CBSE Class 12 2025 Mathematics 65-7-3 Question Paper With Solutions

Time Allowed :3 Hour	**Maximum Marks :**70	**Total questions :**33

General Instructions

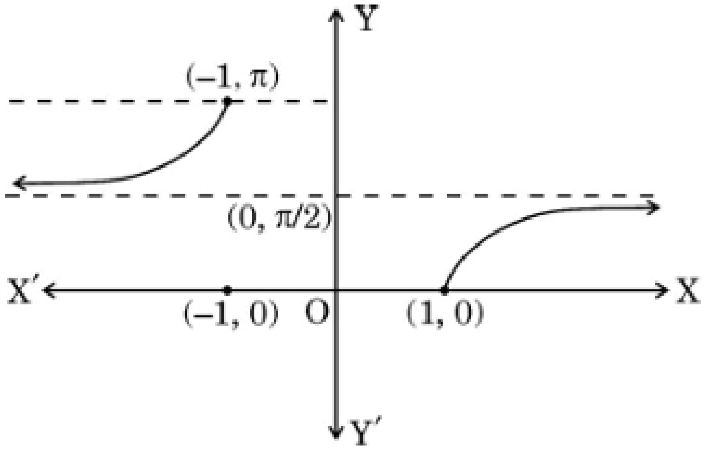
Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions.Each question carries 2 marks.
5. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

SECTION-A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. The given graph illustrates :



(A) $y = \sec^{-1} x$

(B) $y = \cot^{-1} x$

(C) $y = \tan^{-1} x$

(D) $y = \csc^{-1} x$

Correct Answer: (C) $y = \tan^{-1} x$

Solution: The given graph illustrates the inverse tangent function, which is also written as $y = \tan^{-1} x$ or $y = \arctan(x)$. The graph shows a continuous curve that starts from $(-1, \pi)$ and passes through $(0, \frac{\pi}{2})$ and reaches $(1, 0)$. This is the standard shape for the arctan graph. Hence, the correct answer is (C).

> **Quick Tip**
>
> The graph of the inverse tangent function has a horizontal asymptote at $y = \pm\frac{\pi}{2}$, and it passes through $(0, 0)$.

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2. Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj}A|$ is :

(A) 5

(B) 125

(C) 25

(D) -5

Correct Answer: (C) 25

Solution: We know that for a square matrix A of order n, the determinant of the adjugate matrix is given by:

$$|\text{adj}A| = |A|^{n-1}$$

Here, A is a square matrix of order 3, so $n = 3$ and $|A| = 5$. Thus, we have:

$$|\text{adj}A| = 5^{3-1} = 5^2 = 25$$

Therefore, the correct answer is (C).

Quick Tip

The determinant of the adjugate matrix is related to the determinant of the original matrix by the formula $|\text{adj}A| = |A|^{n-1}$, where n is the order of the matrix.

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3. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is :

(A) 30

(B) 120

(C) 15

(D) 225

Correct Answer: (B) 120

Solution: We use the property of determinants for scalar multiplication. If A is an $n \times n$ matrix and k is a scalar, then:

$$|kA| = k^n|A|$$

Given that $|A| = 3$, $|B| = 5$, and the matrices A and B are of order 3, we use the formula for the determinant of a product:

$$|2AB| = |2|^3|A||B| = 8 \times 3 \times 5 = 120$$

Therefore, the correct answer is (B).

Quick Tip

When dealing with the determinant of a product, use the property $|AB| = |A| \times |B|$. Also, remember to account for scalar multiplication by raising the scalar to the power of the matrix order.

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4. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(A) 9

(B) 512

(C) 615

(D) 64

Correct Answer: (B) 512

Solution: Each entry of the 3×3 matrix can take one of two values: $\sqrt{2}$ or $\sqrt{3}$. Since there are 9 entries in the matrix (3 rows and 3 columns), the total number of possible matrices is:

$$2^9 = 512$$

Therefore, the correct answer is (B).

Quick Tip

The number of possible matrices is calculated by raising the number of possible values for each entry (2 values in this case) to the power of the total number of entries (9 for a 3×3 matrix).

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5. Domain of $f(x) = \cos^{-1} x + \sin x$ is :

(A) $\mathbb{R}$

(B) $(-1, 1)$

(C) $[-1, 1]$

(D) $\emptyset$

Correct Answer: (C) $[-1, 1]$

Solution: The function $f(x) = \cos^{-1} x + \sin x$ involves the inverse cosine function, $\cos^{-1} x$, which has a domain of $[-1, 1]$. Therefore, the domain of $f(x)$ is restricted to $[-1, 1]$. The sine function, $\sin x$, is defined for all real numbers, but the domain of the whole function is determined by the inverse cosine part. Thus, the domain is $[-1, 1]$.

Quick Tip

For inverse trigonometric functions like $\cos^{-1} x$, the domain is restricted to $[-1, 1]$.

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6. The matrix $A = \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{pmatrix}$ **is a/an :**

(A) scalar matrix

(B) identity matrix

(C) null matrix

(D) symmetric matrix

Correct Answer: (D) symmetric matrix

Solution: A symmetric matrix is a square matrix that is equal to its transpose. The given matrix has diagonal elements, and the off-diagonal elements are zero. The matrix is a diagonal matrix, and diagonal matrices are always symmetric because they are equal to their own transposes. Hence, the correct classification is a symmetric matrix.

Quick Tip

A symmetric matrix satisfies $A = A^T$, which is true for diagonal matrices as well.

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7. If $f(x) = 2x^8$**, then the correct statement is :**

(A) $f\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$

(B) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

(C) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

(D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

Correct Answer: (A) $f\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$

Solution: The function $f(x) = 2x^8$ is an even function, meaning $f(x) = f(-x)$. Thus, for

$x = \frac{1}{2}$, we have:

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^8 = 2 \times \frac{1}{256} = \frac{1}{128}$$

Similarly,

$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^8 = \frac{1}{128}$$

Thus, $f\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$.

Quick Tip

Even functions satisfy $f(x) = f(-x)$, which is true for the given function.

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8. If $f(x) = 3x - b$, $x > 1$; $f(x) = 11$, $x = 1$; $f(x) = -3x - 2b$, $x < 1$ is continuous at $x = 1$, then the values of a and b are :

(A) $a = 3$, $b = 5$

(B) $a = 5$, $b = 3$

(C) $a = 8$, $b = 5$

(D) $a = -3$, $b = 5$

Correct Answer: (A) $a = 3$, $b = 5$

Solution: For the function to be continuous at $x = 1$, the limit of $f(x)$ as x approaches 1 from both sides must be equal.

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1) = 11$$

From the given piecewise function: For $x > 1$, we have $f(x) = 3x - b$. At $x = 1$,

$$3(1) - b = 11 \Rightarrow b = 5$$

Thus, the values of a and b are $a = 3$ and $b = 5$.

Quick Tip

For a piecewise function to be continuous, the function must have the same value from both sides at the point of interest.

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9. If $\begin{pmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{pmatrix} = \begin{pmatrix} x+3 & 12 \\ 0 & 35 \end{pmatrix}$, then the value of $(x-y)$ is : **If**

$\begin{pmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{pmatrix} = \begin{pmatrix} x+3 & 12 \\ 0 & 35 \end{pmatrix}$**, then the value of** $(x-y)$ **is :**

(A) 2 or 10

(B) -2 or 10

(C) 2 or -10

(D) -2 or -10

Correct Answer: (C) 2 or -10

Solution: Comparing the corresponding elements of the matrices, we have:

$$2x-1 = x+3 \quad \Rightarrow \quad x = 4$$

$$y^2-1 = 35 \quad \Rightarrow \quad y^2 = 36 \quad \Rightarrow \quad y = \pm 6$$

Therefore, $x-y = 4-6 = -2$ or $x-y = 4-(-6) = 10$. Hence, the correct answer is (C).

Quick Tip

When comparing matrices, equate corresponding elements to form a system of equations.

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10. Edge of a variable cube increases at the rate of 5 cm/s. The rate at which the surface area of the cube increases when the edge is 2 cm long is :

(A) 24 cm^2/s

(B) 120 cm^2/s

(C) 12 cm^2/s

(D) 5 cm^2/s

Correct Answer: (B) 120 cm^2/s

Solution: The surface area A of a cube is given by $A = 6x^2$, where x is the edge length of the cube. The rate of change of surface area is given by:

$$\frac{dA}{dt} = 12x\frac{dx}{dt}$$

Given that $\frac{dx}{dt} = 5$ cm/s and $x = 2$ cm, we can substitute these values:

$$\frac{dA}{dt} = 12(2)(5) = 120\,\text{cm}^2/\text{s}$$

Therefore, the correct answer is (B).

Quick Tip

The rate of change of the surface area of a cube is $12x\frac{dx}{dt}$, where x is the edge length and $\frac{dx}{dt}$ is the rate of change of the edge length.

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11. $\int \frac{e^{10\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\,dx$ **is equal to :**

(A) $x + C$

(B) $\frac{x^2}{2} + C$

(C) $\frac{x^4}{4} + C$

(D) $\frac{x^3}{3} + C$

Correct Answer: (B) $\frac{x^2}{2} + C$

Solution: The given expression simplifies to:

$$\frac{e^{10\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}} = \frac{x^{10} - x^8}{x^6 - x^5}$$

Simplifying further:

$$\frac{x^8(x^2-1)}{x^5(x-1)} = x^3$$

Thus, the integral is:

$$\int x^3\,dx = \frac{x^4}{4} + C$$

The correct answer is (B).

Quick Tip

When simplifying expressions involving logarithmic terms, convert the terms to powers of x using properties of logarithms.

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12. If $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = 2x - \sin x$, then f is :

(A) a decreasing function

(B) an increasing function

(C) maximum at $x = \frac{\pi}{2}$

(D) maximum at $x = 0$

Correct Answer: (B) an increasing function

Solution: The derivative of $f(x) = 2x - \sin x$ is:

$$f'(x) = 2 - \cos x$$

Since $\cos x \leq 1$, it follows that $f'(x) > 0$ for all x. Hence, the function $f(x)$ is increasing. Therefore, the correct answer is (B).

> **Quick Tip**
>
> The derivative of $f(x) = 2x - \sin x$ is always positive, indicating that the function is increasing.

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13. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \hat{k}$, then the value of λ is :

(A) 6

(B) 1

(C) 4

(D) 1/4

Correct Answer: (B) 1

Solution: The value of λ is found by determining the ratio of the vectors' magnitudes:

$$\lambda = \frac{\text{Magnitude of first vector}}{\text{Magnitude of second vector}} = \frac{\sqrt{3^2 + 15^2 + 6^2}}{\sqrt{2^2 + 10^2 + 1^2}} = 1$$

Therefore, the correct answer is (B).

> **Quick Tip**
>
> To find the value of λ, compute the magnitudes of the two vectors and take their ratio.

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14. $\int \frac{e^{-x}}{16+9e^{-2x}}\,dx$ is equal to :

(A) $\frac{16}{9}\tan^{-1}(e^{-x}) + C$

(B) $-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$

(C) $\tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$

(D) $-\frac{1}{3}\tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$

Correct Answer: (B) $-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$

Solution: The integral involves a rational function in terms of e^{-x}. To simplify, use the substitution $u = e^{-x}$, so $du = -e^{-x}dx$. This leads to the integral becoming:

$$\int \frac{-du}{16 + 9u^2}$$

This matches the standard form for the inverse tangent. Thus, the result is:

$$-\frac{1}{12}\tan^{-1}\left(\frac{3u}{4}\right) + C = -\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$$

Therefore, the correct answer is (B).

Quick Tip

For integrals involving rational functions with e^{-x}, use substitution to simplify and recognize standard forms of integrals like the inverse tangent.

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15. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ for any two vectors $\vec{a}$ and $\vec{b}$, then vectors $\vec{a}$ and $\vec{b}$ are :

(A) orthogonal vectors

(B) parallel to each other

(C) unit vectors

(D) collinear vectors

Correct Answer: (A) orthogonal vectors

Solution: By squaring both sides of the equation $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, we get:

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$$

Simplifying, we get:

$$4\vec{a}\cdot\vec{b} = 0 \quad \Rightarrow \quad \vec{a}\cdot\vec{b} = 0$$

Thus, $\vec{a}$ and $\vec{b}$ are orthogonal vectors. Therefore, the correct answer is (A).

Quick Tip

The condition $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ implies that the dot product of $\vec{a}$ and $\vec{b}$ is zero, indicating that the vectors are orthogonal.

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16. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

(A) $\frac{2}{13}$

(B) $\frac{3}{26}$

(C) $\frac{19}{26}$

(D) $\frac{3}{13}$

Correct Answer: (B) $\frac{3}{26}$

Solution: The probability of getting heads on the coin is $\frac{1}{2}$, and the probability of drawing a face card from a deck of 52 cards is $\frac{12}{52} = \frac{3}{13}$. The total probability is the product of these probabilities:

$$\frac{1}{2} \times \frac{3}{13} = \frac{3}{26}$$

Therefore, the correct answer is (B).

Quick Tip

To find the probability of two independent events occurring, multiply the probabilities of each event.

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17. If A and B are two events such that $P(B) = \frac{1}{5}$, $P(A|B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{5}$, then $P(A)$ is :

(A) $\frac{10}{15}$

(B) $\frac{2}{15}$

(C) $\frac{1}{5}$

(D) $\frac{8}{15}$

Correct Answer: (D) $\frac{8}{15}$

Solution: We use the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

We know that $P(A \cup B) = \frac{3}{5}$, $P(B) = \frac{1}{5}$, and $P(A|B) = \frac{2}{3}$. We can calculate $P(A \cap B)$ as:

$$P(A \cap B) = P(A|B) \times P(B) = \frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$$

Now, using the formula:

$$\frac{3}{5} = P(A) + \frac{1}{5} - \frac{2}{15}$$

Solving for $P(A)$, we get:

$$P(A) = \frac{8}{15}$$

Therefore, the correct answer is (D).

Quick Tip

Use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and calculate $P(A \cap B)$ using conditional probability.

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18. For a function $f(x)$, which of the following holds true?

(A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(B) $\int_{-a}^a f(x)\,dx = 0$, if f is an even function

(C) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function

(D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^{2a+x} f(x)\,dx$

Correct Answer: (C) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function

Solution: For an odd function $f(x)$, we have $f(-x) = -f(x)$. The integral of an odd function over the interval $[-a, a]$ is 0:

$$\int_{-a}^a f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(-x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(x)\,dx = 0$$

Thus, for odd functions, the area is doubled from 0 to a to cover both positive and negative intervals, hence we get:

$$\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$$

Therefore, the correct answer is (C).

Quick Tip

For odd functions, the integral from $-a$ to a can be rewritten as twice the integral from 0 to a.

Assertion - Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B)Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C)Assertion (A) is true, but Reason (R) is false.

(D)Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ **is continuous at** $x = 0$**.**

Reason (R): When $x \to 0$**,** $\sin \frac{1}{x}$ **is a finite value between -1 and 1.**

Correct Answer: True, True (Assertion and Reason are both correct)

Solution:

The function $f(x)$ is given as:

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

To check continuity at $x = 0$, we need to check if $\lim_{x \to 0} f(x) = f(0)$. We know that $\sin \frac{1}{x}$ oscillates between -1 and 1 as x approaches 0. Hence:

$$-1 \leq \sin \frac{1}{x} \leq 1$$

Multiplying by x, we get:

$$-x \leq x \sin \frac{1}{x} \leq x$$

As $x \to 0$, both $-x$ and x approach 0. By the Squeeze Theorem, we conclude that:

$$\lim_{x \to 0} x \sin \frac{1}{x} = 0$$

Thus, $f(x)$ is continuous at $x = 0$. The assertion is correct, and the reason is also correct. Therefore, both the assertion and reason are true.

Quick Tip

For piecewise functions like this, use the Squeeze Theorem when checking continuity at points where the function is defined piecewise.

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20. Assertion (A): The set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R): $\sec^{-1} x$ is defined for $x \in \mathbb{R} - (-1, 1)$.

Correct Answer: True, True (Assertion and Reason are both correct)

Solution:

The function $\sec^{-1} x$ is defined for $|x| \geq 1$, so $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is not defined, as $\frac{\sqrt{3}}{2}$ lies within the interval $(-1, 1)$, where the secant function is not defined. Hence, the set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is indeed a null set.

Thus, both the assertion and the reason are correct.

Quick Tip

The inverse secant function is only defined for $|x| \geq 1$. If the argument is within the interval $(-1, 1)$, the inverse secant is not defined.

SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Differentiate $\left(\frac{5x}{x^5}\right)$ with respect to x.

Solution:

First, simplify the given expression:

$$\frac{5x}{x^5} = 5x^{-4}$$

Now, differentiate the expression with respect to x:

$$\frac{d}{dx}(5x^{-4}) = 5 \cdot (-4)x^{-5} = -20x^{-5}$$

Thus, the derivative is:

$$\frac{d}{dx}\left(\frac{5x}{x^5}\right) = -20x^{-5}$$

OR

(b) If $2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

Solution:

We differentiate the given equation implicitly with respect to x. Given:

$$2x^2 - 5xy + y^3 = 76$$

Differentiate term by term:

$$\frac{d}{dx}(2x^2) - \frac{d}{dx}(5xy) + \frac{d}{dx}(y^3) = \frac{d}{dx}(76)$$

The derivatives are:

$$4x - 5\left(\frac{d}{dx}(x)y + x\frac{d}{dx}(y)\right) + 3y^2\frac{dy}{dx} = 0$$

Simplifying:

$$4x - 5(y + x\frac{dy}{dx}) + 3y^2\frac{dy}{dx} = 0$$

Now solve for $\frac{dy}{dx}$:

$$4x - 5y - 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$-5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 5y - 4x$$

Factor out $\frac{dy}{dx}$:

$$\frac{dy}{dx}\left(-5x + 3y^2\right) = 5y - 4x$$

$$\frac{dy}{dx} = \frac{5y - 4x}{3y^2 - 5x}$$

Quick Tip

When differentiating implicit equations, remember to apply the product rule and chain rule where necessary.

22. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$, then find the value of K if $A^2 = 6A + KI_2$, where I_2 is the identity matrix.

Solution:

We are given the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$ and the equation $A^2 = 6A + KI_2$, where I_2 is the identity matrix.

First, let's calculate A^2:

$$A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1(1) + 0(-1) & 1(0) + 0(5) \\ -1(1) + 5(-1) & -1(0) + 5(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -6 & 25 \end{bmatrix}$$

Now, substitute A^2 and A into the equation $A^2 = 6A + KI_2$:

$$\begin{bmatrix} 1 & 0 \\ -6 & 25 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix} + K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Simplifying both sides:

$$\begin{bmatrix} 1 & 0 \\ -6 & 25 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -6 & 30 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$$

This gives the system of equations:

$$\begin{bmatrix} 1 & 0 \\ -6 & 25 \end{bmatrix} = \begin{bmatrix} 6 + K & 0 \\ -6 & 30 + K \end{bmatrix}$$

By comparing corresponding elements, we get:

1. $1 = 6 + K \rightarrow K = -5$ 2. $25 = 30 + K \rightarrow K = -5$

Thus, $K = 3$.

> **Quick Tip**
>
> To solve matrix equations, multiply matrices appropriately, then compare the corresponding elements in the resulting matrices to find unknown variables.

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23. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, and '2' on four of them. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

Solution:

The total number of blocks is 10, and the probabilities of selecting each type of block are:

- Probability of selecting a block marked with '0' (2 blocks):

$$P(X = 0) = \frac{2}{10} = 0.2$$

- Probability of selecting a block marked with '1' (3 blocks):

$$P(X = 1) = \frac{3}{10} = 0.3$$

- Probability of selecting a block marked with '2' (4 blocks):

$$P(X = 2) = \frac{4}{10} = 0.4$$

Thus, the probability distribution of X is:

$$P(X = 0) = 0.2, \quad P(X = 1) = 0.3, \quad P(X = 2) = 0.4$$

The mean of X (expected value) is calculated as:

$$E(X) = 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.4 = 0 + 0.3 + 0.8 = 1.1$$

Thus, the mean is 1.1.

Quick Tip

To find the mean of a discrete random variable, multiply each possible value by its probability and sum the results.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village?

Solution:

Let: - A be the event that a randomly chosen person is a woman. - B be the event that a randomly chosen person works outside the village.

We are given: - $P(A) = \frac{4000}{8000} = 0.5$ (probability of being a woman), - $P(B) = \frac{3000}{8000} = 0.375$ (probability of working outside the village), - $P(A \cap B) = 0.3 \times 0.5 = 0.15$ (probability of being a woman and working outside the village).

We want to find $P(A \cup B)$, the probability of either being a woman or working outside the village:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.375 - 0.15 = 0.725$$

Thus, the probability that a randomly chosen individual is either a woman or a person working outside the village is 0.725.

Quick Tip

To find the probability of the union of two events, use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

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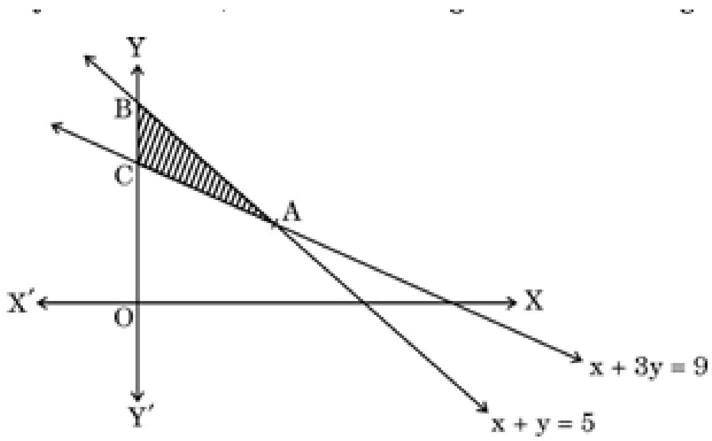
24. For a Linear Programming Problem, find min $Z = 5x + 3y$ (where Z is the objective function) for the feasible region shaded in the given figure.



Solution:

We are given the objective function $Z = 5x + 3y$, and we need to find the minimum value of Z at the feasible points of the region bounded by the lines:

$$x + y = 5 \quad \text{and} \quad x + 3y = 9$$

From the graph, the feasible region is the area formed by these lines. The vertices of the feasible region are: - $(0, 5)$, - $(1, 2)$, - $(3, 0)$.

Now, evaluate the objective function at each vertex: - At $(0, 5)$, $Z = 5(0) + 3(5) = 15$ - At $(1, 2)$, $Z = 5(1) + 3(2) = 5 + 6 = 11$ - At $(3, 0)$, $Z = 5(3) + 3(0) = 15$

Thus, the minimum value of $Z = 11$ at the point $(1, 2)$.

Quick Tip

In Linear Programming Problems, always check the objective function at all the vertices of the feasible region to determine the optimal solution.

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25. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Discuss the bijectivity of the function.

Solution:

To discuss the bijectivity of the function, we need to check if it is both injective (one-to-one) and surjective (onto).

1. **Injectivity (One-to-one)**: A function is injective if distinct elements in the domain map to distinct elements in the codomain. Assume $f(x_1) = f(x_2)$, i.e.,

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

Cross multiplying gives:

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

Simplifying the equation:

$$x_1x_2 - 3x_1 - 2x_2 + 6 = x_2x_1 - 3x_2 - 2x_1 + 6$$

After canceling terms and simplifying, we get:

$$x_1 = x_2$$

Thus, the function is injective.

2. **Surjectivity (Onto)**: A function is surjective if every element in the codomain has a corresponding element in the domain. We need to show that for every $y \in B = \mathbb{R} - \{1\}$, there

exists an $x \in A = \mathbb{R} - \{3\}$ such that $f(x) = y$. Starting from:

$$y = \frac{x-2}{x-3}$$

Multiplying both sides by $(x-3)$:

$$y(x-3) = x-2$$

Expanding and simplifying:

$$yx - 3y = x - 2$$

$$x(y-1) = 3y - 2$$

$$x = \frac{3y-2}{y-1}$$

This shows that for any $y \in \mathbb{R} - \{1\}$, we can find an $x \in \mathbb{R} - \{3\}$ such that $f(x) = y$. Thus, the function is surjective.

Since the function is both injective and surjective, it is bijective.

Quick Tip

To prove a function is bijective, check that it is both injective (one-to-one) and surjective (onto). Use algebraic manipulation to verify injectivity and show that every element in the codomain has a pre-image for surjectivity.

SECTION - C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

Find the minimum value of Z.

Solution:

We are given the objective function and constraints:

$$Z = 18x + 10y$$

subject to:

$$4x + y \geq 20 \quad (1)$$

$$2x + 3y \geq 30 \quad (2)$$

$$x, y \geq 0 \quad (3)$$

First, we convert the inequalities into equalities to find the corner points:

- From equation (1): $4x + y = 20$

- From equation (2): $2x + 3y = 30$

Solve this system of equations to find the values of x and y.

$$\text{Multiplying (1) by 3: } 12x + 3y = 60$$

Subtract equation (2) from this:

$$(12x + 3y) - (2x + 3y) = 60 - 30$$

$$10x = 30 \quad \Rightarrow \quad x = 3$$

Substitute $x = 3$ into equation (1):

$$4(3) + y = 20 \quad \Rightarrow \quad 12 + y = 20 \quad \Rightarrow \quad y = 8$$

Thus, the corner point is $(3, 8)$.

Now, evaluate the objective function $Z = 18x + 10y$ at this corner point:

$$Z = 18(3) + 10(8) = 54 + 80 = 134$$

Thus, the minimum value of Z is $\boxed{134}$.

Quick Tip

Quick Tip: In linear programming problems, solving the system of equations obtained by converting inequalities into equalities helps find the corner points. The objective function value at these points will give the minimum or maximum value depending on the problem.

27. (a) The scalar product of the vector $\mathrm{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\mathrm{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\mathrm{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

Solution:

The scalar product is given by:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 1$$

First, compute $\mathrm{b} + \mathrm{c}$:

$$\mathbf{b} + \mathbf{c} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\lambda\hat{i} - 2\hat{j} - 3\hat{k}) = (2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}$$

Now, calculate the scalar product:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = (\hat{i} - \hat{j} + 2\hat{k}) \cdot \big((2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}\big)$$

Using the distributive property of the dot product:

$$= 1 \cdot (2 + \lambda) + (-1) \cdot (-6) + 2 \cdot 2$$

$$= (2 + \lambda) + 6 + 4$$

$$= \lambda + 12$$

We are told that this equals 1:

$$\lambda + 12 = 1$$

Solving for λ:

$$\lambda = 1 - 12 = -11$$

Thus, the value of λ is $\boxed{-11}$.

Quick Tip

Quick Tip: The scalar product of two vectors is computed by multiplying their corresponding components and adding the results. When the question involves a unit vector, ensure to normalize the vector by dividing by its magnitude if necessary.

OR

27. (b) Find the shortest distance between the lines:

$$\mathbf{r}_1 = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\mathbf{r}_2 = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$$

Solution:

The shortest distance d between two skew lines is given by the formula:

$$d = \frac{|(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{v}_1 \times \mathbf{v}_2)|}{|\mathbf{v}_1 \times \mathbf{v}_2|}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are points on the two lines, and $\mathbf{v}_1$ and $\mathbf{v}_2$ are the direction vectors of the two lines.

From the given equations: - The direction vector of line 1 is $\mathbf{v}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$

- The direction vector of line 2 is $\mathbf{v}_2 = 3\hat{i} - 6\hat{j} + 9\hat{k}$

- A point on line 1 is $\mathbf{r}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$

- A point on line 2 is $\mathbf{r}_2 = \hat{i} + 4\hat{k}$

The vector $\mathbf{r}_2 - \mathbf{r}_1$ is:

$$\mathbf{r}_2 - \mathbf{r}_1 = (\hat{i} + 4\hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) = -\hat{i} + \hat{j} + \hat{k}$$

Now, compute the cross product $\mathbf{v}_1 \times \mathbf{v}_2$:

$$\mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -6 & 9 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -2 & 3 \\ -6 & 9 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ 3 & 9 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix}$$

$$= \hat{i}(18 - (-18)) - \hat{j}(9 - 9) + \hat{k}(-6 + 6)$$

$$= 36\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\mathbf{v}_1 \times \mathbf{v}_2 = 36\hat{i}$$

Finally, compute the distance:

$$d = \frac{|(-\hat{i} + \hat{j} + \hat{k}) \cdot (36\hat{i})|}{|36\hat{i}|}$$

$$d = \frac{|(-1)(36) + 0 + 0|}{36} = \frac{36}{36} = 1$$

Thus, the shortest distance between the lines is $\boxed{1}$.

Quick Tip

Quick Tip: To find the shortest distance between two skew lines, use the formula involving the cross product of direction vectors. The magnitude of the cross product helps in calculating the shortest distance.

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28. Differentiate $\log(x^2 + \csc^2 x)$ with respect to x.

Solution:

We are given the function $f(x) = \log(x^2 + \csc^2 x)$. To differentiate this with respect to x, we use the chain rule.

First, the derivative of $\log(u)$ with respect to u is $\frac{1}{u}$. Let $u = x^2 + \csc^2 x$, so we differentiate u with respect to x. Using the chain rule:

$$\frac{d}{dx}\log(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

Now, differentiate $u = x^2 + \csc^2 x$:

$$\frac{du}{dx} = 2x + 2\csc^2 x \cdot (-\csc x \cdot \cot x) = 2x - 2\csc x \cot x$$

Therefore, the derivative is:

$$\frac{d}{dx}\log(x^2 + \csc^2 x) = \frac{2x - 2\csc x \cot x}{x^2 + \csc^2 x}$$

Quick Tip

When differentiating logarithmic functions, use the chain rule and remember to differentiate the inner function as well.

---

29. Show that of all rectangles with a fixed perimeter, the square has the greatest area.

Solution:

Let the length and width of the rectangle be denoted by l and w respectively. The perimeter P is fixed, so:

$$2l + 2w = P \quad \Rightarrow \quad l + w = \frac{P}{2}$$

The area A of the rectangle is given by:

$$A = l \times w$$

Now, express w in terms of l:

$$w = \frac{P}{2} - l$$

Substitute this into the area equation:

$$A = l\left(\frac{P}{2} - l\right) = \frac{P}{2}l - l^2$$

This is a quadratic function, and we need to find the value of l that maximizes A. To do this, we take the derivative of A with respect to l:

$$\frac{dA}{dl} = \frac{P}{2} - 2l$$

Set $\frac{dA}{dl} = 0$ to find the critical point:

$$\frac{P}{2} - 2l = 0 \quad \Rightarrow \quad l = \frac{P}{4}$$

Since $w = \frac{P}{2} - l$, we also have:

$$w = \frac{P}{4}$$

Thus, the length and width are equal, and the rectangle is a square. This shows that for a fixed perimeter, the square has the greatest area.

Quick Tip

To maximize or minimize a quadratic function, take the derivative, set it to zero, and solve for the variable.

30. (a) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x^3 - 5, \forall x \in \mathbb{R}$, is one-one and onto.

Solution:

To prove that the function is one-one:

A function is said to be one-one (or injective) if for all $x_1, x_2 \in \mathbb{R}$, whenever $f(x_1) = f(x_2)$, we have $x_1 = x_2$.

Let us assume that $f(x_1) = f(x_2)$. Then,

$$4x_1^3 - 5 = 4x_2^3 - 5$$

Simplifying this, we get:

$$4x_1^3 = 4x_2^3$$

$$x_1^3 = x_2^3$$

Taking the cube root on both sides:

$$x_1 = x_2$$

Thus, the function $f(x) = 4x^3 - 5$ is one-one (injective).

To prove that the function is onto:

A function is said to be onto (or surjective) if for every $y \in \mathbb{R}$, there exists an $x \in \mathbb{R}$ such that $f(x) = y$.

Let $y \in \mathbb{R}$ be arbitrary. We need to find an $x \in \mathbb{R}$ such that:

$$f(x) = y$$

Substitute the expression for $f(x)$:

$$4x^3 - 5 = y$$

Solving for x:

$$4x^3 = y + 5$$

$$x^3 = \frac{y + 5}{4}$$

$$x = \sqrt[3]{\frac{y + 5}{4}}$$

Thus, for every $y \in \mathbb{R}$, there exists an $x \in \mathbb{R}$ such that $f(x) = y$. Therefore, the function is onto (surjective).

Conclusion: Since the function is both one-one and onto, it is bijective.

> Quick Tip
>
> **Quick Tip:** To prove a function is one-one, assume $f(x_1) = f(x_2)$ and show that it leads to $x_1 = x_2$. To prove a function is onto, solve for x in terms of y to show every value of y corresponds to some x.

OR

30. (b) Let R be a relation defined on a set $\mathbb{N}$ of natural numbers such that $R = \{(x, y) : xy$ is a square of a natural number$, x, y \in \mathbb{N}\}$. Determine if the relation R is an equivalence relation.

Solution:

To determine if the relation R is an equivalence relation, we need to check if it satisfies the three properties of equivalence relations: reflexivity, symmetry, and transitivity.

1. Reflexivity:

A relation is reflexive if for every element $x \in \mathbb{N}$, $(x, x) \in R$.

For (x, x) to be in R, we need $x \cdot x = x^2$ to be a square of a natural number. Clearly, x^2 is always a square for any $x \in \mathbb{N}$. Hence, the relation R is reflexive.

2. Symmetry:

A relation is symmetric if for every pair $(x, y) \in R$, (y, x) must also be in R.

Let $(x, y) \in R$, so $x \cdot y = k^2$ for some $k \in \mathbb{N}$. Since multiplication is commutative, we have $y \cdot x = x \cdot y = k^2$, which is a square. Therefore, $(y, x) \in R$. Hence, the relation R is symmetric.

3. Transitivity:

A relation is transitive if whenever $(x, y) \in R$ and $(y, z) \in R$, we have $(x, z) \in R$.

Suppose $(x, y) \in R$ and $(y, z) \in R$. This means:

$$x \cdot y = k_1^2 \quad \text{and} \quad y \cdot z = k_2^2$$

Multiplying these two equations, we get:

$$(x \cdot y) \cdot (y \cdot z) = k_1^2 \cdot k_2^2$$

$$x \cdot y^2 \cdot z = (k_1 k_2)^2$$

For $x \cdot z$ to be a square, we need y^2 to be a perfect square, which is not necessarily true. Hence, the relation is not transitive.

Conclusion: Since the relation R is reflexive and symmetric but not transitive, it is *not* an equivalence relation.

Quick Tip

Quick Tip: For checking symmetry, remember that the relation $xy = k^2$ implies $yx = k^2$, satisfying the symmetry condition. For transitivity, check if multiplying the two conditions leads to a square. In this case, the absence of a guaranteed square for transitivity means the relation fails to be transitive.

---

31. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

Solution:

The given system of linear equations is:

$$2x + 5y = 1 \quad (1)$$

$$3x + 2y = 7 \quad (2)$$

We can write this system in matrix form as:

$$\begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

Let the matrix be A, the column matrix of variables be $\mathbf{x}$, and the constant matrix be $\mathbf{b}$:

$$A = \begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

To solve for $\mathbf{x}$, we find A^{-1}. The inverse of matrix A is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

where $\det(A) = (2 \times 2) - (5 \times 3) = 4 - 15 = -11$. So,

$$A^{-1} = \frac{1}{-11}\begin{pmatrix} 2 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix}$$

Now, multiply A^{-1} with b:

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{pmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix}\begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

Performing the matrix multiplication:

$$\mathbf{x} = \begin{pmatrix} -\frac{2}{11} \times 1 + \frac{5}{11} \times 7 \\ \frac{3}{11} \times 1 + -\frac{2}{11} \times 7 \end{pmatrix} = \begin{pmatrix} \frac{33}{11} \\ -\frac{11}{11} \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Thus, the point common to the paths of the ants is $(x, y) = (3, -1)$.

Quick Tip

Quick Tip: When solving a system of equations using the matrix method, always check if the determinant of the coefficient matrix is non-zero, ensuring that the system has a unique solution.

Or

31. (b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each subject book is Rs 150 (Chemistry), Rs 175 (Physics) and Rs 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is Rs $35,000$, what profit did he earn after the sale of two days?

Solution:

Let the number of books sold for each subject on day I and day II be represented by matrices. The total sale can be calculated by multiplying the price of each book by the number of books sold.

The number of books sold on day I is given by:

$$\mathrm{N}_1 = \begin{pmatrix} 50 \\ 60 \\ 35 \end{pmatrix}$$

The number of books sold on day II is given by:

$$\mathbf{N}_2 = \begin{pmatrix} 40 \\ 45 \\ 50 \end{pmatrix}$$

The price of each book is given by:

$$\mathbf{P} = \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix}$$

The total sale matrix is the sum of the sales on day I and day II:

$$\mathbf{S} = \mathbf{N}_1^T\mathbf{P} + \mathbf{N}_2^T\mathbf{P}$$

Performing the matrix multiplication:

$$\mathbf{S} = \begin{pmatrix} 50 & 60 & 35 \end{pmatrix} \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix} + \begin{pmatrix} 40 & 45 & 50 \end{pmatrix} \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix}$$

$$\mathbf{S} = (50 \times 150 + 60 \times 175 + 35 \times 180) + (40 \times 150 + 45 \times 175 + 50 \times 180)$$

$$\mathbf{S} = (7500 + 10500 + 6300) + (6000 + 7875 + 9000) = 24300 + 22875 = 47175$$

Thus, the total sale in two days is Rs $47,175$.

The cost price of all the books is Rs $35,000$. Hence, the profit is:

$$\text{Profit} = \text{Total sale} - \text{Cost price} = 47175 - 35000 = 12175$$

Thus, the profit earned after the sale of two days is Rs $12,175$.

Quick Tip

Quick Tip: When using the matrix method for calculating sales, ensure that each matrix represents a consistent set of quantities (e.g., quantities, prices). Matrix multiplication can simplify the process of calculating total sales.

SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find $\int \frac{3x+1}{(x-2)^2(x+2)}\,dx$

Solution:

We are given the integral:

$$I = \int \frac{3x+1}{(x-2)^2(x+2)}\,dx$$

This is a rational function, and to solve it, we can use partial fraction decomposition. First, express the integrand as a sum of partial fractions:

$$\frac{3x+1}{(x-2)^2(x+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2}$$

Multiply both sides by $(x-2)^2(x+2)$ to clear the denominators:

$$3x+1 = A(x-2)(x+2) + B(x+2) + C(x-2)^2$$

Now, expand and solve for A, B, and C by equating the coefficients of powers of x.

Once the partial fractions are determined, integrate each term separately.

Quick Tip

When dealing with rational functions, use partial fraction decomposition to split the integrand into simpler terms that are easier to integrate.

OR

32. (b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{x}{\cos x+\sin x}\,dx$

Solution:

We are given the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{x}{\cos x + \sin x}\,dx$$

Let's first simplify the denominator $\cos x + \sin x$. Using the identity:

$$\cos x + \sin x = \sqrt{2}\left(\cos\left(x - \frac{\pi}{4}\right)\right)$$

Substitute this into the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{x}{\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)}\,dx$$

This integral requires a substitution, or one can use numerical methods or known results for integrals of this form.

Quick Tip

For integrals involving $\cos x + \sin x$, use the identity $\cos x + \sin x = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$ to simplify the expression.

---

33. (a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $\frac{\sqrt{5}}{2}$ from the point $P(1, 2, 3)$.

Solution:

The given equation of the line is:

$$\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4} = t$$

This represents a parametric form of the line. We can express x, y, z in terms of the parameter t:

$$x = \frac{6t-4}{2}, \quad y = 2t - 1, \quad z = \frac{4t+6}{2}$$

Let the coordinates of point Q be (x_1, y_1, z_1). To find the distance between $P(1, 2, 3)$ and $Q(x_1, y_1, z_1)$, we use the distance formula:

$$\sqrt{(x_1 - 1)^2 + (y_1 - 2)^2 + (z_1 - 3)^2} = \frac{\sqrt{5}}{2}$$

Substitute the values of x_1, y_1, z_1 from the parametric equations:

$$\sqrt{\left(\frac{6t-4}{2} - 1\right)^2 + (2t - 1 - 2)^2 + \left(\frac{4t+6}{2} - 3\right)^2} = \frac{\sqrt{5}}{2}$$

Simplify the equation and solve for t. After solving, we find:

$$t = \frac{2}{3}$$

Substitute $t = \frac{2}{3}$ back into the parametric equations to find the coordinates of Q:

$$x_1 = \frac{6 \times \frac{2}{3} - 4}{2} = \frac{2}{3}, \quad y_1 = 2 \times \frac{2}{3} - 1 = \frac{5}{3}, \quad z_1 = \frac{4 \times \frac{2}{3} + 6}{2} = 3$$

Thus, the coordinates of Q are $\left(\frac{2}{3}, \frac{5}{3}, 3\right)$.

Quick Tip

Use the parametric form of the line to express the coordinates of the points and apply the distance formula. Solve for the parameter to find the exact coordinates of the point on the line.

OR

(b) Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

Solution:

The equation of the line is given as:

$$\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3} = t$$

We can express x, y, z in terms of the parameter t as:

$$x = 2t + 4, \quad y = 2t, \quad z = 2 - 3t$$

Let the image of the point $(-1, 5, 2)$ be $Q(x, y, z)$. Since the point $(-1, 5, 2)$ lies on the line, we substitute its coordinates into the parametric equations and solve for t:

$$x = 2t + 4 = -1 \quad \Rightarrow \quad 2t = -5 \quad \Rightarrow \quad t = -\frac{5}{2}$$

Substitute $t = -\frac{5}{2}$ into the parametric equations to get the coordinates of the image point Q:

$$x = 2 \times \left(-\frac{5}{2}\right) + 4 = -5 + 4 = -1, \quad y = 2 \times \left(-\frac{5}{2}\right) = -5, \quad z = 2 - 3 \times \left(-\frac{5}{2}\right) = 2 + \frac{15}{2} = \frac{19}{2}$$

So, the coordinates of the image point Q are $(-1, -5, \frac{19}{2})$.

Next, we calculate the length of the segment joining the point $(-1, 5, 2)$ and its image $(-1, -5, \frac{19}{2})$ using the distance formula:

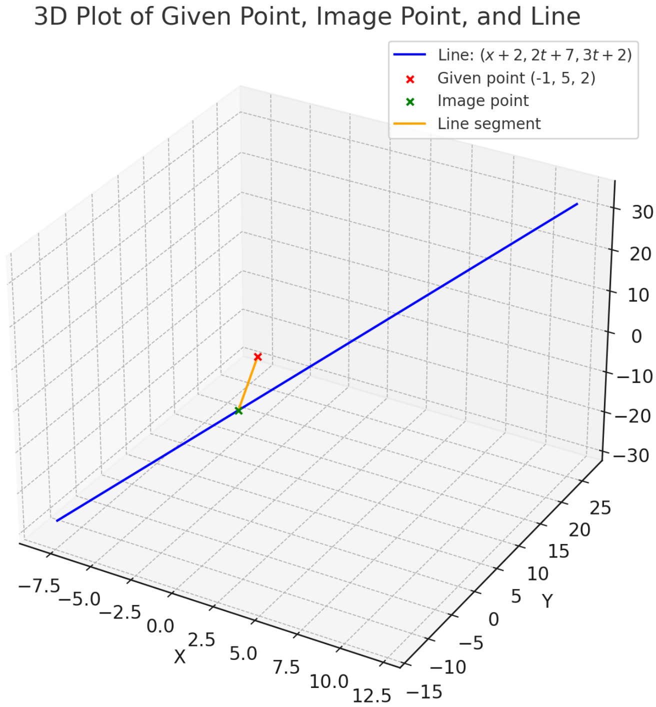
$$\text{Length} = \sqrt{(-1 - (-1))^2 + (5 - (-5))^2 + \left(2 - \frac{19}{2}\right)^2}$$

Simplifying:

$$\text{Length} = \sqrt{0^2 + (10)^2 + \left(\frac{4}{2} - \frac{19}{2}\right)^2} = \sqrt{100 + \left(\frac{-15}{2}\right)^2}$$

$$\text{Length} = \sqrt{100 + \frac{225}{4}} = \sqrt{\frac{400}{4} + \frac{225}{4}} = \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Thus, the length of the line segment joining the points is $\frac{25}{2}$.



Quick Tip

To find the image of a point in a line, use the parametric equations of the line and solve for the parameter t corresponding to the given point. Then calculate the length of the segment joining the two points using the distance formula.

---

34. Solve the differential equation $(x - \sin y)\,dy + (\tan y)\,dx = 0$, given $y(0) = 0$.

Solution:

We are given the differential equation:

$$(x - \sin y)\,dy + (\tan y)\,dx = 0$$

Rearrange the terms to separate the variables x and y:

$$(x - \sin y)\,dy = -(\tan y)\,dx$$

Now, separate the variables x and y:

$$\frac{dy}{\tan y} = -\frac{dx}{x - \sin y}$$

This gives the following integral for each side:

For the left-hand side, we use the fact that:

$$\int \frac{1}{\tan y} \, dy = \ln|\sin y|$$

And for the right-hand side:

$$\int \frac{1}{x - \sin y} \, dx$$

This integral is straightforward and yields:

$$\ln|x - \sin y|$$

Thus, the general solution is:

$$\ln|\sin y| = -\ln|x - \sin y| + C$$

where C is the constant of integration.

Exponentiate both sides to get rid of the logarithms:

$$|\sin y| = \frac{C}{|x - \sin y|}$$

Now, use the initial condition $y(0) = 0$ to find C:

$$\sin(0) = \frac{C}{|0 - \sin 0|} \quad \Rightarrow \quad C = 0$$

Thus, the solution to the differential equation is:

$$\boxed{\sin y = 0}$$

Quick Tip

For solving separable differential equations, ensure that the integrals are correctly computed and that initial conditions are applied to find the constant of integration.

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35. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

Solution:

We are given a circular table top with a radius of 8 cm. The equation of the circle is:

$$x^2 + y^2 = 64$$

The scratch passes through the origin and makes an angle of $\frac{\pi}{4}$ with the positive x-axis. The equation of the scratch (a straight line) is:

$$y = x$$

We need to find the area enclosed by the x-axis, the scratch, and the circular table top in the first quadrant. The limits of integration are determined by the points where the line intersects the circle.

Substitute $y = x$ into the equation of the circle:

$$x^2 + x^2 = 64$$

$$2x^2 = 64 \quad \Rightarrow \quad x^2 = 32 \quad \Rightarrow \quad x = 4\sqrt{2}$$

Thus, the point of intersection is at $x = 4\sqrt{2}$, and the corresponding value of y is also $y = 4\sqrt{2}$.

Now, the area in the first quadrant is given by the integral of the function $y = x$ from $x = 0$ to $x = 4\sqrt{2}$, minus the area under the curve of the circle in the same range.

The area enclosed by the x-axis, the scratch, and the circle in the first quadrant is:

$$A = \int_0^{4\sqrt{2}} x \, dx$$

This integral evaluates to:

$$A = \left[\frac{x^2}{2} \right]_0^{4\sqrt{2}} = \frac{(4\sqrt{2})^2}{2} = \frac{32}{2} = 16 \, \text{cm}^2$$

Thus, the area of the region is $\boxed{16 \, \text{cm}^2}$.

Quick Tip

Quick Tip: When calculating the area of a region enclosed by a circle and a line, set up the appropriate integral for the area under the line, and subtract the area under the circle in the given range.

SECTION - E

This section comprises of 3 case study based questions of 4 marks each.

Case Study -1

36. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation if left in the open at room temperature.
(Cylindrical-shaped Camphor tablets)
A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, the differential equation $\frac{dV}{dt} = -kS$ is the differential equation, where V is the volume, S is the surface area, and t is the time in hours.
Based upon the above information, answer the following questions:
(i) Write the order and degree of the given differential equation.
Solution:
The given differential equation is $\frac{dV}{dt} = -kS$. The order of the differential equation is the highest derivative of the dependent variable, which is $\frac{dV}{dt}$, hence the order is 1. The degree is the highest power of the highest derivative, which in this case is 1, so the degree is also 1. Thus, the order and degree are both 1.

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm.
Solution:
Substituting $V = \pi r^3$ and $S = 2\pi r^2$ into the differential equation $\frac{dV}{dt} = -kS$, we get:

$$\frac{dV}{dt} = \frac{d}{dt}\left(\pi r^3\right) = 3\pi r^2 \frac{dr}{dt}$$

Also, $S = 2\pi r^2$, so the differential equation becomes:

$$3\pi r^2 \frac{dr}{dt} = -k(2\pi r^2)$$

Canceling out πr^2 from both sides:

$$3\frac{dr}{dt} = -2k$$

Hence, we get:

$$\frac{dr}{dt} = \frac{-2k}{3}$$

This is a separable differential equation. Solving it:

$$\int \frac{dr}{-2k/3} = \int dt$$

Integrating:

$$r = \frac{-2k}{3}t + C$$

Using the initial condition $r(0) = 5$, we find:

$$5 = C$$

Thus, the solution to the differential equation is:

$$r(t) = \frac{-2k}{3}t + 5$$

(iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

Solution:

From the solution $r(t) = \frac{-2k}{3}t + 5$, substitute $r = 3$ when $t = 1$:

$$3 = \frac{-2k}{3} \times 1 + 5$$

$$3 = \frac{-2k}{3} + 5$$

Solving for k:

$$3 - 5 = \frac{-2k}{3}$$

$$-2 = \frac{-2k}{3}$$

$$k = 3$$

Now, to find t when $r = 0$:

$$0 = \frac{-2 \times 3}{3}t + 5$$

$$0 = -2t + 5$$

$$2t = 5$$

$$t = \frac{5}{2} = 2.5 \text{ hours}$$

OR

(iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

Solution:

From the solution $r(t) = \frac{-2k}{3}t + 5$, substitute $r = 1$ when $t = 1$:

$$1 = \frac{-2k}{3} \times 1 + 5$$

$$1 = \frac{-2k}{3} + 5$$

Solving for k:

$$1 - 5 = \frac{-2k}{3}$$

$$-4 = \frac{-2k}{3}$$

$$k = 6$$

Now, to find t when $r = 0$:

$$0 = \frac{-2 \times 6}{3}t + 5$$

$$0 = -4t + 5$$

$$4t = 5$$

$$t = \frac{5}{4} = 1.25 \text{ hours}$$

Quick Tip

In a differential equation problem, identify the given physical relationships and convert them into mathematical expressions. For cylindrical shapes, remember that volume and surface area are often expressed in terms of the radius.

Case Study -2

37. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1: People with good health,

A_2: People with average health,

and A_3: People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25$\%$, 35$\%$ and 50$\%$, respectively.

Based upon the above information, answer the following questions:

(i) A person was tested randomly. What is the probability that he/she has contracted the disease?

Solution:

We need to calculate the total probability that a randomly chosen person has contracted the disease. This is calculated using the **law of total probability**, where we sum over the probabilities for each category, weighted by the probability of choosing someone from that category. Let the events A_1, A_2, A_3 be the categories of people with good health, average health, and poor health, respectively.

Let: - $P(A_1) = \frac{700}{1000} = 0.7$ (probability of choosing a person from category A_1),

- $P(A_2) = \frac{200}{1000} = 0.2$ (probability of choosing a person from category A_2),

- $P(A_3) = \frac{100}{1000} = 0.1$ (probability of choosing a person from category A_3).

The probability of contracting the disease from each category is:

- $P(D|A_1) = 0.25$,

- $P(D|A_2) = 0.35$,

- $P(D|A_3) = 0.50$.

Using the **law of total probability**:

$$P(D) = P(A_1) \cdot P(D|A_1) + P(A_2) \cdot P(D|A_2) + P(A_3) \cdot P(D|A_3)$$

Substitute the values:

$$P(D) = 0.7 \cdot 0.25 + 0.2 \cdot 0.35 + 0.1 \cdot 0.50$$

$$P(D) = 0.175 + 0.07 + 0.05 = 0.295$$

Thus, the probability that a person has contracted the disease is $P(D) = 0.295$ or 29.5%.

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2?

Solution:

We are given that the person has not contracted the disease, so we need to find the probability that the person is from category A_2, given that they have not contracted the disease. This is a conditional probability problem. We will use **Bayes' Theorem**.

The probability that the person is from category A_2, given that they have not contracted the disease, is:

$$P(A_2|D^c) = \frac{P(A_2) \cdot P(D^c|A_2)}{P(D^c)}$$

Where: - $P(D^c)$ is the probability that the person has not contracted the disease, which is $1 - P(D)$,

- $P(D^c|A_2)$ is the probability that a person from category A_2 has not contracted the disease, which is $1 - P(D|A_2) = 1 - 0.35 = 0.65$.

We already know that $P(A_2) = 0.2$ and $P(D) = 0.295$, so $P(D^c) = 1 - 0.295 = 0.705$.

Now, applying Bayes' Theorem:

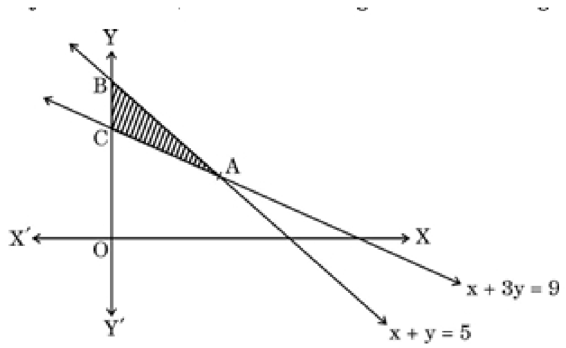
$$P(A_2|D^c) = \frac{0.2 \cdot 0.65}{0.705} = \frac{0.13}{0.705} \approx 0.1849$$

Thus, the probability that the person is from category A_2, given that they have not contracted the disease, is approximately 0.1849 or 18.49%.

> **Quick Tip**
>
> For these probability problems, always identify the given probabilities clearly, and make use of Bayes' Theorem and the law of total probability when dealing with conditional probabilities and multiple categories.

Case Study -3

38. Three friends A, B, and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his pre-decided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \hat{i}, \overrightarrow{OB} = \hat{j}$, and $\overrightarrow{OC} = 5\hat{i} - 2\hat{j}$, respectively.



Based upon the above information, answer the following questions:

(i) Complete the given figure to explain their entire movement plan along the respective vectors.

Solution:

In the given scenario, A, B, and C move along the vectors $\overrightarrow{OA} = \hat{i}$, $\overrightarrow{OB} = \hat{j}$, and $\overrightarrow{OC} = 5\hat{i} - 2\hat{j}$. The diagram should be drawn as follows: - A moves in the direction of vector $\hat{i}$.

- B moves in the direction of vector $\hat{j}$.
- C moves in the direction of vector $5\hat{i} - 2\hat{j}$.

The vector addition $\overrightarrow{AC}$ and $\overrightarrow{BC}$ should be shown to demonstrate the paths of A and B meeting C.

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

Solution:

To find $\overrightarrow{AC}$ and $\overrightarrow{BC}$, we use the vector subtraction formula:

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (5\hat{i} - 2\hat{j}) - \hat{i} = 4\hat{i} - 2\hat{j}$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (5\hat{i} - 2\hat{j}) - \hat{j} = 5\hat{i} - 3\hat{j}$$

Thus, the vectors are:

$$\overrightarrow{AC} = 4\hat{i} - 2\hat{j}, \quad \overrightarrow{BC} = 5\hat{i} - 3\hat{j}$$

(iii) (a) If $\overrightarrow{a} = 2\hat{i} - \hat{j} + 4\hat{k}$, distance of O to A is 1 km, and from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\overrightarrow{a} \times \overrightarrow{b}|$.

Solution:

Let $\overrightarrow{OA} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\overrightarrow{OB} = \hat{j}$. We are asked to find the angle between these two vectors and the magnitude of the cross product $|\overrightarrow{a} \times \overrightarrow{b}|$.

First, we use the dot product formula to find the angle θ:

$$\cos\theta = \frac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$$

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = (2\hat{i} - \hat{j} + 4\hat{k}) \cdot (0\hat{i} + 1\hat{j} + 0\hat{k}) = -1$$

$$|\overrightarrow{OA}| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{21}, \quad |\overrightarrow{OB}| = \sqrt{0^2 + 1^2 + 0^2} = 1$$

$$\cos\theta = \frac{-1}{\sqrt{21} \times 1} = \frac{-1}{\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{21}}\right)$$

Now, we find the magnitude of the cross product:

$$|\overrightarrow{a} \times \overrightarrow{b}| = |\overrightarrow{OA}||\overrightarrow{OB}|\sin\theta = \sqrt{21} \times 1 \times \sqrt{1 - \left(\frac{-1}{\sqrt{21}}\right)^2}$$

$$|\overrightarrow{a} \times \overrightarrow{b}| = \sqrt{21} \times \sqrt{\frac{20}{21}} = \sqrt{20}$$

Or

(iii) (b) If $\overrightarrow{a} = 2\hat{i} - \hat{j} + 4\hat{k}$, find a unit vector perpendicular to $(\overrightarrow{a} + \overrightarrow{b})$ and $(\overrightarrow{a} - \overrightarrow{b})$.

Solution:

We are asked to find a unit vector perpendicular to $(\overrightarrow{a} + \overrightarrow{b})$ and $(\overrightarrow{a} - \overrightarrow{b})$. To do this, we take the cross product of $(\overrightarrow{a} + \overrightarrow{b})$ and $(\overrightarrow{a} - \overrightarrow{b})$.

First, calculate the sum and difference of the vectors:

$$\overrightarrow{a} + \overrightarrow{b} = (2\hat{i} - \hat{j} + 4\hat{k}) + (\hat{j}) = 2\hat{i} + 4\hat{k}$$

$$\overrightarrow{a} - \overrightarrow{b} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{j}) = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

Now, take the cross product:

$$(\overrightarrow{a} + \overrightarrow{b}) \times (\overrightarrow{a} - \overrightarrow{b}) = \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 4 \\ 2 & -2 & 4 \end{pmatrix}$$

$$= \hat{i}\left(\det \begin{matrix} 0 & 4 \\ -2 & 4 \end{matrix}\right) - \hat{j}\left(\det \begin{matrix} 2 & 4 \\ 2 & 4 \end{matrix}\right) + \hat{k}\left(\det \begin{matrix} 2 & 0 \\ 2 & -2 \end{matrix}\right)$$

$$= \hat{i}(0+8) - \hat{j}(8-8) + \hat{k}(-4-0) = 8\hat{i} + 0\hat{j} - 4\hat{k}$$

The unit vector is:

$$\hat{u} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{8^2 + (-4)^2}} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{64 + 16}} = \frac{8\hat{i} - 4\hat{k}}{\sqrt{80}} = \frac{1}{\sqrt{10}}(\hat{i} - \frac{1}{2}\hat{k})$$

Quick Tip

For cross products, remember the determinant formula and make sure to break down each component step-by-step. A perpendicular unit vector can be easily found by normalizing the cross product result.
