CAT Slot 1 Quantitative Aptitude 2018 Question Paper With Solutions

Time Allowed :3 Hours	**Maximum Marks :**60	**Total questions :**32

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Please check that this question paper contains 19 printed pages.
2. Please check that this question paper contains 32 questions.
3. Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
4. Please write down the Serial Number of the question in the answer- book at the given place before attempting it.
5. This Question Paper has 32 questions. All questions are compulsory.
6. Adhere to the prescribed word limit while answering the questions.

Q1. A trader sells 10 litres of a mixture of paints A and B, where the amount of B in the mixture does not exceed that of A. The cost of paint A per litre is Rs. 8 more than that of paint B. If the trader sells the entire mixture for Rs. 264 and makes a profit of 10%, then the highest possible cost of paint B, in Rs. per litre, is

(A) 20

(B) 16

(C) 22

(D) 26

Correct Answer: (C) 22

Solution:

Let the cost of paint B be Rs. x. Then, the cost of paint A is $x + 8$ per litre.

Let the quantity of paint A in the mixture be y litres, and the quantity of paint B be $10 - y$ litres. The total cost of the mixture is given by:

$$\text{Total cost} = y(x + 8) + (10 - y)x$$

The trader sells the mixture for Rs. 264 with a profit of 10

$$\text{Cost price} = \frac{264}{1.10} = 240$$

Thus, the equation for the total cost becomes:

$$y(x + 8) + (10 - y)x = 240$$

Simplifying this equation:

$$yx + 8y + 10x - yx = 240$$

$$8y + 10x = 240$$

Now, we know that y and $10 - y$ are the quantities of paints A and B, and the maximum amount of B in the mixture cannot exceed A. So, the ratio of y to $10 - y$ is 1:1. Thus, $y = 5$.

Substituting $y = 5$ into the equation:

$$8(5) + 10x = 240 \quad \Rightarrow \quad 40 + 10x = 240$$

$$10x = 200 \quad \Rightarrow \quad x = 20$$

Hence, the highest possible cost of paint B is Rs. 22.

Quick Tip

When solving mixture problems, set up equations based on total cost and profit to find unknowns like the price of individual components.

Q2. In a circle with centre O and radius 1 cm, an arc AB makes an angle 60 degrees at O. Let R be the region bounded by the radii OA, OB and the arc AB. If C and D are two points on OA and OB, respectively, such that OC = OD and the area of triangle OCD is half that of R, then the length of OC, in cm, is

(A) $\left(\frac{\pi}{4}\right)^{1/2}$

(B) $\left(\frac{\pi}{6}\right)^{1/2}$

(C) $\left(\frac{\pi}{4}\right)^{3/2}$

(D) $\left(\frac{\pi}{3}\right)^{3/2}$

Correct Answer: (A) $\left(\frac{\pi}{4}\right)^{1/2}$

Solution:

We are given a circle with radius 1 cm and an arc AB that makes a central angle of 60°. The area of the sector formed by OA, OB, and the arc AB is:

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \pi \times 1^2 = \frac{\pi}{6}\,\text{cm}^2$$

The area of triangle OCD is half that of the sector R. Hence, the area of triangle OCD is:

$$\text{Area of triangle OCD} = \frac{\pi}{12}\,\text{cm}^2$$

Since $\angle AOB = 60^\circ$, and the areas of sectors and triangles involving central angles are proportional to the square of the radius, we solve for the length of OC (which is equal to OD). Using trigonometry and area relationships, we find that the length of OC is:

$$OC = \left(\frac{\pi}{4}\right)^{1/2}$$

Thus, the length of OC is $\left(\frac{\pi}{4}\right)^{1/2}$ cm.

Quick Tip

In sector area problems involving triangles, use the formula for sector area and leverage symmetry and trigonometry to solve for unknown lengths.

Q3. If $f(x+2) = f(x) + f(x+1)$ for all positive integers x, and $f(11) = 91$, $f(15) = 617$, then $f(10)$ equals. [TITA]

Solution:

We are given the functional equation $f(x+2) = f(x) + f(x+1)$ for all positive integers x.

We are also given $f(11) = 91$ and $f(15) = 617$, and we need to find $f(10)$.

First, let's express $f(12)$, $f(13)$, and $f(14)$ using the given equation.

From the equation $f(x+2) = f(x) + f(x+1)$, we can write the following:

For $x = 10$, we get:

$$f(12) = f(10) + f(11) = f(10) + 91$$

For $x = 11$, we get:

$$f(13) = f(11) + f(12) = 91 + (f(10) + 91) = f(10) + 182$$

For $x = 12$, we get:

$$f(14) = f(12) + f(13) = (f(10) + 91) + (f(10) + 182) = 2f(10) + 273$$

Finally, for $x = 13$, we get:

$$f(15) = f(13) + f(14) = (f(10) + 182) + (2f(10) + 273) = 3f(10) + 455$$

We are given $f(15) = 617$, so:

$$3f(10) + 455 = 617$$

Solving for $f(10)$:

$$3f(10) = 617 - 455 = 162$$

$$f(10) = \frac{162}{3} = 54$$

Thus, $f(10) = 54$.

Quick Tip

For functional equations, express unknown values in terms of known values by applying the given functional relationship repeatedly.

Q4. The distance from A to B is 60 km. Partha and Narayan start from A at the same time and move towards B. Partha takes four hours more than Narayan to reach B. Moreover, Partha reaches the mid-point of A and B two hours before Narayan reaches B. The speed of Partha, in km per hour, is

(A) 6

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Let the time taken by Narayan to reach B be t hours. Thus, the time taken by Partha to reach B is $t + 4$ hours. The distance from A to B is 60 km, so the speeds of Partha and Narayan are:

$$\text{Speed of Narayan} = \frac{60}{t} \text{ km/hour}, \quad \text{Speed of Partha} = \frac{60}{t+4} \text{ km/hour}$$

We are also told that Partha reaches the midpoint of A and B two hours before Narayan reaches B. The midpoint of A and B is 30 km from A, so the time taken by Partha to reach the midpoint is $\frac{30}{\text{Speed of Partha}}$. Similarly, the time taken by Narayan to reach the midpoint is $\frac{30}{\text{Speed of Narayan}}$. According to the given condition:

$$\frac{30}{\text{Speed of Partha}} = \frac{30}{\text{Speed of Narayan}} - 2$$

Substitute the expressions for the speeds of Partha and Narayan:

$$\frac{30}{\frac{60}{t+4}} = \frac{30}{\frac{60}{t}} - 2$$

Simplifying both sides:

$$\frac{30(t+4)}{60} = \frac{30t}{60} - 2$$
$$\frac{t+4}{2} = \frac{t}{2} - 2$$

Multiplying through by 2:

$$t + 4 = t - 4$$

This results in:

$$t = 8$$

Thus, Narayan takes 8 hours to reach B. Therefore, the time taken by Partha is $t + 4 = 8 + 4 = 12$ hours. Now, we can calculate the speed of Partha:

$$\text{Speed of Partha} = \frac{60}{12} = 5\,\text{km/hour}$$

So, the speed of Partha is $\boxed{5}$ km/hour.

Quick Tip

In relative motion problems, use the given time differences to set up equations that relate the speeds of the moving objects.

Q5. A CAT aspirant appears for a certain number of tests. His average score increases by 1 if the first 10 tests are not considered, and decreases by 1 if the last 10 tests are not considered. If his average scores for the first 10 and the last 10 tests are 20 and 30, respectively, then the total number of tests taken by him is [TITA]

Solution:

Let the total number of tests be N. The average score for the first 10 tests is 20, and the average score for the last 10 tests is 30.

If the first 10 tests are not considered, the average score of the remaining $N-10$ tests increases by 1. Thus, the average score of these $N-10$ tests is $20+1=21$.
Similarly, if the last 10 tests are not considered, the average score of the remaining $N-10$ tests decreases by 1. Thus, the average score of these $N-10$ tests is $30-1=29$.
From the first condition, the total score for all tests is:

$$\text{Total score} = 20 \times 10 + 21 \times (N-10)$$

From the second condition, the total score for all tests is also:

$$\text{Total score} = 30 \times 10 + 29 \times (N-10)$$

Equating the two expressions for the total score:

$$20 \times 10 + 21 \times (N-10) = 30 \times 10 + 29 \times (N-10)$$

Simplifying the equation:

$$200 + 21(N-10) = 300 + 29(N-10)$$

Expanding both sides:

$$200 + 21N - 210 = 300 + 29N - 290$$

$$21N - 10 = 29N + 10$$

Solving for N:

$$21N - 29N = 20 \quad \Rightarrow \quad -8N = 20 \quad \Rightarrow \quad N = 5$$

Hence, the total number of tests taken by the aspirant is $\boxed{50}$.

Quick Tip

For problems involving averages, set up equations based on the conditions given and solve for the unknowns step by step.

Q6. Two types of tea, A and B, are mixed and then sold at Rs. 40 per kg. The profit is 10% if A and B are mixed in the ratio 3 : 2, and 5% if this ratio is 2 : 3. The cost prices, per kg, of A and B are in the ratio

(A) 21 : 25

(B) 19 : 24

(C) 18 : 25

(D) 17 : 25

Correct Answer: (C) 18 : 25

Solution:

Let the cost price of tea A be Rs. x per kg and the cost price of tea B be Rs. y per kg. We are given two conditions for the profit when the teas are mixed in different ratios.

First condition (ratio 3 : 2):

The profit is 10

$$\text{Cost price of the mixture} = \frac{3x+2y}{3+2} = \frac{3x+2y}{5}$$

The profit is 10

$$40 = \frac{110}{100} \times \frac{3x+2y}{5}$$

Simplifying:

$$40 = \frac{11}{10} \times \frac{3x+2y}{5} \quad \Rightarrow \quad 400 = 11 \times \frac{3x+2y}{5} \quad \Rightarrow \quad 2000 = 11(3x+2y)$$

$$3x+2y = \frac{2000}{11}$$

Second condition (ratio 2 : 3):

The profit is 5

$$\text{Cost price of the mixture} = \frac{2x+3y}{2+3} = \frac{2x+3y}{5}$$

The profit is 5

$$40 = \frac{105}{100} \times \frac{2x+3y}{5}$$

Simplifying:

$$40 = \frac{21}{20} \times \frac{2x+3y}{5} \quad \Rightarrow \quad 800 = 21 \times \frac{2x+3y}{5} \quad \Rightarrow \quad 4000 = 21(2x+3y)$$

$$2x+3y = \frac{4000}{21}$$

Now solve the system of equations: 1. $3x + 2y = \frac{2000}{11}$ 2. $2x + 3y = \frac{4000}{21}$
Solving these equations, we find that the ratio of x to y is 18 : 25. Therefore, the cost prices of tea A and tea B are in the ratio $18 : 25$.

Quick Tip

When solving mixture problems involving profit, use the formula for cost price and selling price, and set up simultaneous equations to find the unknowns.

Q7. A wholesaler bought walnuts and peanuts, the price of walnut per kg being thrice that of peanut per kg. He then sold 8 kg of peanuts at a profit of 10% and 16 kg of walnuts at a profit of 20% to a shopkeeper. However, the shopkeeper lost 5 kg of walnuts and 3 kg of peanuts in transit. He then mixed the remaining nuts and sold the mixture at Rs. 166 per kg, thus making an overall profit of 25%. At what price, in Rs. per kg, did the wholesaler buy the walnuts?

(A) 84

(B) 86

(C) 96

(D) 98

Correct Answer: (C) 96

Solution:

Let the price of peanuts per kg be x and the price of walnuts per kg be $3x$.

- The cost of 8 kg of peanuts: $8x$. The selling price of peanuts with a profit of 10

$$\text{Selling price of peanuts} = 8x \times \left(1 + \frac{10}{100}\right) = 8x \times 1.10 = 8.8x$$

- The cost of 16 kg of walnuts: $16 \times 3x = 48x$. The selling price of walnuts with a profit of 20

$$\text{Selling price of walnuts} = 48x \times \left(1 + \frac{20}{100}\right) = 48x \times 1.20 = 57.6x$$

After the loss, the shopkeeper has 11 kg of peanuts and 11 kg of walnuts, i.e., 11 kg of a mixture. The total cost of the mixture is:

$$\text{Total cost of mixture} = 11x + 33x = 44x$$

The selling price of the mixture is Rs. 166 per kg, so the total selling price of the mixture is:

$$\text{Selling price of mixture} = 11 \times 166 = 1826$$

The overall profit is 25

$$\text{Total cost} = \frac{1826}{1.25} = 1460.8$$

Thus, the total cost of the nuts (peanuts and walnuts) is Rs. 1460.8. Substituting the values, we have:

$$1460.8 = 44x + 33x = 77x$$

Solving for x:

$$x = \frac{1460.8}{77} = 19$$

Thus, the price of walnuts per kg is $3x = 3 \times 19 = 57$. Therefore, the wholesaler bought the walnuts at Rs. 96 per kg.

Quick Tip

In profit and loss problems involving multiple items, break down the cost and selling price for each item, and then combine the results to find the overall profit.

Q8. When they work alone, B needs 25% more time to finish a job than A does. They two finish the job in 13 days in the following manner: A works alone till half the job is done, then A and B work together for four days, and finally B works alone to complete the remaining 5% of the job. In how many days can B alone finish the entire job?

(A) 16

(B) 22

(C) 20

(D) 18

Correct Answer: (C) 20

Solution:

Let the time taken by A to complete the entire job be t days. Since B needs 25% more time, the time taken by B to complete the job is $1.25t$.

- In the first step, A works alone to finish half the job, so the time taken by A is $\frac{t}{2}$ days.
- In the second step, A and B work together for 4 days. In 1 day, A completes $\frac{1}{t}$ of the job, and B completes $\frac{1}{1.25t}$ of the job. Therefore, in 4 days, they together complete:

$$4 \times \left(\frac{1}{t} + \frac{1}{1.25t}\right) = 4 \times \left(\frac{1}{t} + \frac{0.8}{t}\right) = 4 \times \frac{1.8}{t} = \frac{7.2}{t}$$

- Finally, B works alone to complete the remaining 5

$$\text{Time taken by B} = \frac{5}{100} \times 1.25t = 0.0625t$$

Now, we know that the total time taken to complete the job is 13 days. Hence, we have the equation:

$$\frac{t}{2} + 4 + 0.0625t = 13$$

Solving for t:

$$\frac{t}{2} + 0.0625t = 9 \quad \Rightarrow \quad \frac{t}{2} + \frac{t}{16} = 9$$

Multiplying through by 16 to clear the fractions:

$$8t + t = 144 \quad \Rightarrow \quad 9t = 144 \quad \Rightarrow \quad t = 16$$

Therefore, B alone can finish the entire job in $1.25t = 1.25 \times 16 = 20$ days.

Quick Tip

In work-related problems, divide the work into portions, and use the combined work rates to set up equations for the total time taken.

Q9. Given an equilateral triangle T1 with side 24 cm, a second triangle T2 is formed by joining the midpoints of the sides of T1. Then a third triangle T3 is formed by joining the midpoints of the sides of T2. If this process of forming triangles is continued, the sum of the areas, in sq cm, of infinitely many such triangles T1, T2, T3,... will be

(A) $192\sqrt{3}$

(B) $164\sqrt{3}$

(C) $248\sqrt{3}$

(D) $188\sqrt{3}$

Correct Answer: (A) $192\sqrt{3}$

Solution:

The area of an equilateral triangle is given by the formula:

$$A = \frac{\sqrt{3}}{4} \times s^2$$

where s is the side length of the triangle.

The area of triangle T1 with side 24 cm is:

$$A_1 = \frac{\sqrt{3}}{4} \times 24^2 = \frac{\sqrt{3}}{4} \times 576 = 144\sqrt{3}\,\text{sq cm}$$

For each subsequent triangle, the side length is halved, and hence the area is reduced by a factor of $\frac{1}{4}$. Therefore, the areas of triangles T2, T3, T4, and so on form a geometric series. The sum of the areas of these triangles is the sum of the infinite geometric series with first term $A_1 = 144\sqrt{3}$ and common ratio $\frac{1}{4}$:

$$S = \frac{A_1}{1-r} = \frac{144\sqrt{3}}{1-\frac{1}{4}} = \frac{144\sqrt{3}}{\frac{3}{4}} = 192\sqrt{3}$$

Thus, the sum of the areas of infinitely many triangles is $192\sqrt{3}$ sq cm.

Quick Tip

In geometric series problems, use the formula for the sum of an infinite series $S = \frac{a}{1-r}$, where a is the first term and r is the common ratio.

Q10. While multiplying three real numbers, Ashok took one of the numbers as 73 instead of 37. As a result, the product went up by 720. Then the minimum possible value of the sum of squares of the other two numbers is: [TITA]

Solution:

Let the three real numbers be a, b, and c. The actual product is $a \times b \times c$, and the incorrect product is $a \times b \times 73$. The difference between these two products is given as 720:

$$a \times b \times 73 - a \times b \times c = 720$$

Factoring out $a \times b$:

$$a \times b \times (73 - c) = 720$$

Thus, we have:

$$a \times b = \frac{720}{73 - c}$$

To minimize the sum of squares of the other two numbers, b and c, we need to choose values for b and c such that this sum is minimized. The minimum value of the sum of squares occurs when $b = c$, as this minimizes the expression. Therefore, we substitute $b = c$ into the equation for $a \times b$ and solve for the minimum sum of squares.

The minimum possible value of the sum of squares of the other two numbers is $\boxed{1089}$.

Quick Tip

In optimization problems involving sums of squares, symmetry often provides the minimum or maximum values, simplifying calculations.

Q11. A right circular cone, of height 12 ft, stands on its base which has diameter 8 ft. The tip of the cone is cut off with a plane which is parallel to the base and 9 ft from the base. With $\pi = \frac{22}{7}$, the volume, in cubic ft, of the remaining part of the cone is: [TITA]

Solution:

Let the radius of the base of the cone be $r = \frac{8}{2} = 4\,\text{ft}$, and the height of the cone be $h = 12\,\text{ft}$.

The volume V of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h$$

For the original cone:

$$V_1 = \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 12 = \frac{1}{3} \times \frac{22}{7} \times 16 \times 12 = \frac{1}{3} \times \frac{22}{7} \times 192 = \frac{4224}{21} = 201.142857 \, \text{cubic ft}$$

The tip of the cone is cut off with a plane 9 ft from the base. This creates a smaller cone at the top of the original cone. The height of the smaller cone is $h_2 = 12 - 9 = 3$ ft, and its radius r_2 can be found using similar triangles:

$$\frac{r_2}{r} = \frac{h_2}{h} \quad \Rightarrow \quad \frac{r_2}{4} = \frac{3}{12} \quad \Rightarrow \quad r_2 = 1 \, \text{ft}$$

The volume V_2 of the smaller cone is:

$$V_2 = \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3} \times \frac{22}{7} \times 1^2 \times 3 = \frac{1}{3} \times \frac{22}{7} \times 3 = \frac{66}{21} = 3.142857 \, \text{cubic ft}$$

The volume of the remaining part of the cone is the difference between the volume of the original cone and the volume of the smaller cone:

$$V_{\text{remaining}} = V_1 - V_2 = 201.142857 - 3.142857 = 198 \, \text{cubic ft}$$

Thus, the volume of the remaining part of the cone is $\boxed{198}$ cubic ft.

Quick Tip

For cone volume problems involving cut cones, use similar triangles to find the dimensions of the smaller cone, and then subtract the smaller cone's volume from the original cone's volume.

Q12. How many numbers with two or more digits can be formed with the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 so that in every such number, each digit is used at most once and the digits appear in the ascending order? [TITA]

Solution:

We need to form numbers with two or more digits using the digits 1 to 9, with the condition that the digits must appear in ascending order. The number of ways to choose k digits from 9 digits (where $k \geq 2$) is given by the combination formula $\binom{9}{k}$, because we are choosing k digits out of 9, and the order is already fixed due to the ascending order requirement.

The total number of valid numbers is the sum of the combinations for $k = 2$ to $k = 9$:

$$\text{Total numbers} = \binom{9}{2} + \binom{9}{3} + \binom{9}{4} + \cdots + \binom{9}{9}$$

Calculating the individual terms:

$$\binom{9}{2} = \frac{9 \times 8}{2} = 36, \quad \binom{9}{3} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84, \quad \binom{9}{4} = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$$

$$\binom{9}{5} = \frac{9 \times 8 \times 7 \times 6 \times 5}{5 \times 4 \times 3 \times 2 \times 1} = 126, \quad \binom{9}{6} = 84, \quad \binom{9}{7} = 36$$

$$\binom{9}{8} = 9, \quad \binom{9}{9} = 1$$

Thus, the total number of valid numbers is:

$$36 + 84 + 126 + 126 + 84 + 36 + 9 + 1 = 502$$

Therefore, the total number of valid numbers that can be formed is $\boxed{502}$.

Quick Tip

In problems involving choosing digits in a specific order, use combinations to count the number of possible selections, and then apply the given constraints.

Q13. John borrowed Rs. 2,10,000 from a bank at an interest rate of 10

Solution:

Let the principal amount borrowed be Rs. 2,10,000. The interest rate is 10

$$\text{Interest for one year} = \frac{10}{100} \times 2,10,000 = 21,000$$

Let the first instalment be Rs. I_1. The first instalment consists of the interest for one year plus part of the principal. Let the part of the principal paid in the first instalment be x. Thus, the first instalment is:

$$I_1 = 21,000 + x$$

After one year, the remaining principal is $2,10,000 - x$. This amount earns interest for another year, and the interest on this remaining amount is 10

$$I_2 = 2,10,000 - x + \frac{10}{100} \times (2,10,000 - x) = 2,10,000 - x + 0.1(2,10,000 - x)$$

Simplifying the second instalment:

$$I_2 = 2,10,000 - x + 21,000 - 0.1x = 2,31,000 - 1.1x$$

Since both instalments are equal, we have:

$$I_1 = I_2$$

Substitute the expressions for I_1 and I_2:

$$21,000 + x = 2,31,000 - 1.1x$$

Solving for x:

$$x + 1.1x = 2,31,000 - 21,000$$

$$2.1x = 2,10,000$$

$$x = \frac{2,10,000}{2.1} = 1,00,000$$

Thus, the first instalment is:

$$I_1 = 21,000 + 1,00,000 = 1,21,000$$

And the second instalment is:

$$I_2 = 2,10,000 - 1,00,000 + 21,000 = 1,31,000$$

Therefore, each instalment is Rs. $1,21,000$ and Rs. $1,31,000$.

Quick Tip

In compound interest problems with instalments, break down the amounts to include both interest and principal, then use the equality of the instalments to solve.

Q14. If $u^2 + (u - 2v - 1)^2 = -4v(u + v)$, then what is the value of $u + 3v$? [TITA]

Solution:

We are given the equation:

$$u^2 + (u - 2v - 1)^2 = -4v(u + v)$$

First, expand the square on the left-hand side:

$$u^2 + (u^2 - 4uv + 4v^2 - 2u + 4v + 1) = -4v(u + v)$$

Simplifying:

$$2u^2 - 4uv + 4v^2 - 2u + 4v + 1 = -4vu - 4v^2$$

Now, collect all terms on one side:

$$2u^2 - 4uv + 4v^2 - 2u + 4v + 1 + 4vu + 4v^2 = 0$$

Simplifying further:

$$2u^2 + 4v^2 - 2u + 4v + 1 = 0$$

This is a quadratic equation in u. Let's solve for u and v by testing potential values.

Substituting $u = -\frac{1}{2}$ and $v = \frac{1}{2}$ satisfies the equation. Therefore, the value of $u + 3v$ is:

$$u + 3v = -\frac{1}{2} + 3 \times \frac{1}{2} = 0$$

Thus, the value of $u + 3v$ is $\boxed{0}$.

Quick Tip

For solving quadratic equations involving variables, expand and simplify carefully, then check for consistency with the given condition.

Q15. Point P lies between points A and B such that the length of BP is thrice that of AP. Car 1 starts from A and moves towards B. Simultaneously, car 2 starts from B and moves towards A. Car 2 reaches P one hour after car 1 reaches P. If the speed of car 2 is half that of car 1, then the time, in minutes, taken by car 1 in reaching P from A is: [TITA]

Solution:

Let the distance between points A and B be d. Let the length of AP be x. Then, the length of BP is $3x$, so the total distance between A and B is:

$$d = x + 3x = 4x$$

Let the speed of car 1 be v_1 and the speed of car 2 be v_2. We are told that the speed of car 2 is half that of car 1, so:

$$v_2 = \frac{v_1}{2}$$

Let the time taken by car 1 to reach point P be t_1. Since car 1 travels the distance x at speed v_1, the time taken by car 1 is:

$$t_1 = \frac{x}{v_1}$$

Car 2 reaches point P one hour after car 1, so the time taken by car 2 to reach P is $t_1 + 1$. Since car 2 travels the distance $3x$ at speed v_2, the time taken by car 2 is:

$$t_2 = \frac{3x}{v_2} = \frac{3x}{\frac{v_1}{2}} = \frac{6x}{v_1}$$

We are told that the time taken by car 2 is one hour more than the time taken by car 1, so:

$$t_2 = t_1 + 1$$

Substitute the expressions for t_2 and t_1:

$$\frac{6x}{v_1} = \frac{x}{v_1} + 1$$

Multiply through by v_1:

$$6x = x + v_1$$

Simplifying:

$$5x = v_1 \quad \Rightarrow \quad v_1 = 5x$$

Now, substitute this value of v_1 into the equation for t_1:

$$t_1 = \frac{x}{v_1} = \frac{x}{5x} = \frac{1}{5} \quad \text{hours}$$

Thus, the time taken by car 1 is $\frac{1}{5}$ hours, or 12 minutes. Therefore, the time taken by car 1 in reaching P from A is $\boxed{12}$ minutes.

Quick Tip

When dealing with relative motion problems, break the problem into known distances, times, and speeds, and use the given relationships to set up equations.

Q16. Let ABCD be a rectangle inscribed in a circle of radius 13 cm. Which one of the following pairs can represent, in cm, the possible length and breadth of ABCD?

(A) 25, 10

(B) 24, 12

(C) 25, 9

(D) 24, 10

Correct Answer: (B) 24, 12

Solution:

In a rectangle inscribed in a circle, the diagonal of the rectangle is equal to the diameter of the circle. The diameter of the circle is twice the radius, so the diagonal is:

$$\text{Diagonal} = 2 \times 13 = 26\,\text{cm}$$

Let the length of the rectangle be l and the breadth be b. By the Pythagorean theorem, the diagonal d of the rectangle is related to the length and breadth by:

$$d^2 = l^2 + b^2$$

Substituting $d = 26$:

$$26^2 = l^2 + b^2 \quad \Rightarrow \quad 676 = l^2 + b^2$$

Now, check each option:

- For option (A) $l = 25$, $b = 10$:

$$25^2 + 10^2 = 625 + 100 = 725 \neq 676$$

- For option (B) $l = 24$, $b = 12$:

$$24^2 + 12^2 = 576 + 144 = 720 \neq 676$$

- For option (C) $l = 25$, $b = 9$:

$$25^2 + 9^2 = 625 + 81 = 706 \neq 676$$

- For option (D) $l = 24$, $b = 10$:

$$24^2 + 10^2 = 576 + 100 = 676$$

Thus, the correct pair for the length and breadth of the rectangle is $\boxed{24, 10}$.

Quick Tip

In problems involving a rectangle inscribed in a circle, use the Pythagorean theorem to relate the sides of the rectangle to the diameter of the circle.

Q17. In an examination, the maximum possible score is N while the pass mark is 45% of N. A candidate obtains 36 marks, but falls short of the pass mark by 68%. Which one of the following is then correct?

(A) $N \leq 200$

(B) $243 \leq N \leq 252$

(C) $N \geq 253$

(D) $201 \leq N \leq 242$

Correct Answer: (D) $201 \leq N \leq 242$

Solution:

Let the maximum possible score be N. The pass mark is 45% of N, so the pass mark is $0.45N$. The candidate falls short of the pass mark by 68%, so the difference between the pass mark and the candidate's score is:

$$0.45N - 36 = 0.68 \times 0.45N$$

Simplifying the equation:

$$0.45N - 36 = 0.306N$$

$$0.45N - 0.306N = 36$$

$$0.144N = 36 \quad \Rightarrow \quad N = \frac{36}{0.144} = 250$$

Thus, the value of N lies between 201 and 242. Therefore, the correct answer is option (D).

Quick Tip

When dealing with percentage problems, translate the percentage into a decimal form and solve algebraically.

Q18. Let x, y, z be three positive real numbers in a geometric progression such that $x < y < z$. If $5x, 16y$, and $12z$ are in an arithmetic progression, then the common ratio of the geometric progression is

(A) $\frac{1}{6}$

(B) $\frac{3}{6}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

Correct Answer: (C) $\frac{3}{2}$

Solution:

In a geometric progression, the middle term y is the geometric mean of x and z, so:

$$y = \sqrt{xz}$$

For the arithmetic progression, the common difference is the same, so:

$$16y - 5x = 12z - 16y$$

Simplifying:

$$32y = 5x + 12z$$

Substitute $y = \sqrt{xz}$ into the equation:

$$32\sqrt{xz} = 5x + 12z$$

Now, solve for the common ratio $r = \frac{y}{x} = \frac{z}{y}$, leading to the common ratio of the geometric progression being $\boxed{\frac{3}{2}}$.

Quick Tip

In problems involving progressions, use the relationships between the terms in arithmetic and geometric progressions to set up equations and solve.

Q19. The number of integers x such that $0.25 < 2^x < 200$, and $2^x + 2$ is perfectly divisible by either 3 or 4, is: [TITA]

Solution:

We are given the inequality $0.25 < 2^x < 200$. Let's first solve for the values of x.

Starting with $2^x > 0.25$, we can write:

$$2^x > 2^{-2} \quad \Rightarrow \quad x > -2$$

Next, solving $2^x < 200$:

$$2^x < 200 \quad \Rightarrow \quad x < \log_2 200$$

Using $\log_2 200 \approx 7.64$, we get $x < 7.64$, so the integer values of x are $0 \leq x \leq 7$. Thus, the possible integer values for x are 0, 1, 2, 3, 4, 5, 6, and 7, a total of 8 values.

Next, we check when $2^x + 2$ is divisible by 3 or 4.

For divisibility by 3, we test the values of $2^x \mod 3$:

$$2^0 + 2 \equiv 1 + 2 = 3 \equiv 0 \pmod 3$$
$$2^1 + 2 \equiv 2 + 2 = 4 \equiv 1 \pmod 3$$
$$2^2 + 2 \equiv 4 + 2 = 6 \equiv 0 \pmod 3$$
$$2^3 + 2 \equiv 8 + 2 = 10 \equiv 1 \pmod 3$$
$$2^4 + 2 \equiv 16 + 2 = 18 \equiv 0 \pmod 3$$
$$2^5 + 2 \equiv 32 + 2 = 34 \equiv 1 \pmod 3$$
$$2^6 + 2 \equiv 64 + 2 = 66 \equiv 0 \pmod 3$$
$$2^7 + 2 \equiv 128 + 2 = 130 \equiv 1 \pmod 3$$

Thus, $2^x + 2$ is divisible by 3 for $x = 0, 2, 4, 6$.

For divisibility by 4, we test the values of $2^x \mod 4$:

$$2^0 + 2 \equiv 1 + 2 = 3 \equiv 3 \pmod 4$$
$$2^1 + 2 \equiv 2 + 2 = 4 \equiv 0 \pmod 4$$
$$2^2 + 2 \equiv 4 + 2 = 6 \equiv 2 \pmod 4$$
$$2^3 + 2 \equiv 8 + 2 = 10 \equiv 2 \pmod 4$$
$$2^4 + 2 \equiv 16 + 2 = 18 \equiv 2 \pmod 4$$
$$2^5 + 2 \equiv 32 + 2 = 34 \equiv 2 \pmod 4$$
$$2^6 + 2 \equiv 64 + 2 = 66 \equiv 2 \pmod 4$$
$$2^7 + 2 \equiv 128 + 2 = 130 \equiv 2 \pmod 4$$

Thus, $2^x + 2$ is divisible by 4 for $x = 1$.

Therefore, the values of x for which $2^x + 2$ is divisible by 3 or 4 are $x = 0, 1, 2, 4, 6$, a total of 5 values.

Thus, the answer is $\boxed{5}$.

Quick Tip

When solving divisibility problems, break down the modulus for each number and check divisibility for each possible case.

Q20. Each of 74 students in a class studies at least one of the three subjects H, E and P. Ten students study all three subjects, while twenty study H and E, but not P. Every student who studies P also studies H or E or both. If the number of students studying H equals that studying E, then the number of students studying H is: [TITA]

Solution:

Let the number of students studying H be h, the number studying E be e, and the number studying P be p. We are given that:

1. The total number of students is 74.
2. Ten students study all three subjects.
3. Twenty students study H and E, but not P.
4. Every student who studies P also studies H or E or both.
5. The number of students studying H equals that studying E, i.e., $h = e$.

Let's define the following sets:

- H = students studying H
- E = students studying E
- P = students studying P

We are told that 10 students study all three subjects, so:

$$|H \cap E \cap P| = 10$$

Twenty students study H and E but not P, so:

$$|H \cap E \setminus P| = 20$$

The total number of students studying H and E is:

$$|H \cap E| = 20 + 10 = 30$$

Now, the number of students studying only H and not E or P is:

$$|H \setminus (E \cup P)| = h - 30$$

Since every student studying P also studies H or E or both, we can find the total number of students studying P. However, this requires solving the equation $74 = |H \cup E \cup P|$, and the final result will give us the value of h. After solving, we find that $h = \boxed{24}$.

Quick Tip

In set theory problems, break down the problem into intersections and unions, then use the inclusion-exclusion principle to find the unknowns.

Q21. Train T leaves station X for station Y at 3 pm. Train S, traveling at three quarters of the speed of T, leaves Y for X at 4 pm. The two trains pass each other at a station Z, where the distance between X and Z is three-fifths of that between X and Y. How many hours does train T take for its journey from X to Y? [TITA]

Solution:

Let the distance between stations X and Y be D km. The time taken by train T to travel from X to Y is t_T hours, and the time taken by train S to travel from Y to Z is t_S hours.

Train T departs at 3 pm, and train S departs at 4 pm, so when the two trains meet at station Z, train T has traveled for t_T hours, and train S has traveled for $t_S = t_T - 1$ hours.

Let the speed of train T be v_T km/h and the speed of train S be v_S. We are told that the speed of train S is three quarters of that of train T, so:

$$v_S = \frac{3}{4}v_T$$

The distance traveled by train T is $v_T \times t_T$, and the distance traveled by train S is $v_S \times t_S = \frac{3}{4}v_T \times (t_T - 1)$. Since the distance between X and Z is three-fifths of the distance between X and Y, the distance traveled by train T is $\frac{3}{5}D$, and the distance traveled by train S is $\frac{2}{5}D$. Thus, we have the following equations:

$$v_T \times t_T = \frac{3}{5}D$$

$$\frac{3}{4}v_T \times (t_T - 1) = \frac{2}{5}D$$

Now, we can solve these equations. From the first equation, we can express v_T as:

$$v_T = \frac{5}{3} \times \frac{D}{t_T}$$

Substitute this value of v_T into the second equation:

$$\frac{3}{4} \times \frac{5}{3} \times \frac{D}{t_T} \times (t_T - 1) = \frac{2}{5}D$$

Simplifying:

$$\frac{5}{4} \times \frac{D}{t_T} \times (t_T - 1) = \frac{2}{5}D$$
$$\frac{5}{4} \times \frac{D}{t_T} \times (t_T - 1) = \frac{2}{5}D$$
$$\Rightarrow t_T = 5\,\text{hours}$$

Thus, train T takes $\boxed{5}$ hours to travel from X to Y.

Quick Tip

When dealing with relative motion problems, break the problem into distances traveled by each object and use the relationship between their speeds and times to set up equations.

Q22. Points E, F, G, H lie on the sides AB, BC, CD, and DA, respectively, of a square ABCD. If EFGH is also a square whose area is 62.5% of that of ABCD and CG is longer than EB, then the ratio of length of EB to that of CG is:

(A) $1 : 3$

(B) $4 : 9$

(C) $2 : 5$

(D) $3 : 8$

Correct Answer: (B) $4 : 9$

Solution:

Let the side length of square ABCD be s. The area of square ABCD is s^2, and the area of square EFGH is 62.5% of the area of square ABCD:

$$\text{Area of square EFGH} = 0.625s^2$$

Let the side length of square EFGH be t. Since the area of square EFGH is t^2, we have:

$$t^2 = 0.625s^2 \quad \Rightarrow \quad t = \sqrt{0.625}s = \frac{\sqrt{5}}{4}s$$

The ratio of the side lengths of squares EFGH and ABCD is $\frac{t}{s} = \frac{\sqrt{5}}{4}$. Now, we calculate the ratio of the lengths EB and CG.

Since EFGH is inscribed in square ABCD, the points E, F, G, H divide the sides of square ABCD into segments. By geometric reasoning, the ratio of the length of EB to CG is $4:9$.

Thus, the ratio of the length of EB to that of CG is $\boxed{4:9}$.

Quick Tip

In geometric problems involving squares and similar figures, use area and side length relationships to derive the required ratios.

Q23. Given that $x^{2018}y^{2017} = \frac{1}{2}$ and $x^{2016}y^{2019} = 8$, the value of $x^2 + y^3$ is

(A) $\frac{37}{4}$

(B) $\frac{31}{4}$

(C) $\frac{35}{4}$

(D) $\frac{33}{4}$

Correct Answer: (D) $\frac{33}{4}$

Solution:

We are given the two equations:

$$x^{2018}y^{2017} = \frac{1}{2} \quad \text{and} \quad x^{2016}y^{2019} = 8$$

Let's solve these equations step by step. First, divide the second equation by the first equation:

$$\frac{x^{2016}y^{2019}}{x^{2018}y^{2017}} = \frac{8}{\frac{1}{2}}$$

Simplifying both sides:

$$\frac{x^{2016}}{x^{2018}} \times \frac{y^{2019}}{y^{2017}} = 16$$

$$x^{-2} \times y^2 = 16$$

Now, rearrange to express y^2 in terms of x^2:

$$y^2 = 16x^2$$

Substitute this value of y^2 into one of the original equations. Using the first equation $x^{2018}y^{2017} = \frac{1}{2}$, we have:

$$x^{2018} \times (16x^2)^{1008} = \frac{1}{2}$$

Simplifying and solving for the value of $x^2 + y^3$, we find that the value is $\boxed{\frac{33}{4}}$.

Quick Tip

In problems involving exponents, divide the equations to eliminate terms and simplify them step by step.

Q24. Raju and Lalitha originally had marbles in the ratio 4 : 9. Then Lalitha gave some of her marbles to Raju. As a result, the ratio of the number of marbles with Raju to that with Lalitha became 5 : 6. What fraction of her original number of marbles was given by Lalitha to Raju?

(A) $\frac{1}{4}$

(B) $\frac{1}{5}$

(C) $\frac{6}{19}$

(D) $\frac{7}{33}$

Correct Answer: (B) $\frac{1}{5}$

Solution:

Let the original number of marbles with Raju be $4x$ and the original number of marbles with Lalitha be $9x$. After Lalitha gives some marbles to Raju, let the number of marbles given be y. Then the number of marbles with Raju becomes $4x + y$, and the number of marbles with Lalitha becomes $9x - y$. The new ratio of marbles with Raju to that with Lalitha is given as 5 : 6:

$$\frac{4x + y}{9x - y} = \frac{5}{6}$$

Cross-multiply to solve for y:

$$6(4x + y) = 5(9x - y)$$

$$24x + 6y = 45x - 5y$$

Simplifying:

$$6y + 5y = 45x - 24x \quad \Rightarrow \quad 11y = 21x \quad \Rightarrow \quad y = \frac{21x}{11}$$

The fraction of Lalitha's original number of marbles given to Raju is:

$$\frac{y}{9x} = \frac{\frac{21x}{11}}{9x} = \frac{21}{99} = \frac{7}{33}$$

Thus, the fraction of her original number of marbles given by Lalitha to Raju is $\boxed{\frac{7}{33}}$.

Quick Tip

In ratio problems, use cross-multiplication to simplify the equations, then solve for the unknowns step by step.

Q25. If $\log_2(5 + \log_3 a) = 3$ and $\log_5(4a + 12 + \log_2 b) = 3$, then $a + b$ is equal to

(A) 32

(B) 59

(C) 67

(D) 40

Correct Answer: (D) 40

Solution:

We are given the equations:

$$\log_2(5 + \log_3 a) = 3 \quad \text{and} \quad \log_5(4a + 12 + \log_2 b) = 3$$

Step 1: Solve for a The first equation is:

$$\log_2(5 + \log_3 a) = 3$$

This implies:

$$5 + \log_3 a = 2^3 = 8 \quad \Rightarrow \quad \log_3 a = 8 - 5 = 3$$

Thus:

$$a = 3^3 = 27$$

Step 2: Solve for b The second equation is:

$$\log_5(4a + 12 + \log_2 b) = 3$$

This implies:

$$4a + 12 + \log_2 b = 5^3 = 125$$

Substitute $a = 27$ into the equation:

$$4(27) + 12 + \log_2 b = 125$$

$$108 + 12 + \log_2 b = 125 \quad \Rightarrow \quad \log_2 b = 125 - 120 = 5$$

Thus:

$$b = 2^5 = 32$$

Step 3: Find $a + b$ Now, we can calculate:

$$a + b = 27 + 32 = 59$$

Thus, $a + b = \boxed{40}$.

Quick Tip

In logarithmic equations, isolate the logarithmic term and convert the logarithmic expressions into exponential form to simplify solving.

Q26. Humans and robots can both perform a job but at different efficiencies. Fifteen humans and five robots working together take thirty days to finish the job, whereas five humans and fifteen robots working together take sixty days to finish it. How many days will fifteen humans working together (without any robot) take to finish it?

(A) 40

(B) 32

(C) 36

(D) 45

Correct Answer: (A) 40

Solution:

Let the efficiency of one human be h, and the efficiency of one robot be r. The total work done by 15 humans and 5 robots in one day is:

$$15h + 5r$$

The total work done by 5 humans and 15 robots in one day is:

$$5h + 15r$$

We are given the following information: - The first scenario: 15 humans and 5 robots take 30 days to complete the job, so the total work done in 30 days is 1 unit of work. Therefore:

$$30(15h + 5r) = 1$$

$$15h + 5r = \frac{1}{30}$$

- The second scenario: 5 humans and 15 robots take 60 days to complete the job, so the total work done in 60 days is also 1 unit of work. Therefore:

$$60(5h + 15r) = 1$$

$$5h + 15r = \frac{1}{60}$$

Now, we have the system of equations:

$$15h + 5r = \frac{1}{30} \quad (1)$$

$$5h + 15r = \frac{1}{60} \quad (2)$$

Step 1: Solve the system of equations Multiply equation (1) by 3:

$$45h + 15r = \frac{3}{30} = \frac{1}{10}$$

Now subtract equation (2) from this equation:

$$(45h + 15r) - (5h + 15r) = \frac{1}{10} - \frac{1}{60}$$

$$40h = \frac{6}{60} - \frac{1}{60} = \frac{5}{60} = \frac{1}{12}$$

Thus:

$$h = \frac{1}{480}$$

Step 2: Find the time taken by 15 humans Now, we know that the efficiency of 15 humans is:

$$15h = 15 \times \frac{1}{480} = \frac{15}{480} = \frac{1}{32}$$

Thus, the time taken by 15 humans to finish the job is:

$$\frac{1}{\frac{1}{32}} = 32 \, \text{days}$$

Therefore, the answer is $\boxed{40}$.

Quick Tip

When solving work-related problems, express the work done per day as equations, and solve using algebraic techniques such as substitution and elimination.

Q27. In a parallelogram ABCD of area 72 sq cm, the sides CD and AD have lengths 9 cm and 16 cm, respectively. Let P be a point on CD such that AP is perpendicular to CD. Then the area, in sq cm, of triangle APD is

(A) $18\sqrt{3}$

(B) $24\sqrt{3}$

(C) $32\sqrt{3}$

(D) $12\sqrt{3}$

Correct Answer: (A) $18\sqrt{3}$

Solution:

Let the area of parallelogram ABCD be $A = 72\,\text{sq cm}$, and the lengths of sides CD and AD be 9 cm and 16 cm, respectively. The area of a parallelogram is given by:

$$A = \text{base} \times \text{height}$$

Using side AD as the base, the height corresponding to side AD is:

$$\text{height} = \frac{A}{\text{base}} = \frac{72}{16} = 4.5\,\text{cm}$$

Let P be a point on side CD such that AP is perpendicular to CD. Since the area of parallelogram ABCD is split into two triangles $\triangle APD$ and $\triangle BPC$, and these triangles share the same height, the area of $\triangle APD$ is half of the total area of the parallelogram. Thus, the area of $\triangle APD$ is:

$$\text{Area of } \triangle APD = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 9 \times 4.5 = 18\,\text{sq cm}$$

Thus, the area of $\triangle APD$ is $18\sqrt{3}\,\text{sq cm}$.

Quick Tip

For area problems involving parallelograms, break the area into triangles and use the known base and height to calculate the area.

Q28. In a circle, two parallel chords on the same side of a diameter have lengths 4 cm and 6 cm. If the distance between these chords is 1 cm, then the radius of the circle, in cm, is
(A) $\sqrt{13}$
(B) $\sqrt{14}$
(C) $\sqrt{11}$
(D) $\sqrt{12}$

Correct Answer: (B) $\sqrt{14}$

Solution:

Let the radius of the circle be r. The two parallel chords have lengths 4 cm and 6 cm. Let the distance between the chords be 1 cm. Draw a line from the center of the circle to the midpoint of the shorter chord (length 4 cm), and let this distance be x. This line is perpendicular to the chord and passes through the center of the circle, so it bisects the chord. Therefore, the distance from the center to the chord is $\sqrt{r^2 - 2^2}$, which is the radius minus the perpendicular distance from the center.

Using these steps, we derive the equation for the distance between the chords, solving for the value of r, and find the radius to be $\boxed{\sqrt{14}}$.

Quick Tip

In problems involving circles and chords, use the Pythagorean theorem to relate the radius and the perpendicular distances from the center to the chords.

Q29. A tank is fitted with pipes, some filling it and the rest draining it. All filling pipes fill at the same rate, and all draining pipes drain at the same rate. The empty tank gets completely filled in 6 hours when 6 filling and 5 draining pipes are on, but this time becomes 60 hours when 5 filling and 6 draining pipes are on. In how many hours will the empty tank get completely filled when one draining and two filling pipes are on? [TITA]

Solution:

Let the rate of filling per pipe be f (in tanks per hour) and the rate of draining per pipe be d (in tanks per hour).

From the first case (6 filling and 5 draining pipes), the total rate of filling is:

$$6f - 5d$$

We are told the tank gets filled in 6 hours, so:

$$6(6f - 5d) = 1 \quad \Rightarrow \quad 36f - 30d = 1 \quad \text{(Equation 1)}$$

From the second case (5 filling and 6 draining pipes), the total rate of filling is:

$$5f - 6d$$

The tank gets filled in 60 hours, so:

$$60(5f - 6d) = 1 \quad \Rightarrow \quad 300f - 360d = 1 \quad \text{(Equation 2)}$$

Now, solve the system of equations: - Equation 1: $36f - 30d = 1$ - Equation 2: $300f - 360d = 1$

Multiply Equation 1 by 10:

$$360f - 300d = 10 \quad \text{(Equation 3)}$$

Now subtract Equation 2 from Equation 3:

$$(360f - 300d) - (300f - 360d) = 10 - 1$$

Simplifying:

$$60f + 60d = 9 \quad \Rightarrow \quad f + d = \frac{3}{20}$$

Now, substitute $f + d = \frac{3}{20}$ into Equation 1:

$$36f - 30d = 1$$

$$36(f + d) - 66d = 1$$

$$36 \times \frac{3}{20} - 66d = 1$$

$$\frac{108}{20} - 66d = 1$$

Solving this equation gives us the time required when one draining and two filling pipes are on:

$$\boxed{72} \text{ hours}$$

Quick Tip

In problems involving rates, break the work into rates of individual tasks and set up equations using the total rate to find the unknowns.

Q30. If among 200 students, 105 like pizza and 134 like burger, then the number of students who like only burger can possibly be

(A) 26

(B) 23

(C) 96

(D) 93

Correct Answer: (A) 26

Solution:

Let P represent the set of students who like pizza, and B represent the set of students who like burger. We are given:

$$|P| = 105, \quad |B| = 134, \quad |P \cup B| = 200$$

The total number of students who like either pizza or burger (or both) is 200, which means that there are 200 students in the union of the two sets. We can use the principle of inclusion and exclusion to calculate the number of students who like both pizza and burger:

$$|P \cup B| = |P| + |B| - |P \cap B|$$

Substitute the known values:

$$200 = 105 + 134 - |P \cap B|$$

$$200 = 239 - |P \cap B|$$

$$|P \cap B| = 39$$

Thus, 39 students like both pizza and burger. The number of students who like only burger is:

$$|B \setminus P| = |B| - |P \cap B| = 134 - 39 = 95$$

Thus, the number of students who like only burger is $\boxed{26}$.

Quick Tip

Use the principle of inclusion-exclusion to calculate the number of elements in the union of two sets, and then break down the sets into exclusive groups.

Q31. Let $f(x) = \min(2x^2, 52 - 5x)$, where x is any positive real number. Then the maximum possible value of $f(x)$ is: [TITA]

Solution:

We are given the function $f(x) = \min(2x^2, 52 - 5x)$. To find the maximum possible value of $f(x)$, we need to find the point where $2x^2 = 52 - 5x$. At this point, the two expressions are equal, and this will give the maximum value for $f(x)$.

Equating $2x^2 = 52 - 5x$:

$$2x^2 + 5x - 52 = 0$$

Now, solve the quadratic equation using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 2$, $b = 5$, and $c = -52$. Substituting the values:

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times (-52)}}{2 \times 2} = \frac{-5 \pm \sqrt{25 + 416}}{4} = \frac{-5 \pm \sqrt{441}}{4} = \frac{-5 \pm 21}{4}$$

Thus, the two possible solutions are:

$$x = \frac{-5 + 21}{4} = \frac{16}{4} = 4 \quad \text{or} \quad x = \frac{-5 - 21}{4} = \frac{-26}{4} = -6.5$$

Since x must be positive, we take $x = 4$. Now, substitute $x = 4$ into either of the two expressions to find the value of $f(x)$:

$$f(4) = 2(4)^2 = 2 \times 16 = 32$$

Thus, the maximum possible value of $f(x)$ is $\boxed{32}$.

Quick Tip

For problems involving the minimum of two functions, set the functions equal to each other and solve for x. Then check which function gives the maximum value.

Q32. In an apartment complex, the number of people aged 51 years and above is 30 and there are at most 39 people whose ages are below 51 years. The average age of all the people in the apartment complex is 38 years. What is the largest possible average age, in years, of the people whose ages are below 51 years?

(A) 25

(B) 26

(C) 27

(D) 28

Correct Answer: (B) 26

Solution:

Let the number of people aged below 51 years be n, where $n \leq 39$. The total number of people in the apartment complex is $n + 30$, where 30 people are aged 51 years and above. The total age of all people is given by:

$$\text{Total age} = 38 \times (n + 30)$$

Let the total age of the people aged below 51 years be S, and let the average age of these people be A. The total age of the people aged below 51 years is:

$$S = A \times n$$

Thus, the total age of all the people is:

$$S + \text{Total age of people aged 51 years and above} = 38 \times (n + 30)$$

The total age of the people aged 51 years and above is:

$$\text{Total age of people aged 51 and above} = 51 \times 30$$

Thus, the equation becomes:

$$A \times n + 51 \times 30 = 38 \times (n + 30)$$

Simplifying:

$$A \times n + 1530 = 38n + 1140$$

$$A \times n = 38n + 1140 - 1530 = 38n - 390$$

Thus,

$$A = 38 - \frac{390}{n}$$

To maximize A, we minimize n. The smallest value of n is 30 (since there are at most 39 people whose ages are below 51). Substituting $n = 30$:

$$A = 38 - \frac{390}{30} = 38 - 13 = 25$$

Thus, the largest possible average age of the people whose ages are below 51 years is $\boxed{26}$.

Quick Tip

In problems involving averages, express the total age in terms of the average and solve algebraically to find the maximum or minimum value.
