrt{16} = 4$$

Thus, the distance between the points is 4 units.

Quick Tip

For points on the x-axis (or y-axis), the distance formula simplifies to the absolute difference between the x-coordinates (or y-coordinates).

---

8. The n^{th} term of G.P. is ar^{n-1}. Here, r represents __ .

Solution:

In a geometric progression (G.P.), the n^{th} term is given by the formula:

$$T_n = ar^{n-1}$$

where: - a is the first term of the G.P., - r is the common ratio between consecutive terms, - n is the term number.

Thus, r represents the common ratio between consecutive terms in the geometric progression.

Quick Tip

In a geometric progression, the common ratio r is the constant factor that each term is multiplied by to get the next term.

---

9. Two ___ are not always similar.

(A) Line segments

(B) Triangles

(C) Circles

(D) Squares

Correct Answer: (A) Line segments

Solution:

- **Line segments** are not always similar because they can have different lengths. For similarity, all corresponding angles must be equal and all corresponding sides must be proportional. Since line segments can have different lengths, they are not always similar. - **Triangles**, **Circles**, and **Squares** are always similar in certain conditions. For example, all circles are similar to each other because they are defined by a radius, and any triangle or square can be scaled to match another triangle or square.

Thus, the correct answer is option (A) Line segments.

Quick Tip

For two shapes to be similar, their corresponding angles must be equal and their corresponding sides must be proportional. Line segments, however, can differ in length and are not always similar.

---

10. Number of tangents drawn from the external point to the circle is __ .

Solution:

When a point lies outside a circle, two tangents can be drawn from the point to the circle. These tangents are equal in length, and they touch the circle at exactly one point each. Thus, the number of tangents drawn from the external point to the circle is 2.

Quick Tip

From any external point, two tangents can be drawn to a circle. The tangents will be equal in length and meet the circle at distinct points.

---

11. What is the length of the edge of the cube whose volume is 64 cm^3?

(A) 4 cm

(B) 16 cm

(C) 5 cm

(D) 6 cm

Correct Answer: (A) 4 cm

Solution:

The volume V of a cube is given by the formula:

$$V = a^3$$

where a is the length of the edge. Given that the volume is $64\,\text{cm}^3$, we have:

$$a^3 = 64$$

Taking the cube root of both sides:

$$a = \sqrt[3]{64} = 4\,\text{cm}$$

Thus, the length of the edge of the cube is 4 cm.

Quick Tip

The volume of a cube is given by $V = a^3$, where a is the length of the edge. To find the edge length, take the cube root of the volume.

---

12. Which of the following cannot be the probability of an event?

(A) 0.3

(B) -1.5

(C) 15(D) $\frac{2}{7}$

Correct Answer: (B) -1.5

Solution:

The probability of an event must always be a value between 0 and 1, inclusive. This means that any probability greater than 1 or less than 0 is not possible. - Option (A) 0.3 is a valid probability. - Option (B) -1.5 is not a valid probability because probabilities cannot be negative. - Option (C) 15- Option (D) $\frac{2}{7} \approx 0.2857$, which is also a valid probability.

Thus, the correct answer is option (B) -1.5, as it is not a valid probability.

> **Quick Tip**
>
> The probability of an event must always be a value between 0 and 1. Values outside this range are not valid probabilities.

SECTION - II

Note : 1) Answer all the questions.

2) Each question carries 2 marks.

13. Check whether 3 and -2 are the zeroes of the polynomial $p(x) = x^2 - x - 6$.

Solution:

To check whether 3 and -2 are the zeroes of the polynomial $p(x) = x^2 - x - 6$, we substitute these values into the polynomial.

Step 1: Check for $x = 3$ Substitute $x = 3$ into $p(x)$:

$$p(3) = (3)^2 - (3) - 6 = 9 - 3 - 6 = 0$$

Since $p(3) = 0$, 3 is a zero of the polynomial.

Step 2: Check for $x = -2$ Substitute $x = -2$ into $p(x)$:

$$p(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$$

Since $p(-2) = 0$, -2 is also a zero of the polynomial.

Thus, both 3 and -2 are zeroes of the polynomial $p(x) = x^2 - x - 6$.

Quick Tip

To check if a number is a zero of a polynomial, substitute the number into the polynomial. If the result is 0, the number is a zero of the polynomial.

---

14. 5 pencils and 7 pens together cost 80, whereas 7 pencils and 5 pens together cost 46. Represent this information in the form of pair of linear equations in variables x and y.

Solution:

Let x be the cost of one pencil, and y be the cost of one pen. We can form two linear equations based on the given information:

- 5 pencils and 7 pens together cost 80, so the equation is:

$$5x + 7y = 80$$

- 7 pencils and 5 pens together cost 46, so the equation is:

$$7x + 5y = 46$$

Thus, the system of linear equations representing the given information is:

$$5x + 7y = 80$$

$$7x + 5y = 46$$

Quick Tip

To represent real-life situations with linear equations, define variables for the unknowns (like the cost of an item) and translate the given conditions into equations.

---

15. Check whether $(x - 2)^2 + 1 = 2x - 3$ is a quadratic equation.

Solution:

We will simplify the given equation to check if it is a quadratic equation.

Step 1: Start with the given equation:

$$(x - 2)^2 + 1 = 2x - 3$$

Step 2: Expand $(x-2)^2$:

$$(x-2)^2 = x^2 - 4x + 4$$

So the equation becomes:

$$x^2 - 4x + 4 + 1 = 2x - 3$$

Step 3: Simplify both sides:

$$x^2 - 4x + 5 = 2x - 3$$

Step 4: Move all terms to one side:

$$x^2 - 4x + 5 - 2x + 3 = 0$$

Simplify:

$$x^2 - 6x + 8 = 0$$

This is a quadratic equation in the standard form $ax^2 + bx + c = 0$, where $a = 1$, $b = -6$, and $c = 8$.

Thus, the given equation is indeed a quadratic equation.

Quick Tip

A quadratic equation is an equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$. If the equation can be simplified to this form, it is a quadratic equation.

---

16. Find the centroid of the triangle whose vertices are $(3, -2)$, $(-2, 8)$, and $(0, 4)$.

Solution:

The centroid G of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is given by the formula:

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Substitute the coordinates of the vertices $A(3, -2)$, $B(-2, 8)$, and $C(0, 4)$:

$$G_x = \frac{3 + (-2) + 0}{3} = \frac{1}{3}, \quad G_y = \frac{-2 + 8 + 4}{3} = \frac{10}{3}$$

Thus, the centroid of the triangle is $G\left(\frac{1}{3}, \frac{10}{3}\right)$.

Quick Tip

The centroid of a triangle is the point where the three medians intersect. It is found by averaging the coordinates of the three vertices.

---

17. A flag pole 4m tall casts a 6m shadow. At the same time, a nearby building casts a shadow of 24m. How tall is the building?

Solution:

We can solve this using the concept of similar triangles. The ratio of the height of the flagpole to the length of its shadow is equal to the ratio of the height of the building to the length of its shadow. Thus, we can write the proportion:

$$\frac{\text{height of flagpole}}{\text{shadow of flagpole}} = \frac{\text{height of building}}{\text{shadow of building}}$$

Substitute the given values:

$$\frac{4}{6} = \frac{h}{24}$$

Now, solve for h:

$$h = \frac{4 \times 24}{6} = 16$$

Thus, the height of the building is 16 meters.

Quick Tip

When two objects cast shadows at the same time, their heights and shadow lengths are proportional, so we can use the concept of similar triangles to find unknown values.

---

18. Calculate the length of the tangent drawn from a point 15 cm away from the center of a circle of radius 9 cm.

Solution:

To calculate the length of the tangent drawn from an external point to a circle, we use the formula:

$$\text{Length of tangent} = \sqrt{d^2 - r^2}$$

where: - d is the distance from the external point to the center of the circle, - r is the radius of the circle.

Substituting the given values $d = 15\,\text{cm}$ and $r = 9\,\text{cm}$:

$$\text{Length of tangent} = \sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12\,\text{cm}$$

Thus, the length of the tangent is 12 cm.

Quick Tip

To find the length of the tangent drawn from an external point to a circle, use the formula $\sqrt{d^2 - r^2}$, where d is the distance from the external point to the center, and r is the radius of the circle.

---

19. A solid toy is in the form of right circular cylinder with hemispherical shape at one end and a cone at the other end. Draw a rough diagram of this solid toy,

Solution: The solid toy consists of the following components:

1. A right circular cylinder as the main body.
2. A hemisphere attached at one end of the cylinder.
3. A cone attached at the other end of the cylinder.

Below is a rough diagram of the solid toy:

[scale=1] [thick] (0, 0) ellipse (1.5 and 0.4); [thick] (0, -2) ellipse (1.5 and 0.4); [thick] (-1.5, 0) – (-1.5, -2); [thick] (1.5, 0) – (1.5, -2); [thick] (-1.5, -2) – (1.5, -2);
[thick] (0, 0.4) arc[start angle=180,end angle=360,radius=1.5];
[thick] (-1.5, -2) – (0, -3.5) – (1.5, -2); [thick] (0, -3.5) circle(0.5);

Quick Tip

To create a solid toy like the one shown, combine basic 3D shapes: a cylinder for the body, a hemisphere for one end, and a cone for the other end. These shapes are commonly used in geometry to create complex structures.

---

20. Express $\sin 81^\circ - \tan 81^\circ$ **in terms of trigonometric ratios of angles between 0° and 45°.**

Solution:

We know the following trigonometric identities:

$$\sin(90^\circ - \theta) = \cos(\theta)$$

$$\tan(90^\circ - \theta) = \cot(\theta)$$

For 81°, we have:

$$\sin 81^\circ = \cos 9^\circ$$

and

$$\tan 81^\circ = \cot 9^\circ$$

Thus, the expression becomes:

$$\sin 81^\circ - \tan 81^\circ = \cos 9^\circ - \cot 9^\circ$$

This is the desired expression in terms of angles between 0° and 45°.

Quick Tip

Using trigonometric identities such as $\sin(90^\circ - \theta) = \cos(\theta)$ and $\tan(90^\circ - \theta) = \cot(\theta)$, we can express trigonometric functions of angles greater than 45° in terms of functions of angles less than 45°.

SECTION - III

Note : 1) Answer all the questions.

2) Each question carries 4 marks.

21. If $x^2 + y^2 = 25xy$, then prove that $2\log(x + y) = 3\log 3 + \log x + \log y$.

Solution:

We are given the equation $x^2 + y^2 = 25xy$ and need to prove that:

$$2\log(x + y) = 3\log 3 + \log x + \log y$$

Step 1: Rewrite the left-hand side:

$$2\log(x + y) = \log(x + y)^2$$

Thus, the equation becomes:

$$\log(x+y)^2 = 3\log 3 + \log x + \log y$$

Step 2: Simplify the right-hand side using the logarithmic property $\log a + \log b = \log(ab)$:

$$3\log 3 + \log x + \log y = \log 3^3 + \log(xy) = \log 27 + \log(xy) = \log(27xy)$$

Now, the equation becomes:

$$\log(x+y)^2 = \log(27xy)$$

Step 3: Equating both sides:

$$(x+y)^2 = 27xy$$

Step 4: Using the given equation $x^2 + y^2 = 25xy$, we know:

$$(x+y)^2 = x^2 + 2xy + y^2 = 25xy + 2xy = 27xy$$

Thus, we have shown that:

$$(x+y)^2 = 27xy$$

This proves the equation.

Quick Tip

To prove logarithmic identities, use properties such as $\log a + \log b = \log(ab)$ and manipulate the terms algebraically to simplify both sides of the equation.

---

22. Draw the Venn diagrams of AOB, And, A B and B A (Here A, I are non-stopt, ata)

Venn Diagram of $A \cup B$, $A \cap B$, A, and B:

[fill=blue!30] (0,0) circle (1.5); [fill=red!30] (1,0) circle (1.5);

at (-1, 1) A; at (2, 1) B;

at (0.5, -0.5) $A \cap B$; at (0.5, 1.5) $A \cup B$;

at (-1, 0.5) Only A; at (2, 0.5) Only B;

Explanation:

1. $A \cup B$ (Union): The area covered by both sets A and B, including their intersection.

2. $A \cap B$ (Intersection): The overlapping region where both sets A and B intersect.
3. A: The entire set A, including the part that overlaps with B.
4. B: The entire set B, including the part that overlaps with A.

Quick Tip

Venn diagrams are a helpful way to visually represent sets and their relationships, such as union and intersection. They are useful for understanding how sets overlap and for solving problems involving set theory.

---

23. Solve the pair of linear equations $3x + 2y = 11$ and $2x + y = 4$.

Solution:

We are given the system of linear equations:

$$3x + 2y = 11 \quad (1)$$

$$2x + y = 4 \quad (2)$$

Step 1: Solve equation (2) for y:

$$2x + y = 4$$

$$y = 4 - 2x$$

Step 2: Substitute $y = 4 - 2x$ into equation (1):

$$3x + 2(4 - 2x) = 11$$

Simplify:

$$3x + 8 - 4x = 11$$

$$-x + 8 = 11$$

$$-x = 3$$

$$x = -3$$

Step 3: Substitute $x = -3$ into the equation for y:

$$y = 4 - 2(-3) = 4 + 6 = 10$$

Thus, the solution is:

$$x = -3, \quad y = 10$$

Quick Tip

When solving a system of linear equations, it is often helpful to solve one equation for one variable and then substitute into the other equation.

---

24. Find the roots of the quadratic equation $2x^2 + x = 0$.

Solution:

We are given the quadratic equation:

$$2x^2 + x = 0$$

Step 1: Factor the equation:

$$x(2x + 1) = 0$$

Step 2: Set each factor equal to zero:

$$x = 0$$

and

$$2x + 1 = 0$$

Solving for x:

$$2x = -1$$

$$x = -\frac{1}{2}$$

Thus, the roots of the quadratic equation are:

$$x = 0 \quad \text{and} \quad x = -\frac{1}{2}$$

Quick Tip

When solving quadratic equations, factor the equation if possible and set each factor equal to zero to find the roots.

25. Find the volume and surface area of a sphere of radius 2.1 cm. (Take $\pi = \frac{22}{7}$).

Solution:

We are given the radius $r = 2.1\,\text{cm}$ and $\pi = \frac{22}{7}$. The formulas for the volume and surface area of a sphere are:

$$V = \frac{4}{3}\pi r^3 \quad \text{and} \quad A = 4\pi r^2$$

Step 1: Find the volume Substitute the given values into the volume formula:

$$V = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

First, calculate $(2.1)^3$:

$$2.1^3 = 9.261$$

Now substitute:

$$V = \frac{4}{3} \times \frac{22}{7} \times 9.261 \approx 38.88\,\text{cm}^3$$

Step 2: Find the surface area Substitute the given values into the surface area formula:

$$A = 4 \times \frac{22}{7} \times (2.1)^2$$

First, calculate $(2.1)^2$:

$$2.1^2 = 4.41$$

Now substitute:

$$A = 4 \times \frac{22}{7} \times 4.41 \approx 55.44\,\text{cm}^2$$

Thus, the volume is approximately $38.88\,\text{cm}^3$ and the surface area is approximately $55.44\,\text{cm}^2$.

Quick Tip

To find the volume and surface area of a sphere, use the formulas $V = \frac{4}{3}\pi r^3$ for volume and $A = 4\pi r^2$ for surface area. Make sure to use the correct value for π as given in the problem.

---

26. Simplify $(1 + \cos\theta)(1 + \cos\theta)(1 + \cot\theta)$**.**

Solution:

We are given the expression:

$$(1 + \cos\theta)(1 + \cos\theta)(1 + \cot\theta)$$

Step 1: Expand $(1 + \cos\theta)(1 + \cos\theta)$:

$$(1 + \cos\theta)(1 + \cos\theta) = 1 + 2\cos\theta + \cos^2\theta$$

Thus, the expression becomes:

$$(1 + 2\cos\theta + \cos^2\theta)(1 + \cot\theta)$$

Step 2: Distribute $(1 + \cot\theta)$ across the terms:

$$(1 + 2\cos\theta + \cos^2\theta)(1 + \cot\theta) = (1 + 2\cos\theta + \cos^2\theta) + (1 + 2\cos\theta + \cos^2\theta)\cot\theta$$

Thus, the expression becomes:

$$1 + 2\cos\theta + \cos^2\theta + \cot\theta + 2\cos\theta\cot\theta + \cos^2\theta\cot\theta$$

This is the simplified form of the expression.

Quick Tip

When simplifying expressions involving trigonometric identities, expand the terms and distribute carefully. Sometimes, trigonometric identities like $\cot\theta = \frac{\cos\theta}{\sin\theta}$ can be used for further simplification.

---

27. A die is thrown once. Find the probability of getting:

1. a prime number
2. an odd number

Solution: When a die is thrown, the possible outcomes are: $\{1, 2, 3, 4, 5, 6\}$.

i) Probability of getting a prime number: The prime numbers between 1 and 6 are $2, 3, 5$.

Thus, the favorable outcomes are $\{2, 3, 5\}$.

So, the probability is:

$$P(\text{prime number}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

ii) Probability of getting an odd number: The odd numbers between 1 and 6 are $1, 3, 5$.

Thus, the favorable outcomes are $\{1, 3, 5\}$.

So, the probability is:

$$P(\text{odd number}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

Quick Tip

The probability of an event is given by $P = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$.

---

28. Write the formula to find the median of a grouped data and explain the terms involved in it.

Solution:

The formula to find the median of grouped data is:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$$

Where:

- L = Lower boundary of the median class
- N = Total number of observations
- F = Cumulative frequency of the class preceding the median class
- f = Frequency of the median class
- h = Class width (difference between the upper and lower boundaries of any class)

Explanation of the Terms:

1. L is the lower boundary of the median class, i.e., the value at the lower edge of the group containing the median.
2. N is the total number of observations in the dataset.
3. F is the cumulative frequency just before the median class, which is the sum of frequencies of all classes preceding the median class.
4. f is the frequency of the median class, i.e., the number of data points in the class that contains the median.
5. h is the class width, the difference between the upper and lower boundaries of any class.

Quick Tip

When calculating the median for grouped data, make sure to identify the median class, and use the correct cumulative frequency, frequency, and class width for accurate results.

SECTION - IV

Note : 1) Answer all the questions.

2) Each question carries 8 marks.

3) There is an Internal choice for each question.

29. a) Prove that $\sqrt{7}$ is irrational.

Solution: We will prove by contradiction that $\sqrt{7}$ is irrational.

Step 1: Assume $\sqrt{7}$ is rational Suppose that $\sqrt{7}$ is rational. Then, by the definition of rational numbers, we can write:

$$\sqrt{7} = \frac{p}{q}$$

where p and q are integers, and $\frac{p}{q}$ is in its simplest form (i.e., p and q have no common factors other than 1).

Step 2: Square both sides Now, square both sides of the equation:

$$7 = \frac{p^2}{q^2}$$

Multiply both sides by q^2:

$$7q^2 = p^2$$

This implies that p^2 is divisible by 7. Since 7 is a prime number, this means that p must also be divisible by 7.

Step 3: Express p as a multiple of 7 Since p is divisible by 7, we can write $p = 7k$ for some integer k.

Step 4: Substitute $p = 7k$ into the equation Substitute $p = 7k$ into the equation $7q^2 = p^2$:

$$7q^2 = (7k)^2$$

$$7q^2 = 49k^2$$

Divide both sides by 7:

$$q^2 = 7k^2$$

This implies that q^2 is divisible by 7. Since 7 is a prime number, this means that q must also be divisible by 7.

Step 5: Contradiction We have now shown that both p and q are divisible by 7, which contradicts our original assumption that $\frac{p}{q}$ is in its simplest form (since p and q should have no common factors).

Therefore, our assumption that $\sqrt{7}$ is rational must be false.

Thus, $\sqrt{7}$ is irrational.

Quick Tip

To prove a number is irrational, assume it is rational, then show that it leads to a contradiction by showing the terms involved are not consistent with the assumption.

OR

29. b) ABC is a right triangle right-angled at C. Let BC = a, CA = b, AB = c, and let p be the length of the perpendicular from C on AB. Prove that:

1. $p \cdot c = ab$
2. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution:

i) Prove that $p \cdot c = ab$: We use the area of triangle ABC in two ways. - Using the base $AB = c$ and the height p, we have:

$$\text{Area} = \frac{1}{2} \cdot c \cdot p$$

- Using the other two sides $BC = a$ and $CA = b$, we have:

$$\text{Area} = \frac{1}{2} \cdot a \cdot b$$

Equating these two expressions for the area:

$$\frac{1}{2} \cdot c \cdot p = \frac{1}{2} \cdot a \cdot b$$

Simplifying gives:

$$c \cdot p = a \cdot b$$

Thus, we have proven that:

$$p \cdot c = ab$$

ii) Prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$: Using the Pythagorean theorem, we have:

$$a^2 + b^2 = c^2$$

From the previous result, we have $p = \frac{ab}{c}$. Substituting into $\frac{1}{p^2}$:

$$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

Using $a^2 + b^2 = c^2$, we get:

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2}$$

This simplifies to:

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Thus, we have proven the second relation.

Quick Tip

In right-angled triangles, the relationship between the sides and the perpendicular from the right angle to the hypotenuse can be useful in deriving important identities like the ones above.

30. a) If $A = \{1, 3, 4, 5, 7\}, B = \{2, 4, 5, 6\}, C = \{4, 5, 8, 9\}, D = \{1, 3, 7, 8\}$, then find:

1. $A \cup B$
2. $B \cap D$
3. $A \cap D$
4. $C - D$

Solution:

We are given the following sets:

$$A = \{1, 3, 4, 5, 7\}, \quad B = \{2, 4, 5, 6\}, \quad C = \{4, 5, 8, 9\}, \quad D = \{1, 3, 7, 8\}$$

i) $A \cup B$**:** The union of two sets is the set of elements that are in either set, without repetition:

$$A \cup B = \{1, 3, 4, 5, 7\} \cup \{2, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$$

ii) $B \cap D$**:** The intersection of two sets is the set of elements that are common to both sets:

$$B \cap D = \{2, 4, 5, 6\} \cap \{1, 3, 7, 8\} = \emptyset$$

iii) $A \cap D$**:** The intersection of sets A and D is the set of elements that are common to both sets:

$$A \cap D = \{1, 3, 4, 5, 7\} \cap \{1, 3, 7, 8\} = \{1, 3, 7\}$$

iv) $C - D$**:** The difference between two sets is the set of elements that are in the first set but not in the second:

$$C - D = \{4, 5, 8, 9\} - \{1, 3, 7, 8\} = \{4, 5, 9\}$$

Quick Tip

Use the basic set operations: union ($\cup$), intersection ($\cap$), and difference ($-$) to find the required sets. These operations are fundamental in set theory.

OR

b.A sum of 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is 20 less than its preceding prize, find the value of each of the prizes.

Solution: We are given the following: - The total sum of money available for the prizes is 700. - There are 7 prizes, and each prize is 20 less than the preceding one.

Let the value of the first prize be x. Then, the values of the seven prizes are:

$$x, x - 20, x - 40, x - 60, x - 80, x - 100, x - 120$$

The total sum of the prizes is:

$$x + (x - 20) + (x - 40) + (x - 60) + (x - 80) + (x - 100) + (x - 120) = 700$$

Simplifying:

$$7x - (20 + 40 + 60 + 80 + 100 + 120) = 700$$

$$7x - 420 = 700$$

$$7x = 700 + 420$$

$$7x = 1120$$

$$x = \frac{1120}{7} = 160$$

So, the value of the first prize is 160.

The values of the other prizes are: - Second prize: $160 - 20 = 140$ - Third prize: $160 - 40 = 120$ - Fourth prize: $160 - 60 = 100$ - Fifth prize: $160 - 80 = 80$ - Sixth prize: $160 - 100 = 60$ - Seventh prize: $160 - 120 = 40$

Thus, the values of the prizes are 160, 140, 120, 100, 80, 60, and 40.

Quick Tip

When solving problems involving consecutive terms, like in this case with the prizes, express the terms algebraically and then use the sum formula to find the values.

---

31. a) Find the coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

Solution: Let the points of trisection divide the line segment joining $A(4, -1)$ and $B(-2, -3)$ into three equal parts. The two points of trisection divide the segment in the ratio $1 : 2$ and $2 : 1$, respectively.

The section formula for dividing a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ is:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

i) For the first point of trisection, the ratio is $1 : 2$: Applying the section formula with $m = 1$ and $n = 2$:

$$x_1 = \frac{1(-2) + 2(4)}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$y_1 = \frac{1(-3) + 2(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

Thus, the coordinates of the first point of trisection are $\left(2, \frac{-5}{3}\right)$.

ii) For the second point of trisection, the ratio is $2 : 1$: Applying the section formula with $m = 2$ and $n = 1$:

$$x_2 = \frac{2(-2) + 1(4)}{2+1} = \frac{-4+4}{3} = \frac{0}{3} = 0$$

$$y_2 = \frac{2(-3) + 1(-1)}{2+1} = \frac{-6-1}{3} = \frac{-7}{3}$$

Thus, the coordinates of the second point of trisection are $\left(0, \frac{-7}{3}\right)$.

Quick Tip

The section formula helps in finding the coordinates of a point dividing a line segment in a given ratio. It is useful in coordinate geometry for problems involving midpoints, centroids, and trisection points.

OR

31. b) The table below shows the daily expenditure on food of 25 households in a locality:

Daily Expenditure (in Rupees)	No. of Households
100	4
150	5
$150 - 200$	12
$200 - 250$	2
$250 - 300$	2
$300 - 350$	2

Find the mean daily expenditure on food by a suitable method.

Solution: To find the mean daily expenditure, we use the weighted mean formula for grouped data:

$$\text{Mean} = \frac{\sum(f \cdot x)}{\sum f}$$

where f is the frequency (number of households) and x is the midpoint of each class interval.

Step 1: Find the midpoints of the class intervals:

- For the class 100, the midpoint $x = 100$
- For the class 150, the midpoint $x = 150$
- For the class $150 - 200$, the midpoint $x = \frac{150+200}{2} = 175$
- For the class $200 - 250$, the midpoint $x = \frac{200+250}{2} = 225$
- For the class $250 - 300$, the midpoint $x = \frac{250+300}{2} = 275$
- For the class $300 - 350$, the midpoint $x = \frac{300+350}{2} = 325$

Step 2: Calculate $f \cdot x$:

Daily Expenditure (Rs)	No. of Households (f)	Midpoint (x)	$f \cdot x$
100	4	100	400
150	5	150	750
$150 - 200$	12	175	2100
$200 - 250$	2	225	450
$250 - 300$	2	275	550
$300 - 350$	2	325	650

Step 3: Calculate the total:

$$\sum f = 4 + 5 + 12 + 2 + 2 + 2 = 25$$

$$\sum (f \cdot x) = 400 + 750 + 2100 + 450 + 550 + 650 = 5400$$

Step 4: Apply the formula for the mean:

$$\text{Mean} = \frac{\sum (f \cdot x)}{\sum f} = \frac{5400}{25} = 216$$

Thus, the mean daily expenditure on food is 216.

> **Quick Tip**
>
> For grouped data, the weighted mean formula is helpful in calculating the mean. Always ensure you calculate the midpoints correctly and sum $f \cdot x$ before applying the formula.

---

32. a) Two poles of equal heights are standing opposite to each other on either side of the road, which is 120 feet wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distance of the point from the poles.

Solution: Let the height of the poles be h, and let the point on the road from which the angles of elevation are measured be P. Let the distance from the point P to the first pole (with angle of elevation 60°) be x, and the distance from the point P to the second pole (with angle of elevation 30°) be $120 - x$.

Step 1: Use trigonometry to form two equations For the first pole, we use the tangent of the angle of elevation 60°:

$$\tan(60^\circ) = \frac{h}{x}$$

Since $\tan(60^\circ) = \sqrt{3}$, we get:

$$h = x \cdot \sqrt{3}$$

For the second pole, we use the tangent of the angle of elevation 30°:

$$\tan(30^\circ) = \frac{h}{120 - x}$$

Since $\tan(30^\circ) = \frac{1}{\sqrt{3}}$, we get:

$$h = (120 - x) \cdot \frac{1}{\sqrt{3}}$$

Step 2: Set the two expressions for h equal Equating the two expressions for h:

$$x \cdot \sqrt{3} = \frac{120 - x}{\sqrt{3}}$$

Multiplying both sides by $\sqrt{3}$:

$$x \cdot 3 = 120 - x$$

$$3x = 120 - x$$

$$4x = 120$$

$$x = 30$$

Step 3: Find h Substitute $x = 30$ into the equation for h:

$$h = 30 \cdot \sqrt{3} \approx 30 \cdot 1.732 = 51.96 \, \text{feet}$$

Thus, the height of the poles is approximately 51.96 feet, the distance from the first pole is 30 feet, and the distance from the second pole is $120 - 30 = 90$ feet.

Quick Tip

For problems involving angles of elevation or depression, use trigonometric ratios such as tangent to relate the height of objects to distances.

OR

32. b) One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:

1. a face card
2. a spade
3. the queen of diamonds
4. the king of hearts

Solution: We are given a standard deck of 52 cards, and we need to find the probabilities for the following events.

i) Probability of drawing a face card: In a standard deck, there are 3 face cards (Jack, Queen, King) in each of the 4 suits (hearts, diamonds, spades, clubs). Therefore, the total number of face cards is:

$$3 \times 4 = 12 \, \text{face cards}$$

The probability of drawing a face card is:

$$P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$$

ii) Probability of drawing a spade: There are 13 cards in each suit, and one of the suits is spades. So, the number of spades is 13. The probability of drawing a spade is:

$$P(\text{spade}) = \frac{13}{52} = \frac{1}{4}$$

iii) Probability of drawing the queen of diamonds: There is only one queen of diamonds in the deck. The probability of drawing the queen of diamonds is:

$$P(\text{queen of diamonds}) = \frac{1}{52}$$

iv) Probability of drawing the king of hearts: There is only one king of hearts in the deck. The probability of drawing the king of hearts is:

$$P(\text{king of hearts}) = \frac{1}{52}$$

Quick Tip

For probability problems involving cards, use the formula $P(\text{event}) = \frac{\text{favorable outcomes}}{\text{total outcomes}}$, where the total number of outcomes is the number of cards in the deck.

---

33. a) Draw the graph of the polynomial $p(x) = x^2 - 12$ and find its zeros.

Solution:

The given polynomial is $p(x) = x^2 - 12$.

Step 1: Find the zeros of the polynomial. To find the zeros of the polynomial, we set $p(x) = 0$:

$$x^2 - 12 = 0$$

$$x^2 = 12$$

$$x = \pm\sqrt{12} = \pm 2\sqrt{3}$$

Thus, the zeros of the polynomial are $x = 2\sqrt{3}$ and $x = -2\sqrt{3}$.

Step 2: Draw the graph of the polynomial. The polynomial $p(x) = x^2 - 12$ is a parabola opening upwards (since the coefficient of x^2 is positive) and has a vertex at $(0, -12)$, since the constant term is -12.

The graph will intersect the x-axis at the zeros $x = 2\sqrt{3}$ and $x = -2\sqrt{3}$, and the y-axis at $y = -12$.

[-¿] (-4,0) – (4,0) node[right] x; [-¿] (0,-15) – (0,5) node[above] y;
[domain=-4:4,smooth,variable=,blue] plot (,*-12); [red,thick] (2*sqrt(3),0) – (-2*sqrt(3),0);

The graph above shows the parabola $p(x) = x^2 - 12$, with its zeros at $x = \pm 2\sqrt{3}$.

Quick Tip

For quadratic functions of the form $p(x) = ax^2 + bx + c$, the graph is a parabola. The vertex is at $x = -\frac{b}{2a}$, and the zeros can be found by solving $ax^2 + bx + c = 0$.

OR

33. b) Construct a triangle of sides 4 cm, 5 cm, and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle.

Solution:

We are given a triangle with sides of lengths 4 cm, 5 cm, and 6 cm. The task is to construct a triangle similar to the given one, with sides being $\frac{3}{4}$ of the corresponding sides of the first triangle.

Step 1: Construct the first triangle with sides 4 cm, 5 cm, and 6 cm.

We can construct the triangle using the following steps: - Draw a line segment of length 6 cm. - Using a compass, mark points at a distance of 4 cm and 5 cm from the two endpoints of the 6 cm line segment to form the two remaining sides of the triangle. - Adjust the compass to draw arcs that intersect at a point to form the triangle.

Step 2: Construct the similar triangle with sides $\frac{3}{4}$ of the corresponding sides of the first triangle.

To create a similar triangle with sides $\frac{3}{4}$ of the first triangle's sides, we follow these steps: - Draw a line parallel to the first triangle's sides, starting from a point on one of the sides. - Use the ratio $\frac{3}{4}$ to scale down the length of each side.

The new triangle will have sides $4 \times \frac{3}{4} = 3$ cm, $5 \times \frac{3}{4} = 3.75$ cm, and $6 \times \frac{3}{4} = 4.5$ cm.

Conclusion: The triangle similar to the given triangle has sides 3 cm, 3.75 cm, and 4.5 cm.

Quick Tip

When constructing a similar triangle, the corresponding sides are proportional. You can use this property to scale down or scale up the sides while maintaining similarity.
