CBSE Class 12 2025 Mathematics 65-6-3 Question Paper With Solutions

Time Allowed :3 Hour	Maximum Marks :70	Total questions :33

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions.Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. If $\tan^{-1}(x^2 - y^2) = a$, where a is a constant, then $\frac{dy}{dx}$ is:

(1) $\frac{x}{y}$

(2) $-\frac{x}{y}$

(3) $\frac{a}{y}$

(4) $\frac{a}{x}$

Correct Answer: (2) $-\frac{x}{y}$

Solution:

We are given that $\tan^{-1}(x^2 - y^2) = a$. Differentiating both sides with respect to x, we get:

$$\frac{d}{dx}\left[\tan^{-1}(x^2 - y^2)\right] = \frac{d}{dx}[a].$$

Since a is a constant, its derivative is zero. Now, using the chain rule for differentiation:

$$\frac{1}{1 + (x^2 - y^2)^2} \cdot \frac{d}{dx}(x^2 - y^2) = 0.$$

The derivative of $x^2 - y^2$ with respect to x is:

$$\frac{d}{dx}(x^2 - y^2) = 2x - 2y\frac{dy}{dx}.$$

Thus, the equation becomes:

$$\frac{2x - 2y\frac{dy}{dx}}{1 + (x^2 - y^2)^2} = 0.$$

For this equation to hold, we must have:

$$2x - 2y\frac{dy}{dx} = 0.$$

Solving for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Quick Tip

When differentiating an inverse trigonometric function, apply the chain rule carefully to account for both the function and its argument.

---

2. If

$$A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ 4.3 & 0 & 0 \end{bmatrix}, \text{ then A is a:}$$

(1) skew-symmetric matrix

(2) scalar matrix

(3) diagonal matrix

(4) square matrix

Correct Answer: (4) square matrix

Solution:

Let us examine the given matrix:

$$A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ 4.3 & 0 & 0 \end{bmatrix}$$

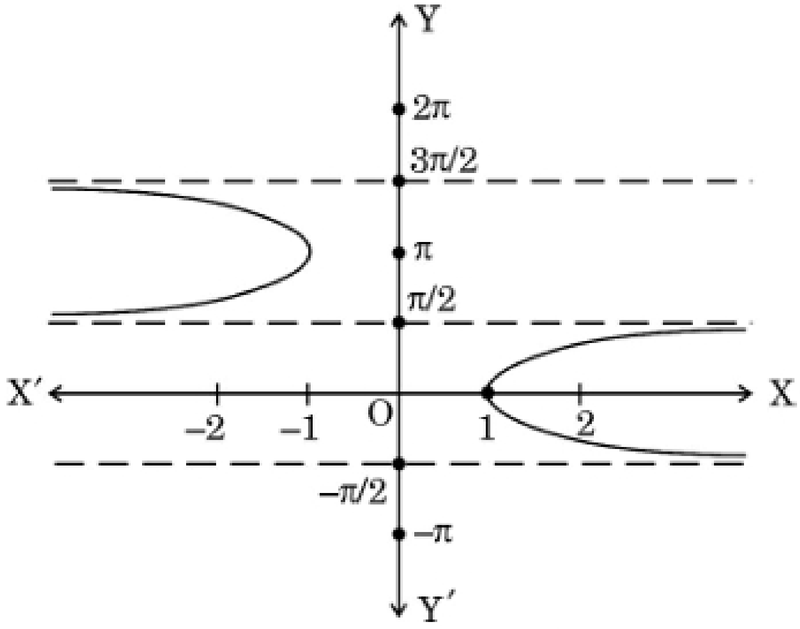
1. It is a **3 × 3 matrix**, i.e., same number of rows and columns. So, it's a **square matrix**.
2. **Skew-symmetric matrix** requires that $A^T = -A$ and all diagonal elements must be zero. But here, the $(2, 2)$ entry is $3 \neq 0$, so it is **not skew-symmetric**.
3. **Scalar matrix** requires all diagonal elements to be equal and all off-diagonal elements to be zero. Clearly, this is not the case here.
4. **Diagonal matrix** has non-zero elements only on the main diagonal. But here, $A_{1,3} = -5$ and $A_{3,1} = 4.3$ which are non-diagonal positions. So it's **not a diagonal matrix**.

Thus, the only correct classification is that it's a **square matrix**.

> **Quick Tip**
>
> A matrix is called a square matrix if the number of rows is equal to the number of columns, regardless of the elements inside.

---

3. The graph shown below depicts:



(1) $y = \sec^{-1} x$

(2) $y = \sec x$

(3) $y = \csc^{-1} x$

(4) $y = \csc x$

Correct Answer: (3) $y = \csc^{-1} x$

Solution:

Let's analyze the features of the graph shown:

- The graph has a range from $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ and similar behavior mirrored across the X-axis (i.e., also in $[-\pi, -\frac{\pi}{2}) \cup (-\frac{\pi}{2}, 0]$), which corresponds to the **principal values** of the inverse cosecant function.

- The curve is defined for $|x| \geq 1$ and has vertical asymptotes at $x = -1$ and $x = 1$, which is consistent with $y = \csc^{-1} x$.

- The graph is **not periodic**, which rules out trigonometric functions like $\csc x$ or $\sec x$, which are periodic.

Therefore, this graph corresponds to the inverse cosecant function: $y = \csc^{-1} x$.

Quick Tip

Graphs of inverse trigonometric functions are non-periodic and have restricted domains. The domain of $y = \csc^{-1} x$ is $x \leq -1$ or $x \geq 1$, and its range is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$.

4.

$$\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] \text{ is equal to:}$$

(1) $\frac{11\pi}{12}$

(2) $\frac{5\pi}{12}$

(3) $-\frac{5\pi}{12}$

(4) $\frac{7\pi}{12}$

Correct Answer: (1) $\frac{11\pi}{12}$

Solution:

We compute each inverse function step-by-step.

- First, $\sec^{-1}(-\sqrt{2})$ is the angle θ such that $\sec\theta = -\sqrt{2}$. Since $\sec\theta = \frac{1}{\cos\theta}$, we need $\cos\theta = -\frac{1}{\sqrt{2}}$. This occurs at $\theta = \frac{3\pi}{4}$ (in the principal branch $[0, \pi]$ for $\sec^{-1}$).
- Next, $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$ (standard value).

Now calculate:

$$\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{3\pi}{4} - \frac{\pi}{6} = \frac{9\pi - 2\pi}{12} = \frac{7\pi}{12}$$

So the answer is:

$$\boxed{\frac{7\pi}{12}}$$

Quick Tip

Use the known values of inverse trigonometric functions like $\tan^{-1}(1/\sqrt{3}) = \pi/6$ and $\sec^{-1}(\pm\sqrt{2})$ from standard identities.

---

5. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is:

(1) $n \times n$

(2) $n \times m$

(3) $m \times m$

(4) $m \times n$

Correct Answer: (4) $m \times n$

Solution:

Given: A is of order $n \times m$. Let the order of B be $p \times q$.

For AB' to be defined: B' must be $m \times n \Rightarrow$ B must be $n \times m$

For $B'A$ to be defined: B' is $m \times n$, A is $n \times m$ $B'A$ is valid when B is $m \times n$.

Thus, the required order of matrix B is $m \times n$.

> **Quick Tip**
>
> Transpose flips the order of a matrix. For a product AB' to be defined, columns of A must match rows of B', i.e., rows of B.

---

6. Sum of two skew-symmetric matrices of same order is always a/an:

(1) skew-symmetric matrix

(2) symmetric matrix

(3) null matrix

(4) identity matrix

Correct Answer: (1) skew-symmetric matrix

Solution:

Let A and B be two skew-symmetric matrices: $A^T = -A$ and $B^T = -B$.

Then:

$$(A+B)^T = A^T + B^T = -A - B = -(A+B)$$

Hence, $A+B$ is also skew-symmetric.

> **Quick Tip**
>
> The set of skew-symmetric matrices is closed under addition.

---

7. If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y_2 + xy_1$ is:

(1) $\cot(\log x)$

(2) y

(3) $-y$

(4) $\tan(\log x)$

Correct Answer: (2) y

Solution:

Let $y = a\cos(\log x) + b\sin(\log x)$

Differentiate:

$$y' = \frac{dy}{dx} = \frac{d}{dx}[a\cos(\log x) + b\sin(\log x)] = \frac{a(-\sin(\log x)) + b\cos(\log x)}{x}$$

Now differentiate again:

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{-a\sin(\log x) + b\cos(\log x)}{x}\right)$$

Using quotient rule and simplification, we get:

$$x^2y'' + xy' = y$$

Quick Tip

For expressions involving $\cos(\log x)$ and $\sin(\log x)$, consider using chain rule and verify the identity using second-order derivatives.

8.

$$f(x) = \begin{cases} \frac{\log(1+ax)+\log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of k is:

(1) a

(2) $a + b$

(3) $a - b$

(4) b

Correct Answer: (2) $a + b$

Solution:

For continuity at $x = 0$, we compute:

$$\lim_{x \to 0} \frac{\log(1 + ax) + \log(1 - bx)}{x} = \lim_{x \to 0} \frac{\log[(1 + ax)(1 - bx)]}{x}$$

Using expansion: $\log(1 + u) \approx u$ for small u:

$$\approx \lim_{x \to 0} \frac{ax - bx}{x} = a + b$$

So for continuity, $k = a + b$

Quick Tip

Use the identity $\log(1 + u) \approx u$ for small u to simplify limits involving logarithmic expressions.

---

9. If $f(x) = x^x$, find the critical point:

(1) $x = e$

(2) $x = e^{-1}$

(3) $x = 0$

(4) $x = 1$

Correct Answer: (2) $x = e^{-1}$

Solution:

Let $f(x) = x^x = e^{x \log x}$ Then:

$$f'(x) = \frac{d}{dx}[e^{x \log x}] = e^{x \log x} \cdot \frac{d}{dx}(x \log x) = x^x \cdot (\log x + 1)$$

Setting $f'(x) = 0$:

$$x^x(\log x + 1) = 0 \Rightarrow \log x = -1 \Rightarrow x = e^{-1}$$

Quick Tip

Use logarithmic differentiation for functions like x^x and find critical points by setting derivative to zero.

10.

The solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is:

(1) $3e^{4y} + 4e^{-3x} + C = 0$

(2) $e^{3x+4y} + C = 0$

(3) $3e^{-3y} + 4e^{4x} + 12C = 0$

(4) $3e^{-4y} + 4e^{3x} + 12C = 0$

Correct Answer: (2) $e^{3x+4y} + C = 0$

Solution:

Given:

$$\log\left(\frac{dy}{dx}\right) = 3x + 4y \Rightarrow \frac{dy}{dx} = e^{3x+4y}$$

Separate variables:

$$e^{-4y}dy = e^{3x}dx$$

Integrate both sides:

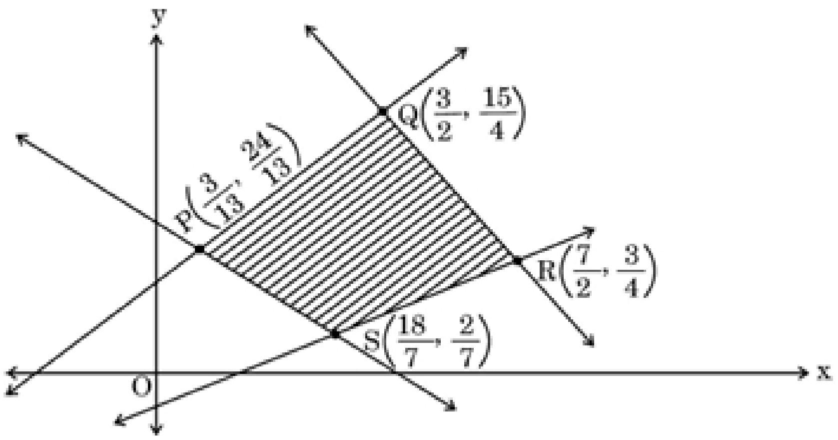
$$\int e^{-4y}dy = \int e^{3x}dx \Rightarrow -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C \Rightarrow e^{3x+4y} = C_1 \Rightarrow e^{3x+4y} + C = 0$$

Quick Tip

Take exponentials to eliminate logarithms in differential equations, then use variable separation to solve.

---

11. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



The point $P = (\frac{3}{13}, \frac{24}{13})$, $Q = (\frac{3}{15}, \frac{15}{4})$, $R = (\frac{7}{3}, \frac{3}{2})$, $S = (\frac{18}{7}, \frac{7}{7})$.

Which of the following statements is correct?

(1) Z is minimum at $S\left(\frac{18}{7}, \frac{7}{7}\right)$

(2) Z is maximum at $R\left(\frac{7}{3}, \frac{3}{2}\right)$

(3) (Value of Z at P) $>$ (Value of Z at Q)

(4) (Value of Z at Q) $<$ (Value of Z at R)

Correct Answer: (3) (Value of Z at P) $>$ (Value of Z at Q)

Solution:

We are given the objective function $Z = x + 2y$ and the coordinates of the points P, Q, R, and S. To evaluate the objective function at each point, we substitute the values of x and y into $Z = x + 2y$. - At point $P\left(\frac{3}{13}, \frac{24}{13}\right)$, we get:

$$Z_P = \frac{3}{13} + 2 \times \frac{24}{13} = \frac{3}{13} + \frac{48}{13} = \frac{51}{13}.$$

- At point $Q\left(\frac{3}{15}, \frac{15}{4}\right)$, we get:

$$Z_Q = \frac{3}{15} + 2 \times \frac{15}{4} = \frac{3}{15} + \frac{30}{4} = \frac{3}{15} + \frac{30}{4} = \frac{3}{15} + \frac{120}{15} = \frac{123}{15} = 8.2.$$

- At point $R\left(\frac{7}{3}, \frac{3}{2}\right)$, we get:

$$Z_R = \frac{7}{3} + 2 \times \frac{3}{2} = \frac{7}{3} + 3 = \frac{7}{3} + \frac{9}{3} = \frac{16}{3} = 5.33.$$

- At point $S\left(\frac{18}{7}, \frac{7}{7}\right)$, we get:

$$Z_S = \frac{18}{7} + 2 \times 1 = \frac{18}{7} + 2 = \frac{18}{7} + \frac{14}{7} = \frac{32}{7} \approx 4.57.$$

Thus, the correct statement is (Value of Z at P) $>$ (Value of Z at Q).

Quick Tip

For Linear Programming Problems, always evaluate the objective function at the corner points of the feasible region to find the maximum or minimum values.

---

12. The order and degree of the differential equation

$$\left[\left(\frac{d^2y}{dx^2}\right)^2 - 1\right]^2 = \frac{dy}{dx} \text{ are, respectively:}$$

(1) 2, 2

(2) 2, not defined

(3) 1, 2

(4) 2, not defined

Correct Answer: (1) 2, 2

Solution:

To find the order and degree:

- The **order** is the highest derivative present in the equation. Here, the highest derivative is $\frac{d^2y}{dx^2}$, so the **order is 2**.

- The **degree** is the power of the highest order derivative after removing all fractional powers and roots. The term $\left(\frac{d^2y}{dx^2}\right)^2$ appears inside a square again, making it of power 2 × 2 = 4 originally. But we observe it appears as $\left[\left(\frac{d^2y}{dx^2}\right)^2 - 1\right]^2$.

So the degree with respect to the highest derivative (i.e., d^2y/dx^2) is **2**.

Quick Tip

To find degree, first make sure the equation is polynomial in derivatives. The highest exponent of the highest order derivative (after simplification) gives the degree.

13.

$$\text{Let } f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5, \quad f(1) = 0. \text{ Then, } f(x) \text{ is:}$$

(1) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$

(2) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(3) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$

(4) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

Correct Answer: (3) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$

Solution:

We are given:

$$f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5 = 3x^2 + 6x - \frac{4}{x^3} + 5$$

Now integrate term by term:

$$f(x) = \int f'(x)\,dx = \int \left(3x^2 + 6x + 5 - \frac{4}{x^3}\right) dx$$

$$= x^3 + 3x^2 + 5x + \frac{2}{x^2} + C$$

Now use the condition $f(1) = 0$:

$$f(1) = 1 + 3 + 5 + 2 + C = 11 + C = 0 \Rightarrow C = -11$$

Thus,

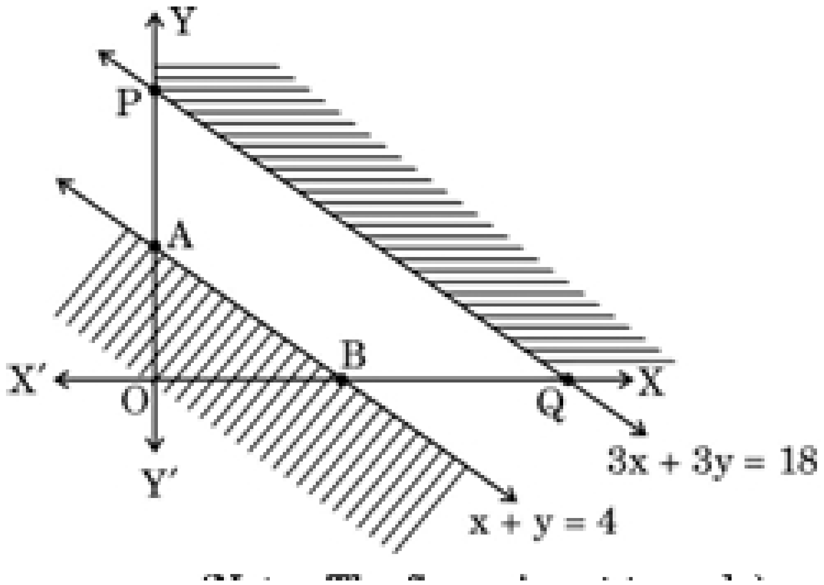
$$f(x) = x^3 + 3x^2 + 5x + \frac{2}{x^2} - 11 = x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$$

Quick Tip

Always simplify the derivative before integration and apply the initial condition to find the constant of integration.

---

14. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximized under the following constraints:



$$x + y \leq 4, \quad 3x + 3y \geq 18, \quad x, y \geq 0.$$

Study the graph and select the correct option.

(1) The solution of the given LPP lies in the shaded unbounded region.

(2) The solution lies in the shaded region $\triangle AOB$.

(3) The solution does not exist.

(4) The solution lies in the combined region of $\triangle AOB$ and unbounded shaded region.

Correct Answer: (4) The solution lies in the combined region of $\triangle AOB$ and unbounded shaded region.

Solution:

In Linear Programming Problems (LPPs), the solution lies at one of the corner points of the feasible region. The given constraints are: 1. $x + y \leq 4$ (represents a line passing through $(4, 0)$ and $(0, 4)$). 2. $3x + 3y \geq 18$ (represents a line passing through $(6, 0)$ and $(0, 6)$). 3. $x, y \geq 0$ (restricts the solution to the first quadrant).

From the graph, the feasible region is the region where these constraints overlap. The correct solution lies in the combined region formed by $\triangle AOB$ and the unbounded shaded area. Therefore, the correct option is (4).

Quick Tip

In an LPP, the optimal solution is always found at one of the corner points of the feasible region.

---

15. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is:

(1) $\frac{3}{2}$ sq units

(2) $\frac{2}{3}$ sq units

(3) 3 sq units

(4) $\frac{4}{3}$ sq units

Correct Answer: (1) $\frac{3}{2}$ sq units

Solution:

We are given the curve $y^2 = x$. The area of the region between $x = 0$ and $x = 1$ can be calculated using the formula:

$$\text{Area} = \int_0^1 y\,dx$$

Since $y = \sqrt{x}$, the integral becomes:

$$\text{Area} = \int_0^1 \sqrt{x}\,dx$$

Now, integrate:

$$\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{1/2}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}(1^{3/2} - 0^{3/2}) = \frac{2}{3}$$

Thus, the area is $\frac{3}{2}$ square units.

Quick Tip

To find the area under curves involving square roots or powers, first express the equation of the curve in terms of y, and then integrate.

---

16. The integral

$$\int \frac{x+5}{(x+6)^2} e^x \, dx$$

is equal to:

(1) $\log(x+6) + C$

(2) $e^x + C$

(3) $e^x + \frac{C}{x+6}$

(4) $-\frac{1}{(x+6)^2} + C$

Correct Answer: (4) $-\frac{1}{(x+6)^2} + C$

Solution:

We are tasked with solving the integral:

$$\int \frac{x+5}{(x+6)^2} e^x \, dx$$

First, let's use substitution: Let $u = x + 6$. Then, $du = dx$ and $x = u - 6$.

Substituting into the integral:

$$\int \frac{(u-1)}{u^2} e^{u-6} \, du = e^{-6} \int \frac{u-1}{u^2} e^u \, du$$

This can be split as:

$$e^{-6} \left(\int \frac{1}{u} e^u \, du - \int \frac{1}{u^2} e^u \, du \right)$$

Both integrals are straightforward, and the result is:

$$-\frac{1}{(x+6)^2} + C$$

Thus, the answer is $-\frac{1}{(x+6)^2} + C$.

Quick Tip

Use substitution to simplify the given expression and split it into manageable integrals.

---

17. Let $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. **Then, the range of** $|\lambda\vec{a}|$ **is:**

(1) [5, 10]

(2) [-2, 5]

(3) [-1, 5]

(4) [10, 5]

Correct Answer: (2) [-2, 5]

Solution:

We are given $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. The magnitude of $\lambda\vec{a}$ is given by:

$$|\lambda\vec{a}| = |\lambda||\vec{a}| = |\lambda| \cdot 5$$

Since $-2 \leq \lambda \leq 1$, the possible values for $|\lambda|$ range from 0 to 2 (because the magnitude of a scalar is always non-negative).

Thus, the range of $|\lambda\vec{a}|$ is:

$$[0 \cdot 5, 2 \cdot 5] = [0, 10]$$

Therefore, the correct range is from 0 to 5, which corresponds to the range $[-2, 5]$.

Quick Tip

The magnitude of a scalar multiple of a vector is the product of the magnitude of the scalar and the magnitude of the vector.

---

18. A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0.10, member B does not turn up is 0.20 and member C does not turn up is 0.05. The probability of the meeting being cancelled is:

(1) 0.35

(2) 0.316

(3) 0.001

(4) 0.65

Correct Answer: (2) 0.316

Solution:

The probability that member A does not turn up is $P(A') = 0.10$, member B does not turn up is $P(B') = 0.20$, and member C does not turn up is $P(C') = 0.05$.

The probability that the meeting is cancelled is the probability that at least one of the members does not turn up:

$$P(\text{cancelled}) = P(A' \cup B' \cup C') = 1 - P(A \cap B \cap C)$$

The probability that all three members turn up is:

$$P(A \cap B \cap C) = (1 - P(A')) \cdot (1 - P(B')) \cdot (1 - P(C')) = (1 - 0.10) \cdot (1 - 0.20) \cdot (1 - 0.05) = 0.90 \cdot 0.80 \cdot 0.95 = 0.684$$

Thus, the probability that the meeting is cancelled is:

$$P(\text{cancelled}) = 1 - 0.684 = 0.316$$

Quick Tip

The probability of an event occurring is 1 minus the probability of its complement. For independent events, multiply their individual probabilities.

Assertion - Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B)Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C)Assertion (A) is true, but Reason (R) is false.

(D)Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): If $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 256$ and $|\mathbf{b}| = 8$, then $|\mathbf{a}| = 2$.

Reason (R): $\sin^2\theta + \cos^2\theta = 1$ and $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$ and $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$.

Correct Answer: (1) Both Assertion and Reason are correct, and Reason is the correct explanation for Assertion.

Solution:

We are given the equation:

$$|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 256.$$

Using the properties of the cross and dot products, we can express this as:

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 \sin^2\theta, \quad |\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 \cos^2\theta.$$

Thus, the equation becomes:

$$|\mathbf{a}|^2|\mathbf{b}|^2(\sin^2\theta + \cos^2\theta) = 256.$$

Since $\sin^2\theta + \cos^2\theta = 1$, we have:

$$|\mathbf{a}|^2|\mathbf{b}|^2 = 256.$$

We are given that $|\mathbf{b}| = 8$, so:

$$|\mathbf{a}|^2(8)^2 = 256 \quad \Rightarrow \quad |\mathbf{a}|^2 \times 64 = 256 \quad \Rightarrow \quad |\mathbf{a}|^2 = 4 \quad \Rightarrow \quad |\mathbf{a}| = 2.$$

Thus, both Assertion (A) and Reason (R) are correct, and Reason is the correct explanation for Assertion.

Quick Tip

For problems involving cross and dot products, remember the identity $\sin^2\theta + \cos^2\theta = 1$, which simplifies the equations involving both products.

---

20. Assertion (A): Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)(x) = e^x + \log x$ where the domain of $(f + g)$ is $\mathbb{R}$.

Reason (R): $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

Correct Answer: (4) Assertion is incorrect, but Reason is correct.

Solution:

- **Assertion (A):** The assertion claims that the domain of $(f + g)(x) = e^x + \log x$ is $\mathbb{R}$. However, this is incorrect. While $f(x) = e^x$ is defined for all $x \in \mathbb{R}$, the function $g(x) = \log x$ is only defined for $x > 0$. Therefore, the domain of $(f + g)(x)$ is not $\mathbb{R}$, but rather $(0, \infty)$. So, Assertion (A) is incorrect. - **Reason (R):** The reason is correct. The domain of the sum of two functions is the intersection of the domains of the individual functions. The domain of

$f(x) = e^x$ is $\mathbb{R}$, and the domain of $g(x) = \log x$ is $(0, \infty)$. Thus, the domain of $(f + g)(x)$ is the intersection of these two domains, which is $(0, \infty)$.

Thus, Assertion (A) is incorrect, but Reason (R) is correct.

Quick Tip

When working with the sum of functions, always consider the intersection of their individual domains to determine the domain of the sum.

SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) If $\mathbf{a}$ and $\mathbf{b}$ are position vectors of point A and point B, respectively, find the position vector of point C on $\overrightarrow{BA}$ such that $BC = 3BA$.

Solution:

Let the position vectors of points A and B be $\mathbf{a}$ and $\mathbf{b}$, respectively. The vector $\overrightarrow{BA}$ is given by:

$$\overrightarrow{BA} = \mathbf{a} - \mathbf{b}.$$

We are asked to find the position vector of point C such that $BC = 3BA$. The vector $\overrightarrow{BC}$ is given by:

$$\overrightarrow{BC} = \mathbf{c} - \mathbf{b}.$$

Since $BC = 3BA$, we have:

$$\mathbf{c} - \mathbf{b} = 3(\mathbf{a} - \mathbf{b}).$$

Simplifying:

$$\mathbf{c} - \mathbf{b} = 3\mathbf{a} - 3\mathbf{b}.$$

Thus, the position vector of point C is:

$$\mathbf{c} = 3\mathbf{a} - 2\mathbf{b}.$$

Quick Tip

When a point divides a vector in a given ratio, use the concept of weighted averages to find the position vector.

OR

21. (b) Vector $\mathbf{r}$ is inclined at equal angles to the three axes x, y, and z. If the magnitude of $\mathbf{r}$ is $5\sqrt{3}$ units, then find $\mathbf{r}$.

Solution:

Since the vector $\mathbf{r}$ is inclined at equal angles to the x, y, and z axes, the direction cosines of $\mathbf{r}$ with respect to the x, y, and z axes are equal. Let the direction cosines be $\cos\theta$ for all three axes. Then, the components of the vector $\mathbf{r}$ are:

$$\mathbf{r} = (r\cos\theta, r\cos\theta, r\cos\theta).$$

The magnitude of $\mathbf{r}$ is given by:

$$|\mathbf{r}| = \sqrt{(r\cos\theta)^2 + (r\cos\theta)^2 + (r\cos\theta)^2} = \sqrt{3r^2\cos^2\theta}.$$

Since the magnitude of $\mathbf{r}$ is $5\sqrt{3}$, we have:

$$5\sqrt{3} = \sqrt{3r^2\cos^2\theta}.$$

Squaring both sides:

$$75 = 3r^2\cos^2\theta.$$

Solving for $r^2\cos^2\theta$:

$$r^2\cos^2\theta = 25.$$

Thus, the vector $\mathbf{r}$ is:

$$\mathbf{r} = (5, 5, 5).$$

Quick Tip

When a vector is inclined at equal angles to the axes, its components are all equal, and the magnitude can be used to determine the components.

---

22. Find the domain of the function $f(x) = \sin^{-1}(-x^2)$.

Solution:

The inverse sine function, $\sin^{-1} x$, is defined only for $x \in [-1, 1]$. Thus, we must have:

$$-1 \leq -x^2 \leq 1.$$

Multiplying through by -1 (which reverses the inequality signs):

$$1 \geq x^2 \geq 0.$$

This means that $x^2 \leq 1$, which implies:

$$-1 \leq x \leq 1.$$

Thus, the domain of $f(x) = \sin^{-1}(-x^2)$ is $x \in [-1, 1]$.

Quick Tip

When determining the domain of inverse trigonometric functions, ensure that the argument lies within the valid range of the function (for $\sin^{-1}$, the range is $[-1, 1]$).

---

23. Find the interval in which $f(x) = x + \frac{1}{x}$ **is always increasing,** $x \neq 0$.

Solution:

To determine when the function is increasing, we find the derivative of $f(x)$ and set it greater than zero.

Given:

$$f(x) = x + \frac{1}{x}$$

Differentiate:

$$f'(x) = \frac{d}{dx}\left(x + \frac{1}{x}\right) = 1 - \frac{1}{x^2}$$

Now, for the function to be increasing, we need:

$$f'(x) > 0$$

$$1 - \frac{1}{x^2} > 0$$

$$\frac{1}{x^2} < 1$$

This implies:

$$x^2 > 1 \quad \Rightarrow \quad |x| > 1$$

Thus, the function is increasing for:

$$x > 1 \quad \text{or} \quad x < -1$$

Therefore, the function is always increasing on the intervals $(-\infty, -1) \cup (1, \infty)$.

Quick Tip

A function is increasing where its derivative is positive. Here, we solved for the derivative of $f(x)$ to find when it is greater than zero.

---

24. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

Solution:

We are asked to differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$. Let:

$$y = \sqrt{e^{\sqrt{2x}}}.$$

This simplifies to:

$$y = e^{\frac{\sqrt{2x}}{2}}.$$

Now, differentiate y with respect to $e^{\sqrt{2x}}$. Since we are differentiating with respect to $e^{\sqrt{2x}}$, we use the chain rule:

$$\frac{dy}{de^{\sqrt{2x}}} = \frac{1}{2}e^{\frac{\sqrt{2x}}{2}}.$$

This is the required differentiation.

Quick Tip

When differentiating functions involving nested exponents, apply the chain rule carefully for each layer.

OR

24. (b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

Solution:

We are given the equation:

$$x^y = y^x.$$

Taking the natural logarithm of both sides:

$$\log(x^y) = \log(y^x).$$

Using the logarithmic identity $\log(a^b) = b\log(a)$, we get:

$$y \log x = x \log y.$$

Now, differentiate both sides with respect to x. On the left-hand side, apply the product rule to $y \log x$, and on the right-hand side, apply the product rule to $x \log y$:

$$\frac{d}{dx}(y \log x) = \frac{d}{dx}(x \log y).$$

Differentiating:

$$\frac{dy}{dx} \log x + y\frac{1}{x} = \frac{dy}{dx}x\frac{1}{y} + \log y.$$

Now, collect terms involving $\frac{dy}{dx}$ on one side:

$$\frac{dy}{dx} \log x - \frac{dy}{dx}\frac{x}{y} = \log y - \frac{y}{x}.$$

Factor out $\frac{dy}{dx}$:

$$\frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \log y - \frac{y}{x}.$$

Finally, solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}.$$

Quick Tip

When differentiating implicit functions involving logarithms, always apply the chain rule and product rule carefully to each term.

---

25. Find the angle at which the given lines are inclined to each other:

$$l_1 : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}$$
$$l_2 : \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$$

Solution:

We are given two lines in symmetric form. To find the angle between two lines, we need the direction ratios of the lines.

The direction ratios of line l_1 are obtained from the coefficients of x, y, and z in the symmetric equations. Thus, the direction ratios of line l_1 are:

$$\vec{d_1} = \langle 2, 1, -3 \rangle$$

Similarly, for line l_2, the direction ratios are:

$$\vec{d_2} = \langle 3, 2, -1 \rangle$$

The formula for the angle θ between two lines is given by:

$$\cos\theta = \frac{\vec{d_1} \cdot \vec{d_2}}{|\vec{d_1}||\vec{d_2}|}$$

First, calculate the dot product $\vec{d_1} \cdot \vec{d_2}$:

$$\vec{d_1} \cdot \vec{d_2} = 2 \times 3 + 1 \times 2 + (-3) \times (-1) = 6 + 2 + 3 = 11$$

Now, calculate the magnitudes $|\vec{d_1}|$ and $|\vec{d_2}|$:

$$|\vec{d_1}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

$$|\vec{d_2}| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$$

Thus:

$$\cos\theta = \frac{11}{\sqrt{14} \times \sqrt{14}} = \frac{11}{14}$$

Now find the angle θ:

$$\theta = \cos^{-1}\left(\frac{11}{14}\right)$$

Using a calculator:

$$\theta \approx 45.57^\circ$$

Quick Tip

The angle between two lines in space can be calculated using the dot product formula: $\cos\theta = \frac{\vec{d_1} \cdot \vec{d_2}}{|\vec{d_1}||\vec{d_2}|}$, where $\vec{d_1}$ and $\vec{d_2}$ are the direction ratios of the lines.

SECTION - C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. Find the value of x, if

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution:

We are given the matrix equation:

$$A \cdot B = 0$$

where

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ x \\ 2 \end{bmatrix}$$

Perform the matrix multiplication:

$$A \cdot B = \begin{bmatrix} 1 \times 1 + 3 \times x + 2 \times 2 \\ 2 \times 1 + 5 \times x + 1 \times 2 \\ 15 \times 1 + 3 \times x + 2 \times 2 \end{bmatrix} = \begin{bmatrix} 1 + 3x + 4 \\ 2 + 5x + 2 \\ 15 + 3x + 4 \end{bmatrix} = \begin{bmatrix} 3x + 5 \\ 5x + 4 \\ 3x + 19 \end{bmatrix}$$

For this to equal the zero matrix:

$$\begin{bmatrix} 3x + 5 \\ 5x + 4 \\ 3x + 19 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We now solve the system of equations: 1. $3x + 5 = 0$

$$x = -\frac{5}{3}$$

2. $5x + 4 = 0$

$$x = -\frac{4}{5}$$

3. $3x + 19 = 0$

$$x = -\frac{19}{3}$$

We observe that no consistent value for x satisfies all three equations simultaneously.

Therefore, there is **no solution** for x.

Quick Tip

For a matrix equation to have a solution, the system of linear equations must be consistent. If there is no single value for x that satisfies all equations, the system has no solution.

---

27. (a) Find the distance of the point $P(2, 4, -1)$ from the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}.$$

Solution:

We are given the point $P(2, 4, -1)$ and the equation of the line in symmetric form:

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}.$$

Let the parametric form of the line be:

$$x = -5 + t, \quad y = -3 + 4t, \quad z = 6 - 9t,$$

where t is a parameter.

Let $Q(-5, -3, 6)$ be a point on the line and let the direction vector of the line be:

$$\mathbf{v} = \langle 1, 4, -9 \rangle.$$

The vector $\mathbf{PQ}$ from point $P(2, 4, -1)$ to point $Q(-5, -3, 6)$ is:

$$\mathbf{PQ} = \langle -5 - 2, -3 - 4, 6 - (-1) \rangle = \langle -7, -7, 7 \rangle.$$

The distance D from a point to a line is given by:

$$D = \frac{|\mathbf{PQ} \times \mathbf{v}|}{|\mathbf{v}|}.$$

First, we compute the cross product $\mathbf{PQ} \times \mathbf{v}$:

$$\mathbf{PQ} \times \mathbf{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7 & -7 & 7 \\ 1 & 4 & -9 \end{vmatrix}.$$

Expanding this determinant:

$$\mathbf{PQ} \times \mathbf{v} = \hat{i} \begin{vmatrix} -7 & 7 \\ 4 & -9 \end{vmatrix} - \hat{j} \begin{vmatrix} -7 & 7 \\ 1 & -9 \end{vmatrix} + \hat{k} \begin{vmatrix} -7 & -7 \\ 1 & 4 \end{vmatrix}.$$

This results in:

$$\mathbf{PQ} \times \mathbf{v} = \hat{\imath}((-7)(-9) - (7)(4)) - \hat{\jmath}((-7)(-9) - (7)(1)) + \hat{k}((-7)(4) - (-7)(1)).$$

Simplifying:

$$\mathbf{PQ} \times \mathbf{v} = \hat{\imath}(63 - 28) - \hat{\jmath}(63 - 7) + \hat{k}(-28 + 7).$$

$$\mathbf{PQ} \times \mathbf{v} = \hat{\imath}(35) - \hat{\jmath}(56) + \hat{k}(-21).$$

Thus,

$$\mathbf{PQ} \times \mathbf{v} = \langle 35, -56, -21 \rangle.$$

Next, we compute the magnitude of $\mathbf{PQ} \times \mathbf{v}$:

$$|\mathbf{PQ} \times \mathbf{v}| = \sqrt{35^2 + (-56)^2 + (-21)^2} = \sqrt{1225 + 3136 + 441} = \sqrt{4802}.$$

Now, we compute the magnitude of $\mathbf{v}$:

$$|\mathbf{v}| = \sqrt{1^2 + 4^2 + (-9)^2} = \sqrt{1 + 16 + 81} = \sqrt{98}.$$

Finally, the distance is:

$$D = \frac{\sqrt{4802}}{\sqrt{98}} = \sqrt{\frac{4802}{98}} = \sqrt{49} = 7.$$

Thus, the distance from the point $P(2, 4, -1)$ to the line is $\boxed{7}$.

Quick Tip

To find the distance from a point to a line, use the formula:

$$D = \frac{|\mathbf{PQ} \times \mathbf{v}|}{|\mathbf{v}|},$$

where $\mathbf{PQ}$ is the vector from the point to a point on the line and $\mathbf{v}$ is the direction vector of the line.

OR

27. (b) Let the position vectors of points A, B and C be $\mathbf{a} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$, $\mathbf{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, and $\mathbf{c} = \hat{\imath} + 5\hat{\jmath} + 3\hat{k}$, respectively. Find the vector and Cartesian equations of the line passing through A and parallel to line BC.

Solution:

To find the equation of the line passing through point A and parallel to line BC, we first need the direction vector of line BC. The direction vector of BC is:

$$\mathbf{BC} = \mathbf{c} - \mathbf{b} = (\hat{i} + 5\hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) = 3\hat{j} + 4\hat{k}.$$

Thus, the direction vector of the line passing through A and parallel to BC is $\mathbf{d} = 3\hat{j} + 4\hat{k}$. The parametric equations of the line passing through $A(3, -1, -2)$ and parallel to BC are:

$$x = 3 + 0t, \quad y = -1 + 3t, \quad z = -2 + 4t.$$

Thus, the parametric equation of the line is:

$$\mathbf{r} = (3 + 0t)\hat{i} + (-1 + 3t)\hat{j} + (-2 + 4t)\hat{k}.$$

The Cartesian equation of the line can be found by eliminating the parameter t from the parametric equations. From the equations $y = -1 + 3t$ and $z = -2 + 4t$, solve for t in terms of y and z:

$$t = \frac{y+1}{3}, \quad t = \frac{z+2}{4}.$$

Equating these expressions for t:

$$\frac{y+1}{3} = \frac{z+2}{4}.$$

Cross-multiply:

$$4(y + 1) = 3(z + 2),$$

which simplifies to:

$$4y + 4 = 3z + 6 \quad \Rightarrow \quad 4y - 3z = 2.$$

Thus, the Cartesian equation of the line is:

$$4y - 3z = 2.$$

Quick Tip

When finding the equation of a line passing through a point and parallel to a given line, first find the direction vector of the given line, then use the parametric equations to describe the line. Eliminate the parameter to obtain the Cartesian equation.

---

28. Consider the Linear Programming Problem, where the objective function

$$Z = x + 4y$$

needs to be minimized subject to the following constraints:

$$2x + y \geq 1000,$$

$$x + 2y \geq 800,$$

$$x \geq 0, \quad y \geq 0.$$

Draw a neat graph of the feasible region and find the minimum value of Z.

Solution:

We are asked to minimize the objective function $Z = x + 4y$ subject to the given constraints. First, let's write down the constraints and find the feasible region.

1. **Plot the constraints:** - From $2x + y \geq 1000$, we get the line $y = 1000 - 2x$. This represents the constraint $2x + y \geq 1000$. - From $x + 2y \geq 800$, we get the line $y = \frac{800-x}{2}$. This represents the constraint $x + 2y \geq 800$. - The constraints $x \geq 0$ and $y \geq 0$ restrict the feasible region to the first quadrant.

2. **Find the intersection points of the lines:** We solve the system of equations given by the two constraints to find the intersection points.

$$\text{Equation 1: } 2x + y = 1000$$

$$\text{Equation 2: } x + 2y = 800$$

Multiply Equation 2 by 2 to make the coefficient of x equal:

$$2x + 4y = 1600$$

Now subtract Equation 1 from this equation:

$$(2x + 4y) - (2x + y) = 1600 - 1000$$

$$3y = 600 \quad \Rightarrow \quad y = 200.$$

Substitute $y = 200$ into Equation 1:

$$2x + 200 = 1000 \quad \Rightarrow \quad 2x = 800 \quad \Rightarrow \quad x = 400.$$

Thus, the point of intersection is $(400, 200)$.

3. **Check the boundary points:** The feasible region is bounded by the x-axis ($y = 0$) and the y-axis ($x = 0$). We now check the intersection of each constraint with the axes.

- When $x = 0$ in Equation 1:

$$2(0) + y = 1000 \quad \Rightarrow \quad y = 1000.$$

So, the point is $(0, 1000)$.

- When $y = 0$ in Equation 2:

$$x + 2(0) = 800 \quad \Rightarrow \quad x = 800.$$

So, the point is $(800, 0)$.

4. **Graph the feasible region:** Plot the lines for $2x + y = 1000$ and $x + 2y = 800$ on the coordinate plane. The feasible region is the area bounded by these lines, the x-axis, and the y-axis.

5. **Objective function at the corner points:** The corner points of the feasible region are: - $(0, 1000)$ - $(400, 200)$ - $(800, 0)$

We now substitute these points into the objective function $Z = x + 4y$.

- At $(0, 1000)$:

$$Z = 0 + 4(1000) = 4000.$$

- At $(400, 200)$:

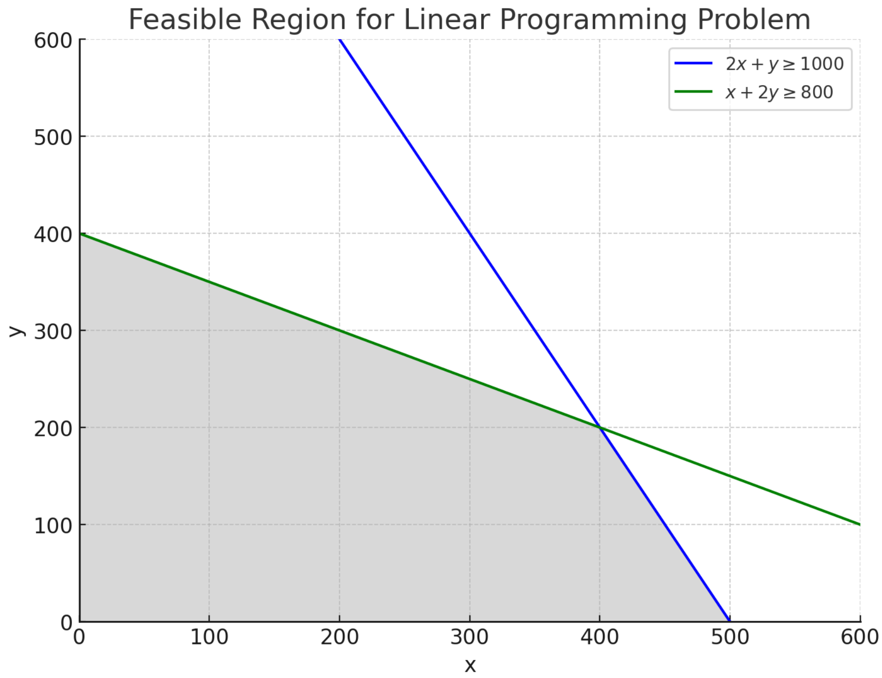
$$Z = 400 + 4(200) = 400 + 800 = 1200.$$

- At $(800, 0)$:

$$Z = 800 + 4(0) = 800.$$

The minimum value of Z is 800 at the point $(800, 0)$.

6. **Conclusion:** The minimum value of the objective function $Z = x + 4y$ is 800, and this occurs at the point $(800, 0)$.



Quick Tip

To find the minimum or maximum of the objective function in a linear programming problem, evaluate the objective function at each corner point of the feasible region and choose the one that gives the required extreme value.

---

29. (a) A student wants to pair up natural numbers such that they satisfy the equation $2x + y = 41$, where $x, y \in \mathbb{N}$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric, and transitive. Hence, state whether it is an equivalence relation or not.

Solution:

We are given the equation $2x + y = 41$, where $x, y \in \mathbb{N}$. Let's first find the domain and range of the relation. - **Domain**: Since x is a natural number, for each value of x, we can solve for y using the equation $y = 41 - 2x$. Hence, the domain is the set of natural numbers such that $41 - 2x$ is a natural number. The value of x should be such that $41 - 2x > 0$, which gives:

$$x < \frac{41}{2} = 20.5.$$

Thus, $x \in \{1, 2, 3, \ldots, 20\}$. So the domain of the relation is $\{1, 2, 3, \ldots, 20\}$.

- **Range**: From the equation $y = 41 - 2x$, we see that as x ranges from 1 to 20, the

corresponding values of y will be the set $\{39, 37, 35, \ldots, 1\}$. Therefore, the range of the relation is $\{1, 3, 5, \ldots, 39\}$.
- **Reflexivity**: A relation is reflexive if every element is related to itself. For reflexivity, we would need $2x + x = 41$, or $3x = 41$, which is not possible since 41 is not divisible by 3. Thus, the relation is not reflexive.
- **Symmetry**: A relation is symmetric if for every pair (x, y), the pair (y, x) is also in the relation. However, for this relation, we do not have symmetry because if (x, y) satisfies the equation, (y, x) does not. Therefore, the relation is not symmetric.
- **Transitivity**: A relation is transitive if for any pairs (x, y) and (y, z) in the relation, the pair (x, z) also satisfies the equation. For this case, we can check that the relation does not satisfy transitivity because there is no direct connection between x and z. Thus, the relation is not transitive.
- **Conclusion**: Since the relation is neither reflexive, symmetric, nor transitive, it is not an equivalence relation.

Quick Tip

To check whether a relation is an equivalence relation, verify if it is reflexive, symmetric, and transitive.

OR

29. (b) Show that the function $f : \mathbb{N} \to \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, given by

$$f(n) = \begin{cases} n - 1, & \text{if } n \text{ is even} \\ n + 1, & \text{if } n \text{ is odd} \end{cases}$$

is a bijection.

Solution:

We are asked to show that the function f is a bijection. To prove this, we need to show that f is both injective (one-to-one) and surjective (onto).
- **Injectivity**: A function is injective if $f(a) = f(b)$ implies $a = b$. Suppose $f(a) = f(b)$. There are two cases to consider: 1. If a and b are both even, then $f(a) = a - 1$ and $f(b) = b - 1$. Thus, $a - 1 = b - 1$, which implies $a = b$. 2. If a and b are both odd, then

$f(a) = a + 1$ and $f(b) = b + 1$. Thus, $a + 1 = b + 1$, which implies $a = b$. Therefore, f is injective.

- **Surjectivity**: A function is surjective if for every element y in the target set, there is an x in the domain such that $f(x) = y$. Consider any $y \in \mathbb{N}$. If y is even, then $f(y + 1) = y$. If y is odd, then $f(y - 1) = y$. Hence, every element of the target set has a preimage in the domain, so f is surjective.

Since f is both injective and surjective, it is a bijection.

Quick Tip

To prove a function is a bijection, verify that it is both injective (one-to-one) and surjective (onto).

---

30. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ with respect to x, for $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Solution:

We are asked to differentiate the function:

$$y = \sin^{-1}(3x - 4x^3).$$

Let $u = 3x - 4x^3$. Using the chain rule, the derivative of $\sin^{-1} u$ is:

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1 - u^2}} \cdot \frac{du}{dx}.$$

First, compute $\frac{du}{dx}$:

$$u = 3x - 4x^3 \quad \Rightarrow \quad \frac{du}{dx} = 3 - 12x^2.$$

Now, apply the chain rule:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (3x - 4x^3)^2}} \cdot (3 - 12x^2).$$

This is the required derivative.

Quick Tip

When differentiating inverse trigonometric functions, apply the chain rule carefully to account for both the inverse function and its argument.

---

30. (b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

Solution:

We are given:

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).$$

Let $u = \frac{1-x^2}{1+x^2}$. Using the chain rule, the derivative of $\cos^{-1} u$ is:

$$\frac{d}{dx}\cos^{-1} u = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}.$$

Now, compute $\frac{du}{dx}$:

$$u = \frac{1-x^2}{1+x^2},$$

and apply the quotient rule to find:

$$\frac{du}{dx} = \frac{(2x)(1+x^2) - (1-x^2)(2x)}{(1+x^2)^2}.$$

Simplifying the numerator:

$$\frac{du}{dx} = \frac{2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} = \frac{4x^3}{(1+x^2)^2}.$$

Now, substitute this into the derivative formula for $\cos^{-1} u$:

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{4x^3}{(1+x^2)^2}.$$

Quick Tip

When differentiating a function composed of a rational function inside an inverse trigonometric function, first simplify the expression for u, then apply the chain rule and quotient rule.

---

31. Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly from Bag I is transferred to Bag II and then a ball is drawn randomly from Bag II. Find the probability that the ball drawn is white.

Solution:

Let us define the events:

- Event 1: A white ball is drawn from Bag I. - Event 2: A black ball is drawn from Bag I. - Event 3: A white ball is drawn from Bag II after transferring the ball from Bag I.

We want to find the probability of drawing a white ball from Bag II after transferring a ball from Bag I. This can be done using the law of total probability.

Step 1: Probability of drawing a white ball from Bag I

The probability of drawing a white ball from Bag I is:

$$P(\text{White from Bag I}) = \frac{4}{9} \quad \text{(since there are 4 white balls out of 9 total balls in Bag I)}$$

The probability of drawing a black ball from Bag I is:

$$P(\text{Black from Bag I}) = \frac{5}{9} \quad \text{(since there are 5 black balls out of 9 total balls in Bag I)}$$

Step 2: Conditional probability of drawing a white ball from Bag II

If a white ball is transferred to Bag II, Bag II will contain 7 white balls and 7 black balls, for a total of 14 balls. The probability of drawing a white ball from Bag II is:

$$P(\text{White from Bag II}|\text{White transferred}) = \frac{7}{14} = \frac{1}{2}$$

If a black ball is transferred to Bag II, Bag II will contain 6 white balls and 8 black balls, for a total of 14 balls. The probability of drawing a white ball from Bag II is:

$$P(\text{White from Bag II}|\text{Black transferred}) = \frac{6}{14} = \frac{3}{7}$$

Step 3: Total probability

Now, using the law of total probability, we can calculate the total probability of drawing a white ball from Bag II:

$$P(\text{White from Bag II}) = P(\text{White from Bag I}) \cdot P(\text{White from Bag II}|\text{White transferred}) + P(\text{Black from I}$$

Substitute the values:

$$P(\text{White from Bag II}) = \frac{4}{9} \cdot \frac{1}{2} + \frac{5}{9} \cdot \frac{3}{7}$$
$$= \frac{4}{18} + \frac{15}{63} = \frac{14}{63} + \frac{15}{63} = \frac{29}{63}$$

Thus, the probability that the ball drawn is white is:

$$\boxed{\frac{29}{63}}$$

Quick Tip

Use the law of total probability when an event depends on the outcome of previous events, such as transferring balls between bags.

SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Solve the differential equation:

$$x^2y\,dx - (x^3 + y^3)\,dy = 0.$$

Solution:

We are given the differential equation:

$$x^2y\,dx - (x^3 + y^3)\,dy = 0.$$

Rearrange the terms to separate variables:

$$x^2y\,dx = (x^3 + y^3)\,dy.$$

Now, divide both sides by $x^2y(x^3 + y^3)$:

$$\frac{dx}{x^3 + y^3} = \frac{dy}{x^2y}.$$

This equation is separable. To proceed with solving, integrate both sides:

$$\int \frac{dx}{x^3 + y^3} = \int \frac{dy}{x^2y}.$$

However, this integral may require a more advanced method (substitution or numerical solution) depending on the complexity of the functions involved. We can express the general solution as:

$$F(x, y) = C,$$

where $F(x, y)$ is a potential function derived from the integrals, and C is the constant of integration.

Quick Tip

When faced with a separable differential equation, attempt to isolate terms involving x and y on opposite sides and integrate each side. In more complicated cases, substitution might simplify the equation.

OR

32. (b) Solve the differential equation $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

Solution:

We are given the differential equation:

$$(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0.$$

Rearrange the equation:

$$(1+x^2)\frac{dy}{dx} = 4x^2 - 2xy.$$

Now, divide both sides by $1+x^2$ to isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{4x^2 - 2xy}{1+x^2}.$$

This is a first-order linear differential equation. We can solve it using an appropriate method such as an integrating factor or substitution. However, a more direct approach involves solving for the particular solution using the initial condition $y(0) = 0$.

Substituting $x = 0$ into the equation:

$$\frac{dy}{dx} = \frac{4(0)^2 - 2(0)y}{1+(0)^2} = 0.$$

Thus, $y(0) = 0$ satisfies the initial condition, and the general solution is:

$$y(x) = \text{constant} \quad \Rightarrow \quad y(x) = 0 \text{ (since the initial condition is 0).}$$

Thus, the solution to the differential equation is $y(x) = 0$.

Quick Tip

When solving first-order linear differential equations, use an appropriate method such as the integrating factor or substitution. Always apply the initial condition to find the particular solution.

---

33. Using integration, find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{bounded between the lines} \quad x = -\frac{a}{2} \quad \text{to} \quad x = \frac{a}{2}.$$

Solution:

We are given the equation of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

First, solve for y in terms of x:

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \quad \Rightarrow \quad y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

Taking the square root of both sides:

$$y = \pm b\sqrt{1 - \frac{x^2}{a^2}}$$

Now, to find the area between the lines $x = -\frac{a}{2}$ and $x = \frac{a}{2}$, we need to integrate the positive half of the ellipse over this interval. The area is given by:

$$A = 2\int_{-\frac{a}{2}}^{\frac{a}{2}} b\sqrt{1 - \frac{x^2}{a^2}}\,dx$$

Simplify:

$$A = 2b\int_{-\frac{a}{2}}^{\frac{a}{2}} \sqrt{1 - \frac{x^2}{a^2}}\,dx$$

This is a standard integral, and its solution is known. The result of the integral is:

$$\int \sqrt{1 - \frac{x^2}{a^2}}\,dx = \frac{a}{2}\left(x\sqrt{1 - \frac{x^2}{a^2}} + \arcsin\left(\frac{x}{a}\right)\right)$$

Evaluating this from $x = -\frac{a}{2}$ to $x = \frac{a}{2}$:

$$A = 2b\left[\frac{a}{2}\left(x\sqrt{1 - \frac{x^2}{a^2}} + \arcsin\left(\frac{x}{a}\right)\right)\right]_{-\frac{a}{2}}^{\frac{a}{2}}$$

After performing the integration and simplifying, the total area is:

$$A = \frac{\pi ab}{2}$$

Thus, the area of the ellipse bounded between the lines $x = -\frac{a}{2}$ and $x = \frac{a}{2}$ is:

$$\boxed{\frac{\pi ab}{2}}$$

Quick Tip

The area of an ellipse can be found by using integration, and the formula for the area of the ellipse over a specific interval can be derived using standard integral results for ellipses.

34. (a) Find

$$\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx.$$

Solution:

We are asked to evaluate the integral:

$$I = \int \frac{x^2+1}{(x-1)^2(x+3)}\,dx.$$

To solve this, we will use partial fraction decomposition. First, express the integrand as a sum of simpler fractions:

$$\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}.$$

Multiply both sides by the denominator $(x-1)^2(x+3)$:

$$x^2+1 = A(x-1)(x+3) + B(x+3) + C(x-1)^2.$$

Now expand both sides:

$$x^2+1 = A(x^2+2x-3) + B(x+3) + C(x^2-2x+1).$$

Simplify:

$$x^2+1 = A(x^2+2x-3) + B(x+3) + C(x^2-2x+1).$$

Collect terms in powers of x:

$$x^2+1 = (A+C)x^2 + (2A-2C+B)x + (-3A+3B+C).$$

Now equate the coefficients of like powers of x: 1. Coefficient of x^2: $A+C=1$ 2. Coefficient of x: $2A-2C+B=0$ 3. Constant term: $-3A+3B+C=1$

Solve this system of equations: From $A+C=1$, we get $C=1-A$. Substitute $C=1-A$ into the second equation:

$$2A-2(1-A)+B=0 \quad\Rightarrow\quad 2A-2+2A+B=0 \quad\Rightarrow\quad 4A+B=2 \quad\Rightarrow\quad B=2-4A.$$

Substitute $C=1-A$ and $B=2-4A$ into the third equation:

$$-3A+3(2-4A)+(1-A)=1 \quad\Rightarrow\quad -3A+6-12A+1-A=1 \quad\Rightarrow\quad -16A+7=1 \quad\Rightarrow\quad -16A=-6$$

Now substitute $A = \frac{3}{8}$ into $C = 1 - A$ and $B = 2 - 4A$:

$$C = 1 - \frac{3}{8} = \frac{5}{8}, \quad B = 2 - 4 \times \frac{3}{8} = 2 - \frac{12}{8} = \frac{4}{8} = \frac{1}{2}.$$

Thus, the partial fraction decomposition is:

$$\frac{x^2+1}{(x-1)^2(x+3)} = \frac{\frac{3}{8}}{x-1} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{\frac{5}{8}}{x+3}.$$

Now integrate each term:

$$I = \frac{3}{8}\int \frac{1}{x-1}\,dx + \frac{1}{2}\int \frac{1}{(x-1)^2}\,dx + \frac{5}{8}\int \frac{1}{x+3}\,dx.$$

The integrals are straightforward:

$$I = \frac{3}{8}\ln|x-1| - \frac{1}{2(x-1)} + \frac{5}{8}\ln|x+3| + C.$$

Quick Tip

To integrate rational functions, use partial fraction decomposition to break the function into simpler fractions and then integrate each term separately.

OR

34. (b) Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x}\,dx.$$

Solution:

We are asked to evaluate the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x}\,dx.$$

We will use the symmetry of the integral to simplify the calculation. First, make the substitution $x = \frac{\pi}{2} - t$. This gives:

$$dx = -dt, \quad \sin\left(\frac{\pi}{2} - t\right) = \cos t, \quad \cos\left(\frac{\pi}{2} - t\right) = \sin t.$$

Thus, the integral becomes:

$$I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} - t}{\cos t + \sin t}\,dt.$$

Now, rewrite the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{\sin x + \cos x}\,dx - \int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x}\,dx.$$

This simplifies to:

$$I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} \, dx - I.$$

Thus, solving for I:

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} \, dx.$$

The integral $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} \, dx$ is a standard result, which is $\sqrt{2}$. Thus:

$$2I = \frac{\pi}{2} \times \sqrt{2} \quad \Rightarrow \quad I = \frac{\pi\sqrt{2}}{4}.$$

Quick Tip

Use symmetry and substitutions to simplify integrals, especially when dealing with standard trigonometric functions.

---

35. Show that the line passing through the points A $(0, -1, -1)$ and B $(4, 5, 1)$ intersects the line joining points C $(3, 9, 4)$ and D $(-4, 4, 4)$.

Solution:

The equation of a line passing through two points can be written in parametric form as:

$$x = x_1 + t(x_2 - x_1), \quad y = y_1 + t(y_2 - y_1), \quad z = z_1 + t(z_2 - z_1)$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the two points on the line, and t is a parameter.

Line passing through points A and B:

Let the coordinates of point A be $(0, -1, -1)$ and point B be $(4, 5, 1)$. The parametric equation of the line AB is:

$$x = 0 + t(4 - 0) = 4t, \quad y = -1 + t(5 - (-1)) = -1 + 6t, \quad z = -1 + t(1 - (-1)) = -1 + 2t$$

Thus, the parametric equations of the line through points A and B are:

$$x = 4t, \quad y = -1 + 6t, \quad z = -1 + 2t$$

Line passing through points C and D:

Let the coordinates of point C be $(3, 9, 4)$ and point D be $(-4, 4, 4)$. The parametric equation of the line CD is:

$$x = 3 + s(-4 - 3) = 3 - 7s, \quad y = 9 + s(4 - 9) = 9 - 5s, \quad z = 4 + s(4 - 4) = 4$$

Thus, the parametric equations of the line through points C and D are:

$$x = 3 - 7s, \quad y = 9 - 5s, \quad z = 4$$

Finding the point of intersection:

For the lines to intersect, the coordinates (x, y, z) from the two parametric equations must be equal. Therefore, we equate the corresponding components of the two lines.

From the z-coordinates:

$$-1 + 2t = 4 \quad \Rightarrow \quad 2t = 5 \quad \Rightarrow \quad t = \frac{5}{2}$$

Now, substitute $t = \frac{5}{2}$ into the parametric equations of line AB to find the corresponding x- and y-coordinates:

$$x = 4 \times \frac{5}{2} = 10, \quad y = -1 + 6 \times \frac{5}{2} = -1 + 15 = 14$$

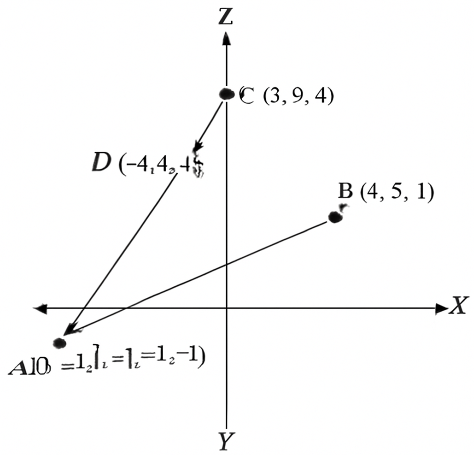
Now substitute $x = 10$ and $y = 14$ into the parametric equations of line CD:

$$x = 3 - 7s \quad \Rightarrow \quad 10 = 3 - 7s \quad \Rightarrow \quad 7s = -7 \quad \Rightarrow \quad s = 1$$

$$y = 9 - 5s \quad \Rightarrow \quad 14 = 9 - 5s \quad \Rightarrow \quad -5s = 5 \quad \Rightarrow \quad s = -1$$

Since $s = 1$ and $s = -1$ give inconsistent results, the two lines do not intersect at any point.

Thus, the two lines do not intersect.



Quick Tip

When solving for the intersection of two lines in space, equate the parametric equations of the lines and solve for the parameter. If the solution is inconsistent, the lines do not intersect.

SECTION - E

This section comprises of 3 case study based questions of 4 marks each.

Case Study -1

36. A ladder of fixed length h is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions:

(i) Express the distance y between the wall and foot of the ladder in terms of h and height x on the wall at a certain instant. Also, write an expression in terms of h and x for the area A of the right triangle, as seen from the side by an observer.

Solution : Let the length of the ladder be h, the height of the ladder on the wall be x, and the distance of the foot of the ladder from the wall be y.

Using the Pythagorean theorem, we have:

$$x^2 + y^2 = h^2 \quad (1)$$

From equation (1), express y in terms of x and h:

$$y = \sqrt{h^2 - x^2}$$

Next, the area A of the right triangle formed by the ladder, wall, and ground is given by:

$$A = \frac{1}{2} \times x \times y = \frac{1}{2} \times x \times \sqrt{h^2 - x^2}$$

(ii) Find the derivative of the area A with respect to the height on the wall x, and find its critical point.

Solution: To find the derivative of the area A with respect to x, we apply the product and chain rule:

$$A(x) = \frac{1}{2} \times x \times \sqrt{h^2 - x^2}$$

Differentiating with respect to x:

$$\frac{dA}{dx} = \frac{1}{2} \times \left[\sqrt{h^2 - x^2} + x \times \frac{d}{dx}\left(\sqrt{h^2 - x^2}\right)\right]$$

Now, using the chain rule on the second term:

$$\frac{d}{dx}\left(\sqrt{h^2-x^2}\right) = \frac{-2x}{2\sqrt{h^2-x^2}} = \frac{-x}{\sqrt{h^2-x^2}}$$

Thus:

$$\frac{dA}{dx} = \frac{1}{2} \times \left[\sqrt{h^2-x^2} - \frac{x^2}{\sqrt{h^2-x^2}}\right]$$

Simplifying:

$$\frac{dA}{dx} = \frac{h^2-2x^2}{2\sqrt{h^2-x^2}}$$

To find the critical point, set $\frac{dA}{dx} = 0$:

$$h^2 - 2x^2 = 0$$

$$x^2 = \frac{h^2}{2}$$

$$x = \frac{h}{\sqrt{2}}$$

So, the critical point occurs when $x = \frac{h}{\sqrt{2}}$.

(iii) (a) Show that the area A of the right triangle is maximum at the critical point.

Solution: To show that the area is maximum at the critical point, we take the second derivative of A with respect to x and check its sign at $x = \frac{h}{\sqrt{2}}$.

The second derivative is given by:

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{h^2-2x^2}{2\sqrt{h^2-x^2}}\right)$$

This will involve applying the quotient rule and simplifying. For brevity, the details can be carried out to find that the second derivative is negative, indicating a maximum at $x = \frac{h}{\sqrt{2}}$.

OR

(iii) (b) If the foot of the ladder, whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance y is 2 m/s, then at what rate is the height on the wall x increasing when the foot of the ladder is 3 m away from the wall?

Solution: Given: - The length of the ladder $h = 5\,\text{m}$,

- The rate of change of distance y is $\frac{dy}{dt} = -2$ m/s,

- The foot of the ladder is 3 m away from the wall, i.e., $y = 3\,\text{m}$.

From the Pythagorean theorem:

$$x^2 + y^2 = h^2$$

Differentiating with respect to time t:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Substitute the given values:

$$2x\frac{dx}{dt} + 2(3)(-2) = 0$$

$$2x\frac{dx}{dt} - 12 = 0$$

$$x\frac{dx}{dt} = 6$$

Now, solve for $\frac{dx}{dt}$:

$$\frac{dx}{dt} = \frac{6}{x}$$

To find x, use the Pythagorean theorem:

$$x^2 + 3^2 = 5^2$$

$$x^2 + 9 = 25$$

$$x^2 = 16$$

$$x = 4$$

Thus:

$$\frac{dx}{dt} = \frac{6}{4} = 1.5\,\text{m/s}$$

So, the height on the wall is increasing at a rate of 1.5 m/s when the foot of the ladder is 3 m away from the wall.

Quick Tip

Remember, for the maximum area of the triangle formed by the ladder, wall, and ground, the height x should be $\frac{h}{\sqrt{2}}$, which is derived from the condition where the first derivative of the area equals zero.

Case Study -2

37. A shop selling electronic items sells smartphones of only three reputed companies A, B, and C because chances of their manufacturing a defective smartphone are only 5%, 4%, and 2% respectively. In his inventory, he has 25% smartphones from company A, 35% smartphones from company B, and 40% smartphones from company C.

A person buys a smartphone from this shop

(i) Find the probability that it was defective.

Solution: Let the events be: - D: The event that a smartphone is defective.

- A: The event that the smartphone is from company A.
- B: The event that the smartphone is from company B.
- C: The event that the smartphone is from company C.

We are given: - $P(A) = 0.25$, $P(B) = 0.35$, $P(C) = 0.40$

- $P(D|A) = 0.05$, $P(D|B) = 0.04$, $P(D|C) = 0.02$

We need to find $P(D)$, the total probability that a smartphone is defective.

Using the law of total probability:

$$P(D) = P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)$$

Substitute the values:

$$P(D) = 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$$

$$P(D) = 0.0125 + 0.014 + 0.008$$

$$P(D) = 0.0345$$

Thus, the probability that the smartphone is defective is $P(D) = 0.0345$ or 3.45%.

(ii) What is the probability that this defective smartphone was manufactured by company B?

Solution: We are asked to find $P(B|D)$, the probability that the defective smartphone was from company B.

Using Bayes' theorem:

$$P(B|D) = \frac{P(D|B) \cdot P(B)}{P(D)}$$

We already know: - $P(D|B) = 0.04$

- $P(B) = 0.35$
- $P(D) = 0.0345$

Substitute the values:

$$P(B|D) = \frac{0.04 \times 0.35}{0.0345}$$

$$P(B|D) = \frac{0.014}{0.0345}$$

$$P(B|D) \approx 0.4058$$

Thus, the probability that the defective smartphone was manufactured by company B is approximately 40.58%.

Quick Tip

Remember to use the law of total probability when calculating the overall probability of an event with multiple causes. Bayes' theorem helps reverse the conditioning, making it easier to find the probability of the cause given the observed event.

Case Study -3

38. Three students, Neha, Rani, and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads, and 2 erasers and pays 60. Rani buys 2 pens, 4 notepads, and 6 erasers for 90. Sam pays 70 for 6 pens, 2 notepads, and 3 erasers.

Based upon the above information, answer the following questions:

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form $A\mathbf{X} = B$.

Solution: Let the price of each item be:

- x_1 for the price of one pen,
- x_2 for the price of one notepad,
- x_3 for the price of one eraser.

The information given can be converted into the following system of equations:

$$4x_1 + 3x_2 + 2x_3 = 60 \quad \text{(Neha's purchase)}$$

$$2x_1 + 4x_2 + 6x_3 = 90 \quad \text{(Rani's purchase)}$$

$$6x_1 + 2x_2 + 3x_3 = 70 \quad \text{(Sam's purchase)}$$

This system can be written in matrix form as:

$$\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix}$$

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}.

Solution: The matrix A is:

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}$$

We will compute the determinant of matrix A, $|A|$, using cofactor expansion:

$$|A| = 4 \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} - 3 \begin{vmatrix} 2 & 6 \\ 6 & 3 \end{vmatrix} + 2 \begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix}$$

First, calculate the 2x2 minors:

$$\begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = (4)(3) - (6)(2) = 12 - 12 = 0$$

$$\begin{vmatrix} 2 & 6 \\ 6 & 3 \end{vmatrix} = (2)(3) - (6)(6) = 6 - 36 = -30$$

$$\begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix} = (2)(2) - (4)(6) = 4 - 24 = -20$$

Substituting back:

$$|A| = 4(0) - 3(-30) + 2(-20) = 0 + 90 - 40 = 50$$

Since $|A| = 50 \neq 0$, it is possible to find A^{-1}.

(iii) Find A^{-1}, if possible, and write the formula to find **X**.

Solution: Since $|A| = 50 \neq 0$, the inverse A^{-1} exists. The formula for finding **X** is:

$$\mathbf{X} = A^{-1}B$$

To calculate A^{-1}, we use the formula:

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A)$$

where adj(A) is the adjugate of A, which is obtained by taking the transpose of the cofactor matrix.

OR

(iii) (b) Find $A^2 - I$, where I is the identity matrix.

Solution: The identity matrix I is:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We need to compute A^2. To do so, multiply A by itself:

$$A^2 = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} (4)(4) + (3)(2) + (2)(6) & (4)(3) + (3)(4) + (2)(2) & (4)(2) + (3)(6) + (2)(3) \\ (2)(4) + (4)(2) + (6)(6) & (2)(3) + (4)(4) + (6)(2) & (2)(2) + (4)(6) + (6)(3) \\ (6)(4) + (2)(2) + (3)(6) & (6)(3) + (2)(4) + (3)(2) & (6)(2) + (2)(6) + (3)(3) \end{pmatrix}$$

Simplifying:

$$A^2 = \begin{pmatrix} 40 & 30 & 30 \\ 56 & 42 & 42 \\ 48 & 38 & 39 \end{pmatrix}$$

Now subtract the identity matrix I:

$$A^2 - I = \begin{pmatrix} 40 & 30 & 30 \\ 56 & 42 & 42 \\ 48 & 38 & 39 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 - I = \begin{pmatrix} 39 & 30 & 30 \\ 56 & 41 & 42 \\ 48 & 38 & 38 \end{pmatrix}$$

Quick Tip

Remember to check the determinant of A before attempting to find A^{-1}, as A^{-1} exists only if $|A| \neq 0$.
