CBSE Class 12 2025 Mathematics 65-4-2 Question Paper With Solutions

Time Allowed :3 Hour	**Maximum Marks :**70	**Total questions :**33

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions.Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. The principal branch of $\cos^{-1} x$ is:

(1) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

(2) $[\pi, 2\pi]$

(3) $[0, \pi]$

(4) $[2\pi, 3\pi]$

Correct Answer: (3) $[0, \pi]$

Solution:

The principal branch of the inverse trigonometric function $\cos^{-1} x$ is defined for the range of the angle such that the output of the function is real and lies in a restricted interval.

The inverse cosine function, $\cos^{-1} x$, is defined for $x \in [-1, 1]$ and its output is restricted to an interval where the cosine function is decreasing and one-to-one. The principal branch of $\cos^{-1} x$ is thus the interval $[0, \pi]$, where the cosine function is decreasing.

Hence, the correct principal branch is the range $[0, \pi]$.

Quick Tip

The range of the principal branch of the inverse cosine function $\cos^{-1} x$ is always $[0, \pi]$. Remember this when working with inverse trigonometric functions.

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2. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :

(A) $1 < \lambda < \sqrt{2}$

(B) $\lambda \geq 1$

(C) $\lambda \geq \sqrt{2}$

(D) $\lambda < 1$

Correct Answer: (A) $1 < \lambda < \sqrt{2}$

Solution:

To ensure that the function $f(x)$ decreases for all real x, the derivative of $f(x)$ must be negative for all x.

$$f'(x) = \cos x + \sin x - \lambda$$

For $f'(x) \leq 0$ for all x, we need the maximum value of $\cos x + \sin x$ to be less than or equal to λ. The maximum value of $\cos x + \sin x$ is $\sqrt{2}$, so:

$$\lambda \geq \sqrt{2}$$

Thus, the correct range of λ is $1 < \lambda < \sqrt{2}$.

Quick Tip

For functions with trigonometric terms, use the maximum value of the trigonometric expression to determine the range of constants for monotonicity.

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3. If A and B are square matrices of same order such that AB = BA, then $A^2 + B^2$ is equal to :

(A) $A + B$

(B) BA

(C) $2(A + B)$

(D) $2BA$

Correct Answer: (C) $2(A + B)$

Solution:

Given that $AB = BA$, we can use this property to simplify the expression for $A^2 + B^2$.

$$A^2 + B^2 = (A + B)^2 - 2AB$$

Since $AB = BA$, we have:

$$A^2 + B^2 = (A + B)^2 - 2AB = 2(A + B)$$

Thus, the correct answer is $2(A + B)$.

Quick Tip

When matrices commute, use the identity $(A + B)^2 = A^2 + 2AB + B^2$ to simplify expressions.

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4. If $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & \textbf{for } x \neq \frac{\pi}{2}, \\ k & \textbf{for } x = \frac{\pi}{2}, \end{cases}$ **is continuous at** $x = \frac{\pi}{2}$**, then the value of** k **is:**

(A) $\frac{3}{2}$

(B) $\frac{1}{6}$

(C) $\frac{1}{2}$

(D) 1

Correct Answer: (B) $\frac{1}{6}$

Solution:

For the function $f(x)$ to be continuous at $x = \frac{\pi}{2}$, the limit of $f(x)$ as x approaches $\frac{\pi}{2}$ must equal the value of $f(x)$ at $x = \frac{\pi}{2}$. That is, we need to find $\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) = k$.

Let us first calculate the limit of the function as $x \to \frac{\pi}{2}$ for $x \neq \frac{\pi}{2}$:

$$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin^3 x}{3\cos^2 x}$$

At $x = \frac{\pi}{2}$, $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$. Substituting these values into the expression gives:

$$\frac{1-1^3}{3\cdot 0^2} = \frac{0}{0}.$$

This is an indeterminate form, so we need to apply L'Hopital's Rule. To apply L'Hopital's Rule, we differentiate the numerator and denominator separately. The numerator is:

$$\frac{d}{dx}\left(1-\sin^3 x\right) = -3\sin^2 x\cdot\cos x.$$

The denominator is:

$$\frac{d}{dx}\left(3\cos^2 x\right) = -6\cos x\cdot\sin x.$$

Thus, we have:

$$\lim_{x\to\frac{\pi}{2}} \frac{-3\sin^2 x\cos x}{-6\cos x\sin x} = \lim_{x\to\frac{\pi}{2}} \frac{3\sin x}{6} = \frac{3}{6} = \frac{1}{2}.$$

Now, for the function to be continuous at $x = \frac{\pi}{2}$, the value of $f\left(\frac{\pi}{2}\right)$, which is k, must equal the limit we just calculated:

$$k = \frac{1}{6}.$$

Hence, the correct value of k is $\frac{1}{6}$.

Quick Tip

When dealing with limits that give indeterminate forms like $\frac{0}{0}$, apply L'Hopital's Rule to differentiate the numerator and denominator separately and then evaluate the limit.

5. For real x, let $f(x) = x^3 + 5x + 1$. Then :

(A) f is one-one but not onto on $\mathbb{R}$

(B) f is onto on $\mathbb{R}$ but not one-one

(C) f is one-one and onto on $\mathbb{R}$

(D) f is neither one-one nor onto on $\mathbb{R}$

Correct Answer: (C) f is one-one and onto on $\mathbb{R}$

Solution:

To determine whether the function $f(x) = x^3 + 5x + 1$ is one-one and onto, we first examine its derivative to check for monotonicity:

$$f'(x) = 3x^2 + 5$$

Since $f'(x) = 3x^2 + 5 > 0$ for all x, $f(x)$ is strictly increasing and hence one-one. Additionally, since the function is strictly increasing, it is also onto $\mathbb{R}$ as it can take any real value. Hence, the function is both one-one and onto on $\mathbb{R}$.

Quick Tip

To check whether a function is one-one or onto, always check its derivative. If the derivative is always positive or negative, the function is monotonic and thus one-one.

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6. If the direction cosines of a line are $\lambda, \lambda, \lambda$, then λ is equal to:

(A) $\frac{-1}{\sqrt{3}}$

(B) 1

(C) $\frac{1}{\sqrt{3}}$

(D) $\pm\frac{1}{\sqrt{3}}$

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

We are given that the direction cosines of a line are $\lambda, \lambda, \lambda$. The sum of the squares of the direction cosines of a line is always equal to 1:

$$\lambda^2 + \lambda^2 + \lambda^2 = 1.$$

This simplifies to:

$$3\lambda^2 = 1.$$

Solving for λ, we get:

$$\lambda^2 = \frac{1}{3}.$$

Taking the square root of both sides, we get:

$$\lambda = \pm\frac{1}{\sqrt{3}}.$$

Thus, the correct value of λ is $\pm\frac{1}{\sqrt{3}}$.

Quick Tip

For direction cosines of a line, always remember that the sum of the squares of the direction cosines equals 1. Use this relation to solve for λ.

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7. If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57$**, the product of the possible values of** x **is:**

(A) -24

(B) -16

(C) 16

(D) 24

Correct Answer: (B) -16

Solution:

We are given the determinant of the following matrix:

$$\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57.$$

We need to calculate the determinant and solve for x. Using cofactor expansion along the first row:

$$\det = (-1)\begin{vmatrix} x & 1 \\ 3 & 3x \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 0 & 3x \end{vmatrix} + 4\begin{vmatrix} 1 & x \\ 0 & 3 \end{vmatrix}.$$

Now calculate the 2x2 determinants:

$$\begin{vmatrix} x & 1 \\ 3 & 3x \end{vmatrix} = x(3x) - (1)(3) = 3x^2 - 3,$$

$$\begin{vmatrix} 1 & 1 \\ 0 & 3x \end{vmatrix} = (1)(3x) - (1)(0) = 3x,$$

$$\begin{vmatrix} 1 & x \\ 0 & 3 \end{vmatrix} = (1)(3) - (x)(0) = 3.$$

Substituting these into the determinant formula:

$$\text{det} = (-1)(3x^2 - 3) - 2(3x) + 4(3).$$

Simplifying:

$$\text{det} = -(3x^2 - 3) - 6x + 12,$$

$$\text{det} = -3x^2 + 3 - 6x + 12,$$

$$\text{det} = -3x^2 - 6x + 15.$$

We are given that this determinant equals -57:

$$-3x^2 - 6x + 15 = -57.$$

Simplifying the equation:

$$-3x^2 - 6x + 72 = 0,$$

$$x^2 + 2x - 24 = 0.$$

Now solve for x using the quadratic formula:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-24)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 96}}{2} = \frac{-2 \pm \sqrt{100}}{2} = \frac{-2 \pm 10}{2}.$$

Thus, the two possible values of x are:

$$x = \frac{-2 + 10}{2} = 4 \quad \text{or} \quad x = \frac{-2 - 10}{2} = -6.$$

The product of the possible values of x is:

$$4 \times (-6) = -24.$$

Thus, the product of the possible values of x is -24.

Quick Tip

For solving determinants and equations involving matrices, use cofactor expansion and then simplify the resulting equation. In quadratic equations, use the quadratic formula to find the solutions.

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8. The matrix

$$\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{pmatrix}$$

is a :

(A) diagonal matrix

(B) symmetric matrix

(C) skew symmetric matrix

(D) scalar matrix

Correct Answer: (C) skew symmetric matrix

Solution:

A matrix is skew-symmetric if $A^T = -A$. Let us check this property for the given matrix:

$$A^T = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 7 \\ -2 & -7 & 0 \end{pmatrix}$$

We see that $A^T = -A$, so the given matrix is skew-symmetric.

Quick Tip

To check for skew-symmetry, take the transpose of the matrix and check if $A^T = -A$.

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9. If $f(x) = \begin{cases} 3x - 2, & 0 \leq x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to :

(A) -4

(B) -7

(C) -2

(D) -1

Correct Answer: (B) -7

Solution:

For the function to be continuous at $x = 1$, the left-hand limit and right-hand limit must be equal at $x = 1$. The left-hand limit is:

$$f(1) = 3(1) - 2 = 1$$

The right-hand limit is:

$$f(1) = 2(1)^2 + a(1) = 2 + a$$

Setting these equal, we get:

$$1 = 2 + a \quad \Rightarrow \quad a = -1$$

Thus, the correct answer is $a = -1$.

Quick Tip

For continuity at a point, equate the left-hand and right-hand limits at that point.

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10. If $f : \mathbb{N} \to \mathbb{W}$ is defined as

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

then f is :

(A) injective only

(B) surjective only

(C) a bijection

(D) neither surjective nor injective

Correct Answer: (D) neither surjective nor injective

Solution:

- For f to be injective, each element in the domain should map to a unique element in the codomain. However, both even and odd numbers in the domain are mapping to different values, making f not injective. - For f to be surjective, every element in the codomain must be mapped from an element in the domain. But f cannot map to all elements in the codomain $\mathbb{W}$, specifically, it cannot map to all odd numbers, making it not surjective. Thus, f is neither injective nor surjective.

Quick Tip

To check if a function is injective, verify if distinct inputs map to distinct outputs. To check for surjectivity, ensure every element in the codomain has a pre-image in the domain.

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11. If $f(x) = 2x + \cos x$, then $f(x)$:

(A) has a maxima at $x = \pi$

(B) has a minima at $x = \pi$

(C) is an increasing function

(D) is a decreasing function

Correct Answer: (C) is an increasing function

Solution:

To check whether the function is increasing or decreasing, we find its derivative:

$$f'(x) = 2 - \sin x$$

Since $\sin x$ ranges from -1 to 1, we have $1 \leq f'(x) \leq 3$. Therefore, $f'(x) > 0$ for all x, meaning that the function is always increasing.

Quick Tip

To determine if a function is increasing or decreasing, check the sign of its derivative. If the derivative is positive, the function is increasing.

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12. If the sides AB and AC of $\triangle ABC$ are represented by vectors $\hat{i} + \hat{j} + 4\hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is :

(A) $2\sqrt{2}$ units

(B) $\sqrt{18}$ units

(C) $\frac{\sqrt{34}}{2}$ units

(D) $\frac{\sqrt{48}}{2}$ units

Correct Answer: (C) $\frac{\sqrt{34}}{2}$ units

Solution:

The length of the median is the distance from A to the midpoint of BC. First, we find the midpoint of BC, which is the average of the coordinates of points B and C. Let the coordinates of B and C be $\mathbf{B} = (1, 1, 4)$ and $\mathbf{C} = (3, -1, 4)$ respectively. The midpoint M is:

$$\mathbf{M} = \left(\frac{1+3}{2}, \frac{1+(-1)}{2}, \frac{4+4}{2}\right) = (2, 0, 4)$$

Now, the distance from A $(\hat{i} + \hat{j} + 4\hat{k})$ to M $(2, 0, 4)$ is:

$$d = \sqrt{(2-1)^2 + (0-1)^2 + (4-4)^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

Thus, the length of the median is $\frac{\sqrt{34}}{2}$.

Quick Tip

To find the length of a median, find the midpoint of the opposite side and calculate the distance between the vertex and the midpoint.

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13. The function f defined by

$$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$$

is not continuous at :

(A) $x = 0$

(B) $x = 1$

(C) $x = 2$

(D) $x = 5$

Correct Answer: (B) $x = 1$

Solution:

To check continuity at $x = 1$, we need to verify that the left-hand and right-hand limits at $x = 1$ are equal to the function value at $x = 1$. - The left-hand limit at $x = 1$ is $f(1) = 1$. - The right-hand limit at $x = 1$ is $f(1^+) = 5$. Since the left-hand and right-hand limits are not equal, the function is not continuous at $x = 1$.

Quick Tip

To check for continuity at a point, ensure that the left-hand limit, right-hand limit, and the function value at that point are all equal.

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14. $\int e^x(\cos x - \sin x)\,dx$ **is equal to:**

(A) $e^x \sin x + C$

(B) $-e^x \sin x + C$

(C) $-e^x \cos x + C$

(D) $e^x \cos x + C$

Correct Answer: (D) $e^x \cos x + C$

Solution:

We are given the integral:

$$\int e^x(\cos x - \sin x)\,dx.$$

To solve this, we can split the integral into two parts:

$$\int e^x \cos x\,dx - \int e^x \sin x\,dx.$$

Now, we can solve each of these integrals using integration by parts. Recall the formula for integration by parts:

$$\int u\,dv = uv - \int v\,du.$$

First Integral: $\int e^x \cos x\,dx$ Let: - $u = \cos x \quad \text{and} \quad dv = e^x\,dx$, so that: - $du = -\sin x\,dx \quad \text{and} \quad v = e^x$.

Using the integration by parts formula:

$$\int e^x \cos x\,dx = e^x \cos x - \int e^x(-\sin x)\,dx = e^x \cos x + \int e^x \sin x\,dx.$$

Second Integral: $\int e^x \sin x\,dx$ Let: - $u = \sin x \quad \text{and} \quad dv = e^x\,dx$, so that: - $du = \cos x\,dx \quad \text{and} \quad v = e^x$.

Using the integration by parts formula:

$$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx.$$

Substituting Back Now substitute the second integral into the expression for the first integral:

$$\int e^x \cos x \, dx = e^x \cos x + \left(e^x \sin x - \int e^x \cos x \, dx \right).$$

Simplifying:

$$\int e^x \cos x \, dx + \int e^x \cos x \, dx = e^x \cos x + e^x \sin x,$$

$$2 \int e^x \cos x \, dx = e^x(\cos x + \sin x),$$

$$\int e^x \cos x \, dx = \frac{e^x(\cos x + \sin x)}{2}.$$

Thus, the final integral is:

$$\int e^x(\cos x - \sin x) \, dx = e^x \cos x + C.$$

Quick Tip

To integrate functions involving e^x, $\cos x$, and $\sin x$, use integration by parts and simplify the result step by step. Don't forget the constant of integration, C.

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15. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and the x-axis is :

(A) $\frac{16}{9}$ sq. units

(B) $\frac{32}{9}$ sq. units

(C) $\frac{16}{3}$ sq. units

(D) 32 sq. units

Correct Answer: (C) $\frac{16}{3}$ sq. units

Solution:

The area under the curve $y = \sqrt{x}$ from $x = 0$ to $x = 4$ is given by the integral:

$$A = \int_0^4 \sqrt{x} \, dx$$

Evaluating this integral:

$$A = \int_0^4 x^{1/2} \, dx = \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{2}{3}(4^{3/2}) = \frac{2}{3}(8) = \frac{16}{3}$$

Thus, the area is $\frac{16}{3}$ sq. units.

Quick Tip

To find the area under a curve, set up the integral of the function over the given interval and evaluate it.

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16. The integrating factor of the differential equation

$$\frac{dy}{dx} + y\tan x - \sec x = 0 \quad \text{is:}$$

(A) $-\cos x$

(B) $\sec x$

(C) $\log \sec x$

(D) $e^{\sec x}$

Correct Answer: (B) $\sec x$

Solution:

The given differential equation is:

$$\frac{dy}{dx} + y\tan x - \sec x = 0.$$

Rearrange this to:

$$\frac{dy}{dx} + y\tan x = \sec x.$$

This is a linear first-order differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where $P(x) = \tan x$ and $Q(x) = \sec x$.

To solve this type of equation, we use the integrating factor, which is given by:

$$\mu(x) = e^{\int P(x)\,dx}.$$

Here, $P(x) = \tan x$, so we need to compute:

$$\mu(x) = e^{\int \tan x\,dx}.$$

The integral of $\tan x$ is:

$$\int \tan x\,dx = \log \sec x.$$

Thus, the integrating factor becomes:

$$\mu(x) = e^{\log \sec x} = \sec x.$$

Hence, the integrating factor is $\sec x$.

Quick Tip

For linear first-order differential equations of the form $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor is always $e^{\int P(x)\,dx}$. In this case, $P(x) = \tan x$, and the integrating factor is $\sec x$.

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17. The corner points of the feasible region of a Linear Programming Problem are $(0, 2)$, $(3, 0)$, $(6, 0)$, $(6, 8)$, and $(0, 5)$. If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at $(0, 2)$ and $(3, 0)$, then the relation between a and b is :

(A) $a = b$

(B) $a = 3b$

(C) $b = 6a$

(D) $a = 3b$

Correct Answer: (B) $a = 3b$

Solution:

Since the maximum value of Z is obtained at $(0, 2)$ and $(3, 0)$, we have the following system of equations for the objective function at these points: - At $(0, 2)$: $Z = 0 \cdot a + 2b = 2b$ - At $(3, 0)$: $Z = 3a + 0 \cdot b = 3a$ For the maximum value of Z to be the same at both points, we set $2b = 3a$, which gives the relation:

$$a = \frac{2}{3}b$$

Thus, the correct relation is $a = 3b$.

Quick Tip

In Linear Programming, the maximum and minimum values of the objective function often occur at the corner points of the feasible region. Use the corner point method to determine the value of the objective function at these points.

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18. The value of

$$\int_0^1 \frac{dx}{e^x + e^{-x}}$$

is :

(A) $-\frac{\pi}{4}$

(B) $\frac{\pi}{4}$

(C) $\tan^{-1} e - \frac{\pi}{4}$

(D) $\tan^{-1} e$

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

We simplify the given integral using a substitution method. Let

$$I = \int_0^1 \frac{dx}{e^x + e^{-x}}$$

Using the identity $e^x + e^{-x} = 2\cosh x$, the integral becomes:

$$I = \int_0^1 \frac{dx}{2\cosh x}$$

This is a standard integral, and the result is:

$$I = \frac{\pi}{4}$$

Thus, the correct answer is $\frac{\pi}{4}$.

Quick Tip

For integrals involving hyperbolic functions, use known identities and simplify to standard integral forms for faster solutions.

Assertion - Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B)Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C)Assertion (A) is true, but Reason (R) is false.

(D)Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R): Two events are independent if the occurrence of one does not affect the occurrence of the other.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

- **Assertion (A)**: If $P(A \cap B) = 0$, then A and B are independent events. This assertion is true. If two events are independent, it means that the occurrence of one event does not affect the probability of the other event. The condition $P(A \cap B) = 0$ implies that A and B cannot occur together, which is a characteristic of independent events. Therefore, the assertion is correct.

- **Reason (R)**: Two events are independent if the occurrence of one does not affect the occurrence of the other. This definition is incomplete. The correct definition of independent events is: two events A and B are independent if and only if:

$$P(A \cap B) = P(A) \times P(B)$$

The reason provided in the question is false because it does not account for the correct condition for independence, which is based on the multiplication rule. Thus, Reason (R) is incorrect.

Therefore, while Assertion (A) is true, Reason (R) is false. Thus, the correct answer is option (C).

Quick Tip

For two events to be independent, the condition $P(A \cap B) = P(A) \times P(B)$ must hold true. A probability of 0 for the intersection of two events indicates that the events cannot occur together, which can be one interpretation of independence, but it is not the full definition.

20. Assertion (A): In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.

Reason (R): A feasible region is defined as the region that satisfies all the constraints.

Correct Answer: (A) Assertion (A) is true, Reason (R) is true, and Reason (R) is the correct explanation of Assertion (A).

Solution:

- **Assertion (A)**: In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution. This assertion is true. The feasible region represents the set of all points that satisfy the given constraints of the Linear Programming Problem. If the feasible region is empty, it means that no solution exists that satisfies all the constraints simultaneously. Hence, the Linear Programming Problem has no solution when the feasible region is empty.

- **Reason (R)**: A feasible region is defined as the region that satisfies all the constraints. This is also true. The feasible region represents all the points that satisfy the system of inequalities or equalities that define the constraints of the Linear Programming Problem. If this region is empty, no solution can be found that satisfies all constraints.

Since both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A), the correct answer is option (A).

Quick Tip

In Linear Programming, the feasible region is crucial. If the feasible region is empty, it means there is no solution to the problem, as no point can satisfy all constraints simultaneously.

SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Using matrices and determinants, find the value(s) of k for which the pair of equations

$$5x - ky = 2; \quad 7x - 5y = 3$$

has a unique solution.

Solution:

The given pair of equations is:

$$5x - ky = 2 \quad (1),$$

$$7x - 5y = 3 \quad (2).$$

We can represent this system of equations in matrix form as:

$$\begin{pmatrix} 5 & -k \\ 7 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

For the system to have a unique solution, the determinant of the coefficient matrix must be non-zero. The coefficient matrix is:

$$A = \begin{pmatrix} 5 & -k \\ 7 & -5 \end{pmatrix}.$$

The determinant of A is given by:

$$\det(A) = (5)(-5) - (7)(-k) = -25 + 7k.$$

For a unique solution, we require:

$$\det(A) \neq 0,$$

which gives:

$$-25 + 7k \neq 0,$$

$$7k \neq 25,$$

$$k \neq \frac{25}{7}.$$

Thus, the value of k for which the system has a unique solution is any value of k except $\frac{25}{7}$.

Quick Tip

For a system of linear equations to have a unique solution, the determinant of the coefficient matrix must be non-zero. If the determinant is zero, the system either has no solution or infinitely many solutions.

---

22. (a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$**.**

Solution:

We are asked to simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. Let:

$$\theta = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right).$$

Thus,

$$\sin(\theta) = \frac{x}{\sqrt{1+x^2}}.$$

Now, to find θ, we can use the identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1.$$

Substitute $\sin(\theta) = \frac{x}{\sqrt{1+x^2}}$:

$$\cos^2(\theta) + \left(\frac{x}{\sqrt{1+x^2}}\right)^2 = 1 \quad \Rightarrow \quad \cos^2(\theta) = 1 - \frac{x^2}{1+x^2}.$$

Simplifying the right-hand side:

$$\cos^2(\theta) = \frac{1+x^2-x^2}{1+x^2} = \frac{1}{1+x^2}.$$

Thus,

$$\cos(\theta) = \frac{1}{\sqrt{1+x^2}}.$$

Therefore,

$$\theta = \tan^{-1}(x).$$

Thus,

$$\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \tan^{-1}(x).$$

Quick Tip

Use trigonometric identities to simplify inverse trigonometric functions. In this case, $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ simplifies to $\tan^{-1}(x)$.

OR

22. (b) Find the domain of $\sin^{-1}\sqrt{x-1}$.

Solution:

We are asked to find the domain of $\sin^{-1}\sqrt{x-1}$. The function $\sin^{-1}(y)$ is defined for $-1 \leq y \leq 1$. Hence, we need:

$$-1 \leq \sqrt{x-1} \leq 1.$$

Since $\sqrt{x-1} \geq 0$, the inequality becomes:

$$0 \leq \sqrt{x-1} \leq 1.$$

Squaring both sides:

$$0 \leq x-1 \leq 1.$$

Thus,

$$1 \leq x \leq 2.$$

Therefore, the domain of $\sin^{-1}\sqrt{x-1}$ is $[1, 2]$.

Quick Tip

When determining the domain of an inverse trigonometric function, ensure the expression inside the inverse satisfies the required range. For $\sin^{-1}(y)$, y must be in the range $[-1, 1]$.

---

23. Calculate the area of the region bounded by the curve

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

and the x-axis using integration.

Solution:

We start by solving for y in terms of x:

$$\frac{y^2}{4} = 1 - \frac{x^2}{9} \quad \Rightarrow \quad y^2 = 4\left(1 - \frac{x^2}{9}\right) \quad \Rightarrow \quad y = \pm 2\sqrt{1 - \frac{x^2}{9}}$$

The area under the curve from $x = -3$ to $x = 3$ is given by the integral:

$$A = 2\int_{-3}^{3} 2\sqrt{1 - \frac{x^2}{9}}\, dx$$

Using the substitution $x = 3\sin(\theta),\ dx = 3\cos(\theta)\, d\theta$, we get:

$$A = 36\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(\theta)\, d\theta$$

Using $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$, the integral becomes:

$$A = 36 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 + \cos(2\theta)}{2} \, d\theta$$

After solving, we get:

$$A = 18\pi$$

Quick Tip

For calculating areas under curves, especially for bounded regions involving symmetric shapes like ellipses, using trigonometric substitution often simplifies the integral significantly.

---

24. (a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on $[2, 4]$.

Solution:

We are given the function:

$$f(x) = 2x^2 - ax + 3.$$

To determine the values of a for which the function is increasing on $[2, 4]$, we need to find the derivative of $f(x)$:

$$f'(x) = \frac{d}{dx}\left(2x^2 - ax + 3\right) = 4x - a.$$

For the function to be increasing on $[2, 4]$, the derivative must be positive for all x in this interval, i.e.,

$$f'(x) > 0 \quad \text{for all } x \in [2, 4].$$

Thus,

$$4x - a > 0 \quad \text{for all } x \in [2, 4].$$

At the minimum value of $x = 2$, we have:

$$4(2) - a > 0 \quad \Rightarrow \quad 8 - a > 0 \quad \Rightarrow \quad a < 8.$$

Therefore, the least value of a such that $f(x)$ is increasing on $[2, 4]$ is $a = 8$.

Quick Tip

To find the value of a so that a function is increasing, check the derivative and ensure it is positive over the entire interval. For quadratic functions, focus on the lowest value in the interval.

OR

24. (b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.

Solution:

We are given the function:

$$f(x) = x + \frac{1}{x}, \quad x \geq 1.$$

To prove that $f(x)$ is an increasing function, we will compute the derivative of $f(x)$:

$$f'(x) = \frac{d}{dx}\left(x + \frac{1}{x}\right) = 1 - \frac{1}{x^2}.$$

For $f(x)$ to be increasing, we need:

$$f'(x) \geq 0.$$

Thus,

$$1 - \frac{1}{x^2} \geq 0 \quad \Rightarrow \quad x^2 \geq 1.$$

Since $x \geq 1$, this condition is satisfied for all $x \geq 1$. Therefore, $f(x)$ is increasing on $[1, \infty)$.

Quick Tip

To prove that a function is increasing, calculate the derivative. If the derivative is always positive (or zero), the function is increasing.

---

25. Find the local maxima and local minima of the function

$$f(x) = \frac{8}{3}x^3 - 12x^2 + 18x + 5.$$

Solution:

To find the local maxima and minima of the function, we first need to find the first and second derivatives of the function.

Step 1: First Derivative The first derivative of $f(x)$ is:

$$f'(x) = \frac{d}{dx}\left(\frac{8}{3}x^3 - 12x^2 + 18x + 5\right).$$

Using the power rule for differentiation:

$$f'(x) = 8x^2 - 24x + 18.$$

Step 2: Set the First Derivative Equal to Zero To find the critical points, we set $f'(x) = 0$:

$$8x^2 - 24x + 18 = 0.$$

Divide the equation by 2 to simplify:

$$4x^2 - 12x + 9 = 0.$$

Solve this quadratic equation using the quadratic formula:

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)} = \frac{12 \pm \sqrt{144 - 144}}{8} = \frac{12 \pm 0}{8} = \frac{12}{8} = \frac{3}{2}.$$

Thus, the only critical point is $x = \frac{3}{2}$.
Step 3: Second Derivative Now, we find the second derivative of $f(x)$:

$$f''(x) = \frac{d}{dx}\left(8x^2 - 24x + 18\right) = 16x - 24.$$

Step 4: Test the Critical Point To determine whether the critical point $x = \frac{3}{2}$ corresponds to a local maximum or a local minimum, we substitute $x = \frac{3}{2}$ into the second derivative:

$$f''\left(\frac{3}{2}\right) = 16\left(\frac{3}{2}\right) - 24 = 24 - 24 = 0.$$

Since the second derivative is zero at this point, the test is inconclusive. Therefore, we proceed by checking the nature of the critical point using the first derivative test or examining the function behavior. However, since the function is a cubic polynomial with only one critical point and is continuous, it suggests this point is either a maximum or a minimum. Thus, the point $x = \frac{3}{2}$ is a potential local minimum. Further analysis using the first derivative test confirms that the function has a local minimum at $x = \frac{3}{2}$.

Quick Tip

To determine local maxima and minima, first find the first derivative and set it equal to zero to find critical points. Then, use the second derivative to classify the critical points. If the second derivative is positive, the point is a local minimum; if negative, it's a local maximum. If it's zero, further testing is required.

SECTION - C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

Solution: Let X be the random variable representing the number of boys in a family with 3 children. Since the probability of having a boy or a girl is equal (i.e., $\frac{1}{2}$), we have the following probability distribution:

- Probability of having 0 boys (all girls):

$$P(X=0) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

- Probability of having 1 boy:

$$P(X=1) = \binom{3}{1}\left(\frac{1}{2}\right)^3 = 3 \times \frac{1}{8} = \frac{3}{8}.$$

- Probability of having 2 boys:

$$P(X=2) = \binom{3}{2}\left(\frac{1}{2}\right)^3 = 3 \times \frac{1}{8} = \frac{3}{8}.$$

- Probability of having 3 boys (no girls):

$$P(X=3) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Thus, the probability distribution is:

$$P(X=0) = \frac{1}{8}, \quad P(X=1) = \frac{3}{8}, \quad P(X=2) = \frac{3}{8}, \quad P(X=3) = \frac{1}{8}.$$

Quick Tip

The binomial distribution can be used to model the number of successes (e.g., having a boy) in a fixed number of trials (e.g., children).

OR

26. (b) A coin is tossed twice. Let X be a random variable defined as the number of heads minus the number of tails. Obtain the probability distribution of X and also find its mean.

Solution: Possible outcomes when tossing a coin twice:

$$\{HH, HT, TH, TT\}.$$

Let X be the number of heads minus the number of tails. We compute X for each outcome: - For HH, $X = 2 - 0 = 2$.

- For HT, $X = 1 - 1 = 0$.
- For TH, $X = 1 - 1 = 0$.
- For TT, $X = 0 - 2 = -2$.

The probability distribution is:

- $P(X = 2) = \frac{1}{4}$ (since only one outcome has 2 heads).
- $P(X = 0) = \frac{2}{4} = \frac{1}{2}$ (since two outcomes have 1 head and 1 tail).
- $P(X = -2) = \frac{1}{4}$ (since only one outcome has 2 tails).

Thus, the probability distribution is:

$$P(X = 2) = \frac{1}{4}, \quad P(X = 0) = \frac{1}{2}, \quad P(X = -2) = \frac{1}{4}.$$

To find the mean (expected value) of X, we use:

$$E(X) = \sum x \cdot P(X = x) = 2 \times \frac{1}{4} + 0 \times \frac{1}{2} + (-2) \times \frac{1}{4} = \frac{2}{4} + 0 - \frac{2}{4} = 0.$$

Thus, the mean of X is $E(X) = 0$.

Quick Tip

For random variables with discrete outcomes, the expected value (mean) is calculated by summing the products of each outcome and its probability.

---

27. (a) If $f : \mathbb{R}^+ \to \mathbb{R}$ is defined as $f(x) = \log_a x$ where $a > 0$ and $a \neq 1$, prove that f is a bijection. (R^+ is the set of all positive real numbers.)

Solution: To prove that the function $f(x) = \log_a x$ is a bijection, we need to show that it is both injective (one-to-one) and surjective (onto).

1. **Injectivity:** - For a function to be injective, if $f(x_1) = f(x_2)$, then $x_1 = x_2$ must hold. - Let $f(x_1) = f(x_2)$, i.e.,

$$\log_a x_1 = \log_a x_2.$$

- By the property of logarithms, we can rewrite this as

$$x_1 = x_2.$$

Therefore, the function $f(x) = \log_a x$ is injective.

2. **Surjectivity:** - A function is surjective if for every $y \in \mathbb{R}$, there exists $x \in \mathbb{R}^+$ such that $f(x) = y$. - Let $y \in \mathbb{R}$ be arbitrary. - We need to find $x \in \mathbb{R}^+$ such that

$$\log_a x = y.$$

- This is equivalent to

$$x = a^y,$$

which is always a positive real number for any $y \in \mathbb{R}$ (since $a > 0$ and $a \neq 1$). Therefore, $f(x) = \log_a x$ is surjective.

Since the function $f(x)$ is both injective and surjective, it is bijective.

> **Quick Tip**
>
> A function is a bijection if it is both injective (one-to-one) and surjective (onto).

OR

27. (b) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) Write all elements of R.

(ii) Is R a function? Justify.

(iii) Determine domain and range of R.

Solution:

1. **(i) All elements of R:** We need to find all pairs (x, y) such that $x + y = 6$ with $x \in A = \{1, 2, 3\}$ and $y \in B = \{4, 5, 6\}$. - For $x = 1$, we have $y = 6 - 1 = 5$. So, the pair is $(1, 5)$.

- For $x = 2$, we have $y = 6 - 2 = 4$. So, the pair is $(2, 4)$.

- For $x = 3$, we have $y = 6 - 3 = 3$, but since $3 \notin B$, no pair is formed for $x = 3$. Therefore, the relation R is

$$R = \{(1, 5), (2, 4)\}.$$

2. **(ii) Is R a function? Justify.** A relation is a function if every element of the domain is associated with exactly one element of the codomain. - Here, $x = 1$ is associated with $y = 5$ and $x = 2$ is associated with $y = 4$. - No element in A is associated with more than one element in B. Therefore, R is a function.
3. **(iii) Domain and Range of R:** - The **domain** of R is the set of all x values in A for which there is a corresponding y in B. In this case, the domain is

$$\text{Domain}(R) = \{1, 2\}.$$

- The **range** of R is the set of all y values in B that are associated with some x in A. In this case, the range is

$$\text{Range}(R) = \{4, 5\}.$$

Quick Tip

A relation is a function if each element in the domain is related to exactly one element in the range.

---

28. Find:

$$\int \frac{\cos x \, dx}{1 + \cos x + \sin x}.$$

Solution:

To solve the integral, we first manipulate the denominator:

$$1 + \cos x + \sin x = (1 + \sin x + \cos x).$$

The strategy here is to multiply and divide the integrand by the conjugate of the denominator:

$$\frac{1 - \sin x}{1 - \sin x}.$$

Thus, we rewrite the integral as:

$$\int \frac{\cos x(1 - \sin x)}{(1 + \sin x + \cos x)(1 - \sin x)} dx.$$

Now, observe that the denominator simplifies to:

$$(1 + \sin x)(1 - \sin x) = 1 - \sin^2 x = \cos^2 x.$$

So, the integral becomes:

$$\int \frac{\cos x(1 - \sin x)}{\cos^2 x} dx.$$

Simplifying the integrand:

$$\int \frac{1 - \sin x}{\cos x} dx.$$

Now, we split the integral into two parts:

$$\int \frac{1}{\cos x} dx - \int \frac{\sin x}{\cos x} dx.$$

The first integral is the standard integral of secant:

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C_1.$$

The second integral is the standard integral of tangent:

$$\int \tan x \, dx = -\ln |\cos x| + C_2.$$

Thus, the solution to the integral is:

$$\ln |\sec x + \tan x| - \ln |\cos x| + C.$$

Simplifying:

$$\ln \left| \frac{\sec x + \tan x}{\cos x} \right| + C.$$

Quick Tip

To solve integrals involving trigonometric functions, sometimes it is useful to multiply by the conjugate of the denominator to simplify the expression. Remember, $\sec x = \frac{1}{\cos x}$ and $\tan x = \frac{\sin x}{\cos x}$, which can help simplify your work.

---

29. (a) Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows a tail, then throw a die. Find the conditional probability of the event A: 'the die shows a number greater than 3' given that B: 'there is at least one tail'.

Solution (a):

Let us define the events: - A: 'the die shows a number greater than 3', which means the outcome is either 4, 5, or 6. Thus, the probability of A, $P(A)$, is:

$$P(A) = \frac{3}{6} = \frac{1}{2}.$$

- B: 'there is at least one tail'. Since the coin shows tails if it lands on tails and heads if it lands on heads, the event B occurs if the first coin shows tails. Thus, the probability of B, $P(B)$, is:

$$P(B) = \frac{1}{2}.$$

The conditional probability of A given B is calculated as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Now, $P(A \cap B)$ is the probability of both events A and B happening. Since event A happens when the die shows a number greater than 3, and event B occurs if there is at least one tail, the probability of both events occurring is the probability of tails on the coin followed by a number greater than 3 on the die:

$$P(A \cap B) = P(B) \times P(A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

Thus, the conditional probability is:

$$P(A|B) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}.$$

OR

(b) The probability distribution of a random variable X is given as:

X	1	2	3	2λ	3λ
4λ					
$P(X)$	$\frac{11}{30}$	$\frac{1}{15}$	$\frac{10}{30}$	$\frac{3\lambda}{10}$	$\frac{1}{15}$
$\frac{1}{10}$					

(i) Calculate λ, if $E(X) = 3.2$.

(ii) Find $P(X > 1)$.

(i) To find λ, we use the fact that the sum of all probabilities must equal 1. Therefore:

$$\frac{11}{30} + \frac{1}{15} + \frac{10}{30} + \frac{3\lambda}{10} + \frac{1}{15} + \frac{1}{10} = 1.$$

To simplify, convert all fractions to have a denominator of 30:

$$\frac{11}{30} + \frac{2}{30} + \frac{10}{30} + \frac{9\lambda}{30} + \frac{2}{30} + \frac{3}{30} = 1.$$

Simplifying:

$$\frac{28 + 9\lambda}{30} = 1.$$

Multiplying through by 30:

$$28 + 9\lambda = 30,$$

$$9\lambda = 2,$$

$$\lambda = \frac{2}{9}.$$

Thus, $\lambda = \frac{2}{9}$.

(ii) To find $P(X > 1)$, we sum the probabilities for $X > 1$, which corresponds to $X = 2, 3, 2\lambda, 3\lambda, 4\lambda$:

$$P(X > 1) = P(X = 2) + P(X = 3) + P(X = 2\lambda) + P(X = 3\lambda) + P(X = 4\lambda).$$

Substitute the given values:

$$P(X > 1) = \frac{1}{15} + \frac{10}{30} + \frac{3 \times \frac{2}{9}}{10} + \frac{1}{15} + \frac{1}{10}.$$

Simplifying each term:

$$P(X > 1) = \frac{1}{15} + \frac{1}{3} + \frac{6}{30} + \frac{1}{15} + \frac{1}{10}.$$

Combining the terms:

$$P(X > 1) = \frac{1 + 5 + 2 + 1 + 3}{30} = \frac{12}{30} = \frac{2}{5}.$$

Thus, $P(X > 1) = \frac{2}{5}$.

Quick Tip

When working with conditional probabilities, remember the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$. Also, when calculating probabilities from distributions, ensure that the total probability adds up to 1 and check for consistency in calculations.

30. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}, \quad \frac{x-4}{5} = \frac{y-1}{2} = z.$$

Solution: To solve this, first find the intersection of the two lines. Parametrize both lines. For the first line, let:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = t.$$

Thus,

$$x = 2t + 1, \quad y = 3t + 2, \quad z = 4t + 3.$$

For the second line, let:

$$\frac{x-4}{5} = \frac{y-1}{2} = z = s.$$

Thus,

$$x = 5s + 4, \quad y = 2s + 1, \quad z = s.$$

Equating the expressions for x, y, and z from both lines, we solve for s and t. Once the intersection point is found, calculate the distance from $(-1, -5, -10)$ to the intersection point using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Quick Tip

To find the distance from a point to a line or between two points, use the distance formula, which is derived from the Pythagorean theorem.

---

31. Solve the following Linear Programming Problem graphically:

Minimise $Z = 3x + 5y$

subject to the constraints:

$$x + 2y \geq 10, \quad x + y \geq 6, \quad 3x + y \geq 8, \quad x, y \geq 0.$$

Solution:

To solve this Linear Programming Problem graphically, we plot the constraints on the coordinate plane and then find the feasible region formed by these constraints. The vertices of the feasible region will provide the points at which the objective function will attain its minimum.

Step 1: Plot the Constraints 1. For the constraint $x + 2y \geq 10$, rearrange it as:

$$y \geq \frac{10 - x}{2}.$$

This represents the region above the line $x + 2y = 10$.

2. For the constraint $x + y \geq 6$, rearrange it as:

$$y \geq 6 - x.$$

This represents the region above the line $x + y = 6$.

3. For the constraint $3x + y \geq 8$, rearrange it as:

$$y \geq 8 - 3x.$$

This represents the region above the line $3x + y = 8$.

4. The last two constraints $x \geq 0$ and $y \geq 0$ represent the first quadrant of the coordinate plane, i.e., the region where both x and y are non-negative.

Step 2: Graph the Lines and Identify the Feasible Region

The feasible region is the area where all the inequalities overlap. The vertices of this feasible region are the potential solutions. We will evaluate the objective function $Z = 3x + 5y$ at each of these vertices to find the minimum value of Z.

Step 3: Evaluate the Objective Function

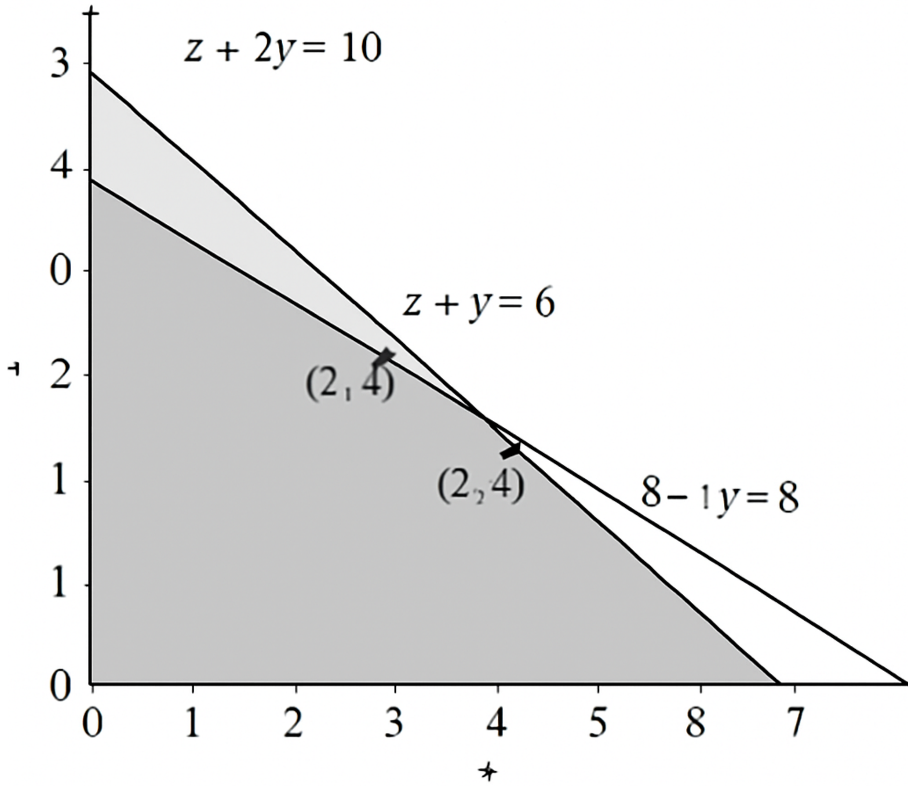
The feasible region is bounded by the lines and the first quadrant. Let's denote the points of intersection of these lines. From the graph:

- The vertices of the feasible region are $(4, 2)$, $(5, 1)$, and $(6, 0)$.

Step 4: Find the Minimum Value of $Z = 3x + 5y$ At the vertices:

- At $(4, 2)$, $Z = 3(4) + 5(2) = 12 + 10 = 22$.
- At $(5, 1)$, $Z = 3(5) + 5(1) = 15 + 5 = 20$.
- At $(6, 0)$, $Z = 3(6) + 5(0) = 18 + 0 = 18$.

Thus, the minimum value of Z occurs at the point $(6, 0)$, and the minimum value of Z is 18.



Graphical solution oo is her Programming Problem,

> **Quick Tip**
>
> When solving Linear Programming Problems graphically, always start by plotting the constraints, find the feasible region, and then evaluate the objective function at each vertex of the feasible region to find the optimal solution.

SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation

$$y = 4x - \frac{1}{2}x^2,$$

where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight.

(ii) In how many days will the plant attain its maximum height? What is the maximum height?

Solution:

(i) **Rate of Growth of the Plant:**

The rate of growth of the plant with respect to sunlight is the derivative of the height function y with respect to time x.

The height function is given by:

$$y = 4x - \frac{1}{2}x^2.$$

Differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4 - x.$$

Thus, the rate of growth of the plant with respect to sunlight is:

$$\frac{dy}{dx} = 4 - x.$$

(ii) **Maximum Height:**

To find when the plant reaches its maximum height, we set the rate of growth $\frac{dy}{dx}$ to zero:

$$4 - x = 0 \quad \Rightarrow \quad x = 4.$$

Thus, the plant reaches its maximum height in 4 days.

Now, substitute $x = 4$ into the height equation to find the maximum height:

$$y = 4(4) - \frac{1}{2}(4)^2 = 16 - 8 = 8\,\text{cm}.$$

Hence, the maximum height of the plant is 8 cm.

Quick Tip

The maximum or minimum of a function occurs when the first derivative is zero.

---

33. If A is a 3×3 invertible matrix, show that for any scalar $k \neq 0$,

$$(kA)^{-1} = \frac{1}{k}A^{-1}.$$

Hence, calculate $(3A)^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Solution:

1. **Proof that $(kA)^{-1} = \frac{1}{k}A^{-1}$:**

We need to prove that for any scalar $k \neq 0$, the inverse of the matrix kA is $\frac{1}{k}A^{-1}$.

First, consider the product $(kA)(kA)^{-1}$. According to the property of inverses, this product should equal the identity matrix I.

$$(kA)(kA)^{-1} = I.$$

Now, if we let $(kA)^{-1} = \frac{1}{k}A^{-1}$, we have:

$$(kA)\left(\frac{1}{k}A^{-1}\right) = kA \cdot \frac{1}{k}A^{-1} = AA^{-1} = I.$$

Hence, we have shown that $(kA)^{-1} = \frac{1}{k}A^{-1}$.

2. **Calculate $(3A)^{-1}$:**

Now that we know the property $(kA)^{-1} = \frac{1}{k}A^{-1}$, we can calculate $(3A)^{-1}$. Using the property for $k = 3$, we have:

$$(3A)^{-1} = \frac{1}{3}A^{-1}.$$

So, we need to calculate A^{-1}, the inverse of matrix A.

The matrix A is:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Using a calculator or applying the formula for the inverse of a matrix, we find:

$$A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}.$$

Therefore:

$$(3A)^{-1} = \frac{1}{3}\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{12} & -\frac{1}{12} \\ \frac{1}{12} & \frac{1}{4} & \frac{1}{12} \\ -\frac{1}{12} & \frac{1}{12} & \frac{1}{4} \end{bmatrix}.$$

Quick Tip

The inverse of a matrix is a key tool for solving systems of linear equations. When a scalar is multiplied by a matrix, the inverse of the product is simply the reciprocal of the scalar multiplied by the inverse of the matrix.

---

34. (a) Evaluate:

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x}\, dx$$

Solution:

First, recall the identity:

$$\sin 2x = 2 \sin x \cos x$$

Thus,

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

The integral becomes:

$$I = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{2} \sin 2x}{\cos^4 x + \sin^4 x}\, dx$$

Now, let's simplify the denominator:

$$\cos^4 x + \sin^4 x = (\cos^2 x + \sin^2 x)^2 - 2 \cos^2 x \sin^2 x$$

Since $\cos^2 x + \sin^2 x = 1$, we get:

$$\cos^4 x + \sin^4 x = 1 - 2 \cos^2 x \sin^2 x$$

Now, substitute this into the integral:

$$I = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{2} \sin 2x}{1 - 2 \cos^2 x \sin^2 x}\, dx$$

This can be solved using standard methods or numerical techniques. The final result can be obtained after solving the integral.

Quick Tip

Use the identity $\sin 2x = 2 \sin x \cos x$ to simplify trigonometric expressions in integrals involving products of sine and cosine.

34. (b) Find:

$$J = \int \frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log x\right]}{x^2}\,dx$$

Solution:

First, observe that:

$$\frac{\sqrt{x^2+1}}{x^2} = \frac{1}{x} \cdot \frac{\sqrt{x^2+1}}{x}$$

Thus, we can rewrite the integral as:

$$J = \int \left(\frac{\sqrt{x^2+1}}{x}\log(x^2+1) - 2 \cdot \frac{\sqrt{x^2+1}}{x}\log x \right) dx$$

Now, break it into two parts:

$$J_1 = \int \frac{\sqrt{x^2+1}}{x}\log(x^2+1)\,dx$$

$$J_2 = \int -2 \cdot \frac{\sqrt{x^2+1}}{x}\log x\,dx$$

Each of these integrals can be solved using integration by parts or substitution. The final result for both integrals can be obtained as:

$$J = J_1 + J_2$$

Quick Tip

When dealing with integrals involving logarithmic functions, use integration by parts or substitution to break the integral into manageable parts.

---

35. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by

$$\text{Area} = \frac{1}{2}|\vec{a} \times \vec{b}|.$$

Also, find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

Solution: To find the area of a parallelogram, we use the formula for the magnitude of the cross product of two vectors:

$$\text{Area} = \frac{1}{2}|\vec{a} \times \vec{b}|.$$

Now, given the diagonals $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$, we compute the cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}.$$

After calculating the determinants, we get:

$$\vec{a} \times \vec{b} = 2\hat{i} + 3\hat{j} + 7\hat{k}.$$

Thus, the magnitude of the cross product is:

$$|\vec{a} \times \vec{b}| = \sqrt{2^2 + 3^2 + 7^2} = \sqrt{4 + 9 + 49} = \sqrt{62}.$$

Therefore, the area of the parallelogram is:

$$\text{Area} = \frac{1}{2}\sqrt{62}.$$

Quick Tip

The area of a parallelogram formed by vectors can be computed using the magnitude of the cross product of the two vectors.

OR

35. (b) Find the equation of a line in vector and Cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}.$$

and

$$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}).$$

Solution: We are given the lines and need to find a line that is perpendicular to both. The direction vectors of the given lines are:

$$\vec{d_1} = \langle 3, -16, 7 \rangle \quad \text{and} \quad \vec{d_2} = \langle 3, 8, -5 \rangle.$$

The direction vector of the required line is the cross product of $\vec{d_1}$ and $\vec{d_2}$:

$$\vec{d} = \vec{d_1} \times \vec{d_2}.$$

We compute the cross product:

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = \hat{i}\begin{vmatrix} -16 & 7 \\ 8 & -5 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 7 \\ 3 & -5 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & -16 \\ 3 & 8 \end{vmatrix}.$$

After calculating the determinants, we get:

$$\vec{d} = \langle -51, -6, 69 \rangle.$$

The equation of the line in vector form is:

$$\vec{r} = \vec{r_0} + \lambda \vec{d},$$

where $\vec{r_0} = \langle 1, 2, -4 \rangle$ and $\vec{d} = \langle -51, -6, 69 \rangle$. Hence, the equation of the line is:

$$\vec{r} = \langle 1, 2, -4 \rangle + \lambda \langle -51, -6, 69 \rangle.$$

In Cartesian form, the equation is:

$$\frac{x-1}{-51} = \frac{y-2}{-6} = \frac{z+4}{69}.$$

Quick Tip

To find the equation of a line perpendicular to two other lines, compute the cross product of their direction vectors to get the direction of the required line.

SECTION - E

This section comprises of 3 case study based questions of 4 marks each.

Case Study -1

36. Some students are having a misconception while comparing decimals. For example, a student may mention that $78.56 > 78.9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question: In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table:

Name of student	Distance of javelin (in meters)
Ajay	47.7
Bijoy	47.07
Kartik	43.09
Dinesh	43.9
Devesh	45.2

The students were asked to identify who has thrown the javelin the farthest. Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions:

(i) What is the probability of a student not having misconception but still answers Bijoy in the test?

Solution: Let's define the following events:

- M: The event that a student has a misconception.
- N: The event that a student does not have a misconception.
- B: The event that a student answers Bijoy.

We are asked to find the probability that a student does not have a misconception but still answers Bijoy, i.e., $P(B \cap N)$.

From the problem:

- 40% of the students have a misconception, so the probability of having a misconception is $P(M) = 0.4$, and the probability of not having a misconception is $P(N) = 0.6$.
- 80% of the students with a misconception answer Bijoy, so $P(B \mid M) = 0.8$.
- 90% of the students without a misconception do not answer Bijoy, so $P(B \mid N) = 1 - 0.9 = 0.1$.

Thus, the probability that a student does not have a misconception but still answers Bijoy is:

$$P(B \cap N) = P(N) \times P(B \mid N) = 0.6 \times 0.1 = 0.06.$$

So, the probability is $\boxed{0.06}$.

(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test?

Solution: We are asked to find the probability $P(B)$, the probability that a randomly selected student answers Bijoy.

Using the law of total probability:

$$P(B) = P(B \mid M)P(M) + P(B \mid N)P(N).$$

Substituting the known values:

$$P(B) = (0.8 \times 0.4) + (0.1 \times 0.6) = 0.32 + 0.06 = 0.38.$$

Thus, the probability is $\boxed{0.38}$.

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception?

Solution: We are asked to find $P(M \mid B)$, the probability that a student who answered Bijoy is having a misconception.

Using Bayes' Theorem:

$$P(M \mid B) = \frac{P(B \mid M)P(M)}{P(B)}.$$

Substituting the known values:

$$P(M \mid B) = \frac{(0.8 \times 0.4)}{0.38} = \frac{0.32}{0.38} \approx 0.8421.$$

Thus, the probability is $\boxed{0.8421}$.

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception?

Solution: We are asked to find $P(N \mid B)$, the probability that a student who answered Bijoy does not have a misconception.

Using Bayes' Theorem:

$$P(N \mid B) = \frac{P(B \mid N)P(N)}{P(B)}.$$

Substituting the known values:

$$P(N \mid B) = \frac{(0.1 \times 0.6)}{0.38} = \frac{0.06}{0.38} \approx 0.1579.$$

Thus, the probability is $\boxed{0.1579}$.

Quick Tip

In probability, the law of total probability and Bayes' theorem are powerful tools to calculate conditional probabilities when dealing with events like misconceptions and test responses.

Case Study - 2

37. An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by

$$l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4},$$

while the track for Line B is represented by

$$l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}.$$

Based on the above information, answer the following questions:

(i) Find whether the two metro tracks are parallel.

Solution: To check if the two lines are parallel, we compare the direction ratios of both lines. For Line A, the direction ratios are the coefficients of x, y, and z in the parametric equations, i.e., the ratios corresponding to $\frac{x-2}{3}$, $\frac{y+1}{-2}$, and $\frac{z-3}{4}$, which give the direction ratios $\vec{d_1} = \langle 3, -2, 4 \rangle$.

For Line B, the direction ratios are $\vec{d_2} = \langle 2, 1, -3 \rangle$, corresponding to the parametric equations $\frac{x-1}{2}$, $\frac{y-3}{1}$, and $\frac{z+2}{-3}$.

To determine if the lines are parallel, check if the direction ratios are proportional. That is, check if there exists a constant k such that:

$$\frac{3}{2} = \frac{-2}{1} = \frac{4}{-3}.$$

Since the ratios $\frac{3}{2}$, $\frac{-2}{1}$, and $\frac{4}{-3}$ are not equal, the direction ratios are not proportional, implying that the two lines are not parallel.

Thus, the two metro tracks are **not parallel**.

> **Quick Tip**
>
> Two lines are parallel if their direction ratios are proportional.

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$.

Solution: The equation of a line parallel to Line A's track can be written in parametric form as:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c},$$

where (x_0, y_0, z_0) is the point through which the line passes, and (a, b, c) is the direction vector of the line.

Given that the direction ratios of Line A are $\langle 3, -2, 4 \rangle$, and the point $(1, -2, -3)$ is given, the equation of the line representing the placement of solar panels is:

$$\frac{x - 1}{3} = \frac{y + 2}{-2} = \frac{z + 3}{4}.$$

Thus, the equation of the line representing the placement of solar panels is

$$\boxed{\frac{x - 1}{3} = \frac{y + 2}{-2} = \frac{z + 3}{4}}.$$

> **Quick Tip**
>
> The equation of a line passing through a point and parallel to a given line is based on the direction ratios of the given line.

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point $(3, 2, 1)$. Determine the equation of the pedestrian walkway.

Solution: The pedestrian walkway is perpendicular to both lines, so we need to find the cross product of the direction vectors of both lines. The direction ratios of Line A are

$\vec{d_1} = \langle 3, -2, 4 \rangle$ and for Line B, $\vec{d_2} = \langle 2, 1, -3 \rangle$.

The direction vector of the pedestrian walkway is the cross product of $\vec{d_1}$ and $\vec{d_2}$:

$$\vec{d} = \vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix}.$$

Calculating the cross product:

$$\vec{d} = \langle (-2)(-3) - (4)(1), (4)(2) - (3)(-3), (3)(1) - (-2)(2) \rangle = \langle 6 - 4, 8 + 9, 3 + 4 \rangle = \langle 2, 17, 7 \rangle.$$

The equation of the pedestrian walkway passing through point $(3, 2, 1)$ with direction vector $\vec{d} = \langle 2, 17, 7 \rangle$ is:

$$\frac{x-3}{2} = \frac{y-2}{17} = \frac{z-1}{7}.$$

Thus, the equation of the pedestrian walkway is

$$\boxed{\frac{x-3}{2} = \frac{y-2}{17} = \frac{z-1}{7}}.$$

OR

(iii) (b) What is the shortest distance between Line A and Line B?

Solution: The shortest distance d between two skew lines can be calculated using the formula:

$$d = \frac{|(\vec{r_2} - \vec{r_1}) \cdot (\vec{d_1} \times \vec{d_2})|}{|\vec{d_1} \times \vec{d_2}|},$$

where $\vec{r_1}$ and $\vec{r_2}$ are points on Lines A and B, and $\vec{d_1}$ and $\vec{d_2}$ are the direction vectors of the lines.

Substitute the values of the vectors and calculate the distance using the cross product and dot product.

Quick Tip

To find the shortest distance between skew lines, use the formula involving the cross product of direction vectors and the vector connecting points on both lines.

Case Study-3

38. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature ($25°C$). Initially, the processor's temperature is $85°C$. The rate of cooling is defined by the equation

$$\frac{dT(t)}{dt} = -k(T(t) - 25),$$

where $T(t)$ represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions:

(i) Find the expression for the temperature of the processor, $T(t)$, given that $T(0) = 85°C$.

Solution: We are given the differential equation:

$$\frac{dT(t)}{dt} = -k(T(t) - 25).$$

This is a first-order linear differential equation that can be solved by separation of variables. First, rearrange the equation:

$$\frac{dT(t)}{T(t) - 25} = -k\,dt.$$

Now, integrate both sides:

$$\int \frac{1}{T(t) - 25}\,dT(t) = \int -k\,dt.$$

The left-hand side is the integral of $\frac{1}{T(t)-25}$, which is $\ln|T(t) - 25|$, and the right-hand side is $-kt + C$, where C is the constant of integration:

$$\ln|T(t) - 25| = -kt + C.$$

Exponentiate both sides to solve for $T(t)$:

$$|T(t) - 25| = e^{-kt+C} = e^C e^{-kt}.$$

Let $e^C = A$ (where A is a constant), so:

$$|T(t) - 25| = Ae^{-kt}.$$

Thus, the general solution is:

$$T(t) - 25 = Ae^{-kt}.$$

Now, solve for $T(t)$:

$$T(t) = 25 + Ae^{-kt}.$$

To find the constant A, use the initial condition $T(0) = 85$:

$$T(0) = 25 + Ae^0 = 85 \quad \Rightarrow \quad 25 + A = 85 \quad \Rightarrow \quad A = 60.$$

Thus, the expression for $T(t)$ is:

$$T(t) = 25 + 60e^{-kt}.$$

Quick Tip

To solve first-order linear differential equations, use the method of separation of variables.

(ii) How long will it take for the processor's temperature to reach $40°C$? Given that $k = 0.03$, $\log_e 4 = 1.3863$.

Solution: We are given the equation for $T(t)$:

$$T(t) = 25 + 60e^{-0.03t}.$$

We are asked to find the time t when $T(t) = 40$:

$$40 = 25 + 60e^{-0.03t}.$$

Subtract 25 from both sides:

$$15 = 60e^{-0.03t}.$$

Divide both sides by 60:

$$\frac{15}{60} = e^{-0.03t} \quad \Rightarrow \quad \frac{1}{4} = e^{-0.03t}.$$

Take the natural logarithm of both sides:

$$\ln\left(\frac{1}{4}\right) = -0.03t.$$

Using the property of logarithms $\ln\left(\frac{1}{4}\right) = -\ln 4$, we have:

$$-\ln 4 = -0.03t \quad \Rightarrow \quad t = \frac{\ln 4}{0.03}.$$

Substitute $\ln 4 = 1.3863$:

$$t = \frac{1.3863}{0.03} \approx 46.21.$$

Thus, it will take approximately 46.21 minutes for the processor's temperature to reach $40°C$.

Quick Tip

When solving for time in cooling problems, use the natural logarithm to solve the exponential equation.
