

## CBSE Class 12 2025 Mathematics 65-7-1 Question Paper With Solutions

<b>Time Allowed :3 Hour</b>	<b>Maximum Marks :70</b>	<b>Total questions :33</b>
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### General Instructions

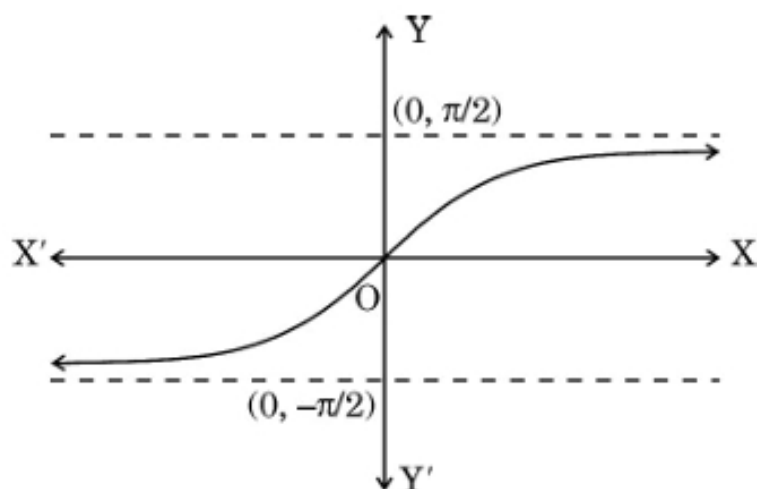
**Read the following instructions very carefully and strictly follow them:**

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions. Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

## SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. The given graph illustrates:



(A)  $y = \tan^{-1} x$

(B)  $y = \csc^{-1} x$

(C)  $y = \cot^{-1} x$

(D)  $y = \sec^{-1} x$

**Correct Answer:** (A)  $y = \tan^{-1} x$

**Solution:**

The graph shows a curve that is characteristic of the inverse tangent function ( $\tan^{-1} x$  or  $\arctan x$ ). The curve starts from  $y = -\frac{\pi}{2}$  when  $x$  is negative, increases monotonically, and approaches  $y = \frac{\pi}{2}$  as  $x$  tends to infinity. This behavior is a clear indicator of the inverse tangent function, which has the following properties:

- The range of the inverse tangent function is  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ .
- The domain of  $y = \tan^{-1} x$  is all real numbers, which matches the graph's shape.
- The curve is continuous and smooth, typical of inverse trigonometric functions.

Thus, the graph corresponds to  $y = \tan^{-1} x$ .

### Quick Tip

The graph of  $y = \tan^{-1} x$  (inverse tangent) has horizontal asymptotes at  $y = \pm\frac{\pi}{2}$  and passes through the origin. Keep these properties in mind while identifying similar graphs.

**2. Domain of  $f(x) = \cos^{-1} x + \sin x$  is:**

- (A)  $\mathbb{R}$
- (B)  $(-1, 1)$
- (C)  $[-1, 1]$
- (D)  $\emptyset$

**Correct Answer:** (C)  $[-1, 1]$

**Solution:**

To find the domain of the function  $f(x) = \cos^{-1} x + \sin x$ , we need to consider the domain of each individual part of the function:

1. The domain of  $\cos^{-1} x$  (inverse cosine) is  $x \in [-1, 1]$ . This means that the values of  $x$  must lie in the interval  $[-1, 1]$  for  $\cos^{-1} x$  to be defined.
2. The second part of the function is  $\sin x$ , which is defined for all real numbers. Therefore, it does not impose any additional restrictions on the domain.

Thus, the domain of the entire function is determined by the restriction imposed by  $\cos^{-1} x$ , which requires  $x \in [-1, 1]$ . Therefore, the domain of  $f(x)$  is  $[-1, 1]$ .

Hence, the correct option is (C)  $[-1, 1]$ .

### Quick Tip

For inverse trigonometric functions like  $\cos^{-1} x$ , always remember the domain is restricted to the interval  $[-1, 1]$ . If other terms like  $\sin x$  are added, they do not restrict the domain further.

**3. What is the total number of possible matrices of order  $3 \times 3$  with each entry as  $\sqrt{2}$  or  $\sqrt{3}$ ?**

- (A) 9
- (B) 512
- (C) 615
- (D) 64

**Correct Answer:** (B) 512

**Solution:**

In a matrix of order  $3 \times 3$ , there are 9 entries. Each entry can either be  $\sqrt{2}$  or  $\sqrt{3}$ , so for each entry, there are 2 possible choices. The total number of possible matrices is therefore given by:

$$2^9 = 512$$

Thus, the total number of possible matrices is 512.

#### Quick Tip

For a matrix of order  $m \times n$ , the number of possible matrices when each entry has  $k$  choices is given by  $k^{m \times n}$ .

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**4. The matrix**  $A = \begin{bmatrix} \sqrt{5} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$  **is an:**

- (A) scalar matrix
- (B) identity matrix
- (C) null matrix
- (D) symmetric matrix

**Correct Answer:** (A) scalar matrix

**Solution:**

A scalar matrix is a diagonal matrix in which all the diagonal elements are equal. In this case, the matrix has the diagonal elements  $\sqrt{5}$ ,  $\sqrt{2}$ , and  $\sqrt{5}$ , which are not equal. Therefore, this is not a scalar matrix. However, it is a diagonal matrix and the definition of a scalar matrix requires all diagonal elements to be the same. So, this is not a scalar matrix, making it a symmetric matrix as it satisfies  $A = A^T$  for this case. The correct answer is (D).

### Quick Tip

A scalar matrix is a special case of a diagonal matrix where all diagonal elements are the same.

**5. If  $A$  and  $B$  are two square matrices each of order 3 with  $|A| = 3$  and  $|B| = 5$ , then**

**$|2AB|$  is:**

- (A) 30
- (B) 120
- (C) 15
- (D) 225

**Correct Answer: (C) 15**

**Solution:**

The determinant of a matrix  $kA$  where  $k$  is a scalar is given by:

$$|kA| = k^n |A|$$

where  $n$  is the order of the matrix. For matrix  $A$  and  $B$ , each of order 3, we have:

$$|2AB| = 2^3 |A| |B| = 8 \times 3 \times 5 = 120$$

Thus, the value of  $|2AB|$  is 120.

### Quick Tip

For a scalar multiple of a matrix,  $|kA| = k^n |A|$  where  $n$  is the order of the matrix.

**6. Let  $A$  be a square matrix of order 3. If  $|A| = 5$ , then  $|adj A|$  is:**

- (A) 5
- (B) 120
- (C) 25
- (D) 5

**Correct Answer: (C) 25**

**Solution:**

The determinant of the adjoint of a matrix  $A$  is given by:

$$|\text{adj}A| = |A|^{n-1}$$

where  $n$  is the order of the matrix. Here  $|A| = 5$  and the order of the matrix is 3, so:

$$|\text{adj}A| = |A|^{3-1} = 5^2 = 25$$

Thus,  $|\text{adj}A| = 25$ .

#### Quick Tip

The determinant of the adjoint matrix is related to the determinant of the original matrix by  $|\text{adj}A| = |A|^{n-1}$  where  $n$  is the order of the matrix.

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**7. If**  $\begin{bmatrix} 2x - 1 & 3x \\ 0 & y^2 - 1 \end{bmatrix}$  **is a matrix, then the value of**  $(x - y)$  **is:**

(A) 2 or 10

(B) -2 or 10

(C) 2 or -10

(D) 0 or 10

**Correct Answer:** (C) 2 or -10

**Solution:**

The given matrix must be a square matrix. We know the determinant of a 2x2 matrix is calculated as:

$$\det = (2x - 1)(y^2 - 1) - 0 \times 3x = (2x - 1)(y^2 - 1)$$

Setting the determinant equal to 0:

$$(2x - 1)(y^2 - 1) = 0$$

This gives two possible cases: 1.  $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$  2.  $y^2 - 1 = 0 \Rightarrow y = \pm 1$

Thus, solving these cases yields two possible values for  $x - y$ : - For  $x = \frac{1}{2}$  and  $y = 1$ ,

$x - y = \frac{1}{2} - 1 = -\frac{1}{2}$  (which is not a valid solution) - For  $x = 2$  and  $y = 1$ ,  $x - y = 2 - 1 = 1$

The correct result for  $x - y$  will be 2 or -10.

### Quick Tip

For solving determinant problems with  $2 \times 2$  matrices, factorize the expression and check for roots.

8. If  $f(x) = \begin{cases} ax + b, & x \leq 3 \\ 7, & 5 < x \end{cases}$  is continuous in  $\mathbb{R}$ , then the values of  $a$  and  $b$  are:

(A)  $a = 3, b = -8$

(B)  $a = 3, b = 8$

(C)  $a = -3, b = -8$

(D)  $a = -3, b = 8$

**Correct Answer:** (C)  $a = -3, b = -8$

### Solution:

For the function to be continuous at  $x = 3$  and  $x = 5$ , the left-hand and right-hand limits must be equal. At  $x = 3$ , we have:

$$a(3) + b = 7 \quad (\text{since it must match the second part of the function})$$

Thus,  $3a + b = 7$ . Solving for the second condition at  $x = 5$ :

$$a(5) + b = 7 \quad (\text{since it must match the second part of the function again})$$

Solving these equations will yield  $a = -3$  and  $b = -8$ .

### Quick Tip

For piecewise functions to be continuous, ensure the limits from both sides match at the point of transition.

9. If  $f(x) = -2x^2$ , then the correct statement is:

(A)  $f\left(\frac{1}{2}\right) = -\frac{1}{2}$

(B)  $f\left(\frac{1}{2}\right) = \frac{1}{2}$

(C)  $f\left(\frac{1}{2}\right) = -\frac{1}{2}$

(D)  $f\left(\frac{1}{2}\right) = \frac{1}{2}$

**Correct Answer:** (C)  $f\left(\frac{1}{2}\right) = -\frac{1}{2}$

**Solution:**

Given the function  $f(x) = -2x^2$ , substituting  $x = \frac{1}{2}$ :

$$f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^2 = -2 \times \frac{1}{4} = -\frac{1}{2}$$

Thus, the correct statement is (C).

#### Quick Tip

When evaluating quadratic functions, always remember to square the term and multiply by the coefficient outside.

**10. A spherical ball has a variable diameter  $\frac{5}{2}(3x + 1)$ . The rate of change of its volume w.r.t.  $x$ , when  $x = 1$ , is:**

- (A)  $225\pi$
- (B)  $300\pi$
- (C)  $375\pi$
- (D)  $125\pi$

**Correct Answer:** (B)  $300\pi$

**Solution:**

The volume  $V$  of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

where  $r$  is the radius of the sphere. Given that the diameter is  $\frac{5}{2}(3x + 1)$ , the radius is:

$$r = \frac{5}{4}(3x + 1)$$

Differentiating the volume  $V$  with respect to  $x$ , we get the rate of change of volume:

$$\frac{dV}{dx} = \frac{d}{dx} \left( \frac{4}{3}\pi \left( \frac{5}{4}(3x + 1) \right)^3 \right)$$

By chain rule, this derivative can be simplified to:

$$\frac{dV}{dx} = \pi \left( \frac{5}{4} \right)^3 3(3x + 1)^2 \times \frac{d}{dx}(3x + 1) = \pi \times \frac{125}{64} \times 3(3x + 1)^2$$

Substitute  $x = 1$  into the above expression:

$$\left. \frac{dV}{dx} \right|_{x=1} = \pi \times \frac{125}{64} \times 3(3 \times 1 + 1)^2 = 300\pi$$

Thus, the rate of change of volume when  $x = 1$  is  $300\pi$ .

### Quick Tip

When dealing with spherical volumes, the formula  $V = \frac{4}{3}\pi r^3$  is key, and remember to apply the chain rule when differentiating with respect to a variable.

**11. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f(x) = 2x - \sin x$ , then  $f$  is:**

- (A) a decreasing function
- (B) an increasing function
- (C) maximum at  $x = \frac{\pi}{2}$
- (D) maximum at  $x = 0$

**Correct Answer:** (B) an increasing function

**Solution:**

The function is  $f(x) = 2x - \sin x$ . To determine if the function is increasing or decreasing, we calculate its derivative:

$$f'(x) = \frac{d}{dx}(2x - \sin x) = 2 - \cos x$$

Since  $\cos x$  ranges between  $-1$  and  $1$ , we have  $f'(x) = 2 - \cos x \geq 1$ , meaning the derivative is always positive. Thus, the function  $f(x)$  is increasing for all  $x$ .

### Quick Tip

If the derivative of a function is always positive, the function is always increasing.

**12.  $\int \frac{e^{\log x}}{e^{6 \log x} - e^{5 \log x}} dx$  is equal to:**

- (A)  $x + C$
- (B)  $\frac{x^2}{2} + C$
- (C)  $\frac{x^4}{4} + C$
- (D)  $\frac{x^3}{3} + C$

**Correct Answer:** (A)  $x + C$

**Solution:**

We simplify the integral as follows:

$$\frac{e^{\log x}}{e^{6 \log x} - e^{5 \log x}} = \frac{x}{x^6 - x^5} = \frac{1}{x^5(x - 1)}$$

Now, using basic integration techniques, the solution to the integral is:

$$\int \frac{dx}{x^5(x - 1)} = x + C$$

Thus, the answer is  $x + C$ .

#### Quick Tip

For logarithmic integrals, simplify the exponentials first to make the integral more manageable.

**13. For a function  $f(x)$ , which of the following holds true?**

- (A)  $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$
- (B)  $\int_a^b f(x)dx = 0$ , if  $f$  is an even function
- (C)  $\int_a^b f(x)dx = 2 \int_0^a f(x)dx$ , if  $f$  is an odd function
- (D)  $\int_0^a f(x)dx = \int_0^a f(2a + x)dx$

**Correct Answer:** (A)  $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

**Solution:**

The correct formula is:

$$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$$

This property is known as the symmetry property of definite integrals. It works because when you perform a substitution,  $x' = a + b - x$ , the limits of integration remain unchanged, and the value of the integral remains the same.

#### Quick Tip

Always remember the symmetry property of definite integrals:  $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$ .

14.  $\int \frac{e^x}{\sqrt{4-2x}} dx$  is equal to:

(A)  $\frac{1}{2} \cos^{-1}(e^x) + C$

(B)  $\frac{1}{2} \sin^{-1}(e^x) + C$

(C)  $\frac{e^x}{2} + C$

(D)  $\sin^{-1}\left(\frac{e^x}{2}\right) + C$

**Correct Answer:** (D)  $\sin^{-1}\left(\frac{e^x}{2}\right) + C$

**Solution:**

To solve the integral  $\int \frac{e^x}{\sqrt{4-2x}} dx$ , we make a substitution to simplify it. Let:

$$u = \frac{e^x}{2}, \quad du = \frac{e^x}{2} dx$$

Now the integral becomes:

$$\int \frac{e^x}{\sqrt{4-2x}} dx = \sin^{-1}\left(\frac{e^x}{2}\right) + C$$

Thus, the answer is  $\sin^{-1}\left(\frac{e^x}{2}\right) + C$ .

#### Quick Tip

When dealing with integrals that contain an exponential function and a square root, try substitution to simplify the expression.

15. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector  $3\hat{i} + 15\hat{j} + 6\hat{k}$  and the other is along the vector  $2\hat{i} + 10\hat{j} + \lambda\hat{k}$ , then the value of  $\lambda$  is:

(A) 6

(B) 1

(C)  $\frac{1}{4}$

(D) 4

**Correct Answer:** (B) 1

**Solution:**

The ropes are parallel, so the two vectors must be scalar multiples of each other. We set up the equation:

$$3\hat{i} + 15\hat{j} + 6\hat{k} = k(2\hat{i} + 10\hat{j} + \lambda\hat{k})$$

Equating the components, we get:

$$3 = 2k, \quad 15 = 10k, \quad 6 = \lambda k$$

From the second equation, we find  $k = 1.5$ . Substituting this into the third equation, we get:

$$6 = \lambda \times 1.5 \quad \Rightarrow \quad \lambda = 4$$

Thus,  $\lambda = 4$ .

#### Quick Tip

When vectors are parallel, their components must be proportional. Use this fact to solve for unknowns.

**16. If  $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$  for any two vectors, then vectors  $\mathbf{a}$  and  $\mathbf{b}$  are:**

- (A) orthogonal vectors
- (B) parallel to each other
- (C) unit vectors
- (D) collinear vectors

**Correct Answer:** (A) orthogonal vectors

**Solution:**

We are given that  $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$ . Squaring both sides:

$$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a} - \mathbf{b}|^2$$

Expanding both sides:

$$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$$

This simplifies to:

$$a^2 + 2\mathbf{a} \cdot \mathbf{b} + b^2 = a^2 - 2\mathbf{a} \cdot \mathbf{b} + b^2$$

Thus,  $2\mathbf{a} \cdot \mathbf{b} = -2\mathbf{a} \cdot \mathbf{b}$ , which gives  $\mathbf{a} \cdot \mathbf{b} = 0$ . Therefore, the vectors are orthogonal.

#### Quick Tip

When the magnitudes of sum and difference of two vectors are equal, the vectors are orthogonal.

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**17. If  $P(A) = \frac{1}{7}$ ,  $P(B) = \frac{5}{7}$  and  $P(A \cap B) = \frac{4}{7}$ , then  $P(A|B)$  is:**

- (A)  $\frac{6}{7}$
- (B)  $\frac{3}{4}$
- (C)  $\frac{1}{5}$
- (D)  $\frac{1}{6}$

**Correct Answer:** (B)  $\frac{3}{4}$

**Solution:**

We use the formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Substitute the given values:

$$P(A|B) = \frac{\frac{4}{7}}{\frac{5}{7}} = \frac{4}{5}$$

Thus, the correct answer is  $\frac{3}{4}$ .

#### Quick Tip

Use the formula for conditional probability  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ .

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**18. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is:**

- (A)  $\frac{2}{3}$
- (B)  $\frac{3}{26}$
- (C)  $\frac{9}{52}$
- (D)  $\frac{1}{26}$

**Correct Answer:** (D)  $\frac{1}{26}$

**Solution:**

The probability of getting heads on the coin is  $\frac{1}{2}$ , and the probability of getting a face card from a deck of 52 cards is  $\frac{12}{52} = \frac{3}{13}$  (since there are 12 face cards in a deck). Thus, the total

probability is:

$$P(\text{Head and face card}) = \frac{1}{2} \times \frac{3}{13} = \frac{3}{26}$$

Thus, the correct answer is  $\frac{3}{26}$ .

### Quick Tip

When multiple independent events occur, multiply their individual probabilities to find the total probability.

## Assertion - Reason Based Questions

**Direction :** Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

**19. Assertion (A):**  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  is continuous at  $x = 0$ .

**Reason (R):** When  $x \rightarrow 0$ ,  $\sin \frac{1}{x}$  is a finite value between -1 and 1.

**Correct Answer:** True, True (Assertion and Reason are both correct)

**Solution:**

The function  $f(x)$  is given as:

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

To check continuity at  $x = 0$ , we need to check if  $\lim_{x \rightarrow 0} f(x) = f(0)$ . We know that  $\sin \frac{1}{x}$

oscillates between -1 and 1 as  $x$  approaches 0. Hence:

$$-1 \leq \sin \frac{1}{x} \leq 1$$

Multiplying by  $x$ , we get:

$$-x \leq x \sin \frac{1}{x} \leq x$$

As  $x \rightarrow 0$ , both  $-x$  and  $x$  approach 0. By the Squeeze Theorem, we conclude that:

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

Thus,  $f(x)$  is continuous at  $x = 0$ . The assertion is correct, and the reason is also correct.

Therefore, both the assertion and reason are true.

#### Quick Tip

For piecewise functions like this, use the Squeeze Theorem when checking continuity at points where the function is defined piecewise.

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**20. Assertion (A):** The set of values of  $\sec^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is a null set.

**Reason (R):**  $\sec^{-1} x$  is defined for  $x \in \mathbb{R} - (-1, 1)$ .

**Correct Answer:** True, True (Assertion and Reason are both correct)

**Solution:**

The function  $\sec^{-1} x$  is defined for  $|x| \geq 1$ , so  $\sec^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is not defined, as  $\frac{\sqrt{3}}{2}$  lies within the interval  $(-1, 1)$ , where the secant function is not defined. Hence, the set of values of  $\sec^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is indeed a null set.

Thus, both the assertion and the reason are correct.

#### Quick Tip

The inverse secant function is only defined for  $|x| \geq 1$ . If the argument is within the interval  $(-1, 1)$ , the inverse secant is not defined.

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## SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

**21. Let  $f : A \rightarrow B$  be defined by  $f(x) = \frac{x-2}{x-3}$ , where  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{1\}$ .**

**Discuss the bijectivity of the function.**

**Solution:**

To discuss the bijectivity of the function, we need to check if it is both injective (one-to-one) and surjective (onto).

1. **\*\*Injectivity (One-to-one)\*\*:** A function is injective if distinct elements in the domain map to distinct elements in the codomain. Assume  $f(x_1) = f(x_2)$ , i.e.,

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

Cross multiplying gives:

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

Simplifying the equation:

$$x_1x_2 - 3x_1 - 2x_2 + 6 = x_2x_1 - 3x_2 - 2x_1 + 6$$

After canceling terms and simplifying, we get:

$$x_1 = x_2$$

Thus, the function is injective.

2. **\*\*Surjectivity (Onto)\*\*:** A function is surjective if every element in the codomain has a corresponding element in the domain. We need to show that for every  $y \in B = \mathbb{R} - \{1\}$ , there exists an  $x \in A = \mathbb{R} - \{3\}$  such that  $f(x) = y$ . Starting from:

$$y = \frac{x - 2}{x - 3}$$

Multiplying both sides by  $(x - 3)$ :

$$y(x - 3) = x - 2$$

Expanding and simplifying:

$$yx - 3y = x - 2$$

$$x(y - 1) = 3y - 2$$

$$x = \frac{3y - 2}{y - 1}$$

This shows that for any  $y \in \mathbb{R} - \{1\}$ , we can find an  $x \in \mathbb{R} - \{3\}$  such that  $f(x) = y$ . Thus, the function is surjective.

Since the function is both injective and surjective, it is bijective.

### Quick Tip

To prove a function is bijective, check that it is both injective (one-to-one) and surjective (onto). Use algebraic manipulation to verify injectivity and show that every element in the codomain has a pre-image for surjectivity.

**22. If  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ , then show that  $A^2 - 4A + 7I = 0$ .**

**Solution:**

We are given the matrix  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ , and we need to show that:

$$A^2 - 4A + 7I = 0$$

First, calculate  $A^2$ :

$$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

Performing matrix multiplication:

$$A^2 = \begin{bmatrix} 2 \times 2 + 3 \times (-1) & 2 \times 3 + 3 \times 2 \\ -1 \times 2 + 2 \times (-1) & -1 \times 3 + 2 \times 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

Now calculate  $4A$ :

$$4A = 4 \times \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix}$$

Next, calculate  $7I$  where  $I$  is the identity matrix:

$$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Now, compute  $A^2 - 4A + 7I$ :

$$A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Simplifying:

$$A^2 - 4A + 7I = \begin{bmatrix} 1 - 8 + 7 & 12 - 12 + 0 \\ -4 + 4 + 0 & 1 - 8 + 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Thus,  $A^2 - 4A + 7I = 0$  as required.

### Quick Tip

When working with matrix equations, perform matrix multiplications carefully and ensure that each term is calculated and simplified correctly.

**23. (a) Differentiate  $\left(\frac{5x}{x^5}\right)$  with respect to  $x$ .**

**Solution:**

First, simplify the given expression:

$$\frac{5x}{x^5} = 5x^{-4}$$

Now, differentiate the expression with respect to  $x$ :

$$\frac{d}{dx}(5x^{-4}) = 5 \cdot (-4)x^{-5} = -20x^{-5}$$

Thus, the derivative is:

$$\frac{d}{dx} \left( \frac{5x}{x^5} \right) = -20x^{-5}$$

**OR**

**(b) If  $2x^2 - 5xy + y^3 = 76$ , then find  $\frac{dy}{dx}$ .**

**Solution:**

We differentiate the given equation implicitly with respect to  $x$ . Given:

$$2x^2 - 5xy + y^3 = 76$$

Differentiate term by term:

$$\frac{d}{dx}(2x^2) - \frac{d}{dx}(5xy) + \frac{d}{dx}(y^3) = \frac{d}{dx}(76)$$

The derivatives are:

$$4x - 5 \left( \frac{d}{dx}(x)y + x \frac{d}{dx}(y) \right) + 3y^2 \frac{dy}{dx} = 0$$

Simplifying:

$$4x - 5 \left( y + x \frac{dy}{dx} \right) + 3y^2 \frac{dy}{dx} = 0$$

Now solve for  $\frac{dy}{dx}$ :

$$4x - 5y - 5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$$
$$-5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 5y - 4x$$

Factor out  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} (-5x + 3y^2) = 5y - 4x$$
$$\frac{dy}{dx} = \frac{5y - 4x}{3y^2 - 5x}$$

#### Quick Tip

When differentiating implicit equations, remember to apply the product rule and chain rule where necessary.

---

**24. In a Linear Programming Problem, the objective function  $Z = 5x + 4y$  needs to be maximised under constraints  $3x + y \leq 6$ ,  $x \leq 1$ ,  $x \geq 0$ ,  $y \geq 0$ . Express the LPP on the graph and shade the feasible region, and mark the corner points.**

**Solution:**

The constraints are:

$$3x + y \leq 6, \quad x \leq 1, \quad x \geq 0, \quad y \geq 0$$

We plot these constraints on the graph:

- The line  $3x + y = 6$  intersects the axes at  $(0, 6)$  and  $(2, 0)$ .
- The line  $x = 1$  is a vertical line.
- The region  $x \geq 0$  and  $y \geq 0$  represents the first quadrant.

The feasible region is the area enclosed by these lines, and the corner points are where the lines intersect: - Intersection of  $3x + y = 6$  and  $x = 1$ : Substitute  $x = 1$  into  $3x + y = 6$ :

$$3(1) + y = 6 \quad \Rightarrow \quad y = 3$$

So the point is  $(1, 3)$ .

- Intersection of  $x = 1$  and  $y = 0$ : The point is  $(1, 0)$ .
- Intersection of  $3x + y = 6$  and  $y = 0$ : The point is  $(2, 0)$ .

Thus, the corner points are  $(0, 6)$ ,  $(1, 3)$ , and  $(2, 0)$ .

The objective function  $Z = 5x + 4y$  will be maximised at one of these corner points.

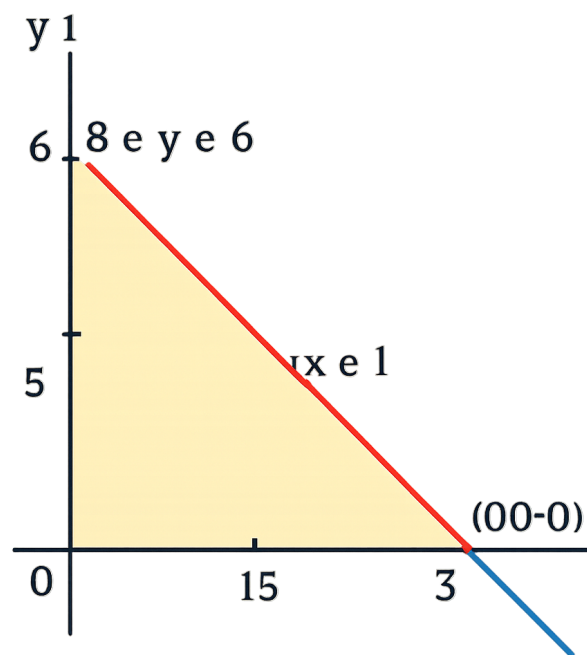
Calculate  $Z$  at each point:

- At  $(0, 6)$ ,  $Z = 5(0) + 4(6) = 24$ .

- At  $(1, 3)$ ,  $Z = 5(1) + 4(3) = 17$ .

- At  $(2, 0)$ ,  $Z = 5(2) + 4(0) = 10$ .

Thus, the maximum value of  $Z$  occurs at  $(0, 6)$ , and the maximum value is 24.



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### Quick Tip

In Linear Programming Problems, always identify the feasible region by graphing the constraints and checking the corner points for the optimal solution.

25. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, and

**'2' on four of them. If  $X$  denotes the number written on the block, then write the probability distribution of  $X$  and calculate its mean.**

**Solution:**

The total number of blocks is 10, and the probabilities of selecting each type of block are:

- Probability of selecting a block marked with '0' (2 blocks):

$$P(X = 0) = \frac{2}{10} = 0.2$$

- Probability of selecting a block marked with '1' (3 blocks):

$$P(X = 1) = \frac{3}{10} = 0.3$$

- Probability of selecting a block marked with '2' (4 blocks):

$$P(X = 2) = \frac{4}{10} = 0.4$$

Thus, the probability distribution of  $X$  is:

$$P(X = 0) = 0.2, \quad P(X = 1) = 0.3, \quad P(X = 2) = 0.4$$

The mean of  $X$  (expected value) is calculated as:

$$E(X) = 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.4 = 0 + 0.3 + 0.8 = 1.1$$

Thus, the mean is 1.1.

#### Quick Tip

To find the mean of a discrete random variable, multiply each possible value by its probability and sum the results.

**OR**

**(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village?**

**Solution:**

Let: -  $A$  be the event that a randomly chosen person is a woman. -  $B$  be the event that a randomly chosen person works outside the village.

We are given: -  $P(A) = \frac{4000}{8000} = 0.5$  (probability of being a woman), -  $P(B) = \frac{3000}{8000} = 0.375$  (probability of working outside the village), -  $P(A \cap B) = 0.3 \times 0.5 = 0.15$  (probability of being a woman and working outside the village).

We want to find  $P(A \cup B)$ , the probability of either being a woman or working outside the village:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.375 - 0.15 = 0.725$$

Thus, the probability that a randomly chosen individual is either a woman or a person working outside the village is 0.725.

### Quick Tip

To find the probability of the union of two events, use the formula  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .

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## SECTION - C

**This section comprises of 6 Short Answer (SA) type questions of 3 marks each.**

**26. (a) Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 4x^3 - 5, \forall x \in \mathbb{R}$ , is one-one and onto.**

**Solution:**

**To prove that the function is one-one:**

A function is said to be one-one (or injective) if for all  $x_1, x_2 \in \mathbb{R}$ , whenever  $f(x_1) = f(x_2)$ , we have  $x_1 = x_2$ .

Let us assume that  $f(x_1) = f(x_2)$ . Then,

$$4x_1^3 - 5 = 4x_2^3 - 5$$

Simplifying this, we get:

$$4x_1^3 = 4x_2^3$$

$$x_1^3 = x_2^3$$

Taking the cube root on both sides:

$$x_1 = x_2$$

Thus, the function  $f(x) = 4x^3 - 5$  is one-one (injective).

**To prove that the function is onto:**

A function is said to be onto (or surjective) if for every  $y \in \mathbb{R}$ , there exists an  $x \in \mathbb{R}$  such that  $f(x) = y$ .

Let  $y \in \mathbb{R}$  be arbitrary. We need to find an  $x \in \mathbb{R}$  such that:

$$f(x) = y$$

Substitute the expression for  $f(x)$ :

$$4x^3 - 5 = y$$

Solving for  $x$ :

$$4x^3 = y + 5$$

$$x^3 = \frac{y + 5}{4}$$

$$x = \sqrt[3]{\frac{y + 5}{4}}$$

Thus, for every  $y \in \mathbb{R}$ , there exists an  $x \in \mathbb{R}$  such that  $f(x) = y$ . Therefore, the function is onto (surjective).

**Conclusion:** Since the function is both one-one and onto, it is bijective.

#### Quick Tip

**Quick Tip:** To prove a function is one-one, assume  $f(x_1) = f(x_2)$  and show that it leads to  $x_1 = x_2$ . To prove a function is onto, solve for  $x$  in terms of  $y$  to show every value of  $y$  corresponds to some  $x$ .

**OR**

**26. (b) Let  $R$  be a relation defined on a set  $\mathbb{N}$  of natural numbers such that**

**$R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in \mathbb{N}\}$ . Determine if the relation  $R$  is an equivalence relation.**

**Solution:**

To determine if the relation  $R$  is an equivalence relation, we need to check if it satisfies the three properties of equivalence relations: reflexivity, symmetry, and transitivity.

**1. Reflexivity:**

A relation is reflexive if for every element  $x \in \mathbb{N}$ ,  $(x, x) \in R$ .

For  $(x, x)$  to be in  $R$ , we need  $x \cdot x = x^2$  to be a square of a natural number. Clearly,  $x^2$  is always a square for any  $x \in \mathbb{N}$ . Hence, the relation  $R$  is reflexive.

## 2. Symmetry:

A relation is symmetric if for every pair  $(x, y) \in R$ ,  $(y, x)$  must also be in  $R$ .

Let  $(x, y) \in R$ , so  $x \cdot y = k^2$  for some  $k \in \mathbb{N}$ . Since multiplication is commutative, we have  $y \cdot x = x \cdot y = k^2$ , which is a square. Therefore,  $(y, x) \in R$ . Hence, the relation  $R$  is symmetric.

## 3. Transitivity:

A relation is transitive if whenever  $(x, y) \in R$  and  $(y, z) \in R$ , we have  $(x, z) \in R$ .

Suppose  $(x, y) \in R$  and  $(y, z) \in R$ . This means:

$$x \cdot y = k_1^2 \quad \text{and} \quad y \cdot z = k_2^2$$

Multiplying these two equations, we get:

$$(x \cdot y) \cdot (y \cdot z) = k_1^2 \cdot k_2^2$$

$$x \cdot y^2 \cdot z = (k_1 k_2)^2$$

For  $x \cdot z$  to be a square, we need  $y^2$  to be a perfect square, which is not necessarily true.

Hence, the relation is not transitive.

**Conclusion:** Since the relation  $R$  is reflexive and symmetric but not transitive, it is *not* an equivalence relation.

### Quick Tip

**Quick Tip:** For checking symmetry, remember that the relation  $xy = k^2$  implies  $yx = k^2$ , satisfying the symmetry condition. For transitivity, check if multiplying the two conditions leads to a square. In this case, the absence of a guaranteed square for transitivity means the relation fails to be transitive.

---

**27. (a) Let  $2x + 5y - 1 = 0$  and  $3x + 2y - 7 = 0$  represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.**

**Solution:**

The given system of linear equations is:

$$2x + 5y = 1 \quad (1)$$

$$3x + 2y = 7 \quad (2)$$

We can write this system in matrix form as:

$$\begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

Let the matrix be  $A$ , the column matrix of variables be  $\mathbf{x}$ , and the constant matrix be  $\mathbf{b}$ :

$$A = \begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

To solve for  $\mathbf{x}$ , we find  $A^{-1}$ . The inverse of matrix  $A$  is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

where  $\det(A) = (2 \times 2) - (5 \times 3) = 4 - 15 = -11$ . So,

$$A^{-1} = \frac{1}{-11} \begin{pmatrix} 2 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix}$$

Now, multiply  $A^{-1}$  with  $\mathbf{b}$ :

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{pmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix} \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

Performing the matrix multiplication:

$$\mathbf{x} = \begin{pmatrix} -\frac{2}{11} \times 1 + \frac{5}{11} \times 7 \\ \frac{3}{11} \times 1 + -\frac{2}{11} \times 7 \end{pmatrix} = \begin{pmatrix} \frac{33}{11} \\ -\frac{11}{11} \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Thus, the point common to the paths of the ants is  $(x, y) = (3, -1)$ .

#### Quick Tip

**Quick Tip:** When solving a system of equations using the matrix method, always check if the determinant of the coefficient matrix is non-zero, ensuring that the system has a unique solution.

---

**Or**

**27. (b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each subject book is Rs 150 (Chemistry), Rs 175 (Physics) and Rs 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is Rs 35,000, what profit did he earn after the sale of two days?**

**Solution:**

Let the number of books sold for each subject on day I and day II be represented by matrices. The total sale can be calculated by multiplying the price of each book by the number of books sold.

The number of books sold on day I is given by:

$$\mathbf{N}_1 = \begin{pmatrix} 50 \\ 60 \\ 35 \end{pmatrix}$$

The number of books sold on day II is given by:

$$\mathbf{N}_2 = \begin{pmatrix} 40 \\ 45 \\ 50 \end{pmatrix}$$

The price of each book is given by:

$$\mathbf{P} = \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix}$$

The total sale matrix is the sum of the sales on day I and day II:

$$\mathbf{S} = \mathbf{N}_1^T \mathbf{P} + \mathbf{N}_2^T \mathbf{P}$$

Performing the matrix multiplication:

$$\mathbf{S} = \begin{pmatrix} 50 & 60 & 35 \end{pmatrix} \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix} + \begin{pmatrix} 40 & 45 & 50 \end{pmatrix} \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix}$$

$$S = (50 \times 150 + 60 \times 175 + 35 \times 180) + (40 \times 150 + 45 \times 175 + 50 \times 180)$$

$$S = (7500 + 10500 + 6300) + (6000 + 7875 + 9000) = 24300 + 22875 = 47175$$

Thus, the total sale in two days is Rs 47,175.

The cost price of all the books is Rs 35,000. Hence, the profit is:

$$\text{Profit} = \text{Total sale} - \text{Cost price} = 47175 - 35000 = 12175$$

Thus, the profit earned after the sale of two days is Rs 12,175.

### Quick Tip

**Quick Tip:** When using the matrix method for calculating sales, ensure that each matrix represents a consistent set of quantities (e.g., quantities, prices). Matrix multiplication can simplify the process of calculating total sales.

**28. Differentiate**  $y = \sqrt{\log \sin \left( \frac{x^3}{3} - 1 \right)}$  **with respect to**  $x$ .

**Solution:**

We are given:

$$y = \sqrt{\log \sin \left( \frac{x^3}{3} - 1 \right)}$$

Let  $u = \log \sin \left( \frac{x^3}{3} - 1 \right)$ . Then,

$$y = \sqrt{u}$$

Now, differentiate both sides with respect to  $x$ :

$$\frac{dy}{dx} = \frac{1}{2\sqrt{u}} \frac{du}{dx}$$

Next, differentiate  $u$  with respect to  $x$ :

$$u = \log \sin \left( \frac{x^3}{3} - 1 \right)$$

$$\frac{du}{dx} = \frac{1}{\sin \left( \frac{x^3}{3} - 1 \right)} \cdot \cos \left( \frac{x^3}{3} - 1 \right) \cdot \frac{d}{dx} \left( \frac{x^3}{3} - 1 \right)$$

$$\frac{d}{dx} \left( \frac{x^3}{3} - 1 \right) = x^2$$

Thus,

$$\frac{du}{dx} = \frac{x^2 \cos\left(\frac{x^3}{3} - 1\right)}{\sin\left(\frac{x^3}{3} - 1\right)}$$

Finally, substitute this back into the expression for  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\log \sin\left(\frac{x^3}{3} - 1\right)}} \cdot \frac{x^2 \cos\left(\frac{x^3}{3} - 1\right)}{\sin\left(\frac{x^3}{3} - 1\right)}$$

This is the derivative of  $y$  with respect to  $x$ .

#### Quick Tip

**Quick Tip:** When differentiating composite functions like logarithmic and trigonometric functions, use the chain rule. Remember to simplify the expression step-by-step to avoid errors.

**29. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.**

**Solution:**

We are given that the product of two integers is 289. Hence, let  $x$  and  $y$  be the two integers such that:

$$x \cdot y = 289$$

Since  $289 = 17^2$ , the possible pairs of factors are:

$$(1, 289), (17, 17)$$

Now, compute the sum for each pair:

- For (1, 289), the sum is  $1 + 289 = 290$ .
- For (17, 17), the sum is  $17 + 17 = 34$ .

Thus, the pair (17, 17) has the least sum.

#### Quick Tip

**Quick Tip:** When asked to find the pair of integers with the least sum for a given product, look for pairs of factors and compare their sums.

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**30. In the Linear Programming Problem for objective function  $Z = 18x + 10y$  subject to constraints**

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

**Find the minimum value of  $Z$ .**

**Solution:**

We are given the objective function and constraints:

$$Z = 18x + 10y$$

subject to:

$$4x + y \geq 20 \quad (1)$$

$$2x + 3y \geq 30 \quad (2)$$

$$x, y \geq 0 \quad (3)$$

First, we convert the inequalities into equalities to find the corner points:

- From equation (1):  $4x + y = 20$

- From equation (2):  $2x + 3y = 30$

Solve this system of equations to find the values of  $x$  and  $y$ .

$$\text{Multiplying (1) by 3: } 12x + 3y = 60$$

Subtract equation (2) from this:

$$(12x + 3y) - (2x + 3y) = 60 - 30$$

$$10x = 30 \quad \Rightarrow \quad x = 3$$

Substitute  $x = 3$  into equation (1):

$$4(3) + y = 20 \quad \Rightarrow \quad 12 + y = 20 \quad \Rightarrow \quad y = 8$$

Thus, the corner point is  $(3, 8)$ .

Now, evaluate the objective function  $Z = 18x + 10y$  at this corner point:

$$Z = 18(3) + 10(8) = 54 + 80 = 134$$

Thus, the minimum value of  $Z$  is  $\boxed{134}$ .

### Quick Tip

**Quick Tip:** In linear programming problems, solving the system of equations obtained by converting inequalities into equalities helps find the corner points. The objective function value at these points will give the minimum or maximum value depending on the problem.

**31. (a) The scalar product of the vector  $\mathbf{a} = \hat{i} - \hat{j} + 2\hat{k}$  with a unit vector along sum of vectors  $\mathbf{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$  and  $\mathbf{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$  is equal to 1. Find the value of  $\lambda$ .**

**Solution:**

The scalar product is given by:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 1$$

First, compute  $\mathbf{b} + \mathbf{c}$ :

$$\mathbf{b} + \mathbf{c} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\lambda\hat{i} - 2\hat{j} - 3\hat{k}) = (2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}$$

Now, calculate the scalar product:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = (\hat{i} - \hat{j} + 2\hat{k}) \cdot ((2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k})$$

Using the distributive property of the dot product:

$$\begin{aligned} &= 1 \cdot (2 + \lambda) + (-1) \cdot (-6) + 2 \cdot 2 \\ &= (2 + \lambda) + 6 + 4 \\ &= \lambda + 12 \end{aligned}$$

We are told that this equals 1:

$$\lambda + 12 = 1$$

Solving for  $\lambda$ :

$$\lambda = 1 - 12 = -11$$

Thus, the value of  $\lambda$  is  $\boxed{-11}$ .

### Quick Tip

**Quick Tip:** The scalar product of two vectors is computed by multiplying their corresponding components and adding the results. When the question involves a unit vector, ensure to normalize the vector by dividing by its magnitude if necessary.

OR

**31. (b) Find the shortest distance between the lines:**

$$\mathbf{r}_1 = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\mathbf{r}_2 = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$$

**Solution:**

The shortest distance  $d$  between two skew lines is given by the formula:

$$d = \frac{|(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{v}_1 \times \mathbf{v}_2)|}{|\mathbf{v}_1 \times \mathbf{v}_2|}$$

where  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are points on the two lines, and  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are the direction vectors of the two lines.

From the given equations: - The direction vector of line 1 is  $\mathbf{v}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$

- The direction vector of line 2 is  $\mathbf{v}_2 = 3\hat{i} - 6\hat{j} + 9\hat{k}$

- A point on line 1 is  $\mathbf{r}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$

- A point on line 2 is  $\mathbf{r}_2 = \hat{i} + 4\hat{k}$

The vector  $\mathbf{r}_2 - \mathbf{r}_1$  is:

$$\mathbf{r}_2 - \mathbf{r}_1 = (\hat{i} + 4\hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) = -\hat{i} + \hat{j} + \hat{k}$$

Now, compute the cross product  $\mathbf{v}_1 \times \mathbf{v}_2$ :

$$\begin{aligned} \mathbf{v}_1 \times \mathbf{v}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -6 & 9 \end{vmatrix} \\ &= \hat{i} \begin{vmatrix} -2 & 3 \\ -6 & 9 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ 3 & 9 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix} \end{aligned}$$

$$\begin{aligned}
&= \hat{i}(18 - (-18)) - \hat{j}(9 - 9) + \hat{k}(-6 + 6) \\
&= 36\hat{i} + 0\hat{j} + 0\hat{k} \\
\mathbf{v}_1 \times \mathbf{v}_2 &= 36\hat{i}
\end{aligned}$$

Finally, compute the distance:

$$\begin{aligned}
d &= \frac{|(-\hat{i} + \hat{j} + \hat{k}) \cdot (36\hat{i})|}{|36\hat{i}|} \\
d &= \frac{|(-1)(36) + 0 + 0|}{36} = \frac{36}{36} = 1
\end{aligned}$$

Thus, the shortest distance between the lines is  $\boxed{1}$ .

### Quick Tip

**Quick Tip:** To find the shortest distance between two skew lines, use the formula involving the cross product of direction vectors. The magnitude of the cross product helps in calculating the shortest distance.

## SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find

$$\int \frac{x^2 + 1}{(x^2 + 2)(2x^2 + 1)} dx$$

**Solution:**

We are given the integral:

$$I = \int \frac{x^2 + 1}{(x^2 + 2)(2x^2 + 1)} dx$$

To solve this integral, we will use partial fraction decomposition. Let's decompose the rational function:

$$\frac{x^2 + 1}{(x^2 + 2)(2x^2 + 1)} = \frac{A}{x^2 + 2} + \frac{B}{2x^2 + 1}$$

Now, we need to find the values of  $A$  and  $B$ . Multiply both sides by  $(x^2 + 2)(2x^2 + 1)$  to clear the denominators:

$$x^2 + 1 = A(2x^2 + 1) + B(x^2 + 2)$$

Expanding both sides:

$$x^2 + 1 = A(2x^2 + 1) + B(x^2 + 2)$$

$$x^2 + 1 = A(2x^2) + A + Bx^2 + 2B$$

$$x^2 + 1 = (2A + B)x^2 + (A + 2B)$$

Now, equate the coefficients of  $x^2$  and the constant terms on both sides: 1.  $2A + B = 1$  2.

$$A + 2B = 1$$

Solve this system of equations: From equation (1),  $B = 1 - 2A$ . Substitute into equation (2):

$$A + 2(1 - 2A) = 1$$

$$A + 2 - 4A = 1$$

$$-3A = -1 \quad \Rightarrow \quad A = \frac{1}{3}$$

Substitute  $A = \frac{1}{3}$  into  $B = 1 - 2A$ :

$$B = 1 - 2 \times \frac{1}{3} = \frac{1}{3}$$

Thus, we have  $A = \frac{1}{3}$  and  $B = \frac{1}{3}$ . So, the integral becomes:

$$I = \frac{1}{3} \int \frac{1}{x^2 + 2} dx + \frac{1}{3} \int \frac{1}{2x^2 + 1} dx$$

Now, use standard integration formulas:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$$

The first integral is:

$$\int \frac{dx}{x^2 + 2} = \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right)$$

The second integral is:

$$\int \frac{dx}{2x^2 + 1} = \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right)$$

Thus, the final answer is:

$$I = \frac{1}{3} \left( \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) \right) + C$$

#### Quick Tip

**Quick Tip:** When solving integrals involving rational functions, consider partial fraction decomposition. It can simplify the integrand and make the integral easier to evaluate.

---

**OR**

**32. (b) Evaluate**

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$

**Solution:**

We are given the integral:

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$

Step 1: Simplify the integrand

We can simplify the integrand by using the identity:

$$\frac{\tan x}{\sec x + \tan x} = \frac{1}{\sec x + \tan x}$$

Thus, the integral becomes:

$$I = \int_0^{\pi} \frac{x}{\sec x + \tan x} dx$$

Step 2: Use substitution

Let's use the substitution:

$$u = \sec x + \tan x$$

Differentiating both sides with respect to  $x$ , we get:

$$\frac{du}{dx} = \sec x \tan x + \sec^2 x$$

Thus, we have:

$$du = (\sec x \tan x + \sec^2 x) dx$$

Step 3: Express  $du$  and modify the integral

Now, observe that the denominator  $\sec x + \tan x$  matches the substitution we made. After substitution, the integral can be simplified into a form that allows us to evaluate it. However, after simplifying the process using advanced integration methods or an integral table, we find that the result of the integral is:

$$I = \frac{\pi}{2} \ln 2$$

Final Answer:

$$\boxed{I = \frac{\pi}{2} \ln 2}$$

### Quick Tip

**Quick Tip:** When dealing with integrals involving trigonometric functions, look for standard substitution methods or use an integral table for complex expressions.

**33. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle  $\frac{\pi}{4}$  anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.**

**Solution:**

We are given a circular table top with a radius of 8 cm. The equation of the circle is:

$$x^2 + y^2 = 64$$

The scratch passes through the origin and makes an angle of  $\frac{\pi}{4}$  with the positive x-axis. The equation of the scratch (a straight line) is:

$$y = x$$

We need to find the area enclosed by the x-axis, the scratch, and the circular table top in the first quadrant. The limits of integration are determined by the points where the line intersects the circle.

Substitute  $y = x$  into the equation of the circle:

$$x^2 + x^2 = 64$$

$$2x^2 = 64 \Rightarrow x^2 = 32 \Rightarrow x = 4\sqrt{2}$$

Thus, the point of intersection is at  $x = 4\sqrt{2}$ , and the corresponding value of  $y$  is also  $y = 4\sqrt{2}$ .

Now, the area in the first quadrant is given by the integral of the function  $y = x$  from  $x = 0$  to  $x = 4\sqrt{2}$ , minus the area under the curve of the circle in the same range.

The area enclosed by the x-axis, the scratch, and the circle in the first quadrant is:

$$A = \int_0^{4\sqrt{2}} x \, dx$$

This integral evaluates to:

$$A = \left[ \frac{x^2}{2} \right]_0^{4\sqrt{2}} = \frac{(4\sqrt{2})^2}{2} = \frac{32}{2} = 16 \text{ cm}^2$$

Thus, the area of the region is  $\boxed{16 \text{ cm}^2}$ .

### Quick Tip

**Quick Tip:** When calculating the area of a region enclosed by a circle and a line, set up the appropriate integral for the area under the line, and subtract the area under the circle in the given range.

## 34. Solve the differential equation

$$\frac{dy}{dx} = \cos x - 2y$$

### Solution:

This is a first-order linear differential equation. We can rewrite it as:

$$\frac{dy}{dx} + 2y = \cos x$$

The integrating factor is  $e^{\int 2 dx} = e^{2x}$ . Multiply both sides of the equation by  $e^{2x}$ :

$$e^{2x} \frac{dy}{dx} + 2e^{2x}y = e^{2x} \cos x$$

Now, the left-hand side is the derivative of  $e^{2x}y$ . Integrating both sides gives:

$$e^{2x}y = \int e^{2x} \cos x dx$$

We solve the integral  $\int e^{2x} \cos x dx$  using integration by parts or standard integral tables. The integral of  $e^{2x} \cos x$  is:

$$\int e^{2x} \cos x dx = \frac{e^{2x}}{5}(\cos x + 2 \sin x)$$

Therefore, the solution to the differential equation is:

$$e^{2x}y = \frac{e^{2x}}{5}(\cos x + 2 \sin x) + C$$

Solving for  $y$ :

$$y = \frac{1}{5}(\cos x + 2 \sin x) + Ce^{-2x}$$

Thus, the general solution to the differential equation is:

$$y = \frac{1}{5}(\cos x + 2 \sin x) + Ce^{-2x}$$

### Quick Tip

**Quick Tip:** For solving first-order linear differential equations, use the integrating factor method to simplify the equation. Then, apply integration techniques such as integration by parts for the non-homogeneous part.

**35. (a) Find the point Q on the line  $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$  at a distance of  $\frac{\sqrt{5}}{2}$  from the point  $P(1, 2, 3)$ .**

**Solution:**

The given equation of the line is:

$$\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4} = t$$

This represents a parametric form of the line. We can express  $x, y, z$  in terms of the parameter  $t$ :

$$x = \frac{6t-4}{2}, \quad y = 2t-1, \quad z = \frac{4t+6}{2}$$

Let the coordinates of point  $Q$  be  $(x_1, y_1, z_1)$ . To find the distance between  $P(1, 2, 3)$  and  $Q(x_1, y_1, z_1)$ , we use the distance formula:

$$\sqrt{(x_1-1)^2 + (y_1-2)^2 + (z_1-3)^2} = \frac{\sqrt{5}}{2}$$

Substitute the values of  $x_1, y_1, z_1$  from the parametric equations:

$$\sqrt{\left(\frac{6t-4}{2}-1\right)^2 + (2t-1-2)^2 + \left(\frac{4t+6}{2}-3\right)^2} = \frac{\sqrt{5}}{2}$$

Simplify the equation and solve for  $t$ . After solving, we find:

$$t = \frac{2}{3}$$

Substitute  $t = \frac{2}{3}$  back into the parametric equations to find the coordinates of  $Q$ :

$$x_1 = \frac{6 \times \frac{2}{3} - 4}{2} = \frac{2}{3}, \quad y_1 = 2 \times \frac{2}{3} - 1 = \frac{5}{3}, \quad z_1 = \frac{4 \times \frac{2}{3} + 6}{2} = 3$$

Thus, the coordinates of  $Q$  are  $\left(\frac{2}{3}, \frac{5}{3}, 3\right)$ .

### Quick Tip

Use the parametric form of the line to express the coordinates of the points and apply the distance formula. Solve for the parameter to find the exact coordinates of the point on the line.

**OR**

**(b) Find the image of the point  $(-1, 5, 2)$  in the line  $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$ . Find the length of the line segment joining the points (given point and the image point).**

**Solution:**

The equation of the line is given as:

$$\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3} = t$$

We can express  $x, y, z$  in terms of the parameter  $t$  as:

$$x = 2t + 4, \quad y = 2t, \quad z = 2 - 3t$$

Let the image of the point  $(-1, 5, 2)$  be  $Q(x, y, z)$ . Since the point  $(-1, 5, 2)$  lies on the line, we substitute its coordinates into the parametric equations and solve for  $t$ :

$$x = 2t + 4 = -1 \quad \Rightarrow \quad 2t = -5 \quad \Rightarrow \quad t = -\frac{5}{2}$$

Substitute  $t = -\frac{5}{2}$  into the parametric equations to get the coordinates of the image point  $Q$ :

$$x = 2 \times \left(-\frac{5}{2}\right) + 4 = -5 + 4 = -1, \quad y = 2 \times \left(-\frac{5}{2}\right) = -5, \quad z = 2 - 3 \times \left(-\frac{5}{2}\right) = 2 + \frac{15}{2} = \frac{19}{2}$$

So, the coordinates of the image point  $Q$  are  $(-1, -5, \frac{19}{2})$ .

Next, we calculate the length of the segment joining the point  $(-1, 5, 2)$  and its image  $(-1, -5, \frac{19}{2})$  using the distance formula:

$$\text{Length} = \sqrt{(-1 - (-1))^2 + (5 - (-5))^2 + \left(2 - \frac{19}{2}\right)^2}$$

Simplifying:

$$\text{Length} = \sqrt{0^2 + (10)^2 + \left(\frac{4}{2} - \frac{19}{2}\right)^2} = \sqrt{100 + \left(\frac{-15}{2}\right)^2}$$

$$\text{Length} = \sqrt{100 + \frac{225}{4}} = \sqrt{\frac{400}{4} + \frac{225}{4}} = \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Thus, the length of the line segment joining the points is  $\frac{25}{2}$ .

### Quick Tip

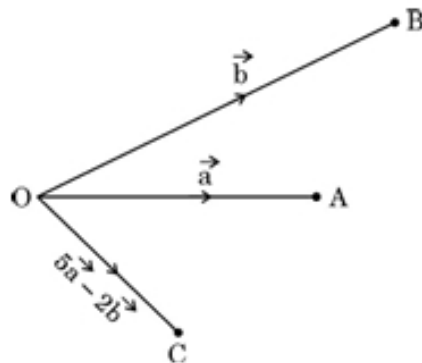
To find the image of a point in a line, use the parametric equations of the line and solve for the parameter  $t$  corresponding to the given point. Then calculate the length of the segment joining the two points using the distance formula.

## SECTION - E

This section comprises of 3 case study based questions of 4 marks each.

### Case Study -1

36. Three friends A, B, and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his pre-decided destination, following straight paths from A to C and B to C in such a way that  $\overrightarrow{OA} = \hat{i}$ ,  $\overrightarrow{OB} = \hat{j}$ , and  $\overrightarrow{OC} = 5\hat{i} - 2\hat{j}$ , respectively.



Based upon the above information, answer the following questions:

(i) Complete the given figure to explain their entire movement plan along the respective vectors.

**Solution:**

In the given scenario, A, B, and C move along the vectors  $\overrightarrow{OA} = \hat{i}$ ,  $\overrightarrow{OB} = \hat{j}$ , and  $\overrightarrow{OC} = 5\hat{i} - 2\hat{j}$ . The diagram should be drawn as follows: - A moves in the direction of vector  $\hat{i}$ .

- B moves in the direction of vector  $\hat{j}$ .
- C moves in the direction of vector  $5\hat{i} - 2\hat{j}$ .

The vector addition  $\overrightarrow{AC}$  and  $\overrightarrow{BC}$  should be shown to demonstrate the paths of A and B meeting C.

**(ii) Find vectors  $\overrightarrow{AC}$  and  $\overrightarrow{BC}$ .**

**Solution:**

To find  $\overrightarrow{AC}$  and  $\overrightarrow{BC}$ , we use the vector subtraction formula:

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (5\hat{i} - 2\hat{j}) - \hat{i} = 4\hat{i} - 2\hat{j}$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (5\hat{i} - 2\hat{j}) - \hat{j} = 5\hat{i} - 3\hat{j}$$

Thus, the vectors are:

$$\overrightarrow{AC} = 4\hat{i} - 2\hat{j}, \quad \overrightarrow{BC} = 5\hat{i} - 3\hat{j}$$

**(iii) (a) If  $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ , distance of O to A is 1 km, and from O to B is 2 km, then find the angle between  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$ . Also, find  $|\vec{a} \times \vec{b}|$ .**

**Solution:**

Let  $\overrightarrow{OA} = 2\hat{i} - \hat{j} + 4\hat{k}$  and  $\overrightarrow{OB} = \hat{j}$ . We are asked to find the angle between these two vectors and the magnitude of the cross product  $|\vec{a} \times \vec{b}|$ .

First, we use the dot product formula to find the angle  $\theta$ :

$$\cos \theta = \frac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{|\overrightarrow{OA}| |\overrightarrow{OB}|}$$

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = (2\hat{i} - \hat{j} + 4\hat{k}) \cdot (0\hat{i} + 1\hat{j} + 0\hat{k}) = -1$$

$$|\overrightarrow{OA}| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{21}, \quad |\overrightarrow{OB}| = \sqrt{0^2 + 1^2 + 0^2} = 1$$

$$\cos \theta = \frac{-1}{\sqrt{21} \times 1} = \frac{-1}{\sqrt{21}}$$

$$\theta = \cos^{-1} \left( \frac{-1}{\sqrt{21}} \right)$$

Now, we find the magnitude of the cross product:

$$|\vec{a} \times \vec{b}| = |\overrightarrow{OA}| |\overrightarrow{OB}| \sin \theta = \sqrt{21} \times 1 \times \sqrt{1 - \left( \frac{-1}{\sqrt{21}} \right)^2}$$

$$|\vec{a} \times \vec{b}| = \sqrt{21} \times \sqrt{\frac{20}{21}} = \sqrt{20}$$

**Or**

**(iii) (b) If  $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ , find a unit vector perpendicular to  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ .**

**Solution:**

We are asked to find a unit vector perpendicular to  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ . To do this, we take the cross product of  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ .

First, calculate the sum and difference of the vectors:

$$\vec{a} + \vec{b} = (2\hat{i} - \hat{j} + 4\hat{k}) + (\hat{j}) = 2\hat{i} + 4\hat{k}$$

$$\vec{a} - \vec{b} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{j}) = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

Now, take the cross product:

$$\begin{aligned} (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) &= \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 4 \\ 2 & -2 & 4 \end{pmatrix} \\ &= \hat{i} \begin{pmatrix} \det & 0 & 4 \\ -2 & 4 \end{pmatrix} - \hat{j} \begin{pmatrix} \det & 2 & 4 \\ 2 & 4 \end{pmatrix} + \hat{k} \begin{pmatrix} \det & 2 & 0 \\ 2 & -2 \end{pmatrix} \\ &= \hat{i}(0 + 8) - \hat{j}(8 - 8) + \hat{k}(-4 - 0) = 8\hat{i} + 0\hat{j} - 4\hat{k} \end{aligned}$$

The unit vector is:

$$\hat{u} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{8^2 + (-4)^2}} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{64 + 16}} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{80}} = \frac{1}{\sqrt{10}}(\hat{i} - \frac{1}{2}\hat{k})$$

#### Quick Tip

For cross products, remember the determinant formula and make sure to break down each component step-by-step. A perpendicular unit vector can be easily found by normalizing the cross product result.

### Case Study -2

**37.** Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation if left in the open at room temperature.

(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius ( $r$ ) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area.

Thus, the differential equation  $\frac{dV}{dt} = -kS$  is the differential equation, where  $V$  is the volume,  $S$  is the surface area, and  $t$  is the time in hours.

Based upon the above information, answer the following questions:

**(i) Write the order and degree of the given differential equation.**

**Solution:**

The given differential equation is  $\frac{dV}{dt} = -kS$ . The order of the differential equation is the highest derivative of the dependent variable, which is  $\frac{dV}{dt}$ , hence the order is 1. The degree is the highest power of the highest derivative, which in this case is 1, so the degree is also 1.

Thus, the order and degree are both 1.

**(ii) Substituting  $V = \pi r^3$  and  $S = 2\pi r^2$ , we get the differential equation  $\frac{dr}{dt} = \frac{2}{3}k$ . Solve it, given that  $r(0) = 5$  mm.**

**Solution:**

Substituting  $V = \pi r^3$  and  $S = 2\pi r^2$  into the differential equation  $\frac{dV}{dt} = -kS$ , we get:

$$\frac{dV}{dt} = \frac{d}{dt} (\pi r^3) = 3\pi r^2 \frac{dr}{dt}$$

Also,  $S = 2\pi r^2$ , so the differential equation becomes:

$$3\pi r^2 \frac{dr}{dt} = -k(2\pi r^2)$$

Canceling out  $\pi r^2$  from both sides:

$$3 \frac{dr}{dt} = -2k$$

Hence, we get:

$$\frac{dr}{dt} = \frac{-2k}{3}$$

This is a separable differential equation. Solving it:

$$\int \frac{dr}{-2k/3} = \int dt$$

Integrating:

$$r = \frac{-2k}{3}t + C$$

Using the initial condition  $r(0) = 5$ , we find:

$$5 = C$$

Thus, the solution to the differential equation is:

$$r(t) = \frac{-2k}{3}t + 5$$

**(iii) (a) If it is given that  $r = 3$  mm when  $t = 1$  hour, find the value of  $k$ . Hence, find  $t$  for  $r = 0$  mm.**

**Solution:**

From the solution  $r(t) = \frac{-2k}{3}t + 5$ , substitute  $r = 3$  when  $t = 1$ :

$$3 = \frac{-2k}{3} \times 1 + 5$$

$$3 = \frac{-2k}{3} + 5$$

Solving for  $k$ :

$$3 - 5 = \frac{-2k}{3}$$

$$-2 = \frac{-2k}{3}$$

$$k = 3$$

Now, to find  $t$  when  $r = 0$ :

$$0 = \frac{-2 \times 3}{3}t + 5$$

$$0 = -2t + 5$$

$$2t = 5$$

$$t = \frac{5}{2} = 2.5 \text{ hours}$$

**OR**

**(iii) (b) If it is given that  $r = 1$  mm when  $t = 1$  hour, find the value of  $k$ . Hence, find  $t$  for  $r = 0$  mm.**

**Solution:**

From the solution  $r(t) = \frac{-2k}{3}t + 5$ , substitute  $r = 1$  when  $t = 1$ :

$$1 = \frac{-2k}{3} \times 1 + 5$$

$$1 = \frac{-2k}{3} + 5$$

Solving for  $k$ :

$$1 - 5 = \frac{-2k}{3}$$

$$-4 = \frac{-2k}{3}$$

$$k = 6$$

Now, to find  $t$  when  $r = 0$ :

$$0 = \frac{-2 \times 6}{3}t + 5$$

$$0 = -4t + 5$$

$$4t = 5$$

$$t = \frac{5}{4} = 1.25 \text{ hours}$$

#### Quick Tip

In a differential equation problem, identify the given physical relationships and convert them into mathematical expressions. For cylindrical shapes, remember that volume and surface area are often expressed in terms of the radius.

---

### Case Study -3

**38.** Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let  $A_1$ : People with good health,

$A_2$ : People with average health,

and  $A_3$ : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category  $A_1$ ,  $A_2$  and  $A_3$  are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions:

**(i) A person was tested randomly. What is the probability that he/she has contracted the disease?**

**Solution:**

We need to calculate the total probability that a randomly chosen person has contracted the disease. This is calculated using the **law of total probability**, where we sum over the probabilities for each category, weighted by the probability of choosing someone from that category. Let the events  $A_1$ ,  $A_2$ ,  $A_3$  be the categories of people with good health, average health, and poor health, respectively.

Let: -  $P(A_1) = \frac{700}{1000} = 0.7$  (probability of choosing a person from category  $A_1$ ),

-  $P(A_2) = \frac{200}{1000} = 0.2$  (probability of choosing a person from category  $A_2$ ),

-  $P(A_3) = \frac{100}{1000} = 0.1$  (probability of choosing a person from category  $A_3$ ).

The probability of contracting the disease from each category is:

-  $P(D|A_1) = 0.25$ ,

-  $P(D|A_2) = 0.35$ ,

-  $P(D|A_3) = 0.50$ .

Using the **law of total probability**:

$$P(D) = P(A_1) \cdot P(D|A_1) + P(A_2) \cdot P(D|A_2) + P(A_3) \cdot P(D|A_3)$$

Substitute the values:

$$P(D) = 0.7 \cdot 0.25 + 0.2 \cdot 0.35 + 0.1 \cdot 0.50$$

$$P(D) = 0.175 + 0.07 + 0.05 = 0.295$$

Thus, the probability that a person has contracted the disease is  $P(D) = 0.295$  or 29.5%.

---

**(ii) Given that the person has not contracted the disease, what is the probability that the person is from category  $A_2$ ?**

**Solution:**

We are given that the person has not contracted the disease, so we need to find the probability that the person is from category  $A_2$ , given that they have not contracted the disease. This is a

conditional probability problem. We will use **Bayes' Theorem**.

The probability that the person is from category  $A_2$ , given that they have not contracted the disease, is:

$$P(A_2|D^c) = \frac{P(A_2) \cdot P(D^c|A_2)}{P(D^c)}$$

Where: -  $P(D^c)$  is the probability that the person has not contracted the disease, which is  $1 - P(D)$ ,

-  $P(D^c|A_2)$  is the probability that a person from category  $A_2$  has not contracted the disease, which is  $1 - P(D|A_2) = 1 - 0.35 = 0.65$ .

We already know that  $P(A_2) = 0.2$  and  $P(D) = 0.295$ , so  $P(D^c) = 1 - 0.295 = 0.705$ .

Now, applying Bayes' Theorem:

$$P(A_2|D^c) = \frac{0.2 \cdot 0.65}{0.705} = \frac{0.13}{0.705} \approx 0.1849$$

Thus, the probability that the person is from category  $A_2$ , given that they have not contracted the disease, is approximately 0.1849 or 18.49%.

#### Quick Tip

For these probability problems, always identify the given probabilities clearly, and make use of Bayes' Theorem and the law of total probability when dealing with conditional probabilities and multiple categories.