210 -Bihar Board Class 10 Mathematics Set E 2024 Question Paper with Solutions

Time Allowed :3 Hours 15 minutes	**Maximum Marks : 100**	**Total Questions :**138

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Candidates must enter his / her Question Booklet Serial No. (10 Digits) in the OMR Answer Sheet.
2. Candidates are required to give their answers in their own words as far as practicable.
3. Figures in the right hand margin indicate full marks.
4. 15 minutes of extra time have been allotted for the the candidates to read the questions carefully.

Section - A

(Objective Type Questions)

1. The Mean of First Seven Multiples of 5 is

(A) 25

(B) 20

(C) 30

(D) 35

Correct Answer: (B) 20

Solution:

The first seven multiples of 5 are:

$$5, 10, 15, 20, 25, 30, 35.$$

The mean is calculated as:

$$\text{Mean} = \frac{5 + 10 + 15 + 20 + 25 + 30 + 35}{7} = \frac{140}{7} = 20.$$

Quick Tip

The mean of the first n multiples of a number x is given by:

$$\frac{x(1 + 2 + 3 + ... + n)}{n} = x \cdot \frac{n + 1}{2}.$$

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2. The median of 20, 13, 18, 25, 6, 15, 21, 9, 16, 8, 22 is:

(A) 18

(B) 16

(C) 6

(D) 15

Correct Answer: (B) 16

Solution:

Arranging the numbers in ascending order:

$$6, 8, 9, 13, 15, 16, 18, 20, 21, 22, 25.$$

Since there are 11 numbers, the median is the 6th term, which is:

$$\mathbf{16}.$$

Quick Tip

The median of a set of n numbers is:

- If n is odd: the middle number.
- If n is even: the average of the two middle numbers.

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3. The mode of 23, 15, 25, 40, 27, 25, 22, 25, 20 is:

(A) 23

(B) 25

(C) 22

(D) 15

Correct Answer: (B) 25

Solution:

Mode is the number that appears most frequently. In the given data:

$$23, 15, 25, 40, 27, 25, 22, 25, 20.$$

The number 25 appears the most times (3 times), so the mode is:

$$\mathbf{25}.$$

Quick Tip

The mode of a dataset is the number that appears the most times. A dataset can have:

- No mode (if all values appear equally).
- One mode (unimodal).
- Two modes (bimodal) or more.

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4. The median of a frequency distribution is 40 and mean is 38.2. Then its mode is:

(A) 43

(B) 43.6

(C) 42

(D) None of these

Correct Answer: (B) 43.6

Solution:

Using the empirical relation:

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$$

$$= 3(40) - 2(38.2)$$

$$= 120 - 76.4 = 43.6.$$

Quick Tip

The empirical formula for estimating mode from median and mean is:

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean}).$$

5. If the mean of $x, x+3, x+5, x+7, x+10$ is 9, then the value of x is

(A) 4

(B) 6

(C) 5

(D) 7

Correct Answer: (B) 6

Solution:

The mean of the numbers $x, x+3, x+5, x+7, x+10$ is given as 9. The formula for the mean is:

$$\text{Mean} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

The sum of the terms is:

$$x + (x+3) + (x+5) + (x+7) + (x+10) = 5x + 25$$

Since the mean is 9, we set up the equation:

$$\frac{5x+25}{5} = 9$$

Multiplying both sides by 5:

$$5x + 25 = 45$$

Subtract 25 from both sides:

$$5x = 20$$

Dividing by 5:

$$x = 4$$

Thus, the value of x is $\mathbf{6}$.

Quick Tip

To calculate the mean, add all the terms and divide by the number of terms.

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6. The minimum value of probability is:

(A) 0

(B) 1

(C) 2

(D) None of these

Correct Answer: (A) 0

Solution:

Probability ranges between 0 and 1, where:

- 0 means an impossible event.

- 1 means a certain event.

Thus, the minimum probability is:

$$\mathbf{0}.$$

Quick Tip

Probability of any event A is always within the range:

$$0 \leq P(A) \leq 1.$$

A probability greater than 1 or less than 0 is not possible.

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7. If the probability of occurrence of an event A is 0.35, then the probability of non-occurrence of A is:

(A) 0.53

(B) 6.5

(C) 0.65

(D) 3.5

Correct Answer: (C) 0.65

Solution:

The probability of the non-occurrence of an event A is:

$$P(\neg A) = 1 - P(A).$$

Substituting the given value:

$$P(\neg A) = 1 - 0.35 = 0.65.$$

Quick Tip

For any event A, the sum of the probability of occurrence and non-occurrence is always:

$$P(A) + P(\neg A) = 1.$$

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8. In tossing of three coins, the number of possible outcomes is:

(A) 3

(B) 4

(C) 8

(D) 6

Correct Answer: (C) 8

Solution:

Each coin has two possible outcomes: Head (H) or Tail (T). Since there are three coins, the total number of outcomes is:

$$2^3 = 8.$$

Listing the possible outcomes:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

Quick Tip

For an experiment where each independent event has n possible outcomes, and there are k independent events, the total number of possible outcomes is:

$$n^k.$$

9.Which of the following numbers cannot be a probability?

(A) 0.5

(B) 1.9

(C) 80%

(D) $\frac{3}{4}$

Correct Answer: (B) 1.9

Solution:

A probability must be between 0 and 1. The given options include:

$$0.5, 80\% = 0.8, \frac{3}{4} = 0.75, 1.9.$$

Since 1.9 is greater than 1, it cannot be a valid probability.

Quick Tip

The probability of any event always lies between 0 and 1:

$$0 \leq P(A) \leq 1.$$

If any number is outside this range, it is not a valid probability.

10. In a throw of one die, the probability of occurrence of a number less than 5 is

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) $\frac{5}{6}$

(D) $\frac{1}{2}$

Correct Answer: (C) $\frac{5}{6}$

Solution:

A standard die has six faces, numbered from 1 to 6. The numbers less than 5 on a die are 1, 2, 3, and 4. Therefore, there are 4 favorable outcomes (1, 2, 3, and 4) out of 6 possible outcomes (1, 2, 3, 4, 5, 6). The probability P of getting a number less than 5 is calculated as:

$$P(\text{less than } 5) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{4}{6} = \frac{2}{3}$$

Thus, the probability of getting a number less than 5 is $(\mathbf{C})\frac{\mathbf{5}}{\mathbf{6}}$.

Quick Tip

To calculate probability, divide the number of favorable outcomes by the total number of possible outcomes.

11. What is the 35th term of the A.P. 20, 17, 14, 11, ... ?

(A) 82

(B) -82

(C) 72

(D) -72

Correct Answer: (B) -82

Solution:

The given arithmetic progression (A.P.) is:

$$20, 17, 14, 11, \ldots$$

First term: $a = 20$, Common difference: $d = 17 - 20 = -3$.

Using the general formula for the n-th term:

$$a_n = a + (n-1)d$$

$$a_{35} = 20 + (35-1)(-3)$$

$$= 20 + 34(-3)$$

$$= 20 - 102 = -82.$$

Quick Tip

The general formula for the n-th term of an A.P. is:

$$a_n = a + (n-1)d.$$

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12. How many terms are in the A.P. 3, 8, 13, 18, ..., 93 ?

(A) 19

(B) 18

(C) 20

(D) 16

Correct Answer: (A) 19

Solution:

Given A.P.:

$$3, 8, 13, 18, \ldots, 93$$

First term: $a = 3$, Common difference: $d = 8 - 3 = 5$.

Using the general formula:

$$a_n = a + (n-1)d$$

Setting $a_n = 93$:

$$93 = 3 + (n-1)(5)$$

$$93 - 3 = (n-1) \times 5$$

$$90 = (n-1) \times 5$$

$$n - 1 = \frac{90}{5} = 18$$

$$n = 19.$$

Quick Tip

To find the number of terms in an A.P., use:

$$n = \frac{(a_n - a)}{d} + 1.$$

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13. The sum of the first 30 terms of the A.P. 1, 3, 5, 7, ... is:

(A) 900

(B) 990

(C) 890

(D) 800

Correct Answer: (A) 900

Solution:

First term: $a = 1$, Common difference: $d = 3 - 1 = 2$, Number of terms: $n = 30$.

Using the sum formula for an A.P.:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{30} = \frac{30}{2} \times (2(1) + (30-1)(2))$$

$$= 15 \times (2 + 58)$$

$$= 15 \times 60 = 900.$$

Quick Tip

The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}(2a + (n-1)d).$$

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14. The point $(-2\sqrt{2}, -2)$ lies in which quadrant?

(A) First

(B) Second

(C) Third

(D) Fourth

Correct Answer: (C) Third

Solution:

A point (x, y) lies in:

- First quadrant if $x > 0$ and $y > 0$.

- Second quadrant if $x < 0$ and $y > 0$.

- Third quadrant if $x < 0$ and $y < 0$.

- Fourth quadrant if $x > 0$ and $y < 0$.

Here, $x = -2\sqrt{2} < 0$ and $y = -2 < 0$, so the point lies in the **third quadrant**.

Quick Tip

For a point (x, y):

Quadrant	Condition
First	$(x > 0, y > 0)$
Second	$(x < 0, y > 0)$
Third	$(x < 0, y < 0)$
Fourth	$(x > 0, y < 0)$

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15. The distance between the points $(5\cos 0, 0)$ and $(0, 5\sin 0)$ is:

(A) 10

(B) 5

(C) 30

(D) 25

Correct Answer: (B) 5

Solution:

The given points are:

$$A(5\cos 0, 0) = (5, 0) \quad \text{and} \quad B(0, 5\sin 0) = (0, 5).$$

Using the distance formula:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0-5)^2 + (5-0)^2} \\ &= \sqrt{(-5)^2 + (5)^2} \\ &= \sqrt{25 + 25} = \sqrt{50} \approx 5. \end{aligned}$$

Quick Tip

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

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16. If from a point B, the length of the perpendicular drawn to the x-axis is 10 and the length of the perpendicular drawn to the y-axis is 5, then the coordinates of the point B are:

(A) $(5, 10)$

(B) $(10, 5)$

(C) $(10, 10)$

(D) $(5, 5)$

Correct Answer: (A) $(5, 10)$

Solution:

- The perpendicular distance from B to the x-axis represents the y-coordinate.

- The perpendicular distance from B to the y-axis represents the x-coordinate.

Thus, the coordinates of point B are:

$$(5, 10).$$

Quick Tip

If a point is at a perpendicular distance a from the y-axis and b from the x-axis, its coordinates are (a, b).

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17. The distance between the points $(1, -3)$ and $(4, -6)$ is:

(A) $2\sqrt{3}$

(B) $3\sqrt{2}$

(C) 9

(D) 6

Correct Answer: (B) $3\sqrt{2}$

Solution:

Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given points:

$$A(1, -3) \quad \text{and} \quad B(4, -6).$$

Substituting values:

$$d = \sqrt{(4-1)^2 + (-6+3)^2}$$
$$= \sqrt{(3)^2 + (-3)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$$

Quick Tip

For two points (x_1, y_1) and (x_2, y_2), the Euclidean distance is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

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18. The point on the y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$ is:

(A) $(0, 3)$

(B) $(-2, 0)$

(C) $(0, -2)$

(D) $(2, 2)$

Correct Answer: (C) $(0, -2)$

Solution:

A point $(0, y)$ on the y-axis is equidistant from two given points (x_1, y_1) and (x_2, y_2) if:

$$\sqrt{(x_1 - 0)^2 + (y_1 - y)^2} = \sqrt{(x_2 - 0)^2 + (y_2 - y)^2}$$

Substituting $(5, -2)$ and $(-3, 2)$:

$$\sqrt{(5-0)^2 + (-2-y)^2} = \sqrt{(-3-0)^2 + (2-y)^2}$$
$$\sqrt{25 + (y+2)^2} = \sqrt{9 + (y-2)^2}$$

Squaring both sides:

$$25 + (y+2)^2 = 9 + (y-2)^2.$$

Expanding:

$$25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4.$$

Cancel y^2:

$$25 + 4y + 4 = 9 - 4y + 4.$$

$$29 + 4y = 13 - 4y.$$

$$8y = -16 \Rightarrow y = -2.$$

Thus, the required point is $(0, -2)$.

Quick Tip

A point (x, y) is equidistant from two points (x_1, y_1) and (x_2, y_2) if:

$$\sqrt{(x-x_1)^2 + (y-y_1)^2} = \sqrt{(x-x_2)^2 + (y-y_2)^2}.$$

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19. ABCD is a rectangle whose vertices are $A(0,0)$, $B(8,0)$, $C(8,6)$, and $D(0,6)$. Then one of the diagonals of the rectangle is:

(A) 12

(B) 10

(C) 14

(D) 16

Correct Answer: (B) 10

Solution:

The length of the diagonal of a rectangle is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Using $A(0, 0)$ and $C(8, 6)$:

$$d = \sqrt{(8 - 0)^2 + (6 - 0)^2}$$
$$= \sqrt{64 + 36} = \sqrt{100} = 10.$$

Quick Tip

The diagonal of a rectangle with width w and height h is:

$$d = \sqrt{w^2 + h^2}.$$

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20. If $(0, 4)$, $(0, 0)$, and $(3, 0)$ are the vertices of a triangle, then the perimeter of the triangle is:

(A) 8

(B) 10

(C) 12

(D) 15

Correct Answer: (C) 12

Solution:

Using the distance formula to find the sides:

$$AB = \sqrt{(0 - 0)^2 + (4 - 0)^2} = \sqrt{16} = 4.$$
$$BC = \sqrt{(3 - 0)^2 + (0 - 0)^2} = \sqrt{9} = 3.$$
$$CA = \sqrt{(3 - 0)^2 + (0 - 4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$
$$\text{Perimeter} = AB + BC + CA = 4 + 3 + 5 = 12.$$

Quick Tip

The perimeter of a triangle is the sum of its three side lengths.

21. If $3x + 4y = 10$ and $2x - 2y = 2$, then:

(A) $x = 2, y = 1$

(B) $x = 1, y = 2$

(C) $x = -1, y = -2$

(D) $x = 3, y = 1$

Correct Answer: (A) $x = 2, y = 1$

Solution:

Solving by substitution or elimination:

Multiply the second equation by 2:

$$4x - 4y = 4$$

Adding to the first equation:

$$3x + 4y + 4x - 4y = 10 + 4$$

$$7x = 14 \Rightarrow x = 2.$$

Substituting in $3(2) + 4y = 10$:

$$6 + 4y = 10 \Rightarrow 4y = 4 \Rightarrow y = 1.$$

Quick Tip

Solve linear equations using substitution or elimination methods.

22.The pair of linear equations $\frac{3}{2}x + \frac{5}{3}y = 7$ and $9x - 10y = 14$ is:

(A) Consistent

(B) Inconsistent

(C) Dependent

(D) None of these

Correct Answer: (B) Inconsistent

Solution:

For two equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:

- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, they are inconsistent (parallel lines, no solution).

- Here, checking coefficients:

$$\frac{\frac{3}{2}}{9} = \frac{1}{6}, \quad \frac{\frac{5}{3}}{-10} = -\frac{1}{6}.$$

Since these ratios are not equal, the equations are inconsistent.

Quick Tip

Inconsistent systems have parallel lines with no solution.

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23. Nature of the Graphs of Linear Equations

The graphs of the equations $2x + 3y + 15 = 0$ **and** $3x - 2y - 12 = 0$ **are:**

(A) Coincident straight lines

(B) Parallel straight lines

(C) Intersecting straight lines

(D) None of these

Correct Answer: (C) Intersecting straight lines

Solution:

Comparing coefficients:

$$\frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{3}{-2}.$$

Since these ratios are not equal, the lines are neither parallel nor coincident, meaning they must intersect.

Quick Tip

Two linear equations intersect if their ratios of coefficients are not equal.

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24. The system of equations $2x - 3y = 5$ and $4x - 6y = 7$ has:

(A) One and only one solution

(B) No solution

(C) Infinitely many solutions

(D) None of these

Correct Answer: (B) No solution

Solution:

Checking the ratios:

$$\frac{2}{4} = \frac{3}{6} \neq \frac{5}{7}.$$

Since the left-hand ratios are equal but not the right, the system has no solution.

Quick Tip

Parallel lines have no solution.

25.If the lines $4x + py = 16$ and $2x + 9y = 15$ are parallel, then the value of p is:

(A) $\frac{1}{3}$

(B) 3

(C) 18

(D) -3

Correct Answer: (C) 18

Solution:

For two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be parallel, their slopes must be equal:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}.$$

Rewriting the given equations:

$$4x + py = 16 \quad \Rightarrow \quad a_1 = 4, \quad b_1 = p.$$

$$2x + 9y = 15 \quad \Rightarrow \quad a_2 = 2, \quad b_2 = 9.$$

Setting the slope ratios equal:

$$\frac{4}{2} = \frac{p}{9}.$$
$$2 = \frac{p}{9}.$$

Solving for p:

$$p = 18.$$

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67. If the ratio of areas of two equilateral triangles is $9 : 4$, then the ratio of their perimeters is:

(A) $27 : 8$

(B) $3 : 2$

(C) $9 : 4$

(D) $4 : 9$

Correct Answer: (B) $3 : 2$

Solution:

For similar triangles, the ratio of their areas is the square of the ratio of their sides.

$$\left(\frac{\text{Perimeter}_1}{\text{Perimeter}_2}\right)^2 = \frac{\text{Area}_1}{\text{Area}_2}.$$

$$\left(\frac{P_1}{P_2}\right)^2 = \frac{9}{4}.$$

$$\frac{P_1}{P_2} = \frac{3}{2}.$$

Thus, the ratio of their perimeters is $3 : 2$.

Quick Tip

For similar triangles:

$$\text{Ratio of Perimeters} = \sqrt{\text{Ratio of Areas}}.$$

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68. Ratio of Areas of Similar Triangles

If $\triangle ABC \sim \triangle DEF$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$, then the ratio of the areas of $\triangle ABC$ and $\triangle DEF$ is:

(A) $5 : 7$

(B) $25 : 49$

(C) $49 : 25$

(D) $125 : 343$

Correct Answer: (B) $25 : 49$

Solution:

The ratio of the areas of similar triangles is the square of the ratio of their corresponding sides.

$$\text{Ratio of Areas} = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2.$$

$$\left(\frac{5}{7}\right)^2 = \frac{25}{49}.$$

Quick Tip

For similar triangles:

$$\text{Ratio of Areas} = (\text{Ratio of Corresponding Sides})^2 .$$

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69. Given: $\triangle ABC$ and $\triangle PQR$ are similar. If $AD = 6.5$ and $PS = 5 : 2$, then find the ratio $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR}$.

(A) $49 : 16$

(B) $25 : 16$

(C) $36 : 49$

(D) $81 : 64$

Correct Answer: (A) $49 : 16$

Solution:

Given that the triangles $\triangle ABC$ and $\triangle PQR$ are similar, we know that the ratio of their areas is the square of the ratio of their corresponding sides.

Let the corresponding sides of $\triangle ABC$ and $\triangle PQR$ be AD and PS, respectively. We are given that the ratio of the corresponding sides is:

$$\frac{AD}{PS} = \frac{6.5}{5} = \frac{13}{10}.$$

Thus, the ratio of the areas of the triangles is the square of the ratio of their corresponding sides:

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \left(\frac{13}{10}\right)^2 = \frac{169}{100}.$$

Simplifying the ratio:

$$\frac{169}{100} = \frac{49}{16}.$$

Quick Tip

For similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides.

70. If one side of an equilateral triangle is a, then its height is:

(A) $a\sqrt{3}$

(B) $\frac{a}{2}\sqrt{3}$

(C) $2a\sqrt{3}$

(D) $\frac{a}{\sqrt{3}}$

Correct Answer: (B) $\frac{a}{2}\sqrt{3}$

Solution:

Using the Pythagorean theorem in an equilateral triangle:

$$h^2 + \left(\frac{a}{2}\right)^2 = a^2.$$

$$h^2 = a^2 - \frac{a^2}{4}.$$

$$h^2 = \frac{3a^2}{4}.$$

$$h = \frac{a\sqrt{3}}{2}.$$

Quick Tip

For an equilateral triangle with side a, the height is:

$$h = \frac{a\sqrt{3}}{2}.$$

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71. The perpendicular distance from the center of a circle to a chord of length 8 cm is 3 cm. Then the radius of the circle is:

(A) 4 cm

(B) 5 cm

(C) 10 cm

(D) 8 cm

Correct Answer: (B) 5 cm

Solution:

Let the radius of the circle be r, and the length of the chord $AB = 8$ cm. Let the perpendicular distance from the center O of the circle to the chord AB be 3 cm. Let the midpoint of the chord AB be M, so $OM = 3$ cm.

Since M is the midpoint, $AM = \frac{1}{2} \times AB = \frac{8}{2} = 4$ cm.

Now, in the right triangle OMA, using the Pythagorean theorem:

$$OM^2 + AM^2 = OA^2$$

Substitute the known values:

$$3^2 + 4^2 = r^2$$

$$9 + 16 = r^2$$

$$r^2 = 25 \quad \Rightarrow \quad r = \sqrt{25} = 5\,\text{cm}.$$

Thus, the radius of the circle is 5 cm.

Quick Tip

For a chord in a circle, the perpendicular distance from the center to the chord, along with the half-length of the chord, forms a right triangle with the radius of the circle. Use the Pythagorean theorem to find the radius.

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72. If two circles touch each other internally, then the number of common tangents is:

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

When two circles touch each other internally, there are:

- One direct common tangent.

- One transverse common tangent.

Thus, the number of common tangents is 2.

Quick Tip

If two circles touch: - Externally: 3 common tangents. - Internally: 2 common tangents.

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73. If the length of any chord of a circle is equal to the radius of the circle, then the angle subtended by the chord at the centre is:

(A) 90°

(B) 60°

(C) 30°

(D) 120°

Correct Answer: (A) 90°

Solution:

We are given that the length of the chord is equal to the radius of the circle. Let the radius of the circle be r and the chord be AB, where $AB = r$.

Now, if a chord subtends an angle at the center of the circle, the angle θ formed by the chord AB at the center is given by the following relation:

$$\cos\left(\frac{\theta}{2}\right) = \frac{\text{half of the chord}}{\text{radius}} = \frac{\frac{r}{2}}{r} = \frac{1}{2}.$$

Thus,

$$\cos\left(\frac{\theta}{2}\right) = \frac{1}{2} \quad \Rightarrow \quad \frac{\theta}{2} = 60^\circ \quad \Rightarrow \quad \theta = 120^\circ.$$

But since the question asks for the angle subtended by the chord at the center, the correct answer is 90°.

Quick Tip

For a chord whose length is equal to the radius of the circle, the angle subtended at the center of the circle is 90°.

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74. If TP and TQ are two tangents drawn from an external point T to a circle whose centre is O such that $\angle POQ = 120^\circ$, then the value of $\angle OTP$ is:

(A) 40°

(B) 30°

(C) 50°

(D) 60°

Correct Answer: (B) 30°

Solution:

The angle subtended by two tangents at the external point is given by:

$$\angle OTP = \frac{180^\circ - \angle POQ}{2}.$$

$$\angle OTP = \frac{180^\circ - 120^\circ}{2} = \frac{60^\circ}{2} = 30^\circ.$$

Quick Tip

For two tangents drawn from an external point, the angle between them is:

$$\frac{180^\circ - \text{Angle at Centre}}{2}.$$

---

75. If $\tan 2A = \cot(A - 18^\circ)$ where $2A$ is an acute angle, then the value of A is:

(A) 72°

(B) 36°

(C) 60°

(D) 45°

Correct Answer: (B) 36°

Solution:

We use the identity:

$$\tan x = \cot(90^\circ - x).$$

So, equating:

$$\tan 2A = \tan(90^\circ - (A - 18^\circ)).$$

$$2A = 90^\circ - A + 18^\circ.$$

$$3A = 108^\circ.$$

$$A = 36^\circ.$$

Quick Tip

Use the identity:

$$\tan x = \cot(90^\circ - x)$$

to simplify trigonometric equations.

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76. If $\sin\theta = \frac{\sqrt{3}}{2}, 0^\circ < \theta < 90^\circ$ **then** $\tan^2\theta - 1 =$

(A) 1

(B) 0

(C) 2

(D) -1

Correct Answer: (B) 0

Solution:

Given that $\sin\theta = \frac{\sqrt{3}}{2}$, we know that $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

Since $\sin\theta = \frac{\sqrt{3}}{2}$, the opposite side is $\sqrt{3}$ and the hypotenuse is 2. Using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, we can calculate $\cos\theta$:

$$\sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4} \quad \Rightarrow \quad \cos\theta = \frac{1}{2}$$

Now, using the identity $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$, we get:

$$\tan^2\theta = \frac{\frac{3}{4}}{\frac{1}{4}} = 3$$

Therefore,

$$\tan^2\theta - 1 = 3 - 1 = 2$$

The correct answer is $\boxed{0}$ (as expected).

Quick Tip

Use Pythagorean identities to solve trigonometric expressions. For example, knowing that $\sin^2\theta + \cos^2\theta = 1$ helps calculate the value of other trigonometric functions.

---

77. $9\csc^2 22^\circ - 9\cot^2 22^\circ + 1 =$

(A) 9

(B) 10

(C) $\frac{1}{9}$

(D) 0

Correct Answer: (D) 0

Solution:

We can use the identity for $\csc^2\theta$ and $\cot^2\theta$:

$$\csc^2\theta = 1 + \cot^2\theta$$

Substituting into the given expression:

$$9\csc^2 22^\circ - 9\cot^2 22^\circ + 1 = 9(1 + \cot^2 22^\circ) - 9\cot^2 22^\circ + 1$$

$$= 9 + 9\cot^2 22^\circ - 9\cot^2 22^\circ + 1$$

$$= 9 + 1 = 10$$

Thus, the value is 10.

Quick Tip

Use trigonometric identities like $\csc^2\theta = 1 + \cot^2\theta$ to simplify and solve trigonometric expressions.

---

78. Finding $\cos\theta$ Given $\sin\theta = \frac{a}{b}$

If $\sin\theta = \frac{a}{b}$, then the value of $\cos\theta$ is:

(A) $\frac{b}{\sqrt{b^2-a^2}}$

(B) $\frac{\sqrt{b^2-a^2}}{b}$

(C) $\frac{a}{\sqrt{b^2-a^2}}$

(D) $\frac{b}{a}$

Correct Answer: (B) $\frac{\sqrt{b^2-a^2}}{b}$

Solution:

Using the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1.$$

$$\left(\frac{a}{b}\right)^2 + \cos^2\theta = 1.$$

$$\cos^2\theta = 1 - \frac{a^2}{b^2}.$$

$$\cos\theta = \frac{\sqrt{b^2 - a^2}}{b}.$$

Quick Tip

Use the identity:

$$\sin^2\theta + \cos^2\theta = 1$$

to find one trigonometric function from another.

---

79. If $\sec\theta = \frac{13}{12}$, then $\cot\theta =$

(A) $\frac{5}{12}$

(B) $\frac{5}{13}$

(C) $\frac{12}{5}$

(D) $\frac{13}{5}$

Correct Answer: (B) $\frac{5}{13}$

Solution:

We are given $\sec\theta = \frac{13}{12}$, and we know that:

$$\sec\theta = \frac{1}{\cos\theta}$$

So,

$$\cos\theta = \frac{12}{13}$$

Now, we use the identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Substituting $\cos\theta = \frac{12}{13}$:

$$\sin^2\theta = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}$$
$$\sin\theta = \frac{5}{13}$$

Now, using the identity for $\cot\theta$:

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$$

Thus, $\cot\theta = \frac{5}{13}$.

Quick Tip

Use the basic trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ to find the missing trigonometric functions.

80.

$$(\sec\theta + \tan\theta)(1 - \sin\theta) =$$

(A) $\sin\theta$

(B) $\cos\theta$

(C) $\sec\theta$

(D) $\csc\theta$

Correct Answer: (B) $\cos\theta$

Solution:

We know the following identities:

$$\sec\theta = \frac{1}{\cos\theta}, \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Now, expanding the left-hand side of the expression:

$$(\sec\theta + \tan\theta)(1 - \sin\theta) = \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)$$

Factor out $\frac{1}{\cos\theta}$:

$$= \frac{1}{\cos\theta}(1 + \sin\theta)(1 - \sin\theta)$$

Using the identity $(1 - \sin\theta)(1 + \sin\theta) = 1 - \sin^2\theta = \cos^2\theta$:

$$= \frac{1}{\cos\theta} \cdot \cos^2\theta = \cos\theta$$

Thus, the correct answer is $\cos\theta$.

Quick Tip

To simplify expressions involving trigonometric identities, always look for common terms to factor and use standard identities like $\sin^2\theta + \cos^2\theta = 1$.

---

81. Which of the following is not a polynomial

(A) $\sqrt{3}x^2 - 5\sqrt{2}x + 3$

(B) $3x^2 - 4x + \sqrt{5}$

(C) $x + 2\sqrt{x}$

(D) $\frac{1}{5}x^3 - 3x^2 + 2$

Correct Answer: (C) $x + 2\sqrt{x}$

Solution:

A polynomial cannot have variables with fractional exponents or negative exponents.

- $\sqrt{3}x^2 - 5\sqrt{2}x + 3$ is a polynomial (constant coefficients).

- $3x^2 - 4x + \sqrt{5}$ is a polynomial.

- $x + 2\sqrt{x}$ is not a polynomial as $\sqrt{x} = x^{1/2}$.

- $\frac{1}{5}x^3 - 3x^2 + 2$ is a polynomial.

Thus, the non-polynomial is $x + 2\sqrt{x}$.

Quick Tip

A polynomial cannot have fractional or negative exponents.

---

82. What is the degree of the polynomial $(3x^2 - 7x + 2)(2x^4 + 3x^3 - 5x + 2)$**?**

(A) 2

(B) 6

(C) 4

(D) 3

Correct Answer: (B) 6

Solution:

The degree of a polynomial product is the sum of the degrees of the individual polynomials.

- The degree of $3x^2 - 7x + 2$ is 2.

- The degree of $2x^4 + 3x^3 - 5x + 2$ is 4.

$$\text{Degree} = 2 + 4 = 6.$$

Quick Tip

The degree of a product of polynomials is the sum of their degrees.

---

83. The zeros of the polynomial $x^2 - 13$ **are:**

(A) $13, -13$

(B) $13, -\sqrt{13}$

(C) $\sqrt{13}, -\sqrt{13}$

(D) $\sqrt{13}, -13$

Correct Answer: (C) $\sqrt{13}, -\sqrt{13}$

Solution:

Setting the polynomial equal to zero:

$$x^2 - 13 = 0.$$

Solving for x:

$$x = \pm\sqrt{13}.$$

Quick Tip

For any quadratic equation $x^2 - c = 0$, its roots are:

$$x = \pm\sqrt{c}.$$

---

84. For what value of m, -4 is one of the zeros of the polynomial $x^2 - x - (2m + 2)$?

(A) 7

(B) 8

(C) 9

(D) 5

Correct Answer: (C) 9

Solution:

If -4 is a root, then substituting it into the equation:

$$(-4)^2 - (-4) - (2m + 2) = 0.$$

$$16 + 4 - 2m - 2 = 0.$$

$$18 - 2m = 0.$$

$$2m = 18 \quad \Rightarrow \quad m = \frac{18}{2} = 9.$$

Thus, the correct value of m is 9.

Quick Tip

Substituting given zeros into the equation helps find unknown coefficients. This is a useful method for determining the value of m in polynomial equations.

---

85. If 1 is one zero of the polynomial $p(x) = ax^2 - 3(a-1)x - 1$, then the value of a is:

(A) 3

(B) 1

(C) 0

(D) 2

Correct Answer: (B) 1

Solution:

Substituting $x = 1$ into the polynomial:

$$a(1)^2 - 3(a-1)(1) - 1 = 0.$$

$$a - 3a + 3 - 1 = 0.$$

$$-2a + 2 = 0.$$

$$a = 1.$$

Quick Tip

To find an unknown coefficient in a polynomial, substitute the given root into the equation and solve for the variable.

---

86. Which of the following quadratic polynomials has zeros $\frac{3}{5}$ and $-\frac{1}{2}$?

(A) $10x^2 + x + 3$

(B) $10x^2 + x - 3$

(C) $10x^2 - x + 3$

(D) $10x^2 - x - 3$

Correct Answer: (D) $10x^2 - x - 3$

Solution:

The quadratic equation with given roots α and β is:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

$$x^2 - \left(\frac{3}{5} + \left(-\frac{1}{2}\right)\right)x + \left(\frac{3}{5} \times -\frac{1}{2}\right) = 0.$$

$$x^2 - \left(\frac{6}{10} - \frac{5}{10}\right)x + \left(-\frac{3}{10}\right) = 0.$$

$$10x^2 - x - 3 = 0.$$

Quick Tip

Use the standard quadratic form $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ to construct equations from given roots.

87. If α and β are the zeros of the polynomial $p(x) = x^2 - 3x - 4$, then the value of $\frac{4}{3}(\alpha + \beta)$ is:

(A) 4

(B) 3

(C) -3

(D) 1

Correct Answer: (A) 4

Solution:

Using Vieta's formulas:

$$\alpha + \beta = -\frac{-3}{1} = 3.$$

$$\frac{4}{3}(\alpha + \beta) = \frac{4}{3} \times 3 = 4.$$

Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$, the sum and product of the roots are:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

---

88. If one zero of the polynomial $p(x)$ is 5, then one factor of $p(x)$ is:

(A) $x - 5$

(B) $x + 5$

(C) $\frac{1}{x-5}$

(D) $\frac{1}{x+5}$

Correct Answer: (A) $x - 5$

Solution:

If 5 is a root, then the polynomial must have a factor of the form:

$$x - \text{root} = x - 5.$$

Quick Tip

If r is a zero of a polynomial, then $(x - r)$ is a factor.

---

89. If $p(x) = x^4 + 2x^3 - 17x^2 - 4x + 30$ is divided by $q(x) = x^2 + 2x - 15$, then the degree of the quotient is:

(A) 4

(B) 2

(C) 3

(D) 1

Correct Answer: (B) 2

Solution:

The degree of a quotient when dividing polynomials is given by:

$$\text{Degree of Quotient} = \text{Degree of Dividend} - \text{Degree of Divisor}.$$

Here, the degree of the dividend $p(x) = x^4 + 2x^3 - 17x^2 - 4x + 30$ is 4 and the degree of the divisor $q(x) = x^2 + 2x - 15$ is 2.

Thus, the degree of the quotient is:

$$4 - 2 = 2.$$

Therefore, the degree of the quotient is 2.

Quick Tip

The degree of a quotient in polynomial division is the difference between the degrees of the dividend and divisor. This can help you quickly determine the degree of the resulting quotient.

90. If α and β are the zeros of the polynomial $x^2 + 5x + 8$, then the value of $\alpha^2 + \beta^2 + 2\alpha\beta$ is:

(A) 25

(B) 5

(C) 8

(D) 64

Correct Answer: (A) 25

Solution:

Using the identity:

$$\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2.$$

From Vieta's formulas:

$$\alpha + \beta = -\frac{5}{1} = -5.$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = (-5)^2 = 25.$$

Quick Tip

For any quadratic equation $ax^2 + bx + c = 0$:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

91. If the areas of three adjacent faces of a cuboid are a, b, c respectively, then the volume of the cuboid is:

(A) abc

(B) $2abc$

(C) $\sqrt{abc}$

(D) $3\sqrt{abc}$

Correct Answer: (C) $\sqrt{abc}$

Solution:

The volume V of a cuboid with three adjacent faces having areas a, b, c is given by:

$$V = \sqrt{abc}$$

Quick Tip

For a cuboid, the volume can be calculated if three adjacent face areas are known using $V = \sqrt{abc}$.

---

92. The total surface area of a cube is 216 cm^2. Find its volume.

(A) 144 cm^3

(B) 196 cm^3

(C) 212 cm^3

(D) 216 cm^3

Correct Answer: (D) 216 cm^3

Solution:

Let the side length of the cube be s. The total surface area is given by:

$$6s^2 = 216$$

$$s^2 = 36 \Rightarrow s = 6$$

$$\text{Volume} = s^3 = 6^3 = 216$$

Quick Tip

For a cube, total surface area is $6s^2$, and volume is s^3.

---

93. If the ratio of volumes of two cubes is 1:64, then the ratio of their total surface areas is:

(A) 1:4

(B) 1:16

(C) 1:18

(D) 1:8

Correct Answer: (B) 1:16

Solution:

Since volume ratio $V_1 : V_2 = 1 : 64$, side length ratio is:

$$s_1 : s_2 = \sqrt[3]{1 : 64} = 1 : 4$$

Total surface area ratio:

$$s_1^2 : s_2^2 = (1^2) : (4^2) = 1 : 16$$

Quick Tip

Volume ratio of cubes is the cube of side ratio, and surface area ratio is the square of side ratio.

---

94. Two circular cylinders of equal volume have their heights in the ratio 1:2. The ratio of their radii is:

(A) $1 : \sqrt{2}$

(B) $\sqrt{2} : 1$

(C) 1:2

(D) 1:4

Correct Answer: (B) $\sqrt{2} : 1$

Solution:

Volume of cylinder:

$$V = \pi r^2 h$$

Since $h_1 : h_2 = 1 : 2$, keeping the volume constant:

$$r_1^2 : r_2^2 = 2 : 1$$

$$r_1 : r_2 = \sqrt{2} : 1$$

Quick Tip

For equal volume, if height ratio is given, square root of the inverse ratio gives the radius ratio.

---

95. If the curved surface area of a cylinder is 1760 cm^2 and its base diameter is 28 cm, find its height.

(A) 10 cm

(B) 15 cm

(C) 20 cm

(D) 40 cm

Correct Answer: (C) 20 cm

Solution:

Curved Surface Area (CSA) of a cylinder:

$$2\pi rh = 1760$$

Given $d = 28 \Rightarrow r = 14$,

$$2\pi(14)h = 1760$$

$$h = \frac{1760}{2\pi \times 14} = 20 \text{ cm}$$

Quick Tip

CSA of a cylinder is given by $2\pi rh$, solve for h when CSA and r are known.

---

96. If O is the center and R is the radius of a circle, and $\angle AOB = \theta$, then the length of arc AB is:

(A) $\frac{2\pi R\theta}{180}$

(B) $\frac{2\pi R\theta}{360}$

(C) $\frac{\pi R^2\theta}{180}$

(D) $\frac{\pi R^2\theta}{360}$

Correct Answer: (B) $\frac{2\pi R\theta}{360}$

Solution:

Arc length formula:

$$\ell = \frac{\theta}{360} \times 2\pi R$$

Quick Tip

Arc length is a fraction of the circumference based on the given angle.

---

97. If l is slant height and r is the radius of a cone, the total surface area is:

(A) $\pi rl + r$

(B) $\pi rl + \pi r^2$

(C) $\pi rl + r^2$

(D) $\pi rl + 2r^2$

Correct Answer: (B) $\pi rl + \pi r^2$

Solution:

Total surface area of a cone:

$$\pi rl + \pi r^2$$

Quick Tip

Total surface area of a cone = Curved surface area + Base area.

---

98. If the volume ratio of two spheres is 125:27, then their surface area ratio is:

(A) 9:25

(B) 25:9

(C) 5:3

(D) 3:5

Correct Answer: (B) 25:9

Solution:

$$r_1 : r_2 = \sqrt[3]{125 : 27} = 5 : 3$$

$$\text{Surface area ratio} = (5^2 : 3^2) = 25 : 9$$

Quick Tip

For spheres, surface area ratio is the square of the radius ratio.

---

99. A sphere of radius 8 cm is melted to form a cone of height 32 cm. The radius of the base of the cone is:

(A) 8 cm

(B) 9 cm

(C) 10 cm

(D) 12 cm

Correct Answer: (A) 8 cm

Solution:

Since the volume remains the same, equating the volumes:

$$\frac{4}{3}\pi r_s^3 = \frac{1}{3}\pi r_c^2 h$$

Given, $r_s = 8$ cm and $h = 32$ cm:

$$\frac{4}{3}\pi(8)^3 = \frac{1}{3}\pi r_c^2(32)$$

Canceling $\frac{1}{3}\pi$ from both sides:

$$4(512) = 32r_c^2$$

$$2048 = 32r_c^2$$

$$r_c^2 = \frac{2048}{32} = 64$$

$$r_c = \sqrt{64} = 8 \text{ cm}$$

Quick Tip

When a solid is reshaped, its volume remains the same. Use volume formulas to find unknown dimensions.

---

100. If the surface area of a sphere is 616 cm^2, then the diameter of the sphere is:

(A) 7 cm

(B) 14 cm

(C) 28 cm

(D) 56 cm

Correct Answer: (B) 14 cm

Solution:

The surface area of a sphere is given by:

$$4\pi r^2 = 616$$

Solving for r:

$$r^2 = \frac{616}{4\pi}$$

Approximating $\pi = 3.14$:

$$r^2 = \frac{616}{12.56} = 49$$

$$r = \sqrt{49} = 7 \text{ cm}$$

$$\text{Diameter} = 2r = 2(7) = 14 \text{ cm}$$

Quick Tip

For spheres, surface area is $4\pi r^2$. Solve for r, then find diameter as $2r$.

Section - B

(Short Answer Type Questions)

1. Using Euclid division algorithm find the HCF of 148 and 185.

Solution:

Using Euclid's division algorithm, we divide 185 by 148:

$$185 = 148 \times 1 + 37$$

Now, divide 148 by 37:

$$148 = 37 \times 4 + 0$$

Since the remainder is 0, the HCF is **37**.

Correct Answer: 37

Quick Tip

HCF using Euclid's Algorithm: Divide the larger number by the smaller number, take the remainder, and repeat until the remainder is 0.

---

2. Find the mean of the following data:

Variable	2	4	6	10	12
Frequency	3	2	3	1	2

Solution:

Mean is given by:

$$\text{Mean} = \frac{\sum(x_i \cdot f_i)}{\sum f_i}$$

Calculating:

$$\sum(x_i \cdot f_i) = (2 \times 3) + (4 \times 2) + (6 \times 3) + (10 \times 1) + (12 \times 2) = 6 + 8 + 18 + 10 + 24 = 66$$

$$\sum f_i = 3 + 2 + 3 + 1 + 2 = 11$$

$$\text{Mean} = \frac{66}{11} = 6$$

Thus, the mean is **6**.

Correct Answer: 6

Quick Tip

Mean of Data: Use the formula Mean $= \frac{\sum(x_i \cdot f_i)}{\sum f_i}$.

---

3. If the angle between two tangents drawn from an external point P to a circle of radius 3 cm and centre O is $60°$, find the length of OP.

Solution:

Using the formula:

$$OP = \frac{r}{\cos\frac{\theta}{2}}$$

$$OP = \frac{3}{\cos 30°}$$

$$= \frac{3}{\frac{\sqrt{3}}{2}} = \frac{6}{\sqrt{3}} = 6\,\text{cm}$$

Thus, the length of OP is **6** cm.

Correct Answer: $6\,\text{cm}$

Quick Tip

Length of OP in Circle: Use $OP = \frac{r}{\cos(\theta/2)}$.

---

4. For what value of k equations $kx + y = 1$ and $(k-1)x + 2y = 3$ have no solution?

Solution:

For no solution, the system of equations should be inconsistent, i.e.,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

From the given equations:

$$\frac{k}{k-1} = \frac{1}{2} \neq \frac{1}{3}$$

Solving:

$$2k = k - 1$$

$$2k - k = -1$$

$$k = -1$$

Thus, for $k = -1$, the system has no solution.

Correct Answer: $k = -1$

Quick Tip

Inconsistent Linear Equations: If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the system has no solution.

---

5. Write the solution of the equation $3x + y = 11$ in natural numbers.

Solution:

We need integer solutions where x, y are natural numbers.

Rearrange:

$$y = 11 - 3x$$

For natural y, $11 - 3x > 0 \Rightarrow x < \frac{11}{3} \approx 3.67$.

Possible integer values:

$$x = 1, y = 8 \quad (1, 8)$$

$$x = 2, y = 5 \quad (2, 5)$$

$$x = 3, y = 2 \quad (3, 2)$$

Thus, the natural number solutions are $(\mathbf{1}, \mathbf{8}), (\mathbf{2}, \mathbf{5}), (\mathbf{3}, \mathbf{2})$.

Correct Answer: $(1, 8), (2, 5), (3, 2)$

Quick Tip

Natural Number Solutions: Solve for integer values that satisfy the equation.

---

6. Find the roots of the equation $\frac{1}{x} - \frac{1}{x-2} = 3$, $x \neq 0, 2$.

Solution:

Rewriting the equation:

$$\frac{(x-2) - x}{x(x-2)} = 3$$

$$\frac{-2}{x(x-2)} = 3$$

$$-2 = 3x(x-2)$$

$$3x^2 - 6x + 2 = 0$$

Using the quadratic formula:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$x = \frac{6 \pm 2\sqrt{3}}{6}$$

$$x = \frac{3 \pm \sqrt{3}}{3}$$

Thus, the roots are $\frac{\mathbf{3+\sqrt{3}}}{\mathbf{3}}, \frac{\mathbf{3-\sqrt{3}}}{\mathbf{3}}$.

Correct Answer: $\frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$

Quick Tip

Roots of a Rational Equation: Convert fractions to quadratic equations and use the quadratic formula.

---

7. In $\triangle ABC$, $\angle C = 90°$ and P, Q are midpoints of CA and CB respectively. Prove that $4AQ^2 = 4AC^2 + BC^2$.

Solution:

Since P and Q are midpoints,

$$AQ = \frac{1}{2}AB, \quad PQ = \frac{1}{2}BC$$

Applying the midpoint theorem:

$$4AQ^2 = AB^2 + 4PQ^2$$

Since $AB^2 = AC^2 + BC^2$ (Pythagoras theorem),

$$4AQ^2 = 4AC^2 + BC^2$$

Thus, the given equation is proved.

Correct Answer: $4AQ^2 = 4AC^2 + BC^2$ is proved.

Quick Tip

Midpoint Theorem in Right Triangle: If P, Q are midpoints of right triangle sides, use $4AQ^2 = 4AC^2 + BC^2$.

---

8. If point $A(x, 2)$ is equidistant from the points $B(8, -2)$ and $C(2, -2)$, find the value of x.

Solution:

Using the distance formula:

$$AB^2 = AC^2$$

$$(x - 8)^2 + (2 + 2)^2 = (x - 2)^2 + (2 + 2)^2$$

$$(x - 8)^2 + 16 = (x - 2)^2 + 16$$

Cancel 16 from both sides:

$$(x - 8)^2 = (x - 2)^2$$

$$x^2 - 16x + 64 = x^2 - 4x + 4$$

$$-16x + 64 = -4x + 4$$

$$-12x = -60$$

$$x = 5$$

Thus, $x = \mathbf{5}$.

Correct Answer: $x = 5$

Quick Tip

Distance Formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

---

9. In what ratio does the point $(-4, 6)$ divide the line segment joining $A(-6, 10)$ and $B(3, -8)$?

Solution:

Using section formula:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Let the ratio be $k : 1$, so

$$-4 = \frac{3k + (-6)}{k + 1}, \quad 6 = \frac{-8k + 10}{k + 1}$$

Solving for k,

$$-4(k + 1) = 3k - 6$$

$$-4k - 4 = 3k - 6$$

$$-7k = -2$$

$$k = \frac{2}{7}$$

Thus, the required ratio is $\mathbf{2} : \mathbf{7}$.

Correct Answer: $2 : 7$

Quick Tip

Section Formula: If a point divides a line in ratio $m : n$, use $x = \frac{mx_2+nx_1}{m+n}$.

---

10. Find the area of the triangle whose vertices are $A(2, 1)$, $B(4, 5)$ and $C(0, 3)$.

Solution:

Using the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2} |2(5 - 3) + 4(3 - 1) + 0(1 - 5)|$$

$$= \frac{1}{2} |2(2) + 4(2) + 0|$$

$$= \frac{1}{2} |4 + 8| = \frac{1}{2} \times 12 = 6$$

Thus, the area is **6** square units.

Correct Answer: 6

Quick Tip

Area of Triangle: Use determinant formula $A = \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$.

---

11. Evaluate $\frac{\sec(90^\circ-\theta)\csc\theta-\tan(90^\circ-\theta)\cot\theta+\cos^2 25^\circ+\cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$.

Solution:

Using trigonometric identities:

$$\sec(90^\circ - \theta) = \csc\theta, \quad \tan(90^\circ - \theta) = \cot\theta$$

$$\Rightarrow \frac{\csc\theta\csc\theta - \cot\theta\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$$

Since,

$$\cos^2 25^\circ + \cos^2 65^\circ = 1$$

$$\Rightarrow \frac{1}{3(1)} = \frac{1}{3}$$

Thus, the value is $\mathbf{\frac{1}{3}}$.

Correct Answer: $\frac{1}{3}$

Quick Tip

Trigonometric Identities: Convert using $\sec(90^\circ - \theta) = \csc\theta$ and $\tan(90^\circ - \theta) = \cot\theta$.

---

12. Prove that $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$.

Solution:

Using identities:

$$\sec\theta + \tan\theta = \frac{1+\sin\theta}{\cos\theta}, \quad \sec\theta - \tan\theta = \frac{1-\sin\theta}{\cos\theta}$$

$$\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}}$$

$$= \frac{1+\sin\theta}{1-\sin\theta}$$

Multiplying numerator and denominator by $1 + \sin\theta$:

$$= \frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$

Since $1 - \sin^2\theta = \cos^2\theta$:

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$$

Thus, the identity is proved.

Correct Answer: $\left(\frac{1+\sin\theta}{\cos\theta}\right)^2$

> **Quick Tip**
>
> Quadratic Discriminant: If $D > 0$, roots are real and distinct; if $D = 0$, equal; if $D < 0$, complex.

---

13. Find the discriminant of the quadratic equation $\sqrt{2}x^2 - x - \sqrt{2} = 0$ and hence find the nature of the roots.

Solution:

Discriminant formula:

$$D = b^2 - 4ac$$

For $\sqrt{2}x^2 - x - \sqrt{2} = 0$,

$$a = \sqrt{2}, \quad b = -1, \quad c = -\sqrt{2}$$

$$D = (-1)^2 - 4(\sqrt{2})(-\sqrt{2})$$

$$= 1 - 4(2)$$

$$= 1 - 8 = 9$$

Since $D > 0$, the roots are real and distinct.

Correct Answer: $9\,\text{cm}$

> **Quick Tip**
>
> Discriminant Calculation: $D = b^2 - 4ac$ helps determine the nature of the roots.

---

14. For what value of k will the equation $kx(x-2) + 6 = 0$ have equal roots?

Solution:

Expanding:

$$kx^2 - 2kx + 6 = 0$$

For equal roots:

$$D = 0$$

$$(-2k)^2 - 4(k)(6) = 0$$

$$4k^2 - 24k = 0$$

$$4k(k - 6) = 0$$

$$k = 0 \quad \text{or} \quad k = 6$$

Thus, $k = 6$ ensures equal roots.

Correct Answer: $k = 6$

Quick Tip

Irrational Proofs: Show that the given number has a term involving an irrational number.

---

15. Check whether 301 is a term of the sequence $5, 11, 17, 23, \ldots$**.**

Solution:

Given A.P.:

$$a = 5, \quad d = 11 - 5 = 6$$

General term:

$$a_n = a + (n - 1)d$$

$$301 = 5 + (n-1)6$$

$$301 - 5 = (n-1)6$$

$$296 = (n-1)6$$

$$n - 1 = \frac{296}{6} = 49.33$$

Since n is not an integer, 301 is not a term.

Correct Answer: 301 is not a term of the A.P.

Quick Tip

To check if a number is a term of an A.P., substitute the number into the general term formula $a_n = a + (n-1)d$. If n is not an integer, the number is not a term of the A.P.

---

16. Prove that $(2+\sqrt{3})^2$ is not a rational number.

Solution:

Expanding:

$$(2+\sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$$

Since $4\sqrt{3}$ is irrational, $7 + 4\sqrt{3}$ is also irrational.

Thus, it is not a rational number.

Correct Answer: $7 + 4\sqrt{3}$ is not a rational number.

Quick Tip

Zeroes of Quadratic Equation: Use the quadratic formula and verify sum-product properties.

---

17. Find the zeroes of the quadratic polynomial $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.

Solution:

Using the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{1 + 48}}{6}$$

$$x = \frac{1 \pm \sqrt{49}}{6}$$

$$x = \frac{1 \pm 7}{6}$$

$$x = \frac{8}{6} = \frac{4}{3}, \quad x = \frac{-6}{6} = -1$$

Verifying:

Sum of zeroes:

$$\frac{4}{3} + (-1) = \frac{4}{3} - \frac{3}{3} = \frac{1}{3} = -\frac{b}{a} = -\frac{-1}{3}$$

Product of zeroes:

$$\frac{4}{3} \times (-1) = -\frac{4}{3} = \frac{c}{a} = \frac{-4}{3}$$

Thus, the relationship is verified.

Correct Answer: $\frac{4}{3}, -1$ are the zeroes, relationship verified.

Quick Tip

Zeroes of Quadratic Equation:Use the quadratic formula and verify sum-product properties.

---

18. Divide polynomial $2x^4 + 3x^3 - 2x^2 - 9x - 12$ by polynomial $x^2 - 3$.

Solution: Using polynomial division:

$$\frac{2x^4 + 3x^3 - 2x^2 - 9x - 12}{x^2 - 3}$$

1. Divide $2x^4$ by x^2:

$$2x^2$$

Multiply:

$$2x^4 - 6x^2$$

Subtract:

$$(2x^4 + 3x^3 - 2x^2 - 9x - 12) - (2x^4 - 6x^2)$$

$$3x^3 + 4x^2 - 9x - 12$$

2. Divide $3x^3$ by x^2:

$$3x$$

Multiply:

$$3x^3 - 9x$$

Subtract:

$$(3x^3 + 4x^2 - 9x - 12) - (3x^3 - 9x)$$

$$4x^2 - 12$$

3. Divide $4x^2$ by x^2:

$$4$$

Multiply:

$$4x^2 - 12$$

Subtract:

$$(4x^2 - 12) - (4x^2 - 12) = 0$$

Thus, the quotient is:

$$2x^2 + 3x + 4$$

Correct Answer: $2x^2 + 3x + 4$

Quick Tip

Polynomial Division: Divide each term successively using $x^2 - 3$.

---

19. Find the sum of 10 terms of A.P. 9, 17, 25, ...

Solution:

Given A.P.:

$$a = 9, \quad d = 17 - 9 = 8, \quad n = 10$$

Sum of first n terms:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(9) + (10-1)(8)]$$

$$= 5[18 + 72] = 5 \times 90 = 450$$

Thus, $S_{10} = \mathbf{450}$.

Correct Answer: 450

Quick Tip

Sum of A.P.: Use $S_n = \frac{n}{2}[2a + (n-1)d]$.

---

20. Find the sum of the first 15 multiples of 8.

Solution:

Multiples of 8 form an A.P.:

$$8, 16, 24, \ldots$$

$$a = 8, \quad d = 8, \quad n = 15$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(8) + (15-1)(8)]$$

$$= \frac{15}{2}[16 + 112] = \frac{15}{2} \times 128 = 960$$

Thus, $S_{15} = \mathbf{960}$.

Correct Answer: 960

Quick Tip

Basic Proportionality Theorem: If $\frac{PS}{SQ} = \frac{PT}{TR}$, then $ST \parallel QR$.

---

21. In triangle $\triangle PQR$, two points S and T are on sides PQ and PR such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that $\triangle PQR$ is an isosceles triangle.

Solution:

Since $\frac{PS}{SQ} = \frac{PT}{TR}$, by the Basic Proportionality Theorem,

$$ST \parallel QR$$

Since $\angle PST = \angle PRQ$, corresponding angles are equal, proving $\triangle PST \sim \triangle PRQ$.

From similarity:

$$\frac{PS}{PQ} = \frac{PT}{PR}$$

Since proportions hold, $PQ = PR$, proving $\triangle PQR$ is isosceles.

Correct Answer: $\triangle PQR$ is an isosceles triangle.

Quick Tip

Reciprocal Roots Condition: If one root is reciprocal of the other, use $\alpha\beta = 1$.

---

22. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find a.

Solution:

If roots are reciprocal:

$$\alpha \cdot \beta = 1$$

Using the product of roots formula:

$$\frac{c}{a} = 1$$

$$\frac{6a}{a^2 + 9} = 1$$

$$6a = a^2 + 9$$

$$a^2 - 6a + 9 = 0$$

$$(a - 3)^2 = 0$$

$$a = 3$$

Thus, $a = \mathbf{3}$.

Correct Answer: $a = 3$

Quick Tip

Equation Formation from Word Problems: Convert conditions into equations and solve them.

---

23. On adding 1 to the numerator of a fraction it becomes $\frac{1}{2}$ and on adding 1 to the denominator it becomes $\frac{1}{3}$. Write the equation for this statement.

Solution:

Let the fraction be $\frac{x}{y}$.

According to the first condition, when 1 is added to the numerator, the fraction becomes $\frac{1}{2}$:

$$\frac{x+1}{y} = \frac{1}{2}$$

Cross-multiply:

$$2(x+1) = y$$

$$2x + 2 = y$$

Thus, $y = 2x + 2$. $\cdots(1)$

According to the second condition, when 1 is added to the denominator, the fraction becomes $\frac{1}{3}$:

$$\frac{x}{y+1} = \frac{1}{3}$$

Cross-multiply:

$$3x = y + 1$$

$$y = 3x - 1$$

Thus, $y = 3x - 1$. $\cdots (2)$

Now, solving equations (1) and (2):

From equation (1), $y = 2x + 2$, and from equation (2), $y = 3x - 1$:

$$2x + 2 = 3x - 1$$

$$2x - 3x = -1 - 2$$

$$-x = -3$$

$$x = 3$$

Substitute $x = 3$ in equation (1):

$$y = 2(3) + 2 = 6 + 2 = 8$$

Thus, the fraction is $\frac{x}{y} = \frac{3}{8}$.

Correct Answer: The fraction is $\frac{3}{8}$.

Quick Tip

To solve such problems, write equations based on the given conditions and solve them step by step.

---

24. Solve the pair of equations $8x + 5y = 9$ and $3x + 2y = 4$ by substitution method.

Solution:

From $3x + 2y = 4$:

$$y = \frac{4 - 3x}{2}$$

Substituting in $8x + 5y = 9$:

$$8x + 5\left(\frac{4 - 3x}{2}\right) = 9$$

$$16x + 20 - 15x = 18$$

$$x = -2$$

Substituting $x = -2$ in $y = \frac{4-3x}{2}$:

$$y = \frac{4 - 3(-2)}{2} = \frac{4 + 6}{2} = 5$$

Thus, $x = -\mathbf{2}, y = \mathbf{5}$.

Correct Answer: $x = -2, y = 5$

Quick Tip

Quick Tips for Solving Linear Equations by Substitution:

- **Step 1:** Solve one equation for one variable in terms of the other.
- **Step 2:** Substitute this expression into the second equation.
- **Step 3:** Solve for the remaining variable.
- **Step 4:** Substitute the obtained value back into the first equation to find the second variable.
- **Step 5:** Always check the solution by substituting it into both original equations.
- **Tip:** Choose the equation where solving for a variable is easiest to simplify calculations.

---

34. Prove that

$$\left(\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}\right)\left(\frac{\sin A}{1 - \cos A} + \frac{1 - \cos A}{\sin A}\right) = 4 \csc A \cot A.$$

Solution:

Consider the first term:

$$\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}$$

Taking LCM:

$$= \frac{\sin^2 A + (1 + \cos A)^2}{\sin A(1 + \cos A)}$$

Expanding:

$$= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{\sin A(1 + \cos A)}$$

$$= \frac{2 + 2\cos A}{\sin A(1 + \cos A)}$$

$$= \frac{2(1 + \cos A)}{\sin A(1 + \cos A)} = \frac{2}{\sin A}$$

Similarly,

$$\frac{\sin A}{1 - \cos A} + \frac{1 - \cos A}{\sin A}$$

Solving similarly, we get:

$$= \frac{2}{\sin A}$$

Multiplying both terms:

$$\frac{2}{\sin A} \times \frac{2}{\sin A} = \frac{4}{\sin^2 A}$$

$$= 4 \csc A \cot A$$

Thus, the identity is proved.

Correct Answer: $\frac{5}{12}$

Quick Tip

Trigonometric Proofs: Use trigonometric identities and simplifications.

---

35. Metallic spheres of radii 6 cm, 8 cm, and 10 cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:

The volume of a sphere is given by the formula:

$$V = \frac{4}{3}\pi r^3$$

The total volume of the three spheres is the sum of their individual volumes. So,

$$V_{\text{total}} = V_1 + V_2 + V_3$$

Where the radii of the three spheres are $r_1 = 6\,\text{cm}, r_2 = 8\,\text{cm}, r_3 = 10\,\text{cm}$.

Thus, the volumes are:

$$V_1 = \frac{4}{3}\pi(6)^3, \quad V_2 = \frac{4}{3}\pi(8)^3, \quad V_3 = \frac{4}{3}\pi(10)^3$$

$$V_1 = \frac{4}{3}\pi \times 216 = 288\pi, \quad V_2 = \frac{4}{3}\pi \times 512 = 682.67\pi, \quad V_3 = \frac{4}{3}\pi \times 1000 = 1333.33\pi$$

Total volume:

$$V_{\text{total}} = 288\pi + 682.67\pi + 1333.33\pi = 2304\pi$$

Now, let the radius of the resulting sphere be R. The volume of the resulting sphere is:

$$V_{\text{new}} = \frac{4}{3}\pi R^3$$

Since the total volume of the material is conserved:

$$\frac{4}{3}\pi R^3 = 2304\pi$$

Canceling π from both sides:

$$\frac{4}{3}R^3 = 2304$$

Multiplying both sides by 3:

$$4R^3 = 6912$$

Now, divide by 4:

$$R^3 = 1728$$

Taking the cube root of both sides:

$$R = \sqrt[3]{1728} = 12\,\text{cm}$$

Thus, the radius of the resulting sphere is $\mathbf{12}\,\text{cm}$.

Quick Tip

To solve this problem, remember that the volume of a sphere is $V = \frac{4}{3}\pi r^3$ and use the conservation of volume when combining multiple spheres into one.

36. The angles of elevation of the top of a tower from two points at distances of 4 m and 9 m from the base of the tower, which are in the same straight line, are complementary. Prove that the height of the tower is 6 m.

Solution:

Let the height of the tower be h.

Given angles:

$$\theta \text{ and } 90^\circ - \theta$$

Using tan formula:

$$\tan\theta = \frac{h}{4}, \quad \tan(90^\circ - \theta) = \cot\theta = \frac{h}{9}$$

$$\frac{h}{4} \times \frac{h}{9} = 1$$

$$\frac{h^2}{36} = 1$$

$$h^2 = 36$$

$$h = 6$$

Thus, the height of the tower is **6** m.

Quick Tip

Height and Distance Problems:Use $\tan$ and $\cot$ formulas.

37. Draw a circle of radius 6 cm. From a point 10 cm away from its center, construct the pair of tangents to the circle and measure their lengths.

Solution:

Using tangent length formula:

$$\text{Length of tangent} = \sqrt{d^2 - r^2}$$

$$= \sqrt{10^2 - 6^2}$$

$$= \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm}$$

Thus, the length of each tangent is 8 cm.

Quick Tip

Tangent Length Formula: $\sqrt{d^2 - r^2}$ where d is distance from center.

---

38. Find the median of the following data:

Class Interval	Frequency
40-45	2
45-50	3
50-55	8
55-60	6
60-65	6
65-70	3
70-75	2

Solution:

To find the median, we first calculate the cumulative frequency (CF).

Class Interval	Frequency (f)	Cumulative Frequency (CF)
$40 - 45$	2	2
$45 - 50$	3	5
$50 - 55$	8	13
$55 - 60$	6	19
$60 - 65$	6	25
$65 - 70$	3	28
$70 - 75$	2	30

The total frequency, $N = 30$.

The median class is the class where the cumulative frequency exceeds $\frac{N}{2} = \frac{30}{2} = 15$.

From the cumulative frequency table, the median class is $50 - 55$ because the cumulative frequency just exceeds 15.

Now, we use the following formula for the median:

$$\text{Median} = L + \frac{\frac{N}{2} - CF}{f} \times h$$

Where: - L is the lower limit of the median class = 50, - N is the total frequency = 30, - CF is the cumulative frequency of the class before the median class = 5, - f is the frequency of the median class = 8, - h is the class width = 5 (since $50 - 45 = 5$).

Substituting the values:

$$\text{Median} = 50 + \frac{15 - 5}{8} \times 5$$

$$= 50 + \frac{10}{8} \times 5$$

$$= 50 + \frac{50}{8}$$

$$= 50 + 6.25$$

$$\text{Median} = 56.25$$

Thus, the median of the data is $\mathbf{56.25}$.

Quick Tip

To find the median of grouped data, use the formula Median $= L + \frac{\frac{N}{2} - CF}{f} \times h$, where N is the total frequency, CF is the cumulative frequency of the class before the median class, f is the frequency of the median class, and h is the class width.