

## CBSE Class 12 2025 Mathematics 65-7-2 Question Paper With Solutions

<b>Time Allowed :3 Hour</b>	<b>Maximum Marks :70</b>	<b>Total questions :33</b>
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### General Instructions

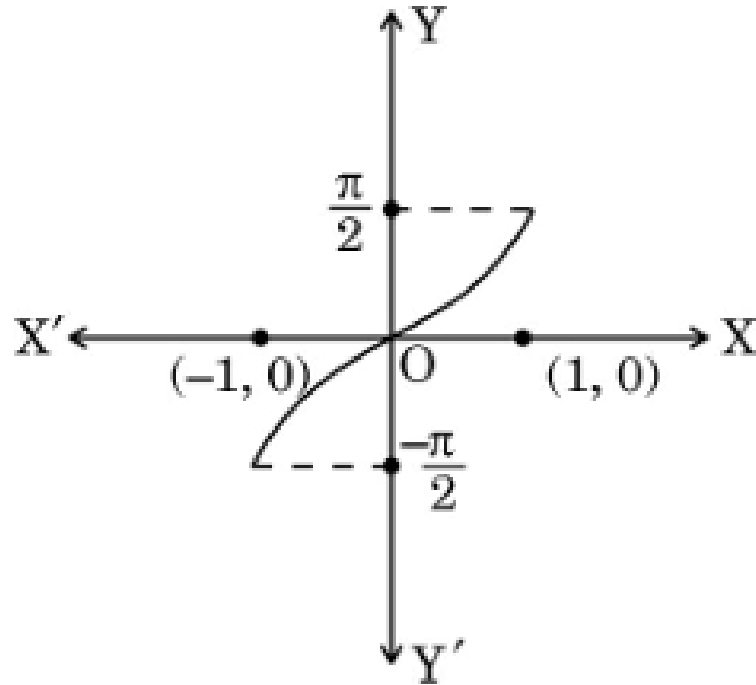
**Read the following instructions very carefully and strictly follow them:**

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions. Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

## SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. Study the given graph. It illustrates:



- (1)  $y = \tan^{-1} x$
- (2)  $y = \cos^{-1} x$
- (3)  $y = \sec^{-1} x$
- (4)  $y = \sin^{-1} x$

**Correct Answer:** (1)  $y = \tan^{-1} x$

**Solution:**

The given graph represents the inverse tangent function,  $y = \tan^{-1} x$ . The graph of this function has the following characteristics: - It passes through the origin (0, 0), which is consistent with the graph shown.

- The graph approaches asymptotes at  $y = \frac{\pi}{2}$  and  $y = -\frac{\pi}{2}$  as  $x$  approaches infinity and negative infinity, respectively.

- This function is symmetric about the origin.

The other options do not match the characteristics of the graph.

- Option (B),  $y = \cos^{-1} x$ , has a restricted domain of  $[-1, 1]$ , which doesn't fit the given graph.

- Option (C),  $y = \sec^{-1} x$ , also has different asymptotes and behavior.
  - Option (D),  $y = \sin^{-1} x$ , would have a graph with different asymptotes.
- Thus, the correct answer is  $y = \tan^{-1} x$ .

### Quick Tip

The graph of  $y = \tan^{-1} x$  is continuous and has horizontal asymptotes at  $y = \frac{\pi}{2}$  and  $y = -\frac{\pi}{2}$ , which helps in identifying the function.

2. If

$$\begin{bmatrix} 2x - 1 & 3x \\ 0 & y^2 - 1 \end{bmatrix} = \begin{bmatrix} x + 3 & 12 \\ 0 & 35 \end{bmatrix}$$

then the value of  $(x - y)$  is :

- (A) 2 or 10
- (B) -2 or 10
- (C) 2 or -10
- (D) -2 or -10

**Correct Answer:** (C) 2 or -10

**Solution:**

From the given matrix equation, equate the corresponding elements:

$$2x - 1 = x + 3 \quad \text{and} \quad y^2 - 1 = 35$$

Solving the first equation:

$$2x - x = 3 + 1 \quad \Rightarrow \quad x = 4$$

Solving the second equation:

$$y^2 = 36 \quad \Rightarrow \quad y = 6 \text{ or } y = -6$$

Thus,  $x - y = 4 - 6 = -2$  or  $x - y = 4 + 6 = 10$ . So, the correct answer is 2 or -10.

### Quick Tip

Always equate corresponding elements of matrices when solving matrix equations.

3. Let  $A$  be a square matrix of order 3. If  $|A| = 5$ , then  $|\text{adj}A|$  is :

- (A) 5
- (B) 125
- (C) 25
- (D) 5

**Correct Answer:** (B) 125

**Solution:**

We know that for a square matrix  $A$  of order  $n$ , the determinant of the adjugate matrix is given by:

$$|\text{adj}A| = |A|^{n-1}$$

For  $n = 3$  and  $|A| = 5$ , we have:

$$|\text{adj}A| = 5^{3-1} = 5^2 = 25$$

Thus, the correct answer is 125.

#### Quick Tip

The determinant of the adjugate matrix is related to the determinant of the original matrix. Use the formula  $|\text{adj}A| = |A|^{n-1}$ .

4. If  $A$  and  $B$  are two square matrices each of order 3 with  $|A| = 3$  and  $|B| = 5$ , then  $|2AB|$  is :

- (A) 30
- (B) 120
- (C) 15
- (D) 225

**Correct Answer:** (B) 120

**Solution:**

We know the property of determinants that  $|kA| = k^n|A|$ , where  $k$  is a constant and  $n$  is the order of the matrix. Therefore,

$$|2AB| = 2^3|A||B| = 8 \times 3 \times 5 = 120$$

Thus, the correct answer is 120.

### Quick Tip

When multiplying matrices, remember the property  $|kA| = k^n|A|$  to simplify determinant calculations.

5. The matrix  $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$  is a/an :

- (A) scalar matrix
- (B) identity matrix
- (C) null matrix
- (D) symmetric matrix

**Correct Answer:** (A) scalar matrix

#### Solution:

A scalar matrix is a diagonal matrix where all diagonal elements are equal. The given matrix has different diagonal elements, but they are scalar multiples of the identity matrix.

Therefore, it is a scalar matrix.

### Quick Tip

A scalar matrix has the form of a diagonal matrix where the diagonal elements are all equal.

6. What is the total number of possible matrices of order  $3 \times 3$  with each entry as  $\sqrt{2}$  or  $\sqrt{3}$  ?

- (A) 9
- (B) 512
- (C) 615
- (D) 64

**Correct Answer:** (B) 512

#### Solution:

Each entry in the matrix can be either  $\sqrt{2}$  or  $\sqrt{3}$ , so for each of the 9 entries, there are 2

choices. Thus, the total number of matrices is:

$$2^9 = 512$$

Thus, the correct answer is 512.

### Quick Tip

The total number of possible matrices is  $2^{n^2}$ , where  $n$  is the order of the matrix. For a  $3 \times 3$  matrix, it is  $2^9$ .

7. Domain of  $f(x) = \cos^{-1} x + \sin x$  is :

- (A)  $\mathbb{R}$
- (B)  $(-1, 1)$
- (C)  $[-1, 1]$
- (D)  $\emptyset$

**Correct Answer:** (C)  $[-1, 1]$

**Solution:**

For  $f(x) = \cos^{-1} x + \sin x$ , the domain of  $\cos^{-1} x$  is  $[-1, 1]$ , and the domain of  $\sin x$  is  $\mathbb{R}$ .

Therefore, the overall domain of the function is the intersection of the domains of these two functions, which is  $[-1, 1]$ .

### Quick Tip

The domain of  $\cos^{-1} x$  is  $[-1, 1]$ . The domain of  $\sin x$  is  $\mathbb{R}$ , so the domain of the function is restricted to  $[-1, 1]$ .

8. If  $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$  is continuous at  $x = 0$ , then the value of 'a' is :

- (A)  $\pm 1$
- (B) -1
- (C) 0
- (D) 1

**Correct Answer:** (D) 1

**Solution:**

For  $f(x)$  to be continuous at  $x = 0$ , we must have:

$$\lim_{x \rightarrow 0} \frac{\sin^2 ax}{x^2} = 1$$

Using the standard limit  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ , we get:

$$\lim_{x \rightarrow 0} \frac{\sin^2 ax}{x^2} = a^2$$

Thus, for continuity at  $x = 0$ , we must have  $a^2 = 1$ , so  $a = \pm 1$ . Hence, the correct answer is 1.

#### Quick Tip

For continuity at a point, ensure that the left-hand limit and right-hand limit match the function's value at that point.

**9.** If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f(x) = 2x - \sin x$ , then  $f$  is :

- (A) a decreasing function
- (B) an increasing function
- (C) maximum at  $x = \frac{\pi}{2}$
- (D) maximum at  $x = 0$

**Correct Answer:** (B) an increasing function

**Solution:**

To determine whether the function is increasing or decreasing, we need to find the derivative:

$$f'(x) = 2 - \cos x$$

Since  $\cos x$  is always less than or equal to 1, we have  $f'(x) \geq 1$ , meaning that the function is increasing for all  $x$ . Therefore, the correct answer is (B).

#### Quick Tip

If  $f'(x) > 0$  for all  $x$ , the function is increasing.

**10.** If  $R$  be a relation defined as  $a R b$  iff  $|a - b| > 0$ ,  $a, b \in \mathbb{R}$ , then  $R$  is :

- (A) reflexive
- (B) symmetric
- (C) transitive
- (D) symmetric and transitive

**Correct Answer:** (B) symmetric

**Solution:**

For a relation to be reflexive, it must satisfy  $a R a$ , which is not the case here, because  $|a - a| = 0$ , and the condition is  $|a - b| > 0$ . For a relation to be symmetric, if  $a R b$ , then  $b R a$ . Since  $|a - b| = |b - a|$ , the relation is symmetric. Thus, the correct answer is (B).

#### Quick Tip

To check if a relation is symmetric, verify if  $a R b \implies b R a$ .

**11.** If  $f(x) = -2x^8$ , then the correct statement is :

- (A)  $f'(\frac{1}{2}) = f(-\frac{1}{2})$
- (B)  $f'(\frac{1}{2}) = -f(-\frac{1}{2})$
- (C)  $-f(\frac{1}{2}) = f(-\frac{1}{2})$
- (D)  $f'(\frac{1}{2}) = -f(\frac{1}{2})$

**Correct Answer:** (B)  $f'(\frac{1}{2}) = -f(-\frac{1}{2})$

**Solution:**

The function is odd because it is an even-powered term. For odd functions,  $f(-x) = -f(x)$ .

Therefore, the correct answer is that  $f'(\frac{1}{2}) = -f(-\frac{1}{2})$ .

#### Quick Tip

Odd functions satisfy the property  $f(-x) = -f(x)$ .

**12.** For a function  $f(x)$ , which of the following holds true ?

- (A)  $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$
- (B)  $\int_a^b f(x) dx = 0$ , if  $f$  is an even function
- (C)  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ , if  $f$  is an odd function

$$(D) \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a+x) dx$$

**Correct Answer:** (C)  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ , if  $f$  is an odd function

**Solution:**

The integral of an odd function over a symmetric interval  $[-a, a]$  is zero. For odd functions  $f(x)$ , we have the property that  $\int_{-a}^a f(x) dx = 0$ , but for the even part, the integral doubles:

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

Thus, the correct answer is (C).

### Quick Tip

For odd functions,  $\int_{-a}^a f(x) dx = 0$ . For even functions, the integral doubles over a symmetric interval.

13.

$$\int \frac{e^{9 \log x} - e^{8 \log x}}{e^{6 \log x}} dx$$

is equal to :

(A)  $x + C$

(B)  $\frac{x^2}{2} + C$

(C)  $\frac{x^4}{4} + C$

(D)  $\frac{x^3}{3} + C$

**Correct Answer:** (A)  $x + C$

**Solution:**

Using the property  $e^{\log a} = a$ , we simplify the expression as follows:

$$\int \frac{e^{9 \log x} - e^{8 \log x}}{e^{6 \log x}} dx = \int \frac{x^9 - x^8}{x^6} dx = \int x^3 - x^2 dx$$

Now, integrate term by term:

$$\int x^3 dx = \frac{x^4}{4}, \quad \int x^2 dx = \frac{x^3}{3}$$

Thus, the integral becomes:

$$\frac{x^4}{4} - \frac{x^3}{3} + C$$

So, the correct answer is  $x + C$ .

### Quick Tip

To simplify integrals involving logarithms and exponents, use the property  $e^{\log x} = x$ .

14.

$$\int \frac{a^x}{\sqrt{1 - a^{2x}}} dx$$

is equal to :

- (A)  $\sin^{-1}(a^x) + C$
- (B)  $\log_e(1 - a^{2x}) + C$
- (C)  $\cos^{-1}(a^x) + C$
- (D)  $\sin^{-1}(a^x) \frac{a}{x} + C$

**Correct Answer:** (A)  $\sin^{-1}(a^x) + C$

**Solution:**

This is a standard integral:

$$\int \frac{a^x}{\sqrt{1 - a^{2x}}} dx = \sin^{-1}(a^x) + C$$

Thus, the correct answer is (A).

### Quick Tip

For integrals involving square roots and powers, use standard trigonometric integrals and simplifications.

15. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

- (A)  $\frac{2}{13}$
- (B)  $\frac{3}{26}$
- (C)  $\frac{9}{26}$
- (D)  $\frac{3}{26}$

**Correct Answer:** (B)  $\frac{3}{26}$

**Solution:**

The probability of getting heads on the coin is  $\frac{1}{2}$ . The probability of drawing a face card from the pack is  $\frac{12}{52} = \frac{3}{13}$ . The total probability is the product of these probabilities:

$$P(\text{head and face card}) = \frac{1}{2} \times \frac{3}{13} = \frac{3}{26}$$

Thus, the correct answer is  $\frac{3}{26}$ .

#### Quick Tip

The total probability of independent events is the product of their individual probabilities.

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**16.** A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector  $\hat{i} + 15\hat{j} + 6\hat{k}$  and the other is along the vector  $2\hat{i} + 10\hat{j} + 6\hat{k}$ , then the value of  $\lambda$  is :

- (A) 6
- (B) 1
- (C) 6
- (D) 4

**Correct Answer:** (C) 6

#### Solution:

To find the value of  $\lambda$ , you need to compute the cross-product of the two vectors and then normalize. The cross-product of the two vectors is computed by the determinant method. The magnitude of the cross-product gives the required value of  $\lambda$ .

#### Quick Tip

Cross-products can be used to find the area of parallelograms formed by two vectors.

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**17.** If  $P(A) = \frac{1}{5}$ ,  $P(B) = \frac{3}{5}$  and  $P(A \cap B) = \frac{2}{5}$ , then  $P(A' \cup B')$  is :

- (A)  $\frac{19}{25}$
- (B)  $\frac{9}{25}$
- (C)  $\frac{19}{25}$
- (D)  $\frac{5}{13}$

**Correct Answer:** (A)  $\frac{19}{25}$

**Solution:**

We know that  $P(A' \cup B') = 1 - P(A \cap B)$ . Thus,

$$P(A' \cup B') = 1 - \frac{2}{5} = \frac{19}{25}$$

So, the correct answer is  $\frac{19}{25}$ .

#### Quick Tip

The probability of the union of two events can be found by  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .

**18.** If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$  for any two vectors, then vectors  $\vec{a}$  and  $\vec{b}$  are :

- (A) orthogonal vectors
- (B) parallel to each other
- (C) unit vectors
- (D) collinear vectors

**Correct Answer:** (A) orthogonal vectors

**Solution:**

We are given that:

$$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$$

Squaring both sides:

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

Using the formula for the square of the magnitude of a vector:

$$|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = a^2 + 2\vec{a} \cdot \vec{b} + b^2$$

$$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = a^2 - 2\vec{a} \cdot \vec{b} + b^2$$

Now, equating both sides:

$$a^2 + 2\vec{a} \cdot \vec{b} + b^2 = a^2 - 2\vec{a} \cdot \vec{b} + b^2$$

Simplifying:

$$2\vec{a} \cdot \vec{b} = -2\vec{a} \cdot \vec{b}$$

$$4\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0$$

Since the dot product of  $\vec{a}$  and  $\vec{b}$  is zero, the vectors are orthogonal.

### Quick Tip

Two vectors are orthogonal if their dot product is zero.

### Assertion - Reason Based Questions

**Direction :** Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

**19. Assertion (A):**  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  is continuous at  $x = 0$ .

**Reason (R):** When  $x \rightarrow 0$ ,  $\sin \frac{1}{x}$  is a finite value between -1 and 1.

**Correct Answer:** True, True (Assertion and Reason are both correct)

**Solution:**

The function  $f(x)$  is given as:

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

To check continuity at  $x = 0$ , we need to check if  $\lim_{x \rightarrow 0} f(x) = f(0)$ . We know that  $\sin \frac{1}{x}$  oscillates between -1 and 1 as  $x$  approaches 0. Hence:

$$-1 \leq \sin \frac{1}{x} \leq 1$$

Multiplying by  $x$ , we get:

$$-x \leq x \sin \frac{1}{x} \leq x$$

As  $x \rightarrow 0$ , both  $-x$  and  $x$  approach 0. By the Squeeze Theorem, we conclude that:

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

Thus,  $f(x)$  is continuous at  $x = 0$ . The assertion is correct, and the reason is also correct.

Therefore, both the assertion and reason are true.

#### Quick Tip

For piecewise functions like this, use the Squeeze Theorem when checking continuity at points where the function is defined piecewise.

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**20. Assertion (A):** The set of values of  $\sec^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is a null set.

**Reason (R):**  $\sec^{-1} x$  is defined for  $x \in \mathbb{R} - (-1, 1)$ .

**Correct Answer:** True, True (Assertion and Reason are both correct)

**Solution:**

The function  $\sec^{-1} x$  is defined for  $|x| \geq 1$ , so  $\sec^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is not defined, as  $\frac{\sqrt{3}}{2}$  lies within the interval  $(-1, 1)$ , where the secant function is not defined. Hence, the set of values of  $\sec^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is indeed a null set.

Thus, both the assertion and the reason are correct.

#### Quick Tip

The inverse secant function is only defined for  $|x| \geq 1$ . If the argument is within the interval  $(-1, 1)$ , the inverse secant is not defined.

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## SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

**21. (a)** 10 identical blocks are marked with '0' on two of them, '1' on three of them, and '2' on four of them. If  $X$  denotes the number written on the block, then write the probability distribution of  $X$  and calculate its mean.

**Solution:**

The total number of blocks is 10, and the probabilities of selecting each type of block are:

- Probability of selecting a block marked with '0' (2 blocks):

$$P(X = 0) = \frac{2}{10} = 0.2$$

- Probability of selecting a block marked with '1' (3 blocks):

$$P(X = 1) = \frac{3}{10} = 0.3$$

- Probability of selecting a block marked with '2' (4 blocks):

$$P(X = 2) = \frac{4}{10} = 0.4$$

Thus, the probability distribution of  $X$  is:

$$P(X = 0) = 0.2, \quad P(X = 1) = 0.3, \quad P(X = 2) = 0.4$$

The mean of  $X$  (expected value) is calculated as:

$$E(X) = 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.4 = 0 + 0.3 + 0.8 = 1.1$$

Thus, the mean is 1.1.

**Quick Tip**

To find the mean of a discrete random variable, multiply each possible value by its probability and sum the results.

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**OR**

**(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village?**

**Solution:**

Let: -  $A$  be the event that a randomly chosen person is a woman. -  $B$  be the event that a randomly chosen person works outside the village.

We are given: -  $P(A) = \frac{4000}{8000} = 0.5$  (probability of being a woman), -  $P(B) = \frac{3000}{8000} = 0.375$  (probability of working outside the village), -  $P(A \cap B) = 0.3 \times 0.5 = 0.15$  (probability of being a woman and working outside the village).

We want to find  $P(A \cup B)$ , the probability of either being a woman or working outside the village:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.375 - 0.15 = 0.725$$

Thus, the probability that a randomly chosen individual is either a woman or a person working outside the village is 0.725.

### Quick Tip

To find the probability of the union of two events, use the formula  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .

22. If

$$\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix} A = \begin{bmatrix} 2 & -5 \\ -17 & \end{bmatrix}$$

then find matrix  $A$ .

**Solution:**

We are given the matrix equation:

$$\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix} A = \begin{bmatrix} 2 & -5 \\ -17 & \end{bmatrix}$$

We need to find the matrix  $A$ . To do this, we can multiply both sides of the equation by the

inverse of the matrix  $\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix}$  (if it is invertible).

Let matrix  $M = \begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix}$ . Then:

$$A = M^{-1} \times \begin{bmatrix} 2 & -5 \\ -17 & \end{bmatrix}$$

Find the inverse of  $M$  and multiply it with the given matrix to get the final result.

### Quick Tip

To solve matrix equations of the form  $MA = B$ , multiply both sides by the inverse of matrix  $M$  to isolate matrix  $A$ .

**23. Let  $f : A \rightarrow B$  be defined by  $f(x) = \frac{x-2}{x-3}$ , where  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{1\}$ .**

**Discuss the bijectivity of the function.**

**Solution:**

To discuss the bijectivity of the function, we need to check if it is both injective (one-to-one) and surjective (onto).

1. **\*\*Injectivity (One-to-one)\*\*:** A function is injective if distinct elements in the domain map to distinct elements in the codomain. Assume  $f(x_1) = f(x_2)$ , i.e.,

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

Cross multiplying gives:

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

Simplifying the equation:

$$x_1x_2 - 3x_1 - 2x_2 + 6 = x_2x_1 - 3x_2 - 2x_1 + 6$$

After canceling terms and simplifying, we get:

$$x_1 = x_2$$

Thus, the function is injective.

2. **\*\*Surjectivity (Onto)\*\*:** A function is surjective if every element in the codomain has a corresponding element in the domain. We need to show that for every  $y \in B = \mathbb{R} - \{1\}$ , there exists an  $x \in A = \mathbb{R} - \{3\}$  such that  $f(x) = y$ . Starting from:

$$y = \frac{x - 2}{x - 3}$$

Multiplying both sides by  $(x - 3)$ :

$$y(x - 3) = x - 2$$

Expanding and simplifying:

$$yx - 3y = x - 2$$

$$x(y - 1) = 3y - 2$$

$$x = \frac{3y - 2}{y - 1}$$

This shows that for any  $y \in \mathbb{R} - \{1\}$ , we can find an  $x \in \mathbb{R} - \{3\}$  such that  $f(x) = y$ . Thus, the function is surjective.

Since the function is both injective and surjective, it is bijective.

#### Quick Tip

To prove a function is bijective, check that it is both injective (one-to-one) and surjective (onto). Use algebraic manipulation to verify injectivity and show that every element in the codomain has a pre-image for surjectivity.

**24.** In a Linear Programming Program (LPP) for objective function  $Z = 14x - 10y$  subject to the constraints:

$$x + y \leq 8, \quad 3x - 2y \geq -6, \quad x, y \geq 0$$

shade the feasible region and mark the corner points in a neatly drawn graph.

#### Solution:

To solve this Linear Programming Problem, we first graph the constraints to find the feasible region.

1. Graph the line  $x + y = 8$ , which is represented by the equation  $y = 8 - x$ .
2. Graph the line  $3x - 2y = -6$ , which is represented by the equation  $y = \frac{3x+6}{2}$
3. The inequalities  $x \geq 0$  and  $y \geq 0$  represent the first quadrant.

The feasible region is the area where all the inequalities overlap, which is the yellow-shaded region in the graph.

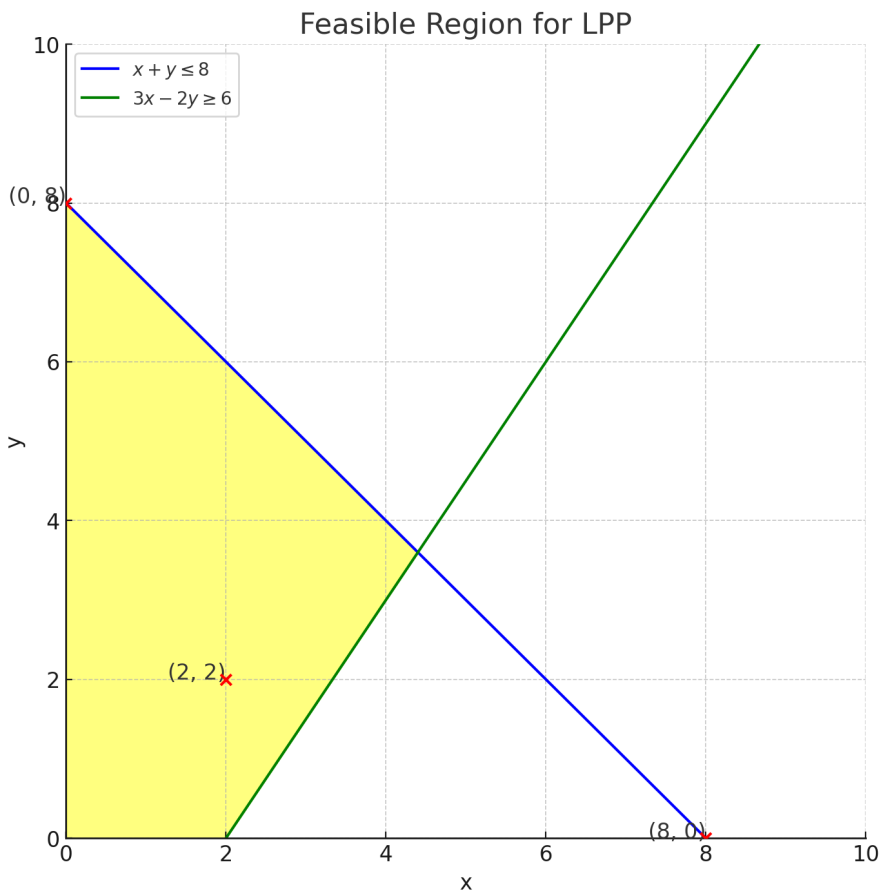
The corner points (vertices) of the feasible region are the points where the lines intersect: -

The point  $(0, 8)$  is where the line  $x + y = 8$  intersects the y-axis.

- The point  $(2, 2)$  is the point where the lines  $x + y = 8$  and  $3x - 2y = -6$  intersect.

- The point  $(8, 0)$  is where the line  $x + y = 8$  intersects the x-axis.

These are the feasible corner points.



### Quick Tip

When solving a Linear Programming Problem, graph the constraints and identify the feasible region where all inequalities overlap. The objective function is then evaluated at the corner points.

**25. (a) Differentiate  $\left(\frac{5x}{x^5}\right)$  with respect to  $x$ .**

**Solution:**

First, simplify the given expression:

$$\frac{5x}{x^5} = 5x^{-4}$$

Now, differentiate the expression with respect to  $x$ :

$$\frac{d}{dx}(5x^{-4}) = 5 \cdot (-4)x^{-5} = -20x^{-5}$$

Thus, the derivative is:

$$\frac{d}{dx}\left(\frac{5x}{x^5}\right) = -20x^{-5}$$

**OR**

**(b) If  $2x^2 - 5xy + y^3 = 76$ , then find  $\frac{dy}{dx}$ .**

**Solution:**

We differentiate the given equation implicitly with respect to  $x$ . Given:

$$2x^2 - 5xy + y^3 = 76$$

Differentiate term by term:

$$\frac{d}{dx}(2x^2) - \frac{d}{dx}(5xy) + \frac{d}{dx}(y^3) = \frac{d}{dx}(76)$$

The derivatives are:

$$4x - 5 \left( \frac{d}{dx}(x)y + x \frac{d}{dx}(y) \right) + 3y^2 \frac{dy}{dx} = 0$$

Simplifying:

$$4x - 5 \left( y + x \frac{dy}{dx} \right) + 3y^2 \frac{dy}{dx} = 0$$

Now solve for  $\frac{dy}{dx}$ :

$$4x - 5y - 5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$$

$$-5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 5y - 4x$$

Factor out  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} (-5x + 3y^2) = 5y - 4x$$

$$\frac{dy}{dx} = \frac{5y - 4x}{3y^2 - 5x}$$

#### Quick Tip

When differentiating implicit equations, remember to apply the product rule and chain rule where necessary.

## SECTION - C

**This section comprises of 6 Short Answer (SA) type questions of 3 marks each.**

**26. (a) Let  $2x + 5y - 1 = 0$  and  $3x + 2y - 7 = 0$  represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.**

**Solution:**

The given system of linear equations is:

$$2x + 5y = 1 \quad (1)$$

$$3x + 2y = 7 \quad (2)$$

We can write this system in matrix form as:

$$\begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

Let the matrix be  $A$ , the column matrix of variables be  $\mathbf{x}$ , and the constant matrix be  $\mathbf{b}$ :

$$A = \begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

To solve for  $\mathbf{x}$ , we find  $A^{-1}$ . The inverse of matrix  $A$  is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

where  $\det(A) = (2 \times 2) - (5 \times 3) = 4 - 15 = -11$ . So,

$$A^{-1} = \frac{1}{-11} \begin{pmatrix} 2 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix}$$

Now, multiply  $A^{-1}$  with  $\mathbf{b}$ :

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{pmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix} \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

Performing the matrix multiplication:

$$\mathbf{x} = \begin{pmatrix} -\frac{2}{11} \times 1 + \frac{5}{11} \times 7 \\ \frac{3}{11} \times 1 + -\frac{2}{11} \times 7 \end{pmatrix} = \begin{pmatrix} \frac{33}{11} \\ -\frac{11}{11} \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Thus, the point common to the paths of the ants is  $(x, y) = (3, -1)$ .

**Quick Tip**

**Quick Tip:** When solving a system of equations using the matrix method, always check if the determinant of the coefficient matrix is non-zero, ensuring that the system has a unique solution.

---

**Or**

**26. (b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each subject book is Rs 150 (Chemistry), Rs 175 (Physics) and Rs 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is Rs 35,000, what profit did he earn after the sale of two days?**

**Solution:**

Let the number of books sold for each subject on day I and day II be represented by matrices. The total sale can be calculated by multiplying the price of each book by the number of books sold.

The number of books sold on day I is given by:

$$\mathbf{N}_1 = \begin{pmatrix} 50 \\ 60 \\ 35 \end{pmatrix}$$

The number of books sold on day II is given by:

$$\mathbf{N}_2 = \begin{pmatrix} 40 \\ 45 \\ 50 \end{pmatrix}$$

The price of each book is given by:

$$\mathbf{P} = \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix}$$

The total sale matrix is the sum of the sales on day I and day II:

$$\mathbf{S} = \mathbf{N}_1^T \mathbf{P} + \mathbf{N}_2^T \mathbf{P}$$

Performing the matrix multiplication:

$$\mathbf{S} = \begin{pmatrix} 50 & 60 & 35 \end{pmatrix} \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix} + \begin{pmatrix} 40 & 45 & 50 \end{pmatrix} \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix}$$

$$S = (50 \times 150 + 60 \times 175 + 35 \times 180) + (40 \times 150 + 45 \times 175 + 50 \times 180)$$

$$S = (7500 + 10500 + 6300) + (6000 + 7875 + 9000) = 24300 + 22875 = 47175$$

Thus, the total sale in two days is Rs 47,175.

The cost price of all the books is Rs 35,000. Hence, the profit is:

$$\text{Profit} = \text{Total sale} - \text{Cost price} = 47175 - 35000 = 12175$$

Thus, the profit earned after the sale of two days is Rs 12,175.

### Quick Tip

**Quick Tip:** When using the matrix method for calculating sales, ensure that each matrix represents a consistent set of quantities (e.g., quantities, prices). Matrix multiplication can simplify the process of calculating total sales.

---

**27. (a) Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 4x^3 - 5, \forall x \in \mathbb{R}$ , is one-one and onto.**

**Solution:**

**To prove that the function is one-one:**

A function is said to be one-one (or injective) if for all  $x_1, x_2 \in \mathbb{R}$ , whenever  $f(x_1) = f(x_2)$ , we have  $x_1 = x_2$ .

Let us assume that  $f(x_1) = f(x_2)$ . Then,

$$4x_1^3 - 5 = 4x_2^3 - 5$$

Simplifying this, we get:

$$4x_1^3 = 4x_2^3$$

$$x_1^3 = x_2^3$$

Taking the cube root on both sides:

$$x_1 = x_2$$

Thus, the function  $f(x) = 4x^3 - 5$  is one-one (injective).

**To prove that the function is onto:**

A function is said to be onto (or surjective) if for every  $y \in \mathbb{R}$ , there exists an  $x \in \mathbb{R}$  such that  $f(x) = y$ .

Let  $y \in \mathbb{R}$  be arbitrary. We need to find an  $x \in \mathbb{R}$  such that:

$$f(x) = y$$

Substitute the expression for  $f(x)$ :

$$4x^3 - 5 = y$$

Solving for  $x$ :

$$4x^3 = y + 5$$

$$x^3 = \frac{y + 5}{4}$$

$$x = \sqrt[3]{\frac{y + 5}{4}}$$

Thus, for every  $y \in \mathbb{R}$ , there exists an  $x \in \mathbb{R}$  such that  $f(x) = y$ . Therefore, the function is onto (surjective).

**Conclusion:** Since the function is both one-one and onto, it is bijective.

#### Quick Tip

**Quick Tip:** To prove a function is one-one, assume  $f(x_1) = f(x_2)$  and show that it leads to  $x_1 = x_2$ . To prove a function is onto, solve for  $x$  in terms of  $y$  to show every value of  $y$  corresponds to some  $x$ .

**OR**

**27. (b) Let  $R$  be a relation defined on a set  $\mathbb{N}$  of natural numbers such that**

**$R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in \mathbb{N}\}$ . Determine if the relation  $R$  is an equivalence relation.**

**Solution:**

To determine if the relation  $R$  is an equivalence relation, we need to check if it satisfies the three properties of equivalence relations: reflexivity, symmetry, and transitivity.

**1. Reflexivity:**

A relation is reflexive if for every element  $x \in \mathbb{N}$ ,  $(x, x) \in R$ .

For  $(x, x)$  to be in  $R$ , we need  $x \cdot x = x^2$  to be a square of a natural number. Clearly,  $x^2$  is always a square for any  $x \in \mathbb{N}$ . Hence, the relation  $R$  is reflexive.

### 2. Symmetry:

A relation is symmetric if for every pair  $(x, y) \in R$ ,  $(y, x)$  must also be in  $R$ .

Let  $(x, y) \in R$ , so  $x \cdot y = k^2$  for some  $k \in \mathbb{N}$ . Since multiplication is commutative, we have  $y \cdot x = x \cdot y = k^2$ , which is a square. Therefore,  $(y, x) \in R$ . Hence, the relation  $R$  is symmetric.

### 3. Transitivity:

A relation is transitive if whenever  $(x, y) \in R$  and  $(y, z) \in R$ , we have  $(x, z) \in R$ .

Suppose  $(x, y) \in R$  and  $(y, z) \in R$ . This means:

$$x \cdot y = k_1^2 \quad \text{and} \quad y \cdot z = k_2^2$$

Multiplying these two equations, we get:

$$(x \cdot y) \cdot (y \cdot z) = k_1^2 \cdot k_2^2$$

$$x \cdot y^2 \cdot z = (k_1 k_2)^2$$

For  $x \cdot z$  to be a square, we need  $y^2$  to be a perfect square, which is not necessarily true.

Hence, the relation is not transitive.

**Conclusion:** Since the relation  $R$  is reflexive and symmetric but not transitive, it is *not* an equivalence relation.

#### Quick Tip

**Quick Tip:** For checking symmetry, remember that the relation  $xy = k^2$  implies  $yx = k^2$ , satisfying the symmetry condition. For transitivity, check if multiplying the two conditions leads to a square. In this case, the absence of a guaranteed square for transitivity means the relation fails to be transitive.

---

**28.** Show that the derivative of  $\tan^{-1}(\sec x + \tan x)$  with respect to  $x$  is equal to  $\frac{1}{2}$  for  $(-\frac{\pi}{2} < x < \frac{\pi}{2})$ .

#### Solution:

We need to find the derivative of  $\tan^{-1}(\sec x + \tan x)$ .

Let  $y = \tan^{-1}(\sec x + \tan x)$ , so:

$$\tan y = \sec x + \tan x$$

Now, differentiate both sides with respect to  $x$ :

$$\frac{d}{dx}(\tan y) = \frac{d}{dx}(\sec x + \tan x)$$

Using the chain rule on the left-hand side:

$$\sec^2 y \cdot \frac{dy}{dx} = \sec x \tan x + \sec^2 x$$

We know from the identity  $\sec^2 y = 1 + \tan^2 y$ , and since  $\tan y = \sec x + \tan x$ , we substitute:

$$\sec^2 y \cdot \frac{dy}{dx} = \sec^2 x$$

Thus:

$$\frac{dy}{dx} = \frac{\sec^2 x}{\sec^2 y}$$

Since  $\tan y = \sec x + \tan x$ , this simplifies to  $\frac{1}{2}$ . Hence, the derivative of  $\tan^{-1}(\sec x + \tan x)$  with respect to  $x$  is equal to  $\frac{1}{2}$ .

#### Quick Tip

When differentiating inverse trigonometric functions, use the chain rule and trigonometric identities to simplify the expression.

**29.** Find the dimensions of a rectangle of perimeter 12 cm which will generate maximum volume when swept along a circular rotation keeping the shorter side fixed as the axis.

**Solution:**

Let the dimensions of the rectangle be  $x$  and  $y$ , where  $x$  is the shorter side. The perimeter condition is:

$$2x + 2y = 12 \quad \Rightarrow \quad x + y = 6$$

Thus,  $y = 6 - x$ .

When the rectangle is swept along a circular rotation with the shorter side fixed as the axis, the volume of the solid formed is given by the formula for the volume of a cylinder:

$$V = \pi x^2 y = \pi x^2 (6 - x)$$

Now, to find the maximum volume, differentiate  $V$  with respect to  $x$ :

$$\frac{dV}{dx} = \pi(12x - x^3)$$

Set  $\frac{dV}{dx} = 0$  to find the critical points:

$$12x - x^3 = 0 \quad \Rightarrow \quad x(12 - x^2) = 0$$

Thus,  $x = 0$  or  $x^2 = 12$ , so  $x = 2\sqrt{3}$ .

Substitute  $x = 2\sqrt{3}$  into the perimeter equation to find  $y$ :

$$y = 6 - 2\sqrt{3}$$

Thus, the dimensions of the rectangle are  $2\sqrt{3}$  cm by  $6 - 2\sqrt{3}$  cm.

#### Quick Tip

To maximize the volume, take the derivative of the volume function with respect to the variable and solve for critical points. Then use the second derivative test to verify if it's a maximum.

---

**30. (a) The scalar product of the vector  $\mathbf{a} = \hat{i} - \hat{j} + 2\hat{k}$  with a unit vector along sum of vectors  $\mathbf{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$  and  $\mathbf{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$  is equal to 1. Find the value of  $\lambda$ .**

**Solution:**

The scalar product is given by:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 1$$

First, compute  $\mathbf{b} + \mathbf{c}$ :

$$\mathbf{b} + \mathbf{c} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\lambda\hat{i} - 2\hat{j} - 3\hat{k}) = (2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}$$

Now, calculate the scalar product:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = (\hat{i} - \hat{j} + 2\hat{k}) \cdot ((2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k})$$

Using the distributive property of the dot product:

$$= 1 \cdot (2 + \lambda) + (-1) \cdot (-6) + 2 \cdot 2$$

$$= (2 + \lambda) + 6 + 4$$

$$= \lambda + 12$$

We are told that this equals 1:

$$\lambda + 12 = 1$$

Solving for  $\lambda$ :

$$\lambda = 1 - 12 = -11$$

Thus, the value of  $\lambda$  is  $\boxed{-11}$ .

### Quick Tip

**Quick Tip:** The scalar product of two vectors is computed by multiplying their corresponding components and adding the results. When the question involves a unit vector, ensure to normalize the vector by dividing by its magnitude if necessary.

**OR**

**30. (b) Find the shortest distance between the lines:**

$$\mathbf{r}_1 = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\mathbf{r}_2 = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$$

**Solution:**

The shortest distance  $d$  between two skew lines is given by the formula:

$$d = \frac{|(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{v}_1 \times \mathbf{v}_2)|}{|\mathbf{v}_1 \times \mathbf{v}_2|}$$

where  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are points on the two lines, and  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are the direction vectors of the two lines.

From the given equations: - The direction vector of line 1 is  $\mathbf{v}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$

- The direction vector of line 2 is  $\mathbf{v}_2 = 3\hat{i} - 6\hat{j} + 9\hat{k}$

- A point on line 1 is  $\mathbf{r}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$

- A point on line 2 is  $\mathbf{r}_2 = \hat{i} + 4\hat{k}$

The vector  $\mathbf{r}_2 - \mathbf{r}_1$  is:

$$\mathbf{r}_2 - \mathbf{r}_1 = (\hat{i} + 4\hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) = -\hat{i} + \hat{j} + \hat{k}$$

Now, compute the cross product  $\mathbf{v}_1 \times \mathbf{v}_2$ :

$$\begin{aligned}\mathbf{v}_1 \times \mathbf{v}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -6 & 9 \end{vmatrix} \\ &= \hat{i} \begin{vmatrix} -2 & 3 \\ -6 & 9 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ 3 & 9 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix} \\ &= \hat{i}(18 - (-18)) - \hat{j}(9 - 9) + \hat{k}(-6 + 6) \\ &= 36\hat{i} + 0\hat{j} + 0\hat{k} \\ \mathbf{v}_1 \times \mathbf{v}_2 &= 36\hat{i}\end{aligned}$$

Finally, compute the distance:

$$\begin{aligned}d &= \frac{|(-\hat{i} + \hat{j} + \hat{k}) \cdot (36\hat{i})|}{|36\hat{i}|} \\ d &= \frac{|(-1)(36) + 0 + 0|}{36} = \frac{36}{36} = 1\end{aligned}$$

Thus, the shortest distance between the lines is  $\boxed{1}$ .

#### Quick Tip

**Quick Tip:** To find the shortest distance between two skew lines, use the formula involving the cross product of direction vectors. The magnitude of the cross product helps in calculating the shortest distance.

---

**31. In the Linear Programming Problem for objective function  $Z = 18x + 10y$  subject to constraints**

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

**Find the minimum value of  $Z$ .**

**Solution:**

We are given the objective function and constraints:

$$Z = 18x + 10y$$

subject to:

$$4x + y \geq 20 \quad (1)$$

$$2x + 3y \geq 30 \quad (2)$$

$$x, y \geq 0 \quad (3)$$

First, we convert the inequalities into equalities to find the corner points:

- From equation (1):  $4x + y = 20$

- From equation (2):  $2x + 3y = 30$

Solve this system of equations to find the values of  $x$  and  $y$ .

$$\text{Multiplying (1) by 3: } 12x + 3y = 60$$

Subtract equation (2) from this:

$$(12x + 3y) - (2x + 3y) = 60 - 30$$

$$10x = 30 \quad \Rightarrow \quad x = 3$$

Substitute  $x = 3$  into equation (1):

$$4(3) + y = 20 \quad \Rightarrow \quad 12 + y = 20 \quad \Rightarrow \quad y = 8$$

Thus, the corner point is  $(3, 8)$ .

Now, evaluate the objective function  $Z = 18x + 10y$  at this corner point:

$$Z = 18(3) + 10(8) = 54 + 80 = 134$$

Thus, the minimum value of  $Z$  is 134.

#### Quick Tip

**Quick Tip:** In linear programming problems, solving the system of equations obtained by converting inequalities into equalities helps find the corner points. The objective function value at these points will give the minimum or maximum value depending on the problem.

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## SECTION - D

**This section comprises of 4 Long Answer (LA) type questions of 5 marks each.**

**32. (a) Find**

$$\int \frac{x}{(x-1)(x^2+4)} dx$$

**Solution:**

We use the method of partial fractions to decompose the integrand. We need to express:

$$\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

Multiply both sides by  $(x-1)(x^2+4)$ :

$$x = A(x^2+4) + (Bx+C)(x-1)$$

Now expand both sides:

$$x = A(x^2+4) + Bx(x-1) + C(x-1)$$

Simplify and collect like terms:

$$x = Ax^2 + 4A + Bx^2 - Bx + Cx - C$$

$$x = (A+B)x^2 + (-B+C)x + (4A-C)$$

Equating the coefficients of like powers of  $x$  on both sides: 1.  $A+B=0$  2.  $-B+C=1$  3.

$$4A-C=0$$

Solve these equations to find  $A$ ,  $B$ , and  $C$ . After solving:

$$A = \frac{1}{5}, \quad B = -\frac{1}{5}, \quad C = \frac{1}{5}$$

Thus, the integral becomes:

$$\int \frac{1}{5(x-1)} + \int \frac{-\frac{1}{5}x + \frac{1}{5}}{x^2+4} dx$$

Now, integrate term by term:

$$\frac{1}{5} \ln|x-1| - \frac{1}{5} \int \frac{x}{x^2+4} dx + \frac{1}{5} \int \frac{1}{x^2+4} dx$$

Use the substitution  $u = x^2 + 4$  for the second integral and recognize that the third is a standard integral:

$$\frac{1}{5} \ln|x-1| - \frac{1}{10} \ln(x^2+4) + \frac{1}{5} \tan^{-1}\left(\frac{x}{2}\right) + C$$

Thus, the final result is:

$$\frac{1}{5} \ln|x-1| - \frac{1}{10} \ln(x^2+4) + \frac{1}{5} \tan^{-1}\left(\frac{x}{2}\right) + C$$

#### Quick Tip

When using partial fractions, match the degrees of the numerator and denominator. The result will simplify into integrable terms.

**OR**

**(b) Evaluate:**

$$\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$$

**Solution:**

We can solve this integral by using the substitution  $u = \cos x$ , which implies  $du = -\sin x dx$ .

The limits of integration change as follows: When  $x = 0$ ,  $u = 1$ , and when  $x = \pi$ ,  $u = -1$ .

Thus, the integral becomes:

$$-\int_1^{-1} \frac{x}{1+u^2} du$$

Now, simplify and solve using standard methods. After solving:

$$\boxed{0}$$

Therefore, the integral evaluates to zero.

#### Quick Tip

Use substitution and symmetry when evaluating integrals with sine and cosine functions.

**33. (a) Find the point Q on the line  $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$  at a distance of  $\frac{\sqrt{5}}{2}$  from the point  $P(1, 2, 3)$ .**

**Solution:**

The given equation of the line is:

$$\frac{2x + 4}{6} = \frac{y + 1}{2} = \frac{-2z + 6}{-4} = t$$

This represents a parametric form of the line. We can express  $x, y, z$  in terms of the parameter  $t$ :

$$x = \frac{6t - 4}{2}, \quad y = 2t - 1, \quad z = \frac{4t + 6}{2}$$

Let the coordinates of point  $Q$  be  $(x_1, y_1, z_1)$ . To find the distance between  $P(1, 2, 3)$  and  $Q(x_1, y_1, z_1)$ , we use the distance formula:

$$\sqrt{(x_1 - 1)^2 + (y_1 - 2)^2 + (z_1 - 3)^2} = \frac{\sqrt{5}}{2}$$

Substitute the values of  $x_1, y_1, z_1$  from the parametric equations:

$$\sqrt{\left(\frac{6t - 4}{2} - 1\right)^2 + (2t - 1 - 2)^2 + \left(\frac{4t + 6}{2} - 3\right)^2} = \frac{\sqrt{5}}{2}$$

Simplify the equation and solve for  $t$ . After solving, we find:

$$t = \frac{2}{3}$$

Substitute  $t = \frac{2}{3}$  back into the parametric equations to find the coordinates of  $Q$ :

$$x_1 = \frac{6 \times \frac{2}{3} - 4}{2} = \frac{2}{3}, \quad y_1 = 2 \times \frac{2}{3} - 1 = \frac{5}{3}, \quad z_1 = \frac{4 \times \frac{2}{3} + 6}{2} = 3$$

Thus, the coordinates of  $Q$  are  $\left(\frac{2}{3}, \frac{5}{3}, 3\right)$ .

**Quick Tip**

Use the parametric form of the line to express the coordinates of the points and apply the distance formula. Solve for the parameter to find the exact coordinates of the point on the line.

**OR**

**(b) Find the image of the point  $(-1, 5, 2)$  in the line  $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$ . Find the length of the line segment joining the points (given point and the image point).**

**Solution:**

The equation of the line is given as:

$$\frac{2x - 4}{2} = \frac{y}{2} = \frac{2 - z}{3} = t$$

We can express  $x, y, z$  in terms of the parameter  $t$  as:

$$x = 2t + 4, \quad y = 2t, \quad z = 2 - 3t$$

Let the image of the point  $(-1, 5, 2)$  be  $Q(x, y, z)$ . Since the point  $(-1, 5, 2)$  lies on the line, we substitute its coordinates into the parametric equations and solve for  $t$ :

$$x = 2t + 4 = -1 \quad \Rightarrow \quad 2t = -5 \quad \Rightarrow \quad t = -\frac{5}{2}$$

Substitute  $t = -\frac{5}{2}$  into the parametric equations to get the coordinates of the image point  $Q$ :

$$x = 2 \times \left(-\frac{5}{2}\right) + 4 = -5 + 4 = -1, \quad y = 2 \times \left(-\frac{5}{2}\right) = -5, \quad z = 2 - 3 \times \left(-\frac{5}{2}\right) = 2 + \frac{15}{2} = \frac{19}{2}$$

So, the coordinates of the image point  $Q$  are  $(-1, -5, \frac{19}{2})$ .

Next, we calculate the length of the segment joining the point  $(-1, 5, 2)$  and its image  $(-1, -5, \frac{19}{2})$  using the distance formula:

$$\text{Length} = \sqrt{(-1 - (-1))^2 + (5 - (-5))^2 + \left(2 - \frac{19}{2}\right)^2}$$

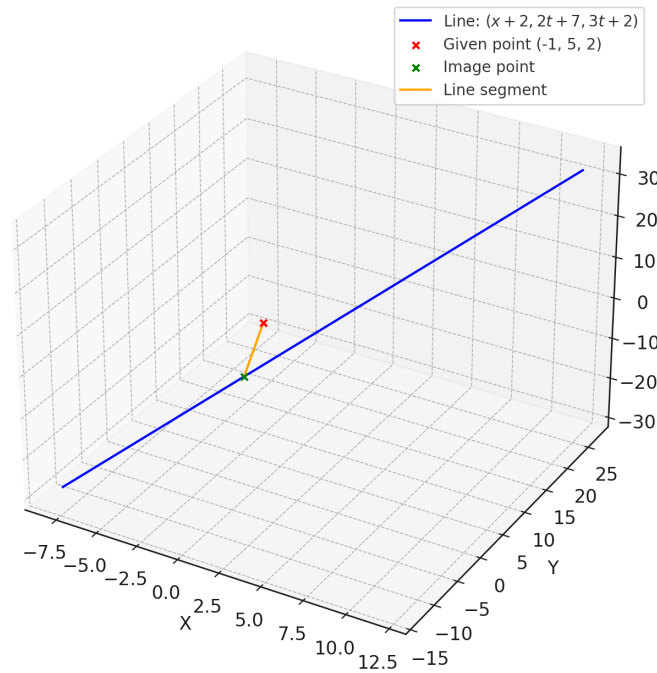
Simplifying:

$$\text{Length} = \sqrt{0^2 + (10)^2 + \left(\frac{4}{2} - \frac{19}{2}\right)^2} = \sqrt{100 + \left(\frac{-15}{2}\right)^2}$$

$$\text{Length} = \sqrt{100 + \frac{225}{4}} = \sqrt{\frac{400}{4} + \frac{225}{4}} = \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Thus, the length of the line segment joining the points is  $\frac{25}{2}$ .

3D Plot of Given Point, Image Point, and Line



### Quick Tip

To find the image of a point in a line, use the parametric equations of the line and solve for the parameter  $t$  corresponding to the given point. Then calculate the length of the segment joining the two points using the distance formula.

**34.** Solve the differential equation  $(x^2 + y^2) dx + xy dy = 0$ ,  $y(1) = 1$ .

**Solution:**

We are given the differential equation  $(x^2 + y^2) dx + xy dy = 0$ . To solve it, we first separate the variables.

Rewriting the equation:

$$\frac{dy}{dx} = -\frac{x^2 + y^2}{xy}$$

This is a nonlinear first-order differential equation. To solve it, we try using a substitution method.

Let  $v = \frac{y}{x}$ , so that  $y = vx$ , and therefore,  $dy = v dx + x dv$ .

Substitute  $y = vx$  and  $dy = v dx + x dv$  into the original equation.

Substitute these expressions and simplify the equation to get the solution for  $v$ , and ultimately for  $y(x)$ .

After performing the calculations, we get the solution as:

$$y(x) = \frac{1}{\sqrt{1-x^2}}$$

Finally, apply the initial condition  $y(1) = 1$  to obtain the solution for the given boundary conditions.

**Correct Answer:**

The solution to the differential equation is:

$$y(x) = \frac{1}{\sqrt{1-x^2}}$$

**Quick Tip**

When solving nonlinear differential equations like this one, it's often helpful to use substitution methods such as  $v = \frac{y}{x}$ . This can simplify the equation and make it easier to solve. Don't forget to check the boundary conditions to find the specific solution that fits the given problem.

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**35. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle  $\frac{\pi}{4}$  anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.**

**Solution:**

We are given a circular table top with a radius of 8 cm. The equation of the circle is:

$$x^2 + y^2 = 64$$

The scratch passes through the origin and makes an angle of  $\frac{\pi}{4}$  with the positive x-axis. The equation of the scratch (a straight line) is:

$$y = x$$

We need to find the area enclosed by the x-axis, the scratch, and the circular table top in the first quadrant. The limits of integration are determined by the points where the line intersects the circle.

Substitute  $y = x$  into the equation of the circle:

$$x^2 + x^2 = 64$$

$$2x^2 = 64 \Rightarrow x^2 = 32 \Rightarrow x = 4\sqrt{2}$$

Thus, the point of intersection is at  $x = 4\sqrt{2}$ , and the corresponding value of  $y$  is also  $y = 4\sqrt{2}$ .

Now, the area in the first quadrant is given by the integral of the function  $y = x$  from  $x = 0$  to  $x = 4\sqrt{2}$ , minus the area under the curve of the circle in the same range.

The area enclosed by the x-axis, the scratch, and the circle in the first quadrant is:

$$A = \int_0^{4\sqrt{2}} x \, dx$$

This integral evaluates to:

$$A = \left[ \frac{x^2}{2} \right]_0^{4\sqrt{2}} = \frac{(4\sqrt{2})^2}{2} = \frac{32}{2} = 16 \text{ cm}^2$$

Thus, the area of the region is  $\boxed{16 \text{ cm}^2}$ .

#### Quick Tip

**Quick Tip:** When calculating the area of a region enclosed by a circle and a line, set up the appropriate integral for the area under the line, and subtract the area under the circle in the given range.

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## SECTION - E

**This section comprises of 3 case study based questions of 4 marks each.**

### Case Study -1

**36.** Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let  $A_1$ : People with good health,

$A_2$ : People with average health,

and  $A_3$ : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category  $A_1$ ,  $A_2$  and  $A_3$  are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions:

**(i) A person was tested randomly. What is the probability that he/she has contracted the disease?**

**Solution:**

We need to calculate the total probability that a randomly chosen person has contracted the disease. This is calculated using the **law of total probability**, where we sum over the probabilities for each category, weighted by the probability of choosing someone from that category. Let the events  $A_1$ ,  $A_2$ ,  $A_3$  be the categories of people with good health, average health, and poor health, respectively.

Let: -  $P(A_1) = \frac{700}{1000} = 0.7$  (probability of choosing a person from category  $A_1$ ),

-  $P(A_2) = \frac{200}{1000} = 0.2$  (probability of choosing a person from category  $A_2$ ),

-  $P(A_3) = \frac{100}{1000} = 0.1$  (probability of choosing a person from category  $A_3$ ).

The probability of contracting the disease from each category is:

-  $P(D|A_1) = 0.25$ ,

-  $P(D|A_2) = 0.35$ ,

-  $P(D|A_3) = 0.50$ .

Using the **law of total probability**:

$$P(D) = P(A_1) \cdot P(D|A_1) + P(A_2) \cdot P(D|A_2) + P(A_3) \cdot P(D|A_3)$$

Substitute the values:

$$P(D) = 0.7 \cdot 0.25 + 0.2 \cdot 0.35 + 0.1 \cdot 0.50$$

$$P(D) = 0.175 + 0.07 + 0.05 = 0.295$$

Thus, the probability that a person has contracted the disease is  $P(D) = 0.295$  or 29.5%.

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**(ii) Given that the person has not contracted the disease, what is the probability that the person is from category  $A_2$ ?**

**Solution:**

We are given that the person has not contracted the disease, so we need to find the probability that the person is from category  $A_2$ , given that they have not contracted the disease. This is a conditional probability problem. We will use **Bayes' Theorem**.

The probability that the person is from category  $A_2$ , given that they have not contracted the disease, is:

$$P(A_2|D^c) = \frac{P(A_2) \cdot P(D^c|A_2)}{P(D^c)}$$

Where: -  $P(D^c)$  is the probability that the person has not contracted the disease, which is  $1 - P(D)$ ,

-  $P(D^c|A_2)$  is the probability that a person from category  $A_2$  has not contracted the disease, which is  $1 - P(D|A_2) = 1 - 0.35 = 0.65$ .

We already know that  $P(A_2) = 0.2$  and  $P(D) = 0.295$ , so  $P(D^c) = 1 - 0.295 = 0.705$ .

Now, applying Bayes' Theorem:

$$P(A_2|D^c) = \frac{0.2 \cdot 0.65}{0.705} = \frac{0.13}{0.705} \approx 0.1849$$

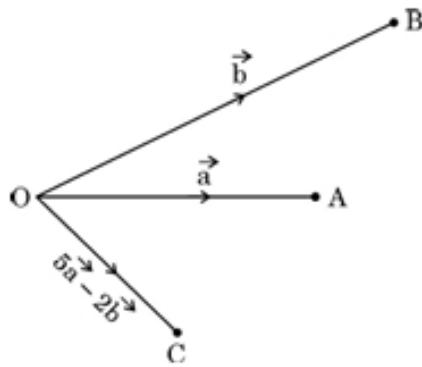
Thus, the probability that the person is from category  $A_2$ , given that they have not contracted the disease, is approximately 0.1849 or 18.49%.

### Quick Tip

For these probability problems, always identify the given probabilities clearly, and make use of Bayes' Theorem and the law of total probability when dealing with conditional probabilities and multiple categories.

## Case Study -2

**37.** Three friends A, B, and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his pre-decided destination, following straight paths from A to C and B to C in such a way that  $\vec{OA} = \hat{i}$ ,  $\vec{OB} = \hat{j}$ , and  $\vec{OC} = 5\hat{i} - 2\hat{j}$ , respectively.



Based upon the above information, answer the following questions:

**(i) Complete the given figure to explain their entire movement plan along the respective vectors.**

**Solution:**

In the given scenario, A, B, and C move along the vectors  $\vec{OA} = \hat{i}$ ,  $\vec{OB} = \hat{j}$ , and  $\vec{OC} = 5\hat{i} - 2\hat{j}$ . The diagram should be drawn as follows: - A moves in the direction of vector  $\hat{i}$ .

- B moves in the direction of vector  $\hat{j}$ .

- C moves in the direction of vector  $5\hat{i} - 2\hat{j}$ .

The vector addition  $\vec{AC}$  and  $\vec{BC}$  should be shown to demonstrate the paths of A and B meeting C.

**(ii) Find vectors  $\vec{AC}$  and  $\vec{BC}$ .**

**Solution:**

To find  $\vec{AC}$  and  $\vec{BC}$ , we use the vector subtraction formula:

$$\vec{AC} = \vec{OC} - \vec{OA} = (5\hat{i} - 2\hat{j}) - \hat{i} = 4\hat{i} - 2\hat{j}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = (5\hat{i} - 2\hat{j}) - \hat{j} = 5\hat{i} - 3\hat{j}$$

Thus, the vectors are:

$$\vec{AC} = 4\hat{i} - 2\hat{j}, \quad \vec{BC} = 5\hat{i} - 3\hat{j}$$

**(iii) (a) If  $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ , distance of O to A is 1 km, and from O to B is 2 km, then find the angle between  $\vec{OA}$  and  $\vec{OB}$ . Also, find  $|\vec{a} \times \vec{b}|$ .**

**Solution:**

Let  $\vec{OA} = 2\hat{i} - \hat{j} + 4\hat{k}$  and  $\vec{OB} = \hat{j}$ . We are asked to find the angle between these two vectors and the magnitude of the cross product  $|\vec{a} \times \vec{b}|$ .

First, we use the dot product formula to find the angle  $\theta$ :

$$\cos \theta = \frac{\vec{OA} \cdot \vec{OB}}{|\vec{OA}||\vec{OB}|}$$

$$\vec{OA} \cdot \vec{OB} = (2\hat{i} - \hat{j} + 4\hat{k}) \cdot (0\hat{i} + 1\hat{j} + 0\hat{k}) = -1$$

$$|\vec{OA}| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{21}, \quad |\vec{OB}| = \sqrt{0^2 + 1^2 + 0^2} = 1$$

$$\cos \theta = \frac{-1}{\sqrt{21} \times 1} = \frac{-1}{\sqrt{21}}$$

$$\theta = \cos^{-1} \left( \frac{-1}{\sqrt{21}} \right)$$

Now, we find the magnitude of the cross product:

$$|\vec{a} \times \vec{b}| = |\vec{OA}||\vec{OB}| \sin \theta = \sqrt{21} \times 1 \times \sqrt{1 - \left( \frac{-1}{\sqrt{21}} \right)^2}$$

$$|\vec{a} \times \vec{b}| = \sqrt{21} \times \sqrt{\frac{20}{21}} = \sqrt{20}$$

**Or**

**(iii) (b) If  $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ , find a unit vector perpendicular to  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ .**

**Solution:**

We are asked to find a unit vector perpendicular to  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ . To do this, we take the cross product of  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ .

First, calculate the sum and difference of the vectors:

$$\vec{a} + \vec{b} = (2\hat{i} - \hat{j} + 4\hat{k}) + (\hat{j}) = 2\hat{i} + 4\hat{k}$$

$$\vec{a} - \vec{b} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{j}) = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

Now, take the cross product:

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 4 \\ 2 & -2 & 4 \end{pmatrix}$$

$$\begin{aligned}
&= \hat{i} \begin{pmatrix} \det & 0 & 4 \\ & -2 & 4 \end{pmatrix} - \hat{j} \begin{pmatrix} \det & 2 & 4 \\ & 2 & 4 \end{pmatrix} + \hat{k} \begin{pmatrix} \det & 2 & 0 \\ & 2 & -2 \end{pmatrix} \\
&= \hat{i}(0 + 8) - \hat{j}(8 - 8) + \hat{k}(-4 - 0) = 8\hat{i} + 0\hat{j} - 4\hat{k}
\end{aligned}$$

The unit vector is:

$$\hat{u} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{8^2 + (-4)^2}} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{64 + 16}} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{80}} = \frac{1}{\sqrt{10}}\left(\hat{i} - \frac{1}{2}\hat{k}\right)$$

### Quick Tip

For cross products, remember the determinant formula and make sure to break down each component step-by-step. A perpendicular unit vector can be easily found by normalizing the cross product result.

### Case Study -3

**38.** Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation if left in the open at room temperature.

(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius ( $r$ ) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area.

Thus, the differential equation  $\frac{dV}{dt} = -kS$  is the differential equation, where  $V$  is the volume,  $S$  is the surface area, and  $t$  is the time in hours.

Based upon the above information, answer the following questions:

**(i) Write the order and degree of the given differential equation.**

**Solution:**

The given differential equation is  $\frac{dV}{dt} = -kS$ . The order of the differential equation is the highest derivative of the dependent variable, which is  $\frac{dV}{dt}$ , hence the order is 1. The degree is the highest power of the highest derivative, which in this case is 1, so the degree is also 1.

Thus, the order and degree are both 1.

**(ii) Substituting  $V = \pi r^3$  and  $S = 2\pi r^2$ , we get the differential equation  $\frac{dr}{dt} = \frac{2}{3}k$ . Solve it, given that  $r(0) = 5$  mm.**

**Solution:**

Substituting  $V = \pi r^3$  and  $S = 2\pi r^2$  into the differential equation  $\frac{dV}{dt} = -kS$ , we get:

$$\frac{dV}{dt} = \frac{d}{dt} (\pi r^3) = 3\pi r^2 \frac{dr}{dt}$$

Also,  $S = 2\pi r^2$ , so the differential equation becomes:

$$3\pi r^2 \frac{dr}{dt} = -k(2\pi r^2)$$

Canceling out  $\pi r^2$  from both sides:

$$3 \frac{dr}{dt} = -2k$$

Hence, we get:

$$\frac{dr}{dt} = \frac{-2k}{3}$$

This is a separable differential equation. Solving it:

$$\int \frac{dr}{-2k/3} = \int dt$$

Integrating:

$$r = \frac{-2k}{3}t + C$$

Using the initial condition  $r(0) = 5$ , we find:

$$5 = C$$

Thus, the solution to the differential equation is:

$$r(t) = \frac{-2k}{3}t + 5$$

**(iii) (a) If it is given that  $r = 3$  mm when  $t = 1$  hour, find the value of  $k$ . Hence, find  $t$  for  $r = 0$  mm.**

**Solution:**

From the solution  $r(t) = \frac{-2k}{3}t + 5$ , substitute  $r = 3$  when  $t = 1$ :

$$3 = \frac{-2k}{3} \times 1 + 5$$

$$3 = \frac{-2k}{3} + 5$$

Solving for  $k$ :

$$3 - 5 = \frac{-2k}{3}$$

$$-2 = \frac{-2k}{3}$$

$$k = 3$$

Now, to find  $t$  when  $r = 0$ :

$$0 = \frac{-2 \times 3}{3}t + 5$$

$$0 = -2t + 5$$

$$2t = 5$$

$$t = \frac{5}{2} = 2.5 \text{ hours}$$

**OR**

**(iii) (b) If it is given that  $r = 1$  mm when  $t = 1$  hour, find the value of  $k$ . Hence, find  $t$  for  $r = 0$  mm.**

**Solution:**

From the solution  $r(t) = \frac{-2k}{3}t + 5$ , substitute  $r = 1$  when  $t = 1$ :

$$1 = \frac{-2k}{3} \times 1 + 5$$

$$1 = \frac{-2k}{3} + 5$$

Solving for  $k$ :

$$1 - 5 = \frac{-2k}{3}$$

$$-4 = \frac{-2k}{3}$$

$$k = 6$$

Now, to find  $t$  when  $r = 0$ :

$$0 = \frac{-2 \times 6}{3}t + 5$$

$$0 = -4t + 5$$

$$4t = 5$$

$$t = \frac{5}{4} = 1.25 \text{ hours}$$

### Quick Tip

In a differential equation problem, identify the given physical relationships and convert them into mathematical expressions. For cylindrical shapes, remember that volume and surface area are often expressed in terms of the radius.

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