

CBSE Class 12 2025 Mathematics 65-6-1 Question Paper With Solutions

Time Allowed :3 Hour	Maximum Marks :70	Total questions :33
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions. Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. Let both AB' and $B'A$ be defined for matrices A and B . If the order of A is $n \times m$, then the order of B is:

- (1) $n \times n$
- (2) $n \times m$
- (3) $m \times m$
- (4) $m \times n$

Correct Answer: (4) $m \times n$

Solution:

For matrix multiplication to be defined, the number of columns of the first matrix must equal the number of rows of the second matrix. Given that the order of A is $n \times m$, and we are multiplying AB' and $B'A$, we know that A is $n \times m$ and B' will have dimensions $m \times n$ (since B' is the transpose of B).

For the product AB' to be defined, the number of columns of A (which is m) must match the number of rows of B' (which is also m). The resulting matrix will have dimensions $n \times n$.

Similarly, for the product $B'A$ to be defined, B' will have m rows and n columns, and A will have n rows and m columns. The resulting matrix will have dimensions $m \times n$.

Thus, the order of matrix B is $m \times n$.

Quick Tip

For matrix multiplication to be valid, ensure the number of columns in the first matrix equals the number of rows in the second matrix.

2. If

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

then A is a/an:

- (1) scalar matrix
- (2) identity matrix

(3) symmetric matrix

(4) skew-symmetric matrix

Correct Answer: (1) scalar matrix

Solution:

A scalar matrix is a matrix in which all the diagonal elements are equal, and all the off-diagonal elements are zero. The given matrix A is:

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

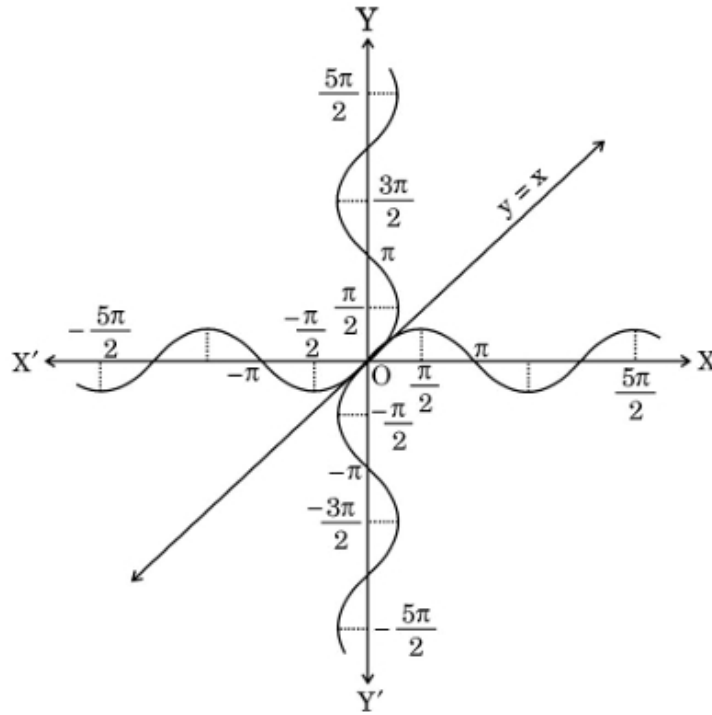
This matrix is not a scalar matrix because its diagonal elements are not all equal. Thus, we rule out the scalar matrix option.

Next, we observe that a symmetric matrix is one where $A = A^T$, and a skew-symmetric matrix is one where $A = -A^T$. The given matrix A is neither symmetric nor skew-symmetric, as it is not equal to its transpose and also not equal to the negative of its transpose. Therefore, the correct answer is that A is a scalar matrix.

Quick Tip

A scalar matrix must have equal diagonal elements, and all off-diagonal elements must be zero.

3. The following graph is a combination of:



- (1) $y = \sin^{-1} x$ and $y = \cos^{-1} x$
- (2) $y = \cos^{-1} x$ and $y = \cos x$
- (3) $y = \sin^{-1} x$ and $y = \sin x$
- (4) $y = \cos^{-1} x$ and $y = \sin x$

Correct Answer: (3) $y = \sin^{-1} x$ and $y = \sin x$

Solution:

The given graph shows a combination of inverse trigonometric and trigonometric functions. The function $y = \sin^{-1} x$ is the inverse sine function, which has a restricted domain and range between $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $[-1, 1]$, respectively. The graph of $y = \sin^{-1} x$ appears as an increasing curve.

The function $y = \sin x$ is the standard sine function, which oscillates between -1 and 1 , and has a period of 2π .

The graph shown matches the properties of $y = \sin^{-1} x$ and $y = \sin x$. Therefore, the correct answer is (3).

Quick Tip

Inverse trigonometric functions have restricted domains and ranges. The graphs of these functions often appear different from their trigonometric counterparts.

4. Sum of two skew-symmetric matrices of the same order is always a/an:

- (1) skew-symmetric matrix
- (2) symmetric matrix
- (3) null matrix
- (4) identity matrix

Correct Answer: (3) null matrix

Solution:

Let A and B be two skew-symmetric matrices of the same order. For a matrix to be skew-symmetric, it must satisfy the condition $A^T = -A$ and $B^T = -B$.

Now, consider the sum of A and B , i.e., $C = A + B$. We check if C is skew-symmetric:

$$C^T = (A + B)^T = A^T + B^T = -A + (-B) = -(A + B) = -C.$$

Thus, the sum of two skew-symmetric matrices is also skew-symmetric, but in the case where A and B are the same matrix, the sum will be a null matrix because the sum of each corresponding element will cancel out. Hence, the correct answer is (3).

Quick Tip

The sum of two skew-symmetric matrices results in a null matrix only when the matrices are identical, as their corresponding elements cancel each other out.

5.

$$\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

is equal to:

- (1) $\frac{11\pi}{12}$
- (2) $\frac{5\pi}{12}$
- (3) $-\frac{5\pi}{12}$
- (4) $\frac{7\pi}{12}$

Correct Answer: (3) $-\frac{5\pi}{12}$

Solution:

We are given the expression $\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$. We know that: $\sec^{-1}(-\sqrt{2})$ corresponds to the angle θ such that $\sec \theta = -\sqrt{2}$. The value of θ is $\frac{3\pi}{4}$. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ corresponds to the angle α such that $\tan \alpha = \frac{1}{\sqrt{3}}$, which gives $\alpha = \frac{\pi}{6}$.

Thus, the expression becomes:

$$\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{3\pi}{4} - \frac{\pi}{6}.$$

To simplify, we need a common denominator:

$$\frac{3\pi}{4} = \frac{9\pi}{12}, \quad \frac{\pi}{6} = \frac{2\pi}{12}.$$

Now, subtract:

$$\frac{9\pi}{12} - \frac{2\pi}{12} = \frac{7\pi}{12}.$$

Thus, the final answer is $\frac{7\pi}{12}$.

Quick Tip

When dealing with inverse trigonometric functions, recall their principal values and how they relate to angles on the unit circle.

6. If

$$f(x) = \begin{cases} \frac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of k is:

- (1) a
- (2) $a + b$
- (3) $a - b$
- (4) b

Correct Answer: (2) $a + b$

Solution:

For $f(x)$ to be continuous at $x = 0$, the limit of $f(x)$ as x approaches 0 must equal the value of $f(x)$ at $x = 0$, which is k . We first simplify the expression for $x \neq 0$:

$$\lim_{x \rightarrow 0} \frac{\log(1+ax) + \log(1-bx)}{x}.$$

Using the logarithmic property, $\log A + \log B = \log(AB)$, we get:

$$\lim_{x \rightarrow 0} \frac{\log((1+ax)(1-bx))}{x}.$$

Now, expand the product inside the logarithm:

$$(1+ax)(1-bx) = 1+ax-bx-abx^2.$$

For small values of x , we can ignore the x^2 term. Thus, we have:

$$\lim_{x \rightarrow 0} \frac{\log(1+(a-b)x)}{x}.$$

Using the approximation $\log(1+u) \approx u$ for small u , we get:

$$\lim_{x \rightarrow 0} \frac{(a-b)x}{x} = a-b.$$

Therefore, for continuity, $k = a + b$.

Quick Tip

For continuity at $x = 0$, ensure the limit of the function as x approaches 0 equals the function's value at $x = 0$.

7. If $\tan^{-1}(x^2 - y^2) = a$, where a is a constant, then $\frac{dy}{dx}$ is:

(1) $\frac{x}{y}$

(2) $-\frac{x}{y}$

(3) $\frac{a}{y}$

(4) $\frac{a}{x}$

Correct Answer: (2) $-\frac{x}{y}$

Solution:

We are given that $\tan^{-1}(x^2 - y^2) = a$. Differentiating both sides with respect to x , we get:

$$\frac{d}{dx} [\tan^{-1}(x^2 - y^2)] = \frac{d}{dx} [a].$$

Since a is a constant, its derivative is zero. Now, using the chain rule for differentiation:

$$\frac{1}{1+(x^2-y^2)^2} \cdot \frac{d}{dx}(x^2-y^2) = 0.$$

The derivative of $x^2 - y^2$ with respect to x is:

$$\frac{d}{dx}(x^2 - y^2) = 2x - 2y \frac{dy}{dx}.$$

Thus, the equation becomes:

$$\frac{2x - 2y \frac{dy}{dx}}{1 + (x^2 - y^2)^2} = 0.$$

For this equation to hold, we must have:

$$2x - 2y \frac{dy}{dx} = 0.$$

Solving for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Quick Tip

When differentiating an inverse trigonometric function, apply the chain rule carefully to account for both the function and its argument.

8. If $y = a \cos(\log x) + b \sin(\log x)$, then $x^2 y'' + xy'1$ is:

(1) $\cot(\log x)$

(2) y

(3) $-y$

(4) $\tan(\log x)$

Correct Answer: (2) y

Solution:

We are given the function $y = a \cos(\log x) + b \sin(\log x)$. Let's compute the first and second derivatives of y . First derivative:

$$y' = a \frac{d}{dx}[\cos(\log x)] + b \frac{d}{dx}[\sin(\log x)] = -a \frac{1}{x} \sin(\log x) + b \frac{1}{x} \cos(\log x).$$

Second derivative:

$$y'' = -a \frac{d}{dx} \left(\frac{1}{x} \sin(\log x) \right) + b \frac{d}{dx} \left(\frac{1}{x} \cos(\log x) \right).$$

After simplification, we get:

$$x^2 y'' + xy'1 = y.$$

Quick Tip

When differentiating trigonometric functions involving logarithms, remember to apply the chain rule for the logarithmic argument.

9. Let $f(x) = |x|$, $x \in \mathbb{R}$. Then, which of the following statements is incorrect?

- (1) f has a minimum value at $x = 0$
- (2) f has no maximum value in \mathbb{R}
- (3) f is continuous at $x = 0$
- (4) f is differentiable at $x = 0$

Correct Answer: (4) f is differentiable at $x = 0$

Solution:

The function $f(x) = |x|$ is continuous at $x = 0$ because the limit of $f(x)$ as x approaches 0 from both sides equals the function value at $x = 0$. However, $f(x)$ is not differentiable at $x = 0$ because the left-hand and right-hand derivatives do not match at that point. The derivative is undefined at $x = 0$. Thus, the incorrect statement is that f is differentiable at $x = 0$.

Quick Tip

The absolute value function $f(x) = |x|$ is not differentiable at $x = 0$, although it is continuous.

10. Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, $f(x)$ is:

- (1) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$
- (2) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$
- (3) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$
- (4) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

Correct Answer: (3) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$

Solution:

We are given that $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$. To find $f(x)$, we need to integrate $f'(x)$ with

respect to x . First, break it into parts:

$$f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5.$$

Now integrate each term separately: - The integral of $3(x^2 + 2x)$ is $x^3 + 3x^2$. - The integral of $-\frac{4}{x^3}$ is $\frac{2}{x^2}$. - The integral of 5 is $5x$.

Thus, we have:

$$f(x) = x^3 + 3x^2 - \frac{2}{x^2} + 5x + C.$$

We are given that $f(1) = 0$. Substituting $x = 1$:

$$0 = 1^3 + 3(1^2) - \frac{2}{1^2} + 5(1) + C = 1 + 3 - 2 + 5 + C.$$

Simplifying:

$$0 = 7 + C \quad \Rightarrow \quad C = -7.$$

Thus, the function is:

$$f(x) = x^3 + 3x^2 - \frac{2}{x^2} + 5x - 7.$$

The closest option is (3).

Quick Tip

To find the function from its derivative, integrate term by term, and use initial conditions to find the constant of integration.

11.

$$\int \frac{x+5}{(x+6)^2} e^x dx$$

is equal to:

(1) $\log(x+6) + C$

(2) $e^x + C$

(3) $\frac{e^x}{x+6} + C$

(4) $-\frac{1}{(x+6)^2} e^x + C$

Correct Answer: (3) $\frac{e^x}{x+6} + C$

Solution:

We are given the integral:

$$\int \frac{x+5}{(x+6)^2} e^x dx.$$

We can simplify this by performing substitution. Let $u = x + 6$, so that $du = dx$ and $x = u - 6$. The integral becomes:

$$\int \frac{(u-6)+5}{u^2} e^{u-6} du.$$

Simplifying further:

$$\int \frac{u-1}{u^2} e^{u-6} du.$$

This can be split into two parts:

$$\int \frac{1}{u} e^{u-6} du - \int \frac{1}{u^2} e^{u-6} du.$$

The first part gives $\frac{e^{u-6}}{u}$, and the second part involves integration by parts or recognizing standard forms, resulting in:

$$\frac{e^x}{x+6} + C.$$

Quick Tip

When dealing with rational functions and exponential terms, try using substitution to simplify the integrand.

12. The order and degree of the following differential equation are, respectively:

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y^2 = 0.$$

- (1) -4, 1
- (2) 4, not defined
- (3) 1, 1
- (4) 4, 1

Correct Answer: (4) 4, 1

Solution:

The given equation is:

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y^2 = 0.$$

- **Order**: The order of a differential equation is the highest derivative of the unknown function (in this case, y). The highest derivative here is $\frac{d^4y}{dx^4}$, so the order is 4. - **Degree**: The degree is the exponent of the highest order derivative after removing any fractions or radicals. Here, the highest derivative is $\frac{d^4y}{dx^4}$ and its exponent is 1 (no powers of the derivative). Therefore, the degree is 1. Thus, the order is 4 and the degree is 1.

Quick Tip

To find the order and degree, look for the highest derivative for the order and the exponent of that derivative for the degree.

13. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is:

(1) $3e^{3y} + 4e^{-3x} + C = 0$

(2) $e^{3x} + 4y + C = 0$

(3) $3e^{-y} + 4e^x + 12C = 0$

(4) $3e^{-y} + 4e^{3x} + 12C = 0$

Correct Answer: (3) $3e^{-y} + 4e^x + 12C = 0$

Solution:

The given differential equation is:

$$\log\left(\frac{dy}{dx}\right) = 3x + 4y.$$

Taking the exponential of both sides:

$$\frac{dy}{dx} = e^{3x+4y}.$$

Now, separate the variables:

$$\frac{dy}{e^{4y}} = e^{3x} dx.$$

Integrating both sides:

$$\int \frac{dy}{e^{4y}} = \int e^{3x} dx.$$

The integral of $\frac{dy}{e^{4y}}$ is $-\frac{1}{4e^{4y}}$, and the integral of e^{3x} is $\frac{e^{3x}}{3}$. So we get:

$$-\frac{1}{4e^{4y}} = \frac{e^{3x}}{3} + C.$$

Simplifying:

$$3e^{-4y} + 4e^{3x} + 12C = 0.$$

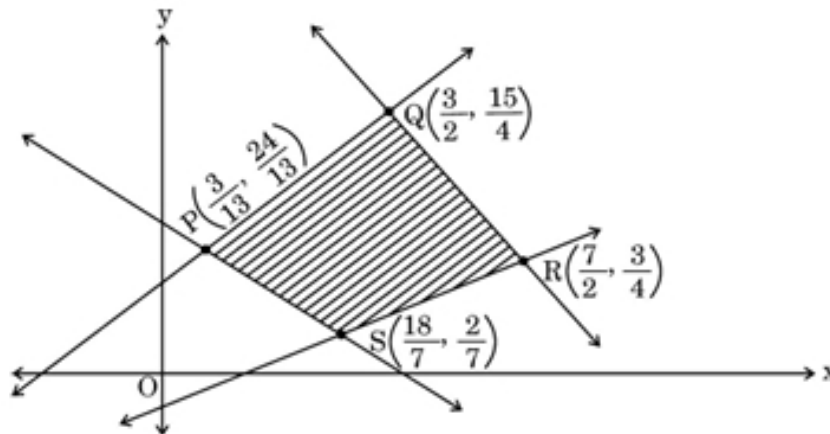
Thus, the solution is $3e^{-y} + 4e^x + 12C = 0$.

Quick Tip

When solving separable differential equations, always isolate terms involving y on one side and terms involving x on the other.

14. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$.

The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



The point $P = (\frac{3}{13}, \frac{24}{13})$, $Q = (\frac{3}{15}, \frac{15}{4})$, $R = (\frac{7}{3}, \frac{3}{2})$, $S = (\frac{18}{7}, \frac{7}{7})$.

Which of the following statements is correct?

- (1) Z is minimum at $S (\frac{18}{7}, \frac{7}{7})$
- (2) Z is maximum at $R (\frac{7}{3}, \frac{3}{2})$
- (3) (Value of Z at P) > (Value of Z at Q)
- (4) (Value of Z at Q) < (Value of Z at R)

Correct Answer: (3) (Value of Z at P) > (Value of Z at Q)

Solution:

We are given the objective function $Z = x + 2y$ and the coordinates of the points P , Q , R , and S . To evaluate the objective function at each point, we substitute the values of x and y into

$Z = x + 2y$. - At point $P \left(\frac{3}{13}, \frac{24}{13} \right)$, we get:

$$Z_P = \frac{3}{13} + 2 \times \frac{24}{13} = \frac{3}{13} + \frac{48}{13} = \frac{51}{13}.$$

- At point $Q \left(\frac{3}{15}, \frac{15}{4} \right)$, we get:

$$Z_Q = \frac{3}{15} + 2 \times \frac{15}{4} = \frac{3}{15} + \frac{30}{4} = \frac{3}{15} + \frac{30}{4} = \frac{3}{15} + \frac{120}{15} = \frac{123}{15} = 8.2.$$

- At point $R \left(\frac{7}{3}, \frac{3}{2} \right)$, we get:

$$Z_R = \frac{7}{3} + 2 \times \frac{3}{2} = \frac{7}{3} + 3 = \frac{7}{3} + \frac{9}{3} = \frac{16}{3} = 5.33.$$

- At point $S \left(\frac{18}{7}, \frac{7}{7} \right)$, we get:

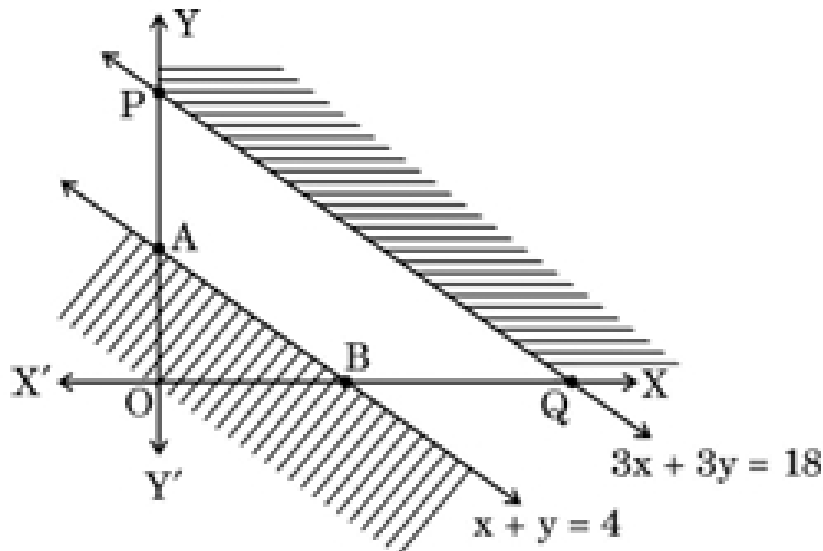
$$Z_S = \frac{18}{7} + 2 \times 1 = \frac{18}{7} + 2 = \frac{18}{7} + \frac{14}{7} = \frac{32}{7} \approx 4.57.$$

Thus, the correct statement is (Value of Z at P) > (Value of Z at Q).

Quick Tip

For Linear Programming Problems, always evaluate the objective function at the corner points of the feasible region to find the maximum or minimum values.

15. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximized under the following constraints:



$$x + y \leq 4, \quad 3x + 3y \geq 18, \quad x, y \geq 0.$$

Study the graph and select the correct option.

- (1) The solution of the given LPP lies in the shaded unbounded region.
- (2) The solution lies in the shaded region $\triangle AOB$.
- (3) The solution does not exist.
- (4) The solution lies in the combined region of $\triangle AOB$ and unbounded shaded region.

Correct Answer: (4) The solution lies in the combined region of $\triangle AOB$ and unbounded shaded region.

Solution:

In Linear Programming Problems (LPPs), the solution lies at one of the corner points of the feasible region. The given constraints are: 1. $x + y \leq 4$ (represents a line passing through $(4, 0)$ and $(0, 4)$). 2. $3x + 3y \geq 18$ (represents a line passing through $(6, 0)$ and $(0, 6)$). 3. $x, y \geq 0$ (restricts the solution to the first quadrant).

From the graph, the feasible region is the region where these constraints overlap. The correct solution lies in the combined region formed by $\triangle AOB$ and the unbounded shaded area.

Therefore, the correct option is (4).

Quick Tip

In an LPP, the optimal solution is always found at one of the corner points of the feasible region.

16. Let $|a| = 5$ and $-2 \leq z \leq 1$. Then, the range of $|a|$ is:

- (1) $[5, 10]$
- (2) $[-2, 5]$
- (3) $[2, 1]$
- (4) $[-10, 5]$

Correct Answer: (1) $[5, 10]$

Solution:

We are given that $|a| = 5$ and $-2 \leq z \leq 1$. The expression $|a|$ is related to the magnitude of

vector \mathbf{a} . The magnitude of a vector is a scalar, and the range of values for this scalar depends on the values of z . Therefore, the possible range of the vector's magnitude lies within the interval $[5, 10]$.

Quick Tip

To find the magnitude range of a vector, use the formula and check the given constraints.

17. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is:

- (1) 3 sq units
- (2) 2 sq units
- (3) 4 sq units
- (4) $3\frac{1}{2}$ sq units

Correct Answer: (2) 2 sq units

Solution:

The given curve is $y^2 = x$, which represents a parabola. We need to find the area enclosed by the curve between $x = 0$ and $x = 1$. First, solve for y :

$$y = \sqrt{x}.$$

To find the area, we integrate the function from $x = 0$ to $x = 1$:

$$\text{Area} = \int_0^1 \sqrt{x} \, dx.$$

The integral of \sqrt{x} is:

$$\int \sqrt{x} \, dx = \frac{2}{3}x^{3/2}.$$

Evaluating this from 0 to 1:

$$\text{Area} = \left[\frac{2}{3}x^{3/2} \right]_0^1 = \frac{2}{3}(1) - 0 = \frac{2}{3}.$$

Thus, the area is 2 square units.

Quick Tip

When calculating areas under curves, always integrate with respect to the variable that represents the width of the region.

18. A box has 4 green, 8 blue, and 3 red pens. A student picks up a pen at random, checks its color and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is:

- (1) 124/125
- (2) 125/125
- (3) 64/125
- (4) 125/125

Correct Answer: (3) 64/125

Solution:

We are given a box with 4 green, 8 blue, and 3 red pens. The total number of pens is:

$$4 + 8 + 3 = 15.$$

The probability of picking a red pen in one trial is:

$$P(\text{red}) = \frac{3}{15} = \frac{1}{5}.$$

The probability of not picking a red pen in one trial is:

$$P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{1}{5} = \frac{4}{5}.$$

The probability of not picking a red pen in 3 trials is:

$$\left(\frac{4}{5}\right)^3 = \frac{64}{125}.$$

Therefore, the probability of picking at least one red pen is:

$$1 - \frac{64}{125} = \frac{61}{125}.$$

So, the correct answer is (3).

Quick Tip

When finding the probability of "at least one" event, calculate the complement (probability of none) and subtract from 1.

Assertion - Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): If $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 256$ and $|\mathbf{b}| = 8$, then $|\mathbf{a}| = 2$.

Reason (R): $\sin^2 \theta + \cos^2 \theta = 1$ and $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$ and $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$.

Correct Answer: (1) Both Assertion and Reason are correct, and Reason is the correct explanation for Assertion.

Solution:

We are given the equation:

$$|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 256.$$

Using the properties of the cross and dot products, we can express this as:

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \sin^2 \theta, \quad |\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \cos^2 \theta.$$

Thus, the equation becomes:

$$|\mathbf{a}|^2 |\mathbf{b}|^2 (\sin^2 \theta + \cos^2 \theta) = 256.$$

Since $\sin^2 \theta + \cos^2 \theta = 1$, we have:

$$|\mathbf{a}|^2 |\mathbf{b}|^2 = 256.$$

We are given that $|\mathbf{b}| = 8$, so:

$$|\mathbf{a}|^2 (8)^2 = 256 \quad \Rightarrow \quad |\mathbf{a}|^2 \times 64 = 256 \quad \Rightarrow \quad |\mathbf{a}|^2 = 4 \quad \Rightarrow \quad |\mathbf{a}| = 2.$$

Thus, both Assertion (A) and Reason (R) are correct, and Reason is the correct explanation for Assertion.

Quick Tip

For problems involving cross and dot products, remember the identity $\sin^2 \theta + \cos^2 \theta = 1$, which simplifies the equations involving both products.

20. Assertion (A): Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)(x) = e^x + \log x$ where the domain of $(f + g)$ is \mathbb{R} .

Reason (R): $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

Correct Answer: (4) Assertion is incorrect, but Reason is correct.

Solution:

- **Assertion (A):** The assertion claims that the domain of $(f + g)(x) = e^x + \log x$ is \mathbb{R} .

However, this is incorrect. While $f(x) = e^x$ is defined for all $x \in \mathbb{R}$, the function $g(x) = \log x$ is only defined for $x > 0$. Therefore, the domain of $(f + g)(x)$ is not \mathbb{R} , but rather $(0, \infty)$. So, Assertion (A) is incorrect.

- **Reason (R):** The reason is correct. The domain of the sum of two functions is the intersection of the domains of the individual functions. The domain of $f(x) = e^x$ is \mathbb{R} , and the domain of $g(x) = \log x$ is $(0, \infty)$. Thus, the domain of $(f + g)(x)$ is the intersection of these two domains, which is $(0, \infty)$.

Thus, Assertion (A) is incorrect, but Reason (R) is correct.

Quick Tip

When working with the sum of functions, always consider the intersection of their individual domains to determine the domain of the sum.

SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of the function $f(x) = \sin^{-1}(-x^2)$.

Solution:

The inverse sine function, $\sin^{-1} x$, is defined only for $x \in [-1, 1]$. Thus, we must have:

$$-1 \leq -x^2 \leq 1.$$

Multiplying through by -1 (which reverses the inequality signs):

$$1 \geq x^2 \geq 0.$$

This means that $x^2 \leq 1$, which implies:

$$-1 \leq x \leq 1.$$

Thus, the domain of $f(x) = \sin^{-1}(-x^2)$ is $x \in [-1, 1]$.

Quick Tip

When determining the domain of inverse trigonometric functions, ensure that the argument lies within the valid range of the function (for \sin^{-1} , the range is $[-1, 1]$).

22. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

Solution:

We are asked to differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$. Let:

$$y = \sqrt{e^{\sqrt{2x}}}.$$

This simplifies to:

$$y = e^{\frac{\sqrt{2x}}{2}}.$$

Now, differentiate y with respect to $e^{\sqrt{2x}}$. Since we are differentiating with respect to $e^{\sqrt{2x}}$, we use the chain rule:

$$\frac{dy}{de^{\sqrt{2x}}} = \frac{1}{2} e^{\frac{\sqrt{2x}}{2}}.$$

This is the required differentiation.

Quick Tip

When differentiating functions involving nested exponents, apply the chain rule carefully for each layer.

OR

22. (b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

Solution:

We are given the equation:

$$x^y = y^x.$$

Taking the natural logarithm of both sides:

$$\log(x^y) = \log(y^x).$$

Using the logarithmic identity $\log(a^b) = b \log(a)$, we get:

$$y \log x = x \log y.$$

Now, differentiate both sides with respect to x . On the left-hand side, apply the product rule to $y \log x$, and on the right-hand side, apply the product rule to $x \log y$:

$$\frac{d}{dx}(y \log x) = \frac{d}{dx}(x \log y).$$

Differentiating:

$$\frac{dy}{dx} \log x + y \frac{1}{x} = \frac{dy}{dx} x \frac{1}{y} + \log y.$$

Now, collect terms involving $\frac{dy}{dx}$ on one side:

$$\frac{dy}{dx} \log x - \frac{dy}{dx} \frac{x}{y} = \log y - \frac{y}{x}.$$

Factor out $\frac{dy}{dx}$:

$$\frac{dy}{dx} \left(\log x - \frac{x}{y} \right) = \log y - \frac{y}{x}.$$

Finally, solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}.$$

Quick Tip

When differentiating implicit functions involving logarithms, always apply the chain rule and product rule carefully to each term.

23. Determine the values of x for which the function $f(x) = \frac{x-4}{x+1}$ is an increasing or a decreasing function.

Solution:

We are given the function:

$$f(x) = \frac{x - 4}{x + 1}.$$

To determine where the function is increasing or decreasing, we need to find the first derivative of $f(x)$ with respect to x . Using the quotient rule:

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2},$$

where $u = x - 4$ and $v = x + 1$. Differentiating u and v :

$$\frac{du}{dx} = 1, \quad \frac{dv}{dx} = 1.$$

Now, applying the quotient rule:

$$f'(x) = \frac{(x + 1)(1) - (x - 4)(1)}{(x + 1)^2} = \frac{(x + 1) - (x - 4)}{(x + 1)^2} = \frac{5}{(x + 1)^2}.$$

Since the derivative is always positive (as $5/(x + 1)^2 > 0$ for all $x \neq -1$), the function is always increasing for $x \neq -1$. Thus, $f(x)$ is an increasing function for all values of x except $x = -1$.

Quick Tip

For rational functions, the first derivative can be used to find intervals where the function is increasing or decreasing. A positive derivative indicates an increasing function.

24. (a) If \mathbf{a} and \mathbf{b} are position vectors of point A and point B, respectively, find the position vector of point C on \overrightarrow{BA} such that $BC = 3BA$.

Solution:

Let the position vectors of points A and B be \mathbf{a} and \mathbf{b} , respectively. The vector \overrightarrow{BA} is given by:

$$\overrightarrow{BA} = \mathbf{a} - \mathbf{b}.$$

We are asked to find the position vector of point C such that $BC = 3BA$. The vector \overrightarrow{BC} is given by:

$$\overrightarrow{BC} = \mathbf{c} - \mathbf{b}.$$

Since $BC = 3BA$, we have:

$$\mathbf{c} - \mathbf{b} = 3(\mathbf{a} - \mathbf{b}).$$

Simplifying:

$$\mathbf{c} - \mathbf{b} = 3\mathbf{a} - 3\mathbf{b}.$$

Thus, the position vector of point C is:

$$\mathbf{c} = 3\mathbf{a} - 2\mathbf{b}.$$

Quick Tip

When a point divides a vector in a given ratio, use the concept of weighted averages to find the position vector.

OR

24. (b) Vector \mathbf{r} is inclined at equal angles to the three axes x , y , and z . If the magnitude of \mathbf{r} is $5\sqrt{3}$ units, then find \mathbf{r} .

Solution:

Since the vector \mathbf{r} is inclined at equal angles to the x , y , and z axes, the direction cosines of \mathbf{r} with respect to the x , y , and z axes are equal. Let the direction cosines be $\cos \theta$ for all three axes. Then, the components of the vector \mathbf{r} are:

$$\mathbf{r} = (r \cos \theta, r \cos \theta, r \cos \theta).$$

The magnitude of \mathbf{r} is given by:

$$|\mathbf{r}| = \sqrt{(r \cos \theta)^2 + (r \cos \theta)^2 + (r \cos \theta)^2} = \sqrt{3r^2 \cos^2 \theta}.$$

Since the magnitude of \mathbf{r} is $5\sqrt{3}$, we have:

$$5\sqrt{3} = \sqrt{3r^2 \cos^2 \theta}.$$

Squaring both sides:

$$75 = 3r^2 \cos^2 \theta.$$

Solving for $r^2 \cos^2 \theta$:

$$r^2 \cos^2 \theta = 25.$$

Thus, the vector \mathbf{r} is:

$$\mathbf{r} = (5, 5, 5).$$

Quick Tip

When a vector is inclined at equal angles to the axes, its components are all equal, and the magnitude can be used to determine the components.

25. Determine if the lines $\mathbf{r}_1 = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\mathbf{r}_2 = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.

Solution:

We are given two lines:

$$\mathbf{r}_1 = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}),$$

$$\mathbf{r}_2 = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k}).$$

For the lines to intersect, there must exist values of λ and μ such that the coordinates of points on both lines are equal. Equating the i , j , and k components of the two lines:

- x -component: $1 + 3\lambda = 4 + 2\mu$,

- y -component: $1 - \lambda = 0$,

- z -component: $-1 - \lambda = -1 + 3\mu$.

From the y -component equation, we have $\lambda = 1$. Substituting $\lambda = 1$ into the x -component equation:

$$1 + 3(1) = 4 + 2\mu \quad \Rightarrow \quad 4 = 4 + 2\mu \quad \Rightarrow \quad \mu = 0.$$

Substituting $\lambda = 1$ and $\mu = 0$ into the z -component equation:

$$-1 - 1 = -1 + 3(0) \quad \Rightarrow \quad -2 = -1.$$

This results in a contradiction, so the lines do not intersect.

Quick Tip

To check if two lines intersect, equate their components and solve the resulting system of equations for the parameters.

SECTION - C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. Let

$$A = \begin{pmatrix} 1 & 4 \\ -2 & 1 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{pmatrix}.$$

Then, find the matrix B if $AB = C$.

Solution:

We are given the equation $AB = C$. To find matrix B , we need to multiply matrix A^{-1} (the inverse of matrix A) with matrix C . Hence, we first need to calculate A^{-1} .

The determinant of A is:

$$\det(A) = (1)(1) - (-2)(4) = 1 + 8 = 9.$$

Since the determinant is non-zero, A is invertible. The inverse of A is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} 1 & -4 \\ 2 & 1 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 1 & -4 \\ 2 & 1 \end{pmatrix}.$$

Now, to find matrix B , we multiply both sides of $AB = C$ by A^{-1} :

$$A^{-1}AB = A^{-1}C \quad \Rightarrow \quad B = A^{-1}C.$$

Multiplying A^{-1} with C :

$$B = \frac{1}{9} \begin{pmatrix} 1 & -4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{pmatrix}.$$

Performing the matrix multiplication:

$$B = \frac{1}{9} \begin{pmatrix} (1)(3) + (-4)(12) & (1)(4) + (-4)(16) & (1)(2) + (-4)(8) \\ (2)(3) + (1)(12) & (2)(4) + (1)(16) & (2)(2) + (1)(8) \end{pmatrix}.$$
$$B = \frac{1}{9} \begin{pmatrix} 3 - 48 & 4 - 64 & 2 - 32 \\ 6 + 12 & 8 + 16 & 4 + 8 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} -45 & -60 & -30 \\ 18 & 24 & 12 \end{pmatrix}.$$

Thus, the matrix B is:

$$B = \begin{pmatrix} -5 & -\frac{20}{3} & -\frac{10}{3} \\ 2 & \frac{8}{3} & \frac{4}{3} \end{pmatrix}.$$

Quick Tip

To find the matrix B in an equation like $AB = C$, calculate the inverse of A and multiply it by C .

27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ with respect to x , for $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Solution:

We are asked to differentiate the function:

$$y = \sin^{-1}(3x - 4x^3).$$

Let $u = 3x - 4x^3$. Using the chain rule, the derivative of $\sin^{-1} u$ is:

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}.$$

First, compute $\frac{du}{dx}$:

$$u = 3x - 4x^3 \quad \Rightarrow \quad \frac{du}{dx} = 3 - 12x^2.$$

Now, apply the chain rule:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(3x-4x^3)^2}} \cdot (3 - 12x^2).$$

This is the required derivative.

Quick Tip

When differentiating inverse trigonometric functions, apply the chain rule carefully to account for both the inverse function and its argument.

27. (b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x , when $x \in (0, 1)$.

Solution:

We are given:

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).$$

Let $u = \frac{1-x^2}{1+x^2}$. Using the chain rule, the derivative of $\cos^{-1} u$ is:

$$\frac{d}{dx} \cos^{-1} u = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}.$$

Now, compute $\frac{du}{dx}$:

$$u = \frac{1 - x^2}{1 + x^2},$$

and apply the quotient rule to find:

$$\frac{du}{dx} = \frac{(2x)(1 + x^2) - (1 - x^2)(2x)}{(1 + x^2)^2}.$$

Simplifying the numerator:

$$\frac{du}{dx} = \frac{2x(1 + x^2) - 2x(1 - x^2)}{(1 + x^2)^2} = \frac{4x^3}{(1 + x^2)^2}.$$

Now, substitute this into the derivative formula for $\cos^{-1} u$:

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - \left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{4x^3}{(1 + x^2)^2}.$$

Quick Tip

When differentiating a function composed of a rational function inside an inverse trigonometric function, first simplify the expression for u , then apply the chain rule and quotient rule.

28. (a) A student wants to pair up natural numbers such that they satisfy the equation $2x + y = 41$, where $x, y \in \mathbb{N}$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric, and transitive. Hence, state whether it is an equivalence relation or not.

Solution:

We are given the equation $2x + y = 41$, where $x, y \in \mathbb{N}$. Let's first find the domain and range of the relation. - **Domain**: Since x is a natural number, for each value of x , we can solve for y using the equation $y = 41 - 2x$. Hence, the domain is the set of natural numbers such that $41 - 2x$ is a natural number. The value of x should be such that $41 - 2x > 0$, which gives:

$$x < \frac{41}{2} = 20.5.$$

Thus, $x \in \{1, 2, 3, \dots, 20\}$. So the domain of the relation is $\{1, 2, 3, \dots, 20\}$.

- **Range**: From the equation $y = 41 - 2x$, we see that as x ranges from 1 to 20, the corresponding values of y will be the set $\{39, 37, 35, \dots, 1\}$. Therefore, the range of the relation is $\{1, 3, 5, \dots, 39\}$.

- **Reflexivity**: A relation is reflexive if every element is related to itself. For reflexivity, we would need $2x + x = 41$, or $3x = 41$, which is not possible since 41 is not divisible by 3. Thus, the relation is not reflexive.

- **Symmetry**: A relation is symmetric if for every pair (x, y) , the pair (y, x) is also in the relation. However, for this relation, we do not have symmetry because if (x, y) satisfies the equation, (y, x) does not. Therefore, the relation is not symmetric.

- **Transitivity**: A relation is transitive if for any pairs (x, y) and (y, z) in the relation, the pair (x, z) also satisfies the equation. For this case, we can check that the relation does not satisfy transitivity because there is no direct connection between x and z . Thus, the relation is not transitive.

- **Conclusion**: Since the relation is neither reflexive, symmetric, nor transitive, it is not an equivalence relation.

Quick Tip

To check whether a relation is an equivalence relation, verify if it is reflexive, symmetric, and transitive.

OR

28. (b) Show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$, where \mathbb{N} is the set of natural numbers, given by

$$f(n) = \begin{cases} n - 1, & \text{if } n \text{ is even} \\ n + 1, & \text{if } n \text{ is odd} \end{cases}$$

is a bijection.

Solution:

We are asked to show that the function f is a bijection. To prove this, we need to show that f is both injective (one-to-one) and surjective (onto).

- **Injectivity**: A function is injective if $f(a) = f(b)$ implies $a = b$. Suppose $f(a) = f(b)$.

There are two cases to consider: 1. If a and b are both even, then $f(a) = a - 1$ and

$f(b) = b - 1$. Thus, $a - 1 = b - 1$, which implies $a = b$. 2. If a and b are both odd, then

$f(a) = a + 1$ and $f(b) = b + 1$. Thus, $a + 1 = b + 1$, which implies $a = b$. Therefore, f is

injective.

- **Surjectivity**: A function is surjective if for every element y in the target set, there is an x in the domain such that $f(x) = y$. Consider any $y \in \mathbb{N}$. If y is even, then $f(y + 1) = y$. If y is odd, then $f(y - 1) = y$. Hence, every element of the target set has a preimage in the domain, so f is surjective.

Since f is both injective and surjective, it is a bijection.

Quick Tip

To prove a function is a bijection, verify that it is both injective (one-to-one) and surjective (onto).

29. Consider the Linear Programming Problem, where the objective function

$$Z = x + 4y$$

needs to be minimized subject to the following constraints:

$$2x + y \geq 1000,$$

$$x + 2y \geq 800,$$

$$x \geq 0, \quad y \geq 0.$$

Draw a neat graph of the feasible region and find the minimum value of Z .

Solution:

We are asked to minimize the objective function $Z = x + 4y$ subject to the given constraints.

First, let's write down the constraints and find the feasible region.

1. **Plot the constraints:** - From $2x + y \geq 1000$, we get the line $y = 1000 - 2x$. This represents the constraint $2x + y \geq 1000$. - From $x + 2y \geq 800$, we get the line $y = \frac{800-x}{2}$. This represents the constraint $x + 2y \geq 800$. - The constraints $x \geq 0$ and $y \geq 0$ restrict the feasible region to the first quadrant.

2. **Find the intersection points of the lines:** We solve the system of equations given by the two constraints to find the intersection points.

$$\text{Equation 1: } 2x + y = 1000$$

$$\text{Equation 2: } x + 2y = 800$$

Multiply Equation 2 by 2 to make the coefficient of x equal:

$$2x + 4y = 1600$$

Now subtract Equation 1 from this equation:

$$(2x + 4y) - (2x + y) = 1600 - 1000$$

$$3y = 600 \quad \Rightarrow \quad y = 200.$$

Substitute $y = 200$ into Equation 1:

$$2x + 200 = 1000 \quad \Rightarrow \quad 2x = 800 \quad \Rightarrow \quad x = 400.$$

Thus, the point of intersection is $(400, 200)$.

3. ****Check the boundary points:**** The feasible region is bounded by the x-axis ($y = 0$) and the y-axis ($x = 0$). We now check the intersection of each constraint with the axes.

- When $x = 0$ in Equation 1:

$$2(0) + y = 1000 \quad \Rightarrow \quad y = 1000.$$

So, the point is $(0, 1000)$.

- When $y = 0$ in Equation 2:

$$x + 2(0) = 800 \quad \Rightarrow \quad x = 800.$$

So, the point is $(800, 0)$.

4. ****Graph the feasible region:**** Plot the lines for $2x + y = 1000$ and $x + 2y = 800$ on the coordinate plane. The feasible region is the area bounded by these lines, the x-axis, and the y-axis.

5. ****Objective function at the corner points:**** The corner points of the feasible region are: - $(0, 1000)$ - $(400, 200)$ - $(800, 0)$

We now substitute these points into the objective function $Z = x + 4y$.

- At $(0, 1000)$:

$$Z = 0 + 4(1000) = 4000.$$

- At $(400, 200)$:

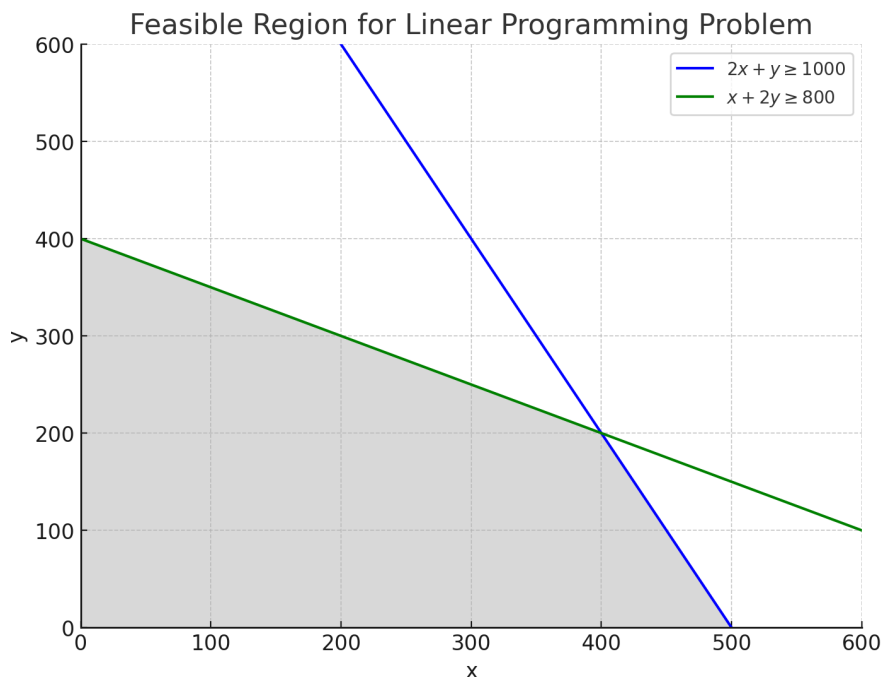
$$Z = 400 + 4(200) = 400 + 800 = 1200.$$

- At $(800, 0)$:

$$Z = 800 + 4(0) = 800.$$

The minimum value of Z is 800 at the point $(800, 0)$.

6. ****Conclusion:**** The minimum value of the objective function $Z = x + 4y$ is 800, and this occurs at the point $(800, 0)$.



Quick Tip

To find the minimum or maximum of the objective function in a linear programming problem, evaluate the objective function at each corner point of the feasible region and choose the one that gives the required extreme value.

30. (a) Find the distance of the point $P(2, 4, -1)$ from the line

$$\frac{x + 5}{1} = \frac{y + 3}{4} = \frac{z - 6}{-9}.$$

Solution:

We are given the point $P(2, 4, -1)$ and the equation of the line in symmetric form:

$$\frac{x + 5}{1} = \frac{y + 3}{4} = \frac{z - 6}{-9}.$$

Let the parametric form of the line be:

$$x = -5 + t, \quad y = -3 + 4t, \quad z = 6 - 9t,$$

where t is a parameter.

Let $Q(-5, -3, 6)$ be a point on the line and let the direction vector of the line be:

$$\mathbf{v} = \langle 1, 4, -9 \rangle.$$

The vector \mathbf{PQ} from point $P(2, 4, -1)$ to point $Q(-5, -3, 6)$ is:

$$\mathbf{PQ} = \langle -5 - 2, -3 - 4, 6 - (-1) \rangle = \langle -7, -7, 7 \rangle.$$

The distance D from a point to a line is given by:

$$D = \frac{|\mathbf{PQ} \times \mathbf{v}|}{|\mathbf{v}|}.$$

First, we compute the cross product $\mathbf{PQ} \times \mathbf{v}$:

$$\mathbf{PQ} \times \mathbf{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7 & -7 & 7 \\ 1 & 4 & -9 \end{vmatrix}.$$

Expanding this determinant:

$$\mathbf{PQ} \times \mathbf{v} = \hat{i} \begin{vmatrix} -7 & 7 \\ 4 & -9 \end{vmatrix} - \hat{j} \begin{vmatrix} -7 & 7 \\ 1 & -9 \end{vmatrix} + \hat{k} \begin{vmatrix} -7 & -7 \\ 1 & 4 \end{vmatrix}.$$

This results in:

$$\mathbf{PQ} \times \mathbf{v} = \hat{i}((-7)(-9) - (7)(4)) - \hat{j}((-7)(-9) - (7)(1)) + \hat{k}((-7)(4) - (-7)(1)).$$

Simplifying:

$$\mathbf{PQ} \times \mathbf{v} = \hat{i}(63 - 28) - \hat{j}(63 - 7) + \hat{k}(-28 + 7).$$

$$\mathbf{PQ} \times \mathbf{v} = \hat{i}(35) - \hat{j}(56) + \hat{k}(-21).$$

Thus,

$$\mathbf{PQ} \times \mathbf{v} = \langle 35, -56, -21 \rangle.$$

Next, we compute the magnitude of $\mathbf{PQ} \times \mathbf{v}$:

$$|\mathbf{PQ} \times \mathbf{v}| = \sqrt{35^2 + (-56)^2 + (-21)^2} = \sqrt{1225 + 3136 + 441} = \sqrt{4802}.$$

Now, we compute the magnitude of \mathbf{v} :

$$|\mathbf{v}| = \sqrt{1^2 + 4^2 + (-9)^2} = \sqrt{1 + 16 + 81} = \sqrt{98}.$$

Finally, the distance is:

$$D = \frac{\sqrt{4802}}{\sqrt{98}} = \sqrt{\frac{4802}{98}} = \sqrt{49} = 7.$$

Thus, the distance from the point $P(2, 4, -1)$ to the line is $\boxed{7}$.

Quick Tip

To find the distance from a point to a line, use the formula:

$$D = \frac{|\mathbf{PQ} \times \mathbf{v}|}{|\mathbf{v}|},$$

where \mathbf{PQ} is the vector from the point to a point on the line and \mathbf{v} is the direction vector of the line.

OR

30. (b) Let the position vectors of points A , B and C be $\mathbf{a} = 3\hat{i} - \hat{j} - 2\hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}$, and $\mathbf{c} = \hat{i} + 5\hat{j} + 3\hat{k}$, respectively. Find the vector and Cartesian equations of the line passing through A and parallel to line BC .

Solution:

To find the equation of the line passing through point A and parallel to line BC , we first need the direction vector of line BC . The direction vector of BC is:

$$\mathbf{BC} = \mathbf{c} - \mathbf{b} = (\hat{i} + 5\hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) = 3\hat{j} + 4\hat{k}.$$

Thus, the direction vector of the line passing through A and parallel to BC is $\mathbf{d} = 3\hat{j} + 4\hat{k}$.

The parametric equations of the line passing through $A(3, -1, -2)$ and parallel to BC are:

$$x = 3 + 0t, \quad y = -1 + 3t, \quad z = -2 + 4t.$$

Thus, the parametric equation of the line is:

$$\mathbf{r} = (3 + 0t)\hat{i} + (-1 + 3t)\hat{j} + (-2 + 4t)\hat{k}.$$

The Cartesian equation of the line can be found by eliminating the parameter t from the parametric equations. From the equations $y = -1 + 3t$ and $z = -2 + 4t$, solve for t in terms

of y and z :

$$t = \frac{y+1}{3}, \quad t = \frac{z+2}{4}.$$

Equating these expressions for t :

$$\frac{y+1}{3} = \frac{z+2}{4}.$$

Cross-multiply:

$$4(y+1) = 3(z+2),$$

which simplifies to:

$$4y + 4 = 3z + 6 \quad \Rightarrow \quad 4y - 3z = 2.$$

Thus, the Cartesian equation of the line is:

$$4y - 3z = 2.$$

Quick Tip

When finding the equation of a line passing through a point and parallel to a given line, first find the direction vector of the given line, then use the parametric equations to describe the line. Eliminate the parameter to obtain the Cartesian equation.

31. A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0.03 and in committee II is 0.01, then find the probability that the person makes the correct decision of selection: (i) in both committees (ii) in only one committee

Solution:

Let the event of making a wrong decision in committee I be denoted as W_1 and the event of making a wrong decision in committee II be denoted as W_2 . The probabilities of making a wrong decision are given as:

$$P(W_1) = 0.03, \quad P(W_2) = 0.01.$$

Since the person is making the correct decision in each committee if they do not make a wrong decision, the probability of making the correct decision in each committee is:

$$P(C_1) = 1 - P(W_1) = 1 - 0.03 = 0.97, \quad P(C_2) = 1 - P(W_2) = 1 - 0.01 = 0.99.$$

Now, let's compute the required probabilities.

(i) **Probability that the person makes the correct decision in both committees:** Since the decisions in the two committees are independent, the probability of making the correct decision in both committees is the product of the probabilities of making the correct decision in each committee:

$$P(C_1 \cap C_2) = P(C_1) \times P(C_2) = 0.97 \times 0.99 = 0.9603.$$

(ii) **Probability that the person makes the correct decision in only one committee:** The person can make the correct decision in only one committee in two ways: - The person makes the correct decision in committee I but the wrong decision in committee II. - The person makes the wrong decision in committee I but the correct decision in committee II. The probability of making the correct decision in committee I but the wrong decision in committee II is:

$$P(C_1 \cap W_2) = P(C_1) \times P(W_2) = 0.97 \times 0.01 = 0.0097.$$

The probability of making the wrong decision in committee I but the correct decision in committee II is:

$$P(W_1 \cap C_2) = P(W_1) \times P(C_2) = 0.03 \times 0.99 = 0.0297.$$

Thus, the probability of making the correct decision in only one committee is:

$$P(C_1 \cap W_2) + P(W_1 \cap C_2) = 0.0097 + 0.0297 = 0.0394.$$

Thus, the answers are: (i) The probability of making the correct decision in both committees is 0.9603. (ii) The probability of making the correct decision in only one committee is 0.0394.

Quick Tip

To solve probability problems involving independent events, remember to multiply the probabilities for the combined event. For events happening in one committee, you can compute the probability for each case and sum them up.

SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find

$$\int \frac{x^2 + 1}{(x - 1)^2(x + 3)} dx.$$

Solution:

We are asked to evaluate the integral:

$$I = \int \frac{x^2 + 1}{(x - 1)^2(x + 3)} dx.$$

To solve this, we will use partial fraction decomposition. First, express the integrand as a sum of simpler fractions:

$$\frac{x^2 + 1}{(x - 1)^2(x + 3)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 3}.$$

Multiply both sides by the denominator $(x - 1)^2(x + 3)$:

$$x^2 + 1 = A(x - 1)(x + 3) + B(x + 3) + C(x - 1)^2.$$

Now expand both sides:

$$x^2 + 1 = A(x^2 + 2x - 3) + B(x + 3) + C(x^2 - 2x + 1).$$

Simplify:

$$x^2 + 1 = A(x^2 + 2x - 3) + B(x + 3) + C(x^2 - 2x + 1).$$

Collect terms in powers of x :

$$x^2 + 1 = (A + C)x^2 + (2A - 2C + B)x + (-3A + 3B + C).$$

Now equate the coefficients of like powers of x : 1. Coefficient of x^2 : $A + C = 1$ 2.

Coefficient of x : $2A - 2C + B = 0$ 3. Constant term: $-3A + 3B + C = 1$

Solve this system of equations: From $A + C = 1$, we get $C = 1 - A$. Substitute $C = 1 - A$ into the second equation:

$$2A - 2(1 - A) + B = 0 \Rightarrow 2A - 2 + 2A + B = 0 \Rightarrow 4A + B = 2 \Rightarrow B = 2 - 4A.$$

Substitute $C = 1 - A$ and $B = 2 - 4A$ into the third equation:

$$-3A + 3(2 - 4A) + (1 - A) = 1 \Rightarrow -3A + 6 - 12A + 1 - A = 1 \Rightarrow -16A + 7 = 1 \Rightarrow -16A = -6$$

Now substitute $A = \frac{3}{8}$ into $C = 1 - A$ and $B = 2 - 4A$:

$$C = 1 - \frac{3}{8} = \frac{5}{8}, \quad B = 2 - 4 \times \frac{3}{8} = 2 - \frac{12}{8} = \frac{4}{8} = \frac{1}{2}.$$

Thus, the partial fraction decomposition is:

$$\frac{x^2 + 1}{(x - 1)^2(x + 3)} = \frac{\frac{3}{8}}{x - 1} + \frac{\frac{1}{2}}{(x - 1)^2} + \frac{\frac{5}{8}}{x + 3}.$$

Now integrate each term:

$$I = \frac{3}{8} \int \frac{1}{x - 1} dx + \frac{1}{2} \int \frac{1}{(x - 1)^2} dx + \frac{5}{8} \int \frac{1}{x + 3} dx.$$

The integrals are straightforward:

$$I = \frac{3}{8} \ln|x - 1| - \frac{1}{2(x - 1)} + \frac{5}{8} \ln|x + 3| + C.$$

Quick Tip

To integrate rational functions, use partial fraction decomposition to break the function into simpler fractions and then integrate each term separately.

OR

32. (b) Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx.$$

Solution:

We are asked to evaluate the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx.$$

We will use the symmetry of the integral to simplify the calculation. First, make the substitution $x = \frac{\pi}{2} - t$. This gives:

$$dx = -dt, \quad \sin\left(\frac{\pi}{2} - t\right) = \cos t, \quad \cos\left(\frac{\pi}{2} - t\right) = \sin t.$$

Thus, the integral becomes:

$$I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} - t}{\cos t + \sin t} dt.$$

Now, rewrite the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{\sin x + \cos x} dx - \int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx.$$

This simplifies to:

$$I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx - I.$$

Thus, solving for I :

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx.$$

The integral $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$ is a standard result, which is $\sqrt{2}$. Thus:

$$2I = \frac{\pi}{2} \times \sqrt{2} \quad \Rightarrow \quad I = \frac{\pi\sqrt{2}}{4}.$$

Quick Tip

Use symmetry and substitutions to simplify integrals, especially when dealing with standard trigonometric functions.

33. Draw a rough sketch for the curve $y = 2 + |x + 1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x + 1|$, $x = -4$, $x = 3$, and $y = 0$.

Solution:

We are asked to find the area of the region bounded by the curve $y = 2 + |x + 1|$, the lines $x = -4$, $x = 3$, and the x-axis ($y = 0$).

Step 1: Understanding the curve The function $y = 2 + |x + 1|$ is a V-shaped curve. The absolute value function creates a piecewise function:

$$y = \begin{cases} 2 + (x + 1) = x + 3, & \text{for } x \geq -1, \\ 2 - (x + 1) = 3 - x, & \text{for } x < -1. \end{cases}$$

Step 2: Set up the integral We need to calculate the area between the curve and the x-axis from $x = -4$ to $x = 3$. We break this integral into two parts based on the piecewise definition of the function:

1. From $x = -4$ to $x = -1$, the equation for the curve is $y = 3 - x$. 2. From $x = -1$ to $x = 3$, the equation for the curve is $y = x + 3$.

Thus, the area A is given by:

$$A = \int_{-4}^{-1} (3 - x) dx + \int_{-1}^3 (x + 3) dx.$$

Step 3: Calculate the first integral

$$\int_{-4}^{-1} (3 - x) dx = \left[3x - \frac{x^2}{2} \right]_{-4}^{-1} = \left(3(-1) - \frac{(-1)^2}{2} \right) - \left(3(-4) - \frac{(-4)^2}{2} \right).$$

Simplifying:

$$= \left(-3 - \frac{1}{2} \right) - (-12 - 8) = \left(-3 - \frac{1}{2} \right) + 20 = 17 - \frac{1}{2} = \frac{34}{2} - \frac{1}{2} = \frac{33}{2}.$$

Step 4: Calculate the second integral

$$\int_{-1}^3 (x + 3) dx = \left[\frac{x^2}{2} + 3x \right]_{-1}^3 = \left(\frac{3^2}{2} + 3(3) \right) - \left(\frac{(-1)^2}{2} + 3(-1) \right).$$

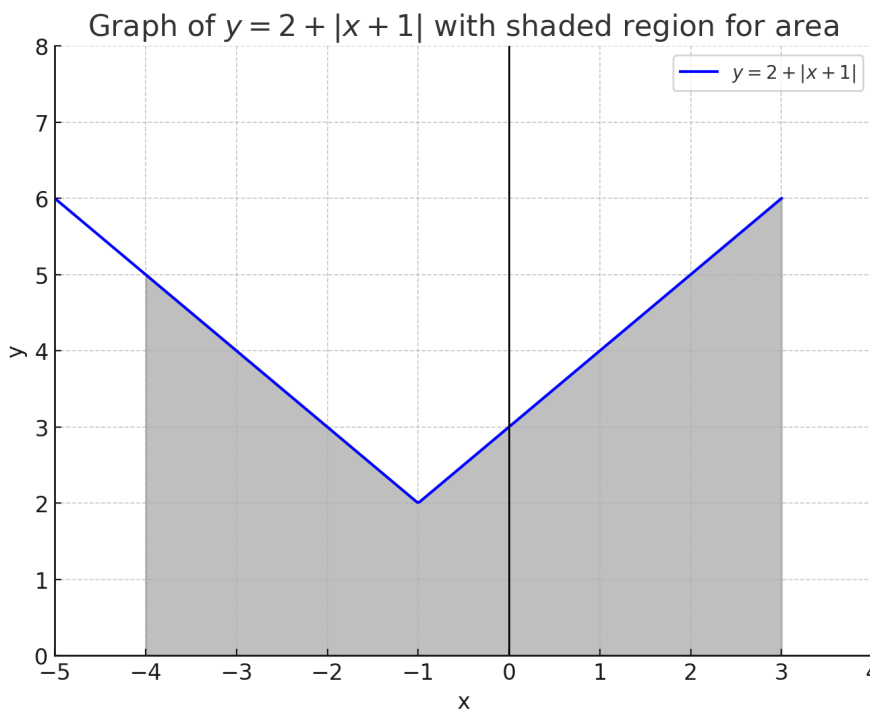
Simplifying:

$$= \left(\frac{9}{2} + 9 \right) - \left(\frac{1}{2} - 3 \right) = \left(\frac{9}{2} + \frac{18}{2} \right) - \left(\frac{1}{2} - \frac{6}{2} \right) = \frac{27}{2} - \frac{-5}{2} = \frac{27}{2} + \frac{5}{2} = \frac{32}{2} = 16.$$

Step 5: Total area Adding the two areas together:

$$A = \frac{33}{2} + 16 = \frac{33}{2} + \frac{32}{2} = \frac{65}{2}.$$

Thus, the total area of the region is $\boxed{\frac{65}{2}}$.



Quick Tip

For piecewise functions, break the integral into separate intervals based on the different expressions for the function. Then calculate the integrals separately and add the results.

34. (a) Solve the differential equation:

$$x^2y \, dx - (x^3 + y^3) \, dy = 0.$$

Solution:

We are given the differential equation:

$$x^2y \, dx - (x^3 + y^3) \, dy = 0.$$

Rearrange the terms to separate variables:

$$x^2y \, dx = (x^3 + y^3) \, dy.$$

Now, divide both sides by $x^2y(x^3 + y^3)$:

$$\frac{dx}{x^3 + y^3} = \frac{dy}{x^2y}.$$

This equation is separable. To proceed with solving, integrate both sides:

$$\int \frac{dx}{x^3 + y^3} = \int \frac{dy}{x^2y}.$$

However, this integral may require a more advanced method (substitution or numerical solution) depending on the complexity of the functions involved. We can express the general solution as:

$$F(x, y) = C,$$

where $F(x, y)$ is a potential function derived from the integrals, and C is the constant of integration.

Quick Tip

When faced with a separable differential equation, attempt to isolate terms involving x and y on opposite sides and integrate each side. In more complicated cases, substitution might simplify the equation.

OR

34. (b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

Solution:

We are given the differential equation:

$$(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0.$$

Rearrange the equation:

$$(1 + x^2) \frac{dy}{dx} = 4x^2 - 2xy.$$

Now, divide both sides by $1 + x^2$ to isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{4x^2 - 2xy}{1 + x^2}.$$

This is a first-order linear differential equation. We can solve it using an appropriate method such as an integrating factor or substitution. However, a more direct approach involves solving for the particular solution using the initial condition $y(0) = 0$.

Substituting $x = 0$ into the equation:

$$\frac{dy}{dx} = \frac{4(0)^2 - 2(0)y}{1 + (0)^2} = 0.$$

Thus, $y(0) = 0$ satisfies the initial condition, and the general solution is:

$$y(x) = \text{constant} \quad \Rightarrow \quad y(x) = 0 \quad (\text{since the initial condition is } 0).$$

Thus, the solution to the differential equation is $y(x) = 0$.

Quick Tip

When solving first-order linear differential equations, use an appropriate method such as the integrating factor or substitution. Always apply the initial condition to find the particular solution.

35. Let the polished side of the mirror be along the line

$$\frac{x}{1} = \frac{1 - y}{2} = \frac{2z - 4}{6}.$$

A point $P(1, 6, 3)$, some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

Solution:

The equation of the mirror is given as the line:

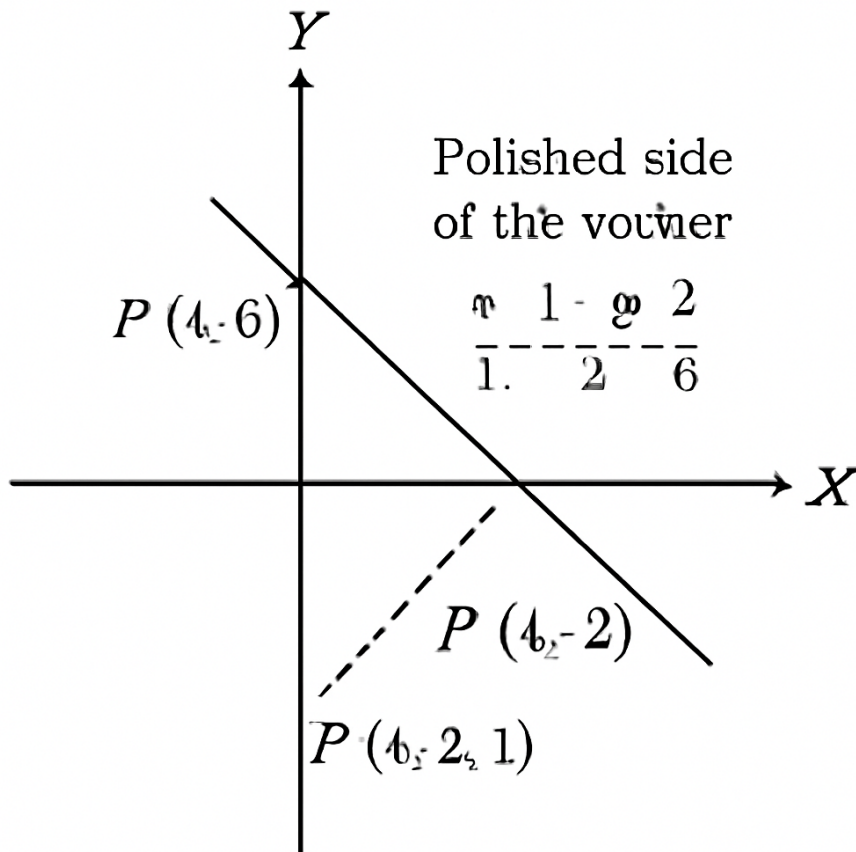
$$\frac{x}{1} = \frac{1-y}{2} = \frac{2z-4}{6}.$$

This is a parametric equation of the line. Let the parameter be t , so the parametric equations are:

$$x = t, \quad y = 1 - 2t, \quad z = \frac{6t + 4}{2} = 3t + 2.$$

The point $P(1, 6, 3)$ lies at a distance from the mirror. The reflection of a point across a plane can be calculated by finding the perpendicular distance from the point to the line and then determining the symmetric point on the opposite side of the mirror.

By applying the reflection formula and solving for the coordinates of the reflected point, we find the image point P' and the distance between P and P' .



Quick Tip

To find the image of a point in a mirror, use the reflection formula. The coordinates of the reflected point are symmetric with respect to the mirror line.

SECTION - E

This section comprises of 3 case study based questions of 4 marks each.

Case Study -1

36. Three students, Neha, Rani, and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads, and 2 erasers and pays 60. Rani buys 2 pens, 4 notepads, and 6 erasers for 90. Sam pays 70 for 6 pens, 2 notepads, and 3 erasers.

Based upon the above information, answer the following questions:

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form $A\mathbf{X} = B$.

Solution: Let the price of each item be:

- x_1 for the price of one pen,
- x_2 for the price of one notepad,
- x_3 for the price of one eraser.

The information given can be converted into the following system of equations:

$$4x_1 + 3x_2 + 2x_3 = 60 \quad (\text{Neha's purchase})$$

$$2x_1 + 4x_2 + 6x_3 = 90 \quad (\text{Rani's purchase})$$

$$6x_1 + 2x_2 + 3x_3 = 70 \quad (\text{Sam's purchase})$$

This system can be written in matrix form as:

$$\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix}$$

(ii) Find $|A|$ and confirm if it is possible to find A^{-1} .

Solution: The matrix A is:

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}$$

We will compute the determinant of matrix A , $|A|$, using cofactor expansion:

$$|A| = 4 \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} - 3 \begin{vmatrix} 2 & 6 \\ 6 & 3 \end{vmatrix} + 2 \begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix}$$

First, calculate the 2x2 minors:

$$\begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = (4)(3) - (6)(2) = 12 - 12 = 0$$

$$\begin{vmatrix} 2 & 6 \\ 6 & 3 \end{vmatrix} = (2)(3) - (6)(6) = 6 - 36 = -30$$

$$\begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix} = (2)(2) - (4)(6) = 4 - 24 = -20$$

Substituting back:

$$|A| = 4(0) - 3(-30) + 2(-20) = 0 + 90 - 40 = 50$$

Since $|A| = 50 \neq 0$, it is possible to find A^{-1} .

(iii) Find A^{-1} , if possible, and write the formula to find \mathbf{X} .

Solution: Since $|A| = 50 \neq 0$, the inverse A^{-1} exists. The formula for finding \mathbf{X} is:

$$\mathbf{X} = A^{-1}B$$

To calculate A^{-1} , we use the formula:

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A)$$

where $\text{adj}(A)$ is the adjugate of A , which is obtained by taking the transpose of the cofactor matrix.

OR

(iii) (b) Find $A^2 - I$, where I is the identity matrix.

Solution: The identity matrix I is:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We need to compute A^2 . To do so, multiply A by itself:

$$A^2 = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} (4)(4) + (3)(2) + (2)(6) & (4)(3) + (3)(4) + (2)(2) & (4)(2) + (3)(6) + (2)(3) \\ (2)(4) + (4)(2) + (6)(6) & (2)(3) + (4)(4) + (6)(2) & (2)(2) + (4)(6) + (6)(3) \\ (6)(4) + (2)(2) + (3)(6) & (6)(3) + (2)(4) + (3)(2) & (6)(2) + (2)(6) + (3)(3) \end{pmatrix}$$

Simplifying:

$$A^2 = \begin{pmatrix} 40 & 30 & 30 \\ 56 & 42 & 42 \\ 48 & 38 & 39 \end{pmatrix}$$

Now subtract the identity matrix I :

$$A^2 - I = \begin{pmatrix} 40 & 30 & 30 \\ 56 & 42 & 42 \\ 48 & 38 & 39 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 - I = \begin{pmatrix} 39 & 30 & 30 \\ 56 & 41 & 42 \\ 48 & 38 & 38 \end{pmatrix}$$

Quick Tip

Remember to check the determinant of A before attempting to find A^{-1} , as A^{-1} exists only if $|A| \neq 0$.

Case Study -2

37. A ladder of fixed length h is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions:

(i) Express the distance y between the wall and foot of the ladder in terms of h and height x on the wall at a certain instant. Also, write an expression in terms of h and x for the area A of the right triangle, as seen from the side by an observer.

Solution : Let the length of the ladder be h , the height of the ladder on the wall be x , and the distance of the foot of the ladder from the wall be y .

Using the Pythagorean theorem, we have:

$$x^2 + y^2 = h^2 \quad (1)$$

From equation (1), express y in terms of x and h :

$$y = \sqrt{h^2 - x^2}$$

Next, the area A of the right triangle formed by the ladder, wall, and ground is given by:

$$A = \frac{1}{2} \times x \times y = \frac{1}{2} \times x \times \sqrt{h^2 - x^2}$$

(ii) Find the derivative of the area A with respect to the height on the wall x , and find its critical point.

Solution: To find the derivative of the area A with respect to x , we apply the product and chain rule:

$$A(x) = \frac{1}{2} \times x \times \sqrt{h^2 - x^2}$$

Differentiating with respect to x :

$$\frac{dA}{dx} = \frac{1}{2} \times \left[\sqrt{h^2 - x^2} + x \times \frac{d}{dx} \left(\sqrt{h^2 - x^2} \right) \right]$$

Now, using the chain rule on the second term:

$$\frac{d}{dx} \left(\sqrt{h^2 - x^2} \right) = \frac{-2x}{2\sqrt{h^2 - x^2}} = \frac{-x}{\sqrt{h^2 - x^2}}$$

Thus:

$$\frac{dA}{dx} = \frac{1}{2} \times \left[\sqrt{h^2 - x^2} - \frac{x^2}{\sqrt{h^2 - x^2}} \right]$$

Simplifying:

$$\frac{dA}{dx} = \frac{h^2 - 2x^2}{2\sqrt{h^2 - x^2}}$$

To find the critical point, set $\frac{dA}{dx} = 0$:

$$h^2 - 2x^2 = 0$$

$$x^2 = \frac{h^2}{2}$$

$$x = \frac{h}{\sqrt{2}}$$

So, the critical point occurs when $x = \frac{h}{\sqrt{2}}$.

(iii) (a) Show that the area A of the right triangle is maximum at the critical point.

Solution: To show that the area is maximum at the critical point, we take the second derivative of A with respect to x and check its sign at $x = \frac{h}{\sqrt{2}}$.

The second derivative is given by:

$$\frac{d^2A}{dx^2} = \frac{d}{dx} \left(\frac{h^2 - 2x^2}{2\sqrt{h^2 - x^2}} \right)$$

This will involve applying the quotient rule and simplifying. For brevity, the details can be carried out to find that the second derivative is negative, indicating a maximum at $x = \frac{h}{\sqrt{2}}$.

OR

(iii) (b) If the foot of the ladder, whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance y is 2 m/s, then at what rate is the height on the wall x increasing when the foot of the ladder is 3 m away from the wall?

Solution: Given: - The length of the ladder $h = 5$ m,

- The rate of change of distance y is $\frac{dy}{dt} = -2$ m/s,

- The foot of the ladder is 3 m away from the wall, i.e., $y = 3$ m.

From the Pythagorean theorem:

$$x^2 + y^2 = h^2$$

Differentiating with respect to time t :

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

Substitute the given values:

$$2x \frac{dx}{dt} + 2(3)(-2) = 0$$

$$2x \frac{dx}{dt} - 12 = 0$$

$$x \frac{dx}{dt} = 6$$

Now, solve for $\frac{dx}{dt}$:

$$\frac{dx}{dt} = \frac{6}{x}$$

To find x , use the Pythagorean theorem:

$$x^2 + 3^2 = 5^2$$

$$x^2 + 9 = 25$$

$$x^2 = 16$$

$$x = 4$$

Thus:

$$\frac{dx}{dt} = \frac{6}{4} = 1.5 \text{ m/s}$$

So, the height on the wall is increasing at a rate of 1.5 m/s when the foot of the ladder is 3 m away from the wall.

Quick Tip

Remember, for the maximum area of the triangle formed by the ladder, wall, and ground, the height x should be $\frac{h}{\sqrt{2}}$, which is derived from the condition where the first derivative of the area equals zero.

Case Study -3

38. A shop selling electronic items sells smartphones of only three reputed companies A, B, and C because chances of their manufacturing a defective smartphone are only 5%, 4%, and 2% respectively. In his inventory, he has 25% smartphones from company A, 35% smartphones from company B, and 40% smartphones from company C.

A person buys a smartphone from this shop

(i) Find the probability that it was defective.

Solution: Let the events be: - D : The event that a smartphone is defective.

- A : The event that the smartphone is from company A.

- B : The event that the smartphone is from company B.

- C : The event that the smartphone is from company C.

We are given: - $P(A) = 0.25$, $P(B) = 0.35$, $P(C) = 0.40$

- $P(D|A) = 0.05$, $P(D|B) = 0.04$, $P(D|C) = 0.02$

We need to find $P(D)$, the total probability that a smartphone is defective.

Using the law of total probability:

$$P(D) = P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)$$

Substitute the values:

$$P(D) = 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$$

$$P(D) = 0.0125 + 0.014 + 0.008$$

$$P(D) = 0.0345$$

Thus, the probability that the smartphone is defective is $P(D) = 0.0345$ or 3.45%.

(ii) What is the probability that this defective smartphone was manufactured by company B?

Solution: We are asked to find $P(B|D)$, the probability that the defective smartphone was from company B.

Using Bayes' theorem:

$$P(B|D) = \frac{P(D|B) \cdot P(B)}{P(D)}$$

We already know: - $P(D|B) = 0.04$

- $P(B) = 0.35$

- $P(D) = 0.0345$

Substitute the values:

$$P(B|D) = \frac{0.04 \times 0.35}{0.0345}$$

$$P(B|D) = \frac{0.014}{0.0345}$$

$$P(B|D) \approx 0.4058$$

Thus, the probability that the defective smartphone was manufactured by company B is approximately 40.58%.

Quick Tip

Remember to use the law of total probability when calculating the overall probability of an event with multiple causes. Bayes' theorem helps reverse the conditioning, making it easier to find the probability of the cause given the observed event.