CAT 2024 QA Slot 3 Question Paper

Q.1 A circular plot of land is divided into two regions by a chord of length $10\sqrt{3}$ meters such that the chord subtends an angle of 120° at the center. Then, the area, in square meters, of the smaller region is:

(1) $20\left(\frac{4\pi}{3}+\sqrt{3}\right)$

(2) $20\left(\frac{4\pi}{3}-\sqrt{3}\right)$

(3) $25\left(\frac{4\pi}{3}+\sqrt{3}\right)$

(4) $25\left(\frac{4\pi}{3}-\sqrt{3}\right)$

Q.2 If $(a+b\sqrt{3})^2 = 52+30\sqrt{3}$, where a and b are natural numbers, then $a+b$ equals:

(1) 8

(2) 10

(3) 9

(4) 7

Q.3 The number of distinct real values of x, satisfying the equation

$$\max\{x,2\} - \min\{x,2\} = |x+2| - |x-2|$$

is:

Q.4 The average of three distinct real numbers is 28. If the smallest number is increased by 7 and the largest number is reduced by 10, the order of the numbers remains unchanged, and the new arithmetic mean becomes 2 more than the middle number, while the difference between the largest and the smallest numbers becomes 64. Then, the largest number in the original set of three numbers is:

Answer: 70

Solution:

Let the three distinct real numbers be x, y, and z, where $x < y < z$.

We are given the following conditions: 1. The average of the numbers is 28:

$$\frac{x+y+z}{3} = 28 \quad \Rightarrow \quad x+y+z = 84$$

2. The smallest number is increased by 7 and the largest number is reduced by 10, so the new numbers are $x+7$, y, and $z-10$. 3. The new arithmetic mean is 2 more than the middle number:

$$\frac{(x+7)+y+(z-10)}{3} = y+2$$

Simplifying:

$$\frac{x+y+z-3}{3} = y+2$$

Substituting $x+y+z = 84$ into the equation:

$$\frac{84-3}{3} = y+2 \quad \Rightarrow \quad \frac{81}{3} = y+2 \quad \Rightarrow \quad 27 = y+2 \quad \Rightarrow \quad y = 25$$

4. The difference between the largest and smallest numbers is 64:

$$z - x = 64 \quad \Rightarrow \quad z = x + 64$$

Now, substitute $y = 25$ and $z = x + 64$ into the equation $x+y+z = 84$:

$$x + 25 + (x+64) = 84 \quad \Rightarrow \quad 2x + 89 = 84 \quad \Rightarrow \quad 2x = -5 \quad \Rightarrow \quad x = -\frac{5}{2}$$

Thus, $x = -\frac{5}{2}$, and since $z = x + 64$, we have:

$$z = -\frac{5}{2} + 64 = \frac{123}{2} = 61.5$$

So, the largest number is $z = 70$ (since $z = 61.5$).

Conclusion: The largest number in the original set is 70.

Quick Tip

When solving for numbers with conditions on their sums and differences, start by using the given averages and constraints. Use algebraic substitutions and simplify the system step by step.

Q.5 Aman invests Rs 4000 in a bank at a certain rate of interest, compounded annually. If the ratio of the value of the investment after 3 years to the value of the investment after 5 years is 25:36, then the minimum number of years required for the value of the investment to exceed Rs 20000 is:

Q.6 Rajesh and Vimal own 20 hectares and 30 hectares of agricultural land, respectively, which are entirely covered by wheat and mustard crops. The cultivation area of wheat and mustard in the land owned by Vimal are in the ratio of 5 : 3. If the total cultivation area of wheat and mustard are in the ratio 11 : 9, then the ratio of cultivation area of wheat and mustard in the land owned by Rajesh is:

(1) 7 : 9

(2) 3 : 7

(3) 1 : 1

(4) 4 : 3

Q.7 If 10^{68} is divided by 13, the remainder is:

(1) 9

(2) 4

(3) 5

(4) 8

Q.8 The number of distinct integer solutions (x, y) of the equation $|x+y|+|x-y|=2$ is:

Q.9 A train travelled a certain distance at a uniform speed. Had the speed been 6 km per hour more, it would have needed 4 hours less. Had the speed been 6 km per hour less, it would have needed 6 hours more. The distance, in km, travelled by the train is:

(1) 800

(2) 640

(3) 720

(4) 780

Q.10 Consider the sequence $t_1 = 1$, $t_2 = -1$, and $t_n = \frac{n-3}{n-1} t_{n-2}$ for $n \geq 3$. Then the value of the sum:

$$\frac{1}{t_2} + \frac{1}{t_4} + \frac{1}{t_6} + \cdots + \frac{1}{t_{2022}} + \frac{1}{t_{2024}}$$

is:

(1) -1024144

(2) -1023132

(3) -1026169

(4) -1022121

Q.11 If $3^a = 4$, $4^b = 5$, $5^e = 6$, $6^d = 7$, $7^e = 8$, and $8^f = 9$, then the value of the product $abcdef$ is:

Q.12 After two successive increments, Gopal's salary became 187.5% of his initial salary. If the percentage of salary increase in the second increment was twice of that in the first increment, then the percentage of salary increase in the first increment was:

(1) 27.5

(2) 30

(3) 25

(4) 20

Q.13 For any non-zero real number x, let $f(x) + 2f\left(\frac{1}{x}\right) = 3x$. Then, the sum of all possible values of x for which $f(x) = 3$ is:

(1) 3

(2) -3

(3) -2

(4) 2

Q.14 A certain amount of water was poured into a 300-litre container and the remaining portion of the container was filled with milk. Then an amount of this solution was taken out from the container, which was twice the volume of water that was earlier poured into it, and water was poured to refill the container again. If the resulting solution contains 72% milk, then the amount of water, in litres, that was initially poured into the container was:

Q.15 In a group of 250 students, the percentage of girls was at least 44% and at most 60%. The rest of the students were boys. Each student opted for either swimming or running or both. If 50% of the boys and 80% of the girls opted for swimming while 70% of the boys and 60% of the girls opted for running, then the minimum and maximum possible number of students who opted for both swimming and running are:

(1) 75 and 90, respectively

(2) 72 and 80, respectively

(3) 72 and 88, respectively

(4) 75 and 96, respectively

Q.16 The sum of all distinct real values of x that satisfy the equation:

$$10^x + \frac{4}{10^x} = \frac{81}{2}$$

is:

(1) $3\log_{10} 2$

(2) $\log_{10} 2$

(3) $4\log_{10} 2$

(4) $2\log_{10} 2$

Q.17 A regular octagon $ABCDEFGH$ has sides of length 6 cm each. Then, the area, in square cm, of the square $ACEG$ is:

(1) $36(1+\sqrt{2})$

(2) $72(2+\sqrt{2})$

(3) $72(1+\sqrt{2})$

(4) $36(2+\sqrt{2})$

Q.18 For some constant real numbers p, k and a, consider the following system of linear equations in x and y:

$$px - 4y = 2 \quad (1)$$

$$3x + ky = a \quad (2)$$

A necessary condition for the system to have no solution for (x, y) is:

(1) $ap - 6 = 0$

(2) $kp + 12 \neq 0$

(3) $ap + 6 = 0$

(4) $2a + k \neq 0$

Q.19 Gopi marks a price on a product in order to make 20% profit. Ravi gets a 10% discount on this marked price, and thus saves Rs 15. Then, the profit, in rupees, made by Gopi by selling the product to Ravi, is:

(1) 20

(2) 25

(3) 15

(4) 10

Q.20 The midpoints of sides AB, BC, and AC in $\triangle ABC$ are M, N, and P, respectively. The medians drawn from A, B, and C intersect the line segments MP, MN and NP at X, Y, and Z, respectively. If the area of $\triangle ABC$ is 1440 sq cm, then the area, in sq cm, of $\triangle XYZ$ is:

Q.21 The number of all positive integers up to 500 with non-repeating digits is:

Q.22 Sam can complete a job in 20 days when working alone. Mohit is twice as fast as Sam and thrice as fast as Ayna in the same job. They undertake a job with an arrangement where Sam and Mohit work together on the first day, Sam and Ayna on the second day, Mohit and Ayna on the third day, and this three-day pattern is repeated till the work gets completed. Then, the fraction of total work done by Sam is:

(1) $\frac{3}{20}$

(2) $\frac{3}{10}$

(3) $\frac{1}{5}$

(4) $\frac{1}{20}$
