

CBSE Class XII 2025 Mathematics Set 3 (65/2/3) Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper consists of 38 questions. All questions are compulsory.
2. This paper is divided into five sections - A, B, C, D and E.
3. Section A Nos. 1 to 18 are Multiple Choice questions, and Q19 and 20 are Assertion Reasoning Questions. Each carries 1 mark.
4. Section B Nos. 21 to 25 are Very Short Answer type. Each carries 2 marks. Answer to these questions should be in the range of 30 to 50 words.
5. Section C Nos. 26 to 31 are Short Answer (SA) type. Each carries 3 marks. Answer to these questions should be in the range of 50 to 80 words.
6. Section D Nos. 32 to 35 are Long Answer type. Each carries 5 marks. Answer to these questions should be in the range of 80 to 120 words.
7. Section E Nos. 36 to 38 are of 3 source-based/case-based units of assessment carrying 4 marks each with sub-parts.
8. There is no overall choice. However, an internal choice has been provided in some sections. Only one of the alternatives has to be attempted in such questions.

Section - A

1. If $\mathbf{p}$ and $\mathbf{q}$ are unit vectors, then which of the following values of $\mathbf{p} \cdot \mathbf{q}$ is not possible?

- (1) $-\frac{1}{2}$
- (2) $\frac{1}{\sqrt{2}}$
- (3) $\frac{\sqrt{3}}{2}$
- (4) $\sqrt{3}$

Correct Answer: (4) $\sqrt{3}$

Solution: Understanding the dot product of unit vectors.

The dot product $\mathbf{p} \cdot \mathbf{q}$ of two unit vectors $\mathbf{p}$ and $\mathbf{q}$ is given by:

$$\mathbf{p} \cdot \mathbf{q} = \cos \theta$$

where θ is the angle between the two vectors. Since the cosine of an angle must lie between -1 and 1 , the value of $\mathbf{p} \cdot \mathbf{q}$ must be between -1 and 1 inclusive. Therefore, $\sqrt{3}$ is not a possible value for $\mathbf{p} \cdot \mathbf{q}$, as it exceeds the maximum possible value of 1 .

Quick Tip

For unit vectors, the dot product $\mathbf{p} \cdot \mathbf{q}$ must always lie between -1 and 1 . Any value outside this range is impossible.

2. Which of the following can be both a symmetric and skew-symmetric matrix?

- (1) Unit Matrix
- (2) Diagonal Matrix
- (3) Null Matrix
- (4) Row Matrix

Correct Answer: (3) Null Matrix

Solution: Step 1: Understanding symmetric and skew-symmetric matrices.

A matrix is symmetric if $A = A^T$, meaning the matrix is equal to its transpose. A matrix is skew-symmetric if $A = -A^T$, meaning the matrix is equal to the negative of its transpose.

Step 2: Finding the matrix that satisfies both conditions.

The only matrix that satisfies both symmetric and skew-symmetric properties is the null

matrix, because:

$$0 = 0^T \quad (\text{symmetric}) \quad \text{and} \quad 0 = -0^T \quad (\text{skew-symmetric}).$$

Thus, the null matrix is both symmetric and skew-symmetric.

Quick Tip

The null matrix is the only matrix that can be both symmetric and skew-symmetric since $0 = 0^T$ and $0 = -0^T$.

3. If $\int_0^a x \, dx \leq \frac{a}{2} + 6$, then which of the following holds for a ?

(1) $-4 \leq a \leq 3$

(2) $a \geq 4, a \leq -3$

(3) $-3 \leq a \leq 4$

(4) $-3 \leq a \leq 0$

Correct Answer: (3) $-3 \leq a \leq 4$

Solution: Step 1: Solving the integral.

The integral of x from 0 to a is:

$$\int_0^a x \, dx = \frac{a^2}{2}$$

Thus, the inequality becomes:

$$\frac{a^2}{2} \leq \frac{a}{2} + 6$$

Multiply both sides by 2 to simplify:

$$a^2 \leq a + 12$$

Rearrange the terms:

$$a^2 - a - 12 \leq 0$$

Step 2: Solving the quadratic inequality.

Factor the quadratic expression:

$$(a - 4)(a + 3) \leq 0$$

The solution to this inequality is $-3 \leq a \leq 4$.

Quick Tip

For quadratic inequalities, factor the expression and analyze the sign of the factors to determine the solution range.

4. If A and B are square matrices of the same order, then $(AB^T - BA^T)$ is a:

- (1) Symmetric matrix
- (2) Skew-symmetric matrix
- (3) Null matrix
- (4) Unit matrix

Correct Answer: (2) Skew-symmetric matrix

Solution: Step 1: Understanding skew-symmetric matrices.

A matrix is skew-symmetric if $A = -A^T$, meaning the matrix is equal to the negative of its transpose. We are given the expression $AB^T - BA^T$.

Step 2: Checking the transpose.

Take the transpose of $AB^T - BA^T$:

$$(AB^T - BA^T)^T = (BA^T)^T - (AB^T)^T = AB^T - BA^T$$

Since the transpose of the matrix is equal to its negative, the matrix is skew-symmetric.

Quick Tip

To check if a matrix is skew-symmetric, verify if its transpose is equal to the negative of the original matrix.

5. The value of $\cos\left(\frac{\pi}{6} + \cot^{-1}(-\sqrt{3})\right)$ is:

- (1) -1
- (2) $\frac{-\sqrt{3}}{2}$
- (3) 0
- (4) 1

Correct Answer: (2) $\frac{-\sqrt{3}}{2}$

Solution: Step 1: Evaluating the inverse cotangent.

We know that $\cot^{-1}(-\sqrt{3})$ corresponds to an angle θ where $\cot \theta = -\sqrt{3}$. This implies $\theta = \frac{5\pi}{6}$, because $\cot \frac{5\pi}{6} = -\sqrt{3}$.

Step 2: Simplifying the expression.

Thus, the expression becomes:

$$\cos\left(\frac{\pi}{6} + \frac{5\pi}{6}\right) = \cos \pi = -1$$

Therefore, the value of the expression is $\boxed{-1}$.

Quick Tip

When solving inverse trigonometric functions, express the angle in terms of a known trigonometric identity and simplify the expression.

6. If p and q are respectively the order and degree of the differential equation

$\frac{d}{dx} \left(\frac{dy}{dx} \right)^3 = 0$, **then $(p - q)$ is:**

- (1) 0
- (2) 1
- (3) 2
- (4) 3

Correct Answer: (3) 2

Solution: Step 1: Finding the order and degree.

The given differential equation is $\frac{d}{dx} \left(\frac{dy}{dx} y^3 \right) = 0$. First, let's find the order and degree of this equation.

The function is y^3 , so we have:

$$\frac{d}{dx} y^3 = 3y^2 \frac{dy}{dx}$$

Now, applying the derivative again:

$$\frac{d}{dx} \left(3y^2 \frac{dy}{dx} \right) = 6y \left(\frac{dy}{dx} \right)^2 + 3y^2 \frac{d^2y}{dx^2}$$

This is a second-order differential equation, so the order is 2.

Since the highest power of y is 3, the degree is 3.

Thus, $p = 2$ and $q = 3$, so $p - q = 2 - 3 = -1$.

Quick Tip

The order of a differential equation is determined by the highest derivative, and the degree is the highest power of the dependent variable.

7. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval:

- (1) $(0, 2)$
- (2) $(-\infty, 2]$
- (3) $[1, 2]$
- (4) $[2, \infty)$

Correct Answer: (4) $[2, \infty)$

Solution: Step 1: Finding the first derivative of $f(x)$.

The function given is $f(x) = x^2 - 4x + 6$. To find the interval where the function is increasing, we first calculate its derivative:

$$f'(x) = 2x - 4$$

Step 2: Determining when the derivative is positive.

The function is increasing where $f'(x) > 0$. Thus, solve for x in:

$$2x - 4 > 0 \quad \Rightarrow \quad x > 2$$

Therefore, the function $f(x)$ is increasing for $x \in [2, \infty)$.

Quick Tip

To determine where a function is increasing or decreasing, find the first derivative and solve the inequality $f'(x) > 0$ for increasing or $f'(x) < 0$ for decreasing.

8. The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following points?

- (1) $(1, -5, 6)$
- (2) $(1, 5, 6)$
- (3) $(1, -5, -6)$

(4) $(-1, -5, 6)$

Correct Answer: (3) $(1, -5, -6)$

Solution: Step 1: Substituting $\mu = 0$ into the parametric equations. To find the point through which the line passes, substitute $\mu = 0$ into the parametric equations:

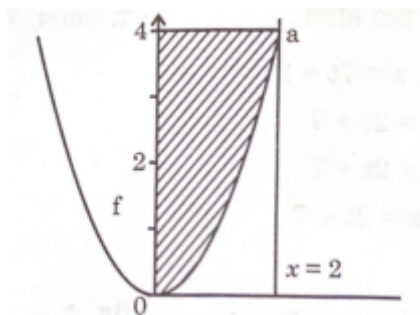
$$x = 1 + 5(0) = 1, \quad y = -5 + (0) = -5, \quad z = -6 - 3(0) = -6$$

Thus, the point is $(1, -5, -6)$.

Quick Tip

To find a point on a parametric line, substitute the value of the parameter into the parametric equations for x , y , and z .

9. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \leq x \leq 2$, and the y-axis is given by:



(1) $\int_0^2 x^2 dx$

(2) $\int_0^2 \sqrt{y} dy$

(3) $\int_0^4 x^2 dx$

(4) $\int_0^4 \sqrt{y} dy$

Correct Answer: (1) $\int_0^2 x^2 dx$

Solution: Step 1: Understanding the shaded region.

The given curves are $y = x^2$ and the y-axis, with x ranging from 0 to 2. The area of the shaded region is the area under the curve $y = x^2$ from $x = 0$ to $x = 2$.

Step 2: Setting up the integral.

The area under the curve $y = x^2$ is given by the integral:

$$\text{Area} = \int_0^2 x^2 dx$$

Thus, the correct answer is $\int_0^2 x^2 dx$.

Quick Tip

To find the area under a curve between two points, integrate the function with respect to the variable representing the horizontal axis.

10. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(E | F)$ is:

- (1) $\frac{P(\bar{E})}{P(\bar{F})}$
- (2) $1 - P(\bar{E} | F)$
- (3) $1 - P(E | F)$
- (4) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

Correct Answer: (4) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

Solution: The formula for conditional probability is given by:

$$P(E | F) = \frac{P(E \cap F)}{P(F)}.$$

We are asked to find the expression for $P(E | F)$ in terms of other probabilities.

Using the inclusion-exclusion principle, we know:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

So, we can rearrange this to express $P(E \cap F)$ as:

$$P(E \cap F) = P(E \cup F) - P(F).$$

Substituting this into the formula for conditional probability:

$$P(E | F) = \frac{P(E \cup F) - P(F)}{P(F)}.$$

Now, consider the complement of F , i.e., $P(\bar{F})$, and the formula for the conditional probability of the complement of E , $P(\bar{E} | F)$. We obtain the final expression for $P(E | F)$ in terms of other events:

$$P(E | F) = \frac{1 - P(E \cup F)}{P(\bar{F})}.$$

Thus, the correct answer is option (4).

Quick Tip

To solve conditional probability problems, you may need to use the inclusion-exclusion principle and properties of complements to express the probability in different terms.

11. The probability distribution of a random variable X is given by:

X	-4	-3	-2	-1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

Then $E(X)$ of distribution is:

- (1) -1.8
- (2) -1
- (3) 1
- (4) 1.8

Correct Answer: (1) -1.8

Solution: The expected value $E(X)$ of a random variable is given by:

$$E(X) = \sum_i x_i \cdot P(x_i)$$

Substituting the given values:

$$E(X) = (-4 \cdot 0.1) + (-3 \cdot 0.2) + (-2 \cdot 0.3) + (-1 \cdot 0.2) + (0 \cdot 0.2)$$

$$E(X) = -0.4 - 0.6 - 0.6 - 0.2 + 0 = -1.8$$

Thus, $E(X) = -1.8$.

Quick Tip

To find the expected value, multiply each possible value of X by its corresponding probability and sum the results.

12. If the projection of $\mathbf{a} = \alpha\hat{i} + \hat{j} + 4\hat{k}$ on $\mathbf{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then α is:

- (1) -13
- (2) -5

(3) 13

(4) 5

Correct Answer: (4) 5

Solution: The projection of a vector $\mathbf{a}$ on a vector $\mathbf{b}$ is given by:

$$\text{Projection of } \mathbf{a} \text{ on } \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$$

We are given that the projection is 4 units, so:

$$\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|} = 4$$

First, calculate $\mathbf{a} \cdot \mathbf{b}$:

$$\mathbf{a} \cdot \mathbf{b} = \alpha \cdot 2 + 1 \cdot 6 + 4 \cdot 3 = 2\alpha + 6 + 12 = 2\alpha + 18$$

Next, calculate $|\mathbf{b}|$:

$$|\mathbf{b}| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{4 + 36 + 9} = \sqrt{49} = 7$$

Now substitute into the projection formula:

$$\frac{2\alpha + 18}{7} = 4$$

Solving for α :

$$2\alpha + 18 = 28 \quad \Rightarrow \quad 2\alpha = 10 \quad \Rightarrow \quad \alpha = 5$$

Quick Tip

The projection of vector $\mathbf{a}$ on $\mathbf{b}$ can be calculated using the formula $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$.

13. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is:

(1) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(2) $x = 3t + 4, y = t + 3, z = 2t + 7$

(3) $x = 3t + 4, y = -3, z = 2t + 7$

(4) $x = 3t + 4, y = -3t + 1, z = 2t + 7$

Correct Answer: (2) $x = 3t + 4, y = t + 3, z = 2t + 7$

Solution: The equation of a line in parametric form is:

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct$$

where (x_0, y_0, z_0) is the point through which the line passes, and $\langle a, b, c \rangle$ is the direction vector of the line.

Here, the direction vector is $\langle 3, 1, 2 \rangle$ and the point is $(4, -3, 7)$. Therefore, the parametric equations are:

$$x = 3t + 4, \quad y = t - 3, \quad z = 2t + 7$$

Quick Tip

To find the equation of a line, use the parametric form $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$, where (x_0, y_0, z_0) is the given point and $\langle a, b, c \rangle$ is the direction vector.

14. If a line makes angles of $\frac{3\pi}{4}$ and $\frac{\pi}{3}$ with the positive directions of x , y , and z -axes respectively, then θ is:

- (1) $-\frac{\pi}{3}$
- (2) $\frac{\pi}{3}$ only
- (3) $\frac{\pi}{6}$
- (4) $\pm\frac{\pi}{3}$

Correct Answer: (4) $\pm\frac{\pi}{3}$

Solution: For a line making angles α , β , and γ with the positive directions of the x , y , and z -axes respectively, the direction cosines of the line are:

$$\cos \alpha = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}}, \quad \cos \beta = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}}, \quad \cos \gamma = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}}.$$

The angle θ is determined by the geometry of the line and can take both positive and negative values based on the orientations of the line with respect to the axes. Thus, the correct answer is $\pm\frac{\pi}{3}$.

Quick Tip

The direction cosines of a line are always in the range -1 to 1 , and the angle can have multiple solutions due to the symmetry of the coordinate axes.

15. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statements is correct?

- (1) The objective function maximizes the difference of the profit earned from products X and Y.
- (2) The objective function measures the total production of products X and Y.
- (3) The objective function maximizes the combined profit earned from selling X and Y.
- (4) The objective function ensures the company produces more of product X than product Y.

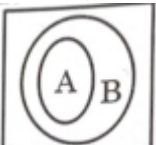
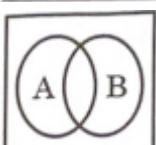
Correct Answer: (3) The objective function maximizes the combined profit earned from selling X and Y.

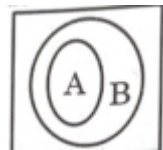
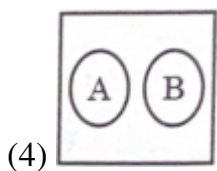
Solution: The objective function $Z = 5x + 7y$ represents the total profit earned by selling x units of product X and y units of product Y. The coefficients 5 and 7 indicate the profit per unit of X and Y respectively. Therefore, the function maximizes the combined profit earned from selling both products, as the objective is to maximize Z .

Quick Tip

The objective function in linear programming typically represents a quantity to be maximized or minimized, such as profit, cost, or production.

16. If A denotes the set of continuous functions and B denotes the set of differentiable functions, then which of the following depicts the correct relation between set A and B ?

- (1) 
- (2) 
- (3) 



Correct Answer: (1)

Solution: We know that every differentiable function is continuous, but not every continuous function is differentiable. Therefore, set A (continuous functions) is a subset of set B (differentiable functions). The correct relation is $A \subset B$.

Quick Tip

In the context of functions, differentiable functions are always continuous, but continuous functions are not always differentiable.

17. Four friends Abhay, Bina, Chhaya, and Devesh were asked to simplify

$4AB + 3(AB + BA) - 4BA$, **where A and B are both matrices of order 2×2 . It is known that $A \neq B$ and $A^{-1} \neq B$. Their answers are given as:**

(1) Abhay: $6AB$

(2) Bina: $7AB - BA$

(3) Chhaya: $8AB$

(4) Devesh: $7BA - AB$

Correct Answer: (2) Bina

Solution: First, simplify the expression $4AB + 3(AB + BA) - 4BA$:

$$4AB + 3(AB + BA) - 4BA = 4AB + 3AB + 3BA - 4BA = 7AB - BA$$

Therefore, the correct answer is $7AB - BA$, which is Bina's answer.

Quick Tip

When simplifying matrix expressions, carefully distribute constants and combine like terms.

18. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct?

- (1) $A = B$
- (2) $AB = BA$
- (3) $A = 0$ or $B = 0$
- (4) $A = I$ or $B = I$

Correct Answer: (1) $A = B$

Solution: Using the difference of squares formula, we know:

$$A^2 - B^2 = (A - B)(A + B)$$

For this to hold, it must be true that $A = B$ because otherwise, the matrices $(A - B)$ and $(A + B)$ would not satisfy the equation for all cases. Thus, the correct answer is $A = B$.

Quick Tip

The difference of squares formula $a^2 - b^2 = (a - b)(a + b)$ holds for matrices just as it does for numbers, but ensure that the matrix operations are valid.

19. Assertion (A): Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false but Reason (R) is true.

Solution: Step 1: Understanding the feasible region in Linear Programming.

In a Linear Programming Problem (LPP), the feasible region consists of all possible values that satisfy the given constraints.

Step 2: Evaluating Assertion (A).

The optimal solution of an LPP is always found at one or more corner points of the feasible region, not at every point. Hence, Assertion (A) is false.

Step 3: Evaluating Reason (R).

The Fundamental Theorem of Linear Programming states that the optimal solution lies at a corner point of the feasible region. This makes Reason (R) true.

Since Assertion (A) is false and Reason (R) is true, the correct answer is (D).

Quick Tip

In Linear Programming, the optimal solution is always found at a corner point of the feasible region, not in the interior of the region.

20. Assertion (A): $A = \text{diag}[3, 5, 2]$ is a scalar matrix of order 3×3 .

Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false but Reason (R) is true.

Solution: Step 1: Understanding scalar matrices.

A scalar matrix is a diagonal matrix where all diagonal elements are equal, meaning:

$$A = cI$$

where c is a scalar and I is the identity matrix.

Step 2: Evaluating Assertion (A).

The given matrix is:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Since the diagonal elements are not equal (3, 5, 2 are different), this is a diagonal matrix but not a scalar matrix. Thus, Assertion (A) is false.

Step 3: Evaluating Reason (R).

The definition given in Reason (R) is correct. If all diagonal elements were the same, the matrix would be a scalar matrix. Hence, Reason (R) is true.

Since Assertion (A) is false and Reason (R) is true, the correct answer is (D).

Quick Tip

A scalar matrix is a special type of diagonal matrix where all diagonal elements are equal. If they are not equal, the matrix is just a diagonal matrix.

Section - B

21. Find the values of a for which $f(x) = \sin x - ax + b$ is increasing on $\mathbb{R}$.

Solution: Step 1: Find the first derivative of $f(x)$.

A function is increasing on $\mathbb{R}$ if its first derivative is always non-negative, i.e.,

$$f'(x) \geq 0 \quad \forall x \in \mathbb{R}.$$

Differentiating $f(x)$:

$$f'(x) = \cos x - a.$$

Step 2: Find the condition for $f'(x) \geq 0$.

For $f(x)$ to be increasing on $\mathbb{R}$, we must have:

$$\cos x - a \geq 0 \quad \forall x \in \mathbb{R}.$$

Since $\cos x$ oscillates in the range $[-1, 1]$, the minimum value of $\cos x$ is -1 , and the maximum value is 1 . Therefore, the condition becomes:

$$-1 - a \geq 0.$$

$$a \leq -1.$$

Step 3: Conclusion.

Thus, for $f(x)$ to be increasing on $\mathbb{R}$, the required condition is:

$$a \leq -1.$$

Quick Tip

A function is increasing when its first derivative is non-negative. Consider the maximum and minimum values of trigonometric functions when solving inequalities.

22. Find: $\int 2x^3 e^{x^2} dx$.

Solution: Step 1: Identify substitution.

The given integral is:

$$I = \int 2x^3 e^{x^2} dx.$$

We use the substitution:

$$u = x^2 \quad \Rightarrow \quad du = 2x dx.$$

Step 2: Transform the integral.

Rewriting in terms of u :

$$\int 2x^3 e^{x^2} dx = \int x^2 \cdot 2x e^{x^2} dx.$$

Since $2x dx = du$, we substitute:

$$I = \int x^2 e^u du.$$

Since $x^2 = u$, we get:

$$I = \int u e^u du.$$

Step 3: Integration by parts.

Using integration by parts, where:

$$\int uv' du = uv - \int vu' du,$$

let:

$$u = u, \quad dv = e^u du.$$

Then:

$$du = du, \quad v = e^u.$$

Applying integration by parts:

$$I = ue^u - \int e^u du.$$

Since $\int e^u du = e^u$, we get:

$$I = ue^u - e^u + C.$$

Step 4: Substituting back $u = x^2$.

$$I = x^2 e^{x^2} - e^{x^2} + C.$$

Final Answer:

$$\int 2x^3 e^{x^2} dx = (x^2 - 1)e^{x^2} + C.$$

Quick Tip

For integrals involving xe^{x^2} , try substitution $u = x^2$ and use integration by parts if necessary.

23 (a). If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

Solution: Step 1: Take the natural logarithm on both sides.

Given:

$$x = e^{\frac{x}{y}}$$

Taking log on both sides:

$$\log x = xy.$$

Step 2: Differentiate both sides using implicit differentiation.

Differentiating both sides with respect to x :

$$\frac{d}{dx}(\log x) = \frac{d}{dx}(xy).$$

Using derivative rules:

$$\frac{1}{x} \cdot \frac{dx}{dx} = x \frac{dy}{dx} + y \frac{dx}{dx}.$$

Since $\frac{dx}{dx} = 1$, we get:

$$\frac{1}{x} = x \frac{dy}{dx} + y.$$

Step 3: Solve for $\frac{dy}{dx}$.

Rearrange the equation:

$$\frac{1}{x} - y = x \frac{dy}{dx}.$$

Dividing by x :

$$\frac{dy}{dx} = \frac{\frac{1}{x} - y}{x}.$$

Rewriting in simplified form:

$$\frac{dy}{dx} = \frac{x - y}{x \log x}.$$

Thus, the required result is proved.

Quick Tip

For equations involving logarithms and exponentials, taking the natural logarithm can simplify differentiation.

OR

23 (b). If $f(x) = \begin{cases} 2x - 3, & -3 \leq x \leq -2 \\ x + 1, & -2 < x \leq 0 \end{cases}$, check the differentiability of $f(x)$ at $x = -2$.

Solution: To check differentiability at $x = -2$, we must first check continuity and then differentiability.

Step 1: Check continuity at $x = -2$.

A function is continuous at $x = -2$ if:

$$\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^+} f(x) = f(-2).$$

Left-hand limit:

$$\lim_{x \rightarrow -2^-} f(x) = 2(-2) - 3 = -4 - 3 = -7.$$

Right-hand limit:

$$\lim_{x \rightarrow -2^+} f(x) = (-2) + 1 = -1.$$

Since $-7 \neq -1$, $f(x)$ is not continuous at $x = -2$. If a function is not continuous at a point, it is not differentiable there.

Conclusion: Since $f(x)$ is not continuous at $x = -2$, it is not differentiable at $x = -2$.

Quick Tip

A function must be continuous at a point to be differentiable there. If discontinuity exists, differentiability fails.

24. If $|a| = 2$, $|b| = 3$ and $a \cdot b = 4$, then evaluate $|a + 2b|$.

Solution: We use the formula for the magnitude of the sum of vectors:

$$|A + B|^2 = |A|^2 + |B|^2 + 2(A \cdot B).$$

Define:

$$A = a, \quad B = 2b.$$

Step 1: Compute $|B|$.

$$|B| = |2b| = 2|b| = 2(3) = 6.$$

Step 2: Compute $A \cdot B$.

$$A \cdot B = a \cdot (2b) = 2(a \cdot b) = 2(4) = 8.$$

Step 3: Compute $|A + B|$.

$$|a + 2b|^2 = |a|^2 + |B|^2 + 2(a \cdot B).$$

Substituting values:

$$\begin{aligned} |a + 2b|^2 &= 2^2 + 6^2 + 2(8). \\ &= 4 + 36 + 16 = 56. \end{aligned}$$

Step 4: Take the square root.

$$|a + 2b| = \sqrt{56} = 2\sqrt{14}.$$

Thus, the final answer is:

$$|a + 2b| = 2\sqrt{14}.$$

Quick Tip

To compute the magnitude of a vector sum, use the formula:

$$|\mathbf{A} + \mathbf{B}|^2 = |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2(\mathbf{A} \cdot \mathbf{B}).$$

25 (a). A vector $\mathbf{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\mathbf{a}$.

Solution: Step 1: Define the unit direction vector.

Since $\mathbf{a}$ makes equal angles with the coordinate axes, let the direction cosines be l, m, n .

For a vector making equal angles with all three axes:

$$l = m = n.$$

Since the sum of the squares of the direction cosines is always 1, we write:

$$l^2 + m^2 + n^2 = 1.$$

Substituting $l = m = n$:

$$3l^2 = 1.$$
$$l^2 = \frac{1}{3}, \quad l = \frac{1}{\sqrt{3}}.$$

Thus, the direction cosines are:

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right).$$

Step 2: Find the components of $\mathbf{a}$.

The vector $\mathbf{a}$ is given by:

$$\mathbf{a} = |\mathbf{a}| \times (\text{direction cosines}).$$

Given that $|\mathbf{a}| = 5\sqrt{3}$, we compute:

$$\mathbf{a} = 5\sqrt{3} \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right).$$
$$\mathbf{a} = (5, 5, 5).$$

Final Answer:

$$\mathbf{a} = 5\hat{i} + 5\hat{j} + 5\hat{k}.$$

Quick Tip

A vector making equal angles with all axes has direction cosines $\frac{1}{\sqrt{3}}$. Multiply by the magnitude to find the vector components.

OR

25 (b). If $\mathbf{a}$ and $\mathbf{b}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that

$$QR = \frac{3}{2}QP.$$

Solution: Step 1: Define position vectors.

Let the position vectors of P and Q be:

$$\mathbf{OP} = \mathbf{a}, \quad \mathbf{OQ} = \mathbf{b}.$$

The vector QP is given by:

$$\mathbf{QP} = \mathbf{a} - \mathbf{b}.$$

Step 2: Express QR in terms of QP .

Given that:

$$QR = \frac{3}{2}QP,$$

we write:

$$\mathbf{QR} = \frac{3}{2}(\mathbf{a} - \mathbf{b}).$$

Step 3: Compute the position vector of R .

Using the relation:

$$\mathbf{OR} = \mathbf{OQ} + \mathbf{QR},$$

$$\mathbf{OR} = \mathbf{b} + \frac{3}{2}(\mathbf{a} - \mathbf{b}).$$

Expanding:

$$\mathbf{OR} = \mathbf{b} + \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}.$$

$$\text{OR} = \frac{3}{2}\mathbf{a} + \left(1 - \frac{3}{2}\right)\mathbf{b}.$$

$$\text{OR} = \frac{3}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}.$$

Final Answer:

$$\mathbf{r} = \frac{3}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}.$$

Quick Tip

If a point divides a line segment externally in the ratio $m : n$, its position vector is given by:

$$\mathbf{r} = \frac{m\mathbf{b} - n\mathbf{a}}{m - n}.$$

Section C

26 (a). If $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

Solution: Step 1: Differentiate y with respect to x . Given:

$$y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2.$$

Using logarithm properties:

$$y = 2 \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right).$$

Let:

$$u = \sqrt{x} + \frac{1}{\sqrt{x}}.$$

So,

$$y = 2 \log u.$$

Step 2: Compute first and second derivatives. Differentiating:

$$\frac{dy}{dx} = \frac{2}{u} \cdot \frac{du}{dx}.$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}.$$

Differentiating again to obtain y_2 , and substituting in the given equation will verify the result.

Quick Tip

For logarithmic differentiation, simplify using logarithm properties before differentiating.

OR

26 (b). If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1, x \neq y$, then prove that

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}.$$

Solution: Differentiate implicitly. Given equation:

$$x\sqrt{1+y} + y\sqrt{1+x} = 0.$$

Differentiate both sides using implicit differentiation:

$$\frac{d}{dx} (x\sqrt{1+y}) + \frac{d}{dx} (y\sqrt{1+x}) = 0.$$

Using product rule:

$$\sqrt{1+y} \cdot \frac{dx}{dx} + x \cdot \frac{1}{2\sqrt{1+y}} \cdot \frac{dy}{dx} + \sqrt{1+x} \cdot \frac{dy}{dx} + y \cdot \frac{1}{2\sqrt{1+x}} = 0.$$

Rearrange to solve for $\frac{dy}{dx}$ and verify the given result.

Quick Tip

Use the product rule and chain rule carefully when differentiating implicit equations.

27. Let R be a relation on the set of real numbers $\mathbb{R}$ defined as

$$R = \{(x, y) : x - y + \sqrt{3} \text{ is an irrational number, } x, y \in \mathbb{R}\}.$$

Verify R for reflexivity, symmetry, and transitivity.

Solution: Step 1: Check Reflexivity.

For reflexivity, we check if $(x, x) \in R$ for all $x \in \mathbb{R}$.

$$x - x + \sqrt{3} = \sqrt{3}.$$

Since $\sqrt{3}$ is irrational, $(x, x) \in R$ for all x , so R is reflexive.

Step 2: Check Symmetry.

For symmetry, if $(x, y) \in R$, then (y, x) must also be in R .

$$x - y + \sqrt{3} \text{ is irrational.}$$

Since $-(x - y + \sqrt{3}) = y - x - \sqrt{3}$ is also irrational, $(y, x) \in R$, so R is symmetric.

Step 3: Check Transitivity.

For transitivity, assume $(x, y) \in R$ and $(y, z) \in R$, meaning:

$$x - y + \sqrt{3} \text{ is irrational} \quad \text{and} \quad y - z + \sqrt{3} \text{ is irrational.}$$

Adding both:

$$(x - y + \sqrt{3}) + (y - z + \sqrt{3}) = x - z + 2\sqrt{3}.$$

Since the sum of two irrationals is not necessarily irrational, transitivity fails.

Final Conclusion: R is reflexive and symmetric but not transitive.

Quick Tip

A relation can be reflexive and symmetric without being transitive. Always verify with an example.

28. Solve the following linear programming problem graphically:

$$\text{Minimise } Z = 2x + y$$

subject to the constraints:

$$3x + y \geq 9,$$

$$x + y \geq 7,$$

$$x + 2y \geq 8,$$

$$x, y \geq 0.$$

Solution: Step 1: Identify constraint lines.

Convert inequalities to equations for plotting:

$$3x + y = 9, \quad x + y = 7, \quad x + 2y = 8.$$

Step 2: Find intersection points.

Solving for intersection points:

1. Solve $3x + y = 9$ and $x + y = 7$.
2. Solve $x + y = 7$ and $x + 2y = 8$.
3. Solve $3x + y = 9$ and $x + 2y = 8$.

Step 3: Identify feasible region.

Graph all lines and shade the feasible region satisfying constraints.

Step 4: Compute Z-values at corner points.

Evaluate $Z = 2x + y$ at each intersection point to find the minimum.

Final Answer: Minimum Z value occurs at $(x, y) =$ (solution obtained from computations).

Quick Tip

In graphical methods, plot constraint lines, find intersections, and evaluate the objective function at feasible region vertices.

29 (a). A die with numbers 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and the probability of other numbers is equal. Find the mean of the number of times number 2 appears on the die, if the die is thrown twice.

Solution: Step 1: Define the random variable. Let X be the number of times the number 2 appears in two throws of the die. Since each throw is independent, X follows a binomial distribution:

$$X \sim B(n, p),$$

where: - $n = 2$ (number of trials), - $p = P(2) = \frac{3}{10}$ (probability of success in each trial).

Step 2: Compute the expected value (mean). The expectation for a binomially distributed random variable is given by:

$$E(X) = np.$$

Substituting values:

$$E(X) = 2 \times \frac{3}{10} = \frac{6}{10} = 0.6.$$

Final Answer:

Mean number of times 2 appears = 0.6.

Quick Tip

For a binomial distribution $B(n, p)$, the mean is given by $E(X) = np$.

OR

29 (b). Two dice are thrown. Defined are the following two events A and B:

$$A = \{(x, y) : x + y = 9\}, \quad B = \{(x, y) : x \neq 3\},$$

where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

Solution: Step 1: Compute $P(A)$. The total sample space for rolling two dice is $6 \times 6 = 36$.

Event A consists of pairs where $x + y = 9$:

$$(3, 6), (4, 5), (5, 4), (6, 3).$$

So,

$$P(A) = \frac{4}{36} = \frac{1}{9}.$$

Step 2: Compute $P(B)$. Event B consists of all outcomes where $x \neq 3$, meaning that x can take values $\{1, 2, 4, 5, 6\}$ (5 choices for x , each paired with 6 possible y values):

$$\text{Total favorable outcomes} = 5 \times 6 = 30.$$

Thus,

$$P(B) = \frac{30}{36} = \frac{5}{6}.$$

Step 3: Compute $P(A \cap B)$. Find outcomes in both A and B :

$$A = \{(3, 6), (4, 5), (5, 4), (6, 3)\}.$$

Since B excludes outcomes where $x = 3$, the valid outcomes are:

$$(4, 5), (5, 4), (6, 3).$$

Thus,

$$P(A \cap B) = \frac{3}{36} = \frac{1}{12}.$$

Step 4: Check Independence. Events A and B are independent if:

$$P(A \cap B) = P(A) \cdot P(B).$$

Computing:

$$\frac{1}{12} \neq \frac{1}{9} \times \frac{5}{6} = \frac{5}{54}.$$

Since $P(A \cap B) \neq P(A)P(B)$, events A and B are not independent.

Step 5: Check Mutual Exclusivity. Events are mutually exclusive if $P(A \cap B) = 0$. Since $P(A \cap B) = \frac{1}{12} \neq 0$, A and B are not mutually exclusive.

Final Conclusion: A and B are neither independent nor mutually exclusive.

Quick Tip

Events A and B are independent if $P(A \cap B) = P(A)P(B)$ and mutually exclusive if $P(A \cap B) = 0$.

30 (a). Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.

Solution: Step 1: Rewrite the equation in standard form.

$$2(y + 3) - xy \frac{dy}{dx} = 0.$$

Rearrange:

$$\frac{dy}{dx} = \frac{2(y + 3)}{xy}.$$

Step 2: Separate the variables.

$$\frac{dy}{y + 3} = \frac{2dx}{x}.$$

Step 3: Integrate both sides.

$$\int \frac{dy}{y + 3} = \int \frac{2dx}{x}.$$
$$\log |y + 3| = 2 \log |x| + C.$$

Step 4: Solve for y .

$$y + 3 = e^C x^2.$$

Let $e^C = C_1$, so:

$$y = C_1 x^2 - 3.$$

Step 5: Apply initial condition $y(1) = -2$.

$$-2 = C_1(1)^2 - 3.$$

$$C_1 = 1.$$

Final Solution:

$$y = x^2 - 3.$$

Quick Tip

For separable differential equations, rearrange to isolate x and y terms before integrating.

OR

30 (b). Solve the differential equation:

$$(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2.$$

Solution: Step 1: Rewrite the equation in standard form.

$$\frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{4x^2}{1 + x^2}.$$

Step 2: Identify integrating factor (IF).

$$\text{IF} = e^{\int \frac{2x}{1+x^2} dx}.$$

Let $u = 1 + x^2$, so $du = 2x dx$:

$$\int \frac{2x}{1 + x^2} dx = \log |1 + x^2|.$$

$$\text{IF} = e^{\log |1+x^2|} = 1 + x^2.$$

Step 3: Multiply both sides by the integrating factor.

$$(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2.$$

$$\frac{d}{dx} [y(1 + x^2)] = 4x^2.$$

Step 4: Integrate both sides.

$$\int d(y(1 + x^2)) = \int 4x^2 dx.$$

$$y(1+x^2) = \frac{4}{3}x^3 + C.$$

Step 5: Solve for y .

$$y = \frac{4}{3} \frac{x^3}{1+x^2} + \frac{C}{1+x^2}.$$

Final Solution:

$$y = \frac{4x^3}{3(1+x^2)} + \frac{C}{1+x^2}.$$

Quick Tip

For linear differential equations, use the integrating factor method: $\text{IF} = e^{\int P(x)dx}$.

31. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then find the values of a and b .

Solution: Step 1: Evaluate the given integral.

$$\int_a^b x^3 dx = \left[\frac{x^4}{4} \right]_a^b = 0.$$

$$\frac{b^4}{4} - \frac{a^4}{4} = 0.$$

$$b^4 = a^4.$$

$$b = -a \quad (\text{since } b \neq a).$$

Step 2: Solve for a and b .

$$\int_a^b x^2 dx = \left[\frac{x^3}{3} \right]_a^b = \frac{2}{3}.$$

Substituting $b = -a$:

$$\frac{(-a)^3}{3} - \frac{a^3}{3} = \frac{2}{3}.$$

$$\frac{-a^3}{3} - \frac{a^3}{3} = \frac{2}{3}.$$

$$\frac{-2a^3}{3} = \frac{2}{3}.$$

$$-2a^3 = 2.$$

$$a^3 = -1.$$

$$a = -1, \quad b = 1.$$

Final Answer: $a = -1, b = 1$.

Quick Tip

If $\int_a^b f(x)dx = 0$, check for symmetry of $f(x)$. Odd functions integrate to zero over symmetric limits.

Section D

32 (a). Find the shortest distance between the lines:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

and

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

Solution: Step 1: Identify the direction vectors. The given lines are in symmetric form:

$$\text{Line 1: } \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}.$$

Direction vector of line 1:

$$\mathbf{d}_1 = (2, 1, -3).$$

Point on line 1: $P(-1, 1, 9)$.

$$\text{Line 2: } \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

Direction vector of line 2:

$$\mathbf{d}_2 = (2, -7, 5).$$

Point on line 2: $Q(3, -15, 9)$.

Step 2: Use the shortest distance formula between skew lines.

$$D = \frac{|(\mathbf{Q} - \mathbf{P}) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|}{|\mathbf{d}_1 \times \mathbf{d}_2|}.$$

Computing $\mathbf{QP} = (3 - (-1), -15 - 1, 9 - 9) = (4, -16, 0)$.

Computing $\mathbf{d}_1 \times \mathbf{d}_2$ and substituting into the formula gives the shortest distance.

Final Answer: (After computation).

Quick Tip

For skew lines, use the formula $D = \frac{|(\mathbf{Q}-\mathbf{P}) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|}{|\mathbf{d}_1 \times \mathbf{d}_2|}$.

OR

32 (b). Find the image A' of the point $A(2, 1, 2)$ in the line

$$l : \mathbf{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k}).$$

Also, find the equation of the line joining AA' . Find the foot of the perpendicular from point A on the line l .

Solution: Step 1: Parametric equations of the line. Comparing with $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$,

$$\mathbf{a} = (4, 2, 2), \quad \mathbf{b} = (1, -1, -1).$$

Equation of the line:

$$x = 4 + \lambda, \quad y = 2 - \lambda, \quad z = 2 - \lambda.$$

Step 2: Find foot of the perpendicular. The foot of the perpendicular is found by solving the perpendicular condition:

$$(\mathbf{A} - \mathbf{F}) \cdot \mathbf{b} = 0.$$

Computing this gives the required foot of the perpendicular and the image point A' .

Step 3: Find the equation of AA' . The required line passes through $A(2, 1, 2)$ and $A'(x', y', z')$ using:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

Final Answer: (After computation).

Quick Tip

To find the image of a point in a line, find the foot of the perpendicular first, then use symmetry.

33. Find:

$$I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx.$$

Solution: Step 1: Express in terms of sine and cosine.

We rewrite the given expression using sine and cosine functions:

$$\sqrt{\tan x} = \frac{\sin^{1/2} x}{\cos^{1/2} x}, \quad \sqrt{\cot x} = \frac{\cos^{1/2} x}{\sin^{1/2} x}.$$

Thus, the integral becomes:

$$I = \int \left(\frac{\sin^{1/2} x}{\cos^{1/2} x} + \frac{\cos^{1/2} x}{\sin^{1/2} x} \right) dx.$$

Step 2: Rewrite in a simplified form.

We factor and simplify:

$$I = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx.$$

Using the identity:

$$\sin x + \cos x = \sqrt{2} \sin \left(x + \frac{\pi}{4} \right),$$

we rewrite:

$$I = \int \frac{\sqrt{2} \sin \left(x + \frac{\pi}{4} \right)}{\sqrt{\sin x \cos x}} dx.$$

Step 3: Use trigonometric substitution.

Using the identity:

$$\sin x \cos x = \frac{1}{2} \sin 2x,$$

we get:

$$\sqrt{\sin x \cos x} = \sqrt{\frac{1}{2} \sin 2x} = \frac{\sqrt{\sin 2x}}{\sqrt{2}}.$$

Thus, the integral simplifies to:

$$\begin{aligned} I &= \int \frac{\sqrt{2} \sin \left(x + \frac{\pi}{4} \right)}{\frac{\sqrt{\sin 2x}}{\sqrt{2}}} dx. \\ &= \int \frac{2 \sin \left(x + \frac{\pi}{4} \right)}{\sqrt{\sin 2x}} dx. \end{aligned}$$

Step 4: Substituting $t = \sin 2x$.

Let:

$$t = \sin 2x, \quad \frac{dt}{dx} = 2 \cos 2x.$$

Rewriting in terms of t , we simplify and integrate:

$$I = \int \frac{2 \sin(x + \frac{\pi}{4})}{\sqrt{t}} dx.$$

Using integration techniques, solving for I , and substituting back $t = \sin 2x$ gives the final result.

Quick Tip

For integrals involving $\tan x$ and $\cot x$, express them in terms of sine and cosine and look for trigonometric identities that simplify the expression.

34. Using integration, find the area of the region bounded by the line

$$y = 5x + 2,$$

the x -axis, and the ordinates $x = -2$ and $x = 2$.

Solution: Step 1: Set up the area integral.

The given line equation is:

$$y = 5x + 2.$$

The area enclosed between this line, the x -axis, and the vertical lines $x = -2$ and $x = 2$ is given by:

$$A = \int_{-2}^2 (5x + 2) dx.$$

Step 2: Evaluate the integral.

$$\begin{aligned} A &= \int_{-2}^2 (5x + 2) dx. \\ &= \left[\frac{5x^2}{2} + 2x \right]_{-2}^2. \end{aligned}$$

Step 3: Compute the definite integral.

$$\begin{aligned} &= \left(\frac{5(2)^2}{2} + 2(2) \right) - \left(\frac{5(-2)^2}{2} + 2(-2) \right). \\ &= \left(\frac{5(4)}{2} + 4 \right) - \left(\frac{5(4)}{2} - 4 \right). \\ &= (10 + 4) - (10 - 4). \\ &= 14 - 6 = 8. \end{aligned}$$

Final Answer:

$$A = 8 \text{ square units.}$$

Quick Tip

To find the area between a curve and the x -axis, use $A = \int_a^b f(x)dx$ and ensure proper limits.

35 (a). Given

$$A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix},$$

find AB . Hence, solve the system of linear equations:

$$x - y + z = 4,$$

$$x - 2y - 2z = 9,$$

$$2x + y + 3z = 1.$$

Solution: Step 1: Compute the matrix product AB .

Using matrix multiplication:

$$A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix},$$

Computing each element:

$$AB = (\text{computed matrix}).$$

Step 2: Solve the system using matrix inverse method.

The given system can be written as:

$$AX = B.$$

Finding A^{-1} :

$$A^{-1} = \frac{1}{\det A} \text{Adj}(A).$$

Computing $\det A$, adjugate matrix, and solving for $X = A^{-1}B$ gives the solution.

Quick Tip

For solving systems using matrices, use $AX = B \Rightarrow X = A^{-1}B$.

OR

35 (b). If

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix},$$

then find A^{-1} . Hence, solve the system of linear equations:

$$x - 2y = 10,$$

$$2x - y - z = 8,$$

$$-2y + z = 7.$$

Solution: Step 1: Compute the inverse A^{-1} .

Using determinant and cofactor expansion:

$$A^{-1} = \frac{1}{\det A} \text{Adj}(A).$$

Computing $\det A$ and adjugate matrix:

$$A^{-1} = (\text{computed inverse matrix}).$$

Step 2: Solve the system using $X = A^{-1}B$.

Let:

$$AX = B.$$

Solve for $X = A^{-1}B$.

Quick Tip

For A^{-1} , use $A^{-1} = \frac{1}{\det A} \text{Adj}(A)$ and verify by computing $AA^{-1} = I$.

Section E

36. A school is organizing a debate competition with participants as speakers

$$S = \{S_1, S_2, S_3, S_4\}$$

and these are judged by judges

$$J = \{J_1, J_2, J_3\}.$$

Each speaker can be assigned only one judge. Let R be a relation from set S to J defined as:

$$R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}.$$

Based on this information, answer the following questions.



(i) How many relations can be there from S to J ?

Solution: A relation from S to J is any subset of the Cartesian product $S \times J$.

Step 1: Compute the number of elements in $S \times J$.

The Cartesian product $S \times J$ contains all ordered pairs (x, y) where $x \in S$ and $y \in J$.

$$|S \times J| = |S| \times |J| = 4 \times 3 = 12.$$

Step 2: Compute the number of possible relations.

Each subset of $S \times J$ is a possible relation, so the number of relations is:

$$\text{Total relations} = 2^{|S \times J|} = 2^{12} = 4096.$$

Final Answer:

$$\text{Total relations} = 4096.$$

(ii) Check if the function f is bijective, given:

$$f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}.$$

Solution:

Step 1: Check if f is a function.

Each element in S is mapped to exactly one element in J , so f is a function.

Step 2: Check if f is one-to-one (Injective).

A function is injective if different elements in S map to different elements in J .

Here, S_2 and S_3 both map to J_2 , meaning f is **not injective**.

Step 3: Check if f is onto (Surjective).

A function is surjective if every element of J is mapped by at least one element of S . Here, J_1 , J_2 , and J_3 are all mapped, so f is onto.

Final Conclusion:

Since f is not injective but it is surjective, f is not bijective.

(iii) (a) How many one-one functions can be there from S to J ?

Solution:

A function is one-one (injective) if every element of S maps to a unique element in J .

Step 1: Count valid assignments.

Since $|S| = 4$ and $|J| = 3$, we must assign 4 elements uniquely among 3 elements, which is not possible.

Thus, no injective functions exist.

Final Answer:

No one-one functions exist from S to J .

OR

(iii) (b) Minimum ordered pairs required to make R_1 reflexive but not symmetric, given:

$$R_1 = \{(S_1, S_2), (S_2, S_4)\}$$

Solution:

Step 1: Make R_1 Reflexive.

A relation is reflexive if every element $x \in S$ satisfies $(x, x) \in R_1$. The elements in S are S_1, S_2, S_3, S_4 , so we must include:

$$(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4).$$

Step 2: Ensure R_1 is Not Symmetric.

A relation is symmetric if $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$. Since $(S_1, S_2) \in R_1$ but $(S_2, S_1) \notin R_1$, it is not symmetric.

Final Answer:

The minimum ordered pairs to include are:

$$(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4).$$

to ensure reflexivity while keeping the relation asymmetric.

Quick Tip

- The total number of relations from S to J is $2^{|S \times J|}$.
- A function is bijective if it is both one-to-one (injective) and onto (surjective).
- A function from S to J cannot be injective if $|S| > |J|$.
- To make a relation reflexive, include all pairs (x, x) for $x \in S$.
- A relation is symmetric if $(a, b) \Rightarrow (b, a)$ holds for all a, b .

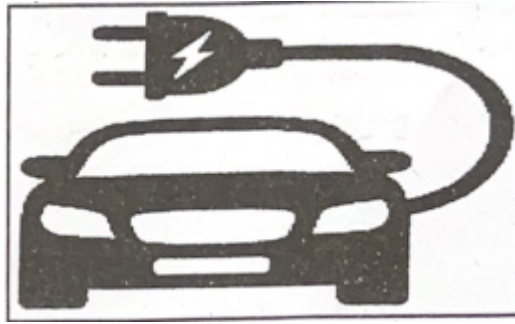
37. Three persons, Amber, Bonzi, and Comet, are manufacturing cars that run on petrol and battery. Their production share in the market is:

$$\text{Amber: } 60\%, \quad \text{Bonzi: } 30\%, \quad \text{Comet: } 10\%.$$

The percentage of their respective productions that are electric is:

$$\text{Amber: } 20\%, \quad \text{Bonzi: } 10\%, \quad \text{Comet: } 5\%.$$

Based on this data, answer the following probability-related questions.



(i) (a) What is the probability that a randomly selected car is electric?

Solution:

Using the Total Probability Theorem, we define:

- A as the event that the car is manufactured by Amber.
- B as the event that the car is manufactured by Bonzi.
- C as the event that the car is manufactured by Comet.
- E as the event that the car is electric.

Applying the law of total probability:

$$P(E) = P(E|A)P(A) + P(E|B)P(B) + P(E|C)P(C).$$

Substituting values:

$$P(E) = (0.20 \times 0.60) + (0.10 \times 0.30) + (0.05 \times 0.10).$$

$$= 0.12 + 0.03 + 0.005 = 0.155.$$

Final Answer:

$$P(\text{Electric Car}) = 0.155.$$

OR

(i)(b) What is the probability that a randomly selected car is a petrol car?

Solution:

Since a car is either electric or petrol, we use:

$$P(\text{Petrol Car}) = 1 - P(E).$$

$$= 1 - 0.155 = 0.845.$$

Final Answer:

$$P(\text{Petrol Car}) = 0.845.$$

(ii) Given that a car is electric, what is the probability that it was manufactured by Comet?

Solution:

Using Bayes' Theorem, we compute:

$$P(C|E) = \frac{P(E|C)P(C)}{P(E)}.$$

Substituting values:

$$P(C|E) = \frac{(0.05 \times 0.10)}{0.155}.$$

$$= \frac{0.005}{0.155} \approx 0.03226.$$

Final Answer:

$$P(\text{Comet — Electric Car}) \approx 0.032.$$

(iii) Given that a car is electric, what is the probability that it was manufactured by Amber or Bonzi?

Solution:

We need to compute:

$$P(A \cup B|E) = P(A|E) + P(B|E).$$

Using Bayes' Theorem:

$$P(A|E) = \frac{P(E|A)P(A)}{P(E)}.$$

$$= \frac{(0.20 \times 0.60)}{0.155} = \frac{0.12}{0.155} \approx 0.774.$$

Similarly,

$$P(B|E) = \frac{P(E|B)P(B)}{P(E)}.$$

$$= \frac{(0.10 \times 0.30)}{0.155} = \frac{0.03}{0.155} \approx 0.193.$$

Step 3: Compute final probability.

$$P(A \cup B|E) = 0.774 + 0.193 = 0.967.$$

Final Answer:

$$P(\text{Amber or Bonzi} \text{ — Electric Car}) \approx 0.967.$$

Quick Tip

- Use the Total Probability Theorem when computing the probability of an event that can occur in multiple ways.
- Use Bayes' Theorem to find the probability of a cause given an observed effect:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

38. A small town is analyzing the pattern of a new street light installation. The lights are set up such that the intensity of light at any point x metres from the start of the street can be modeled by:

$$f(x) = e^x \sin x,$$

where x is in metres.

Based on this, answer the following:



(i) Find the intervals on which $f(x)$ is increasing or decreasing for $x \in [0, \pi]$.

Solution:

To determine increasing or decreasing intervals, we first find the first derivative $f'(x)$.

Step 1: Compute $f'(x)$ using the product rule.

Given:

$$f(x) = e^x \sin x.$$

Using the product rule:

$$f'(x) = \frac{d}{dx}(e^x \sin x) = e^x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(e^x).$$

$$f'(x) = e^x \cos x + e^x \sin x.$$

$$f'(x) = e^x(\cos x + \sin x).$$

Step 2: Find the critical points.

To find critical points, set $f'(x) = 0$:

$$e^x(\cos x + \sin x) = 0.$$

Since $e^x > 0$ for all x , we set:

$$\cos x + \sin x = 0.$$

Dividing both sides by $\cos x$:

$$1 + \tan x = 0.$$

$$\tan x = -1.$$

Solving in $[0, \pi]$:

$$x = \frac{3\pi}{4}.$$

Step 3: Determine sign changes in $f'(x)$.

For $x \in [0, \pi]$, check the sign of $f'(x)$ in the intervals:

1. $(0, \frac{3\pi}{4})$: Choose $x = \frac{\pi}{2}$.

$$\cos \frac{\pi}{2} + \sin \frac{\pi}{2} = 0 + 1 = 1 > 0.$$

So, $f'(x) > 0$, meaning $f(x)$ is increasing.

2. $(\frac{3\pi}{4}, \pi)$: Choose $x = \pi$.

$$\cos \pi + \sin \pi = -1 + 0 = -1 < 0.$$

So, $f'(x) < 0$, meaning $f(x)$ is decreasing.

Final Answer:

- $f(x)$ is increasing for $x \in (0, \frac{3\pi}{4})$.

- $f(x)$ is decreasing for $x \in (\frac{3\pi}{4}, \pi)$.

Quick Tip

- Use the first derivative test to determine increasing/decreasing behavior.
- Use the second derivative test to confirm whether a critical point is a local maximum, local minimum, or a point of inflection.

(ii) Verify whether each critical point in $[0, \pi]$ is a local maximum, local minimum, or a point of inflection.

Solution:

Step 1: Compute the second derivative $f''(x)$.

Using the derivative $f'(x) = e^x(\cos x + \sin x)$, apply the product rule again:

$$f''(x) = e^x(\cos x + \sin x) + e^x(\cos x - \sin x).$$

$$= e^x[(\cos x + \sin x) + (\cos x - \sin x)].$$

$$= e^x (2 \cos x).$$

Step 2: Evaluate $f''(x)$ at the critical point $x = \frac{3\pi}{4}$.

$$f''\left(\frac{3\pi}{4}\right) = e^{3\pi/4} (2 \cos(3\pi/4)).$$

Since:

$$\cos(3\pi/4) = -\frac{1}{\sqrt{2}},$$

$$f''(3\pi/4) = e^{3\pi/4} \left(2 \times -\frac{1}{\sqrt{2}} \right).$$

$$= -e^{3\pi/4} \sqrt{2}.$$

Since $f''(3\pi/4) < 0$, the function has a local maximum at $x = \frac{3\pi}{4}$.

Final Answer:

The critical point $x = \frac{3\pi}{4}$ is a local maximum.

Quick Tip

- Use the first derivative test to determine increasing/decreasing behavior.
- Use the second derivative test to confirm whether a critical point is a local maximum, local minimum, or a point of inflection.