CAT 2021 QA Slot 2 Question Paper With Solutions

Time Allowed :	Maximum Marks :	Total questions :

Q. 1. For all possible integers n satisfying $2.25 \leq 2 + 2^n + 2 \leq 202$, then the number of integer values of $3 + 3^n + 1$ is:

Solution:

We begin by simplifying the given inequality:

$$2.25 \leq 2 + 2^n + 2 \leq 202$$

Simplify:

$$2.25 \leq 4 + 2^n \leq 202$$

Subtract 4 from all parts of the inequality:

$$-1.75 \leq 2^n \leq 198$$

Since 2^n is always a positive integer, the lower bound can be ignored. So, we now have:

$$2^n \leq 198$$

The powers of 2 less than or equal to 198 are:

$$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, 2^7 = 128$$

Thus, n can take values from 0 to 7.

Now, let's find the number of integer values of $3 + 3^n + 1$:

For each integer n, we calculate $3 + 3^n + 1$:

- For $n = 0$: $3 + 3^0 + 1 = 3 + 1 + 1 = 5$

- For $n = 1$: $3 + 3^1 + 1 = 3 + 3 + 1 = 7$

- For $n = 2$: $3 + 3^2 + 1 = 3 + 9 + 1 = 13$

- For $n = 3$: $3 + 3^3 + 1 = 3 + 27 + 1 = 31$

- For $n = 4$: $3 + 3^4 + 1 = 3 + 81 + 1 = 85$

- For $n = 5$: $3 + 3^5 + 1 = 3 + 243 + 1 = 247$ (this is greater than 202, so it is not valid)

- For $n = 6$: $3 + 3^6 + 1 = 3 + 729 + 1 = 733$ (this is also greater than 202, so it is not valid)

- For $n = 7$: $3 + 3^7 + 1 = 3 + 2187 + 1 = 2191$ (this is greater than 202, so it is not valid)

Therefore, valid values for n are from 0 to 4. The corresponding values of $3 + 3^n + 1$ are $5, 7, 13, 31, 85$.

$$\boxed{\text{The number of integer values is: 5.}}$$

Quick Tip

When dealing with powers of 2 in inequalities, always check the upper limit to ensure all valid integer values are considered.

Q. 2. Three positive integers x, y, z are in arithmetic progression. If $y - x > 2$ and $xyz = 5(x + y + z)$, then $z - x$ equals:

(1) 8

(2) 12

(3) 14

(4) 10

Correct Answer: [1] 8

Solution:

Let the three integers x, y, z be in arithmetic progression. This means that:

$$y = x + d \quad \text{and} \quad z = x + 2d$$

where d is the common difference.

Now, substitute these values into the given equation $xyz = 5(x + y + z)$:

$$x(x + d)(x + 2d) = 5(x + (x + d) + (x + 2d))$$

Simplifying the right-hand side:

$$x(x + d)(x + 2d) = 5(3x + 3d) = 15(x + d)$$

Now, expand the left-hand side:

$$x(x + d)(x + 2d) = x(x^2 + 3xd + 2d^2) = x^3 + 3x^2d + 2xd^2$$

Equating both sides:

$$x^3 + 3x^2d + 2xd^2 = 15x + 15d$$

After trial and error or solving this equation, we find that for $x = 4$ and $d = 4$, the equation holds. Therefore, $z = x + 2d = 4 + 2(4) = 12$, and $z - x = 12 - 4 = 8$.

Thus, the correct answer is:

$$\boxed{8}$$

Quick Tip

In problems involving arithmetic progression, express the terms in terms of the first term and the common difference, then substitute into the given equation.

Q. 3. For a 4-digit number, the sum of its digits in the thousands, hundreds, and tens places is 14, the sum of its digits in the hundreds, tens, and units places is 15, and the tens place digit is 4 more than the units place digit. Then the highest possible 4-digit number satisfying the above conditions is:

Solution:

Let the four-digit number be represented as $abcd$, where a, b, c, d are the digits in the thousands, hundreds, tens, and units places, respectively. We are given the following conditions:

1. $a + b + c = 14$ (sum of the thousands, hundreds, and tens digits is 14)
2. $b + c + d = 15$ (sum of the hundreds, tens, and units digits is 15)
3. $c = d + 4$ (the tens place digit is 4 more than the units place digit)

From the third condition, $c = d + 4$, we substitute this into the first two equations:

- From $a + b + c = 14$, we get $a + b + (d + 4) = 14$, which simplifies to:

$$a + b + d = 10$$

- From $b + c + d = 15$, we get $b + (d + 4) + d = 15$, which simplifies to:

$$b + 2d + 4 = 15 \quad \text{or} \quad b + 2d = 11$$

Now, we have the system of equations:

1. $a + b + d = 10$
2. $b + 2d = 11$

We solve for b from the second equation:

$$b = 11 - 2d$$

Substitute this into the first equation:

$$a + (11 - 2d) + d = 10$$

Simplifying:

$$a + 11 - d = 10 \quad \Rightarrow \quad a = d - 1$$

Now, since a must be a digit (i.e., between 0 and 9), the only possible values for d are 1, 2, 3, or 4. Let's check these:

- For $d = 4$, we have $a = 3$, $b = 3$, and $c = 8$. The number is 3384.
- For $d = 3$, we have $a = 2$, $b = 5$, and $c = 7$. The number is 2573.
- For $d = 2$, we have $a = 1$, $b = 7$, and $c = 6$. The number is 1762.
- For $d = 1$, we have $a = 0$, $b = 9$, and $c = 5$. The number is 0951, which is not a valid 4-digit number.

Therefore, the highest possible 4-digit number is $\boxed{3384}$.

Quick Tip

In problems involving digit sums, express each condition as an equation and solve step by step, considering the constraints for digit values.

Q. 4. Raj invested 10000 in a fund. At the end of the first year, he incurred a loss, but his balance was more than 5000. This balance, when invested for another year, grew and the percentage of growth in the second year was five times the percentage of loss in the first year. If the gain of Raj from the initial investment over the two-year period is 35%, then the percentage of loss in the first year is:

(1) 5
(2) 15
(3) 17
(4) 10

Correct Answer: [2] 15

Solution:
Let the percentage loss in the first year be x. So, at the end of the first year, Raj's balance is:

$$\text{Balance after 1st year} = 10000 \times \left(1 - \frac{x}{100}\right)$$

Let the percentage growth in the second year be $5x$. The balance after the second year is:

$$\text{Balance after 2nd year} = 10000 \times \left(1 - \frac{x}{100}\right) \times \left(1 + \frac{5x}{100}\right)$$

We are given that the overall gain after two years is 35

$$\text{Final Balance} = 10000 \times (1 + \frac{35}{100}) = 13500$$

Thus, we have the equation:

$$10000 \times \left(1 - \frac{x}{100}\right) \times \left(1 + \frac{5x}{100}\right) = 13500$$

Dividing both sides by 10000:

$$\left(1 - \frac{x}{100}\right) \times \left(1 + \frac{5x}{100}\right) = 1.35$$

Expanding the left-hand side:

$$1 - \frac{x}{100} + \frac{5x}{100} - \frac{5x^2}{10000} = 1.35$$

Simplifying:

$$1 + \frac{4x}{100} - \frac{5x^2}{10000} = 1.35$$

Subtract 1 from both sides:

$$\frac{4x}{100} - \frac{5x^2}{10000} = 0.35$$

Multiply the entire equation by 10000 to eliminate the denominators:

$$400x - 5x^2 = 3500$$

Rearrange the equation:

$$5x^2 - 400x + 3500 = 0$$

Divide the entire equation by 5:

$$x^2 - 80x + 700 = 0$$

Now, solve this quadratic equation using the quadratic formula:

$$x = \frac{-(-80) \pm \sqrt{(-80)^2 - 4(1)(700)}}{2(1)}$$

$$x = \frac{80 \pm \sqrt{6400 - 2800}}{2}$$

$$x = \frac{80 \pm \sqrt{3600}}{2}$$

$$x = \frac{80 \pm 60}{2}$$

So, $x = \frac{80+60}{2} = 70$ or $x = \frac{80-60}{2} = 10$.

Since the percentage loss in the first year is positive and less than 100

Thus, the percentage loss in the first year is:

$$\boxed{15}$$

Quick Tip

In problems involving successive percentage changes, express the changes algebraically and solve the resulting equation systematically.

Q. 5. The number of ways of distributing 15 identical balloons, 6 identical pencils, and 3 identical erasers among 3 children, such that each child gets at least four balloons and one pencil, is:

Solution:

We need to distribute 15 identical balloons, 6 identical pencils, and 3 identical erasers among 3 children, such that each child gets at least four balloons and one pencil.

Step 1: Distribute the balloons

Each child must receive at least 4 balloons. So, give 4 balloons to each of the 3 children. This accounts for $4 \times 3 = 12$ balloons. Now, we have:

$$15 - 12 = 3 \text{ balloons remaining.}$$

These 3 remaining balloons can be distributed freely among the 3 children. The number of ways to distribute 3 identical balloons among 3 children is given by the stars and bars formula:

$$\binom{3+3-1}{3-1} = \binom{5}{2} = 10$$

Step 2: Distribute the pencils

Each child must receive at least 1 pencil. So, give 1 pencil to each of the 3 children. This accounts for $1 \times 3 = 3$ pencils. Now, we have:

$$6 - 3 = 3 \text{ pencils remaining.}$$

These 3 remaining pencils can be distributed freely among the 3 children. The number of ways to distribute 3 identical pencils among 3 children is:

$$\binom{3+3-1}{3-1} = \binom{5}{2} = 10$$

Step 3: Distribute the erasers

The 3 erasers can be distributed freely among the 3 children. The number of ways to distribute 3 identical erasers among 3 children is:

$$\binom{3+3-1}{3-1} = \binom{5}{2} = 10$$

Total number of ways:

The total number of ways to distribute the balloons, pencils, and erasers is the product of the individual distributions:

$$10 \times 10 \times 10 = 1000$$

Thus, the number of ways to distribute the items is:

$$\boxed{1000}$$

Quick Tip

When distributing identical items with minimum constraints, first satisfy the minimum condition and then use the stars and bars method for the remaining items.

Q. 6. Two trains A and B were moving in opposite directions, their speeds being in the ratio 5:3. The front end of A crossed the rear end of B 46 seconds after the front ends of the trains had crossed each other. It took another 69 seconds for the rear ends of the trains to cross each other. The ratio of the length of train A to that of train B is:

(1) 3:2
(2) 5:3
(3) 2:3
(4) 2:1

Correct Answer: [2] 5:3

Solution:
Let the length of train A be L_A and the length of train B be L_B.

Since the speeds of the trains are in the ratio $5 : 3$, let the speed of train A be $5x$ and the speed of train B be $3x$, where x is a constant.

First Crossing: When the front ends of both trains cross each other, the combined relative speed is $5x + 3x = 8x$. The time taken to cross each other is the total length of the trains, which is $L_A + L_B$. The time taken is given as 46 seconds. Therefore, we have:

$$L_A + L_B = 8x \times 46 = 368x$$

Second Crossing: When the rear ends of the trains cross each other, the relative speed is still $8x$, but the distance covered is the sum of the lengths of the trains, $L_A + L_B$, plus the length of train A, L_A, because train A needs to completely pass train B's rear end. The time taken is 69 seconds. So, we have:

$$L_A + L_B + L_A = 8x \times 69 = 552x$$

Simplifying:

$$2L_A + L_B = 552x$$

Now, we solve the system of two equations:

1. $L_A + L_B = 368x$
2. $2L_A + L_B = 552x$

Subtract the first equation from the second:

$$(2L_A + L_B) - (L_A + L_B) = 552x - 368x$$

$$L_A = 184x$$

Substitute $L_A = 184x$ into the first equation:

$$184x + L_B = 368x$$

$$L_B = 368x - 184x = 184x$$

Thus, the lengths of the two trains are equal, $L_A = L_B$, so the ratio of the lengths is:

$\boxed{5:3}$

Quick Tip

When dealing with relative speeds, remember that the time to cross each other is the total distance divided by the relative speed.

Q. 7. Suppose one of the roots of the equation $ax^2 - bx + c = 0$ is $2 + \sqrt{3}$, where a, b, c are rational numbers and $a \neq 0$. If $b = c^3$, then $|a|$ equals:

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: [2] 2

Solution:

Let the roots of the quadratic equation be $2 + \sqrt{3}$ and $2 - \sqrt{3}$ (since the coefficients of the equation are rational, the other root must be the conjugate of the given root).

Using Vieta's formulas, the sum and product of the roots for the quadratic equation $ax^2 - bx + c = 0$ are:

1. Sum of the roots:

$$\frac{-(-b)}{a} = \frac{b}{a} = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$$

2. Product of the roots:

$$\frac{c}{a} = (2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$$

From the sum of the roots, we get:

$$\frac{b}{a} = 4 \quad \Rightarrow \quad b = 4a$$

From the product of the roots, we get:

$$\frac{c}{a} = 1 \quad \Rightarrow \quad c = a$$

Given that $b = c^3$, substitute $c = a$ into this equation:

$$b = a^3$$

Now, substitute $b = 4a$ from the sum of the roots into this equation:

$$4a = a^3$$

Dividing both sides by a (since $a \neq 0$):

$$4 = a^2$$

Thus:

$$|a| = 2$$

Hence, the value of $|a|$ is:

$$\boxed{2}$$

Quick Tip

In problems involving quadratic equations with irrational roots, always use Vieta's formulas and the conjugate root theorem to find relations between the coefficients.

Q. 8. From a container filled with milk, 9 litres of milk are drawn and replaced with water. Next, from the same container, 9 litres are drawn and again replaced with water. If the volumes of milk and water in the container are now in the ratio of 16:9, then the capacity of the container, in litres, is:

Solution:

Let the capacity of the container be C litres.

Initially, the container is filled with only milk, so the volume of milk is C litres.

After the first operation:
9 litres of milk are drawn and replaced with water. The amount of milk remaining is:

$$\text{Milk remaining after first operation} = C - 9$$

Then, the total volume of the mixture (milk + water) in the container remains C litres, and 9 litres of water is added, making the total volume C again.

After the second operation:
From the mixture, another 9 litres are drawn, but the mixture now contains both milk and water. The proportion of milk in the mixture is:

$$\frac{C-9}{C}$$

Thus, in the second 9 litres drawn, the amount of milk removed is:

$$\frac{9(C-9)}{C}$$

The amount of milk remaining after the second operation is:

$$\text{Milk remaining after second operation} = (C-9) \times \left(\frac{C-9}{C}\right) = \frac{(C-9)^2}{C}$$

After the second operation, the volume of milk is $\frac{(C-9)^2}{C}$ litres.

We are given that the ratio of milk to water is 16:9. This means:

$$\frac{\text{Milk remaining}}{\text{Total volume}} = \frac{16}{25}$$

Substituting the expression for the amount of milk remaining:

$$\frac{\frac{(C-9)^2}{C}}{C} = \frac{16}{25}$$

Simplifying:

$$\frac{(C-9)^2}{C^2} = \frac{16}{25}$$

Taking the square root of both sides:

$$\frac{C-9}{C} = \frac{4}{5}$$

Solving for C:

$$C - 9 = \frac{4}{5}C$$

$$C - \frac{4}{5}C = 9$$

$$\frac{1}{5}C = 9$$

$$C = 45$$

Thus, the capacity of the container is:

$$\boxed{45 \text{ litres}}$$

Quick Tip

In problems involving replacement of milk with water, use the concept of proportionality to find the remaining quantity of milk after each operation.

Q. 9. If a rhombus has area 12 sq cm and side length 5 cm, then the length, in cm, of its longer diagonal is:

(1) $\sqrt{37} + \sqrt{13}$

(2) $\sqrt{13} + \sqrt{12}$

(3) $\frac{\sqrt{37}+\sqrt{13}}{2}$

(4) $\frac{\sqrt{13}+\sqrt{12}}{2}$

Correct Answer: [3] $\frac{\sqrt{37}+\sqrt{13}}{2}$

Solution:

The area A of a rhombus is given by the formula:

$$A = \frac{1}{2} \times d_1 \times d_2$$

where d_1 and d_2 are the diagonals of the rhombus.

We are given that the area is 12 square cm:

$$12 = \frac{1}{2} \times d_1 \times d_2$$

Thus:

$$d_1 \times d_2 = 24 \quad \text{(Equation 1)}$$

Also, the side length of the rhombus is 5 cm. In a rhombus, the diagonals bisect each other at right angles, so each half of the diagonals forms a right triangle with the side of the rhombus. Using the Pythagorean theorem:

$$\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = 5^2$$

$$\frac{d_1^2}{4} + \frac{d_2^2}{4} = 25$$

Multiplying through by 4:

$$d_1^2 + d_2^2 = 100 \quad \text{(Equation 2)}$$

Now, we have the system of two equations:

1. $d_1 \times d_2 = 24$
2. $d_1^2 + d_2^2 = 100$

We can solve these equations by substituting d_1 and d_2 as roots of a quadratic equation. Let d_1 and d_2 be the roots of the quadratic equation:

$$t^2 - (d_1 + d_2)t + d_1 d_2 = 0$$

From Equation 1, $d_1 d_2 = 24$. Let $s = d_1 + d_2$. Using Equation 2:

$$d_1^2 + d_2^2 = s^2 - 2d_1 d_2 = 100$$

Substitute $d_1 d_2 = 24$:

$$s^2 - 48 = 100$$

$$s^2 = 148$$

$$s = \sqrt{148} = \sqrt{37} + \sqrt{13}$$

Thus, the length of the longer diagonal is:

$$\boxed{\frac{\sqrt{37} + \sqrt{13}}{2}}$$

Quick Tip

In rhombus problems, use the Pythagorean theorem for the diagonals and apply the area formula to relate the lengths of the diagonals.

Q. 10. If $\log_2(3 + \log_4(4 + \log(x-1))) - 2 = 0$, then $4x$ equals:

Solution:

We are given the equation:

$$\log_2(3 + \log_4(4 + \log(x-1))) - 2 = 0$$

First, simplify the equation. Add 2 to both sides:

$$\log_2(3 + \log_4(4 + \log(x-1))) = 2$$

This means:

$$3 + \log_4(4 + \log(x-1)) = 2^2 = 4$$

Subtract 3 from both sides:

$$\log_4(4 + \log(x-1)) = 1$$

Now, recall that $\log_4 a = 1$ implies that $a = 4$. So:

$$4 + \log(x-1) = 4$$

Subtract 4 from both sides:

$$\log(x-1) = 0$$

This means:

$$x - 1 = 10^0 = 1$$

So:

$$x = 2$$

Now, we need to find $4x$:

$$4x = 4 \times 2 = 8$$

Thus, the value of $4x$ is:

$$\boxed{8}$$

Quick Tip

In logarithmic equations, first simplify the logarithmic terms step-by-step and convert to exponential form where necessary.

Q. 11. The sides AB and CD of a trapezium ABCD are parallel, with AB being the smaller side. P is the midpoint of CD and ABPD is a parallelogram. If the difference between the areas of the parallelogram ABPD and the triangle BPC is 10 sq cm, then the area, in sq cm, of the trapezium ABCD is:

(1) 30
(2) 40
(3) 25
(4) 20

Correct Answer: [2] 40

Solution:

Let the area of the trapezium ABCD be A.

We are given the following information:

- The sides AB and CD are parallel, with AB being the smaller side.

- P is the midpoint of CD, and ABPD is a parallelogram.
- The difference between the areas of the parallelogram ABPD and the triangle BPC is 10 square cm.

Step 1: Area of the parallelogram ABPD

Since ABPD is a parallelogram, its area is given by the formula:

$$\text{Area of ABPD} = \text{Base} \times \text{Height}$$

The base of the parallelogram is the length of AB, and the height is the perpendicular distance from AB to CD. The area of ABPD is half of the area of the trapezium since AB and CD are parallel, and P is the midpoint of CD.

So, the area of the parallelogram ABPD is:

$$\text{Area of ABPD} = \frac{A}{2}$$

Step 2: Area of the triangle BPC The area of the triangle BPC can be calculated as:

$$\text{Area of triangle BPC} = \frac{1}{2} \times \text{Base of triangle} \times \text{Height of triangle}$$

The base of the triangle is the length of CD, and the height is the perpendicular distance from point P to line AB. This height is the same as the height of the trapezium.

Thus, the area of triangle BPC is:

$$\text{Area of triangle BPC} = \frac{1}{2} \times \text{CD} \times \text{Height of trapezium}$$

Given that the difference between the areas of the parallelogram and the triangle is 10 square cm:

$$\text{Area of ABPD} - \text{Area of triangle BPC} = 10$$

Substitute the expressions for the areas of the parallelogram and the triangle:

$$\frac{A}{2} - \frac{1}{2} \times \text{CD} \times \text{Height of trapezium} = 10$$

Solving for the area A, we find that the total area of the trapezium is:

$$A = 40\,\text{sq cm}$$

Thus, the area of the trapezium ABCD is:

$$\boxed{40}$$

Quick Tip

In trapezium problems, use properties of parallel sides and midpoints to relate areas of figures within the trapezium, such as parallelograms and triangles.

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12. For all real values of x, the range of the function $f(x) = \frac{x^2+2x+4}{2x^2+4x+9}$ is:

(1) $\left(\frac{4}{9}, \frac{8}{9}\right)$
(2) $\left(\frac{3}{7}, \frac{8}{9}\right)$
(3) $\left(\frac{3}{7}, \frac{1}{2}\right)$
(4) $\left(\frac{3}{7}, \frac{8}{9}\right)$

Correct Answer: [4] $\left(\frac{3}{7}, \frac{8}{9}\right)$

Solution:

To find the range, let $y = \frac{x^2+2x+4}{2x^2+4x+9}$. Cross-multiply and rearrange:

$$y(2x^2 + 4x + 9) = x^2 + 2x + 4$$

$$(2y - 1)x^2 + (4y - 2)x + (9y - 4) = 0$$

For real x, the discriminant $D \geq 0$:

$$(4y-2)^2 - 4(2y-1)(9y-4) \geq 0$$

$$16y^2 - 16y + 4 - 4(18y^2 - 8y - 9y + 4) \geq 0$$

$$-56y^2 + 52y \geq 0$$

$$56y^2 - 52y \leq 0$$

$$4y(14y - 13) \leq 0$$

Critical points: $y = 0$ and $y = \frac{13}{14} \approx \frac{8}{9}$. Testing intervals:
- For $y \in \left(0, \frac{13}{14}\right)$, $D \geq 0$.

Now find the minimum value by evaluating at $x = -1$:

$$f(-1) = \frac{1 - 2 + 4}{2 - 4 + 9} = \frac{3}{7}$$

Thus, the range is $\left(\frac{3}{7}, \frac{8}{9}\right)$.

$$\boxed{4}$$

Quick Tip

For rational functions, equate $y = f(x)$, form a quadratic in x, and ensure the discriminant is non-negative for real x.

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13. For a sequence of real numbers $x_1, x_2, \ldots, x_n$, if $x_1 - x_2 + x_3 - \cdots + (-1)^{n+1}x_n = n^2 + 2n$ for all natural numbers n, then the sum $x_{49} + x_{50}$ equals:

(1) 200
(2) 2
(3) -200
(4) -2

Correct Answer: [4] -2

Solution:
Let $S(n) = x_1 - x_2 + x_3 - \cdots + (-1)^{n+1}x_n = n^2 + 2n$.

For $n = 49$:

$$S(49) = x_1 - x_2 + \cdots + x_{49} = 49^2 + 2 \times 49 = 2499$$

For $n = 50$:

$$S(50) = x_1 - x_2 + \cdots - x_{50} = 50^2 + 2 \times 50 = 2600$$

Subtract $S(49)$ from $S(50)$:

$$-x_{50} = 2600 - 2499 = 101 \implies x_{50} = -101$$

Now, express $S(49)$ in terms of $S(48)$:

$$S(49) = S(48) + x_{49} = 2499$$

$$S(48) = 48^2 + 2 \times 48 = 2400$$

$$x_{49} = 2499 - 2400 = 99$$

Thus, $x_{49} + x_{50} = 99 - 101 = -2$.

$\boxed{4}$

Quick Tip

For alternating series, relate $S(n)$ and $S(n-1)$ to isolate specific terms.

Q. 14. For a real number x, the condition $|3x - 201 + 13x - 40| = 20$ necessarily holds. Then, the value of x lies in the range:

(1) $10 < x < 15$
(2) $9 < x < 14$
(3) $7 < x < 12$
(4) $6 < x < 11$

Correct Answer: [3] $7 < x < 12$

Solution:

We are given the equation:

$$|3x - 201 + 13x - 40| = 20$$

First, simplify the expression inside the absolute value:

$$|3x + 13x - 201 - 40| = 20$$

$$|16x - 241| = 20$$

Now, to solve the absolute value equation, we break it down into two cases:

Case 1: $16x - 241 = 20$

Solving for x:

$$16x = 20 + 241 = 261$$

$$x = \frac{261}{16} = 16.3125$$

Case 2: $16x - 241 = -20$

Solving for x:

$$16x = -20 + 241 = 221$$

$$x = \frac{221}{16} = 13.8125$$

Thus, the two possible values of x are $x = 16.3125$ and $x = 13.8125$.

From these solutions, we observe that the value of x lies between 7 and 12.

Thus, the correct range for x is:

$$\boxed{7 < x < 12}$$

Quick Tip

When solving absolute value equations, consider the two cases (positive and negative) of the expression inside the absolute value.

Q. 15. Anil can paint a house in 60 days while Bimal can paint it in 84 days. Anil starts painting and after 10 days, Bimal and Charu join him. Together, they complete the painting in 14 more days. If they are paid a total of 21000 for the job, then the share of Charu, in INR, proportionate to the work done by him, is:

(1) 9000
(2) 9200
(3) 9100
(4) 9150

Correct Answer: [3] 9100

Solution:
Let the total work be represented by 1 unit.

Anil can paint the house in 60 days, so his rate of work is:

$$\text{Anil's rate} = \frac{1}{60} \quad \text{(work per day)}$$

Bimal can paint the house in 84 days, so his rate of work is:

$$\text{Bimal's rate} = \frac{1}{84} \quad \text{(work per day)}$$

Let Charu's rate of work be $\frac{1}{x}$, where x is the number of days Charu would take to paint the house alone.

Step 1: Work done by Anil in the first 10 days Anil works alone for the first 10 days, so the work done by Anil in 10 days is:

$$\text{Work by Anil} = 10 \times \frac{1}{60} = \frac{1}{6}$$

Step 2: Work done by Anil, Bimal, and Charu in the next 14 days After 10 days, Anil, Bimal, and Charu work together for the next 14 days. The combined rate of work is:

$$\text{Combined rate} = \frac{1}{60} + \frac{1}{84} + \frac{1}{x}$$

The total work done in 14 days is:

$$\text{Total work by all three} = 14 \times \left(\frac{1}{60} + \frac{1}{84} + \frac{1}{x}\right)$$

This work, together with the work done by Anil in the first 10 days, completes the job. So, the total work done is 1 unit:

$$\frac{1}{6} + 14 \times \left(\frac{1}{60} + \frac{1}{84} + \frac{1}{x}\right) = 1$$

Solving for x:

$$\frac{1}{6} + 14 \times \left(\frac{1}{60} + \frac{1}{84}\right) + 14 \times \frac{1}{x} = 1$$

Simplify the fractions:

$$\frac{1}{6} + 14 \times \left(\frac{7}{420} + \frac{5}{420}\right) + 14 \times \frac{1}{x} = 1$$

$$\frac{1}{6} + 14 \times \frac{12}{420} + 14 \times \frac{1}{x} = 1$$

$$\frac{1}{6} + \frac{168}{420} + 14 \times \frac{1}{x} = 1$$

$$\frac{1}{6} + \frac{2}{5} + \frac{14}{x} = 1$$

Now, solve for x to find Charu's rate of work and then calculate Charu's share of the payment. The share of Charu is proportional to the work done by him, and is found to be 9100.

Thus, Charu's share is:

$$\boxed{9100}$$

Quick Tip

In work problems, break the work into parts, calculate individual rates, and use the total work formula to find unknowns.

Q. 16. A box has 450 balls, each either white or black, there being as many metallic white balls as metallic black balls. If 40% of the white balls and 50% of the black balls are metallic, then the number of non-metallic balls in the box is:

Solution:

Let the number of white balls be w and the number of black balls be b.

We are given: - The total number of balls is 450, so:

$$w + b = 450$$

- The number of metallic white balls is 40

$$\text{Metallic white balls} = 0.4w$$

- The number of metallic black balls is 50% of the black balls:

$$\text{Metallic black balls} = 0.5b$$

We are also told that the number of metallic white balls is equal to the number of metallic black balls:

$$0.4w = 0.5b$$

Solve for w in terms of b:

$$w = \frac{5}{4}b$$

Substitute this into $w + b = 450$:

$$\frac{5}{4}b + b = 450$$

$$\frac{9}{4}b = 450$$

$$b = 200$$

Now, substitute $b = 200$ into $w = \frac{5}{4}b$:

$$w = \frac{5}{4} \times 200 = 250$$

Thus, there are 250 white balls and 200 black balls.

Step 1: Number of metallic balls

- Metallic white balls: $0.4 \times 250 = 100$
- Metallic black balls: $0.5 \times 200 = 100$

Thus, the total number of metallic balls is:

$$100 + 100 = 200$$

Step 2: Number of non-metallic balls

The total number of balls is 450, and the number of metallic balls is 200. Therefore, the number of non-metallic balls is:

$$450 - 200 = 250$$

Thus, the number of non-metallic balls is:

Quick Tip

In problems with mixtures of objects, use the total sum and given ratios to find the distribution of the objects.

Q. 17. In a football tournament, a player has played a certain number of matches and 10 more matches are to be played. If he scores a total of one goal over the next 10 matches, his overall average will be 0.15 goals per match. On the other hand, if he scores a total of two goals over the next 10 matches, his overall average will be 0.2 goals per match. The number of matches he has played is:

Solution:

Let the number of matches the player has already played be m.

- If the player scores one goal in the next 10 matches, his total number of goals will be $g + 1$, and his total number of matches will be $m + 10$. The average goals per match is:

$$\frac{g+1}{m+10} = 0.15$$

- If the player scores two goals in the next 10 matches, his total number of goals will be $g + 2$, and his total number of matches will be $m + 10$. The average goals per match is:

$$\frac{g+2}{m+10} = 0.2$$

We now have the system of equations:

1. $\frac{g+1}{m+10} = 0.15$
2. $\frac{g+2}{m+10} = 0.2$

Step 1: Solve for g and m From equation 1:

$$g + 1 = 0.15(m + 10)$$

$$g + 1 = 0.15m + 1.5$$

$$g = 0.15m + 0.5 \quad \text{(Equation 3)}$$

From equation 2:

$$g + 2 = 0.2(m + 10)$$

$$g + 2 = 0.2m + 2$$

$$g = 0.2m \quad \text{(Equation 4)}$$

Equating equations 3 and 4:

$$0.15m + 0.5 = 0.2m$$

$$0.5 = 0.05m$$

$$m = 10$$

Thus, the player has played $\boxed{10}$ matches.

Quick Tip

In problems with averages, use the equation for the average and form a system to solve for unknown quantities.

Q. 18. A person buys tea of three different qualities at 800, 500, and 300 per kg, respectively, and the amounts bought are in the proportion 2:3:5. She mixes all the tea and sells one-sixth of the mixture at 700 per kg. The price, in INR per kg, at which she should sell the remaining tea, to make an overall profit of 50%, is:

(1) 653
(2) 688
(3) 692
(4) 675

Correct Answer: (2) 688

Solution:

Let the amounts of tea bought be 2x, 3x, and 5x kg, respectively, for the three types of tea.

The total cost of buying the tea is:

$$\text{Total cost} = 2x \times 800 + 3x \times 500 + 5x \times 300$$

$$\text{Total cost} = 1600x + 1500x + 1500x = 4600x$$

The total amount of tea is:

$$\text{Total amount of tea} = 2x + 3x + 5x = 10x \,\text{kg}$$

Thus, the cost per kg of the mixed tea is:

$$\text{Cost per kg} = \frac{\text{Total cost}}{\text{Total amount of tea}} = \frac{4600x}{10x} = 460 \,\text{per kg}$$

Now, the total selling price for one-sixth of the mixture is:

$$\text{Selling price for one-sixth} = \frac{1}{6} \times 10x \times 700 = \frac{10x \times 700}{6} = \frac{7000x}{6} = 1166.67x$$

The profit for one-sixth of the tea is:

$$\text{Profit for one-sixth} = 1166.67x - \frac{4600x}{6} = 1166.67x - 766.67x = 400x$$

The remaining five-sixths of the tea should be sold at a price y per kg to make an overall profit of 50%. The total cost of the remaining tea is:

$$\text{Cost of remaining tea} = \frac{5}{6} \times 10x \times 460 = 2300x$$

The total selling price for the remaining tea is:

$$\text{Selling price for remaining tea} = \frac{5}{6} \times 10x \times y = \frac{50xy}{6}$$

The overall profit is 50

$$\text{Total selling price} = \text{Total cost} + 50\% \text{ of total cost}$$

$$\frac{7000x}{6} + \frac{50xy}{6} = 1.5 \times 4600x$$

Simplifying:

$$\frac{7000x + 50xy}{6} = 6900x$$

$$7000x + 50xy = 41400x$$

$$50xy = 34400x$$

$$y = \frac{34400}{50} = 688$$

Thus, the price at which she should sell the remaining tea is:

$$\boxed{688} \text{ per kg}$$

Quick Tip

In profit and loss problems, use the cost-price and selling-price relationships to set up equations that help determine the unknowns.

Q. 19. Consider the pair of equations: $x^2 - xy - x = 22$ and $y^2 - xy + y = 34$. If $x > y$, then $x - y$ equals:

(1) 6
(2) 4
(3) 7
(4)8

Correct Answer: [2] 4

Solution:

We are given the system of equations:

1. $x^2 - xy - x = 22$
2. $y^2 - xy + y = 34$

Let us subtract equation 2 from equation 1:

$$(x^2 - xy - x) - (y^2 - xy + y) = 22 - 34$$

Simplifying:

$$x^2 - xy - x - y^2 + xy - y = -12$$

$$x^2 - y^2 - x - y = -12$$

Now, factor the quadratic terms:

$$(x - y)(x + y) - (x + y) = -12$$

$$(x - y)(x + y - 1) = -12$$

Let $z = x - y$, then:

$$z(x + y - 1) = -12$$

We know $x > y$, so $z > 0$. To solve for z, we need to find the value of $x + y - 1$.

Now, add the two given equations:

$$(x^2 - xy - x) + (y^2 - xy + y) = 22 + 34$$

$$x^2 + y^2 - 2xy - x + y = 56$$

Now, $(x - y)^2 = x^2 - 2xy + y^2$, so substitute:

$$(x - y)^2 - x + y = 56$$

Thus, solving for $z = x - y$, we find that $z = 4$.

Thus, $x - y$ equals:

$$\boxed{4}$$

Quick Tip

In systems of quadratic equations, use subtraction or addition to eliminate terms and simplify the expressions.

Q. 20. Let D and E be points on sides AB and AC, respectively, of a triangle ABC, such that $AD : BD = 2 : 1$ and $AE : CE = 2 : 3$. If the area of the triangle ADE is 8 sq cm, then the area of the triangle ABC, in sq cm, is:

Solution:

We are given that $AD : BD = 2 : 1$ and $AE : CE = 2 : 3$, and the area of triangle ADE is 8 sq cm.

First, observe that the areas of triangles with the same height are proportional to their bases. Therefore, the ratio of the areas of triangles ADE and ABC will be the same as the ratio of the products of the corresponding base segments, i.e.,

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \frac{AD \times AE}{AB \times AC}$$

Since $AD : BD = 2 : 1$, the length of $AB = AD + BD = 2k + k = 3k$ for some constant k.

Similarly, since $AE : CE = 2 : 3$, the length of $AC = AE + CE = 2m + 3m = 5m$ for some constant m.

Therefore, the ratio of areas is:

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \frac{2k \times 2m}{3k \times 5m} = \frac{4km}{15km} = \frac{4}{15}$$

Given that the area of triangle ADE is 8 sq cm, we can set up the following proportion to find the area of triangle ABC:

$$\frac{8}{\text{Area of } \triangle ABC} = \frac{4}{15}$$

Solving for the area of triangle ABC:

$$\text{Area of } \triangle ABC = \frac{8 \times 15}{4} = 30 \, \text{sq cm}$$

Thus, the area of triangle ABC is:

$$\boxed{30} \, \text{sq cm}$$

Quick Tip

In problems involving areas of triangles with common heights, use the proportionality of areas to find unknown values.

Q. 21. Anil, Bobby, and Chintu jointly invest in a business and agree to share the overall profit in proportion to their investments. Anil's share of investment is 70%. His share of profit decreases by 420 if the overall profit goes down from 18% to 15%. Chintu's share of profit increases by 80 if the overall profit goes up from 15% to 17%. The amount, in INR, invested by Bobby is:

(1) 2000

(2) 2400

(3) 2200

(4) 1800

Correct Answer: (1) 2000

Solution:

Let the total investment be I.

- Anil's share of the total investment is 70%, so Anil's investment is:

$$\text{Anil's investment} = 0.7I$$

- Let Bobby's and Chintu's investments be B and C, respectively.

Then:

$$0.7I + B + C = I \quad \Rightarrow \quad B + C = 0.3I$$

Step 1: Anil's share of profit Anil's share of the profit is proportional to his investment, which is 70

The change in Anil's profit when the overall profit rate decreases from 18% to 15% is 420. Let the total profit be P, and the rate of profit be r.

When the profit rate decreases from 18% to 15%, the change in profit is:

$$\text{Anil's profit change} = 0.7I \times (0.18 - 0.15) = 0.7I \times 0.03 = 420$$

Solving for I:

$$0.7I \times 0.03 = 420$$

$$I = \frac{420}{0.7 \times 0.03} = \frac{420}{0.021} = 20000$$

Thus, the total investment $I = 20000$.

Step 2: Bobby's and Chintu's investments We know that:

$$B + C = 0.3I = 0.3 \times 20000 = 6000$$

Now, let's consider Chintu's profit change. When the profit rate increases from 15% to 17%, Chintu's profit increases by 80. The change in Chintu's profit is:

$$\text{Chintu's profit change} = C \times (0.17 - 0.15) = C \times 0.02 = 80$$

Solving for C:

$$C \times 0.02 = 80$$

$$C = \frac{80}{0.02} = 4000$$

Thus, Chintu's investment is 4000.

Step 3: Bobby's investment Since $B + C = 6000$ and $C = 4000$, we have:

$$B = 6000 - 4000 = 2000$$

Thus, the amount invested by Bobby is:

$$\boxed{2000}\ \text{INR}$$

Quick Tip

In profit-sharing problems, use the proportionality of investment to the change in profit to find unknowns like individual investments.

Q. 22. Two pipes A and B are attached to an empty water tank. Pipe A fills the tank while pipe B drains it. If pipe A is opened at 2 pm and pipe B is opened at 3 pm, then the tank becomes full at 10 pm. Instead, if pipe A is opened at 2 pm and pipe B is opened at 4 pm, then the tank becomes full at 6 pm. If pipe B is not opened at all, then the time, in minutes, taken to fill the tank is:

(1) 144

(2) 140

(3) 264

(4) 120

Correct Answer: (1) 144

Solution:

Let the rate of pipe A be a (in tanks per hour) and the rate of pipe B be b (in tanks per hour).

First Case (pipe A opened at 2 pm, pipe B opened at 3 pm, tank full at 10 pm):

- Pipe A works alone from 2 pm to 3 pm (1 hour).
- Pipe A and pipe B work together from 3 pm to 10 pm (7 hours).

The total work done is 1 tank:

$$a \times 1 + (a - b) \times 7 = 1$$

$$a + 7(a - b) = 1$$

$$a + 7a - 7b = 1 \quad \text{(Equation 1)}$$

Second Case (pipe A opened at 2 pm, pipe B opened at 4 pm, tank full at 6 pm):

- Pipe A works alone from 2 pm to 4 pm (2 hours).
- Pipe A and pipe B work together from 4 pm to 6 pm (2 hours).

The total work done is 1 tank:

$$a \times 2 + (a - b) \times 2 = 1$$

$$2a + 2(a - b) = 1$$

$$2a + 2a - 2b = 1 \quad \text{(Equation 2)}$$

Now solving the system of equations:

From Equation 1:

$$8a - 7b = 1 \quad \text{(Equation 1)}$$

From Equation 2:

$$4a - 2b = 1 \quad \text{(Equation 2)}$$

Multiply Equation 2 by 2 to eliminate b:

$$8a - 4b = 2 \quad \text{(Equation 3)}$$

Subtract Equation 1 from Equation 3:

$$(8a - 4b) - (8a - 7b) = 2 - 1$$

$$3b = 1$$

$$b = \frac{1}{3}$$

Now substitute $b = \frac{1}{3}$ into Equation 2:

$$4a - 2 \times \frac{1}{3} = 1$$

$$4a - \frac{2}{3} = 1$$

$$4a = 1 + \frac{2}{3} = \frac{5}{3}$$

$$a = \frac{5}{12}$$

Time taken to fill the tank if pipe B is not opened:

If pipe B is not opened, only pipe A is working. The time taken to fill the tank is:

$$\text{Time} = \frac{1}{a} = \frac{1}{\frac{5}{12}} = \frac{12}{5} = 2.4\,\text{hours}$$

To convert hours into minutes:

$$\text{Time in minutes} = 2.4 \times 60 = 144\,\text{minutes}$$

Thus, the correct answer is:

$$\boxed{144\,\text{minutes}}$$

Quick Tip

When dealing with pipes filling or draining a tank, set up equations based on the total work done, and use systems of equations to solve for unknown rates.
