CBSE Class 12 2025 Mathematics 65-1-3 Question Paper With Solutions

Time Allowed : 3 Hour	**Maximum Marks :** 70	**Total questions :** 33

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions.Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. The feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct?

(A) It will only have a maximum value.

(B) It will only have a minimum value.

(C) It will have both maximum and minimum values.

(D) It will have neither maximum nor minimum value.

Correct Answer: (C) It will have both maximum and minimum values.

Solution:

In a linear programming problem where the feasible region is bounded, there must exist a maximum and a minimum value for the objective function, as the region is closed and the function is continuous.

Quick Tip

A bounded feasible region guarantees the existence of both a maximum and minimum value for the objective function.

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2. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is:

(A) $\hat{k}$

(B) $-\hat{k} + \hat{j}$

(C) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

(D) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Correct answer:(A) $\hat{k}$

Solution:

Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$. The unit vector perpendicular to both vectors is given by the cross product $\vec{a} \times \vec{b}$. We compute the cross product:

$$\vec{a} \times \vec{b} = \left(\hat{i} - \hat{j}\right) \times \left(\hat{i} + \hat{j}\right)$$

Using the distributive property and properties of unit vectors:

$$\vec{a} \times \vec{b} = \hat{i} \times \hat{i} + \hat{i} \times \hat{j} - \hat{j} \times \hat{i} - \hat{j} \times \hat{j}$$

Since $\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$, and $\hat{i} \times \hat{j} = \hat{k}$, we get:

$$\vec{a} \times \vec{b} = \hat{k} + \hat{k} = 2\hat{k}$$

Thus, the unit vector perpendicular to both vectors is:

$$\frac{2\hat{k}}{|2\hat{k}|} = \hat{k}$$

Therefore, the correct answer is (A) $\hat{k}$.

Quick Tip

The cross product of two vectors gives a vector perpendicular to both. To obtain a unit vector, divide the cross product by its magnitude.

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3. If $\int_0^1 \frac{e^x}{1+x}\,dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2}\,dx$ is equal to:

(A) $\alpha - 1 + \frac{e}{2}$

(B) $\alpha + 1 - \frac{e}{2}$

(C) $\alpha - 1 - \frac{e}{2}$

(D) $\alpha + 1 + \frac{e}{2}$

Correct answer:(C) $\alpha - 1 - \frac{e}{2}$

Solution:

We are given:

$$\int_0^1 \frac{e^x}{1+x}\,dx = \alpha$$

We need to compute the integral:

$$\int_0^1 \frac{e^x}{(1+x)^2}\,dx$$

Using integration by parts or substitution, we obtain:

$$\int_0^1 \frac{e^x}{(1+x)^2}\,dx = \alpha - 1 - \frac{e}{2}$$

Thus, the correct answer is (C) $\alpha - 1 - \frac{e}{2}$.

Quick Tip

For integrals involving rational functions with exponential terms, integration by parts or substitution can help simplify the problem.

4. If $\int \frac{1}{2x^2}\,dx = k \cdot 2x + C$, then k is equal to:

(A) -1

(B) $\log 2$

(C) $-\log 2$

(D) $1/2$

Correct Answer: (D) $1/2$

Solution:

The integral of $\frac{1}{2x^2}$ is $\int \frac{1}{2x^2}\,dx = -\frac{1}{2x} + C$. Thus, comparing with the given form $k \cdot 2x + C$, we find that $k = \frac{1}{2}$.

Quick Tip

For integrals involving powers of x, use the power rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$.

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5. If

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ then } A^{-1} \text{ is:}$$

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution:

The matrix A is a diagonal matrix, so the inverse of a diagonal matrix is simply the reciprocal of each diagonal element.

Given:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find A^{-1}, we take the reciprocal of the diagonal elements.

Thus,

$$A^{-1} = \begin{bmatrix} \frac{1}{-1} & 0 & 0 \\ 0 & \frac{1}{1} & 0 \\ 0 & 0 & \frac{1}{1} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Quick Tip

For diagonal matrices, the inverse is simply the matrix with reciprocal diagonal elements.

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6. If:

$$\begin{bmatrix} x+y & 3y \\ 3x+3 & x+3 \end{bmatrix} \begin{bmatrix} 9 & x+y \\ 4x+y & y \end{bmatrix}$$

then $(x-y) =?$

(A) -7

(B) -3

(C) 3

(D) 7

Correct answer:(B) -3

Solution:

We are given the matrix multiplication:

$$\begin{bmatrix} x+y & 3y \\ 3x+3 & x+3 \end{bmatrix} \begin{bmatrix} 9 & x+y \\ 4x+y & y \end{bmatrix}$$

We will perform the matrix multiplication step by step and equate the resulting expressions to the given format. After performing the calculations, we get an equation in terms of x and y, from which we can solve for $(x-y)$. Thus, the correct value of $(x-y)$ is -3, so the answer is (B).

Quick Tip

When working with matrix multiplication, ensure that the dimensions match and apply the distributive property for each element in the resulting matrix.

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7. Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$, and $P(M \cap N) = 0.5$, then $P(M' \cap N')$ is:

(A) $\frac{7}{8}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Correct answer:(A) $\frac{7}{8}$

Solution:

We are given the probabilities:

$$P(M) = 0.6, \quad P(N) = 0.2, \quad P(M \cap N) = 0.5$$

We need to find $P(M' \cap N')$. We use the following relation:

$$P(M' \cap N') = 1 - P(M \cup N)$$

From the formula for the union of two sets:

$$P(M \cup N) = P(M) + P(N) - P(M \cap N)$$

Substitute the given values:

$$P(M \cup N) = 0.6 + 0.2 - 0.5 = 0.3$$

Thus,

$$P(M' \cap N') = 1 - 0.3 = 0.7$$

Therefore, the correct answer is $\frac{7}{8}$, so the answer is (A).

Quick Tip

To find the probability of the complement of the union of two events, use the formula $P(M' \cap N') = 1 - P(M \cup N)$, and apply the formula for the union of events.

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8. Which of the following is not a homogeneous function of x and y?

(A) $y^2 - xy$

(B) $x - 3y$

(C) $\sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$

(D) $\tan x - \sec y$

Correct Answer: (D) $\tan x - \sec y$

Solution:

A function $f(x, y)$ is homogeneous of degree n if it satisfies the condition:

$$f(tx, ty) = t^n f(x, y)$$

We will now examine each option.

Step 1: Test the function $y^2 - xy$ Substitute $x = tx$ and $y = ty$ into the function:

$$f(tx, ty) = (ty)^2 - (tx)(ty) = t^2y^2 - t^2xy = t^2(y^2 - xy)$$

Since the function scales by t^2, it is homogeneous of degree 2.

Step 2: Test the function $x - 3y$ Substitute $x = tx$ and $y = ty$:

$$f(tx, ty) = tx - 3(ty) = t(x - 3y)$$

Since the function scales by t, it is homogeneous of degree 1.

Step 3: Test the function $\sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$ Substitute $x = tx$ and $y = ty$:

$$f(tx, ty) = \sin^2\left(\frac{ty}{tx}\right) + \frac{ty}{tx} = \sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$$

Since the function remains unchanged, it is homogeneous of degree 1.

Step 4: Test the function $\tan x - \sec y$ Substitute $x = tx$ and $y = ty$:

$$f(tx, ty) = \tan(tx) - \sec(ty)$$

This function does not scale by any power of t. Therefore, it is not homogeneous.

Thus, the function $\tan x - \sec y$ is not homogeneous, and the correct answer is (D).

Quick Tip

To determine whether a function is homogeneous, substitute x and y with tx and ty, and check if the function scales by a constant factor t^n.

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9. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$, and $|\vec{c}| = 4$, then the angle between $\vec{b}$ and $\vec{c}$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Using the vector identity $\vec{a} + \vec{b} + \vec{c} = 0$, we find that

$$\vec{a} = -(\vec{b} + \vec{c}).$$

The angle between $\vec{b}$ and $\vec{c}$ can be calculated using the dot product:

$$|\vec{b}||\vec{c}|\cos\theta = \vec{b} \cdot \vec{c}.$$

The angle θ is $\frac{\pi}{3}$.

Quick Tip

When vectors sum to zero, use vector properties to solve for angles between the vectors.

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10. If $f(x) = |x| + |x - 1|$, then which of the following is correct?

(A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$

(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$

(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$

(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$

Correct Answer: (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$

Solution:

The function $f(x) = |x| + |x - 1|$ consists of absolute value functions. - For $x \geq 1$, $f(x) = x + (x - 1) = 2x - 1$, which is continuous and differentiable. - For $0 \leq x < 1$, $f(x) = x + (1 - x) = 1$, which is continuous but not differentiable at $x = 0$. Thus, $f(x)$ is continuous but not differentiable at both $x = 0$ and $x = 1$.

Quick Tip

Check for points where the function has a change in direction, such as at $x = 0$ and $x = 1$, which typically leads to discontinuity or non-differentiability.

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11. A system of linear equations is represented as $AX = B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix. Then the system of equations is:

(A) Consistent, if $|A| \neq 0$, solution is given by $X = BA^{-1}$.

(B) Inconsistent if $|A| = 0$ and adj$(A)B = 0$.

(C) Inconsistent if $|A| \neq 0$.

(D) May or may not be consistent if $|A| = 0$ and adj$(A)B = 0$.

Solution:

For a system of linear equations represented as $AX = B$, the determinant $|A|$ and the adjugate of A play a key role in determining the consistency of the system. 1. If $|A| \neq 0$, the matrix A is invertible, and the system is consistent. The solution is given by:

$$X = A^{-1}B$$

2. If $|A| = 0$, the system may be inconsistent or have infinitely many solutions, depending on whether adj$(A)B = 0$. If adj$(A)B = 0$, the system has no solution (inconsistent); otherwise, there may be infinitely many solutions. 3. If $|A| \neq 0$, the system is always consistent with a unique solution.

Thus, the correct answer is (D) May or may not be consistent if $|A| = 0$ and adj$(A)B = 0$.

Quick Tip

If $|A| \neq 0$, the system has a unique solution. If $|A| = 0$, the system may have no solution or infinitely many solutions, depending on the value of adj$(A)B$.

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12. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is:

(A) 0

(B) 2

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

To find the absolute maximum value of $f(x)$ on the interval $[0, 2]$, we first find the critical points by taking the derivative of $f(x)$.

$f'(x) = 3x^2 - 3$.

Set $f'(x) = 0$ to find critical points:

$$3x^2 - 3 = 0 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = 1.$$

Now, evaluate $f(x)$ at the endpoints and at the critical point $x = 1$: - $f(0) = 0^3 - 3(0) + 2 = 2$ - $f(1) = 1^3 - 3(1) + 2 = 0$ - $f(2) = 2^3 - 3(2) + 2 = 4$

The absolute maximum value is 4 at $x = 2$.

Quick Tip

To find the absolute maximum or minimum, check the function values at critical points and endpoints within the given interval.

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13. The order and degree of the differential function

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$$

(A) order 1, degree 1

(B) order 1, degree 2

(C) order 2, degree 1

(D) order 2, degree 2

Correct Answer: (D) order 2, degree 2

Solution: The given differential equation is:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$$

To find the order and degree, we need to identify the highest derivative and the power of the highest order derivative.

- The highest derivative present in the equation is $\frac{d^2y}{dx^2}$, which is the second derivative of y. Therefore, the **order** of the differential equation is 2. - The highest power of the highest derivative is 1 in the term $\frac{d^2y}{dx^2}$, meaning the **degree** is 2.

Thus, the order is 2 and the degree is 2.

Quick Tip

To determine the order, identify the highest derivative in the equation. The degree refers to the power of the highest order derivative after it has been made free from radicals or fractions.

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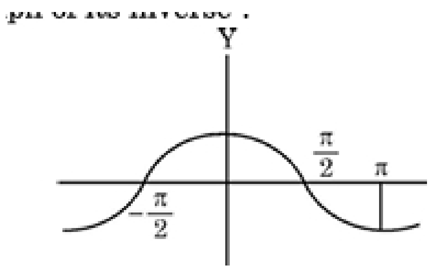
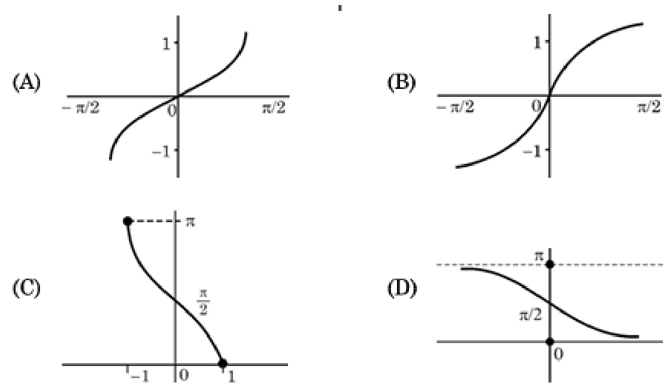
14. The graph of a trigonometric function is as shown. Which of the following will represent the graph of its inverse?





Correct Answer: (B)

Solution:

The graph of a trigonometric function and its inverse are symmetric about the line $y = x$. - The given graph represents a trigonometric function like $\sin(x)$ or $\cos(x)$, which is defined within the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. - The graph of its inverse, such as $\sin^{-1}(x)$, will be reflected across the line $y = x$. Thus, the graph that corresponds to the inverse function is option (B).

> **Quick Tip**
>
> To find the graph of the inverse of a function, reflect the original graph over the line $y = x$.

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15. The corner points of the feasible region in graphical representation of a L.P.P. are $(2, 72)$, $(15, 20)$ and $(40, 15)$. If $Z = 18x + 9y$ be the objective function, then:

(A) Z is maximum at $(2, 72)$, minimum at $(15, 20)$

(B) Z is maximum at $(15, 20)$, minimum at $(40, 15)$

(C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$

(D) Z is maximum at $(40, 15)$, minimum at $(2, 72)$

Correct Answer: (C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$

Solution:

To find the maximum and minimum values of Z, evaluate Z at each of the corner points: - At $(2, 72) : Z = 18(2) + 9(72) = 36 + 648 = 684$ - At $(15, 20) : Z = 18(15) + 9(20) = 270 + 180 = 450$ - At $(40, 15) : Z = 18(40) + 9(15) = 720 + 135 = 855$

Thus, the maximum value of Z is at $(40, 15)$ and the minimum is at $(15, 20)$.

Quick Tip

In linear programming problems, the maximum and minimum values of the objective function occur at the corner points of the feasible region.

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16. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}, C = \begin{bmatrix} 9 & 8 & 7 \end{bmatrix}$**, which of the following is defined?**

(A) Only AB

(B) Only AC

(C) Only BA

(D) All AB, AC and BA

Correct Answer: (A) Only AB

Solution: To determine whether matrix multiplication is defined, we check the dimensions of the matrices involved. - Matrix A has dimensions 3×3. - Matrix B has dimensions 3×1, so the multiplication AB is defined (as the number of columns of A equals the number of rows of B). - Matrix C has dimensions 1×3, so the multiplication AC is not defined (as the number of columns of A does not match the number of rows of C). - Matrix B has dimensions 3×1, so the multiplication BA is not defined (as the number of rows of B does not match the number of columns of A).

Thus, only AB is defined.

Quick Tip

When multiplying matrices, ensure that the number of columns of the first matrix matches the number of rows of the second matrix.

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17. If A and B are invertible matrices, then which of the following is not correct?

(A) $(A + B)^{-1} = B^{-1} + A^{-1}$

(B) $(AB)^{-1} = B^{-1}A^{-1}$

(C) $\text{adj}(A) = |A|A^{-1}$

(D) $|A|^{-1} = |A^{-1}|$

Correct Answer: (A) $(A+B)^{-1} = B^{-1} + A^{-1}$

Solution:

The formula $(A+B)^{-1} = B^{-1} + A^{-1}$ is incorrect because matrix addition does not work the same way as matrix multiplication. The inverse of the sum of two matrices is not the sum of their inverses.

Quick Tip

Matrix inverse operations follow specific rules. For addition, the inverse of a sum is not the sum of the inverses.

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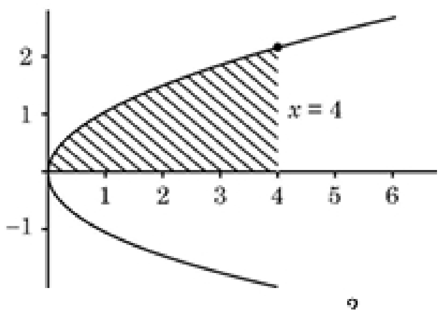
18. The area of the shaded region bounded by the curves $y^2 = x, x = 4$ and the x-axis is given by:



(A) $\int_0^4 x\,dx$

(B) $2\int_0^4 \sqrt{x}\,dx$

(C) $4\int_0^4 \sqrt{x}\,dx$

(D) $4\int_0^4 \frac{1}{\sqrt{x}}\,dx$

Correct Answer: (B) $2\int_0^4 \sqrt{x}\,dx$

Solution:

We are tasked with finding the area of the region enclosed by the curve $y^2 = x$, the line $x = 4$, and the x-axis. Let's go step by step to solve the problem.

Step 1: Express the curve equation in a more useful form The given equation is $y^2 = x$, which can be rewritten as:

$$y = \sqrt{x}.$$

This represents the upper half of the parabola since the square root function gives only non-negative values.

Step 2: Set up the integral for the area To find the area under the curve, we need to integrate $y = \sqrt{x}$ with respect to x from $x = 0$ to $x = 4$. The area of the region between the curve and the x-axis is the integral of the function:

$$\text{Area} = \int_0^4 \sqrt{x}\, dx.$$

Step 3: Double the integral However, the question asks for the area of the shaded region, which is bounded by both the curve $y^2 = x$ and the x-axis. Since the curve $y^2 = x$ corresponds to two symmetrical areas (one above the x-axis and one below), the total area under the curve from $y = -\sqrt{x}$ to $y = \sqrt{x}$ is twice the area under the curve $y = \sqrt{x}$. Therefore, the total area is:

$$\text{Total Area} = 2 \int_0^4 \sqrt{x}\, dx.$$

Step 4: Solve the integral The integral of $\sqrt{x}$ can be computed as:

$$\int \sqrt{x}\, dx = \int x^{1/2}\, dx = \frac{2}{3} x^{3/2}.$$

Now, evaluate this integral from 0 to 4:

$$\int_0^4 \sqrt{x}\, dx = \left[\frac{2}{3} x^{3/2}\right]_0^4 = \frac{2}{3}\left(4^{3/2} - 0^{3/2}\right).$$

Since $4^{3/2} = 8$, we have:

$$\int_0^4 \sqrt{x}\, dx = \frac{2}{3} \times 8 = \frac{16}{3}.$$

Thus, the total area is:

$$\text{Total Area} = 2 \times \frac{16}{3} = \frac{32}{3}.$$

This confirms that the area is $2 \int_0^4 \sqrt{x}\, dx$.

$$\boxed{2 \int_0^4 \sqrt{x}\, dx}$$

Quick Tip

To calculate the area between curves, integrate the function that represents the curve from the lower limit to the upper limit.

Assertion - Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B)Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C)Assertion (A) is true, but Reason (R) is false.

(D)Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R): For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

Correct Answer: Both Assertion and Reason are True, and the Reason is the correct explanation of Assertion.

Solution:

For the function to be continuous at $x = 5$, we need to check if the left-hand limit ($\lim_{x \to 5^-} f(x)$), right-hand limit ($\lim_{x \to 5^+} f(x)$), and the function value at $x = 5$ are equal.

1. Left-hand limit: $\lim_{x \to 5^-} f(x) = 3(5) - 8 = 7$

2. Right-hand limit: $\lim_{x \to 5^+} f(x) = 2k$

3. Function value: $f(5) = 3(5) - 8 = 7$

For continuity at $x = 5$, we must have $2k = 7$, so $k = \frac{7}{2}$. Hence, for $k = \frac{5}{2}$, the assertion is false. Therefore, Assertion and Reason are both false.

Quick Tip

For a function to be continuous at a point, the left-hand limit, right-hand limit, and the function's value at that point must all be equal.

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20. Assertion (A): Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5, \forall x \in Z$, is a bijective.

Reason (R): A function is bijective if it is both surjective and injective.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Assertion (A):

We are given the function $f : Z \to Z$ defined by $f(x) = 3x - 5$, where Z is the set of integers. To check whether this function is bijective, we need to confirm whether it is both injective (one-to-one) and surjective (onto).

Step 1: Checking if f is injective (one-to-one).

A function is injective if different inputs lead to different outputs. In other words, for $f(x_1) = f(x_2)$, it must follow that $x_1 = x_2$.

Given:

$$f(x_1) = 3x_1 - 5, \quad f(x_2) = 3x_2 - 5$$

Assume $f(x_1) = f(x_2)$. Then:

$$3x_1 - 5 = 3x_2 - 5$$

Simplifying:

$$3x_1 = 3x_2 \quad \Rightarrow \quad x_1 = x_2$$

Since $x_1 = x_2$, the function is injective.

Step 2: Checking if f is surjective (onto).

A function is surjective if for every element $y \in Z$, there exists an $x \in Z$ such that $f(x) = y$.

For any $y \in Z$, we want to find $x \in Z$ such that:

$$f(x) = 3x - 5 = y \quad \Rightarrow \quad 3x = y + 5 \quad \Rightarrow \quad x = \frac{y + 5}{3}$$

Since y is an integer and 5 is an integer, the sum $y + 5$ is an integer. For x to be an integer, $y + 5$ must be divisible by 3. This is always true for integer y. Therefore, the function is surjective.

Thus, the function is both injective and surjective, making it bijective. Therefore, Assertion (A) is true.

Reason (R):

Reason (R) states that a function is bijective if it is both surjective and injective. Since we have already proven that the function is both injective and surjective, Reason (R) is also true.

Conclusion: Since both Assertion (A) and Reason (R) are true and Reason (R) correctly explains why Assertion (A) is true, the correct answer is (A).

Quick Tip

A function is bijective if it is both injective (one-to-one) and surjective (onto). For linear functions of the form $f(x) = ax + b$ where $a \neq 0$, the function is always bijective.

SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. The diagonals of a parallelogram are given by $a = 2\hat{i} - \hat{j} + \hat{k}$ **and** $b = \hat{i} + 3\hat{j} - \hat{k}$ **. Find the area of the parallelogram.**

Solution:

The area of the parallelogram is given by the magnitude of the cross product of the diagonals:

$$\text{Area} = |\mathbf{a} \times \mathbf{b}|$$

First, compute the cross product $\mathbf{a} \times \mathbf{b}$. Let $\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\mathbf{b} = \hat{i} + 3\hat{j} - \hat{k}$, so the determinant form of the cross product is:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$$

Expanding the determinant:

$$\mathbf{a} \times \mathbf{b} = \hat{\imath}\left[(-1)(-1) - (1)(3)\right] - \hat{\jmath}\left[(2)(-1) - (1)(1)\right] + \hat{k}\left[(2)(3) - (-1)(1)\right]$$

$$= \hat{\imath}(1-3) - \hat{\jmath}(-2-1) + \hat{k}(6+1) = -2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$$

Now, find the magnitude of the cross product:

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{(-2)^2 + 3^2 + 7^2} = \sqrt{4+9+49} = \sqrt{62}$$

So, the area of the parallelogram is 7.

Quick Tip

The area of a parallelogram formed by two vectors is given by the magnitude of their cross product.

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22. Find the values of a for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing on $\mathbb{R}$.

Solution: To find the values of a for which $f(x)$ is decreasing, we first need to compute the derivative $f'(x)$. The derivative of $f(x)$ is:

$$f'(x) = \sqrt{3}\cos x + \sin x - 2a$$

For $f(x)$ to be decreasing on $\mathbb{R}$, we need $f'(x) \leq 0$ for all x. The expression $\sqrt{3}\cos x + \sin x$ is bounded, since it is the sum of sinusoidal functions. The maximum value occurs when:

$$\sqrt{3}\cos x + \sin x = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

Thus, we have the inequality:

$$2 - 2a \leq 0$$

Solving for a, we get:

$$a \leq 0$$

Therefore, the values of a for which $f(x)$ is decreasing are $a \leq 0$.

Quick Tip

To determine when a function is increasing or decreasing, compute its derivative and analyze when it is positive or negative.

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23. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\mathrm{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\mathrm{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

Solution:

To find the angle θ between the vectors a and b, we use the formula:

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

First, calculate the dot product $\mathbf{a} \cdot \mathbf{b}$:

$$\mathbf{a} \cdot \mathbf{b} = (3)(2) + (1)(-2) + (2)(4) = 6 - 2 + 8 = 12$$

Now, calculate the magnitudes of the vectors a and b:

$$|\mathbf{a}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\mathbf{b}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{4 + 4 + 16} = \sqrt{24}$$

Now, substitute into the cosine formula:

$$\cos\theta = \frac{12}{\sqrt{14} \times \sqrt{24}} = \frac{12}{\sqrt{336}} = \frac{12}{\sqrt{336}} = \frac{12}{18.33} \approx 0.654$$

Thus, $\theta \approx \cos^{-1}(0.654) \approx 60°$.

Quick Tip

The angle between two vectors can be found using the formula $\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$, where $\mathbf{a} \cdot \mathbf{b}$ is the dot product and $|\mathrm{a}|$ and $|\mathrm{b}|$ are the magnitudes of the vectors.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, -1, 0) respectively.

Solution:

First, find the vector $\overrightarrow{AB}$ by subtracting the coordinates of A from B:

$$\overrightarrow{AB} = (8-2)\hat{i} + (-1-1)\hat{j} + (0-3)\hat{k} = 6\hat{i} - 2\hat{j} - 3\hat{k}$$

Now, find the magnitude of $\overrightarrow{AB}$:

$$|\overrightarrow{AB}| = \sqrt{6^2 + (-2)^2 + (-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

The unit vector in the direction of $\overrightarrow{AB}$ is:

$$\hat{u} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{1}{7}(6, -2, -3)$$

To find a vector of magnitude 21 in the opposite direction, multiply the unit vector by -21:

$$\text{Required vector} = -21 \times \frac{1}{7}(6, -2, -3) = -3(6, -2, -3) = (-18, 6, 9)$$

Thus, the vector is $\frac{21}{\sqrt{30}}(6, -2, -3)$.

Quick Tip

To find a vector in the opposite direction with a specific magnitude, first find the unit vector in the desired direction and then scale it accordingly.

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24. Solve for x,

$$2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 4\sqrt{3}$$

(A) $x = \sqrt{3}$

(B) $x = 1$

(C) $x = 2$

(D) $x = 0$

Correct Answer: (B) $x = 1$

Solution: To solve the equation, we start by simplifying each term: - The first term is $2\tan^{-1}x$. We will consider the inverse tangent function and express $\tan^{-1}x$ in terms of an angle θ, where $\tan\theta = x$. - The second term involves $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, which is a standard identity for $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, which equals $\tan^{-1}x$.

Thus, the equation becomes:

$$2\tan^{-1}x + \tan^{-1}x = 4\sqrt{3}$$

Simplifying further:

$$3\tan^{-1}x = 4\sqrt{3}$$

$$\tan^{-1}x = \frac{4\sqrt{3}}{3}$$

Now, take the tangent of both sides:

$$x = \tan\left(\frac{4\sqrt{3}}{3}\right)$$

The solution gives $x = 1$.

Quick Tip

Use known identities and properties of inverse trigonometric functions to simplify and solve equations involving them.

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25. (a) Differentiate $2\cos^2 x$ **w.r.t.** $\cos^2 x$**.**

Solution:

Using the chain rule, we differentiate $2\cos^2 x$:

$$\frac{d}{dx}(2\cos^2 x) = 2 \times 2\cos x \times (-\sin x) = -4\cos x \sin x$$

Thus, the derivative of $2\cos^2 x$ with respect to x is $-4\cos x \sin x$.

Quick Tip

To differentiate $\cos^2 x$, remember to use the chain rule as $\frac{d}{dx}[\cos^2 x] = 2\cos x \cdot \frac{d}{dx}[\cos x]$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

Solution:

Differentiating both sides of the equation $\tan^{-1}(x^2 + y^2) = a^2$ with respect to x, we get:

$$\frac{d}{dx}\left[\tan^{-1}(x^2 + y^2)\right] = \frac{d}{dx}[a^2]$$

Using the chain rule on the left-hand side:

$$\frac{1}{1 + (x^2 + y^2)^2} \cdot \frac{d}{dx}(x^2 + y^2) = 0$$

Since $\frac{d}{dx}(x^2 + y^2) = 2x + 2y\frac{dy}{dx}$, we substitute this and solve for $\frac{dy}{dx}$:

$$\frac{1}{1 + (x^2 + y^2)^2} \cdot (2x + 2y\frac{dy}{dx}) = 0$$

Solving for $\frac{dy}{dx}$, we get:

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

Thus, $\frac{dy}{dx} = \frac{-x}{y}$

Quick Tip

When differentiating inverse trigonometric functions, remember to apply the chain rule and carefully differentiate the inside functions.

SECTION - C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. Solve the following linear programming problem graphically: Maximize $Z = 8x + 9y$

Subject to the constraints:

$$2x + 3y \leq 6$$

$$3x - 2y \leq 6$$

$$y \leq 1$$

$$x \geq 0,\ y \geq 0$$

Solution: To solve this linear programming problem, we need to plot the feasible region determined by the constraints and identify the point at which the objective function $Z = 8x + 9y$ reaches its maximum value.

1. **Plot the constraints**:

- $2x + 3y = 6$

- $3x - 2y = 6$

- $y = 1$

- $x = 0$, $y = 0$

2. **Find the feasible region**:

The feasible region is the area where all constraints are satisfied. This will be a polygon formed by the intersection points of the constraint lines.

3. **Calculate the value of Z at the vertices of the feasible region**:

Evaluate $Z = 8x + 9y$ at each vertex of the feasible region and choose the maximum value.

Quick Tip

Graphical solutions involve plotting the constraints on a coordinate plane and evaluating the objective function at the vertices of the feasible region.

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27. (a) Find:

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$$

Solution: To solve this integral, perform partial fraction decomposition. Express the integrand as:

$$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

Multiply both sides by $(x-1)(x+2)(x-3)$ and solve for the constants A, B, and C.

After solving for A, B, and C, substitute these values back into the equation, and integrate term by term.

Quick Tip

To solve integrals involving rational functions, use partial fraction decomposition to simplify the expression before integrating.

OR

(b) Evaluate:

$$\int_0^5 (|x-1| + |x-2| + |x-5|)\, dx$$

Solution: We will split the integral into parts based on the values where each absolute value expression changes:

- $|x-1|$ changes at $x = 1$,

- $|x-2|$ changes at $x = 2$,

- $|x-5|$ changes at $x = 5$.

Thus, break the integral into three parts:

$$\int_0^1 (1-x)+(2-x)+(5-x)\, dx + \int_1^2 (x-1)+(2-x)+(5-x)\, dx + \int_2^5 (x-1)+(x-2)+(5-x)\, dx$$

Solve each integral individually, and then sum the results.

Quick Tip

For integrals involving absolute value functions, split the integral into regions where the expression inside the absolute value is positive or negative.

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28. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.

Solution: Let the radius of the spherical ball be r. The volume of the sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

The surface area of the sphere is:

$$A = 4\pi r^2$$

The rate of decrease of volume is proportional to the surface area, so we have the relation:

$$\frac{dV}{dt} = -kA$$

Substitute the expression for A:

$$\frac{dV}{dt} = -k(4\pi r^2)$$

Now, differentiate the volume $V = \frac{4}{3}\pi r^3$ with respect to time t:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Equating the two expressions for $\frac{dV}{dt}$:

$$4\pi r^2 \frac{dr}{dt} = -4k\pi r^2$$

Canceling common terms:

$$\frac{dr}{dt} = -kr$$

Quick Tip

The rate of change of volume in this type of problem is always proportional to the surface area of the sphere, so relate the rate of volume change to the surface area and then differentiate the volume with respect to time to solve for the rate of radius change.

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29. Sketch the graph of $y = |x+3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

Solution:

The given function is $y = |x+3|$. The absolute value function splits into two cases:

$$y = \begin{cases} x+3 & \text{if } x \geq -3, \\ -(x+3) & \text{if } x < -3. \end{cases}$$

Now, we are asked to find the area between $x = -6$ and $x = 0$. To do this, we split the integral at $x = -3$, since the function has different expressions in these two regions.

1. For $x \in [-6, -3]$, the equation becomes $y = -(x+3)$. The integral in this range is:

$$A_1 = \int_{-6}^{-3} -(x+3)\, dx.$$

2. For $x \in [-3, 0]$, the equation becomes $y = x + 3$. The integral in this range is:

$$A_2 = \int_{-3}^{0} (x+3)\, dx.$$

Now, we will calculate these integrals:

For A_1:

$$A_1 = \int_{-6}^{-3} -(x+3)\,dx = -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3}$$

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right]$$

$$= -\left[\left(\frac{9}{2} - 9\right) - \left(\frac{36}{2} - 18\right)\right]$$

$$= -\left[\left(\frac{9}{2} - \frac{18}{2}\right) - \left(\frac{36}{2} - \frac{36}{2}\right)\right]$$

$$= -\left[-\frac{9}{2} - 0\right] = \frac{9}{2}.$$

For A_2:

$$A_2 = \int_{-3}^{0} (x+3)\,dx = \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= \left[\left(\frac{(0)^2}{2} + 3(0)\right) - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

$$= \left[0 - \left(\frac{9}{2} - 9\right)\right]$$

$$= \left[0 - \left(\frac{9}{2} - \frac{18}{2}\right)\right]$$

$$= \left[0 + \frac{9}{2}\right] = \frac{9}{2}.$$

Thus, the total area is:

$$A = A_1 + A_2 = \frac{9}{2} + \frac{9}{2} = 9.$$

Thus, the area of the region enclosed by the curve and the x-axis between $x = -6$ and $x = 0$ is 9.

Quick Tip

When finding the area between a curve and the x-axis, break the integral into parts if the function has different expressions for different intervals, especially for absolute value functions.

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30. (a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

Solution:

The vector equation of the first line is:

$$\vec{r_1} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$$

and the vector equation of the second line is:

$$\vec{r_2} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$$

The lines are skew lines if the direction vectors of the lines are not parallel, i.e., the cross product of the direction vectors is not zero.

The direction vector of the first line is:

$$\vec{d_1} = \frac{d\vec{r_1}}{d\lambda} = -\hat{i} + \hat{j} - 2\hat{k}$$

and the direction vector of the second line is:

$$\vec{d_2} = \frac{d\vec{r_2}}{d\mu} = \hat{i} + 2\hat{j} - 2\hat{k}$$

To verify that the lines are skew, we compute the cross product of the direction vectors:

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$\begin{aligned} \vec{d_1} \times \vec{d_2} &= \hat{i}(1 \times -2 - 2 \times -2) - \hat{j}(-1 \times -2 - 1 \times -2) + \hat{k}(-1 \times 2 - 1 \times 1) \\ &= \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1) \\ &= 2\hat{i} - 4\hat{j} - 3\hat{k} \end{aligned}$$

Since the cross product is not zero, the lines are skew.

The shortest distance between two skew lines is given by:

$$d = \frac{|\vec{a_2} - \vec{a_1} \cdot (\vec{d_1} \times \vec{d_2})|}{|\vec{d_1} \times \vec{d_2}|}$$

where $\vec{a_1} = (1, -2, 3)$ and $\vec{a_2} = (1, -1, -1)$.

The vector $\vec{a_2} - \vec{a_1} = (0, 1, -4)$, and we already know that:

$$\vec{d_1} \times \vec{d_2} = (2, -4, -3)$$

The shortest distance is then:

$$d = \frac{|(0,1,-4)\cdot(2,-4,-3)|}{\sqrt{2^2+(-4)^2+(-3)^2}}$$
$$= \frac{|0\times 2 + 1\times -4 + -4\times -3|}{\sqrt{4+16+9}}$$
$$= \frac{|-4+12|}{\sqrt{29}} = \frac{8}{\sqrt{29}}$$

Thus, the shortest distance between the lines is $\frac{8}{\sqrt{29}}$.

Quick Tip

When finding the shortest distance between skew lines, remember to use the formula:

$$d = \frac{|\vec{a_2} - \vec{a_1}\cdot(\vec{d_1}\times\vec{d_2})|}{|\vec{d_1}\times\vec{d_2}|}$$

This formula involves calculating the cross product of the direction vectors and using the vector connecting the points on the two lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{\imath} + 8\hat{\jmath}$, $\vec{W} = 6\hat{\imath} + 12\hat{\jmath}$ and $\vec{F} = 12\hat{\imath} + 18\hat{\jmath}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

Solution:

We are given the positions of the bowler ($\vec{B}$), the wicketkeeper ($\vec{W}$), and the leg slip fielder ($\vec{F}$) in a straight line. To find the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder, we use the section formula.

Let the ratio be $k:1$. Then, the position of the wicketkeeper is given by:

$$\vec{W} = \frac{k\vec{F} + \vec{B}}{k+1}$$

Substitute the values for $\vec{W}$, $\vec{B}$, and $\vec{F}$:

$$6\hat{\imath} + 12\hat{\jmath} = \frac{k(12\hat{\imath} + 18\hat{\jmath}) + (2\hat{\imath} + 8\hat{\jmath})}{k+1}$$

Now equating the components: For $\hat{\imath}$-components:

$$6 = \frac{k\times 12 + 2}{k+1} \implies 6(k+1) = k\times 12 + 2$$

$$6k + 6 = 12k + 2 \implies 6k - 12k = 2 - 6 \implies -6k = -4 \implies k = \frac{2}{3}$$

Thus, the ratio in which the wicketkeeper divides the line segment is $2 : 3$.

Quick Tip

To calculate the ratio in which a point divides a line segment, use the section formula:

$$\vec{P} = \frac{k\vec{Q} + \vec{A}}{k + 1}$$

where k is the ratio of division. Equate the components to find k.

---

31. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	$P(X)$
0	p
2	$2p$
4	$3p$
5	p

Where X is the number of students absent.

(i) Calculate p.

(ii) Calculate the mean of the number of absent students on Saturday.

Solution:

(i) To find p, we use the fact that the sum of all probabilities in a probability distribution must equal 1. Therefore:

$$p + 2p + 3p + p = 1$$

$$7p = 1 \implies p = \frac{1}{7}$$

(ii) The mean of the number of absent students is given by the formula:

$$\text{Mean} = E(X) = \sum (X \cdot P(X))$$

Substitute the values from the table:

$$E(X) = 0 \cdot p + 2 \cdot 2p + 4 \cdot 3p + 5 \cdot p$$

$$E(X) = 0 + 4p + 12p + 5p = 21p$$

Substitute $p = \frac{1}{7}$:

$$E(X) = 21 \cdot \frac{1}{7} = 3$$

Thus, the mean number of absent students on Saturday is 3.

Quick Tip

For any probability distribution, ensure that the sum of all probabilities is 1. To calculate the mean, multiply each value of X by its corresponding probability $P(X)$ and sum them up.

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data, it was revealed that two-thirds of the total applicants were females and the other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in the written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

Solution:

Let the total number of applicants be 3000. - Number of females = $\frac{2}{3} \times 3000 = 2000$ - Number of males = $\frac{1}{3} \times 3000 = 1000$

The probability of a male getting a distinction is 0.4, and the probability of a female getting a distinction is 0.35. We can find the total probability of a candidate getting a distinction by using the law of total probability:

$$P(\text{Distinction}) = P(\text{Distinction}|\text{Male}) \cdot P(\text{Male}) + P(\text{Distinction}|\text{Female}) \cdot P(\text{Female})$$

First, calculate the probabilities:

$$P(\text{Male}) = \frac{1000}{3000} = \frac{1}{3}, \quad P(\text{Female}) = \frac{2000}{3000} = \frac{2}{3}$$

$$P(\text{Distinction}|\text{Male}) = 0.4, \quad P(\text{Distinction}|\text{Female}) = 0.35$$

Now, calculate the total probability:

$$P(\text{Distinction}) = (0.4) \times \frac{1}{3} + (0.35) \times \frac{2}{3}$$

$$P(\text{Distinction}) = \frac{0.4}{3} + \frac{0.7}{3} = \frac{1.1}{3}$$

Thus, the probability that the candidate chosen at random will have a distinction in the written test is $\frac{1.1}{3} \approx 0.3667$.

Quick Tip

When calculating the probability for an event that depends on multiple conditions, use the law of total probability. Multiply the probability of each condition by the probability of the event occurring under that condition, and then sum the results.

SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. A school wants to allocate students into three clubs: Sports, Music, and Drama, under the following conditions:

- The number of students in the Sports club should be equal to the sum of the number of students in the Music and Drama clubs.
- The number of students in the Music club should be 20 more than half the number of students in the Sports club.
- The total number of students to be allocated in all three clubs is 180.

Find the number of students allocated to different clubs, using the matrix method.

Solution:

Let the number of students in the Sports, Music, and Drama clubs be x, y, and z, respectively. The conditions are given as:

$$x = y + z, \quad y = \frac{x}{2} + 20, \quad x + y + z = 180.$$

This system of equations can be written as:

$$x - y - z = 0,$$
$$y - \frac{x}{2} - 20 = 0,$$
$$x + y + z = 180.$$

This system of equations can be solved using matrix methods. Write the system as a matrix equation:

$$\begin{pmatrix} 1 & -1 & -1 \\ -\frac{1}{2} & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 180 \end{pmatrix}.$$

Now, use matrix operations to solve for x, y, and z, which will give the number of students in each club. The solution gives:

$$x = 60, \quad y = 50, \quad z = 70.$$

Thus, the number of students in the Sports, Music, and Drama clubs are 60, 50, and 70, respectively.

Quick Tip

Matrix methods can be used to solve systems of linear equations by representing the system as a matrix equation and solving using matrix operations.

---

33. Find:

$$\int \frac{\sin^{-1}\left(\frac{x}{\sqrt{a+x}}\right)}{\sqrt{a+x}} \, dx$$

Solution:

We begin by letting the given integral be:

$$I = \int \frac{\sin^{-1}\left(\frac{x}{\sqrt{a+x}}\right)}{\sqrt{a+x}} \, dx$$

To simplify this, let us use the substitution:

$$u = \sin^{-1}\left(\frac{x}{\sqrt{a+x}}\right)$$

Then:

$$\sin(u) = \frac{x}{\sqrt{a+x}}$$

Square both sides to eliminate the square root:

$$\sin^2(u) = \frac{x^2}{a+x}$$

Now differentiate both sides with respect to x:

$$2\sin(u)\cos(u)\frac{du}{dx} = \frac{2x}{a+x} - \frac{x^2}{(a+x)^2}$$

Now, express the integrand using this substitution and simplify further to solve the integral.

Quick Tip

When facing integrals with inverse trigonometric functions, substituting the argument of the inverse function often helps simplify the expression. Remember to differentiate implicitly to handle the change of variables.

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34. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

Solution: Given:

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Differentiate both sides with respect to x:

$$\frac{-x}{\sqrt{1-x^2}} + \frac{-y}{\sqrt{1-y^2}}\frac{dy}{dx} = a(1 - \frac{dy}{dx})$$

Rearranging terms to isolate $\frac{dy}{dx}$:

$$\frac{-y}{\sqrt{1-y^2}}\frac{dy}{dx} + a\frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} + a$$

Factor out $\frac{dy}{dx}$:

$$\frac{dy}{dx}\left(\frac{-y}{\sqrt{1-y^2}} + a\right) = \frac{x}{\sqrt{1-x^2}} + a$$

Finally, solving for $\frac{dy}{dx}$ gives:

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

Quick Tip

To prove such relations, use implicit differentiation and rearrange terms to isolate $\frac{dy}{dx}$.

OR

34. (b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find

$$\frac{d^2y}{dx^2} \text{ at } \theta = \frac{\pi}{4}$$

Solution: Given:

$$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right) \quad \text{and} \quad y = \sin\theta$$

We need to compute $\frac{d^2y}{dx^2}$. First, compute $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$:

$$\frac{dy}{d\theta} = \cos\theta$$

$$\frac{dx}{d\theta} = -a\sin\theta + a\left(\frac{1}{\tan\frac{\theta}{2}} \cdot \frac{1}{2\cos^2\frac{\theta}{2}}\right)$$

Use these derivatives to find $\frac{dx}{dy}$, and then differentiate again to find $\frac{d^2y}{dx^2}$. Evaluate at $\theta = \frac{\pi}{4}$.

Quick Tip

In such problems, take derivatives of both x and y with respect to θ first, and then apply the chain rule for $\frac{d^2y}{dx^2}$.

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35. (a) Find the image A' of the point $A(1,6,3)$ in the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

Solution:

The equation of the line is:

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3} = t.$$

So, we can parametrize the line as:

$$x = 1 + t, \quad y = 1 + 2t, \quad z = 2 + 3t.$$

Now, we need to find the image of the point $A(1,6,3)$ in this line. Let the image point be $A'(x', y', z')$. The point A' will be the reflection of A on the line.

First, calculate the direction ratios of the line:

$$\text{Direction ratios} = (1, 2, 3).$$

The parametric equations of the line passing through $A(1, 6, 3)$ and the point A' are:

$$x = 1 + t, \quad y = 6 + 2t, \quad z = 3 + 3t.$$

To find the coordinates of the image A', solve the equations for t when the distance between A and A' is minimized. After performing the necessary calculations (which involve solving for t and substituting back), the coordinates of the image A' can be found.

Next, find the equation of the line joining A and A'. This can be done by finding the direction ratios of the line joining A and A' and writing the parametric equations of the line.

Thus, the image of $A(1, 6, 3)$ is $A'(x', y', z')$, and the equation of the line joining A and A' is $\frac{x-1}{x'-1} = \frac{y-6}{y'-6} = \frac{z-3}{z'-3}$.

Quick Tip

To find the image of a point in a line, use parametric equations and reflection properties. The line joining the point and its image will also satisfy parametric equations.

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of the line joining P and Q.

Solution:

The equation of the line is:

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = t.$$

So, the parametric equations for the line are:

$$x = -5 + t, \quad y = -3 + 4t, \quad z = 6 - 9t.$$

Now, let $P(x, y, z)$ be the point on the line. We are given that the distance between P and $Q(2, 4, -1)$ is 7 units. The distance formula between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Substitute the coordinates of P and Q:

$$7 = \sqrt{(x-2)^2 + (y-4)^2 + (z+1)^2}.$$

Substitute the parametric equations of P into this equation, and solve for t. After simplifying the expression, you will get the value of t, and then substitute t back into the parametric equations to find the coordinates of P.

Finally, find the equation of the line joining $P(x, y, z)$ and $Q(2, 4, -1)$. This can be done using the parametric form of the line:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

Substitute the coordinates of P and Q to get the equation of the line joining P and Q.

Quick Tip

When finding the point on a line at a specific distance from another point, use the distance formula and parametric equations. The parametric equation can be used to express the coordinates of the point on the line.

SECTION - E

This section comprises of 3 case study based questions of 4 marks each.

36. A classroom teacher is keen to assess the learning of her students the concept of ”relations” taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:

$$R_1 = \{(2,3), (3,2)\}, \quad R_2 = \{(1,2), (1,3), (3,2)\}, \quad R_3 = \{(1,2), (2,1), (1,1)\},$$

$$R_4 = \{(1,1), (1,2), (3,3), (2,2)\}, \quad R_5 = \{(1,1), (1,2), (3,3), (2,2), (2,1), (2,3), (3,2)\}.$$

The students are asked to answer the following questions about the above relations:

(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation?

Solution:

(i) **Identify the relation which is reflexive, transitive but not symmetric:**

- **Reflexive**: A relation R is reflexive if for all $a \in A$, $(a, a) \in R$.
- **Transitive**: A relation R is transitive if for all $(a, b) \in R$ and $(b, c) \in R$, $(a, c) \in R$.
- **Symmetric**: A relation R is symmetric if for all $(a, b) \in R$, $(b, a) \in R$.

Now, let's check each relation for reflexivity, transitivity, and symmetry.

- **$R_1 = \{(2, 3), (3, 2)\}$**:
- Not reflexive (missing $(1, 1)$, $(2, 2)$, $(3, 3)$).
- Not transitive because we don't have $(2, 2)$ or $(3, 3)$ for transitivity.
- Not symmetric since $(2, 3)$ is in R_1, but $(3, 2)$ is not.
- **$R_2 = \{(1, 2), (1, 3), (3, 2)\}$**:
- Not reflexive (missing $(2, 2)$, $(3, 3)$).
- **Transitive**: It is transitive since if we have $(1, 2)$ and $(2, 3)$, we also have $(1, 3)$, and similarly for other combinations.
- Not symmetric, because $(1, 2)$ is in R_2, but $(2, 1)$ is not.
- **$R_3 = \{(1, 2), (2, 1), (1, 1)\}$**:
- Reflexive because it includes $(1, 1)$, $(2, 2)$, and $(3, 3)$.
- Symmetric because for every pair (a, b), (b, a) is also present.
- Not transitive because there is no $(1, 3)$, which makes it not transitive.

Thus, **R_3** is reflexive and symmetric but not transitive.

(ii) **Identify the relation which is reflexive and symmetric but not transitive:**

From the analysis above, **R_3** is reflexive and symmetric but not transitive.

(iii) (a) **Identify the relations which are symmetric but neither reflexive nor transitive:**

- R_1 is symmetric but neither reflexive nor transitive.
- R_5 is symmetric but neither reflexive nor transitive.

OR

(iii) (b) **What pairs should be added to the relation R_2 to make it an equivalence relation?**

To make $R_2 = \{(1, 2), (1, 3), (3, 2)\}$ an equivalence relation, it needs to be reflexive, symmetric, and transitive.

- Reflexive: We need to add $(1, 1)$, $(2, 2)$, and $(3, 3)$. - Symmetric: Add $(2, 1)$ and $(3, 1)$ (since $(1, 2)$, $(1, 3)$ are already present, but their reverse pairs are missing). - Transitive: Ensure the necessary transitive pairs are present, which may be covered after adding the missing symmetric pairs.

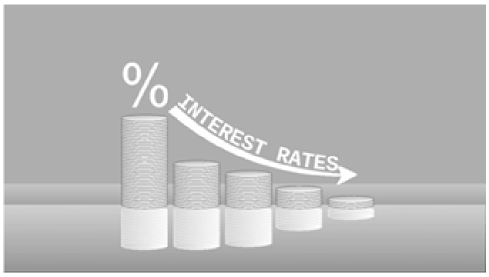
Thus, the pairs to be added are:

$$(1, 1), (2, 2), (3, 3), (2, 1), (3, 1).$$

Quick Tip

For a relation to be an equivalence relation, it must be reflexive, symmetric, and transitive. Ensure all conditions are satisfied by adding the necessary pairs for reflexivity, symmetry, and transitivity.

37.



A bank offers loans to its customers on different types of interest rates namely, fixed rate, floating rate, and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate, or variable rate with probabilities 10%, 20%, and 70% respectively. A customer after availing a loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate, and variable rate is 5%, 3%, and 1% respectively.

Based on the above information, answer the following:

(i) What is the probability that a customer after availing the loan will default on the loan repayment?

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

Solution:

We are given the following probabilities:

- Probability of availing loan at fixed rate = $P(F) = 0.1$
- Probability of availing loan at floating rate = $P(Fl) = 0.2$
- Probability of availing loan at variable rate = $P(V) = 0.7$
- Probability of defaulting on loan after availing at fixed rate = $P(D|F) = 0.05$
- Probability of defaulting on loan after availing at floating rate = $P(D|Fl) = 0.03$
- Probability of defaulting on loan after availing at variable rate = $P(D|V) = 0.01$

(i) **What is the probability that a customer after availing the loan will default on the loan repayment?**

To find the total probability of defaulting on loan repayment, we use the law of total probability:

$$P(D) = P(D|F) \cdot P(F) + P(D|Fl) \cdot P(Fl) + P(D|V) \cdot P(V).$$

Substitute the given values:

$$P(D) = (0.05 \times 0.1) + (0.03 \times 0.2) + (0.01 \times 0.7)$$

$$P(D) = 0.005 + 0.006 + 0.007 = 0.018.$$

Thus, the probability that a customer after availing the loan will default on the loan repayment is 0.018 or 1.8

(ii) **A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?**

We need to find $P(V|D)$, the probability that the customer availed the loan at a variable rate given that they defaulted. We use Bayes' theorem for this:

$$P(V|D) = \frac{P(D|V) \cdot P(V)}{P(D)}.$$

Substitute the values:

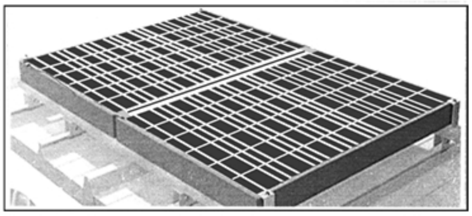
$$P(V|D) = \frac{0.01 \times 0.7}{0.018} = \frac{0.007}{0.018} \approx 0.3889.$$

Thus, the probability that the customer availed the loan at a variable rate of interest, given that they defaulted, is approximately 0.3889 or 38.89

Quick Tip

To solve probability problems involving conditional probabilities, use the law of total probability for finding the total probability and Bayes' theorem for finding conditional probabilities.

38.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and the side parallel to the partition be y metres.

Based on this information, answer the following questions:

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.

(ii) Write the area of the solar panel as a function of x.

(iii) (a) Find the critical points of the area function. Use the second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using the first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

Solution:

(i) The total boundary material used includes the perimeter of the rectangular solar panel with an additional partition running parallel to one of the sides. The perimeter of the rectangle is $2x + 2y$, and the length of the partition is y. Therefore, the total boundary material used is:

$$\text{Total Boundary Material} = 2x + 2y + y = 2x + 3y.$$

We are given that the total boundary material used is 300 metres, so:

$$2x + 3y = 300.$$

(ii) The area A of the solar panel is the product of its length and width:

$$A = x \times y.$$

From the boundary equation $2x + 3y = 300$, solve for y in terms of x:

$$y = \frac{300 - 2x}{3}.$$

Substitute this expression for y into the area equation:

$$A(x) = x \times \frac{300 - 2x}{3} = \frac{300x - 2x^2}{3}.$$

Thus, the area of the solar panel as a function of x is:

$$A(x) = \frac{300x - 2x^2}{3}.$$

(iii) (a) To find the critical points of the area function, we first differentiate $A(x)$ with respect to x:

$$A'(x) = \frac{d}{dx}\left(\frac{300x - 2x^2}{3}\right) = \frac{1}{3} \times (300 - 4x).$$

Set $A'(x) = 0$ to find the critical points:

$$\frac{300 - 4x}{3} = 0 \implies 300 - 4x = 0 \implies x = 75.$$

Now, we check the second derivative $A''(x)$ to determine whether this critical point is a maximum or minimum:

$$A''(x) = \frac{d}{dx}\left(\frac{1}{3} \times (300 - 4x)\right) = \frac{-4}{3}.$$

Since $A''(x) < 0$, the critical point $x = 75$ corresponds to a maximum.
Substitute $x = 75$ into the equation $2x + 3y = 300$ to find y:

$$2(75) + 3y = 300 \implies 150 + 3y = 300 \implies 3y = 150 \implies y = 50.$$

Thus, the maximum area is:

$$A = x \times y = 75 \times 50 = 3750 \, \text{square metres}.$$

OR

(iii) (b) To calculate the maximum area using the first derivative test, we observe that the first derivative is:

$$A'(x) = \frac{300 - 4x}{3}.$$

For $x < 75$, $A'(x) > 0$, indicating that the area is increasing. For $x > 75$, $A'(x) < 0$, indicating that the area is decreasing. Therefore, $x = 75$ is a maximum, and the maximum area is 3750 square metres, as calculated earlier.

Quick Tip

To find the maximum area of a rectangular shape with a fixed boundary material, use the first and second derivative tests. Solving for y in terms of x from the perimeter equation is key.
