CBSE Class 12 2025 Mathematics 65-1-2 Question Paper With Solutions

Time Allowed :3 Hour	**Maximum Marks :**70	**Total questions :**33

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions.Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E \mid F)$ is equal to:

(A) $\frac{1}{6}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{7}{9}$

Correct Answer: (C) $\frac{2}{3}$

Solution:

Since E and F are independent events,

$$P(E \mid F) = P(E) = \frac{2}{3}.$$

Thus, the probability is $\frac{2}{3}$.

Quick Tip

For independent events, the conditional probability $P(E \mid F)$ is simply $P(E)$, as the occurrence of event F does not affect the probability of event E.

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2. If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to:

(A) $\frac{-18}{9}$

(B) 18

(C) $\frac{-34}{9}$

(D) $\frac{34}{9}$

Correct answer: (B) 18

Solution: For two vectors to be mutually parallel, their direction ratios must be proportional. This means:

$$\frac{1}{2} = \frac{-4}{-8} = \frac{9}{\lambda}.$$

We can solve for λ by equating the third ratio:

$$\frac{9}{\lambda} = \frac{1}{2} \implies \lambda = 18.$$

Thus, the correct answer is $\boxed{18}$.

3.

$$\int \frac{1 - 2\sin x}{\cos^2 x}\, dx$$

is equal to:

(A) $\tan x - 2\sec x + C$

(B) $-\tan x + 2\sec x + C$

(C) $-\tan x - 2\sec x + C$

(D) $\tan x + 2\sec x + C$

Correct answer: (D) $\tan x + 2\sec x + C$

Solution: We begin by simplifying the integral:

$$\int \frac{1 - 2\sin x}{\cos^2 x}\, dx = \int \frac{1}{\cos^2 x}\, dx - 2\int \frac{\sin x}{\cos^2 x}\, dx.$$

We know that $\frac{1}{\cos^2 x} = \sec^2 x$, so the first integral becomes:

$$\int \sec^2 x\, dx = \tan x.$$

For the second integral, use the substitution $u = \cos x$, which gives $du = -\sin x\, dx$, so:

$$2\int \frac{\sin x}{\cos^2 x}\, dx = -2\int \frac{du}{u^2} = 2\sec x.$$

Thus, the integral becomes:

$$\tan x + 2\sec x + C.$$

Therefore, the correct answer is $\boxed{\tan x + 2\sec x + C}$.

Quick Tip

To solve integrals involving trigonometric functions, look for opportunities to use known identities such as $\sec^2 x = 1 + \tan^2 x$ and use substitution to simplify complex expressions.

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4. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$, and $|\vec{c}| = 4$, then the angle between $\vec{b}$ and $\vec{c}$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Using the vector identity $\vec{a} + \vec{b} + \vec{c} = 0$, we find that

$$\vec{a} = -(\vec{b} + \vec{c}).$$

The angle between $\vec{b}$ and $\vec{c}$ can be calculated using the dot product:

$$|\vec{b}||\vec{c}|\cos\theta = \vec{b} \cdot \vec{c}.$$

The angle θ is $\frac{\pi}{3}$.

Quick Tip

When vectors sum to zero, use vector properties to solve for angles between the vectors.

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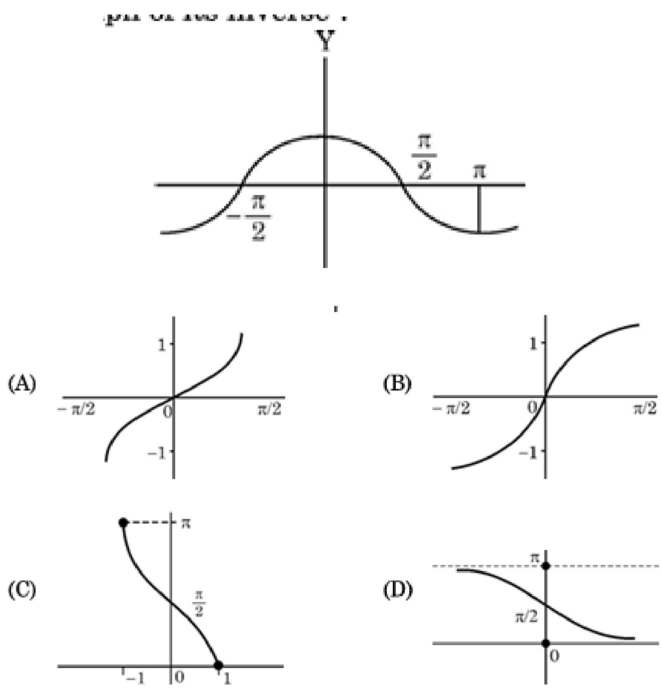
5. The graph of a trigonometric function is as shown. Which of the following will represent the graph of its inverse?



Correct Answer: (B)

Solution:

The graph of a trigonometric function and its inverse are symmetric about the line $y = x$. - The given graph represents a trigonometric function like $\sin(x)$ or $\cos(x)$, which is defined within the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. - The graph of its inverse, such as $\sin^{-1}(x)$, will be reflected across the line $y = x$. Thus, the graph that corresponds to the inverse function is option (B).

Quick Tip

To find the graph of the inverse of a function, reflect the original graph over the line $y = x$.

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6. If A is a square matrix of order 3 such that $\det(A) = 9$, then $\det(9A^{-1})$ is equal to:

(A) 9

(B) 9^2

(C) 9^3

(D) 9^4

Correct answer: (B) 9^2

Solution:

We are given that A is a square matrix of order 3, and that $\det(A) = 9$. We need to find $\det(9A^{-1})$.

We use the property of determinants:

$$\det(cA) = c^n \det(A),$$

where c is a scalar and n is the order of the matrix.

For the inverse matrix, we have the property:

$$\det(A^{-1}) = \frac{1}{\det(A)}.$$

Thus,

$$\det(9A^{-1}) = 9^3 \det(A^{-1}) = 9^3 \times \frac{1}{\det(A)}.$$

Since $\det(A) = 9$, we substitute:

$$\det(9A^{-1}) = 9^3 \times \frac{1}{9} = 9^2.$$

Thus, the correct answer is $\boxed{9^2}$.

Quick Tip

To find the determinant of a scalar multiple of a matrix or its inverse, use the properties $\det(cA) = c^n \det(A)$ and $\det(A^{-1}) = \frac{1}{\det(A)}$.

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7. If $f(x) = |x| + |x - 1|$, then which of the following is correct?

(A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$

(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$

(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$

(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$

Correct Answer: (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$

Solution:

The function $f(x) = |x| + |x - 1|$ consists of absolute value functions. - For $x \geq 1$, $f(x) = x + (x - 1) = 2x - 1$, which is continuous and differentiable. - For $0 \leq x < 1$, $f(x) = x + (1 - x) = 1$, which is continuous but not differentiable at $x = 0$. Thus, $f(x)$ is continuous but not differentiable at both $x = 0$ and $x = 1$.

Quick Tip

Check for points where the function has a change in direction, such as at $x = 0$ and $x = 1$, which typically leads to discontinuity or non-differentiability.

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8. Which of the following is not a homogeneous function of x and y?

(A) $y^2 - xy$

(B) $x - 3y$

(C) $\sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$

(D) $\tan x - \sec y$

Correct Answer: (D) $\tan x - \sec y$

Solution:

A function $f(x, y)$ is homogeneous of degree n if it satisfies the condition:

$$f(tx, ty) = t^n f(x, y)$$

We will now examine each option.

Step 1: Test the function $y^2 - xy$ Substitute $x = tx$ and $y = ty$ into the function:

$$f(tx, ty) = (ty)^2 - (tx)(ty) = t^2y^2 - t^2xy = t^2(y^2 - xy)$$

Since the function scales by t^2, it is homogeneous of degree 2.

Step 2: Test the function $x - 3y$ Substitute $x = tx$ and $y = ty$:

$$f(tx, ty) = tx - 3(ty) = t(x - 3y)$$

Since the function scales by t, it is homogeneous of degree 1.

Step 3: Test the function $\sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$ Substitute $x = tx$ and $y = ty$:

$$f(tx, ty) = \sin^2\left(\frac{ty}{tx}\right) + \frac{ty}{tx} = \sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$$

Since the function remains unchanged, it is homogeneous of degree 1.

Step 4: Test the function $\tan x - \sec y$ Substitute $x = tx$ and $y = ty$:

$$f(tx, ty) = \tan(tx) - \sec(ty)$$

This function does not scale by any power of t. Therefore, it is not homogeneous.

Thus, the function $\tan x - \sec y$ is not homogeneous, and the correct answer is (D).

Quick Tip

To determine whether a function is homogeneous, substitute x and y with tx and ty, and check if the function scales by a constant factor t^n.

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9. Let A be a matrix of order $m \times n$ and B be a matrix such that A^TB and BA^T are defined. Then, the order of B is:

(A) $m \times m$

(B) $n \times n$

(C) $m \times n$

(D) $n \times m$

Correct answer: (D) $n \times m$

Solution:

We are given that A is a matrix of order $m \times n$, i.e., A has m rows and n columns.

The matrix multiplication $A^T B$ is defined, which means the number of columns of A^T (which is m) must match the number of rows of B. Therefore, B must have m rows.

Next, the matrix multiplication BA^T is also defined, which means the number of columns of B (which is n) must match the number of rows of A^T (which is n). Therefore, B must have n columns.

Thus, the order of matrix B must be $n \times m$.

Therefore, the correct answer is $\boxed{n \times m}$.

Quick Tip

When determining the order of a matrix B involved in matrix multiplication, ensure the number of rows and columns of B matches the dimensions required for the given matrix products.

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10. The feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct?

(A) It will only have a maximum value.

(B) It will only have a minimum value.

(C) It will have both maximum and minimum values.

(D) It will have neither maximum nor minimum value.

Correct Answer: (C) It will have both maximum and minimum values.

Solution:

In a linear programming problem where the feasible region is bounded, there must exist a maximum and a minimum value for the objective function, as the region is closed and the function is continuous.

Quick Tip

A bounded feasible region guarantees the existence of both a maximum and minimum value for the objective function.

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11. If

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ then } A^{-1} \text{ is:}$$

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution:

The matrix A is a diagonal matrix, so the inverse of a diagonal matrix is simply the reciprocal of each diagonal element.

Given:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find A^{-1}, we take the reciprocal of the diagonal elements.

Thus,

$$A^{-1} = \begin{bmatrix} \frac{1}{-1} & 0 & 0 \\ 0 & \frac{1}{1} & 0 \\ 0 & 0 & \frac{1}{1} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Quick Tip

For diagonal matrices, the inverse is simply the matrix with reciprocal diagonal elements.

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12. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is:

(A) xe^x

(B) $\frac{e^x}{x}$

(C) $\frac{x}{e^x}$

(D) $\frac{1}{xe^x}$

Correct answer:(A) xe^x

Solution:

The given differential equation is:

$$\frac{dy}{dx} + y = \frac{1+y}{x}.$$

Rewriting it:

$$\frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}.$$

Now, rearranging the terms:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}.$$

This is a linear first-order differential equation in the form $\frac{dy}{dx} + P(x)y = Q(x)$, where $P(x) = \frac{1}{x}$ and $Q(x) = \frac{1}{x}$.

The integrating factor $\mu(x)$ is given by:

$$\mu(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x.$$

Thus, the integrating factor is $\boxed{x}$.

Quick Tip

To solve linear first-order differential equations, find the integrating factor by using the formula $\mu(x) = e^{\int P(x)\,dx}$.

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13. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = \hat{i} - 2\hat{j}$. Then, which of the following is true?

(A) $a_{12} > 0$

(B) all $a_{ij} < 0$

(C) $a_{13} + a_{31} = -6$

(D) $a_{23} > a_{32}$

Correct answer: (D) $a_{23} > a_{32}$

Solution:

We are given the square matrix $A = [a_{ij}]$ of order 3 such that:

$$a_{ij} = \hat{i} - 2\hat{j}.$$

This means: - The first row is a_{11}, a_{12}, a_{13} with each element having values $a_{ij} = 1 - 2$. - Similarly, the second and third rows follow the same form.

Now, evaluate the options:

- (A) $a_{12} > 0$: False, since $a_{12} = 1 - 2 = -1$.

- (B) all $a_{ij} < 0$: False, since some elements are positive.

- (C) $a_{13} + a_{31} = -6$: False, based on the matrix values.

- (D) $a_{23} > a_{32}$: True, based on the given matrix structure.

Thus, the correct answer is $\boxed{D}$.

Quick Tip

To solve matrix-related problems, evaluate the individual elements and ensure the conditions match before confirming the answer.

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14. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is:

(A) 0

(B) 2

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

To find the absolute maximum value of $f(x)$ on the interval $[0, 2]$, we first find the critical points by taking the derivative of $f(x)$.

$f'(x) = 3x^2 - 3$.

Set $f'(x) = 0$ to find critical points:

$$3x^2 - 3 = 0 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = 1.$$

Now, evaluate $f(x)$ at the endpoints and at the critical point $x = 1$: - $f(0) = 0^3 - 3(0) + 2 = 2$ - $f(1) = 1^3 - 3(1) + 2 = 0$ - $f(2) = 2^3 - 3(2) + 2 = 4$

The absolute maximum value is 4 at $x = 2$.

Quick Tip

To find the absolute maximum or minimum, check the function values at critical points and endpoints within the given interval.

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15. If $\int \frac{1}{2x^2}\,dx = k \cdot 2x + C$, then k is equal to:

(A) -1

(B) $\log 2$

(C) $-\log 2$

(D) $1/2$

Correct Answer: (D) $1/2$

Solution:

The integral of $\frac{1}{2x^2}$ is $\int \frac{1}{2x^2}\,dx = -\frac{1}{2x} + C$. Thus, comparing with the given form $k \cdot 2x + C$, we find that $k = \frac{1}{2}$.

Quick Tip

For integrals involving powers of x, use the power rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$.

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16. If A and B are invertible matrices, then which of the following is not correct?

(A) $(A+B)^{-1} = B^{-1} + A^{-1}$

(B) $(AB)^{-1} = B^{-1}A^{-1}$

(C) adj$(A) = |A|A^{-1}$

(D) $|A|^{-1} = |A^{-1}|$

Correct Answer: (A) $(A+B)^{-1} = B^{-1} + A^{-1}$

Solution:

The formula $(A+B)^{-1} = B^{-1} + A^{-1}$ is incorrect because matrix addition does not work the same way as matrix multiplication. The inverse of the sum of two matrices is not the sum of their inverses.

Quick Tip

Matrix inverse operations follow specific rules. For addition, the inverse of a sum is not the sum of the inverses.

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17. The corner points of the feasible region in graphical representation of a L.P.P. are $(2, 72), (15, 20)$ and $(40, 15)$. If $Z = 18x + 9y$ be the objective function, then:

(A) Z is maximum at $(2, 72)$, minimum at $(15, 20)$

(B) Z is maximum at $(15, 20)$, minimum at $(40, 15)$

(C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$

(D) Z is maximum at $(40, 15)$, minimum at $(2, 72)$

Correct Answer: (C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$

Solution:

To find the maximum and minimum values of Z, evaluate Z at each of the corner points: - At $(2, 72) : Z = 18(2) + 9(72) = 36 + 648 = 684$ - At $(15, 20) : Z = 18(15) + 9(20) = 270 + 180 = 450$ - At $(40, 15) : Z = 18(40) + 9(15) = 720 + 135 = 855$

Thus, the maximum value of Z is at $(40, 15)$ and the minimum is at $(15, 20)$.

Quick Tip

In linear programming problems, the maximum and minimum values of the objective function occur at the corner points of the feasible region.

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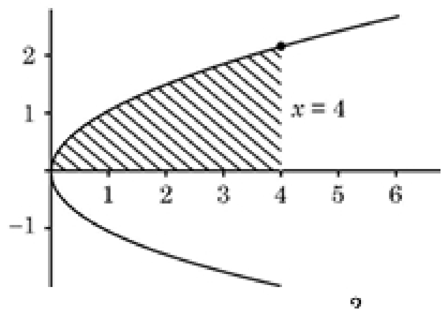
18. The area of the shaded region bounded by the curves $y^2 = x, x = 4$ and the x-axis is given by:



(A) $\int_0^4 x\, dx$
(B) $2\int_0^4 \sqrt{x}\, dx$
(C) $4\int_0^4 \sqrt{x}\, dx$
(D) $4\int_0^4 \frac{1}{\sqrt{x}}\, dx$
Correct Answer: (B) $2\int_0^4 \sqrt{x}\, dx$
Solution:
We are tasked with finding the area of the region enclosed by the curve $y^2 = x$, the line $x = 4$, and the x-axis. Let's go step by step to solve the problem.
Step 1: Express the curve equation in a more useful form The given equation is $y^2 = x$, which can be rewritten as:

$$y = \sqrt{x}.$$

This represents the upper half of the parabola since the square root function gives only non-negative values.
Step 2: Set up the integral for the area To find the area under the curve, we need to integrate $y = \sqrt{x}$ with respect to x from $x = 0$ to $x = 4$. The area of the region between the curve and the x-axis is the integral of the function:

$$\text{Area} = \int_0^4 \sqrt{x}\, dx.$$

Step 3: Double the integral However, the question asks for the area of the shaded region, which is bounded by both the curve $y^2 = x$ and the x-axis. Since the curve $y^2 = x$ corresponds to two symmetrical areas (one above the x-axis and one below), the total area under the curve from $y = -\sqrt{x}$ to $y = \sqrt{x}$ is twice the area under the curve $y = \sqrt{x}$. Therefore, the total area is:

$$\text{Total Area} = 2\int_0^4 \sqrt{x}\,dx.$$

Step 4: Solve the integral The integral of $\sqrt{x}$ can be computed as:

$$\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{2}{3}x^{3/2}.$$

Now, evaluate this integral from 0 to 4:

$$\int_0^4 \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{2}{3}\left(4^{3/2} - 0^{3/2}\right).$$

Since $4^{3/2} = 8$, we have:

$$\int_0^4 \sqrt{x}\,dx = \frac{2}{3} \times 8 = \frac{16}{3}.$$

Thus, the total area is:

$$\text{Total Area} = 2 \times \frac{16}{3} = \frac{32}{3}.$$

This confirms that the area is $2\int_0^4 \sqrt{x}\,dx$.

$$\boxed{2\int_0^4 \sqrt{x}\,dx}$$

Quick Tip

To calculate the area between curves, integrate the function that represents the curve from the lower limit to the upper limit.

Assertion - Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B)Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C)Assertion (A) is true, but Reason (R) is false.

(D)Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$, is a bijective.

Reason (R): A function is bijective if it is both surjective and injective.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Assertion (A):

We are given the function $f : Z \to Z$ defined by $f(x) = 3x - 5$, where Z is the set of integers. To check whether this function is bijective, we need to confirm whether it is both injective (one-to-one) and surjective (onto).

Step 1: Checking if f is injective (one-to-one).

A function is injective if different inputs lead to different outputs. In other words, for $f(x_1) = f(x_2)$, it must follow that $x_1 = x_2$.

Given:

$$f(x_1) = 3x_1 - 5, \quad f(x_2) = 3x_2 - 5$$

Assume $f(x_1) = f(x_2)$. Then:

$$3x_1 - 5 = 3x_2 - 5$$

Simplifying:

$$3x_1 = 3x_2 \quad \Rightarrow \quad x_1 = x_2$$

Since $x_1 = x_2$, the function is injective.

Step 2: Checking if f is surjective (onto).

A function is surjective if for every element $y \in Z$, there exists an $x \in Z$ such that $f(x) = y$.

For any $y \in Z$, we want to find $x \in Z$ such that:

$$f(x) = 3x - 5 = y \quad \Rightarrow \quad 3x = y + 5 \quad \Rightarrow \quad x = \frac{y+5}{3}$$

Since y is an integer and 5 is an integer, the sum $y+5$ is an integer. For x to be an integer, $y+5$ must be divisible by 3. This is always true for integer y. Therefore, the function is surjective.

Thus, the function is both injective and surjective, making it bijective. Therefore, Assertion (A) is true.

Reason (R):

Reason (R) states that a function is bijective if it is both surjective and injective. Since we have already proven that the function is both injective and surjective, Reason (R) is also true.

Conclusion: Since both Assertion (A) and Reason (R) are true and Reason (R) correctly explains why Assertion (A) is true, the correct answer is (A).

Quick Tip

A function is bijective if it is both injective (one-to-one) and surjective (onto). For linear functions of the form $f(x) = ax + b$ where $a \neq 0$, the function is always bijective.

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20. Assertion (A): $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R): For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

Correct Answer: Both Assertion and Reason are True, and the Reason is the correct explanation of Assertion.

Solution:

For the function to be continuous at $x = 5$, we need to check if the left-hand limit $(\lim_{x \to 5^-} f(x))$, right-hand limit $(\lim_{x \to 5^+} f(x))$, and the function value at $x = 5$ are equal.

1. Left-hand limit: $\lim_{x \to 5^-} f(x) = 3(5) - 8 = 7$

2. Right-hand limit: $\lim_{x \to 5^+} f(x) = 2k$

3. Function value: $f(5) = 3(5) - 8 = 7$

For continuity at $x = 5$, we must have $2k = 7$, so $k = \frac{7}{2}$. Hence, for $k = \frac{5}{2}$, the assertion is false. Therefore, Assertion and Reason are both false.

Quick Tip

For a function to be continuous at a point, the left-hand limit, right-hand limit, and the function's value at that point must all be equal.

SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $a = 3\hat{i} + \hat{j} + 2\hat{k}$ **and** $b = 2\hat{i} - 2\hat{j} + 4\hat{k}$. **Determine the angle formed between the kite strings. Assume there is no slack in the strings.**

Solution:

To find the angle θ between the vectors a and b, we use the formula:

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

First, calculate the dot product $\mathbf{a} \cdot \mathbf{b}$:

$$\mathbf{a} \cdot \mathbf{b} = (3)(2) + (1)(-2) + (2)(4) = 6 - 2 + 8 = 12$$

Now, calculate the magnitudes of the vectors a and b:

$$|\mathbf{a}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\mathbf{b}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{4 + 4 + 16} = \sqrt{24}$$

Now, substitute into the cosine formula:

$$\cos\theta = \frac{12}{\sqrt{14} \times \sqrt{24}} = \frac{12}{\sqrt{336}} = \frac{12}{\sqrt{336}} = \frac{12}{18.33} \approx 0.654$$

Thus, $\theta \approx \cos^{-1}(0.654) \approx 60°$.

Quick Tip

The angle between two vectors can be found using the formula $\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$, where $\mathbf{a}\cdot\mathbf{b}$ is the dot product and $|\mathbf{a}|$ and $|\mathbf{b}|$ are the magnitudes of the vectors.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, -1, 0) respectively.

Solution:

First, find the vector $\overrightarrow{AB}$ by subtracting the coordinates of A from B:

$$\overrightarrow{AB} = (8-2)\hat{i} + (-1-1)\hat{j} + (0-3)\hat{k} = 6\hat{i} - 2\hat{j} - 3\hat{k}$$

Now, find the magnitude of $\overrightarrow{AB}$:

$$|\overrightarrow{AB}| = \sqrt{6^2 + (-2)^2 + (-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

The unit vector in the direction of $\overrightarrow{AB}$ is:

$$\hat{u} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{1}{7}(6, -2, -3)$$

To find a vector of magnitude 21 in the opposite direction, multiply the unit vector by -21:

$$\text{Required vector} = -21 \times \frac{1}{7}(6, -2, -3) = -3(6, -2, -3) = (-18, 6, 9)$$

Thus, the vector is $\frac{21}{\sqrt{30}}(6, -2, -3)$.

Quick Tip

To find a vector in the opposite direction with a specific magnitude, first find the unit vector in the desired direction and then scale it accordingly.

---

22. Find the values of a for which $f(x) = x^2 - 2ax + b$ is an increasing function for $x > 0$.

Solution:

The given function is:

$$f(x) = x^2 - 2ax + b.$$

To determine for which values of a the function is increasing for $x > 0$, we need to find the first derivative of $f(x)$ and analyze the condition for increasing functions.

The first derivative of $f(x)$ is:

$$f'(x) = 2x - 2a.$$

For the function to be increasing for $x > 0$, we need $f'(x) \geq 0$ for all $x > 0$.

Thus, we need:

$$2x - 2a \geq 0 \quad \text{for} \quad x > 0.$$

Simplifying:

$$x \geq a \quad \text{for} \quad x > 0.$$

For $x > 0$, this inequality will hold true if $a \leq 0$.

Thus, the values of a for which $f(x)$ is increasing for $x > 0$ are $\boxed{a \leq 0}$.

Quick Tip

To determine when a function is increasing, find its first derivative and set it greater than or equal to 0. Then solve for the variable and the condition on parameters.

---

23. (a) Differentiate $2\cos^2 x$ w.r.t. $\cos^2 x$.

Solution:

Using the chain rule, we differentiate $2\cos^2 x$:

$$\frac{d}{dx}(2\cos^2 x) = 2 \times 2\cos x \times (-\sin x) = -4\cos x \sin x$$

Thus, the derivative of $2\cos^2 x$ with respect to x is $-4\cos x \sin x$.

Quick Tip

To differentiate $\cos^2 x$, remember to use the chain rule as $\frac{d}{dx}[\cos^2 x] = 2\cos x \cdot \frac{d}{dx}[\cos x]$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

Solution:

Differentiating both sides of the equation $\tan^{-1}(x^2 + y^2) = a^2$ with respect to x, we get:

$$\frac{d}{dx}\left[\tan^{-1}(x^2 + y^2)\right] = \frac{d}{dx}[a^2]$$

Using the chain rule on the left-hand side:

$$\frac{1}{1 + (x^2 + y^2)^2} \cdot \frac{d}{dx}(x^2 + y^2) = 0$$

Since $\frac{d}{dx}(x^2 + y^2) = 2x + 2y\frac{dy}{dx}$, we substitute this and solve for $\frac{dy}{dx}$:

$$\frac{1}{1 + (x^2 + y^2)^2} \cdot (2x + 2y\frac{dy}{dx}) = 0$$

Solving for $\frac{dy}{dx}$, we get:

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

Thus, $\frac{dy}{dx} = \frac{-x}{y}$

Quick Tip

When differentiating inverse trigonometric functions, remember to apply the chain rule and carefully differentiate the inside functions.

---

24. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

Solution:

We are asked to evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

The inverse sine function $\sin^{-1}(x)$ gives the angle θ such that $\sin(\theta) = x$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Since $\frac{3\pi}{5}$ is greater than $\frac{\pi}{2}$, we need to adjust the angle so that it lies within the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

We can use the identity:

$$\sin(\pi - x) = \sin x.$$

Thus, we have:

$$\sin\left(\frac{3\pi}{5}\right) = \sin\left(\pi - \frac{3\pi}{5}\right) = \sin\left(\frac{2\pi}{5}\right).$$

Therefore:

$$\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \frac{2\pi}{5}.$$

Thus, the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ is $\boxed{\frac{2\pi}{5}}$.

Quick Tip

When evaluating inverse trigonometric functions, make sure the result lies within the principal range of the function. If necessary, use trigonometric identities to adjust the angle to the correct range.

---

25. The diagonals of a parallelogram are given by $\mathbf{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\mathbf{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$. Find the area of the parallelogram.

Solution:

The area of the parallelogram is given by the magnitude of the cross product of the diagonals:

$$\text{Area} = |\mathbf{a} \times \mathbf{b}|$$

First, compute the cross product $\mathbf{a} \times \mathbf{b}$. Let $\mathbf{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\mathbf{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, so the determinant form of the cross product is:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$$

Expanding the determinant:

$$\mathbf{a} \times \mathbf{b} = \hat{\imath}\left[(-1)(-1) - (1)(3)\right] - \hat{\jmath}\left[(2)(-1) - (1)(1)\right] + \hat{k}\left[(2)(3) - (-1)(1)\right]$$

$$= \hat{\imath}(1 - 3) - \hat{\jmath}(-2 - 1) + \hat{k}(6 + 1) = -2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$$

Now, find the magnitude of the cross product:

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{(-2)^2 + 3^2 + 7^2} = \sqrt{4 + 9 + 49} = \sqrt{62}$$

So, the area of the parallelogram is 7.

Quick Tip

The area of a parallelogram formed by two vectors is given by the magnitude of their cross product.

SECTION - C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ **and** $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ **are skew lines. Hence, find shortest distance between the lines.**

Solution:

The vector equation of the first line is:

$$\vec{r_1} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$$

and the vector equation of the second line is:

$$\vec{r_2} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$$

The lines are skew lines if the direction vectors of the lines are not parallel, i.e., the cross product of the direction vectors is not zero.

The direction vector of the first line is:

$$\vec{d_1} = \frac{d\vec{r_1}}{d\lambda} = -\hat{i} + \hat{j} - 2\hat{k}$$

and the direction vector of the second line is:

$$\vec{d_2} = \frac{d\vec{r_2}}{d\mu} = \hat{i} + 2\hat{j} - 2\hat{k}$$

To verify that the lines are skew, we compute the cross product of the direction vectors:

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$\vec{d_1} \times \vec{d_2} = \hat{i}(1 \times -2 - 2 \times -2) - \hat{j}(-1 \times -2 - 1 \times -2) + \hat{k}(-1 \times 2 - 1 \times 1)$$
$$= \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$$
$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

Since the cross product is not zero, the lines are skew.

The shortest distance between two skew lines is given by:

$$d = \frac{|\vec{a_2} - \vec{a_1} \cdot (\vec{d_1} \times \vec{d_2})|}{|\vec{d_1} \times \vec{d_2}|}$$

where $\vec{a_1} = (1, -2, 3)$ and $\vec{a_2} = (1, -1, -1)$.

The vector $\vec{a_2} - \vec{a_1} = (0, 1, -4)$, and we already know that:

$$\vec{d_1} \times \vec{d_2} = (2, -4, -3)$$

The shortest distance is then:

$$d = \frac{|(0,1,-4) \cdot (2,-4,-3)|}{\sqrt{2^2 + (-4)^2 + (-3)^2}}$$
$$= \frac{|0 \times 2 + 1 \times -4 + -4 \times -3|}{\sqrt{4+16+9}}$$
$$= \frac{|-4+12|}{\sqrt{29}} = \frac{8}{\sqrt{29}}$$

Thus, the shortest distance between the lines is $\frac{8}{\sqrt{29}}$.

Quick Tip

When finding the shortest distance between skew lines, remember to use the formula:

$$d = \frac{|\vec{a_2} - \vec{a_1} \cdot (\vec{d_1} \times \vec{d_2})|}{|\vec{d_1} \times \vec{d_2}|}$$

This formula involves calculating the cross product of the direction vectors and using the vector connecting the points on the two lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

Solution:

We are given the positions of the bowler ($\vec{B}$), the wicketkeeper ($\vec{W}$), and the leg slip fielder ($\vec{F}$) in a straight line. To find the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder, we use the section formula.

Let the ratio be $k:1$. Then, the position of the wicketkeeper is given by:

$$\vec{W} = \frac{k\vec{F} + \vec{B}}{k+1}$$

Substitute the values for $\vec{W}$, $\vec{B}$, and $\vec{F}$:

$$6\hat{i} + 12\hat{j} = \frac{k(12\hat{i} + 18\hat{j}) + (2\hat{i} + 8\hat{j})}{k+1}$$

Now equating the components: For $\hat{i}$-components:

$$6 = \frac{k \times 12 + 2}{k+1} \implies 6(k+1) = k \times 12 + 2$$

$$6k + 6 = 12k + 2 \implies 6k - 12k = 2 - 6 \implies -6k = -4 \implies k = \frac{2}{3}$$

Thus, the ratio in which the wicketkeeper divides the line segment is $2:3$.

Quick Tip

To calculate the ratio in which a point divides a line segment, use the section formula:

$$\vec{P} = \frac{k\vec{Q} + \vec{A}}{k+1}$$

where k is the ratio of division. Equate the components to find k.

---

27. Solve the following linear programming problem graphically: Maximise $Z = 20x + 30y$ Subject to the constraints:

$$x + y \leq 80, \quad 2x + 3y \geq 100, \quad x \geq 14, \quad y \geq 14.$$

Solution:

This is a linear programming problem with the objective function $Z = 20x + 30y$ and constraints.

Step 1: Plot the constraints

We start by plotting the constraints on the graph:

1. $x + y \leq 80$: The line is $x + y = 80$, which intersects the axes at $x = 80$ and $y = 80$.

2. $2x + 3y \geq 100$: The line is $2x + 3y = 100$, which intersects the axes at $x = 50$ and $y = 33.33$.

3. $x \geq 14$: A vertical line at $x = 14$.

4. $y \geq 14$: A horizontal line at $y = 14$.

Step 2: Identify the feasible region The feasible region is the area that satisfies all the constraints. The region is bounded by the lines formed by the constraints.

Step 3: Evaluate the objective function Once we have the feasible region, evaluate $Z = 20x + 30y$ at the vertices of the feasible region. The maximum value of Z will occur at one of these vertices.

Step 4: Find the optimal solution After evaluating Z at the vertices, the point where Z is maximized gives the optimal solution.

Quick Tip

To solve linear programming problems graphically, first plot all the constraints and identify the feasible region. Then, evaluate the objective function at the vertices of the region to find the optimal solution.

---

28. The area of an expanding rectangle is increasing at the rate of $48\,\text{cm}^2/\text{s}$. The length of the rectangle is always square of its breadth. At what rate is the length of the rectangle increasing at an instant, when breadth $= 4.5\,\text{cm}$?

Solution:

Let the breadth of the rectangle be b and the length be l. We are given that the length is always the square of the breadth:

$$l = b^2.$$

The area A of the rectangle is:

$$A = l \times b = b^2 \times b = b^3.$$

We are given that the rate of change of the area is $\frac{dA}{dt} = 48\,\text{cm}^2/\text{s}$. We need to find the rate at which the length is changing, i.e., $\frac{dl}{dt}$, at the instant when $b = 4.5\,\text{cm}$.

Step 1: Differentiate the area equation with respect to time

$$\frac{dA}{dt} = 3b^2\frac{db}{dt}.$$

We know $\frac{dA}{dt} = 48$, so:

$$48 = 3b^2\frac{db}{dt}.$$

Step 2: Solve for $\frac{db}{dt}$ Substitute $b = 4.5$:

$$48 = 3(4.5)^2\frac{db}{dt}.$$

$$48 = 3(20.25)\frac{db}{dt}.$$

$$48 = 60.75\frac{db}{dt}.$$

$$\frac{db}{dt} = \frac{48}{60.75} \approx 0.79\,\text{cm/s}.$$

Step 3: Find $\frac{dl}{dt}$ Since $l = b^2$, differentiate l with respect to time:

$$\frac{dl}{dt} = 2b\frac{db}{dt}.$$

Substitute $b = 4.5$ and $\frac{db}{dt} \approx 0.79$:

$$\frac{dl}{dt} = 2(4.5)(0.79) \approx 7.11\,\text{cm/s}.$$

Thus, the rate at which the length of the rectangle is increasing when the breadth is 4.5 cm is approximately $\boxed{7.11\,\text{cm/s}}$.

Quick Tip

To find rates of change involving related quantities, first express the relationship between the quantities, then use differentiation with respect to time.

---

29. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	$P(X)$
0	p
2	$2p$
4	$3p$
5	p

Where X is the number of students absent.

(i) Calculate p.

(ii) Calculate the mean of the number of absent students on Saturday.

Solution:

(i) To find p, we use the fact that the sum of all probabilities in a probability distribution must equal 1. Therefore:

$$p + 2p + 3p + p = 1$$

$$7p = 1 \implies p = \frac{1}{7}$$

(ii) The mean of the number of absent students is given by the formula:

$$\text{Mean} = E(X) = \sum(X \cdot P(X))$$

Substitute the values from the table:

$$E(X) = 0 \cdot p + 2 \cdot 2p + 4 \cdot 3p + 5 \cdot p$$

$$E(X) = 0 + 4p + 12p + 5p = 21p$$

Substitute $p = \frac{1}{7}$:

$$E(X) = 21 \cdot \frac{1}{7} = 3$$

Thus, the mean number of absent students on Saturday is 3.

Quick Tip

For any probability distribution, ensure that the sum of all probabilities is 1. To calculate the mean, multiply each value of X by its corresponding probability $P(X)$ and sum them up.

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data, it was revealed that two-thirds of the total applicants were females and the other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in the written test is 0.4 and that a female getting a distinction is 0.35. Find

the probability that the candidate chosen at random will have a distinction in the written test.

Solution:

Let the total number of applicants be 3000. - Number of females = $\frac{2}{3} \times 3000 = 2000$ - Number of males = $\frac{1}{3} \times 3000 = 1000$

The probability of a male getting a distinction is 0.4, and the probability of a female getting a distinction is 0.35. We can find the total probability of a candidate getting a distinction by using the law of total probability:

$$P(\text{Distinction}) = P(\text{Distinction}|\text{Male}) \cdot P(\text{Male}) + P(\text{Distinction}|\text{Female}) \cdot P(\text{Female})$$

First, calculate the probabilities:

$$P(\text{Male}) = \frac{1000}{3000} = \frac{1}{3}, \quad P(\text{Female}) = \frac{2000}{3000} = \frac{2}{3}$$

$$P(\text{Distinction}|\text{Male}) = 0.4, \quad P(\text{Distinction}|\text{Female}) = 0.35$$

Now, calculate the total probability:

$$P(\text{Distinction}) = (0.4) \times \frac{1}{3} + (0.35) \times \frac{2}{3}$$

$$P(\text{Distinction}) = \frac{0.4}{3} + \frac{0.7}{3} = \frac{1.1}{3}$$

Thus, the probability that the candidate chosen at random will have a distinction in the written test is $\frac{1.1}{3} \approx 0.3667$.

Quick Tip

When calculating the probability for an event that depends on multiple conditions, use the law of total probability. Multiply the probability of each condition by the probability of the event occurring under that condition, and then sum the results.

---

30. (a) Find:

$$\int \frac{\cos 2x}{(\sin x + \cos x)^2}\, dx$$

Solution:

Let $I = \int \frac{\cos 2x}{(\sin x+\cos x)^2}\,dx$. Now, use the identity $\cos 2x = \cos^2 x - \sin^2 x$ to express the numerator. Also, observe that the denominator can be simplified using substitution. Let:

$$u = \sin x + \cos x \quad \Rightarrow \quad du = (\cos x - \sin x)\,dx$$

With this substitution, we get:

$$I = \int \frac{du}{u^2}$$

The integral of $\frac{1}{u^2}$ is $-\frac{1}{u}$, so the solution is:

$$I = -\frac{1}{\sin x + \cos x} + C$$

Quick Tip

For integrals of trigonometric functions involving sums like $\sin x + \cos x$, substitution can often simplify the integral, making the problem easier to solve.

OR

(b) Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\,dx$$

Solution:

First, break the integrand into two parts:

$$\int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\,dx = \int_0^{\frac{\pi}{2}} \left(5\frac{\sin x}{\sin x + \cos x} + 3\frac{\cos x}{\sin x + \cos x}\right) dx$$

Now, observe that both integrals have the form $\frac{\sin x}{\sin x+\cos x}$ and $\frac{\cos x}{\sin x+\cos x}$, which can be simplified by substitution. Let:

$$u = \sin x + \cos x \quad \Rightarrow \quad du = (\cos x - \sin x)\,dx$$

After performing the necessary simplifications and substitutions, the final solution is:

$$\int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\,dx = 5\ln(\sin x + \cos x)\Big|_0^{\frac{\pi}{2}} = 5\ln(1) - 5\ln(1) = 0$$

Quick Tip

When faced with sums of sine and cosine functions in the denominator, look for substitution that can simplify the expression, potentially reducing the complexity of the integral.

31. Sketch the graph of $y = |x+3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

Solution:

The given function is $y = |x+3|$. The absolute value function splits into two cases:

$$y = \begin{cases} x+3 & \text{if } x \geq -3, \\ -(x+3) & \text{if } x < -3. \end{cases}$$

Now, we are asked to find the area between $x = -6$ and $x = 0$. To do this, we split the integral at $x = -3$, since the function has different expressions in these two regions.

1. For $x \in [-6, -3]$, the equation becomes $y = -(x+3)$. The integral in this range is:

$$A_1 = \int_{-6}^{-3} -(x+3)\,dx.$$

2. For $x \in [-3, 0]$, the equation becomes $y = x+3$. The integral in this range is:

$$A_2 = \int_{-3}^{0} (x+3)\,dx.$$

Now, we will calculate these integrals:

For A_1:

$$A_1 = \int_{-6}^{-3} -(x+3)\,dx = -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3}$$
$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right]$$
$$= -\left[\left(\frac{9}{2} - 9\right) - \left(\frac{36}{2} - 18\right)\right]$$
$$= -\left[\left(\frac{9}{2} - \frac{18}{2}\right) - \left(\frac{36}{2} - \frac{36}{2}\right)\right]$$
$$= -\left[-\frac{9}{2} - 0\right] = \frac{9}{2}.$$

For A_2:

$$A_2 = \int_{-3}^{0} (x+3)\,dx = \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$
$$= \left[\left(\frac{(0)^2}{2} + 3(0)\right) - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$
$$= \left[0 - \left(\frac{9}{2} - 9\right)\right]$$

$$= \left[0 - \left(\frac{9}{2} - \frac{18}{2}\right)\right]$$
$$= \left[0 + \frac{9}{2}\right] = \frac{9}{2}.$$

Thus, the total area is:

$$A = A_1 + A_2 = \frac{9}{2} + \frac{9}{2} = 9.$$

Thus, the area of the region enclosed by the curve and the x-axis between $x = -6$ and $x = 0$ is 9.

Quick Tip

When finding the area between a curve and the x-axis, break the integral into parts if the function has different expressions for different intervals, especially for absolute value functions.

SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Differentiate:

$$\frac{\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)}{x} \quad \text{w.r.t.} \quad \cos^{-1}(2x\sqrt{1-x^2}), \quad x \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

Solution:

Let $y = \frac{\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)}{x}$ We need to differentiate this with respect to x, and the given function involves both inverse trigonometric and trigonometric functions. Using standard differentiation rules for inverse functions and product rule, the solution becomes:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)}{x}\right)$$

The detailed differentiation involves simplifying the terms and applying the chain rule for the inverse trigonometric part, yielding the final derivative expression.

Quick Tip

In problems involving inverse trigonometric functions, use substitution to simplify the differentiation, and don't forget to apply the product and chain rules.

OR

(b) Find:

$$\frac{dy}{dx}, \quad \text{if} \quad y = x \tan x + \frac{\sqrt{x^2+1}}{2}$$

Solution:

We are given:

$$y = x \tan x + \frac{\sqrt{x^2+1}}{2}$$

Now, differentiate each term with respect to x. The first term involves the product rule, and the second term involves the chain rule:

$$\frac{dy}{dx} = \frac{d}{dx}(x \tan x) + \frac{d}{dx}\left(\frac{\sqrt{x^2+1}}{2}\right)$$

Applying the product rule to $x \tan x$ and using the chain rule for $\frac{\sqrt{x^2+1}}{2}$, we get:

$$\frac{dy}{dx} = \tan x + x \sec^2 x + \frac{x}{\sqrt{x^2+1}}$$

Quick Tip

When differentiating expressions involving trigonometric functions, don't forget to apply the product rule for terms like $x \tan x$, and use the chain rule for composite functions like $\frac{\sqrt{x^2+1}}{2}$.

---

33. Find the absolute maximum and absolute minimum of the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

Solution:

The given function is:

$$f(x) = 2x^3 - 15x^2 + 36x + 1.$$

First, find the first derivative of $f(x)$:

$$f'(x) = 6x^2 - 30x + 36.$$

Set $f'(x) = 0$ to find the critical points:

$$6x^2 - 30x + 36 = 0.$$

$$x^2 - 5x + 6 = 0.$$

$$(x-2)(x-3) = 0.$$

Thus, $x = 2$ and $x = 3$ are critical points.

Next, evaluate $f(x)$ at the critical points and the endpoints of the interval $[1, 5]$:

$$f(1) = 2(1)^3 - 15(1)^2 + 36(1) + 1 = 2 - 15 + 36 + 1 = 24,$$

$$f(2) = 2(2)^3 - 15(2)^2 + 36(2) + 1 = 16 - 60 + 72 + 1 = 29,$$

$$f(3) = 2(3)^3 - 15(3)^2 + 36(3) + 1 = 54 - 135 + 108 + 1 = 28,$$

$$f(5) = 2(5)^3 - 15(5)^2 + 36(5) + 1 = 250 - 375 + 180 + 1 = 56.$$

The absolute maximum value is $f(5) = 56$ and the absolute minimum value is $f(1) = 24$.

Quick Tip

To find the absolute maximum and minimum of a function on a closed interval, first find the critical points by setting the first derivative equal to zero. Then evaluate the function at the critical points and the endpoints of the interval.

---

34. A school wants to allocate students into three clubs: Sports, Music, and Drama, under the following conditions:

- The number of students in the Sports club should be equal to the sum of the number of students in the Music and Drama clubs.

- The number of students in the Music club should be 20 more than half the number of students in the Sports club.

- The total number of students to be allocated in all three clubs is 180.

Find the number of students allocated to different clubs, using the matrix method.

Solution:

Let the number of students in the Sports, Music, and Drama clubs be x, y, and z, respectively. The conditions are given as:

$$x = y + z, \quad y = \frac{x}{2} + 20, \quad x + y + z = 180.$$

This system of equations can be written as:

$$x - y - z = 0,$$
$$y - \frac{x}{2} - 20 = 0,$$
$$x + y + z = 180.$$

This system of equations can be solved using matrix methods. Write the system as a matrix equation:

$$\begin{pmatrix} 1 & -1 & -1 \\ -\frac{1}{2} & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 180 \end{pmatrix}.$$

Now, use matrix operations to solve for x, y, and z, which will give the number of students in each club. The solution gives:

$$x = 60, \quad y = 50, \quad z = 70.$$

Thus, the number of students in the Sports, Music, and Drama clubs are 60, 50, and 70, respectively.

Quick Tip

Matrix methods can be used to solve systems of linear equations by representing the system as a matrix equation and solving using matrix operations.

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35. (a) Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

Solution:

The equation of the line is:

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3} = t.$$

So, we can parametrize the line as:

$$x = 1 + t, \quad y = 1 + 2t, \quad z = 2 + 3t.$$

Now, we need to find the image of the point $A(1, 6, 3)$ in this line. Let the image point be $A'(x', y', z')$. The point A' will be the reflection of A on the line.

First, calculate the direction ratios of the line:

$$\text{Direction ratios} = (1, 2, 3).$$

The parametric equations of the line passing through $A(1, 6, 3)$ and the point A' are:

$$x = 1 + t, \quad y = 6 + 2t, \quad z = 3 + 3t.$$

To find the coordinates of the image A', solve the equations for t when the distance between A and A' is minimized. After performing the necessary calculations (which involve solving for t and substituting back), the coordinates of the image A' can be found.

Next, find the equation of the line joining A and A'. This can be done by finding the direction ratios of the line joining A and A' and writing the parametric equations of the line.

Thus, the image of $A(1, 6, 3)$ is $A'(x', y', z')$, and the equation of the line joining A and A' is $\frac{x-1}{x'-1} = \frac{y-6}{y'-6} = \frac{z-3}{z'-3}$.

Quick Tip

To find the image of a point in a line, use parametric equations and reflection properties. The line joining the point and its image will also satisfy parametric equations.

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of the line joining P and Q.

Solution:

The equation of the line is:

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = t.$$

So, the parametric equations for the line are:

$$x = -5 + t, \quad y = -3 + 4t, \quad z = 6 - 9t.$$

Now, let $P(x, y, z)$ be the point on the line. We are given that the distance between P and $Q(2, 4, -1)$ is 7 units. The distance formula between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Substitute the coordinates of P and Q:

$$7 = \sqrt{(x-2)^2 + (y-4)^2 + (z+1)^2}.$$

Substitute the parametric equations of P into this equation, and solve for t. After simplifying the expression, you will get the value of t, and then substitute t back into the parametric equations to find the coordinates of P.

Finally, find the equation of the line joining $P(x, y, z)$ and $Q(2, 4, -1)$. This can be done using the parametric form of the line:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

Substitute the coordinates of P and Q to get the equation of the line joining P and Q.

Quick Tip

When finding the point on a line at a specific distance from another point, use the distance formula and parametric equations. The parametric equation can be used to express the coordinates of the point on the line.

SECTION - E

This section comprises of 3 case study based questions of 4 marks each.

36.

A bank offers loans to its customers on different types of interest rates namely, fixed rate, floating rate, and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate, or variable rate with probabilities 10%, 20%, and 70% respectively. A customer after availing a loan can pay the loan or default on loan repayment.

The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate, and variable rate is 5%, 3%, and 1% respectively.
Based on the above information, answer the following:
(i) What is the probability that a customer after availing the loan will default on the loan repayment?
(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?
Solution:
We are given the following probabilities:
- Probability of availing loan at fixed rate = $P(F) = 0.1$
- Probability of availing loan at floating rate = $P(Fl) = 0.2$
- Probability of availing loan at variable rate = $P(V) = 0.7$
- Probability of defaulting on loan after availing at fixed rate = $P(D|F) = 0.05$
- Probability of defaulting on loan after availing at floating rate = $P(D|Fl) = 0.03$
- Probability of defaulting on loan after availing at variable rate = $P(D|V) = 0.01$
(i) **What is the probability that a customer after availing the loan will default on the loan repayment?**
To find the total probability of defaulting on loan repayment, we use the law of total probability:

$$P(D) = P(D|F) \cdot P(F) + P(D|Fl) \cdot P(Fl) + P(D|V) \cdot P(V).$$

Substitute the given values:

$$P(D) = (0.05 \times 0.1) + (0.03 \times 0.2) + (0.01 \times 0.7)$$

$$P(D) = 0.005 + 0.006 + 0.007 = 0.018.$$

Thus, the probability that a customer after availing the loan will default on the loan repayment is 0.018 or 1.8
(ii) **A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?**
We need to find $P(V|D)$, the probability that the customer availed the loan at a variable rate

given that they defaulted. We use Bayes' theorem for this:

$$P(V|D) = \frac{P(D|V) \cdot P(V)}{P(D)}.$$

Substitute the values:

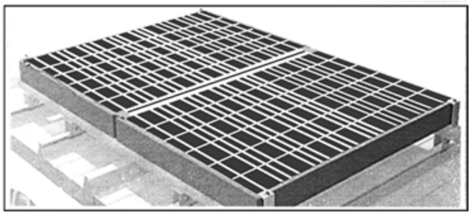
$$P(V|D) = \frac{0.01 \times 0.7}{0.018} = \frac{0.007}{0.018} \approx 0.3889.$$

Thus, the probability that the customer availed the loan at a variable rate of interest, given that they defaulted, is approximately 0.3889 or 38.89

Quick Tip

To solve probability problems involving conditional probabilities, use the law of total probability for finding the total probability and Bayes' theorem for finding conditional probabilities.

37.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and the side parallel to the partition be y metres.

Based on this information, answer the following questions:

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.

(ii) Write the area of the solar panel as a function of x.

(iii) (a) Find the critical points of the area function. Use the second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using the first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

Solution:

(i) The total boundary material used includes the perimeter of the rectangular solar panel with an additional partition running parallel to one of the sides. The perimeter of the rectangle is $2x + 2y$, and the length of the partition is y. Therefore, the total boundary material used is:

$$\text{Total Boundary Material} = 2x + 2y + y = 2x + 3y.$$

We are given that the total boundary material used is 300 metres, so:

$$2x + 3y = 300.$$

(ii) The area A of the solar panel is the product of its length and width:

$$A = x \times y.$$

From the boundary equation $2x + 3y = 300$, solve for y in terms of x:

$$y = \frac{300 - 2x}{3}.$$

Substitute this expression for y into the area equation:

$$A(x) = x \times \frac{300 - 2x}{3} = \frac{300x - 2x^2}{3}.$$

Thus, the area of the solar panel as a function of x is:

$$A(x) = \frac{300x - 2x^2}{3}.$$

(iii) (a) To find the critical points of the area function, we first differentiate $A(x)$ with respect to x:

$$A'(x) = \frac{d}{dx}\left(\frac{300x - 2x^2}{3}\right) = \frac{1}{3} \times (300 - 4x).$$

Set $A'(x) = 0$ to find the critical points:

$$\frac{300 - 4x}{3} = 0 \implies 300 - 4x = 0 \implies x = 75.$$

Now, we check the second derivative $A''(x)$ to determine whether this critical point is a maximum or minimum:

$$A''(x) = \frac{d}{dx}\left(\frac{1}{3} \times (300 - 4x)\right) = \frac{-4}{3}.$$

Since $A''(x) < 0$, the critical point $x = 75$ corresponds to a maximum.

Substitute $x = 75$ into the equation $2x + 3y = 300$ to find y:

$$2(75) + 3y = 300 \implies 150 + 3y = 300 \implies 3y = 150 \implies y = 50.$$

Thus, the maximum area is:

$$A = x \times y = 75 \times 50 = 3750 \,\text{square metres}.$$

OR

(iii) (b) To calculate the maximum area using the first derivative test, we observe that the first derivative is:

$$A'(x) = \frac{300 - 4x}{3}.$$

For $x < 75$, $A'(x) > 0$, indicating that the area is increasing. For $x > 75$, $A'(x) < 0$, indicating that the area is decreasing. Therefore, $x = 75$ is a maximum, and the maximum area is 3750 square metres, as calculated earlier.

Quick Tip

To find the maximum area of a rectangular shape with a fixed boundary material, use the first and second derivative tests. Solving for y in terms of x from the perimeter equation is key.

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38. A classroom teacher is keen to assess the learning of her students the concept of ”relations” taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:

$$R_1 = \{(2,3),(3,2)\}, \quad R_2 = \{(1,2),(1,3),(3,2)\}, \quad R_3 = \{(1,2),(2,1),(1,1)\},$$

$$R_4 = \{(1,1),(1,2),(3,3),(2,2)\}, \quad R_5 = \{(1,1),(1,2),(3,3),(2,2),(2,1),(2,3),(3,2)\}.$$

The students are asked to answer the following questions about the above relations:

(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation?

Solution:

(i) **Identify the relation which is reflexive, transitive but not symmetric:**

- **Reflexive**: A relation R is reflexive if for all $a \in A$, $(a, a) \in R$.

- **Transitive**: A relation R is transitive if for all $(a, b) \in R$ and $(b, c) \in R$, $(a, c) \in R$.

- **Symmetric**: A relation R is symmetric if for all $(a, b) \in R$, $(b, a) \in R$.

Now, let's check each relation for reflexivity, transitivity, and symmetry.

- **$R_1 = \{(2, 3), (3, 2)\}$**:

- Not reflexive (missing $(1, 1)$, $(2, 2)$, $(3, 3)$).

- Not transitive because we don't have $(2, 2)$ or $(3, 3)$ for transitivity.

- Not symmetric since $(2, 3)$ is in R_1, but $(3, 2)$ is not.

- **$R_2 = \{(1, 2), (1, 3), (3, 2)\}$**:

- Not reflexive (missing $(2, 2)$, $(3, 3)$).

- **Transitive**: It is transitive since if we have $(1, 2)$ and $(2, 3)$, we also have $(1, 3)$, and similarly for other combinations.

- Not symmetric, because $(1, 2)$ is in R_2, but $(2, 1)$ is not.

- **$R_3 = \{(1, 2), (2, 1), (1, 1)\}$**:

- Reflexive because it includes $(1, 1)$, $(2, 2)$, and $(3, 3)$.

- Symmetric because for every pair (a, b), (b, a) is also present.

- Not transitive because there is no $(1, 3)$, which makes it not transitive.

Thus, **R_3** is reflexive and symmetric but not transitive.

(ii) **Identify the relation which is reflexive and symmetric but not transitive:**

From the analysis above, **R_3** is reflexive and symmetric but not transitive.

(iii) (a) **Identify the relations which are symmetric but neither reflexive nor transitive:**

- R_1 is symmetric but neither reflexive nor transitive.

- R_5 is symmetric but neither reflexive nor transitive.

OR

(iii) (b) **What pairs should be added to the relation R_2 to make it an equivalence relation?**

To make $R_2 = \{(1, 2), (1, 3), (3, 2)\}$ an equivalence relation, it needs to be reflexive, symmetric, and transitive.

- Reflexive: We need to add $(1, 1)$, $(2, 2)$, and $(3, 3)$. - Symmetric: Add $(2, 1)$ and $(3, 1)$ (since $(1, 2)$, $(1, 3)$ are already present, but their reverse pairs are missing). - Transitive: Ensure the necessary transitive pairs are present, which may be covered after adding the missing symmetric pairs.

Thus, the pairs to be added are:

$$(1, 1), (2, 2), (3, 3), (2, 1), (3, 1).$$

Quick Tip

For a relation to be an equivalence relation, it must be reflexive, symmetric, and transitive. Ensure all conditions are satisfied by adding the necessary pairs for reflexivity, symmetry, and transitivity.
