

CAT 2017 quant Slot-2 Question Paper with Solutions

Time Allowed :3 Hours

Maximum Marks :390

Total questions :130

General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Duration of Section:** 40 Minutes
2. **Total Number of Questions:** 22 Questions (as per latest pattern, may vary slightly)
3. **Section Covered:** Quantitative Aptitude (QA)
4. **Type of Questions:**
 - Multiple Choice Questions (MCQs)
 - Type In The Answer (TITA) Questions – No options given, answer to be typed in
5. **Marking Scheme:**
 - +3 marks for each correct answer
 - -1 mark for each incorrect MCQ
 - No negative marking for TITA questions
6. **Syllabus Coverage:** Arithmetic, Algebra, Geometry, Number System, Modern Math, and Mensuration
7. **Skills Tested:** Numerical ability, analytical thinking, and problem-solving

67. The numbers 1, 2, ..., 9 are arranged in a 3 X 3 square grid in such a way that each number occurs once and the entries along each column, each row, and each of the two diagonals add up to the same value.

If the top left and the top right entries of the grid are 6 and 2, respectively, then the bottom middle entry is

- (A) 3
- (B) 5
- (C) 6
- (D) 8

Correct Answer: (A) 3

Solution:

The square grid is filled with the numbers from 1 to 9. The sum of these numbers, $1 + 2 + 3 + \dots + 9$, equals 45. Since the sum of the numbers in each row, each column, and each diagonal must be equal, the sum of terms in each row and column (and each diagonal) must be 15.

To satisfy this condition, the middle element in the 2nd row and the 2nd column must be the middle term of the 9 terms, which is 5. The corner elements in the first row are 6 and 2 (given), so the middle element in the first row must be 7.

In the second column, the topmost element is 7, and the middle element is 5, so the middle element in the bottom row must be 3.

Quick Tip

In a 3x3 magic square, the sum of each row, column, and diagonal is always the same. For a square where the numbers 1 through 9 are used, the common sum for each row, column, and diagonal is always 15.

68. In a 10 km race, A, B, and C, each running at uniform speed, get the gold, silver, and bronze medals, respectively. If A beats B by 1 km and B beats C by 1 km, then by how many meters does A beat C?

- (A) 1900
- (B) 1700
- (C) 1600
- (D) 2000

Correct Answer: (A) 1900

Solution:

In the 10 km race, we have the following:

- A beats B by 1 km, so when A finishes the race, B has covered 9 km. - B beats C by 1 km, so when B finishes the race, C has covered 9 km.

Therefore, when A finishes the race, C has covered a distance of:

$$\frac{9}{10} \times 9 = 8.1 \text{ km.}$$

Thus, A beats C by:

$$10 - 8.1 = 1.9 \text{ km} = 1900 \text{ meters.}$$

Thus, the correct answer is 1900 meters.

Quick Tip

In relative speed problems, convert the race lengths to the same scale and use proportionality to calculate the distance by which one competitor beats the other.

69. Bottle 1 contains a mixture of milk and water in 7:2 ratio and Bottle 2 contains a mixture of milk and water in 9:4 ratio. In what ratio of volumes should the liquids in Bottle 1 and Bottle 2 be combined to obtain a mixture of milk and water in 3:1 ratio?

- (A) 27 : 14
- (B) 27 : 13
- (C) 27 : 16
- (D) 27 : 18

Correct Answer: (B) 27 : 13

Solution:

Let the required volumes of liquids from Bottle 1 and Bottle 2 be x and y , respectively.

In Bottle 1, milk to water ratio is 7:2, so the amount of milk in Bottle 1 = $\frac{7}{9}x$ and the amount of water in Bottle 1 = $\frac{2}{9}x$.

In Bottle 2, milk to water ratio is 9:4, so the amount of milk in Bottle 2 = $\frac{9}{13}y$ and the amount of water in Bottle 2 = $\frac{4}{13}y$.

We need the final mixture to have a milk to water ratio of 3:1. Therefore, the amount of milk in the mixture must be three times the amount of water.

Hence, the equation becomes:

$$\frac{7}{9}x + \frac{9}{13}y = 3 \times \left(\frac{2}{9}x + \frac{4}{13}y \right)$$

Simplifying the equation will give us the ratio $x : y = 27 : 13$.

Thus, the correct answer is 27 : 13.

Quick Tip

In mixture problems, use the rule of allegation to quickly find the ratio of the quantities to be mixed in order to get the desired concentration.

70. Arun drove from home to his hostel at 60 miles per hour. While returning home he drove half way along the same route at a speed of 25 miles per hour and then took a bypass road which increased his driving distance by 5 miles, but allowed him to drive at 50 miles per hour along this bypass road. If his return journey took 30 minutes more than his onward journey, then the total distance traveled by him is

- (A) 55 miles
- (B) 60 miles
- (C) 65 miles
- (D) 70 miles

Correct Answer: (C) 65 miles

Solution:

Let the total distance from Arun's home to the hostel be D . Therefore, the distance for the return journey becomes $D + 5$ due to the bypass.

Onward journey: Time for onward journey = $\frac{D}{60}$.

Return journey: Time for first half of return journey = $\frac{D}{2 \times 25}$, Time for second half of return journey = $\frac{D+5}{50}$.

We are told that the return journey took 30 minutes (or 0.5 hours) more than the onward journey. Hence, we have the equation:

$$\frac{D}{2 \times 25} + \frac{D+5}{50} - \frac{D}{60} = 0.5$$

Solving for D , we get $D = 65$.

Thus, the total distance traveled by Arun is 65 miles.

Quick Tip

When solving for time-based problems involving different speeds, carefully separate the journey into segments and use the time difference equation to find the unknown distance.

71. Out of the shirts produced in a factory, 15

- (A) 13600
- (B) 13000
- (C) 13400
- (D) 14000

Correct Answer: (B) 13000

Solution:

Let the total number of shirts produced in the factory be x .

- 15- The remaining shirts = $x - 0.15x = 0.85x$. - 20- The remaining shirts left for export = $0.85x - 0.17x = 0.68x$.

We are told that the remaining shirts for export are 8840, so:

$$0.68x = 8840$$

Solving for x , we get:

$$x = \frac{8840}{0.68} = 13000$$

Thus, the number of shirts produced in the factory is 13000.

Quick Tip

In problems involving percentages and divisions, always express the remaining quantity as a fraction of the total and solve using simple equations to find the total number.

72. The average height of 22 toddlers increases by 2 inches when two of them leave this group. If the average height of these two toddlers is one-third the average height of the original 22, then the average height, in inches, of the remaining 20 toddlers is

- (A) 30
- (B) 28
- (C) 32
- (D) 26

Correct Answer: (C) 32

Solution:

Let the original average height of the 22 toddlers be x . Therefore, the total height of all 22 toddlers is $22x$.

- The average height of the two toddlers who leave is $\frac{x}{3}$, and the total height of these two toddlers is $2 \times \frac{x}{3} = \frac{2x}{3}$.

- After these two toddlers leave, the total height of the remaining 20 toddlers is:

$$22x - \frac{2x}{3} = \frac{66x}{3} - \frac{2x}{3} = \frac{64x}{3}.$$

- The average height of the remaining 20 toddlers is:

$$\frac{\frac{64x}{3}}{20} = \frac{64x}{60} = \frac{8x}{7.5}.$$

We are told that the average height increases by 2 inches, so:

$$\frac{8x}{7.5} = x + 2.$$

Solving for x , we get:

$$8x = 7.5(x + 2)$$

$$8x = 7.5x + 15$$

$$0.5x = 15$$

$$x = 30.$$

Thus, the average height of the remaining 20 toddlers is $30 + 2 = 32$ inches.

Therefore, the correct answer is 32.

Quick Tip

When dealing with average-related problems, set up an equation based on the total sum before and after the change, then solve for the unknown quantity.

73. The manufacturer of a table sells it to a wholesale dealer at a profit of 10%. The wholesale dealer sells it to a retailer at a profit of 30

- (A) 1500
- (B) 2000
- (C) 2500
- (D) 3000

Correct Answer: (B) 2000

Solution:

Let the manufacturing cost be x .

1. The manufacturer sells it to the wholesale dealer at a profit of 10

$$\text{Price for wholesale dealer} = x + 0.10x = 1.10x.$$

2. The wholesale dealer sells it to the retailer at a profit of 30

$$\text{Price for retailer} = 1.10x + 0.30(1.10x) = 1.10x \times 1.30 = 1.43x.$$

3. The retailer sells it to the customer at a profit of 50

$$\text{Price for customer} = 1.43x + 0.50(1.43x) = 1.43x \times 1.50 = 2.145x.$$

We are given that the customer pays Rs 4290, so:

$$2.145x = 4290.$$

Solving for x :

$$x = \frac{4290}{2.145} = 2000.$$

Thus, the manufacturing cost is 2000.

Quick Tip

For problems involving successive percentage profits, multiply the original cost by each profit factor step by step to find the final price.

74. A tank has an inlet pipe and an outlet pipe. If the outlet pipe is closed then the inlet pipe fills the empty tank in 8 hours. If the outlet pipe is open then the inlet pipe fills the empty tank in 10 hours. If only the outlet pipe is open then in how many hours the full tank becomes half-full?

- (A) 20
- (B) 30
- (C) 40
- (D) 45

Correct Answer: (A) 20

Solution:

Let the capacity of the tank be C and the rate at which the inlet pipe fills the tank be $\frac{C}{8}$, (because it fills the tank in 8 hours).

- When both the inlet and outlet pipes are working, the inlet pipe fills the tank at a rate of $\frac{C}{8}$, and the outlet pipe empties the tank at a rate of $\frac{C}{10}$ (since the tank fills in 10 hours when the inlet pipe alone is open).

The net rate at which the tank fills when both pipes are open is:

$$\text{Net rate} = \frac{C}{8} - \frac{C}{10} = \frac{5C}{40} - \frac{4C}{40} = \frac{C}{40}.$$

Now, the time taken to fill half the tank (i.e., $\frac{C}{2}$) is:

$$\text{Time} = \frac{\frac{C}{2}}{\frac{C}{40}} = 20 \text{ hours.}$$

Thus, the tank becomes half-full in 20 hours.

Quick Tip

When two pipes are filling and emptying a tank simultaneously, subtract the emptying rate from the filling rate to find the net rate. Use this to calculate the time required for a specific fill level.

75. Mayank buys some candies for Rs 15 a dozen and an equal number of different candies for Rs 12 a dozen. He sells all for Rs 16.50 a dozen and makes a profit of Rs 150. How many dozens of candies did he buy altogether?

- (A) 50
- (B) 30
- (C) 25
- (D) 45

Correct Answer: (A) 50

Solution:

Let the number of dozens of each type of candy be x .

- The cost of the first type of candies = $15x$ (since each dozen costs Rs 15).
- The cost of the second type of candies = $12x$ (since each dozen costs Rs 12).
- The selling price of all candies = $16.50 \times 2x = 33x$.

The total profit made by Mayank is:

$$\text{Profit} = \text{Selling Price} - \text{Cost Price} = 33x - (15x + 12x) = 33x - 27x = 6x.$$

We are told that the profit is Rs 150, so:

$$6x = 150.$$

Solving for x :

$$x = \frac{150}{6} = 25.$$

Thus, the total number of dozens of candies = $2x = 2 \times 25 = 50$.

Therefore, Mayank bought 50 dozens of candies.

Quick Tip

To find the total number of items, use the formula for profit: Profit = Selling Price - Cost Price. This helps when multiple items are involved with different costs and selling prices.

76. In a village, the production of food grains increased by 40

- (A) 16
- (B) 13
- (C) 10
- (D) 7

Correct Answer: (C) 10

Solution:

Let the initial population be P , the initial production of food grains be F , and the total food grains produced initially be $P \times F$.

- After the increase, the new production of food grains = $1.40 \times F$. - After the increase, the new per capita production = $1.27 \times F$.

Now, let the population after the increase be P' . We have:

$$P' \times 1.27F = 1.40 \times F.$$

Dividing both sides by F :

$$P' \times 1.27 = 1.40.$$

Solving for P' :

$$P' = \frac{1.40}{1.27} \approx 1.102.$$

So, the percentage increase in population is:

$$\frac{P' - P}{P} \times 100 = (1.102 - 1) \times 100 = 10.2\% \approx 10\%.$$

Thus, the population of the village increased by approximately 10%.

Quick Tip

To solve percentage changes involving multiple factors, use the formula for percentage change: $\text{New Value} = \text{Old Value} \times (1 + \text{Percentage Increase})$.

77. If a, b, c are three positive integers such that a and b are in the ratio 3 : 4 while b and c are in the ratio 2 : 1, then which one of the following is a possible value of $(a + b + c)$?

- (A) 201
- (B) 205
- (C) 207
- (D) 210

Correct Answer: (A) 201

Solution:

From the ratios $\frac{a}{b} = \frac{3}{4}$ and $\frac{b}{c} = \frac{2}{1}$, we can express a, b, c in terms of a common variable.

- Let $b = 4k$ for some integer k . - Then $a = 3k$ and $c = 2k$.

Thus, $a + b + c = 3k + 4k + 2k = 9k$.

Now, we check the possible values: - For $k = 201/9 = 201$, the value of

$a + b + c = 9 \times 201 = 201$.

Thus, the possible value of $a + b + c$ is 201.

Quick Tip

When ratios are given, express each variable in terms of a common factor, then solve for the unknowns by checking the possible values for the total sum.

78. A motorbike leaves point A at 1 pm and moves towards point B at a uniform speed. A car leaves point B at 2 pm and moves towards point A at a uniform speed which is double that of the motorbike. They meet at 3:40 pm at a point which is 168 km away from A. What is the distance, in km, between A and B?

- (A) 364
- (B) 378
- (C) 380
- (D) 388

Correct Answer: (B) 378

Solution:

The motorbike that left point A traveled 168 km from 1:00 p.m. to 3:40 p.m., which means it traveled for $2\frac{2}{3}$ hours or $\frac{8}{3}$ hours.

The car that left point B started at 2:00 p.m. and traveled until 3:40 p.m., which means it traveled for $1\frac{1}{2}$ hours or $\frac{5}{3}$ hours.

The car would have traveled $\frac{5}{8}$ of the distance covered by the bike. However, since the car traveled at twice the speed, it would have covered $\frac{10}{8}$ of the distance. Thus, the car would have traveled $\frac{10}{8} \times 168 = 210$ km.

Therefore, the total distance between A and B is $168 + 210 = 378$ km.

Quick Tip

When solving relative speed problems, first calculate the total time each object travels and then sum the distances covered by each object.

79. Amal can complete a job in 10 days and Bimal can complete it in 8 days. Amal, Bimal and Kamal together complete the job in 4 days and are paid a total amount of Rs 1000 as remuneration. If this amount is shared by them in proportion to their work, then Kamal's share, in rupees, is

- (A) 100
- (B) 200
- (C) 300
- (D) 400

Correct Answer: (A) 100

Solution:

The team completes the job in 4 days.

Since Amol requires 10 days to complete the job, in 4 days he completes $\frac{4}{10}$ or 40% of the work.

Since Bimal requires 8 days to complete the job, in 4 days he completes $\frac{4}{8}$ or 50% of the work.

Therefore, Kamal completed 10% of the work. As a result, for completing 10% of the work, he receives 10% of the payment, which is Rs. 100.

Quick Tip

In problems with work and time, combine the work rates and solve for the individual contributions. Proportions can be applied to share the total remuneration.

80. Consider three mixtures - the first having water and liquid A in the ratio 1 : 3, the second having water and liquid B in the ratio 1 : 2, and the third having water and liquid C in the ratio 1 : 4. These three mixtures of A, B, and C respectively, are further mixed in the proportion 4:3:2. Then the resulting mixture has

- (A) The same amount of water and liquid B
- (B) The same amount of liquids B and C
- (C) More water than liquid B
- (D) More water than liquid A

Correct Answer: (C) More water than liquid B

Solution:

Let us assume that 4 parts of the first mixture, 3 parts of the second mixture, and 2 parts of the third mixture are taken.

- In the first mixture (water : liquid A = 1 : 3), for every 4 parts, water = $\frac{1}{4} \times 4 = 1$, and liquid A = $\frac{3}{4} \times 4 = 3$.

- In the second mixture (water : liquid B = 1 : 2), for every 3 parts, water = $\frac{1}{3} \times 3 = 1$, and liquid B = $\frac{2}{3} \times 3 = 2$.

- In the third mixture (water : liquid C = 1 : 4), for every 2 parts, water = $\frac{1}{5} \times 2 = 2$, and liquid C = $\frac{4}{5} \times 2 = 4$.

The total water in the mixture is $1 + 1 + 2 = 4$ parts, while the total of liquids B and C is $2 + 4 = 6$ parts. Hence, there is more water than liquid B.

Thus, the correct answer is (C) More water than liquid B.

Quick Tip

When dealing with mixtures, break down the quantities based on the given ratios and compare the total quantities of the ingredients after mixing.

81. Let ABCDEF be a regular hexagon with each side of length 1 cm. The area (in sq cm) of a square with AC as one side is

- (A) $\sqrt{3}$
- (B) 3
- (C) 4
- (D) $\sqrt{3}$

Correct Answer: (B) 3

Solution:

In triangle $\triangle ABC$, we are given that $AB = 1$ cm and $BC = 1$ cm. Additionally, angle $\angle B = 120^\circ$.

By applying the cosine rule:

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cos(\angle B)$$

Substituting the values:

$$AC^2 = 1^2 + 1^2 - 2(1)(1) \cos(120^\circ)$$

$$AC^2 = 1 + 1 - 2 \times 1 \times 1 \times \left(-\frac{1}{2}\right) \quad (\text{since } \cos(120^\circ) = -\frac{1}{2})$$

$$AC^2 = 2 + 1 = 3$$

Thus,

$$AC = \sqrt{3} \text{ cm}$$

The square of the side $\sqrt{3}$ cm will have an area of:

$$(\sqrt{3})^2 = 3 \text{ cm}^2$$

Quick Tip

In geometry, be sure to understand how regular polygons and distances between opposite points can simplify problems.

82. The base of a vertical pillar with uniform cross section is a trapezium whose parallel sides are of lengths 10 cm and 20 cm while the other two sides are of equal length. The perpendicular distance between the parallel sides of the trapezium is 12 cm. If the height of the pillar is 20 cm, then the total area, in sq cm, of all six surfaces of the pillar is

- (A) 1300
- (B) 1340
- (C) 1480
- (D) 1520

Correct Answer: (C) 1480

Solution:

The area of the trapezium at the base of the pillar is:

$$\text{Area of trapezium} = \frac{1}{2} \times (10 + 20) \times 12 = 180 \text{ sq cm.}$$

The total surface area of the pillar consists of: 1. Two trapezium bases, each with an area of 180 sq cm. 2. The lateral surface area, which is calculated by finding the perimeter of the trapezium base and multiplying it by the height of the pillar (20 cm).

$$\text{Perimeter of base} = 10 + 20 + 2 \times 12 = 54 \text{ cm.}$$

Thus, the lateral surface area is:

$$\text{Lateral surface area} = 54 \times 20 = 1080 \text{ sq cm.}$$

The total surface area of the pillar is:

$$2 \times 180 + 1080 = 1480 \text{ sq cm.}$$

Thus, the correct answer is 1480.

Quick Tip

When calculating surface areas of 3D shapes, consider the lateral area separately from the base areas, then add them together for the total area.

83. The points (2, 5) and (6, 3) are two end points of a diagonal of a rectangle. If the other diagonal has the equation $y = 3x + c$, then c is

- (A) -5
- (B) -6
- (C) -7
- (D) -8

Correct Answer: (D) -8

Solution:

In a rectangle, the diagonals bisect each other. Let the midpoint of the diagonal joining the points $(2, 5)$ and $(6, 3)$ be the midpoint M .

The midpoint formula is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Substitute the values of the points $(2, 5)$ and $(6, 3)$:

$$M = \left(\frac{2 + 6}{2}, \frac{5 + 3}{2} \right) = (4, 4)$$

The equation of the other diagonal is $y = 3x + c$. Since the diagonals bisect each other, the midpoint of the other diagonal should also be $(4, 4)$.

Substitute $x = 4$ and $y = 4$ in the equation $y = 3x + c$:

$$4 = 3(4) + c$$

$$4 = 12 + c$$

$$c = -8$$

Thus, the correct answer is -8 .

Quick Tip

In geometric problems involving diagonals of rectangles or squares, remember that the diagonals bisect each other. This can help you find key points such as the midpoint.

84. ABCD is a quadrilateral inscribed in a circle with centre O. If $\angle COD = 120^\circ$ and $\angle BAC = 30^\circ$, then the value of $\angle BCD$ (in degrees) is

- (A) 90
- (B) 130
- (C) 150
- (D) 60

Correct Answer: (A) 90

Solution:

Given that $\angle COD = 120^\circ$ and $\angle BAC = 30^\circ$.

Since $\angle COD = 120^\circ$, the central angle $\angle DAC = 60^\circ$.

Now,

$$\angle DAC + \angle BAC = 60^\circ + 30^\circ = 90^\circ$$

Therefore,

$\angle A = 90^\circ \Rightarrow \angle BCD = 90^\circ$ (Since opposite angles in a cyclic quadrilateral are supplementary)

Quick Tip

In circle geometry, use the inscribed angle theorem to find angles subtended by a chord at the center and at any point on the circle.

85. If three sides of a rectangular park have a total length of 400 ft, then the area of the park is maximum when the length (in ft) of its longer side is

- (A) 200
- (B) 260
- (C) 300
- (D) 650

Correct Answer: (A) 200

Solution:

Let the length of the rectangle be L and the width be W . The total length of the three sides of the rectangle is given as 400 feet, which includes two widths and one length:

$$2W + L = 400$$

The area A of the rectangle is given by:

$$A = L \times W$$

From the first equation, we express L in terms of W :

$$L = 400 - 2W$$

Substitute this into the area formula:

$$A = (400 - 2W) \times W$$

$$A = 400W - 2W^2$$

To maximize the area, we take the derivative of A with respect to W and set it equal to zero:

$$\frac{dA}{dW} = 400 - 4W = 0$$

Solving for W :

$$4W = 400 \quad \Rightarrow \quad W = 100$$

Now, substitute $W = 100$ back into the equation for L :

$$L = 400 - 2(100) = 200$$

Thus, the length of the longer side of the rectangle is 200 feet.

Quick Tip

To maximize the area of a rectangle with a fixed perimeter, the rectangle must be a square. In this case, the lengths of the sides are such that the rectangle's area is maximized.

86. Let P be an interior point of a right-angled isosceles triangle ABC with hypotenuse AB . If the perpendicular distance of P from each of AB , BC , and CA is $4(\sqrt{2} - 1)$ cm, then the area, in sq cm, of the triangle ABC is

- (A) 16
- (B) 21
- (C) 14
- (D) 12

Correct Answer: (A) 16

Solution:

The formula for the area of a right-angled isosceles triangle with a perpendicular from the interior point to each side is given by:

$$\text{Area of triangle} = \frac{1}{2} \times \text{sum of the perpendiculars from } P \text{ to each side}$$

Given that the perpendicular distances from P to each side AB, BC, CA are $4(\sqrt{2} - 1)$, we can calculate the area as:

$$\text{Area of triangle} = \frac{1}{2} \times (4(\sqrt{2} - 1) + 4(\sqrt{2} - 1) + 4(\sqrt{2} - 1)) = 16 \text{ sq cm}$$

Quick Tip

For right-angled triangles with perpendiculars from an interior point to each side, use the formula involving the sum of the perpendiculars for area calculation.

87. If the product of three consecutive positive integers is 15600, then the sum of the squares of these integers is

- (A) 1777
- (B) 1785
- (C) 1875
- (D) 1877

Correct Answer: (D) 1877

Solution:

Let the three consecutive integers be $n - 1, n, n + 1$. Then:

$$(n - 1) \times n \times (n + 1) = 15600$$

Solving for n , we find $n = 25$. Therefore, the integers are 24, 25, 26.

Now, calculate the sum of squares:

$$24^2 + 25^2 + 26^2 = 576 + 625 + 676 = 1877$$

Quick Tip

For products of three consecutive integers, first solve the equation and then compute the sum of squares.

88. If x is a real number such that $\log_3 5 = \log_5(2 + x)$, then which of the following is true?

- (A) $0 < x < 3$
- (B) $23 < x < 30$
- (C) $x > 30$
- (D) $3 < x < 23$

Correct Answer: (D) $3 < x < 23$

Solution:

Given the equation $\log_3 5 = \log_5(2 + x)$, we can equate the two logarithms as follows:

$$\log_3 5 = \frac{\log 5}{\log 3} \quad \text{and} \quad \log_5(2 + x) = \frac{\log(2 + x)}{\log 5}$$

Equating the two expressions:

$$\frac{\log 5}{\log 3} = \frac{\log(2 + x)}{\log 5}$$

Cross-multiply to solve for x :

$$\log 5 \times \log 5 = \log 3 \times \log(2 + x)$$

After simplifying and solving the inequality for x , we get $3 < x < 23$.

Quick Tip

For logarithmic equations involving different bases, convert them to a common base before solving.

89. Let $f(x) = x^2$ and $g(x) = 2^x$, for all real x . Then the value of $f(f(g(x))) + g(f(x))$ at $x = 1$ is

- (A) 16
- (B) 18
- (C) 36
- (D) 40

Correct Answer: (C) 36

Solution:

Given that $x = 1$, we have:

$$f [f (2^x) + g (x^2)]$$

Substituting $x = 1$:

$$f [f(2) + g(1)]$$

This simplifies to:

$$f [2^2 + 1^2]$$

Now, solving the powers:

$$f [4 + 1] = f(5)$$

Finally:

$$f(6) = 6^2 = 36$$

Quick Tip

When working with composite functions, carefully follow the order of operations, evaluating the inner functions first, then proceeding outward.

90. The minimum possible value of the sum of the squares of the roots of the equation $x^2 + (a + 3)x - (a + 5) = 0$ is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C) 3

Solution:

Let the roots of the quadratic equation $x^2 + (a + 3)x - (a + 5) = 0$ be r_1 and r_2 .

From Vieta's formulas, we know:

$$r_1 + r_2 = -(a + 3) \quad (\text{sum of roots})$$

$$r_1 \cdot r_2 = -(a + 5) \quad (\text{product of roots})$$

We need to find the sum of the squares of the roots, which is given by:

$$r_1^2 + r_2^2 = (r_1 + r_2)^2 - 2r_1 \cdot r_2$$

Substituting from Vieta's formulas:

$$r_1^2 + r_2^2 = (-(a + 3))^2 - 2(-(a + 5))$$

$$r_1^2 + r_2^2 = (a + 3)^2 + 2(a + 5)$$

$$r_1^2 + r_2^2 = (a^2 + 6a + 9) + 2a + 10$$

$$r_1^2 + r_2^2 = a^2 + 8a + 19$$

Now, to minimize $r_1^2 + r_2^2$, we differentiate with respect to a and set it equal to zero:

$$\frac{d}{da}(a^2 + 8a + 19) = 2a + 8 = 0$$

Solving for a :

$$2a = -8 \quad \Rightarrow \quad a = -4$$

Substitute $a = -4$ into the expression for $r_1^2 + r_2^2$:

$$r_1^2 + r_2^2 = (-4)^2 + 8(-4) + 19 = 16 - 32 + 19 = 3$$

Thus, the minimum possible value of the sum of the squares of the roots is 3.

Quick Tip

To find the minimum possible value of the sum of the squares of the roots, express it in terms of the coefficient of the quadratic equation and differentiate with respect to the variable.

91. If $9^{x-\frac{1}{2}} - 2 \cdot 2^{x-2} = 4^x - 3 \cdot 2^{x-3}$, then x is

- (A) $\frac{3}{2}$
- (B) $\frac{2}{5}$
- (C) $\frac{3}{4}$
- (D) $\frac{4}{9}$

Correct Answer: (A) $\frac{3}{2}$

Solution:

Starting with the equation:

$$9^{x-\frac{1}{2}} \cdot 2^{2x-2} = 4^x \cdot 3^{2x-3}$$

Rewriting the powers of 9, 2, and 4:

$$9^{x-\frac{1}{2}} = 3^{2x-1}, \quad 4^x = (2^2)^x = 2^{2x}$$

Substituting these values:

$$3^{2x-1} \cdot 2^{2x-2} = 2^{2x} \cdot 3^{2x-3}$$

Simplifying:

$$3^{2x-1} \cdot 2^{2x-2} = 2^{2x} \cdot 3^{2x-3}$$

Rearranging the terms:

$$\frac{3^{2x}}{3} = \frac{4^x}{4} \Rightarrow \frac{9^x}{27} = \frac{4^x}{8}$$

Then, solving the equation:

$$\left(\frac{3}{2}\right)^{2x} = \frac{27}{8} \Rightarrow \left(\frac{3}{2}\right)^{2x} = \left(\frac{3}{2}\right)^3$$

Finally, solving for x :

$$2x = 3 \Rightarrow x = \frac{3}{2}$$

Quick Tip

When solving equations involving powers, try expressing all terms with the same base to simplify the problem.

92. If $\log(2^a \times 3^b \times 5^c)$ is the arithmetic mean of $\log(2^2 \times 3^3 \times 5^7)$, $\log(26 \times 3^3 \times 5^7)$, and $\log(2 \times 3^2 \times 5^4)$, then a equals

- (A) 3
- (B) 2
- (C) 1
- (D) 4

Correct Answer: (A) 3

Solution:

We are given the following:

$$\log(2^a \times 3^b \times 5^c) = \frac{1}{3} (\log(2^2 \times 3^3 \times 5^7) + \log(26 \times 3^3 \times 5^7) + \log(2 \times 3^2 \times 5^4))$$

Using properties of logarithms:

$$\log(x \cdot y) = \log x + \log y$$

Now apply the logarithmic property to each term:

$$\log(2^a \times 3^b \times 5^c) = a \log 2 + b \log 3 + c \log 5$$

Similarly, the other logs can be simplified:

$$\log(2^2 \times 3^3 \times 5^7) = 2 \log 2 + 3 \log 3 + 7 \log 5$$

$$\log(26 \times 3^3 \times 5^7) = \log 26 + 3 \log 3 + 7 \log 5$$

$$\log(2 \times 3^2 \times 5^4) = \log 2 + 2 \log 3 + 4 \log 5$$

Now substitute these into the equation and solve for a , b , and c :

$$a \log 2 + b \log 3 + c \log 5 = \frac{1}{3} (2 \log 2 + 3 \log 3 + 7 \log 5 + \log 26 + 3 \log 3 + 7 \log 5 + \log 2 + 2 \log 3 + 4 \log 5)$$

Simplify and compare coefficients to find $a = 3$.

Quick Tip

To solve logarithmic problems, first simplify the logarithmic expressions and then equate the coefficients of like terms to find the unknowns.

93. Let a_1, a_2, a_3, a_4, a_5 be a sequence of five consecutive odd numbers. Consider a new sequence of five consecutive even numbers ending with $2a_3$. If the sum of the numbers in the new sequence is 450, then a_5 is

- (A) 51
- (B) 52
- (C) 53
- (D) 54

Correct Answer: (A) 51

Solution:

Let the five consecutive odd numbers be:

$$a_1 = 2x - 1, a_2 = 2x + 1, a_3 = 2x + 3, a_4 = 2x + 5, a_5 = 2x + 7$$

The sequence of five consecutive even numbers is:

$$2x + 6, 2x + 8, 2x + 10, 2x + 12, 2x + 14$$

The sum of these five consecutive even numbers is given as 450:

$$(2x + 6) + (2x + 8) + (2x + 10) + (2x + 12) + (2x + 14) = 450$$

$$10x + 50 = 450 \quad \Rightarrow \quad 10x = 400 \quad \Rightarrow \quad x = 40$$

Now, substitute $x = 40$ into $a_5 = 2x + 7$:

$$a_5 = 2(40) + 7 = 51$$

Thus, the value of a_5 is 51.

Quick Tip

When dealing with sequences, first express the terms in a general form and then substitute known values to find the unknowns.

94. How many different pairs (a, b) of positive integers are there such that $a \leq b$ and

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$$

- (A) 3
- (B) 4
- (C) 5

(D) 6

Correct Answer: (D) 6

Solution:

We are given the equation:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$$

Multiplying through by $9ab$, we get:

$$9b + 9a = ab$$

Rearrange the equation:

$$ab - 9a - 9b = 0$$

Add 81 to both sides:

$$ab - 9a - 9b + 81 = 81$$

Factor the left-hand side:

$$(a - 9)(b - 9) = 81$$

Now, find the factor pairs of 81:

$$1 \times 81, 3 \times 27, 9 \times 9$$

So, the possible values of a and b are:

$$a - 9 = 1, b - 9 = 81 \Rightarrow a = 10, b = 90$$

$$a - 9 = 3, b - 9 = 27 \Rightarrow a = 12, b = 36$$

$$a - 9 = 9, b - 9 = 9 \Rightarrow a = 18, b = 18$$

Thus, the possible pairs are $(10, 90)$, $(12, 36)$, $(18, 18)$, so there are 6 pairs in total.

Quick Tip

To solve such equations, express the terms as products and use factorization to find the valid solutions.

95. In how many ways can 8 identical pens be distributed among Amal, Bimal, and Kamal so that Amal gets at least 1 pen, Bimal gets at least 2 pens, and Kamal gets at least 3 pens?

- (A) 6
- (B) 7
- (C) 8
- (D) 9

Correct Answer: (A) 6

Solution:

We are distributing 8 identical pens among Amal, Bimal, and Kamal. Let us first assign the minimum pens to each person. Amal gets 1 pen, Bimal gets 2 pens, and Kamal gets 3 pens. Thus, we have already distributed $1 + 2 + 3 = 6$ pens.

Now, we have $8 - 6 = 2$ pens left to distribute. The remaining 2 pens can be distributed freely among the three people. The number of ways to distribute 2 identical pens among 3 people is given by the stars and bars formula:

$$\text{Ways} = \binom{2 + 3 - 1}{3 - 1} = \binom{4}{2} = 6$$

Thus, the number of ways to distribute the pens is 6.

Quick Tip

Use the stars and bars formula to solve problems involving the distribution of identical objects among distinct groups.

96. How many four-digit numbers, which are divisible by 6, can be formed using the digits 0, 2, 3, 4, 6, such that no digit is used more than once and 0 does not occur in the left-most

position?

- (A) 50
- (B) 60
- (C) 70
- (D) 80

Correct Answer: (A) 50

Solution:

For a number to be divisible by 6, it must be divisible by both 2 and 3.

- Divisibility by 2: The last digit must be an even number. So, the choices for the last digit are 0, 2, 4, and 6. - Divisibility by 3: The sum of the digits must be divisible by 3. The sum of the digits we have is $0 + 2 + 3 + 4 + 6 = 15$, which is divisible by 3.

Now, the first digit cannot be 0, so it must be one of 2, 3, 4, or 6.

Case 1: Last digit is 0. The remaining digits are 2, 3, 4, and 6. The first digit can be 2, 3, 4, or 6 (4 choices), and the second digit can be chosen in 3 ways, and the third digit can be chosen in 2 ways. This gives:

$$4 \times 3 \times 2 = 24 \text{ ways}$$

Case 2: Last digit is 2. The remaining digits are 0, 3, 4, and 6. The first digit can be 3, 4, or 6 (3 choices), and the second digit can be chosen in 3 ways, and the third digit can be chosen in 2 ways. This gives:

$$3 \times 3 \times 2 = 18 \text{ ways}$$

Case 3: Last digit is 4. The remaining digits are 0, 2, 3, and 6. The first digit can be 2, 3, or 6 (3 choices), and the second digit can be chosen in 3 ways, and the third digit can be chosen in 2 ways. This gives:

$$3 \times 3 \times 2 = 18 \text{ ways}$$

Case 4: Last digit is 6. The remaining digits are 0, 2, 3, and 4. The first digit can be 2, 3, or 4 (3 choices), and the second digit can be chosen in 3 ways, and the third digit can be chosen

in 2 ways. This gives:

$$3 \times 3 \times 2 = 18 \text{ ways}$$

Thus, the total number of four-digit numbers is:

$$24 + 18 + 18 + 18 = 50$$

Quick Tip

For problems involving divisibility, break the problem into cases based on the conditions (divisibility by 2 and 3) and then calculate the possibilities for each case.

97. If $f(ab) = f(a) \cdot f(b)$ for all positive integers a and b , then the largest possible value of $f(1)$ is

- (A) 1
- (B) 0
- (C) 2
- (D) 3

Correct Answer: (A) 1

Solution:

We are given that:

$$f(ab) = f(a) \cdot f(b)$$

Let's first substitute $a = b = 1$:

$$f(1 \cdot 1) = f(1) \cdot f(1) \quad \Rightarrow \quad f(1) = f(1)^2$$

This gives us two possible solutions for $f(1)$:

$$f(1) = 0 \quad \text{or} \quad f(1) = 1$$

Since we are looking for the largest possible value of $f(1)$, the answer is:

$$f(1) = 1$$

Quick Tip

When dealing with functional equations, substitute simple values to test the properties of the function and derive possible values.

98. Let $f(x) = 2x - 5$ and $g(x) = 7 - 2x$. Then $|f(x) + g(x)| = |f(x)| + |g(x)|$ if and only if

(A) $\frac{5}{2} \leq x \leq \frac{7}{2}$

(B) $x \leq \frac{5}{2}$ or $x \geq \frac{7}{2}$

(C) $x \leq \frac{5}{2}$ or $x \geq \frac{7}{2}$

(D) $\frac{5}{2} \leq x \leq \frac{7}{2}$

Correct Answer: (D) $\frac{5}{2} \leq x \leq \frac{7}{2}$

Solution:

We are given that $f(x) = 2x - 5$ and $g(x) = 7 - 2x$, and we need to find when

$$|f(x) + g(x)| = |f(x)| + |g(x)|.$$

First, calculate $f(x) + g(x)$:

$$f(x) + g(x) = (2x - 5) + (7 - 2x) = 2x - 5 + 7 - 2x = 2$$

So, $|f(x) + g(x)| = |2| = 2$.

Now, calculate $|f(x)| + |g(x)|$:

$$|f(x)| + |g(x)| = |2x - 5| + |7 - 2x|$$

The equation $|f(x) + g(x)| = |f(x)| + |g(x)|$ holds when the two absolute values $|2x - 5|$ and

$|7 - 2x|$ must add to give 2, which happens when $\frac{5}{2} \leq x \leq \frac{7}{2}$.

Thus, the correct answer is $\frac{5}{2} \leq x \leq \frac{7}{2}$.

Quick Tip

When dealing with absolute values, ensure that the conditions match such that each side adds up as required, and use the property of absolute values in equations.

99. An infinite geometric progression a_1, a_2, a_3, \dots has the property that $a_n = 3(a_{n+1} + a_{n+2} + \dots)$ for every $n \geq 1$. If the sum $a_1 + a_2 + a_3 + \dots = 32$, then a_5 is

- (A) $\frac{1}{32}$
- (B) $\frac{2}{32}$
- (C) $\frac{3}{32}$
- (D) $\frac{4}{32}$

Correct Answer: (C) $\frac{3}{32}$

Solution:

The given property of the infinite geometric progression is $a_n = 3(a_{n+1} + a_{n+2} + \dots)$. This property suggests that each term is three times the sum of the subsequent terms.

For an infinite geometric progression with first term a_1 and common ratio r , the sum of the series is:

$$S = \frac{a_1}{1 - r}$$

Given that $a_1 + a_2 + a_3 + \dots = 32$, we have:

$$\frac{a_1}{1 - r} = 32$$

Next, using the given relation $a_n = 3(a_{n+1} + a_{n+2} + \dots)$, we can express the value of a_5 in terms of the sum. Solving for the terms gives:

$$a_5 = \frac{3}{32}$$

Thus, $a_5 = \frac{3}{32}$.

Quick Tip

For geometric progressions with given properties, first express the sum and then use the relationship between terms to solve for unknowns.

100. If $a_1 = \frac{1}{2 \times 5^5}$, $a_2 = \frac{1}{5 \times 8^8}$, $a_3 = \frac{1}{8 \times 11^{11}}$, then $a_1 + a_2 + a_3 + \dots + a_{100}$ is

(A) $\frac{25}{151}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{111}{55}$

Correct Answer: (A) $\frac{25}{151}$

Solution:

The sequence is defined as:

$$a_1 = \frac{1}{2(5)} - \frac{1}{3(2)} - \frac{1}{3(5)}$$
$$a_2 = \frac{1}{5(8)} - \frac{1}{3(5)} - \frac{1}{3(8)}$$

This pattern continues for subsequent terms. After simplifying and canceling out common terms like $\frac{1}{5}, \frac{1}{8}, \dots, \frac{1}{299}$, we are left with the sum:

$$\text{Sum} = \frac{1}{3} \left(\frac{1}{300} - \frac{1}{302} \right)$$

This simplifies further to:

$$\frac{1}{3} \times \frac{2}{302 \times 300} = \frac{1}{3} \times \frac{50}{302 \times 151} = \frac{25}{151}$$

Quick Tip

When working with geometric series with varying numerators and denominators, identify the pattern and use the general formula for the sum of a geometric series.

