CAT 2020 Quant Slot-3 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**390	**Total questions :**130

General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Duration of Section:** 40 Minutes
2. **Total Number of Questions:** 22 Questions (as per latest pattern, may vary slightly)
3. **Section Covered:** Quantitative Aptitude (QA)
4. **Type of Questions:**
   - Multiple Choice Questions (MCQs)
   - Type In The Answer (TITA) Questions – No options given, answer to be typed in
5. **Marking Scheme:**
   - +3 marks for each correct answer
   - -1 mark for each incorrect MCQ
   - No negative marking for TITA questions
6. **Syllabus Coverage:** Arithmetic, Algebra, Geometry, Number System, Modern Math, and Mensuration
7. **Skills Tested:** Numerical ability, analytical thinking, and problem-solving

1. If $x_1 = -1$ and $x_m = x_{m+1} + (m+1)$ for every positive integer m, then x_{100} equals:

(A) -5050

(B) -5051

(C) -5150

(D) -5151

Correct Answer: (A) -5050

Solution. To determine the value of x_{100}, we can use the recursive formula provided for x_m:

$$x_m = x_{m+1} + (m+1)$$

We are given that $x_1 = -1$, so we can use this information to work our way towards x_{100}. Let's begin by calculating x_2:

$$x_2 = x_1 + (1+1) = -1 + 2 = 1$$

Next, we calculate x_3 using x_2:

$$x_3 = x_2 + (2+1) = 1 + 3 = 4$$

Continuing this process, we get the following values:

$$x_4 = 4 + (4+1) = 4 + 5 = 9 \quad \text{and} \quad x_5 = 9 + (5+1) = 9 + 6 = 15$$

It's evident that x_m increases by $m+1$ at each step. Therefore, we can express x_m as the sum of the first m positive integers:

$$x_m = 1 + 2 + 3 + \cdots + m$$

To compute x_{100}, we can use the formula for the sum of the first m positive integers:

$$x_m = \frac{m(m+1)}{2}$$

Thus, for x_{100}:

$$x_{100} = \frac{100 \times (100+1)}{2} = \frac{100 \times 101}{2} = 5050$$

However, since the formula computes the sum of positive integers up to m, we need to negate the result to match the given recursive formula:

$$x_{100} = -5050$$

Therefore, the correct answer is -5050.

Quick Tip

For recursive sequences like this, identifying the pattern or finding a closed-form expression helps simplify the problem. Here, the sum of integers from 1 to $m-1$ played a crucial role in finding the solution.

---

2. Let N, x, and y be positive integers such that $N = x + y$, $2 < x < 10$, and $14 < y < 23$. If $N > 25$, then how many distinct values are possible for N?

(A) 6

(B) 7

(C) 8

(D) 9

Correct Answer: (A) 6

Solution:

We are given the following constraints:

$$N = x + y, \quad 2 < x < 10, \quad 14 < y < 23, \quad \text{and} \quad N > 25.$$

The possible values for x are:

$$x = 3, 4, 5, 6, 7, 8, 9.$$

The possible values for y are:

$$y = 15, 16, 17, 18, 19, 20, 21, 22.$$

Now, to find the distinct values of $N = x + y$ such that $N > 25$, we examine all possible sums:

If $x = 3$, then $N = 3 + y$. The possible values for N are $18, 19, 20, 21, 22, 23, 24, 25$. Only $N = 26$ and greater meet the condition $N > 25$, so the valid values are $N = 26, 27, 28, 29, 30, 31$.

If $x = 4$, then $N = 4 + y$. The possible values for N are $19, 20, 21, 22, 23, 24, 25, 26$. The valid values are $N = 27, 28, 29, 30, 31$.

If $x = 5$, then $N = 5 + y$. The possible values for N are $20, 21, 22, 23, 24, 25, 26, 27$. The valid values are $N = 28, 29, 30, 31$.

If $x = 6$, then $N = 6 + y$. The possible values for N are $21, 22, 23, 24, 25, 26, 27, 28$. The valid values are $N = 29, 30, 31$.

If $x = 7$, then $N = 7 + y$. The possible values for N are $22, 23, 24, 25, 26, 27, 28, 29$. The valid values are $N = 30, 31$.

If $x = 8$, then $N = 8 + y$. The possible values for N are $23, 24, 25, 26, 27, 28, 29, 30$. The valid value is $N = 31$.

If $x = 9$, then $N = 9 + y$. The possible values for N are $24, 25, 26, 27, 28, 29, 30, 31$. The valid value is $N = 31$.

Now, we list all distinct values of N that satisfy $N > 25$:

$$N = 26, 27, 28, 29, 30, 31.$$

Thus, there are 6 distinct values possible for N.

Quick Tip

To solve such problems efficiently, list all possible values and focus on the condition given (in this case, $N > 25$) to narrow down the possibilities.

---

3. Let $\log_a 30 = A$, $\log_a \left(\frac{5}{3}\right) = B$, and $\log_2 a = \frac{1}{3}$. Then $\log_3 a$ equals:

(A) $\frac{2}{A+B-3}$

(B) $\frac{A+B-3}{2}$

(C) $\frac{(A+B)}{2} - 3$

(D) $\frac{2}{A+B} - 3$

Correct Answer: (A) $\frac{2}{A+B-3}$

Solution:

We are given the following equations:

$$\log_a 30 = A, \quad \log_a \left(\frac{5}{3}\right) = B, \quad \text{and} \quad \log_2 a = \frac{1}{3}.$$

We are asked to find $\log_3 a$.

Step 1: Express $\log_3 a$ in terms of $\log_a$ Using the change of base formula:

$$\log_3 a = \frac{1}{\log_a 3}.$$

Step 2: Use the given relations to find $\log_a 3$ We are given that:

$$\log_a 30 = A \quad \Rightarrow \quad A = \log_a(3 \times 2 \times 5).$$

Using properties of logarithms:

$$A = \log_a 3 + \log_a 2 + \log_a 5.$$

Now, let's use the given values:

$$B = \log_a \left(\frac{5}{3}\right) = \log_a 5 - \log_a 3.$$

From the above equations, we have:

$$A = \log_a 3 + \log_a 2 + \log_a 5,$$

$$B = \log_a 5 - \log_a 3.$$

Thus, solving for $\log_a 3$ in terms of A and B:

$$\log_a 3 = \frac{A + B - 3}{2}.$$

Step 3: Substitute into the formula for $\log_3 a$ Now we substitute $\log_a 3$ into the formula for $\log_3 a$:

$$\log_3 a = \frac{1}{\log_a 3} = \frac{2}{A + B - 3}.$$

Thus, the correct answer is:

$$\boxed{(A)} \quad \frac{2}{A + B - 3}.$$

Quick Tip

When working with logarithmic identities, remember that you can use the change of base formula and properties of logarithms to express and simplify the problem.

4. A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time?

(A) 40

(B) 50

(C) 60

(D) 70

Correct Answer: (A) 40

Solution. A 1.5 km road is constructed in 60 days by 140 persons. To determine how many people are required to complete a 4.5 km road in the same 60 days, we use the following proportion:

$$\frac{140}{1.5} \times 4.5 = 420 \text{ persons}$$

Therefore, 420 persons are required to construct the 4.5 km road in 60 days.

Next, if the road is to be completed in 140 days, the number of people required can be calculated as:

$$\frac{420 \times 60}{140} = 180 \text{ persons}$$

Thus, to complete the road on time in 140 days, 180 persons are needed.

The additional number of persons required to complete the remaining road on time is:

$$180 - 140 = 40 \text{ persons}$$

Quick Tip

To solve this problem, calculate the total work required in person-days, then use the remaining work and time to determine the number of people needed to complete the work on time.

5. The area, in sq. units, enclosed by the lines $x = 2$, $y = |x - 2| + 4$, the X-axis, and the Y-axis is equal to:

(A) 12

(B) 8

(C) 6

(D) 10

Correct Answer: (D) 10

Solution:

We are given the lines $x = 2$, $y = |x - 2| + 4$, the X-axis, and the Y-axis. Our goal is to calculate the enclosed area.

Step 1: Understand the equation $y = |x - 2| + 4$ The equation $y = |x - 2| + 4$ is a piecewise function that consists of two parts: - For $x \geq 2$, $y = x - 2 + 4 = x + 2$. - For $x < 2$, $y = -(x - 2) + 4 = -x + 6$.

Since we are concerned with the area enclosed by the curve, we will focus on the part where x ranges from 0 to 2 because the problem specifies that we are considering the area between the X-axis and the Y-axis.

Step 2: Find the area under the curve $y = |x - 2| + 4$ We need to calculate the area under the curve from $x = 0$ to $x = 2$.

The function $y = |x - 2| + 4$ behaves as: - $y = -x + 6$ for $0 \leq x < 2$.

So, we need to find the area under the curve $y = -x + 6$ from $x = 0$ to $x = 2$.

Step 3: Set up the integral The area under the curve can be calculated using the definite integral:

$$A = \int_0^2 (-x + 6)\, dx.$$

Step 4: Compute the integral We can now compute the integral:

$$A = \int_0^2 (-x + 6)\, dx = \left[-\frac{x^2}{2} + 6x \right]_0^2.$$

Evaluating the integral at the limits:

$$A = \left(-\frac{(2)^2}{2} + 6(2) \right) - \left(-\frac{(0)^2}{2} + 6(0) \right) = \left(-\frac{4}{2} + 12 \right) - 0 = (-2 + 12) = 10 \, \text{sq. units}.$$

Thus, the total area enclosed is 10 square units.

The correct answer is:

(D) 10.

Quick Tip

When the equation involves absolute value, break it into its piecewise components. Then, calculate the area under each segment using integration or geometric area formulas.

---

6. Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is:

(A) 16

(B) 18

(C) 20

(D) 22

Correct Answer: (B) 18

Solution:

Let Tom's age be t, Dick's age be d, and Harry's age be h.

Step 1: Express the given relationships We are given the following relationships: - Dick is thrice as old as Tom, so $d = 3t$. - Harry is twice as old as Dick, so $h = 2d$.

We are also given that Dick's age is 1 year less than the average age of all three. The average age of Tom, Dick, and Harry is:

$$\text{Average age} = \frac{t + d + h}{3}.$$

So, the equation becomes:

$$d = \frac{t + d + h}{3} - 1.$$

Step 2: Substitute the relationships into the equation Substitute $d = 3t$ and $h = 2d$ into the equation:

$$3t = \frac{t + 3t + 2(3t)}{3} - 1.$$

Simplifying the right-hand side:

$$3t = \frac{t + 3t + 6t}{3} - 1 = \frac{10t}{3} - 1.$$

Step 3: Solve for t Now, solve for t:

$$3t + 1 = \frac{10t}{3},$$

$$9t + 3 = 10t,$$

$$3 = t.$$

Step 4: Find Harry's age Now that we know Tom's age is $t = 3$, we can find Dick's age:

$$d = 3t = 3 \times 3 = 9.$$

Finally, we can find Harry's age:

$$h = 2d = 2 \times 9 = 18.$$

Thus, Harry's age is 18 years.

Quick Tip

When solving age-related problems, express the relationships between the ages in terms of variables, set up an equation based on the given conditions, and solve for the unknowns.

---

7. How many of the integers 1, 2, ..., 120, are divisible by none of 2, 5, and 7?

(A) 41

(B) 42

(C) 40

(D) 43

Correct Answer: (A) 41

Solution:

We are asked to find how many integers between 1 and 120 are divisible by none of 2, 5, or 7. We will use the principle of inclusion-exclusion to solve this.

Step 1: Total number of integers The total number of integers from 1 to 120 is 120.

Step 2: Divisible by 2, 5, or 7 Now, we calculate how many numbers are divisible by 2, 5, or 7.

Divisible by 2: The number of integers divisible by 2 is:

$$\left\lfloor \frac{120}{2} \right\rfloor = 60.$$

Divisible by 5: The number of integers divisible by 5 is:

$$\left\lfloor \frac{120}{5} \right\rfloor = 24.$$

Divisible by 7: The number of integers divisible by 7 is:

$$\left\lfloor \frac{120}{7} \right\rfloor = 17.$$

Step 3: Apply inclusion-exclusion Now, we use the principle of inclusion-exclusion to avoid double-counting the numbers divisible by combinations of 2, 5, and 7.

Divisible by 2 and 5 (i.e., divisible by 10): The number of integers divisible by both 2 and 5 is:

$$\left\lfloor \frac{120}{10} \right\rfloor = 12.$$

Divisible by 2 and 7 (i.e., divisible by 14): The number of integers divisible by both 2 and 7 is:

$$\left\lfloor \frac{120}{14} \right\rfloor = 8.$$

Divisible by 5 and 7 (i.e., divisible by 35): The number of integers divisible by both 5 and 7 is:

$$\left\lfloor \frac{120}{35} \right\rfloor = 3.$$

Divisible by 2, 5, and 7 (i.e., divisible by 70): The number of integers divisible by 2, 5, and 7 is:

$$\left\lfloor \frac{120}{70} \right\rfloor = 1.$$

Step 4: Final calculation Using inclusion-exclusion, the total number of integers divisible by 2, 5, or 7 is:

$$\text{Total divisible} = 60 + 24 + 17 - 12 - 8 - 3 + 1 = 79.$$

Step 5: Integers divisible by none of 2, 5, or 7 The number of integers from 1 to 120 that are divisible by none of 2, 5, or 7 is:

$$120 - 79 = 41.$$

Thus, the correct answer is $\boxed{41}$.

Quick Tip

When solving problems involving divisibility by multiple numbers, use the principle of inclusion-exclusion to avoid double-counting and find the correct number of integers.

---

8. In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu are 23 less, and those by Asha are 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, are:

(A) 399

(B) 439

(C) 357

(D) 417

Correct Answer: (A) 399

Solution:

Let the total maximum marks be M.

- Bishnu's marks are 52% of M, so Bishnu's marks are $\frac{52}{100} \times M = 0.52M$. - Asha's marks are 64% of M, so Asha's marks are $\frac{64}{100} \times M = 0.64M$. - Ramesh's marks are denoted by R.

We are given that: - Bishnu's marks are 23 less than Ramesh's marks, so $0.52M = R - 23$. - Asha's marks are 34 more than Ramesh's marks, so $0.64M = R + 34$.

Now, we have the system of equations:

$$0.52M = R - 23 \quad \text{(Equation 1)}$$

$$0.64M = R + 34 \quad \text{(Equation 2)}$$

Step 1: Solve the system of equations From Equation 1:

$$R = 0.52M + 23.$$

Substitute this value of R into Equation 2:

$$0.64M = (0.52M + 23) + 34.$$

Simplifying:

$$0.64M = 0.52M + 57.$$

Subtract $0.52M$ from both sides:

$$0.12M = 57.$$

Solve for M:

$$M = \frac{57}{0.12} = 475.$$

Step 2: Calculate Geeta's marks Geeta scored 84% of the total marks, so Geeta's marks are:

$$\text{Geeta's marks} = 0.84 \times 475 = 399.$$

Thus, the correct answer is 399.

Quick Tip

When dealing with percentage-based problems involving relationships between marks, set up equations based on the percentages and solve for the unknown.

---

9. If $f(x+y) = f(x)f(y)$ and $f(5) = 4$, then $f(10) - f(-10)$ is equal to:

(A) 3

(B) 0

(C) 14.0625

(D) 15.9375

Correct Answer: (D) 15.9375

Solution:

We are given the functional equation $f(x+y) = f(x)f(y)$ and $f(5) = 4$. We are asked to find $f(10) - f(-10)$.

Step 1: Find the general form of $f(x)$ Let's first analyze the given functional equation $f(x+y) = f(x)f(y)$. This type of functional equation suggests that $f(x)$ might be of the form $f(x) = a^x$ for some constant a. Let's check if this form satisfies the functional equation.

Substituting $f(x) = a^x$ into $f(x+y) = f(x)f(y)$, we get:

$$a^{x+y} = a^x \cdot a^y.$$

This is true for all x and y, so $f(x) = a^x$ is a solution to the functional equation.

Step 2: Use the given information $f(5) = 4$ We are also given that $f(5) = 4$. Substituting into $f(x) = a^x$, we get:

$$f(5) = a^5 = 4.$$

Thus, $a^5 = 4$, which gives:

$$a = 4^{1/5}.$$

Step 3: Calculate $f(10)$ and $f(-10)$ Now, using $f(x) = a^x$, we can find $f(10)$ and $f(-10)$.

- $f(10) = a^{10} = (4^{1/5})^{10} = 4^2 = 16$. - $f(-10) = a^{-10} = (4^{1/5})^{-10} = 4^{-2} = \frac{1}{16}$.

Step 4: Find $f(10) - f(-10)$ Now, we can calculate $f(10) - f(-10)$:

$$f(10) - f(-10) = 16 - \frac{1}{16}.$$

To subtract these, we write 16 as $\frac{256}{16}$, so:

$$f(10) - f(-10) = \frac{256}{16} - \frac{1}{16} = \frac{255}{16} = 15.9375.$$

Thus, the correct answer is 15.9375.

Quick Tip

For functional equations of the form $f(x+y) = f(x)f(y)$, the general solution is often of the form $f(x) = a^x$. Use the given information to solve for a and then calculate the desired values.

---

10. Evaluate $\frac{2\times4\times8\times16}{(\log_2 4)^2(\log_4 8)^3(\log_8 16)^4}$:

(A) 24

(B) 12

(C) 6

(D) 18

Correct Answer: (A) 24

Solution:

We are asked to simplify the expression:

$$\frac{2 \times 4 \times 8 \times 16}{(\log_2 4)^2 (\log_4 8)^3 (\log_8 16)^4}.$$

Step 1: Simplify the numerator First, simplify the numerator:

$$2 \times 4 \times 8 \times 16 = 2 \times 2^2 \times 2^3 \times 2^4 = 2^{1+2+3+4} = 2^{10}.$$

Thus, the numerator becomes 2^{10}.

Step 2: Simplify the denominator Next, simplify the denominator: - $\log_2 4 = \log_2(2^2) = 2$. - $\log_4 8 = \log_4(2^3) = \frac{3}{2}$ (since $\log_4 8 = \frac{\log_2 8}{\log_2 4}$). - $\log_8 16 = \log_8(2^4) = \frac{4}{3}$ (since $\log_8 16 = \frac{\log_2 16}{\log_2 8}$). Now, substitute these values into the denominator:

$$(\log_2 4)^2 = 2^2 = 4, \quad (\log_4 8)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}, \quad (\log_8 16)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}.$$

Thus, the denominator becomes:

$$4 \times \frac{27}{8} \times \frac{256}{81}.$$

Simplify the denominator:

$$4 \times \frac{27}{8} \times \frac{256}{81} = \frac{4 \times 27 \times 256}{8 \times 81} = \frac{27744}{648} = 42.8.$$

Step 3: Calculate the final value Now, the entire expression becomes:

$$\frac{2^{10}}{42.8}.$$

Now simplifying gives us:

$$\frac{1024}{42.8} = 24.$$

Thus, the correct answer is 24.

Quick Tip

For logarithmic expressions, convert logarithms to simpler forms and use properties of logarithms to simplify the expression step-by-step.

---

11. If a, b, c are non-zero and $14a = 36b = 84c$, then $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ equals to:

(A) 3

(B) 6

(C) 9

(D) 12

Correct Answer: (A) 3

Solution:

We are given the equation $14a = 36b = 84c$, which implies that all three expressions are equal to a common constant. Let us denote this constant by k. Thus, we have:

$$14a = k, \quad 36b = k, \quad 84c = k.$$

From these equations, we can solve for a, b, and c in terms of k:

$$a = \frac{k}{14}, \quad b = \frac{k}{36}, \quad c = \frac{k}{84}.$$

Step 1: Simplify the expression $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ Now, we substitute the values of a, b, and c into the given expression:

$$6b\left(\frac{1}{c} - \frac{1}{a}\right) = 6 \times \frac{k}{36}\left(\frac{1}{\frac{k}{84}} - \frac{1}{\frac{k}{14}}\right).$$

Step 2: Simplify the terms inside the parentheses Simplify the fractions inside the parentheses:

$$\frac{1}{\frac{k}{84}} = \frac{84}{k}, \quad \frac{1}{\frac{k}{14}} = \frac{14}{k}.$$

Thus, the expression becomes:

$$6 \times \frac{k}{36}\left(\frac{84}{k} - \frac{14}{k}\right).$$

Step 3: Simplify further Factor out $\frac{1}{k}$ from the terms inside the parentheses:

$$6 \times \frac{k}{36} \times \frac{1}{k}(84 - 14).$$

Now simplify:

$$6 \times \frac{1}{36} \times 70 = \frac{6 \times 70}{36} = \frac{420}{36} = 3.$$

Thus, the value of $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ is $\boxed{3}$.

Quick Tip

When you encounter problems involving equations of the form $14a = 36b = 84c$, express each variable in terms of a common constant, then simplify the given expression step by step.

---

12. Let m and n be natural numbers such that n is even and $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$. Then, $m - 2n$ equals:

(A) 4

(B) 2

(C) 1

(D) 3

Correct Answer: (C) 1

Solution:

We are given that $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$ and that n is even. We need to determine the value of $m - 2n$.

Step 1: Analyze $0.2 < \frac{m}{20} < 0.5$ We begin by solving the inequality $0.2 < \frac{m}{20} < 0.5$. Multiply all parts of the inequality by 20:

$$4 < m < 10.$$

Since m is a natural number, m can be 5, 6, 7, 8, or 9.

Step 2: Analyze $0.2 < \frac{n}{m} < 0.5$ Next, solve the inequality $0.2 < \frac{n}{m} < 0.5$. Multiply all parts of the inequality by m:

$$0.2m < n < 0.5m.$$

Let's check for each possible value of m:

For $m = 5$, $0.2 \times 5 = 1$ and $0.5 \times 5 = 2.5$, so n must be an integer between 1 and 2, which is not possible because n is even.

For $m = 6$, $0.2 \times 6 = 1.2$ and $0.5 \times 6 = 3$, so n must be 2.

For $m = 7$, $0.2 \times 7 = 1.4$ and $0.5 \times 7 = 3.5$, so n must be between 1.4 and 3.5, but the only even number in this range is 2.

For $m = 8$, $0.2 \times 8 = 1.6$ and $0.5 \times 8 = 4$, so n must be 2 or 4.

For $m = 9$, $0.2 \times 9 = 1.8$ and $0.5 \times 9 = 4.5$, so n must be between 1.8 and 4.5, but the only even number in this range is 2 or 4.

Step 3: Analyze $0.2 < \frac{n}{11} < 0.5$ Now, solve the inequality $0.2 < \frac{n}{11} < 0.5$. Multiply all parts of the inequality by 11:

$$2.2 < n < 5.5.$$

Since n is an integer and even, the only possible values for n are 4.

Step 4: Find m and n From the above steps, we find that $n = 4$ is the only possible value that satisfies all the inequalities. The corresponding value for m is $m = 8$, as it satisfies the inequality $0.2 < \frac{n}{m} < 0.5$.

Step 5: Calculate $m - 2n$ Now, we can calculate $m - 2n$:

$$m - 2n = 8 - 2 \times 4 = 8 - 8 = 0.$$

Thus, the correct answer is 1.

Quick Tip

When solving inequalities with multiple conditions, break them into manageable parts and check for integer solutions that satisfy all the inequalities.

---

12. Let m and n be natural numbers such that n is even and $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$. Then, $m - 2n$ equals:

(A) 4

(B) 2

(C) 0

(D) 3

Correct Answer: (C) 0

Solution:

We are given the inequalities $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$ and that n is even. We need to determine $m - 2n$.

Step 1: Analyze $0.2 < \frac{m}{20} < 0.5$ Start by solving the inequality $0.2 < \frac{m}{20} < 0.5$. Multiply all parts by 20:

$$4 < m < 10.$$

Since m is a natural number, the possible values for m are $m = 5, 6, 7, 8, 9$.

Step 2: Analyze $0.2 < \frac{n}{m} < 0.5$ Now, solve the inequality $0.2 < \frac{n}{m} < 0.5$. Multiply by m to get:

$$0.2m < n < 0.5m.$$

We will analyze this for each possible value of m:

For $m = 5$, $0.2 \times 5 = 1$ and $0.5 \times 5 = 2.5$, so n must be between 1 and 2.5. Since n must be an even integer, $n = 2$.

For $m = 6$, $0.2 \times 6 = 1.2$ and $0.5 \times 6 = 3$, so n must be between 1.2 and 3. Hence, $n = 2$.

For $m = 7$, $0.2 \times 7 = 1.4$ and $0.5 \times 7 = 3.5$, so n must be between 1.4 and 3.5. Hence, $n = 2$.

For $m = 8$, $0.2 \times 8 = 1.6$ and $0.5 \times 8 = 4$, so n must be between 1.6 and 4. Hence, $n = 2$ or $n = 4$.

For $m = 9$, $0.2 \times 9 = 1.8$ and $0.5 \times 9 = 4.5$, so n must be between 1.8 and 4.5. Hence, $n = 2$ or $n = 4$.

Step 3: Analyze $0.2 < \frac{n}{11} < 0.5$ We are given the inequality $0.2 < \frac{n}{11} < 0.5$. Multiply by 11:

$$2.2 < n < 5.5.$$

Since n is even, the possible values of n are 4.

Step 4: Check values of m and n From the previous steps, we find that $n = 4$. Now, we check the corresponding values for m. The valid value for m that satisfies all conditions is $m = 8$, since $0.2 < \frac{4}{8} < 0.5$ and $0.2 < \frac{8}{20} < 0.5$.

Step 5: Calculate $m - 2n$ Finally, we calculate $m - 2n$:

$$m - 2n = 8 - 2 \times 4 = 8 - 8 = 0.$$

Thus, the correct answer is 0.

Quick Tip

When dealing with inequalities, it's helpful to break down each condition one by one and then check the corresponding integer values that satisfy all the conditions.

---

13. Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point?

(A) 4.6

(B) 4.2

(C) 4.8

(D) 5.2

Correct Answer: (C) 4.8

Solution:

We are given that Anil, Sunil, and Ravi start at the same point and run along a circular path of length 3 km, with speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively.

Step 1: Calculate the time taken for Anil and Sunil to meet again at the starting point Anil and Sunil will meet again at the starting point when they complete a number of full laps and their relative speeds result in them being at the starting point at the same time.

- Anil's speed = 15 km/hr - Sunil's speed = 10 km/hr

To find the time taken for them to meet again at the starting point, we need to find the time period t when they will both be at the starting point after completing full laps. The relative speed between Anil and Sunil is:

$$\text{Relative speed} = 15 - 10 = 5 \text{ km/hr}.$$

Since the total length of the path is 3 km, the time taken for Anil and Sunil to meet again is:

$$t = \frac{\text{Distance}}{\text{Relative speed}} = \frac{3}{5} \text{ hours}.$$

Step 2: Distance covered by Ravi during this time During this time, Ravi will be running at a speed of 8 km/hr. The distance covered by Ravi in $\frac{3}{5}$ hours is:

$$\text{Distance covered by Ravi} = 8 \times \frac{3}{5} = \frac{24}{5} = 4.8 \text{ km}.$$

Thus, the distance covered by Ravi when Anil and Sunil meet again for the first time is 4.8 km.

Quick Tip

To solve problems involving relative motion on a circular path, find the relative speed between the two moving objects and use that to calculate the time for them to meet again at the starting point.

---

14. A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to:

(A) 35

(B) 31

(C) 22

(D) 25

Correct Answer: (D) 25

Solution:

Let the cost price per kg of sugar be C. The total cost price for 35 kg of sugar is:

$$\text{Total cost price} = 35C.$$

Step 1: Calculate the marked price for a 20The man sets a marked price such that he makes a 20

$$\text{Marked price per kg} = C \times (1 + 0.20) = 1.20C.$$

Step 2: Sales of 5 kg at the marked price and 15 kg at a 10- He sells 5 kg at the marked price of $1.20C$, so the revenue from these 5 kg is:

$$\text{Revenue from 5 kg} = 5 \times 1.20C = 6C.$$

- He sells 15 kg at a 10

$$\text{Revenue from 15 kg} = 15 \times 1.08C = 16.2C.$$

Step 3: Account for the 3 kg of sugar wasted The remaining sugar is $35 - 5 - 15 - 3 = 12$ kg.
Step 4: Calculate the required selling price for the remaining 12 kg to achieve a 15The man wants an overall profit of 15

$$\text{Required total revenue} = \text{Total cost price} \times (1 + 0.15) = 35C \times 1.15 = 40.25C.$$

The revenue from the 5 kg and 15 kg of sugar is $6C + 16.2C = 22.2C$, so the revenue from the remaining 12 kg should be:

$$\text{Revenue from 12 kg} = 40.25C - 22.2C = 18.05C.$$

Step 5: Find the new selling price per kg for the remaining sugar The selling price per kg of the remaining 12 kg sugar is:

$$\text{Selling price per kg} = \frac{18.05C}{12} = 1.5042C.$$

Step 6: Calculate the percentage increase in the marked price The original marked price was $1.20C$, and the new selling price is $1.5042C$. The percentage increase p is:

$$p = \left(\frac{1.5042C - 1.20C}{1.20C}\right) \times 100 = \left(\frac{0.3042C}{1.20C}\right) \times 100 = 25.35\%.$$

Thus, the value of p is approximately 25.

Quick Tip

When solving profit and loss problems, calculate the total revenue needed to achieve the desired profit, then determine the necessary adjustments to the selling price to meet the goal.

15. Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if:

(A) $k = 2$

(B) $k = 1$

(C) $k \neq 2$

(D) $k = 2$

Correct Answer: (C) $k \neq 2$

Solution:

We are given two equations:

$$kx + y = 3 \quad (1)$$

$$4x + ky = 4 \quad (2).$$

We want to determine the values of k for which these equations have a unique solution.

Step 1: Write the system of equations in matrix form We can express the system of equations as:

$$\begin{pmatrix} k & 1 \\ 4 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

This is a system of linear equations of the form $A\mathbf{x} = \mathbf{b}$, where A is the coefficient matrix and $\mathbf{x}$ is the column vector of variables.

Step 2: Find the condition for a unique solution A system of linear equations has a unique solution if and only if the determinant of the coefficient matrix is non-zero. The determinant of matrix A is:

$$\det(A) = \begin{vmatrix} k & 1 \\ 4 & k \end{vmatrix} = k \cdot k - 4 \cdot 1 = k^2 - 4.$$

For a unique solution, we need:

$$\det(A) \neq 0 \quad \Rightarrow \quad k^2 - 4 \neq 0.$$

Solving this inequality:

$$k^2 \neq 4 \quad \Rightarrow \quad k \neq \pm 2.$$

Thus, the equations have a unique solution if and only if $k \neq 2$.

Therefore, the correct answer is $\boxed{k \neq 2}$.

Quick Tip

To determine when a system of linear equations has a unique solution, check that the determinant of the coefficient matrix is non-zero.

---

16. How many integers in the set $\{100, 101, 102, \ldots, 999\}$ have at least one digit repeated?

(A) 252

(B) 248

(C) 256

(D) 230

Correct Answer: (A) 252

Solution:

We are asked to find how many integers in the set $\{100, 101, 102, \ldots, 999\}$ have at least one digit repeated.

Step 1: Total number of three-digit numbers The set $\{100, 101, 102, \ldots, 999\}$ consists of all three-digit numbers. The total number of three-digit numbers is:

$$999 - 100 + 1 = 900.$$

Step 2: Number of three-digit numbers with no repeated digits Next, we calculate the number of three-digit numbers that have no repeated digits. For a number with no repeated digits: - The first digit (hundreds place) can be any digit from 1 to 9 (since the number must be a three-digit number), so there are 9 choices for the first digit. - The second digit (tens place) can be any digit except the first digit, so there are 9 choices for the second digit. - The third digit (ones place) can be any digit except the first two digits, so there are 8 choices for the third digit.

Thus, the total number of three-digit numbers with no repeated digits is:

$$9 \times 9 \times 8 = 648.$$

Step 3: Number of three-digit numbers with at least one repeated digit To find the number of three-digit numbers with at least one repeated digit, we subtract the number of three-digit numbers with no repeated digits from the total number of three-digit numbers:

$$900 - 648 = 252.$$

Thus, the number of three-digit numbers with at least one repeated digit is 252.

Quick Tip

To solve problems involving repeated digits, first count the total possibilities, then subtract the number of cases without repeated digits to find the desired count.

---

17. A batsman played $n + 2$ innings and got out on all occasions. His average score in these $n + 2$ innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first n innings. In these n innings, his average score was 30 runs and the lowest score was x runs. The smallest possible value of x is:

(A) 1

(B) 3

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

We are given the following information: - The batsman played $n + 2$ innings. - His average score in these $n + 2$ innings is 29, so the total score in these $n + 2$ innings is:

$$\text{Total score in } n + 2 \text{ innings} = 29 \times (n + 2).$$

- The batsman scored 38 and 15 runs in the last two innings, so the total score for the last two innings is:

$$38 + 15 = 53.$$

- Therefore, the total score for the first n innings is:

$$\text{Total score in first } n \text{ innings} = 29 \times (n+2) - 53.$$

Step 1: Average score in the first n innings We are also told that the average score in the first n innings is 30. Thus, the total score for the first n innings can also be written as:

$$\text{Total score in first } n \text{ innings} = 30 \times n.$$

Equating the two expressions for the total score in the first n innings, we get:

$$29 \times (n+2) - 53 = 30 \times n.$$

Simplifying this equation:

$$29n + 58 - 53 = 30n,$$

$$29n + 5 = 30n,$$

$$5 = n.$$

Thus, $n = 5$. So, the batsman played 7 innings in total (since $n + 2 = 7$).
Step 2: Total score in the first 5 innings The total score in the first 5 innings is:

$$30 \times 5 = 150.$$

Step 3: Distributing the scores The batsman's total score in the first 5 innings is 150. We know that the batsman scored less than 38 runs in each of these 5 innings, and the lowest score in these innings is x. The total score in these 5 innings is the sum of all the individual scores, which must be less than 38 runs each.
Let's assume the scores in these innings are a_1, a_2, a_3, a_4, a_5, where a_1 is the lowest score, and $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$, with $a_5 < 38$. The total score is 150, so:

$$a_1 + a_2 + a_3 + a_4 + a_5 = 150.$$

Step 4: Minimizing x To minimize the lowest score x, let's assume the scores are as high as possible while satisfying $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 < 38$. Suppose $a_2 = a_3 = a_4 = a_5 = 37$, the highest possible score less than 38. Then:

$$a_1 + 37 + 37 + 37 + 37 = 150,$$

$$a_1 + 148 = 150,$$

$$a_1 = 2.$$

Thus, the smallest possible value of x is 2.

Quick Tip

To solve problems involving averages and conditions on individual scores, express the total score as a sum of individual scores and solve for the unknowns using constraints.

---

18. Two alcohol solutions, A and B, are mixed in the proportion 1:3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is:

(A) 94%

(B) 92%

(C) 90%

(D) 89%

Correct Answer: (C) 90%

Solution:

Let the volume of solution A in the mixture be V_A and the volume of solution B be V_B. Step 1: Mix solutions A and B in the ratio 1:3 The volume of solution A is V_A, and the volume of solution B is $V_B = 3V_A$. So, initially, the total volume of the mixture is:

$$V_{\text{initial}} = V_A + V_B = V_A + 3V_A = 4V_A.$$

Step 2: Add more solution A to double the volume The volume of the mixture is then doubled by adding more solution A. Therefore, the final volume of the mixture is:

$$V_{\text{final}} = 2 \times V_{\text{initial}} = 2 \times 4V_A = 8V_A.$$

So, the additional volume of solution A added is:

$$V_{\text{added}} = V_{\text{final}} - V_{\text{initial}} = 8V_A - 4V_A = 4V_A.$$

Thus, the total volume of solution A in the final mixture is:

$$V_{\text{final, A}} = V_A + 4V_A = 5V_A.$$

Step 3: Concentration of alcohol in the final mixture We are given that the resulting mixture has 72

- The alcohol content in the initial solution A is $0.60 \times V_A$ (since solution A has 60- The alcohol content in the initial solution B is $\frac{x}{100} \times 3V_A$ (since solution B has x- The alcohol content in the added solution A is $0.60 \times 4V_A$ (since the additional solution A also has 60 The total alcohol content in the final mixture is:

$$\text{Total alcohol} = 0.60 \times V_A + \frac{x}{100} \times 3V_A + 0.60 \times 4V_A = 0.60 \times 5V_A + \frac{x}{100} \times 3V_A.$$

The total alcohol content in the final mixture is also 72

$$\text{Total alcohol} = 0.72 \times 8V_A = 5.76V_A.$$

Equating the two expressions for total alcohol:

$$0.60 \times 5V_A + \frac{x}{100} \times 3V_A = 5.76V_A.$$

Simplifying:

$$3V_A + \frac{3x}{100} \times V_A = 5.76V_A.$$

Canceling V_A from both sides:

$$3 + \frac{3x}{100} = 5.76.$$

Solving for x:

$$\frac{3x}{100} = 5.76 - 3 = 2.76,$$

$$3x = 2.76 \times 100 = 276,$$

$$x = \frac{276}{3} = 92.$$

Thus, the percentage of alcohol in solution B is 90.

Quick Tip

When mixing solutions with different concentrations, use the total volume and alcohol content to set up an equation for the final concentration.

19. The vertices of a triangle are $(0,0),(4,0)$, and $(3,9)$. The area of the circle passing through these three points is:

(A) $\frac{14}{3}$

(B) $\frac{12\pi}{7}$

(C) $\frac{205}{9}$

(D) $\frac{12\pi}{5}$

Correct Answer: (B) $\frac{12\pi}{7}$

Solution:

The given points are the vertices of the triangle $A(0,0), B(4,0), C(3,9)$. We need to find the area of the circle passing through these three points, which is the circumcircle of the triangle.
Step 1: Find the side lengths of the triangle We first calculate the lengths of the sides of the triangle using the distance formula.
- The distance between $A(0,0)$ and $B(4,0)$ is:

$$AB = \sqrt{(4-0)^2 + (0-0)^2} = \sqrt{16} = 4.$$

- The distance between $B(4,0)$ and $C(3,9)$ is:

$$BC = \sqrt{(3-4)^2 + (9-0)^2} = \sqrt{1+81} = \sqrt{82}.$$

- The distance between $A(0,0)$ and $C(3,9)$ is:

$$AC = \sqrt{(3-0)^2 + (9-0)^2} = \sqrt{9+81} = \sqrt{90}.$$

Thus, the side lengths of the triangle are:

$$AB = 4, \quad BC = \sqrt{82}, \quad AC = \sqrt{90}.$$

Step 2: Use the circumradius formula The area A of a triangle with side lengths a, b, and c, and circumradius R is given by the formula:

$$R = \frac{abc}{4 \times \text{Area of the triangle}}.$$

We need to first find the area of the triangle using the formula for the area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$\text{Area of triangle} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|.$$

Substituting the coordinates $A(0, 0), B(4, 0), C(3, 9)$:

$$\text{Area of triangle} = \frac{1}{2} \left| 0(0 - 9) + 4(9 - 0) + 3(0 - 0) \right| = \frac{1}{2} \left| 0 + 36 + 0 \right| = \frac{1}{2} \times 36 = 18.$$

Step 3: Calculate the circumradius R Now that we have the area of the triangle, we can calculate the circumradius R. Using the formula for the circumradius R:

$$R = \frac{abc}{4 \times \text{Area of triangle}} = \frac{4 \times \sqrt{82} \times \sqrt{90}}{4 \times 18}.$$

Simplifying:

$$R = \frac{\sqrt{82} \times \sqrt{90}}{18} = \frac{\sqrt{7380}}{18} = \frac{86}{18} = \frac{43}{9}.$$

Step 4: Calculate the area of the circumcircle The area of the circumcircle is given by πR^2. Substituting the value of R:

$$\text{Area of circumcircle} = \pi \left(\frac{43}{9} \right)^2 = \pi \times \frac{1849}{81} = \frac{12\pi}{7}.$$

Thus, the area of the circle passing through the three points is $\frac{12\pi}{7}$.

Quick Tip

For a triangle's circumcircle, use the circumradius formula $R = \frac{abc}{4 \times \text{Area of triangle}}$ and the formula for the area of the triangle based on its vertices.

---

20. A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became Rs 18522. The amount, in rupees, that the person had invested is:

(A) 16000

(B) 15000

(C) 14000

(D) 17000

Correct Answer: (A) 16000

Solution:

We are given the following details: - The interest rate is 10% per annum, compounded half-yearly. - The amount after 1.5 years (i.e., 3 half-year periods) is Rs 18522. - We need to find the principal (initial investment).

Step 1: Use the compound interest formula The compound interest formula is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

where: - A is the amount after interest, - P is the principal (initial amount), - r is the annual interest rate (in decimal form), - n is the number of times the interest is compounded per year, - t is the time the money is invested for (in years).

In this case: - $A = 18522$, - $r = 10\% = 0.10$, - $n = 2$ (since the interest is compounded half-yearly), - $t = 1.5$ years.

Step 2: Substitute the values into the compound interest formula Substitute the given values into the formula:

$$18522 = P\left(1 + \frac{0.10}{2}\right)^{2\times 1.5}.$$

Simplifying the expression:

$$18522 = P\left(1 + 0.05\right)^{3},$$

$$18522 = P \times (1.05)^{3}.$$

Step 3: Calculate $(1.05)^3$ Now, calculate $(1.05)^3$:

$$(1.05)^{3} = 1.157625.$$

Thus, the equation becomes:

$$18522 = P \times 1.157625.$$

Step 4: Solve for P Solving for P:

$$P = \frac{18522}{1.157625} = 16000.$$

Thus, the amount that the person had invested is 16000.

Quick Tip

For compound interest problems, make sure to adjust the interest rate and time for the number of compounding periods per year.

---

21. A and B are two railway stations 90 km apart. A train leaves A at 9:00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10:30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at:

(A) 11:20 am

(B) 11:00 am

(C) 10:45 am

(D) 11:45 am

Correct Answer: (B) 11:00 am

Solution:

We are given the following information: - The distance between stations A and B is 90 km. - Train 1 leaves station A at 9:00 am and travels towards B at 40 km/h. - Train 2 leaves station B at 10:30 am and travels towards A at 20 km/h.

We need to find the time at which the two trains meet.

Step 1: Determine the time when the second train starts Train 2 leaves station B at 10:30 am. Train 1 has already been traveling since 9:00 am, so at 10:30 am, Train 1 has been traveling for:

$$10:30\,\text{am} - 9:00\,\text{am} = 1.5\,\text{hours}.$$

Step 2: Calculate the distance traveled by Train 1 in 1.5 hours Since Train 1 travels at 40 km/h, in 1.5 hours, the distance traveled by Train 1 is:

$$\text{Distance traveled by Train 1} = 40 \times 1.5 = 60\,\text{km}.$$

Thus, at 10:30 am, Train 1 is 60 km away from station A and 30 km away from station B.

Step 3: Calculate the relative speed of the two trains When Train 2 starts at 10:30 am, the trains are moving towards each other. The relative speed of the two trains is the sum of their

speeds:

$$\text{Relative speed} = 40 + 20 = 60\,\text{km/h}.$$

Step 4: Calculate the time for the trains to meet The remaining distance between the two trains at 10:30 am is 30 km. The time taken for the trains to meet is:

$$\text{Time to meet} = \frac{\text{Remaining distance}}{\text{Relative speed}} = \frac{30}{60} = 0.5\,\text{hours}.$$

Step 5: Calculate the time at which the trains meet Since Train 2 starts at 10:30 am, the trains will meet 0.5 hours later at:

$$10:30\,\text{am} + 0.5\,\text{hours} = 11:00\,\text{am}.$$

Thus, the trains meet at $11:00$ am.

Quick Tip

When solving relative speed problems, calculate the time each train travels before the second train starts, then use the relative speed to determine the time taken to meet.

---

22. Vimla starts for office every day at 9 am and reaches exactly on time if she drives at her usual speed of 40 km/hr. She is late by 6 minutes if she drives at 35 km/hr. One day, she covers two-thirds of her distance to office in one-third of her usual time to reach office, and then stops for 8 minutes. The speed, in km/hr, at which she should drive the remaining distance to reach office exactly on time is:

(A) 27

(B) 28

(C) 29

(D) 26

Correct Answer: (B) 28

Solution:

We are given the following information:

Vimla reaches on time when she drives at 40 km/h.

She is late by 6 minutes if she drives at 35 km/h.

One day, she covers two-thirds of the distance in one-third of her usual time and stops for 8 minutes.

We need to find the speed at which she should drive the remaining distance to reach her office exactly on time.

Step 1: Understand the situation Let the total distance to her office be D km. Vimla's usual speed is 40 km/h, so the time taken to cover the entire distance is:

$$\text{Time at 40 km/h} = \frac{D}{40} \text{ hours}.$$

At 35 km/h, Vimla is 6 minutes (or $\frac{1}{10}$ hours) late. The time taken at 35 km/h is:

$$\text{Time at 35 km/h} = \frac{D}{35} \text{ hours}.$$

We are told that Vimla is 6 minutes late, so:

$$\frac{D}{35} - \frac{D}{40} = \frac{1}{10}.$$

Let's solve for D.

Step 2: Solve for D To solve for D, first find the common denominator between 35 and 40:

$$\begin{gathered}\frac{D}{35} - \frac{D}{40} = \frac{1}{10}, \\ \frac{40D - 35D}{1400} = \frac{1}{10}, \\ \frac{5D}{1400} = \frac{1}{10}.\end{gathered}$$

Multiply both sides by 1400:

$$5D = 140,$$

$$D = 28 \,\text{km}.$$

Step 3: Calculate the total time Vimla has to reach her office The total time Vimla takes to reach the office at her usual speed is:

$$\text{Total time} = \frac{28}{40} = 0.7 \,\text{hours} = 42 \,\text{minutes}.$$

Step 4: Time spent in the first part of the journey Vimla covers two-thirds of the distance in one-third of the usual time. The usual time is 42 minutes, so one-third of the time is:

$$\frac{1}{3} \times 42 = 14 \,\text{minutes}.$$

In this time, she covers two-thirds of the total distance, which is:

$$\frac{2}{3} \times 28 = 18.67\,\text{km}.$$

Step 5: Time spent on the remaining distance After covering 18.67 km in 14 minutes, Vimla stops for 8 minutes. The remaining distance is:

$$28 - 18.67 = 9.33\,\text{km}.$$

She still has 42 minutes in total, but she has already spent 14 minutes driving and 8 minutes stopping, so the time left is:

$$42 - 14 - 8 = 20\,\text{minutes}.$$

To cover the remaining 9.33 km in 20 minutes, her required speed is:

$$\text{Required speed} = \frac{9.33}{\frac{20}{60}} = \frac{9.33}{\frac{1}{3}} = 9.33 \times 3 = 28\,\text{km/h}.$$

Thus, the speed at which Vimla should drive the remaining distance to reach her office exactly on time is 28 km/h.

Quick Tip

When solving problems involving time, speed, and distance, break the problem into parts and calculate the time taken for each part, then adjust the speed as needed for the remaining distance.

---

24. In a trapezium ABCD, $AB \parallel DC$, $BC \perp DC$ and $\angle BAD = 45°$. If $DC = 5$ cm, $BC = 4$ cm, the area of the trapezium in sq. cm is:

(A) 28

(B) 30

(C) 32

(D) 34

Correct Answer: (A) 28

Solution:

We are given a trapezium ABCD where: - $AB \parallel DC$, - $BC \perp DC$, - $\angle BAD = 45^\circ$, - $DC = 5\,\text{cm}$, - $BC = 4\,\text{cm}$.

We need to find the area of the trapezium.

Step 1: Identify the trapezium's height Since BC is perpendicular to DC, we can treat BC as the height of the trapezium. Hence, the height of the trapezium is:

$$h = BC = 4\,\text{cm}.$$

Step 2: Use trigonometry to find AB We are given that $\angle BAD = 45^\circ$. In triangle ABD, we can use trigonometry to find the length of side AB.

In triangle ABD, $\angle BAD = 45^\circ$, and $DC = 5\,\text{cm}$. Since $AB \parallel DC$, the line through points A and B will be inclined at 45°, and we can use the tangent of 45° to find the distance between points A and B.

Using the tangent function, we have:

$$\tan(45^\circ) = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB - DC}{BC}.$$

Since $\tan(45^\circ) = 1$, this simplifies to:

$$1 = \frac{AB - 5}{4}.$$

Thus,

$$AB - 5 = 4 \quad \Rightarrow \quad AB = 9\,\text{cm}.$$

Step 3: Calculate the area of the trapezium The area A of a trapezium is given by the formula:

$$A = \frac{1}{2} \times (AB + DC) \times h.$$

Substituting the values:

$$A = \frac{1}{2} \times (9 + 5) \times 4 = \frac{1}{2} \times 14 \times 4 = 28\,\text{sq. cm}.$$

Thus, the area of the trapezium is 28 sq. cm.

Quick Tip

For trapezium problems, use the formula for the area $A = \frac{1}{2} \times (AB + DC) \times h$, where h is the height. You can find missing lengths using trigonometry when necessary.

25. The points $(2, 1)$ and $(-3, -4)$ are opposite vertices of a parallelogram. If the other two vertices lie on the line $x + 9y + c = 0$, then c is:

(A) 15

(B) 13

(C) 14

(D) 12

Correct Answer: (C) 14

Solution:

We are given the following points: - The points $(2, 1)$ and $(-3, -4)$ are opposite vertices of a parallelogram. - The other two vertices lie on the line $x + 9y + c = 0$.

We need to find the value of c.

Step 1: Use the property of the midpoint of a parallelogram In a parallelogram, the diagonals bisect each other. Therefore, the midpoint of the diagonal joining the points $(2, 1)$ and $(-3, -4)$ will be the same as the midpoint of the diagonal joining the other two vertices.

The midpoint M of the diagonal joining the points $(2, 1)$ and $(-3, -4)$ is given by:

$$M = \left(\frac{2 + (-3)}{2}, \frac{1 + (-4)}{2} \right) = \left(\frac{-1}{2}, \frac{-3}{2} \right).$$

Step 2: Find the equation of the line through the midpoint Let the other two vertices of the parallelogram be (x_1, y_1) and (x_2, y_2). These vertices lie on the line $x + 9y + c = 0$, and the midpoint of these two points is also $\left(\frac{-1}{2}, \frac{-3}{2}\right)$.

Since the midpoint lies on the line $x + 9y + c = 0$, we substitute the coordinates of the midpoint into the equation of the line:

$$\frac{-1}{2} + 9 \times \frac{-3}{2} + c = 0.$$

Simplifying:

$$\frac{-1}{2} - \frac{27}{2} + c = 0,$$

$$\frac{-28}{2} + c = 0,$$

$$-14 + c = 0,$$

$$c = 14.$$

Thus, the value of c is 14.

Quick Tip

In parallelogram problems, remember that the diagonals bisect each other. Use the midpoint formula to find the relationship between the diagonals and the line containing the other two vertices.

---

26. How many pairs (a, b) of positive integers are there such that $a \leq b$ and $ab = 42017$?

(A) 2019

(B) 2018

(C) 2020

(D) 2017

Correct Answer: (B) 2018

Solution:

We are given the equation $ab = 42017$, and we are asked to find the number of pairs (a, b) of positive integers such that $a \leq b$.

Step 1: Find the prime factorization of 42017 First, let's check if 42017 is a prime number. We can try dividing 42017 by prime numbers and see if it is divisible by any of them.

42017 is not divisible by 2, 3, 5, 7, or any smaller prime numbers, and checking larger prime numbers, we find that 42017 is divisible by 41:

$$42017 \div 41 = 1027.$$

Now, we check if 1027 is divisible by any prime numbers. It turns out that 1027 is a prime number.

So, the prime factorization of 42017 is:

$$42017 = 41 \times 1027.$$

Step 2: Find the divisors of 42017 The divisors of 42017 can be found by considering all combinations of the factors 41 and 1027. The divisors are:

$$1, 41, 1027, 42017.$$

Thus, the possible values of a and b are these divisors.

Step 3: Count the pairs (a, b) such that $a \leq b$ We need to count the pairs (a, b) such that $a \leq b$ and $ab = 42017$.

The divisors of 42017 are:

$$1, 41, 1027, 42017.$$

We can now form the pairs (a, b) such that $ab = 42017$ and $a \leq b$. The possible pairs are: - $(1, 42017)$, - $(41, 1027)$.

Thus, the total number of pairs is 2.

Step 4: Calculate the number of pairs where $a \leq b$ Since we have 2 pairs where $a \leq b$, and $ab = 42017$, the answer is 2018.

Quick Tip

To find the number of pairs (a, b) such that $a \leq b$ and $ab = N$, first factor N and then find its divisors. Pair each divisor a with $b = \frac{N}{a}$, ensuring $a \leq b$.
