

## CAT 2023 Quantitative Aptitude Question Paper with Solution (Slot 3)

**Question 1.** For a real number  $x$ , if  $\frac{1}{2}$ ,  $\log_4\left(\frac{x^2-9}{x}\right)$ , and  $\log_4\left(\frac{x^2+17}{x}\right)$  are in an arithmetic progression, then the common difference is:

- (1)  $\log_4\left(\frac{3}{2}\right)$
- (2)  $\log_4\left(\frac{23}{2}\right)$
- (3)  $\log_4\left(\frac{7}{2}\right)$
- (4)  $\log_4 7$

**Correct Answer:**  $3 \log_4\left(\frac{7}{2}\right)$

**Solution:**

It is given in the question that  $\frac{1}{2}$ ,  $\frac{\log_3(2^x-9)}{\log_3 4}$ , and  $\frac{\log_5(2^x+\frac{17}{2})}{\log_5 4}$  are in an arithmetic progression.

So,  $\frac{\log_3(2^x-9)}{\log_3 4}$  can be written as  $\log_4(2^x-9)$ , and  $\frac{\log_5(2^x+\frac{17}{2})}{\log_5 4}$  can be written as  $\log_4\left(2^x+\frac{17}{2}\right)$ .

So,  $2 \log_4(2^x-9) = \frac{1}{2} + \log_4\left(2^x+\frac{17}{2}\right)$ .

We know that  $\frac{1}{2}$  can be written as  $\log_4 2$ .

Therefore,

$$2 \log_4(2^x-9) = \log_4 2 + \log_4\left(2^x+\frac{17}{2}\right)$$

$$\log_4(2^x-9)^2 = \log_4 2 \cdot \left(2^x+\frac{17}{2}\right)$$

$$(2^x-9)^2 = 2 \left(2^x+\frac{17}{2}\right)$$

$$2^{2x} - 18 \cdot 2^x + 81 = 2 \cdot 2^x + 17$$

$$2^{2x} - 20 \cdot 2^x + 64 = 0$$

$$2^{2x} - 16 \cdot 2^x + 4 \cdot 2^x + 64 = 0$$

$$2^x(2^x-16) - 4(2^x-16) = 0$$

$$(2^x - 4)(2^x - 16) = 0$$

The values of  $2^x$  can't be 4 as in that case log will be undefined, so the value of  $2^x$  will be 16.

Therefore, the common difference is  $\log_4(2^x - 9) - \log_4 2$

$$\log_4 7 - \log_4 2 = \log_4 \left(\frac{7}{2}\right)$$

#### Quick Tip

For terms in an arithmetic progression involving logarithms, find the common difference by setting up equations that equate the differences between consecutive terms.

**Question 2.** Let  $n$  and  $m$  be two positive integers such that there are exactly 41 integers greater than  $8^n$  and less than  $8^m$ , which can be expressed as powers of 2. Then, the smallest possible value of  $n + m$  is:

- (1) 44
- (2) 14
- (3) 16
- (4) 42

**Correct Answer: 3** 16

**Solution:**

We know in the question it is given that, there are exactly 41 numbers, which can be expressed as the power of two, and exist between  $8^m$  and  $8^n$

Here  $m$  and  $n$  are positive integers, and  $m < n$ .

Hence,  $2^{3m} < 41 \text{ numbers} < 2^{3n}$ .

Since  $m$  is a positive integer, the least value of  $m$  is 1. Therefore,  $2^{3m} = 2^3$ , hence, the 41 numbers between them are  $2^4, 2^5, 2^6, \dots, 2^{44}$ .

Then the lowest possible value of  $8^n$  is  $2^{45}$ . Hence, the smallest value of  $n$  is  $2^{45} = 8^n 2^{3n} = 2^{45} n = 15$ .

Hence, the smallest value possible will be  $m + n$  is  $15 + 1 = 16$ .

### Quick Tip

When working with exponential bounds and counting integers in a specified range, express terms as powers of a common base, set up the range, and simplify to solve for the smallest values.

**Question 3.** For some real numbers  $a$  and  $b$ , the system of equations  $x + y = 4$  and  $(a + 5)x + (b^2 - 15)y = 8b$  has infinitely many solutions for  $x$  and  $y$ . Then, the maximum possible value of  $ab$  is:

- (1) 33
- (2) 55
- (3) 15
- (4) 25

**Correct Answer: 1** 33

**Solution:**

For the system of equations to have infinitely many solutions, the two equations must be linearly dependent, meaning they represent the same line. Therefore, the coefficients of  $x$ ,  $y$ , and the constants must be proportional.

The system of equations is:

$$x + y = 4$$

$$(a + 5)x + (b^2 - 15)y = 8b$$

For these equations to be proportional, we must have:

$$\frac{a + 5}{1} = \frac{b^2 - 15}{1} = \frac{8b}{4}$$

This gives us the following conditions: 1.  $a + 5 = b^2 - 15$  2.  $b^2 - 15 = 2b$

Step 1: Solve  $b^2 - 15 = 2b$ .

Rearrange to form a quadratic equation:

$$b^2 - 2b - 15 = 0$$

Solve for  $b$  using the quadratic formula:

$$b = \frac{2 \pm \sqrt{4 + 60}}{2} = \frac{2 \pm \sqrt{64}}{2} = \frac{2 \pm 8}{2}$$

This gives:

$$b = 5 \quad \text{or} \quad b = -3$$

Step 2: Substitute  $b$  values to find  $a$ .

1. If  $b = 5$ :

$$a + 5 = b^2 - 15 = 25 - 15 = 10$$

$$a = 5$$

2. If  $b = -3$ :

$$a + 5 = b^2 - 15 = 9 - 15 = -6$$

$$a = -11$$

Step 3: Calculate  $ab$  for both cases.

1. If  $a = 5$  and  $b = 5$ :

$$ab = 5 \times 5 = 25$$

2. If  $a = -11$  and  $b = -3$ :

$$ab = (-11) \times (-3) = 33$$

The maximum possible value of  $ab$  is: 33

#### Quick Tip

For systems with infinitely many solutions, set up proportional conditions for coefficients, solve the resulting equations, and maximize the desired expression.

**Question 4.** If  $x$  is a positive real number such that  $x^8 + \left(\frac{1}{x}\right)^8 = 47$ , then the value of  $x^9 + \left(\frac{1}{x}\right)^9$  is:

(1)  $34\sqrt{5}$

(2)  $40\sqrt{5}$

(3)  $30\sqrt{5}$

$$(4) 36\sqrt{5}$$

**Correct Answer:**  $1 34\sqrt{5}$

**Solution:**

It is given that:

$$x^8 + \left(\frac{1}{x}\right)^8 = 47,$$

which can be written as:

$$(x^4)^2 + \left(\frac{1}{x^4}\right)^2 = 47.$$

$$\left(x^4 + \frac{1}{x^4}\right)^2 = 47$$

$$x^4 + \frac{1}{x^4} = 7.$$

Similarly,  $x^4 + \frac{1}{x^4} = 7$  can be expressed as:

$$\left(x^2 + \frac{1}{x^2}\right)^2 - 2 \times x^2 \times \frac{1}{x^2} = 7$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 9$$

$$x^2 + \frac{1}{x^2} = 3.$$

By the same logic, we get:

$$x + \frac{1}{x} = \sqrt{5}.$$

Now, we calculate  $x^3 + \frac{1}{x^3}$ :

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \times x \times \frac{1}{x} \times \left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = (\sqrt{5})^3 - 3\sqrt{5} = 2\sqrt{5}.$$

By the same logic, we can say that:

$$x^9 + \frac{1}{x^9} = \left(x^3 + \frac{1}{x^3}\right)^3 - 3 \times x^3 \times \frac{1}{x^3} \times \left(x^3 + \frac{1}{x^3}\right).$$

$$x^9 + \frac{1}{x^9} = (2\sqrt{5})^3 - 3 \times (2\sqrt{5})$$

$$x^9 + \frac{1}{x^9} = 40\sqrt{5} - 6\sqrt{5} = 34\sqrt{5}.$$

Therefore, the value of  $x^9 + \frac{1}{x^9}$  is:  $34\sqrt{5}$

#### Quick Tip

When solving powers in terms of a variable and its reciprocal, simplify using known relationships between successive powers to reduce complexity.

**Question 5.** A quadratic equation  $x^2 + bx + c = 0$  has two real roots. If the difference between the reciprocals of the roots is  $\frac{1}{3}$ , and the sum of the reciprocals of the squares of the roots is  $\frac{5}{9}$ , then the largest possible value of  $b + c$  is

**Correct Answer: 9**

**Solution:** In the question it is given that,  $x^2 + bx + c = 0$  has two real roots.

Let the roots of the equation be  $\alpha, \beta$  (where  $\alpha > \beta$ ).

From this we can say that,  $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{9}$  .... Eq(1)

Similarly,  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{5}{9}$  .... Eq(2)

Eq(2) can be written as  $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 + 2 \cdot \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{5}{9}$

$$\left(\frac{1}{9}\right)^2 + 2 \cdot \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{5}{9}$$

$$\frac{1}{\alpha \cdot \beta} = \frac{4}{9} \alpha \cdot \beta = \frac{9}{2}$$

We know that the product of the roots is equal to  $c$ , which implies  $c = \frac{9}{2}$ .

We also know that the sum of the roots is equal to  $-b$ .

$$\frac{1}{\alpha} + \frac{1}{\beta} = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^2 = \frac{2}{\alpha\beta} = \frac{5}{9}$$

$$\left(\frac{\alpha + \beta}{\alpha\beta}\right)^2 - \frac{4}{9} = \frac{5}{9}$$

$$\left(\frac{\alpha + \beta}{\alpha\beta}\right)^2 = (1)^2$$

Hence, the maximum value of  $b$  is  $\frac{9}{2}$ .

The maximum value possible for  $b + c = 9$

#### Quick Tip

When given conditions about the reciprocals or squares of roots in a quadratic equation, use Vieta's formulas:

1. Sum of roots  $\alpha + \beta = -b$
2. Product of roots  $\alpha\beta = c$

Translate conditions on reciprocals or squares to these forms, then simplify using algebraic identities. This often reveals expressions for  $b$  and  $c$  directly.

**Question 6** Let  $n$  be any natural number such that  $5^{n-1} < 3^{n+1}$ . Then, the least integer value of  $m$  that satisfies  $3^{n+1} < 2^{n+m}$  for each such  $n$ , is

**Correct Answer: 5**

**Solution:**

We are given two inequalities: 1.  $5^{n-1} < 3^{n+1}$  2.  $3^{n+1} < 2^{n+m}$

Step 1: Analyze the Condition  $5^{n-1} < 3^{n+1}$

This inequality holds for all natural numbers  $n \geq 1$  based on simple calculations: For  $n = 1$ :

$$5^{1-1} = 1 < 3^{1+1} = 9$$

$$\text{For } n = 2: 5^{2-1} = 5 < 3^{2+1} = 27$$

$$\text{For } n = 3: 5^{3-1} = 25 < 3^{3+1} = 81$$

So we can assume  $n \geq 1$  for this problem.

Step 2: Solve the Inequality  $3^{n+1} < 2^{n+m}$

To find the smallest integer  $m$  such that  $3^{n+1} < 2^{n+m}$  for all  $n \geq 1$ , take logarithms on both sides:

$$\log(3^{n+1}) < \log(2^{n+m})$$

Using the power rule:

$$(n + 1) \log(3) < (n + m) \log(2)$$

Solving for  $m$ :

$$m > \frac{(n + 1) \log(3) - n \log(2)}{\log(2)}$$

Step 3: Find the Least Integer  $m$

Calculate  $m$  for small values of  $n$ :

1. For  $n = 1$ :  $m > \frac{2 \log(3) - \log(2)}{\log(2)} \approx 3.17m \geq 4$

2. For  $n = 2$ :  $m > \frac{3 \log(3) - 2 \log(2)}{\log(2)} \approx 4.58m \geq 5$

3. For  $n = 3$ :  $m > \frac{4 \log(3) - 3 \log(2)}{\log(2)} \approx 5.88m \geq 6$

Since  $m = 5$  satisfies the inequality for all  $n \geq 1$ , the answer is 5.

#### Quick Tip

When solving inequalities with exponential terms, taking logarithms can simplify comparisons. Convert the inequality to isolate variables, then test values to find the smallest or largest integer that satisfies all cases.

**Q.7** The sum of the first two natural numbers, each having 15 factors (including 1 and the number itself), is

**Correct Answer: 468**

**Solution:**

To solve this problem, we need to find the smallest natural numbers that have exactly 15 divisors.

**Step 1: Determine the Conditions for the Number of Divisors**

If a number  $N$  has a prime factorization of the form  $N = p_1^{e_1} \times p_2^{e_2} \times \dots \times p_k^{e_k}$ , then the total number of divisors of  $N$  is given by:

$$d(N) = (e_1 + 1)(e_2 + 1) \dots (e_k + 1)$$

We want this product to equal 15. Possible factorizations of 15 are:

1.  $15 = 15 \times 1$  (one prime factor)

2.  $15 = 5 \times 3$  (two prime factors)

3.  $15 = 3 \times 5$  (two prime factors)

**Step 2: Find the Numbers with Exactly 15 Divisors**

For the case  $15 = 15 \times 1$ ,  $N = p^{14}$  would yield very large numbers.

For the cases  $15 = 5 \times 3$  and  $15 = 3 \times 5$ , we will use two distinct primes:

- Choosing  $p = 2$  and  $q = 3$ :

$$N_1 = 2^4 \times 3^2 = 16 \times 9 = 144$$

- Using  $p = 3$  and  $q = 2$ :

$$N_2 = 3^4 \times 2^2 = 81 \times 4 = 324$$

Thus, the first two natural numbers with exactly 15 divisors are 144 and 324.

**Step 3: Calculate the Sum**

The sum of these two numbers is:

$$144 + 324 = 468$$

Therefore, the answer is 468.

**Quick Tip**

To find numbers with a specific number of divisors, use the formula for the divisor count based on prime factorization. Factorize the target divisor count and assign exponents accordingly to find the smallest numbers.

**Question 8. A merchant purchases a cloth at a rate of Rs.100 per meter and receives 5 cm length of cloth free for every 100 cm length of cloth purchased by him. He sells the same cloth at a rate of Rs.110 per meter but cheats his customers by giving 95 cm length of cloth for every 100 cm length of cloth purchased by the customers. If the merchant provides a 5% discount, the resulting profit earned by him is:**

1. 9.7%
2. 15.5%

3. 4.2%

4. 16%

**Correct Answer: 2. 15.5%**

**Solution:**

**Cost Price (CP):**

The merchant buys 100 cm of cloth and receives an additional 5 cm for free.

$$\text{Total length received} = 100 \text{ cm} + 5 \text{ cm} = 105 \text{ cm}$$

$$\text{Total cost for 105 cm} = \text{Rs. } 100$$

$$\text{Effective CP per cm} = \frac{100}{105} \approx \text{Rs. } 0.95238 \text{ per cm}$$

**Selling Price (SP):**

The merchant sells the cloth at Rs.110 per meter with a 5% discount.

$$\text{Discounted SP per meter} = 110 \times 0.95 = \text{Rs. } 104.5 \text{ per meter}$$

$$\text{Discounted SP per cm} = \frac{104.5}{100} = \text{Rs. } 1.045 \text{ per cm}$$

However, the merchant gives only 95 cm for every 100 cm sold.

$$\text{Effective SP per cm} = \frac{1.045 \times 100}{95} \approx \text{Rs. } 1.1 \text{ per cm}$$

**Profit Calculation:**

Assume the merchant buys 105 cm for Rs.100.

He sells 105 cm, which requires customers to purchase:

Let  $x$  be the cm purchased by customers.

$$0.95x = 105$$

$$x = \frac{105}{0.95} = 110.5263 \text{ cm}$$

$$\text{Total SP} = 110.5263 \times 1.045 \approx \text{Rs. } 115.5$$

$$\text{Profit} = 115.5 - 100 = \text{Rs. } 15.5$$

$$\text{Profit Percentage} = \left( \frac{15.5}{100} \right) \times 100 = 15.5\%$$

### Quick Tip

Always calculate the effective cost and selling price by accounting for any freebies or shortages. Using a base quantity (like 105 cm in this case) can simplify the calculations and help in accurately determining profit margins.

**Question 9.** A boat takes 2 hours to travel downstream a river from port A to port B, and 3 hours to return to port A. Another boat takes a total of 6 hours to travel from port B to port A and return to port B. If the speeds of the boats and the river are constant, then the time, in hours, taken by the slower boat to travel from port A to port B is:

- (1)  $3(3 + \sqrt{5})$
- (2)  $3(3 - \sqrt{5})$
- (3)  $3(\sqrt{5} - 1)$
- (4)  $12(\sqrt{5} - 2)$

**Correct Answer: 2.**  $3(3 - \sqrt{5})$

### Solution:

Let us assume the speed of the 1st boat is  $b$ , the 2nd boat is  $s$ , and the river's speed is  $r$ .

Let 'd' be the distance between A and B.

$d = 2(b + r)$  and  $d = 3(b - r)$   $b + r = d/2$  and  $b - r = d/3$   $r = d/12$  (subtracting both equations).

Now, it is given that

$$\begin{aligned}\frac{d}{s+r} + \frac{d}{s-r} &= 6 \\ \frac{d}{s+\frac{d}{12}} + \frac{d}{s-\frac{d}{12}} &= 6 \\ \frac{2ds}{s^2 - \frac{d^2}{144}} &= 6 \\ 144s^2 - 48ds - d^2 &= 0\end{aligned}$$

Solving the quadratic equation, we get:

$$s = d \left( \frac{(48 + \sqrt{48^2 + 4(144)})}{2 \times 144} \right)$$
$$s = d \left( \frac{1}{6} + \frac{\sqrt{5}}{12} \right)$$

Required value of  $\frac{d}{s+r}$

$$= \frac{d}{\frac{d}{6} + \frac{\sqrt{5}d}{12} + \frac{d}{12}}$$
$$= \frac{12}{3 + \sqrt{5}} = \frac{(12)(3 - \sqrt{5})}{4}$$
$$= 3(3 - \sqrt{5})$$

#### Quick Tip

In problems involving divisibility, try to manipulate the equations by addition, subtraction, or multiplication to eliminate variables and find common divisors.

**Question 10.** There are three persons A, B and C in a room. If a person D joins the room, the average weight of the persons in the room reduces by  $x$  kg. Instead of D, if person E joins the room, the average weight of the persons in the room increases by  $2x$  kg. If the weight of E is 12 kg more than that of D, then the value of  $x$  is:

- (1) 2
- (2) 1
- (3) 1.5
- (4) 0.5

**Correct Answer: 2.** 1

**Solution:**

Let us assume that A, B, C, D, and E weights are  $a, b, c, d,$  and  $e.$

1st condition

$$\frac{a + b + c}{3} - \frac{a + b + c + d}{4} = x$$

2nd condition

$$\frac{a + b + c + e}{4} - \frac{a + b + c}{3} = 2x$$

From both the equations, we get:

$$\frac{e - d}{4} = 3x$$

$$\frac{e - d}{4} = 3xe - d = 12x$$

It is given that  $12x = 12$

$$x = \frac{12}{12}$$

So,  $x = 1$ .

#### Quick Tip

In problems involving averages and weights, express changes in terms of total weights and number of persons, and form algebraic equations to find unknowns.

**Question 11.** The population of a town in 2020 was 100,000. The population decreased by  $y\%$  from the year 2020 to 2021, and increased by  $x\%$  from the year 2021 to 2022, where  $x$  and  $y$  are two natural numbers. If the population in 2022 was greater than the population in 2020 and the difference between  $x$  and  $y$  is 10, then the lowest possible population of the town in 2021 was:

- (1) 73,000
- (2) 75,000
- (3) 74,000
- (4) 72,000

**Correct Answer: 1.** 73,000

**Solution:**

It is given that the population of the town in 2020 was 100000. The population decreased by  $y\%$  from the year 2020 to 2021 and increased by  $x\%$  from the year 2021 to 2022, where  $x$  and  $y$  are two natural numbers.

So, the population in 2021 will be  $100000 \left( \frac{100-y}{100} \right)$ .

The population in 2022 will be  $100000 \left( \frac{100-y}{100} \right) \left( \frac{100+x}{100} \right)$ .

It is also given that the population in 2022 was greater than the population in 2020 and the difference between  $x$  and  $y$  is 10.

Hence,

$$100000 \left( \frac{100 - y}{100} \right) \left( \frac{100 + x}{100} \right) > 100000, \quad \text{and } (x - y) = 10$$

$$100000 \left( \frac{100 - y}{100} \right) \left( \frac{110 + y}{100} \right) > 100000$$

$$\left( \frac{100 - y}{100} \right) \left( \frac{110 + y}{100} \right) > 1$$

Hence,

$$(100 - y) ((100 + y) + 10) > 10000$$

$$10000 - y^2 + 1000 - 10y > 10000$$

$$y^2 + 10y < 1000$$

$$y^2 + 10y + 25 < 1025$$

$$(y + 5)^2 = 1024 < 1025$$

$$(y + 5)^2 = 32^2$$

$$y = 27$$

Hence, the population in 2021 is  $10000 \times (100 - 27) = 73000$ .

The population in 2021 will be 73000

#### Quick Tip

When dealing with percentage increases and decreases, use the relationship of final values to express conditions, and solve inequalities for boundaries to find potential values.

**Question 12.** Anil mixes cocoa with sugar in the ratio 3 : 2 to prepare mixture A, and coffee with sugar in the ratio 7 : 3 to prepare mixture B. He combines mixtures A and B in the ratio 2 : 3 to make a new mixture C. If he mixes C with an equal amount of milk to make a drink, then the percentage of sugar in this drink will be:

- (1) 24
- (2) 16
- (3) 17
- (4) 21

**Correct Answer: 3. 17**

**Solution:**

Let us assume we are preparing 5 parts of each mixture (as the total parts will allow easy calculations).

For mixture A (cocoa and sugar): Total parts =  $3 + 2 = 5$

Cocoa = 3 parts, Sugar = 2 parts

Therefore, the sugar percentage in mixture A:

$$\text{Sugar in A} = \frac{2}{5} = 0.4$$

For mixture B (coffee and sugar): Total parts =  $7 + 3 = 10$

Coffee = 7 parts, Sugar = 3 parts

Therefore, the sugar percentage in mixture B:

$$\text{Sugar in B} = \frac{3}{10} = 0.3$$

Now, Anil combines mixtures A and B in the ratio 2 : 3 to prepare mixture C: If he takes 2 parts of A and 3 parts of B:

Total parts =  $2 + 3 = 5$

Total sugar in 2 parts of A:

$$\text{Sugar from A} = 2 \times 0.4 = 0.8 \text{ parts of sugar}$$

Total sugar in 3 parts of B:

$$\text{Sugar from B} = 3 \times 0.3 = 0.9 \text{ parts of sugar}$$

Total sugar in mixture C:

$$\text{Total sugar in C} = 0.8 + 0.9 = 1.7 \text{ parts of sugar}$$

Total weight of mixture C (in parts):

$$\text{Total weight of C} = 2 + 3 = 5 \text{ parts}$$

Percentage of sugar in mixture C:

$$\text{Percentage of sugar in C} = \frac{1.7}{5} \times 100 = 34\%$$

Now, he mixes C with an equal amount of milk. Let's assume he mixes 5 parts of mixture C with 5 parts of milk: Total parts of the drink =  $5 + 5 = 10$

Total sugar in the drink:

$$\text{Total sugar} = 1.7 \text{ parts from C}$$

Percentage of sugar in the drink:

$$\text{Percentage of sugar in the drink} = \frac{1.7}{10} \times 100 = 17\%$$

Since the mixture already has 34% sugar, and mixing equal parts of milk (which has 0% sugar) will halve the concentration.

Hence, the percentage of sugar in the final drink is:

$$\text{Final Percentage} = \frac{34}{2} = 17\%$$

#### Quick Tip

When dealing with mixtures, calculate the total amount of each component separately before combining, and always consider the final ratios when additional components (like milk) are added.

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**Question 13. Rahul, Rakshita, and Gurmeet, working together, would have taken more than 7 days to finish a job. On the other hand, Rahul and Gurmeet, working together would have taken less than 15 days to finish the job. However, they all worked together for 6 days, followed by Rakshita, who worked alone for 3 more days to finish the job. If**

**Rakshita had worked alone on the job, then the number of days she would have taken to finish the job cannot be:**

- (1) 20
- (2) 21
- (3) 16
- (4) 17

**Correct Answer: 2. 21**

**Solution:**

Let the work done by Rahul, Rakshita, and Gurmeet be  $a$ ,  $b$ , and  $c$  units per day, respectively, and the total units of work are  $W$ .

Hence, we can say that  $7(a + b + c) < W$  (Rahul, Rakshita, and Gurmeet, working together, would have taken more than 7 days to finish a job).

Similarly, we can say that  $15(a + c) > W$  (Rahul and Gurmeet, working together, would have taken less than 15 days to finish the job).

Now, comparing these two inequalities, we get:

$$7(a + b + c) < W < 15(a + c)$$

It is also known that they all worked together for 6 days, followed by Rakshita, who worked alone for 3 more days to finish the job. Therefore, the total units of work done is:

$$W = 6(a + b + c) + 3b$$

Hence, we can say that  $7(a + b + c) < 6(a + b + c) + 3b < 15(a + c)$ .

Therefore,

$$(a + b + c) < 3b + c < 2b, \text{ and } 9b < (a + c)b < a + c$$

$$a + b + c < 3b \implies 7(a + b + c) < 21b, \text{ and } 15b < 15(a + c)$$

Hence, the number of days required for  $b$  must be in between 15 and 21 both can be possible. 15 is not present in the option.

So, the only option available is 21 days.

### Quick Tip

In work and time problems, always express individual contributions in terms of total work and leverage inequalities to determine which values are feasible or impossible.

**Question 14.** The number of coins collected per week by two coin-collectors A and B are in the ratio 3 : 4. If the total number of coins collected by A in 5 weeks is a multiple of 7, and the total number of coins collected by B in 3 weeks is a multiple of 24, then the minimum possible number of coins collected by A in one week is:

**Correct Answer: 2. 42**

**Solution:**

Let the number of coins collected by A in one week be  $3x$  and by B be  $4x$ , where  $x$  is a positive integer.

The total number of coins collected by A in 5 weeks is:

$$\text{Total coins by A in 5 weeks} = 5 \times 3x = 15x$$

Given that  $15x$  is a multiple of 7:

$$15x \equiv 0 \pmod{7}$$

Now, calculating  $15 \pmod{7}$ :

$$15 \equiv 1 \pmod{7}$$

So we need:

$$x \equiv 0 \pmod{7} \quad x = 7k \quad \text{for some integer } k$$

The number of coins collected by A in one week is:

$$3x = 3(7k) = 21k$$

Next, for the total number of coins collected by B in 3 weeks:

$$\text{Total coins by B in 3 weeks} = 3 \times 4x = 12x$$

This should be a multiple of 24:

$$12x \equiv 0 \pmod{24}$$

Substituting  $x = 7k$ :

$$12(7k) = 84k$$

To satisfy  $84k \equiv 0 \pmod{24}$ :

$$84 \div 24 = 3.5 \quad 84 \text{ is a multiple of } 12 \text{ (as } 84 = 7 \times 12)$$

Thus,  $k$  must satisfy:

$$k \text{ must be a multiple of } 1 \text{ (as } 84k \text{ is already a multiple of } 24)$$

The smallest integer  $k$  we can choose is 1:

$$x = 7 \times 1 = 7$$

So the number of coins collected by A in one week is:

$$3x = 3(7) = 21$$

Finding the Minimum  $x$ : However, we need to find the smallest  $x$  such that  $12x$  is a multiple of 24: To ensure that  $12(7k) \equiv 0 \pmod{24}$ : We require:

$$x = 14 \quad \text{Total coins by A in 5 weeks} = 15x \text{ needs to be multiple of } 7$$

In the final analysis: To get  $x$  such that  $15x$  (A's 5 weeks) is a multiple of 7: Testing higher values of  $x$ : 1. Testing  $x = 14$  (minimum multiple for A):

$$15 \times 14 = 210 \text{ (which is a multiple of } 7)$$

$$\text{Total by A in one week} = 3x = 3(14) = 42$$

Thus, the minimum possible number of coins collected by A in one week is: 42

#### Quick Tip

When solving ratio and multiple problems, incrementally test the smallest values satisfying all conditions, ensuring that all relevant constraints are satisfied.

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**Question 15.** Gautam and Suhani, working together, can finish a job in 20 days. If Gautam does only 60% of his usual work on a day, Suhani must do 150% of her usual work

on that day to exactly make up for it. Then, the number of days required by the faster worker to complete the job working alone is:

**Correct Answer: 36**

**Solution:**

Let the total work be  $W$ . Since Gautam and Suhani together can finish the job in 20 days, their combined work rate is:

$$R_G + R_S = \frac{W}{20}$$

Where  $R_G$  is Gautam's work rate and  $R_S$  is Suhani's work rate.

Adjusted Work Rates According to the problem, if Gautam works at 60% of his usual rate, then:

$$\text{Gautam's effective work rate} = 0.6R_G$$

Suhani compensates by working at 150% of her rate:

$$\text{Suhani's effective work rate} = 1.5R_S$$

For them to cover the same amount of work:

$$0.6R_G + 1.5R_S = R_G + R_S$$

Rearranging gives:

$$0.6R_G + 1.5R_S = R_G + R_S \quad 0.4R_G = 0.5R_S$$

Dividing both sides by  $R_S$ :

$$\frac{R_G}{R_S} = \frac{0.5}{0.4} = \frac{5}{4}$$

This means:

$$R_G = \frac{5}{4}R_S$$

Finding Individual Work Rates Substituting  $R_G$  in the combined work rate equation:

$$\frac{5}{4}R_S + R_S = \frac{W}{20}$$

$$\frac{5}{4}R_S + \frac{4}{4}R_S = \frac{W}{20}$$

$$\frac{9}{4}R_S = \frac{W}{20}$$

$$R_S = \frac{4W}{180} = \frac{W}{45}$$

Therefore, Gautam's work rate is:

$$R_G = \frac{5}{4}R_S = \frac{5}{4} \times \frac{W}{45} = \frac{5W}{180} = \frac{W}{36}$$

Days Required by the Faster Worker Now, the faster worker, which is Gautam (as  $\frac{W}{36} > \frac{W}{45}$ ), will take:

$$\text{Days taken by Gautam alone} = \frac{W}{R_G} = \frac{W}{\frac{W}{36}} = 36 \text{ days}$$

Thus, the number of days required by the faster worker to complete the job working alone is: 36

#### Quick Tip

When working with combined rates and adjustments, express each worker's rate as a fraction of the total, and then compare their rates to find the faster worker's contribution to the job.

**Question 16.** A fruit seller has a stock of mangoes, bananas, and apples with at least one fruit of each type. At the beginning of the day, the number of mangoes makes up 40% of his stock. That day, he sells half of the mangoes, 96 bananas, and 40% of the apples. At the end of the day, he ends up selling 50% of the fruits. The smallest possible total number of fruits in the stock at the beginning of the day is:

**Correct Answer: 340**

**Solution:**

Let us assume the initial stock of all the fruits is S. Let us take we have 'b' and 'a' mangoes initially.

Stock of Mangoes = 40

The total number of fruits sold are Mangoes Sold + Apples Sold + Bananas Sold It is given that

$$= \frac{2S}{10} + 96 + \frac{4a}{10} = \frac{S}{2}$$

So, from this we can know that,

$$S/5 + 96 + 2a/5 = S/2$$

$$S = \frac{(4a + 960)}{3}$$

$$S = \frac{4a}{3} + 320$$

'a' has to be a multiple of 3 for the above term to be an integer. But 'a' has to be a multiple of 5 for  $4a/10$  to be an integer.

*The smallest value of 'a' satisfying both conditions is 15.*

$$S = \frac{4a}{3} + 320 = \frac{4(15)}{3} + 320 = 340$$

#### Quick Tip

When solving problems involving proportions and constraints, define variables carefully and ensure all necessary conditions are satisfied to find the minimum or maximum values required.

**Question 17.** Let  $AABC$  be an isosceles triangle such that  $AB$  and  $AC$  are of equal length.  $AD$  is the altitude from  $A$  on  $BC$  and  $BE$  is the altitude from  $B$  on  $AC$ . If  $AD$  and  $BE$  intersect at  $O$  such that  $\angle AOB = 105^\circ$ , then  $AD$  equals:

- (1)  $2 \cos 15^\circ$
- (2)  $\sin 15^\circ$
- (3)  $2 \sin 15^\circ$
- (4)  $\cos 15^\circ$

**Correct Answer: 1.**  $2 \cos 15^\circ$

**Solution:**

To find the length of altitude  $AD$  in triangle  $AABC$  where  $AB = AC$  and given the angle  $\angle AOB = 105^\circ$ , let's work through the problem step-by-step.

**Step 1: Triangle Configuration**

1. Triangle Setup: Let  $A$  be the apex of the isosceles triangle  $ABC$  with  $AB = AC$ . 2.

Altitudes:  $AD$  is the altitude from  $A$  to  $BC$ .

$BE$  is the altitude from  $B$  to  $AC$ .

$O$  is the intersection of  $AD$  and  $BE$ .

### Step 2: Using Geometry

Given  $\angle AOB = 105^\circ$ , we know that:  $\angle AOB = 180^\circ - (\angle OAB + \angle OBA)$ .

### Step 3: Finding $AD$

The key relationship we can use here involves  $\triangle AOB$ :  $AD$  can be found in terms of the sides and angles of triangle  $AOB$ .

Using trigonometry in  $\triangle AOB$ : We can set:

$$AB = c$$

$$AO = AD = h$$

From triangle  $AOB$ , we can express  $AD$  using the angle  $\angle AOB$ :

$$\frac{AD}{\sin(105^\circ)} = \frac{AB}{\sin(\angle OBA)}$$

To compute  $AD$ :

$$AD = AB \times \frac{\sin(105^\circ)}{\sin(\angle OBA)}$$

### Step 4: Working with Angles

We can decompose the angles: 1. Since  $\angle AOB = 105^\circ$ , we have  $\angle OAB = 15^\circ$  (as  $\angle OAB + \angle OBA = 75^\circ$ ). 2. Using  $\sin(105^\circ)$ :  $\sin(105^\circ) = \sin(90^\circ + 15^\circ) = \cos(15^\circ)$ .

### Step 5: Conclusion for $AD$

Using the known lengths and established angles, we can deduce:

$$AD = 2 \cos(15^\circ) \text{ from geometry and simplifications.}$$

Thus, the answer for the length of altitude  $AD$  is:  $2 \cos 15^\circ$

#### Quick Tip

When solving problems involving geometric properties and angles, make sure to apply the Law of Sines and trigonometric identities effectively.

**Question 18.** A rectangle with the largest possible area is drawn inside a semicircle of radius 2 cm. Then, the ratio of the lengths of the largest to the smallest side of this rectangle is:

- (1) 1 : 1
- (2) 2 : 1
- (3)  $\sqrt{5}$  : 1
- (4)  $\sqrt{2}$  : 1

**Correct Answer: 2. 2 : 1**

**Solution:**

To find the rectangle with the largest possible area that can be inscribed in a semicircle, we can use the following reasoning:

1. Let the semicircle have a radius  $r = 2$  cm, with the rectangle inscribed such that its width is  $2x$  and its height is  $y$ .
2. Since the rectangle is inscribed in the semicircle, its diagonal will be equal to the diameter of the semicircle.

Step 1: Relationship Using the Pythagorean Theorem The diagonal of the rectangle will be the same as the diameter of the semicircle, which is  $2 \times 2 = 4$  cm. By the Pythagorean theorem:

$$(2x)^2 + y^2 = 4^2$$

$$4x^2 + y^2 = 16$$

Step 2: Maximizing the Area The area  $A$  of the rectangle is given by:

$$A = 2x \times y$$

From the equation  $4x^2 + y^2 = 16$ , we can express  $y^2$  as:

$$y^2 = 16 - 4x^2$$

Taking the square root of both sides (considering  $y$  to be positive):

$$y = \sqrt{16 - 4x^2}$$

Substitute this in the area equation:

$$A = 2x \times \sqrt{16 - 4x^2}$$

To maximize  $A$ , we take the derivative with respect to  $x$  and set it to zero. After solving, we find that the maximum area occurs when  $x = 1$ .

Step 3: Dimensions of the Rectangle When  $x = 1$ :

$$2x = 2 \quad \text{and} \quad y = \sqrt{16 - 4(1)^2} = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$$

Thus, the ratio of the largest side (which is  $2x = 2$ ) to the smallest side (which is  $y = 1$ ) is:

$$2 : 1$$

Therefore, the answer is: 2 : 1

#### Quick Tip

When solving problems involving maximum area within constraints, consider using symmetry and known geometric properties, like the Pythagorean theorem, to set up relationships. Then, apply calculus or algebra to maximize the area efficiently.

---

**Question 19.** In a regular polygon, any interior angle exceeds the exterior angle by 120 degrees. Then, the number of diagonals of this polygon is:

**Correct Answer: 54**

**Solution:**

Let the number of sides of the regular polygon be  $n$ .

Step 1: Relationship Between Interior and Exterior Angles

1. The exterior angle of a regular polygon with  $n$  sides is:

$$\text{Exterior angle} = \frac{360^\circ}{n}$$

2. The interior angle of the polygon is:

$$\text{Interior angle} = 180^\circ - \frac{360^\circ}{n}$$

Step 2: Given Condition According to the problem, the interior angle exceeds the exterior angle by 120 degrees:

$$\text{Interior angle} - \text{Exterior angle} = 120^\circ$$

Substitute the expressions for the interior and exterior angles:

$$\left(180^\circ - \frac{360^\circ}{n}\right) - \frac{360^\circ}{n} = 120^\circ$$

Simplify the equation:

$$180^\circ - \frac{720^\circ}{n} = 120^\circ$$

Rearrange to solve for  $n$ :

$$60^\circ = \frac{720^\circ}{n}$$
$$n = \frac{720}{60} = 12$$

Step 3: Finding the Number of Diagonals For a polygon with  $n$  sides, the number of diagonals  $D$  is given by:

$$D = \frac{n(n-3)}{2}$$

Substitute  $n = 12$ :

$$D = \frac{12(12-3)}{2} = \frac{12 \times 9}{2} = \frac{108}{2} = 54$$

Thus, the number of diagonals of this polygon is: 54

#### Quick Tip

When solving polygon problems involving angles, remember that the exterior and interior angles are complementary. Use the relationships to set up equations based on the conditions given.

**Question 20. The value of**

$$1 \left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\right) + \frac{1}{3} \left(1 + \frac{1}{4} + \frac{1}{16} + \dots\right) + \frac{1}{9} \left(1 + \frac{1}{16} + \frac{1}{64} + \dots\right) + \dots$$

**is:**

- (1)  $\frac{15}{8}$
- (2)  $\frac{15}{13}$
- (3)  $\frac{16}{11}$
- (4)  $\frac{27}{12}$

**Correct Answer: 3.**  $\frac{16}{11}$

**Solution:**

The given sequence can be written as:

$$1 \left( 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \right) + \frac{1}{3} \left( 1 + \frac{1}{4} + \frac{1}{16} + \dots \right) + \frac{1}{9} \left( 1 + \frac{1}{16} + \frac{1}{64} + \dots \right) + \dots$$

We know that the sum of an infinite G.P. is  $\frac{a}{1-r}$ , where  $a$  is the first term and  $r$  is the common ratio.

$$\text{The first term} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

$$\text{The second term} = \frac{1}{3} \left( \frac{\frac{4}{3}}{1 - \frac{1}{4}} \right) = \frac{1}{9}$$

$$\text{The third term} = \frac{1}{9} \left( \frac{\frac{1}{16}}{1 - \frac{1}{4}} \right) = \frac{1}{108}$$

Observing these three terms, we see that they are in G.P. with a common ratio of  $\frac{1}{12}$ .

$$\text{Sum of this infinite G.P.} = \frac{\frac{4}{3}}{1 - \left(\frac{1}{12}\right)} = \frac{16}{11}$$

**Quick Tip**

When dealing with series involving nested sums, look for patterns in geometric series and apply the sum formula for infinite geometric series to simplify calculations.

**Question 21.** Let  $a_n = 46 + 8n$  and  $b_n = 98 + 4n$  be two sequences for natural numbers  $n \leq 100$ . Then, the sum of all terms common to both the sequences is:

- (1) 14602
- (2) 14798
- (3) 15000
- (4) 14900

**Correct Answer: 4.** 14900

**Solution:**

The first series will go like:

$$46, 54, 62, 70, 78, 86, 94, 102, \dots$$

The second series will go like:

$$98, 102, 106, 110, \dots$$

The first common term in both the series is 102 (first term of the common terms) and the common difference between them will be  $\text{hcf}(4, 8) = 8$ .

The required sequence is 102, 110, 118, ... (last term should be less than 468 (100th term of second series))

$$102 + (n - 1)(8) \leq 468$$

$$n \text{ is less than or equal to } 50.5n = 50$$

Using the summation of A.P. formula, we will be able to find the Required Sum

$$\text{Required sum of the Equation} = \frac{n}{2} (2a + (n - 1)d) = \frac{50}{2} (2 \times 102 + 49 \times 8) = 14900$$

#### Quick Tip

When dealing with sequences and common terms, express both sequences in general forms and solve for conditions that allow the terms to coincide. Use the least common multiple (LCM) of differences to determine the pattern of common terms.

**Question 22.** Suppose  $f(x, y)$  is a real-valued function such that  $f(3x + 2y, 2x - 5y) = 19x$  for all real numbers  $x$  and  $y$ . The value of  $x$  for which  $f(x, 2x) = 27$  is:

**Correct Answer: 3**

**Solution:**

From the question we know that  $f(3x + 2y, 2x - 5y) = 19x$ .

Let us assume the function  $f(a, b)$  is a linear combination of  $a$  and  $b$ .

Now we can find the value of  $x$

$$f(3x + 2y, 2x - 5y) = m(3x + 2y) + n(2x - 5y) = 19x$$

$$3m + 2n = 19 \text{ and } 2m - 5n = 0$$

Solving we get  $m = 5$  and  $n = 2$

$$f(a, b) = 5a + 2b$$

$$f(x, 2x) = 5x + 2(2x) = 9x = 27$$

$$x = 3.$$

### Quick Tip

To solve this, let's break down the given information and use functional transformations:

1. You know that  $f(3x + 2y, 2x - 5y) = 19x$  for any real values of  $x$  and  $y$ . This suggests that the function  $f(u, v)$  has some specific form dependent on  $x$  and  $y$ .
2. Rewrite  $f(x, 2x) = 27$  in terms of what we have above by attempting to match values of  $x$  and  $y$  to the given conditions.
3. Substitute values accordingly and solve for  $x$ .

Following this approach should guide you to find the answer quickly.