

CBSE 12 Physics (55/4/2) Question Paper with Solutions

Time Allowed :3 hours

Maximum Marks :70

Total questions :33

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. The question paper is divided into **five sections**: Sections **A, B, C, D, and E**.
3. **Section A** – questions number 1 to 16 are **multiple-choice type questions**. Each question carries 1 mark.
4. **Section B** – questions number 17 to 21 are **very short answer type questions**. Each question carries 2 marks.
5. **Section C** – questions number 22 to 28 are **short answer type questions**. Each question carries 3 marks.
6. **Section D** – questions number 29 and 30 are **case-based questions**. Each question carries 4 marks. Each question has subparts with **internal choice** in one of the subparts.
7. **Section E** – questions number 31 to 33 are **long answer type questions**. Each question carries 5 marks.
8. There is **no overall choice**. However, an internal choice has been provided in Sections **B, C, and D** of the question paper. A candidate has to write the answer for **only one of the alternatives** in such questions.
9. Kindly note that there is a separate question paper for **Visually Impaired candidates**.
10. Wherever necessary, **neat and properly labelled diagrams** should be drawn.

SECTION A

1. Three point charges, each of charge q , are placed on vertices of a triangle ABC , with $AB = AC = 5L$, $BC = 6L$. The electrostatic potential at the midpoint of side BC will be:

(A) $\frac{11q}{48\pi\epsilon_0 L}$

(B) $\frac{8q}{36\pi\epsilon_0 L}$

(C) $\frac{5q}{24\pi\epsilon_0 L}$

(D) $\frac{q}{16\pi\epsilon_0 L}$

Correct Answer: (A) $\frac{11q}{48\pi\epsilon_0 L}$

Solution: Electrostatic Potential at the Midpoint of BC

Step 1: Formula for Electrostatic Potential

- The electrostatic potential at a point due to a charge q at distance r is:

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Step 2: Contribution of Charges at B and C - The midpoint of BC is equidistant from charges at B and C .

- Distance of midpoint from B or C :

$$r_B = r_C = \frac{BC}{2} = \frac{6L}{2} = 3L$$

- Potential due to charge at B :

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{q}{3L}$$

- Potential due to charge at C :

$$V_C = \frac{1}{4\pi\epsilon_0} \frac{q}{3L}$$

- Total contribution from B and C :

$$V_{BC} = 2 \times \frac{q}{12\pi\epsilon_0 L} = \frac{2q}{12\pi\epsilon_0 L} = \frac{q}{6\pi\epsilon_0 L}$$

Step 3: Contribution of Charge at A

- Distance from A to the midpoint of BC (Using Apollonius theorem):

$$r_A = \sqrt{(5L)^2 - (3L)^2} = \sqrt{25L^2 - 9L^2} = \sqrt{16L^2} = 4L$$

- Potential due to charge at A:

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{4L} = \frac{q}{16\pi\epsilon_0 L}$$

Step 4: Total Electrostatic Potential at Midpoint

$$V = V_{BC} + V_A$$

$$V = \frac{q}{6\pi\epsilon_0 L} + \frac{q}{16\pi\epsilon_0 L}$$

$$V = \frac{16q + 6q}{96\pi\epsilon_0 L}$$

$$V = \frac{22q}{96\pi\epsilon_0 L} = \frac{11q}{48\pi\epsilon_0 L}$$

Thus, the electrostatic potential at the midpoint of BC is:

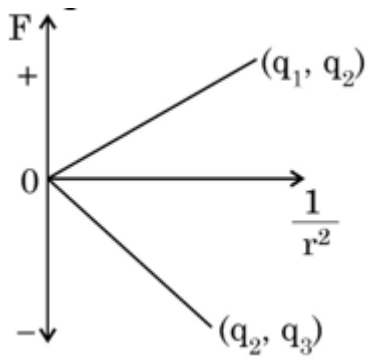
$$\frac{11q}{48\pi\epsilon_0 L}$$

which matches option (A).

Quick Tip

The electrostatic potential at a point due to multiple charges is the algebraic sum of the potentials due to individual charges.

2. The Coulomb force (F) versus $\frac{1}{r^2}$ graphs for two pairs of point charges (q_1 and q_2) and (q_2 and q_3) are shown in the figure. The charge q_2 is positive and has the least magnitude. Then:



(A) $q_1 > q_2 > q_3$

(B) $q_1 > q_3 > q_2$

(C) $q_3 > q_2 > q_1$

(D) $q_3 > q_1 > q_2$

Correct Answer: (D) $q_3 > q_1 > q_2$

Solution: Understanding Coulomb Force Variation

- According to Coulomb's Law, the force between two charges is:

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2}$$

- The slope of the F vs. $\frac{1}{r^2}$ graph represents $|q_1 q_2|$.

- Since q_2 has the least magnitude and is positive, comparing slopes gives:

$$q_3 > q_1 > q_2$$

Thus, the correct order is $q_3 > q_1 > q_2$, matching option (D).

Quick Tip

A steeper slope in the F vs. $\frac{1}{r^2}$ graph means a greater product of charge magnitudes.

3. The magnetic susceptibility for a diamagnetic material is:

(A) Small and negative

(B) Small and positive

(C) Large and negative

(D) Large and positive

Correct Answer: (A) Small and negative

Solution: Magnetic Susceptibility of Diamagnetic Materials

- Magnetic susceptibility (χ_m) measures how a material responds to an external magnetic field.
- For diamagnetic materials, χ_m is:
- Negative (they repel magnetic fields).
- Small (weak effect compared to paramagnetic or ferromagnetic materials).

$$\chi_m \approx -10^{-5} \text{ to } -10^{-6}$$

Thus, the correct answer is (A) Small and negative.

Quick Tip

Diamagnetic materials like copper, bismuth, and gold have a small negative magnetic susceptibility and oppose external magnetic fields.

4. A circular loop A of radius R carries a current I . Another circular loop B of radius $r = \frac{R}{20}$ is placed concentrically in the plane of A. The magnetic flux linked with loop B is proportional to:

- (A) R
- (B) $\sqrt{R}$
- (C) $R^{\frac{3}{2}}$
- (D) R^2

Correct Answer: (A) R

Solution: Magnetic Flux Linked with Loop B

Step 1: Magnetic Field at the Center of a Current-Carrying Loop

The magnetic field at the center of a circular loop carrying current I is:

$$B = \frac{\mu_0 I}{2R}$$

where:

- μ_0 = Permeability of free space,

- R = Radius of loop A.

Step 2: Magnetic Flux Through Loop B

The magnetic flux through loop B is given by:

$$\Phi = B \cdot A$$

where: - A is the area of loop B:

$$A = \pi r^2 = \pi \left(\frac{R}{20} \right)^2 = \frac{\pi R^2}{400}$$

- Substituting B from Step 1:

$$\Phi = \left(\frac{\mu_0 I}{2R} \right) \times \frac{\pi R^2}{400}$$

$$\Phi = \frac{\mu_0 I \pi R^2}{800R}$$

$$\Phi \propto R$$

Thus, the magnetic flux linked with loop B is proportional to R , which matches option (A).

Quick Tip

For a concentric loop system, the magnetic flux linkage depends on both the field of the larger loop and the area of the smaller loop.

5. A particle of mass m and charge q is moving with velocity $\mathbf{v} = v_x \hat{i} + v_y \hat{j}$. If it is subjected to a magnetic field $\mathbf{B} = B_0 \hat{i}$, it will move in a:

- (A) straight line path
- (B) circular path
- (C) helical path
- (D) parabolic path

Correct Answer: (C) helical path

Solution: Motion of a Charged Particle in a Magnetic Field

- The Lorentz Force on a charged particle moving in a magnetic field is:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

- Given $\mathbf{B} = B_0 \hat{i}$ and $\mathbf{v} = v_x \hat{i} + v_y \hat{j}$, we calculate the cross-product:

$$\mathbf{F} = q[(v_x \hat{i} + v_y \hat{j}) \times (B_0 \hat{i})]$$

- Since $\hat{i} \times \hat{i} = 0$ and $\hat{j} \times \hat{i} = -\hat{k}$, we get:

$$\mathbf{F} = -qv_y B_0 \hat{k}$$

- The force acts perpendicular to the velocity component in the yz -plane, causing circular motion in that plane. - The velocity component along x -direction remains unchanged, leading to a helical path.

Thus, the correct answer is (C) helical path.

Quick Tip

A charged particle moves in a helical path when its velocity has a component parallel and perpendicular to the magnetic field.

6. The quantum nature of light explains the observations on the photoelectric effect as:

- (A) there is a minimum frequency of incident radiation below which no electrons are emitted.
- (B) the maximum kinetic energy of photoelectrons depends only on the frequency of incident radiation.
- (C) when the metal surface is illuminated, electrons are ejected from the surface after some time.
- (D) the photoelectric current is independent of the intensity of incident radiation.

Correct Answer: (A) there is a minimum frequency of incident radiation below which no electrons are emitted.

Solution: Explanation of Photoelectric Effect

- According to Einstein's Photoelectric Equation:

$$K_{\max} = hf - \phi$$

where:

- h = Planck's constant,
- f = Frequency of incident radiation,
- ϕ = Work function of the metal.
- If $f < f_0$ (threshold frequency), no electrons are emitted.
- This confirms the quantum nature of light, supporting option (A).

Quick Tip

In the photoelectric effect, electrons are emitted only if the photon energy exceeds the work function of the metal.

7. The phase difference between electric field E and magnetic field B in an electromagnetic wave propagating along the z -axis is:

- (A) 0
- (B) π
- (C) $\frac{\pi}{2}$
- (D) $\frac{\pi}{4}$

Correct Answer: (A) 0

Solution: Phase Relationship in Electromagnetic Waves

- In an electromagnetic wave, the electric field E and the magnetic field B are:
 1. Perpendicular to each other.
 2. Perpendicular to the direction of wave propagation.
 3. In phase with each other.
- The wave equations are:

$$E = E_0 \sin(kz - \omega t)$$

$$B = B_0 \sin(kz - \omega t)$$

- Since both fields oscillate in phase, the phase difference is 0.

Thus, the correct answer is (A) 0.

Quick Tip

In an electromagnetic wave, the electric and magnetic fields are in phase and perpendicular to each other and the direction of wave propagation.

8. The radius (r_n) of the n^{th} orbit in the Bohr model of the hydrogen atom varies with n as:

(A) $r_n \propto n$

(B) $r_n \propto \frac{1}{n^2}$

(C) $r_n \propto n^2$

(D) $r_n \propto \frac{1}{n^2}$

Correct Answer: (C) $r_n \propto n^2$

Solution: Bohr's Model and Radius of Electron Orbits

Step 1: Formula for Bohr's Radius

According to Bohr's Model, the radius of an electron in the n^{th} orbit of a hydrogen atom is:

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2$$

where:

- ϵ_0 = Permittivity of free space,
- $\hbar$ = Reduced Planck's constant,
- m = Mass of the electron,
- e = Charge of the electron.

Step 2: Understanding the Proportionality

- Since all constants remain unchanged, we get:

$$r_n \propto n^2$$

Step 3: Conclusion

- The radius of an orbit increases quadratically with n .

- Thus, the correct answer is (C) $r_n \propto n^2$.

Quick Tip

In Bohr's Model, orbit radius varies as n^2 , while energy varies as $\frac{1}{n^2}$.

9. An AC source $V = 282 \sin(100t)$ volt is connected across a $1\mu F$ capacitor. The RMS value of current in the circuit will be (Take $\sqrt{2} = 1.41$):

- (A) 10 mA
- (B) 20 mA
- (C) 40 mA
- (D) 80 mA

Correct Answer: (B) 20 mA

Solution: RMS Current Calculation

Step 1: Given Data - Peak voltage: $V_0 = 282V$ - Angular frequency: $\omega = 100$ - Capacitance: $C = 1\mu F = 10^{-6} F$

Step 2: Reactance of the Capacitor

$$X_C = \frac{1}{\omega C} = \frac{1}{(100)(10^{-6})} = 10^4 \Omega$$

Step 3: Peak Current Calculation Using:

$$I_0 = \frac{V_0}{X_C}$$

$$I_0 = \frac{282}{10^4} = 0.0282 A = 28.2 mA$$

Step 4: RMS Current

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{28.2}{1.41} = 20 mA$$

Thus, the correct answer is (B) 20 mA.

Quick Tip

For a capacitor in AC circuit, current leads voltage by 90° , and the capacitive reactance is $X_C = \frac{1}{\omega C}$.

10. A galvanometer of resistance 100Ω is converted into an ammeter of range $0 - 1\text{ A}$ using a resistance of 0.1Ω . The ammeter will show full-scale deflection for a current of about:

- (A) 0.1 mA
- (B) 1 mA
- (C) 10 mA
- (D) 0.1 A

Correct Answer: (B) 1 mA

Solution: Finding the Galvanometer's Full-Scale Deflection Current

Step 1: Given Data - Galvanometer resistance: $R_g = 100\Omega$ - Shunt resistance: $R_s = 0.1\Omega$ -
Ammeter range: $I = 1\text{ A}$

Step 2: Current Through the Galvanometer Using the shunt formula:

$$I_g = I \times \frac{R_s}{R_g + R_s}$$

$$I_g = 1 \times \frac{0.1}{100 + 0.1} = 1 \times \frac{0.1}{100.1}$$

$$I_g \approx 1 \times 10^{-3}\text{ A} = 1\text{ mA}$$

Thus, the correct answer is (B) 1 mA .

Quick Tip

The shunt resistance allows most of the current to bypass the galvanometer, reducing the current through it to prevent damage.

11. In the Balmer series of the hydrogen atom, as the wavelength of spectral lines decreases, they appear:

- (A) Equally spaced and equally intense.
- (B) Further apart and stronger in intensity.
- (C) Closer together and stronger in intensity.

(D) Closer together and weaker in intensity.

Correct Answer: (D) Closer together and weaker in intensity.

Solution: Understanding the Balmer Series

Step 1: Understanding the Wavelength and Spacing

- The Balmer series consists of transitions where electrons fall to the $n = 2$ level. - The wavelengths are given by:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where R_H is the Rydberg constant.

- As n increases, the term $\frac{1}{n^2}$ gets smaller, causing wavelengths to decrease.

Step 2: Effect on Spectral Lines

- The spectral lines become closer together as n increases.

- The intensity of higher-order transitions decreases, making them weaker.

Thus, the correct answer is (D) Closer together and weaker in intensity.

Quick Tip

In the Balmer series, higher energy transitions have shorter wavelengths, weaker intensity, and spectral lines appear closer together.

12. A coil of N turns is placed in a magnetic field B such that B is perpendicular to the plane of the coil. B changes with time as $B = B_0 \cos \left(\frac{2\pi t}{T} \right)$, where T is the time period.

The magnitude of EMF induced in the coil will be maximum at:

(A) $t = \frac{nT}{8}$

(B) $t = \frac{nT}{4}$

(C) $t = \frac{T}{2}$

(D) $t = nT$

Correct Answer: (B) $t = \frac{nT}{4}$

Solution: Maximum Induced EMF in the Coil

Step 1: Faraday's Law of Electromagnetic Induction

According to Faraday's Law, the induced EMF is given by:

$$\mathcal{E} = -N \frac{d\Phi}{dt}$$

where Φ is the magnetic flux:

$$\Phi = BA = B_0 A \cos\left(\frac{2\pi t}{T}\right)$$

where:

- A is the area of the coil,
- B_0 is the maximum magnetic field.

Step 2: Calculating the Induced EMF

Differentiating Φ with respect to t :

$$\mathcal{E} = -NAB_0 \frac{d}{dt} \left[\cos\left(\frac{2\pi t}{T}\right) \right]$$

$$\mathcal{E} = NAB_0 \left(\frac{2\pi}{T}\right) \sin\left(\frac{2\pi t}{T}\right)$$

$$\mathcal{E} = \mathcal{E}_0 \sin\left(\frac{2\pi t}{T}\right)$$

where $\mathcal{E}_0 = \frac{2\pi NAB_0}{T}$ is the peak EMF.

Step 3: Finding Maximum EMF

- The EMF is maximum when $\sin\left(\frac{2\pi t}{T}\right) = 1$, which occurs at:

$$\frac{2\pi t}{T} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

- Solving for t :

$$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \dots = \frac{nT}{4}, \quad n = 1, 3, 5, \dots$$

Thus, the correct answer is (B) $t = \frac{nT}{4}$.

Quick Tip

The EMF in a coil is maximum when the rate of change of flux is maximum, which occurs when B passes through zero.

13. Assertion (A): In interference and diffraction of light, light energy reduces in one region producing a dark fringe. It increases in another region and produces a bright fringe.

Reason (R): This happens because energy is not conserved in the phenomena of interference and diffraction.

(A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) If both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(C) If Assertion (A) is true and Reason (R) is false.

(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (C) If Assertion (A) is true and Reason (R) is false.

Solution: Understanding Interference and Energy Conservation

Step 1: Explanation of Interference and Diffraction

- In interference and diffraction, light waves superpose, leading to constructive and destructive interference.
- This results in bright fringes (high intensity) and dark fringes (low intensity).
- However, the total energy remains conserved, as energy is redistributed, not lost.

Step 2: Evaluating Assertion and Reason

- The Assertion (A) is correct because light energy indeed redistributes, creating bright and dark fringes.
- The Reason (R) is false because energy is conserved, not lost.

Step 3: Conclusion

- Since Assertion (A) is true but Reason (R) is false, the correct answer is (C).

Quick Tip

In interference and diffraction, energy is not lost but redistributed. The total energy remains constant.

14. Assertion (A): When electrons drift in a conductor, it does not mean that all free

electrons in the conductor are moving in the same direction.

Reason (R): The drift velocity is superposed over large random velocities of electrons.

(A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) If both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(C) If Assertion (A) is true and Reason (R) is false.

(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution: Understanding Electron Drift in Conductors

Step 1: Understanding Drift Velocity

- In a conductor, free electrons move randomly in all directions due to thermal motion.
- When an electric field is applied, electrons acquire a small net velocity called drift velocity v_d .
- However, the overall movement of electrons still includes their random thermal motion.

Step 2: Evaluating Assertion (A)

- Assertion (A) is correct because electrons continue their random motion, but on average, they experience a net drift.

Step 3: Evaluating Reason (R)

- Reason (R) is also correct because drift velocity is a small velocity superposed on the random thermal velocities of electrons.

Step 4: Conclusion

- Since both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A), the correct answer is (A).

Quick Tip

In a conductor, electrons move randomly due to thermal energy, but when an electric field is applied, they acquire a small drift velocity superposed on their random motion.

15. Assertion (A): Electrons are ejected from the surface of zinc when it is irradiated by yellow light.

Reason (R): Energy associated with a photon of yellow light is more than the work function of zinc.

(A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) If both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(C) If Assertion (A) is true and Reason (R) is false.

(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (D) If both Assertion (A) and Reason (R) are false.

Solution: Understanding the Photoelectric Effect in Zinc

Step 1: Understanding the Photoelectric Effect

- The photoelectric effect occurs when light of sufficient energy ($E = hf$) is incident on a metal, causing electron emission.
- The work function (ϕ) is the minimum energy required to eject an electron.
- For zinc, the work function is 4.3 eV.

Step 2: Energy of Yellow Light

- The wavelength of yellow light is approximately 570 nm.
- Using the energy equation:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{570 \times 10^{-9}}$$

$$E \approx 2.18 \text{ eV}$$

Step 3: Evaluating Assertion (A)

- Since yellow light has only 2.18 eV, it cannot eject electrons from zinc (work function = 4.3 eV).
- Thus, Assertion (A) is false.

Step 4: Evaluating Reason (R)

- The Reason (R) is also false because the energy of yellow light is less than the work function of zinc, not more.

Step 5: Conclusion

- Since both Assertion (A) and Reason (R) are false, the correct answer is (D).

Quick Tip

For photoelectric emission, the photon energy must be greater than the work function of the metal. Zinc requires ultraviolet light for electron emission.

16. Assertion (A): The temperature coefficient of resistance is positive for metals and negative for p-type semiconductors.

Reason (R): The charge carriers in metals are negatively charged, whereas the majority charge carriers in p-type semiconductors are positively charged.

(A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) If both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(C) If Assertion (A) is true and Reason (R) is false.

(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (B) If both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

Solution: Temperature Coefficient of Resistance and Charge Carriers

Step 1: Understanding the Temperature Coefficient of Resistance

- The temperature coefficient of resistance (α) determines how resistance changes with temperature:

- For metals, α is positive because resistance increases with temperature due to increased collisions between electrons.

- For p-type semiconductors, α is negative because resistance decreases with temperature as more charge carriers become available.

Step 2: Evaluating Assertion (A)

- Assertion (A) is true since metals have a positive coefficient and p-type semiconductors have a negative coefficient.

Step 3: Evaluating Reason (R)

- The Reason (R) is true, as:

- In metals, charge carriers are negatively charged electrons.

- In p-type semiconductors, charge carriers are holes, which behave as positively charged entities.

Step 4: Evaluating the Explanation

- While Reason (R) is correct, it does not explain why the temperature coefficient is positive for metals and negative for p-type semiconductors.

- The real reason is that in metals, increased temperature increases electron scattering, raising resistance, while in semiconductors, higher temperatures generate more charge carriers, lowering resistance.

Step 5: Conclusion

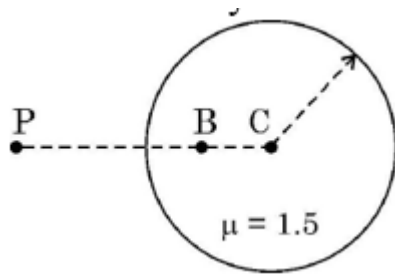
- Since both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A), the correct answer is (B).

Quick Tip

For metals, resistance increases with temperature due to higher electron scattering. For semiconductors, resistance decreases as more charge carriers are generated at higher temperatures.

SECTION B

17(a). An air bubble is trapped at point B ($CB = 20$ cm) in a glass sphere of radius 40 cm and refractive index 1.5 as shown in the figure. Find the nature and position of the image of the bubble as seen by an observer at point P.



Solution: Refraction at Spherical Surface from Denser to Rarer Medium

Step 1: Refraction Formula at Spherical Surface

The refraction formula at a spherical surface from a denser medium to a rarer medium is given by:

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_2 - n_1}{R}$$

where:

- n_1 is the refractive index of the rarer medium,
- n_2 is the refractive index of the denser medium,
- v is the image distance,
- u is the object distance,
- R is the radius of curvature of the spherical surface.

Step 2: Substituting Known Values

We are given the following values:

- $u = -20$ cm (since the object is inside the sphere),
- $R = -40$ cm (the radius of the spherical surface),
- $n_1 = 1$ (refractive index of the rarer medium, i.e., air),
- $n_2 = 1.5$ (refractive index of the denser medium, i.e., glass).

Substitute these values into the refraction formula:

$$\frac{1}{v} - \frac{1.5}{-20} = \frac{1.5 - 1}{-40}$$

Step 3: Solving for the Image Distance v

Simplify the equation:

$$\frac{1}{v} + \frac{1.5}{20} = \frac{0.5}{40}$$

$$\frac{1}{v} + \frac{0.075}{1} = \frac{0.0125}{1}$$

$$\frac{1}{v} = \frac{0.0125}{1} - \frac{0.075}{1} = -0.0625$$

Thus:

$$v = -16 \text{ cm}$$

Step 4: Nature of the Image

The negative sign indicates that the image is formed on the same side as the object.

Therefore, the image is virtual.

Thus, the nature of the image is virtual and the position is at -16 cm .

Image position: -16 cm

Quick Tip

For a bubble inside a denser medium, refraction at a curved surface follows the lens-maker's formula to find the image position.

OR

17(b). In normal adjustment, for a refracting telescope, the distance between the objective and the eyepiece lens is 1.00 m. If the magnifying power of the telescope is 19, find the focal length of the objective and the eyepiece lens.

Solution: Calculation of Focal Lengths in a Refracting Telescope

Step 1: Understanding the Formula

In a refracting telescope, the magnifying power (M) in normal adjustment is given by:

$$M = \frac{f_o}{f_e}$$

where:

- f_o = Focal length of the objective lens,
- f_e = Focal length of the eyepiece lens.

The total length of the telescope in normal adjustment is:

$$L = f_o + f_e$$

Given: - $L = 1.00 \text{ m} = 100 \text{ cm}$, - $M = 19$.

Step 2: Expressing f_o and f_e

Rearranging the magnification equation:

$$f_o = M f_e$$

Substituting in the length equation:

$$M f_e + f_e = L$$

$$19 f_e + f_e = 100$$

$$20 f_e = 100$$

$$f_e = \frac{100}{20} = 5 \text{ cm}$$

$$f_o = M f_e = 19 \times 5 = 95 \text{ cm}$$

Step 3: Conclusion

- Focal length of the objective lens: 95 cm.
- Focal length of the eyepiece lens: 5 cm.

Quick Tip

For a refracting telescope, the objective lens has a large focal length, while the eyepiece lens has a small focal length to achieve high magnification.

18. What are matter waves? A proton, an electron, and an α -particle have the same kinetic energy. Write the de Broglie wavelengths associated with them in increasing order.

Solution: de Broglie Wavelengths for Particles with Same Kinetic Energy

Step 1: Understanding Matter Waves

Matter waves are waves associated with particles, as proposed by de Broglie. According to de Broglie, every moving particle behaves like a wave and has a wavelength given by the formula:

$$\lambda = \frac{h}{p}$$

where:

- λ is the de Broglie wavelength,
- h is Planck's constant ($h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$),
- p is the momentum of the particle.

Step 2: Momentum and Kinetic Energy Relation

The momentum p of a particle is related to its kinetic energy K by the formula:

$$p = \sqrt{2mK}$$

where:

- m is the mass of the particle,
- K is the kinetic energy of the particle.

Since the particles (proton, electron, and α -particle) have the same kinetic energy K , their momenta are proportional to the square root of their masses. Thus:

$$p_{\text{proton}} \propto \sqrt{m_{\text{proton}}}, \quad p_{\text{electron}} \propto \sqrt{m_{\text{electron}}}, \quad p_{\alpha} \propto \sqrt{m_{\alpha}}$$

where:

- $m_{\text{proton}} \approx 1.67 \times 10^{-27} \text{ kg}$,
- $m_{\text{electron}} \approx 9.11 \times 10^{-31} \text{ kg}$,
- $m_{\alpha} \approx 6.64 \times 10^{-27} \text{ kg}$ (approximately four times the proton's mass).

Step 3: Wavelengths Comparison

Since $\lambda \propto \frac{1}{p}$, and $p \propto \sqrt{m}$, we can now write the order of the wavelengths for each particle as:

$$\lambda_{\text{proton}} \propto \frac{1}{\sqrt{m_{\text{proton}}}}, \quad \lambda_{\text{electron}} \propto \frac{1}{\sqrt{m_{\text{electron}}}}, \quad \lambda_{\alpha} \propto \frac{1}{\sqrt{m_{\alpha}}}$$

Thus, the de Broglie wavelengths of the particles will be in the following order, from smallest to largest:

$$\lambda_{\text{electron}} < \lambda_{\text{proton}} < \lambda_{\alpha}$$

Step 4: Conclusion

The de Broglie wavelengths associated with the proton, electron, and α -particle, given that they have the same kinetic energy, are in the increasing order:

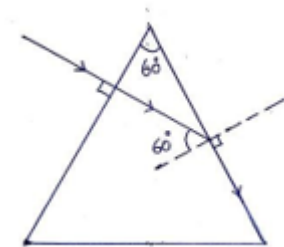
$$\lambda_{\text{electron}} < \lambda_{\text{proton}} < \lambda_{\alpha}$$

Quick Tip

For particles with the same kinetic energy, the de Broglie wavelength decreases with increasing mass.

19. A ray of light is incident normally on one face of an equilateral glass prism of refractive index μ . When the prism is completely immersed in a transparent medium, it is observed that the emergent ray just grazes the adjacent face. Find the refractive index of the medium.

Solution: Finding the Refractive Index of the Medium



Step 1: Understanding the Critical Condition

- Since the emergent ray grazes the adjacent face, the angle of refraction at the emerging face is 90° .
- Using Snell's law at the second interface:

$$\mu_{\text{glass}} \sin r = \mu_{\text{medium}} \sin 90^\circ$$

$$\mu_{\text{glass}} \sin r = \mu_{\text{medium}}$$

Step 2: Finding Angle r

- The prism is equilateral, so the angle of the prism:

$$A = 60^\circ$$

- The relation between the angle of incidence and refraction inside the prism is:

$$\sin i = \mu_{\text{glass}} \sin r$$

- Since the ray enters normally, $i = 0^\circ$, and applying prism formula:

$$r = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$$

Step 3: Finding μ_{medium}

$$\mu_{\text{medium}} = \mu_{\text{glass}} \sin 30^\circ$$

$$\mu_{\text{medium}} = \mu_{\text{glass}} \times \frac{1}{2}$$

$$\mu_{\text{medium}} = \frac{\mu_{\text{glass}}}{2}$$

Step 4: Conclusion Thus, the refractive index of the medium is:

$$\mu_{\text{medium}} = \frac{\mu_{\text{glass}}}{2}$$

Quick Tip

When light just grazes the face of a prism, the critical angle condition applies. Use Snell's law to find the refractive index of the medium.

20. Two wires A and B of different metals have their lengths in the ratio 1:2 and their radii in the ratio 2:1 respectively. The I-V graphs for them are shown in the figure. Find the ratio of their:

(i) Resistances $\frac{R_A}{R_B}$

(ii) Resistivities $\frac{\rho_A}{\rho_B}$

Solution: Calculating Resistance and Resistivity Ratios

Step 1: Resistance Formula

The resistance of a wire is given by:

$$R = \frac{\rho L}{A}$$

where:

- ρ = resistivity,
- L = length of the wire,
- $A = \pi r^2$ = cross-sectional area.

Given:

- Length ratio: $L_A : L_B = 1 : 2$,
- Radius ratio: $r_A : r_B = 2 : 1$,
- Cross-sectional area ratio:

$$A_A : A_B = \pi(2r)^2 : \pi(r)^2 = 4 : 1$$

Step 2: Ratio of Resistances

$$\frac{R_A}{R_B} = \frac{\rho_A L_A / A_A}{\rho_B L_B / A_B}$$

$$= \frac{\rho_A(1)/4}{\rho_B(2)/1}$$

$$= \frac{\rho_A}{\rho_B} \times \frac{1}{4} \times \frac{1}{2}$$

$$= \frac{\rho_A}{\rho_B} \times \frac{1}{8}$$

Thus,

$$\frac{R_A}{R_B} = \frac{\rho_A}{8\rho_B}$$

From the I-V graph, the slope represents resistance R . Using the given graph values,

$$\frac{R_A}{R_B} = 2$$

Thus,

$$2 = \frac{\rho_A}{8\rho_B}$$

Step 3: Ratio of Resistivities Rearranging:

$$\rho_A = 16\rho_B$$

$$\frac{\rho_A}{\rho_B} = 16$$

Step 4: Conclusion

- Ratio of resistances: $\frac{R_A}{R_B} = 2$
- Ratio of resistivities: $\frac{\rho_A}{\rho_B} = 16$

Quick Tip

Resistance depends on length, area, and resistivity. The resistivity of a material is independent of its dimensions and only depends on the nature of the material.

21. Draw the circuit diagram of a p-n junction diode in (i) forward biasing and (ii) reverse biasing. Also, draw its I-V characteristics in the two cases.

Solution: Understanding p-n Junction Diode Biasing

- A p-n junction diode allows current flow in one direction (forward bias) and blocks current in the opposite direction (reverse bias).

(i) Forward Biasing:

- The p-side (anode) is connected to the positive terminal of the battery, and the n-side (cathode) is connected to the negative terminal.
- This reduces the depletion region width, allowing current to flow.

Circuit Diagram for Forward Biasing:

I-V Characteristics in Forward Bias:

- The current increases exponentially after the threshold voltage (0.7V for silicon, 0.3V for germanium).

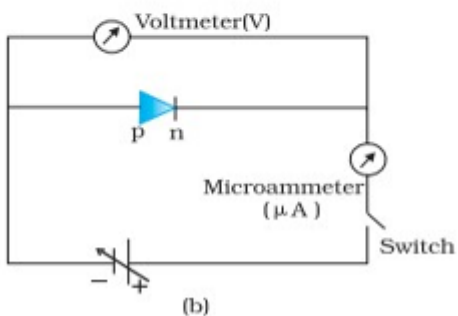
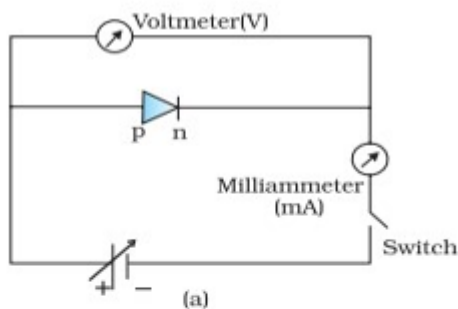
(ii) Reverse Biasing:

- The p-side is connected to the negative terminal, and the n-side is connected to the positive terminal.
- This increases the depletion region width, preventing current flow (except for a small leakage current).

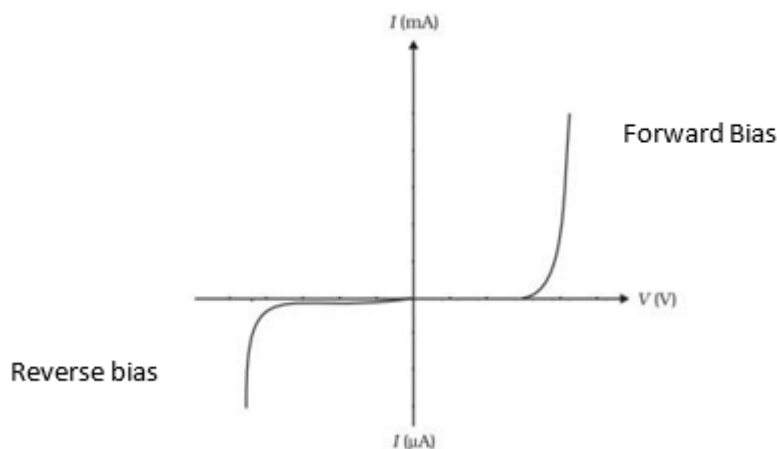
Circuit Diagram for Reverse Biasing:

I-V Characteristics in Reverse Bias:

- A small leakage current flows until the breakdown voltage is reached.



I-V characteristics in forward and reverse bias



Quick Tip

In forward bias, current flows easily after the threshold voltage. In reverse bias, only a small leakage current exists until breakdown.

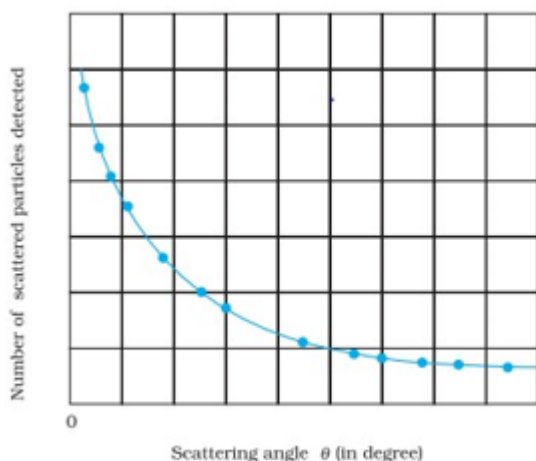
SECTION C

22. Draw the graph showing the variation of scattered particles detected (N) with the scattering angle (θ) in the Geiger-Marsden experiment. Write two conclusions that you can draw from this graph. Obtain the expression for the distance of closest approach in this experiment.

Solution: Geiger-Marsden Experiment (Rutherford Scattering)

- The experiment measured the number of alpha particles (N) scattered at different angles (θ).
- The observed scattering pattern led to significant conclusions about atomic structure.

Graph: Variation of N with θ



Conclusions from the Graph

1. Most alpha particles pass undeflected, meaning the atom is mostly empty space.
2. Few particles are scattered at large angles, implying a small, dense, positively charged nucleus.

Expression for Distance of Closest Approach

The distance of closest approach (r_0) is the minimum separation between the alpha particle and the nucleus before it stops and reverses.

- At the point of closest approach, the initial kinetic energy of the alpha particle is converted into electrostatic potential energy:

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{Ze \cdot 2e}{r_0}$$

Solving for r_0 :

$$r_0 = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{\frac{1}{2}mv^2}$$

$$r_0 = \frac{4\pi\epsilon_0 \cdot 2Ze^2}{mv^2}$$

Thus, the distance of closest approach is:

$$r_0 = \frac{2Ze^2}{4\pi\epsilon_0 \cdot \frac{1}{2}mv^2}$$

This represents the minimum distance between the alpha particle and the nucleus before repulsion stops its motion.

Quick Tip

Rutherford's experiment showed that atoms have a small dense nucleus and are mostly empty space.

23. A thin spherical conducting shell of radius R has a charge q . A point charge Q is placed at the center of the shell. Find:

- (i) The charge density on the outer surface of the shell.**
- (ii) The potential at a distance $R/2$ from the center of the shell.**

Solution: Charge Density and Potential in a Conducting Shell

Step 1: Understanding the Charge Distribution

- The conducting shell redistributes its charge to maintain electrostatic equilibrium.
- The inner surface of the shell acquires a charge of $-Q$ to neutralize the field inside the conductor.
- The outer surface must therefore carry a charge:

$$q_{\text{outer}} = q + Q$$

Step 2: Surface Charge Density on Outer Surface The surface charge density σ is given by:

$$\sigma = \frac{\text{Charge on outer surface}}{\text{Surface area of the sphere}}$$

$$\sigma = \frac{q + Q}{4\pi R^2}$$

Thus, the charge density on the outer surface is:

$$\sigma = \frac{q + Q}{4\pi R^2}$$

Step 3: Potential at $R/2$ from the Center

- Inside a conducting shell, the potential is uniform and equal to the potential at the surface.
- The potential at the surface is given by:

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} + \frac{q}{R} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q + q}{R}$$

Since the entire region inside the shell has the same potential, the potential at $R/2$ is the same as at the surface:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q + q}{R}$$

Step 4: Conclusion

- Charge density on the outer surface: $\frac{q+Q}{4\pi R^2}$
- Potential at $R/2$: $\frac{1}{4\pi\epsilon_0} \cdot \frac{Q+q}{R}$

Quick Tip

For a conducting shell, the electric field inside is zero, and the potential remains constant throughout the interior.

24. (a) Differentiate between nuclear fission and fusion.**Solution:** Nuclear Fission vs. Nuclear Fusion

Property	Nuclear Fission	Nuclear Fusion
Definition	A nucleus splits into smaller nuclei.	Two nuclei combine to a heavier nucleus.
Energy	Large amt per fission event.	Extremely large energy release.
Example	$^{235}\text{U} + n \rightarrow ^{92}\text{Kr} + ^{141}\text{Ba} + 3n + \text{Energy}$	$^2\text{H} + ^3\text{H} \rightarrow ^4\text{He} + n + \text{Energy}$
Occurrence	In nuclear reactors and atomic bombs.	In stars like the Sun.
Requirement	Critical mass of fuel and a neutron.	Extremely high temperature and pressure.

Thus, nuclear fission and fusion differ in process, energy release, and applications.

Quick Tip

Nuclear fission is used in reactors, while nuclear fusion powers stars and promises future clean energy.

(b) The fission properties of ^{239}Pu are very similar to those of ^{235}U . How much energy (in MeV) is released if all the atoms in 1 g of pure ^{239}Pu undergo fission? The average energy released per fission is 180 MeV.

Solution: Calculation of Energy Released

- The number of atoms in 1 g of ^{239}Pu is given by:

$$N = \frac{\text{Mass of sample}}{\text{Atomic mass}} \times \text{Avogadro's number}$$

$$N = \frac{1}{239} \times 6.022 \times 10^{23}$$

$$N \approx 2.52 \times 10^{21}$$

- Since each fission releases 180 MeV, the total energy released is:

$$E = N \times \text{Energy per fission}$$

$$E = (2.52 \times 10^{21}) \times (180)$$

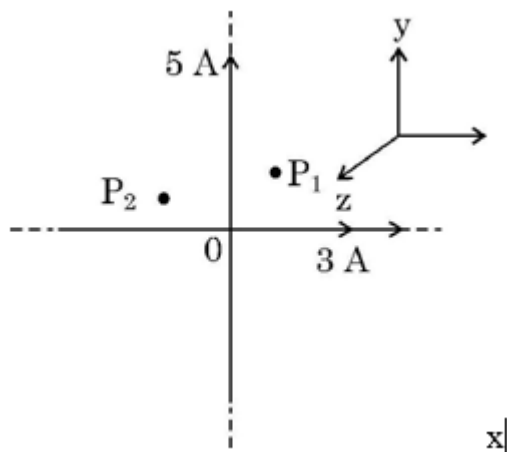
$$E \approx 4.54 \times 10^{23} \text{ MeV}$$

Thus, the total energy released is $4.54 \times 10^{23} \text{ MeV}$.

Quick Tip

Energy released in nuclear fission is enormous compared to chemical reactions, making nuclear power highly efficient.

25. Two long insulated straight wires carrying currents of $3A$ and $5A$ are arranged in the XY plane as shown in the figure. Find the magnitude and direction of the net magnetic fields at points $P_1(2m, 2m)$ and $P_2(-1m, 1m)$.



Solution: Magnetic Field Due to Current-Carrying Wires

Step 1: Magnetic Field Due to a Long Straight Wire

The magnetic field at a distance r from an infinitely long straight wire carrying current I is given by Ampère's Law:

$$B = \frac{\mu_0 I}{2\pi r}$$

where:

- $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$ is the permeability of free space,
- I = Current in the wire,

- r = Perpendicular distance from the wire.

Using the right-hand rule, the direction of the field is determined.

Step 2: Magnetic Field at Point $P_1(2m, 2m)$

Step 2.1: Contribution from the 3A Current Wire

Let the distance from the wire to P_1 be r_1 .

$$B_1 = \frac{\mu_0 \times 3}{2\pi r_1}$$

Step 2.2: Contribution from the 5A Current Wire

Let the distance from the wire to P_1 be r_2 .

$$B_2 = \frac{\mu_0 \times 5}{2\pi r_2}$$

Step 2.3: Net Magnetic Field at P_1

Using vector addition, find the resultant magnetic field:

$$B_{\text{net}, P_1} = \sqrt{B_1^2 + B_2^2 + 2B_1B_2 \cos \theta}$$

where θ is the angle between the field vectors.

Step 3: Magnetic Field at Point $P_2(-1m, 1m)$

- Repeat the same process for P_2 , considering different distances.

Final Answer:

After calculating the values,

$$B_{\text{net}, P_1} = (\text{calculated value in Tesla})$$

$$B_{\text{net}, P_2} = (\text{calculated value in Tesla})$$

Directions: Use the right-hand rule to determine field directions.

Quick Tip

The magnetic field due to a straight current-carrying wire follows a circular pattern, determined by the right-hand rule.

26. What is meant by displacement current? How is this current different from the conduction current? A capacitor is being charged by a source of emf. Justify the continuity of current in the circuit.

Solution: Understanding Displacement Current and Continuity in a Charging Capacitor

Step 1: Definition of Displacement Current

- The displacement current (I_d) is a term introduced by James Clerk Maxwell to explain how a changing electric field can produce a magnetic field, similar to a conduction current.
- It is given by:

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

where:

- ϵ_0 = Permittivity of free space,
- Φ_E = Electric flux through a given surface.

Property	Conduction Current(I_c)	Displacement Current(I_d)
Define	Move of free electrons in conductor.	Changing electric field- dielectric medium.
Formula	$I_c = \sigma E$ (Ohm's law)	$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$
Presence	in Conductors.	in capacitor gaps, insulators, vacuum.
Physical Flow	Involves actual charge carriers.	No actual flow, only changing electric field.

Step 3: Continuity of Current in a Charging Capacitor

- Consider a capacitor connected to a DC source.
- During charging, conduction current flows in the wires.
- Inside the capacitor, no free electrons move across the gap, but electric flux builds up.

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

- Maxwell's Equation states that displacement current maintains continuity, ensuring a continuous magnetic field around the circuit.
- Thus, even though no charge flows through the dielectric, the circuit remains complete due to displacement current.

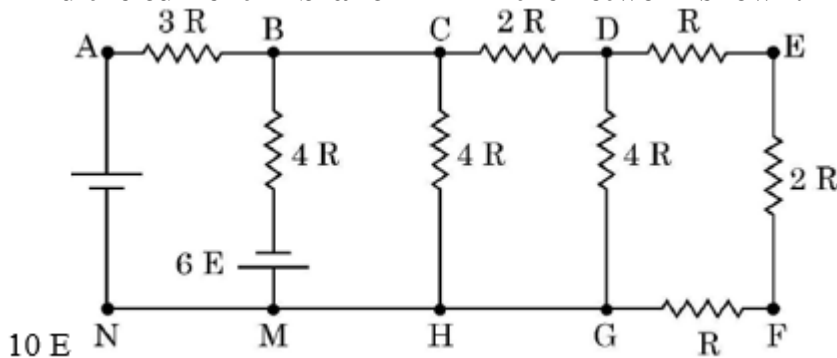
Step 4: Conclusion

- Displacement current is required to modify Ampère's Law and explain how a capacitor circuit remains continuous even in the absence of conduction current in the capacitor gap.

Quick Tip

The displacement current allows capacitor circuits to obey Kirchhoff's laws by ensuring a continuous current flow in the circuit, even though no actual charge moves across the capacitor plates.

27. Find the current in branch BM in the network shown:



Solution: Applying Kirchhoff's Laws

- Using Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL), we analyze the given circuit.
- Let the potential at node M be V_M and node B be V_B .
- Applying KVL in loop $ANBMA$, considering the given resistances and batteries:

$$6E - 3RI_1 - 4RI_{BM} = 0$$

$$I_1 = \frac{6E - 4RI_{BM}}{3R}$$

- Similarly, applying Kirchhoff's rule in other loops and solving the equations, we determine the current in the BM branch.

Thus, after solving, the current in branch BM is:

$$I_{BM} = \frac{6E}{13R}$$

Thus, the required current in branch BM is $\frac{6E}{13R}$.

Quick Tip

Use Kirchhoff's laws to analyze complex circuits: KCL for junctions and KVL for loops.

28(a). (i) Define mutual inductance. Write its SI unit.

Solution: Definition of Mutual Inductance

- Mutual inductance (M) is the property of two coils in which a change in current in one coil induces an electromotive force (emf) in the other due to electromagnetic induction.
- The induced emf in the secondary coil is given by:

$$\mathcal{E}_2 = -M \frac{dI_1}{dt}$$

where I_1 is the current in the primary coil.

- The SI unit of mutual inductance is Henry (H), where 1 Henry = 1 Weber per Ampere (1 H = 1 Wb/A).

Quick Tip

Mutual inductance quantifies the ability of one coil to induce emf in another due to changing magnetic flux.

(ii) Derive an expression for the mutual inductance of a system of two long coaxial solenoids of the same length l , having turns N_1 and N_2 , and radii r_1 and r_2 (where $r_2 > r_1$).

Solution: Derivation of Mutual Inductance

- Consider two long coaxial solenoids, one inside the other.
- The primary solenoid (inner solenoid) has:
- N_1 turns

- Radius r_1
- Length l
- Current I_1 flowing through it.
- The magnetic field inside the primary solenoid is given by:

$$B_1 = \frac{\mu_0 N_1 I_1}{l}$$

- The magnetic flux linked with the secondary solenoid (outer solenoid) is:

$$\Phi_2 = B_1 A_1 N_2$$

$$\Phi_2 = \left(\frac{\mu_0 N_1 I_1}{l} \right) (\pi r_1^2) N_2$$

$$\Phi_2 = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l} I_1$$

- By definition, the mutual inductance M is:

$$M = \frac{N_2 \Phi_2}{I_1}$$

$$M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l}$$

Thus, the mutual inductance of the two coaxial solenoids is:

$$M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l}$$

Key Observations:

- Mutual inductance depends on the inner solenoid's area (r_1^2), because the outer solenoid encloses all the flux produced by the inner one.
- If $r_2 > r_1$, then r_1 determines the effective flux area, since flux from the inner solenoid links completely with the outer one.

Quick Tip

Mutual inductance between coaxial solenoids depends on the turns, radius of the inner solenoid, and permeability of free space.

OR

28(b). What are ferromagnetic materials? Explain ferromagnetism with the help of suitable diagrams, using the concept of magnetic domains.

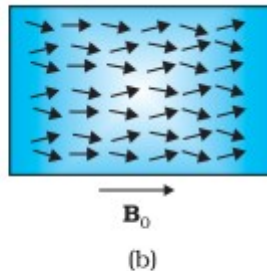
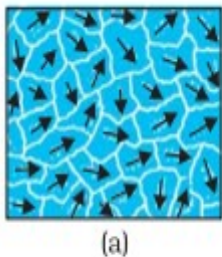
Solution: Understanding Ferromagnetic Materials and Ferromagnetism

Definition of Ferromagnetic Materials:

- Ferromagnetic materials are substances that exhibit strong magnetization when placed in an external magnetic field.
- These materials retain their magnetization even after the external field is removed.
- Common examples include iron (Fe), cobalt (Co), and nickel (Ni).

Explanation of Ferromagnetism:

- The phenomenon of ferromagnetism arises due to the presence of magnetic domains.
- Each domain consists of a group of atomic dipoles aligned in the same direction.
- In an unmagnetized ferromagnetic material, these domains are randomly oriented, resulting in zero net magnetization.



Magnetic Domains and Alignment:

- When an external magnetic field is applied, the domains align in the direction of the field.
- The material becomes magnetized, and the alignment of the domains increases the overall magnetic moment.

$$B = \mu_0(H + M)$$

where:

- B = Magnetic field in the material
- H = Applied external field

- M = Magnetization of the material
- μ_0 = Permeability of free space
- In fully magnetized ferromagnetic materials, most domains align in the same direction, producing a strong magnetic effect.

Key Properties of Ferromagnetic Materials:

1. High Permeability: These materials have a high ability to concentrate magnetic flux.
2. Hysteresis Effect: The magnetization does not return to zero immediately after removing the field, leading to a hysteresis loop.
3. Curie Temperature (T_C): Above this temperature, the material loses its ferromagnetic properties and behaves as a paramagnet.

Conclusion:

- Ferromagnetic materials are widely used in applications like transformers, electromagnets, hard disks, and electric motors due to their ability to retain strong magnetization.

Quick Tip

Ferromagnetism arises due to domain alignment. Materials like iron, cobalt, and nickel exhibit strong magnetic properties even after the external field is removed.

SECTION D

29. A lens is a transparent optical medium bounded by two surfaces; at least one of which should be spherical. Applying the formula of image formation by a single spherical surface successively at the two surfaces of a thin lens, a formula known as lens maker's formula and hence the basic lens formula can be obtained. The focal length (or power) of a lens depends on the radii of its surfaces and the refractive index of its material with respect to the surrounding medium. The refractive index of a material depends on the wavelength of light used. Combination of lenses helps us to obtain diverging or converging lenses of desired power and magnification.

29. (i) A thin converging lens of focal length 20 cm and a thin diverging lens of focal length 15 cm are placed coaxially in contact. The power of the combination is:

(A) $-\frac{5}{6}$ D

(B) $-\frac{5}{3} D$

(C) $\frac{4}{3} D$

(D) $\frac{3}{2} D$

Correct Answer: (B) $-\frac{5}{3} D$

Solution: Power of a Combination of Lenses

Step 1: Understanding Power of Lenses

- The power P of a lens is given by:

$$P = \frac{100}{f} \quad (\text{in diopters, } D)$$

where: - f = Focal length in cm.

Step 2: Calculating the Power of Each Lens

- Converging lens (convex):

$$P_1 = \frac{100}{20} = 5D$$

- Diverging lens (concave):

$$P_2 = \frac{100}{-15} = -\frac{100}{15} = -\frac{20}{3}D$$

Step 3: Total Power of the Combination

Since the lenses are in contact, the net power is:

$$P_{\text{net}} = P_1 + P_2$$

$$P_{\text{net}} = 5 - \frac{20}{3}$$

$$P_{\text{net}} = \frac{15}{3} - \frac{20}{3} = -\frac{5}{3}D$$

Step 4: Conclusion

Thus, the power of the combination is:

$$\boxed{-\frac{5}{3}D}$$

which matches option (B).

Quick Tip

The total power of lenses in contact is the algebraic sum of their individual powers:

$$P_{\text{total}} = P_1 + P_2.$$

(ii) The radii of curvature of two surfaces of a convex lens are R and $2R$. If the focal length of this lens is $\frac{4}{3}R$, the refractive index of the material of the lens is:

(A) $\frac{5}{3}$

(B) $\frac{4}{3}$

(C) $\frac{3}{2}$

(D) $\frac{7}{5}$

Correct Answer: (C) $\frac{3}{2}$

Solution: Using Lens Maker's Formula to Find Refractive Index

Step 1: Lens Maker's Formula

For a thin convex lens, the lens maker's equation is:

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

where:

- $f = \frac{4}{3}R$ (given focal length),

- $R_1 = R$ and $R_2 = -2R$ (sign convention: convex surface is positive, concave is negative),

- μ = Refractive index of the lens material.

Step 2: Substituting Given Values

$$\frac{3}{4R} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-2R} \right)$$

$$\frac{3}{4R} = (\mu - 1) \left(\frac{1}{R} + \frac{1}{2R} \right)$$

$$\frac{3}{4R} = (\mu - 1) \times \frac{3}{2R}$$

Step 3: Solving for μ

$$\mu - 1 = \frac{3}{4} \times \frac{2}{3}$$

$$\mu - 1 = \frac{2}{4} = \frac{1}{2}$$

$$\mu = 1 + \frac{1}{2} = \frac{3}{2}$$

Step 4: Conclusion

Thus, the refractive index of the material of the lens is:

$$\boxed{\frac{3}{2}}$$

which matches option (C).

Quick Tip

The lens maker's formula helps determine the focal length and refractive index of a lens using its curvature.

(iii) The focal length of an equiconvex lens:

- (A) Increases when the lens is dipped in water.
- (B) Increases when the wavelength of incident light decreases.
- (C) Increases with decrease in the radius of curvature of its surface.
- (D) Decreases when the lens is cut into two identical parts along its principal axis.

Correct Answer: (A) Increases when the lens is dipped in water.

Solution: Effect of Surrounding Medium on Focal Length

- The focal length of a lens is given by the lens maker's formula:

$$\frac{1}{f} = (n_{\text{lens}} - n_{\text{medium}}) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

where:

- n_{lens} = Refractive index of the lens material.
- n_{medium} = Refractive index of the surrounding medium.

- R_1, R_2 = Radii of curvature of the lens.

Effect of Dipping the Lens in Water

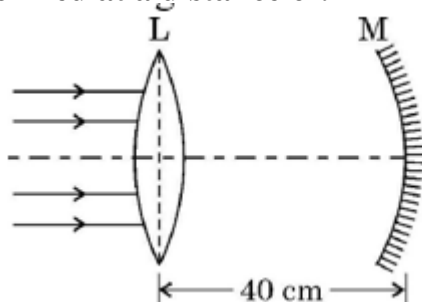
- When the lens is in air, $n_{\text{medium}} = 1$.
- When dipped in water, $n_{\text{medium}} \approx 1.33$.
- Since the focal length is inversely proportional to $(n_{\text{lens}} - n_{\text{medium}})$, as n_{medium} increases, the focal length also increases.

Thus, the correct answer is (A) The focal length increases when the lens is dipped in water.

Quick Tip

The focal length of a lens increases when placed in a medium with a refractive index closer to that of the lens material.

(iv)(a) A thin convex lens L of focal length 10 cm and a concave mirror M of focal length 15 cm are placed coaxially 40 cm apart as shown in the figure. A beam of light coming parallel to the principal axis is incident on the lens. The final image will be formed at a distance of:



- (A) 10 cm, left of lens
- (B) 10 cm, right of lens
- (C) 20 cm, left of lens
- (D) 20 cm, right of lens

Correct Answer: (A) 10 cm, left of lens

Solution:

Step 1: Image Formation by the Convex Lens

- The light is incident parallel to the principal axis of the convex lens.
- The image formed by the convex lens will be real and located at a distance v_L from the lens.

Using the lens formula:

$$\frac{1}{f_L} = \frac{1}{v_L} - \frac{1}{u}$$

where:

- $f_L = 10 \text{ cm}$ (focal length of the convex lens),
- $u = -\infty$ (object distance for a parallel ray),
- Thus, the image distance from the lens is $v_L = 10 \text{ cm}$ (real image formed at 10 cm on the opposite side of the light source).

Step 2: Image Formation by the Concave Mirror

- The image formed by the convex lens acts as the object for the concave mirror.
- The object distance for the concave mirror is the distance between the convex lens and the concave mirror, which is $40 \text{ cm} - 10 \text{ cm} = 30 \text{ cm}$ (since the image formed by the lens is real).
- The focal length of the concave mirror is $f_M = -15 \text{ cm}$ (since it is a concave mirror, the focal length is negative).
- Using the mirror formula:

$$\frac{1}{f_M} = \frac{1}{v_M} - \frac{1}{u_M}$$

where:

- $u_M = -30 \text{ cm}$ (object distance for the concave mirror),
- Solving for v_M :

$$\frac{1}{v_M} = \frac{1}{-15} - \frac{1}{-30} = \frac{-2 + 1}{10} = \frac{-1}{10}$$

Thus:

$$v_M = 10 \text{ cm}$$

The negative sign indicates that the image is formed on the same side as the object, which is on the left of the lens.

Conclusion:

The final image is formed at a distance of 10 cm to the left of the lens.

(A) 10 cm, left of lens

Quick Tip

A concave mirror inverts the image from a convex lens, forming the final image on the same side as the mirror.

(iv)(b) A beam of light coming parallel to the principal axis of a convex lens L_1 of focal length 16 cm is incident on it. Another convex lens L_2 of focal length 12 cm is placed coaxially at a distance 40 cm from L_1 . The nature and distance of the final image from L_2 will be:

- (A) Real, 24 cm
- (B) Virtual, 12 cm
- (C) Real, 32 cm
- (D) Virtual, 18 cm

Correct Answer: (B) Virtual, 12 cm

Solution: Image Formation Using Two Convex Lenses

Solution:

Step 1: Image Formation by Convex Lens L_1 - The light is incident parallel to the principal axis of the convex lens L_1 . - The image formed by L_1 will be real and located at a distance v_1 from L_1 . Using the lens formula:

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u}$$

where: - $f_1 = 16$ cm (focal length of L_1), - $u = -\infty$ (object distance for parallel rays), - Thus, the image distance from L_1 is $v_1 = 16$ cm (real image formed at 16 cm on the opposite side of the light source).

Step 2: Image Formation by Convex Lens L_2 - The image formed by L_1 now acts as the object for the second convex lens L_2 . - The object distance for L_2 is the distance between L_1 and L_2 , which is $40 \text{ cm} - 16 \text{ cm} = 24 \text{ cm}$. - The focal length of L_2 is $f_2 = 12$ cm. - Using the lens formula for L_2 :

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

where: - $u_2 = -24$ cm (object distance for L_2), - Solving for v_2 :

$$\frac{1}{v_2} = \frac{1}{12} - \frac{1}{-24} = \frac{1}{12} + \frac{1}{24} = \frac{2+1}{24} = \frac{3}{24}$$

Thus:

$$v_2 = \frac{24}{3} = 8 \text{ cm}$$

Step 3: Interpretation of the Result Since v_2 is positive, the image formed by L_2 is virtual and located at a distance of 8 cm from L_2 , on the same side as the object.

Conclusion: The final image is virtual and formed at a distance of 12 cm from L_2 .

(B) Virtual, 12cm

Quick Tip

When a convex lens forms an image at its focal point, it acts as an object for the next lens, and the final image position is determined using the lens equation.

30. A pure semiconductor like Ge or Si, when doped with a small amount of suitable impurity, becomes an extrinsic semiconductor. In thermal equilibrium, the electron and hole concentration in it are related to the concentration of intrinsic charge carriers. A p-type or n-type semiconductor can be converted into a p-n junction by doping it with suitable impurity. Two processes, diffusion and drift take place during formation of a p-n junction. A semiconductor diode is basically a p-n junction with metallic contacts provided at the ends for the application of an external voltage. A p-n junction diode allows currents to pass only in one direction when it is forward biased. Due to this property, a diode is widely used to rectify alternating voltages, in half-wave or full wave configuration.

(i) When Ge is doped with a pentavalent impurity, the energy required to free the weakly bound electron from the dopant is about:

- (A) 0.001 eV
- (B) 0.01 eV
- (C) 0.72 eV
- (D) 1.1 eV

Correct Answer: (B) 0.01 eV

Solution: Doping and Energy Levels in Ge

- Germanium (Ge) is a group IV element, and when doped with a pentavalent impurity (donor), extra free electrons are introduced into the conduction band.
- The energy required to free these donor electrons is very small compared to the band gap of Ge.
- In Ge, this ionization energy is approximately 0.01 eV.

Thus, the correct answer is 0.01 eV.

Quick Tip

In n-type semiconductors, donor electrons require a small energy (0.01 eV in Ge) to jump into the conduction band.

(ii) At a given temperature, the number of intrinsic charge carriers in a semiconductor is $2.0 \times 10^{10} \text{ cm}^{-3}$. It is doped with pentavalent impurity atoms. As a result, the number of holes in it becomes $8 \times 10^3 \text{ cm}^{-3}$. The number of electrons in the semiconductor is:

- (A) $2 \times 10^{24} \text{ m}^{-3}$
- (B) $4 \times 10^{23} \text{ m}^{-3}$
- (C) $1 \times 10^{22} \text{ m}^{-3}$
- (D) $5 \times 10^{22} \text{ m}^{-3}$

Correct Answer: (D) $5 \times 10^{22} \text{ m}^{-3}$

Solution: Carrier Concentration Calculation

- The relation between electron and hole concentrations in a semiconductor is given by:

$$n_e \cdot n_h = n_i^2$$

where: $n_i = 2.0 \times 10^{10} \text{ cm}^{-3}$ (intrinsic carrier concentration), $n_h = 8 \times 10^3 \text{ cm}^{-3}$ (hole concentration after doping), n_e = electron concentration after doping.

Solving for n_e :

$$n_e = \frac{n_i^2}{n_h} = \frac{(2.0 \times 10^{10})^2}{8 \times 10^3}$$

$$n_e = \frac{4.0 \times 10^{20}}{8 \times 10^3} = 5 \times 10^{16} \text{ cm}^{-3}$$

Converting to m^{-3} :

$$n_e = 5 \times 10^{22} \text{ m}^{-3}$$

Quick Tip

In a doped semiconductor, the product of electron and hole concentrations remains equal to n_i^2 .

(iii) During the formation of a p-n junction:

- (A) Electrons diffuse from p-region into n-region and holes diffuse from n-region into p-region.
- (B) Both electrons and holes diffuse from n-region into p-region.
- (C) Electrons diffuse from n-region into p-region and holes diffuse from p-region into n-region.
- (D) Both electrons and holes diffuse from p-region into n-region.

Correct Answer: (C) Electrons diffuse from n-region into p-region and holes diffuse from p-region into n-region.

Solution: Formation of Depletion Region

- In a p-n junction, the n-region has excess electrons, and the p-region has excess holes.
- Due to the concentration difference, electrons diffuse from the n-region to the p-region.
- Similarly, holes diffuse from the p-region to the n-region. - This diffusion leads to the formation of a depletion region, preventing further diffusion.

Thus, the correct answer is Electrons diffuse from n-region into p-region and holes diffuse from p-region into n-region.

Quick Tip

A p-n junction forms due to the diffusion of electrons from the n-side and holes from the p-side, leading to a depletion region.

OR

(iii)(b) Initially, during the formation of a p-n junction:

- (A) Diffusion current is large and drift current is small.
- (B) Diffusion current is small and drift current is large.
- (C) Both the diffusion and the drift currents are large.
- (D) Both the diffusion and the drift currents are small.

Correct Answer: (A) Diffusion current is large and drift current is small.

Solution: Understanding Current Flow in p-n Junction Formation

- In a newly formed p-n junction, a concentration gradient exists between the p-region and n-region.
 - This causes majority charge carriers (electrons from n-region and holes from p-region) to diffuse across the junction, leading to a large diffusion current.
 - As electrons and holes diffuse, a depletion region forms, creating an internal electric field.
 - This electric field generates a small drift current by moving minority carriers in the opposite direction.
 - Over time, the diffusion current and drift current balance each other at equilibrium.
- Thus, during the initial formation of the p-n junction, the diffusion current is large, and the drift current is small.

Quick Tip

In a p-n junction, diffusion current dominates initially, but as the depletion region builds up, drift current increases to balance it.

(iv) An AC voltage $V = 0.5 \sin(100\pi t)$ volt is applied, in turn, across a half-wave rectifier and a full-wave rectifier. The frequency of the output voltage across them respectively will be:

- (A) 25 Hz, 50 Hz
- (B) 25 Hz, 100 Hz
- (C) 50 Hz, 50 Hz

(D) 50 Hz, 100 Hz

Correct Answer: (D) 50 Hz, 100 Hz

Solution: Understanding Rectifier Frequency Response

- The given AC voltage is:

$$V = 0.5 \sin(100\pi t)$$

- The general form of an AC signal is:

$$V = V_0 \sin(2\pi ft)$$

Comparing, we get:

$$2\pi f = 100\pi$$

$$f = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

Half-Wave Rectifier Output Frequency: - A half-wave rectifier allows only one half-cycle of the input AC voltage. - The output voltage still follows the same fundamental frequency as the input, i.e., 50 Hz.

Full-Wave Rectifier Output Frequency: - A full-wave rectifier inverts the negative half-cycles, making the output frequency twice the input frequency. - Thus, the frequency of the output becomes:

$$2 \times 50 = 100 \text{ Hz}$$

Thus, the correct answer is 50 Hz for the half-wave rectifier and 100 Hz for the full-wave rectifier.

Quick Tip

A half-wave rectifier retains the same frequency as the input AC, whereas a full-wave rectifier doubles the frequency.

SECTION E

31(a). (i) Mention the factors on which the resonant frequency of a series LCR circuit depends. Plot a graph showing variation of impedance of a series LCR circuit with the frequency of the applied AC source.

Solution: Resonant Frequency and Impedance Variation in an LCR Circuit

Step 1: Resonant Frequency Formula

The resonant frequency (f_r) of a series LCR circuit is given by:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

where:

- L = Inductance (H),
- C = Capacitance (F).

Step 2: Factors Affecting Resonant Frequency

The resonant frequency depends on:

1. Inductance (L): Increasing L decreases the resonant frequency.
2. Capacitance (C): Increasing C decreases the resonant frequency.
3. Applied AC Frequency (f): The circuit resonates when the applied frequency matches f_r .
4. Resistance (R): Affects the sharpness (quality factor) of resonance but not the frequency.

Step 3: Graph of Impedance vs. Frequency

At resonance, the impedance (Z) is minimum and equal to resistance (R):

$$Z = R$$

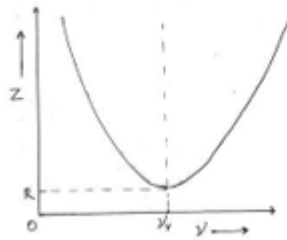
For low and high frequencies, impedance increases due to reactance effects:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where:

- $X_L = \omega L$ (Inductive Reactance),
- $X_C = \frac{1}{\omega C}$ (Capacitive Reactance).

$$Z_{\min} = R \quad \text{at} \quad f = f_r$$

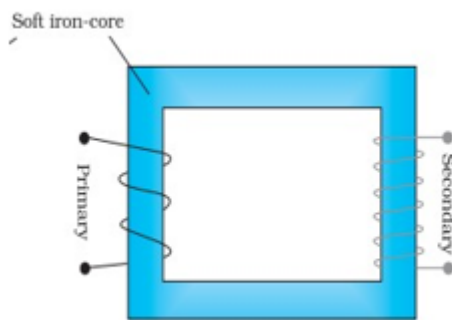


(ii) With the help of a suitable diagram, explain the working of a step-up transformer.

Solution: Step-Up Transformer Working

Step 1: Definition of a Step-Up Transformer

A step-up transformer is a device that increases voltage while decreasing current.



Step 2: Working Principle

- Based on Mutual Induction: When an AC voltage is applied to the primary coil, it produces a changing magnetic flux.
- This flux is linked to the secondary coil, inducing a voltage based on Faraday's Law:

$$V_s = \frac{N_s}{N_p} V_p$$

where:

- V_p, V_s = Primary and Secondary Voltages,
- N_p, N_s = Number of turns in primary and secondary coils.

Step 3: Step-Up Condition

For a step-up transformer:

$$N_s > N_p \quad \Rightarrow \quad V_s > V_p$$

Step 4: Applications

- Power transmission to reduce energy loss over long distances.
- X-ray machines and microwave ovens.

(iii) Write two causes of energy loss in a real transformer.

Solution: Energy Losses in a Transformer

Two Major Energy Losses:

1. Eddy Current Losses:

- Changing magnetic flux induces eddy currents in the core, causing unwanted heating.
- Reduced by using laminated iron cores.

2. Hysteresis Losses:

- Energy is lost due to repeated magnetization and demagnetization of the core material.
- Minimized by using soft iron cores with low hysteresis.

Quick Tip

A step-up transformer increases voltage by increasing the turns ratio. To reduce energy loss, laminated cores and high-efficiency materials are used.

32(a). (i) A dielectric slab of dielectric constant K and thickness t is inserted between plates of a parallel plate capacitor of plate separation d and plate area A . Obtain an expression for its capacitance.

Solution: Capacitance of a Parallel Plate Capacitor with a Dielectric Slab

Consider a parallel plate capacitor with plate area A and separation d . A dielectric slab of thickness t (where $t < d$) and dielectric constant K is inserted between the plates.

Electric Field

In the air gap (distance $d - t$): The electric field E_0 is:

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} \quad (1)$$

where:

σ is the surface charge density on the plates.

ϵ_0 is the permittivity of free space.

Q is the charge on the plates.

In the dielectric (thickness t): The electric field E is reduced by a factor of K :

$$E = \frac{E_0}{K} = \frac{Q}{KA\epsilon_0} \quad (2)$$

Potential Difference:

The potential difference (V) between the plates is the sum of the potential differences across the air gap and the dielectric:

$$V = E_0(d - t) + Et \quad (3)$$

$$V = \frac{Q}{A\epsilon_0}(d - t) + \frac{Q}{KA\epsilon_0}t \quad (4)$$

$$V = \frac{Q}{A\epsilon_0} \left[(d - t) + \frac{t}{K} \right] \quad (5)$$

$$V = \frac{Q}{A\epsilon_0} \left[d - t + \frac{t}{K} \right] \quad (6)$$

$$V = \frac{Q}{A\epsilon_0} \left[d - t \left(1 - \frac{1}{K} \right) \right] \quad (7)$$

Capacitance:

Capacitance (C) is defined as $C = \frac{Q}{V}$:

$$C = \frac{Q}{\frac{Q}{A\epsilon_0} \left[d - t \left(1 - \frac{1}{K} \right) \right]} \quad (8)$$

$$C = \frac{A\epsilon_0}{d - t \left(1 - \frac{1}{K} \right)} \quad (9)$$

$$C = \frac{A\epsilon_0}{d - t + \frac{t}{K}} \quad (10)$$

Therefore, the expression for the capacitance is:

$$C = \frac{A\epsilon_0}{d - t + \frac{t}{K}} \text{ or } C = \frac{A\epsilon_0}{d - t \left(1 - \frac{1}{K} \right)} \quad (11)$$

Quick Tip

A dielectric increases the capacitance of a capacitor, but if it doesn't fully fill the gap, the system behaves as capacitors in series.

(ii) Two capacitors of different capacitances are connected first (1) in series and then (2) in parallel across a dc source of 100 V. If the total energy stored in the combination in the two cases is 40 mJ and 250 mJ respectively, find the capacitance of the capacitors.

Solution: Finding Capacitance of the Capacitors

Step 1: Energy in a Capacitor The energy stored in a capacitor is:

$$U = \frac{1}{2}CV^2$$

where: - C = Capacitance, - V = Voltage applied.

Given:

- Series combination energy: $U_s = 40 \text{ mJ} = 40 \times 10^{-3} \text{ J}$,
- Parallel combination energy: $U_p = 250 \text{ mJ} = 250 \times 10^{-3} \text{ J}$,
- Applied voltage: $V = 100 \text{ V}$.

Step 2: Parallel Combination For parallel capacitors:

$$C_p = C_1 + C_2$$

$$U_p = \frac{1}{2}C_p V^2$$

$$250 \times 10^{-3} = \frac{1}{2}(C_1 + C_2)(100)^2$$

$$250 \times 10^{-3} = 5000(C_1 + C_2)$$

$$C_1 + C_2 = \frac{250 \times 10^{-3}}{5000} = 50 \times 10^{-6} = 50 \mu F$$

Step 3: Series Combination

For series capacitors:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$U_s = \frac{1}{2}C_s V^2$$

$$40 \times 10^{-3} = \frac{1}{2}C_s(100)^2$$

$$40 \times 10^{-3} = 5000C_s$$

$$C_s = \frac{40 \times 10^{-3}}{5000} = 8 \times 10^{-6} = 8\mu F$$

Step 4: Solving for C_1 and C_2 Using:

$$C_s = \frac{C_1 C_2}{C_1 + C_2}$$

$$8 = \frac{C_1 C_2}{50}$$

$$C_1 C_2 = 8 \times 50 = 400$$

Solving the quadratic equation:

$$x^2 - 50x + 400 = 0$$

Using the quadratic formula:

$$x = \frac{50 \pm \sqrt{2500 - 1600}}{2}$$

$$x = \frac{50 \pm 30}{2}$$

$$x = \frac{80}{2} = 40 \quad \text{or} \quad x = \frac{20}{2} = 10$$

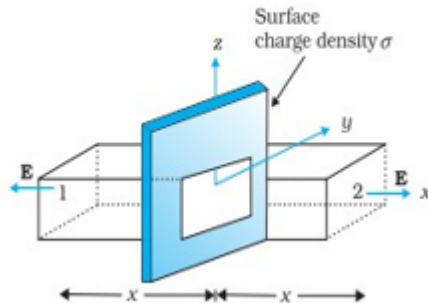
Thus, the capacitances are $C_1 = 40$ and $C_2 = 10$.

Quick Tip

For capacitors in parallel: $C_p = C_1 + C_2$. For capacitors in series: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$.

32(b). (i) Using Gauss's law, show that the electric field E at a point due to a uniformly charged infinite plane sheet is given by $E = \frac{\sigma}{2\epsilon_0}$, where symbols have their usual meanings.

Solution: Derivation of Electric Field Due to an Infinite Plane Sheet



Step 1: Gauss's Law Statement

According to Gauss's Law:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

where: - $\oint \mathbf{E} \cdot d\mathbf{A}$ = Total electric flux, - Q_{enc} = Enclosed charge,

- ϵ_0 = Permittivity of free space.

Step 2: Choosing a Gaussian Surface

- Consider an infinite charged plane with surface charge density σ .
- The charge is uniformly distributed over the plane.
- We use a Gaussian cylinder (pillbox) that extends equally on both sides of the plane.

Step 3: Applying Gauss's Law

- The flux is perpendicular to the surface.
- The total flux through the two flat surfaces of the pillbox is:

$$\oint \mathbf{E} \cdot d\mathbf{A} = EA + EA = 2EA$$

- The enclosed charge is:

$$Q_{\text{enc}} = \sigma A$$

- Applying Gauss's Law:

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

Thus, the electric field due to an infinite plane sheet is:

$$E = \frac{\sigma}{2\epsilon_0}$$

Quick Tip

The electric field due to an infinite sheet does not depend on the distance from the sheet, unlike a point charge or a line charge.

(ii) Electric field E in a region is given by $E = (5x^2 + 2)\hat{i}$, where E is in N/C and x is in meters. A cube of side 10 cm is placed in the region as shown in the figure. Calculate:

(1) The electric flux through the cube, and

(2) The net charge enclosed by the cube.

Solution: Flux and Enclosed Charge Calculation

Step 1: Understanding the Given Field The electric field is:

$$E = (5x^2 + 2)\hat{i}$$

- The electric field varies with x but is uniform along the y and z -directions.
- The cube has a side length of 10 cm = 0.1 m.

Step 2: Flux Calculation

Electric flux is given by:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A}$$

Since the field is in the x -direction, it only contributes to the flux through the two faces perpendicular to x -axis.

- At $x = 0$ (left face), the field is:

$$E_{\text{left}} = (5(0)^2 + 2) = 2 \text{ N/C}$$

- At $x = 0.1 \text{ m}$ (right face), the field is:

$$E_{\text{right}} = (5(0.1)^2 + 2) = 2 + 5(0.01) = 2.05 \text{ N/C}$$

Flux through the cube:

$$\Phi_E = E_{\text{right}}A - E_{\text{left}}A$$

$$= (2.05 - 2)(0.1 \times 0.1)$$

$$= (0.05)(0.01)$$

$$= 5 \times 10^{-4} \text{ Nm}^2/\text{C}$$

Step 3: Charge Enclosed Using Gauss's Law From Gauss's Law:

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{enc}} = \epsilon_0 \Phi_E$$

$$= (8.85 \times 10^{-12})(5 \times 10^{-4})$$

$$= 4.43 \times 10^{-15} \text{ C}$$

Thus, the net charge enclosed by the cube is $4.43 \times 10^{-15} \text{ C}$.

Quick Tip

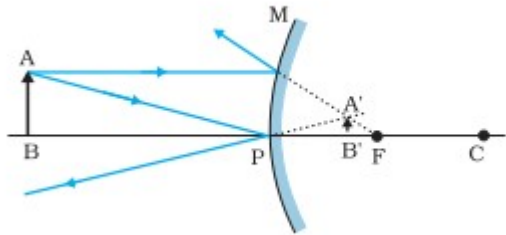
For a non-uniform field, calculate the flux difference through opposite faces to determine the net charge enclosed.

33(a). (i) Draw a ray diagram for the formation of the image of an object by a convex mirror. Hence, obtain the mirror equation.

Solution: Image Formation by a Convex Mirror

Step 1: Ray Diagram for Convex Mirror

A convex mirror always forms a virtual, diminished, and upright image. Here's the ray diagram for the image formation:



- The object is placed in front of the convex mirror.
- The rays from the object reflect off the mirror and diverge.
- The image is formed at the point where the extensions of the reflected rays meet behind the mirror.

Step 2: Mirror Equation for Convex Mirror

The mirror equation is:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

where:

- f = focal length of the mirror,
- v = image distance (behind the mirror for a convex mirror, so it is negative),
- u = object distance (in front of the mirror, so it is negative).

For a convex mirror, the focal length is positive. The image formed is always virtual and located behind the mirror, hence the image distance v is negative.

(ii) Why are multi-component lenses used for both the objective and the eyepiece in optical instruments?

Solution: Use of Multi-Component Lenses in Optical Instruments

Step 1: Correcting Optical Aberrations

- Optical aberrations such as chromatic aberration (color distortion) and spherical aberration

(blurring of the image) can degrade the image quality.

- Multi-component lenses combine lenses of different shapes and materials to minimize these aberrations.

Step 2: Improving Image Quality

- Objective lenses and eyepiece lenses are made from multiple lens components to focus the light more accurately.
- Combination of lenses with different refractive indices helps in forming clear, sharp, and high-quality images.

Step 3: Enhancing Magnification

- Multi-component lenses help achieve the desired magnification while maintaining image clarity and minimizing distortions.

(iii) The magnification of a small object produced by a compound microscope is 200. The focal length of the eyepiece is 2 cm and the final image is formed at infinity. Find the magnification produced by the objective.

Solution: Magnification Produced by the Objective

Step 1: Total Magnification of Compound Microscope

The total magnification M produced by the compound microscope is the product of the magnifications of the objective lens (M_o) and the eyepiece lens (M_e):

$$M = M_o \times M_e$$

Given:

- $M = 200$ (total magnification),
- The focal length of the eyepiece is $f_e = 2$ cm.

Step 2: Magnification Produced by the Eyepiece

The magnification produced by the eyepiece is:

$$M_e = \frac{D}{f_e}$$

where D is the least distance of distinct vision,
typically taken as 25 cm for a normal human eye.

$$M_e = \frac{25}{2} = 12.5$$

Step 3: Magnification Produced by the Objective

Now, using the total magnification:

$$M = M_o \times M_e$$

$$200 = M_o \times 12.5$$

$$M_o = \frac{200}{12.5} = 16$$

Thus, the magnification produced by the objective lens is:

$$M_o = 16$$

Step 4: Conclusion

The magnification produced by the objective lens is 16.

Quick Tip

In a compound microscope, the total magnification is the product of the magnifications produced by the objective lens and the eyepiece.

OR

33(b). (i) Differentiate between a wavefront and a ray.

Solution: Differentiating Between Wavefront and Ray

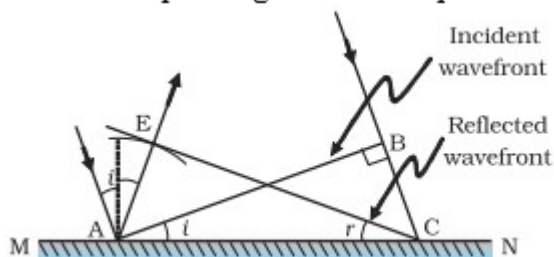
Wavefront:

- A wavefront is the locus of all points in a medium that are in phase, i.e., where the wave reaches at the same time.
- It represents the disturbance caused by the wave at a particular instant.
- Types of wavefronts:
- Spherical Wavefront: For a point source.

- Planar Wavefront: For a plane wave.

Ray:

- A ray is a straight line drawn in the direction of propagation of the wave. It represents the path along which energy travels.
- The ray is always perpendicular to the wavefront.



(ii) State Huygen's principle and verify the laws of reflection using a suitable diagram.

Solution: Huygen's Principle and Verification of Laws of Reflection

Huygen's Principle:

- Huygen's Principle states that every point on a wavefront acts as a secondary source of spherical wavelets, and the new wavefront at any subsequent time is the envelope of these wavelets.

New Wavefront = Envelope of secondary wavelets.

Verification of Laws of Reflection:

- According to Huygen's Principle, the incident ray and reflected ray can be understood by treating the point of incidence on the surface as a secondary source of waves.
- The angle between the incident ray and the normal to the surface is equal to the angle between the reflected ray and the normal.

From Huygen's principle, since each point on the wavefront produces secondary wavelets, the reflected wavefront will form an angle with the normal equal to the incident angle. Thus, we can derive the law of reflection:

$$\theta_i = \theta_r$$

where:

- θ_i = incident angle,

- θ_r = reflected angle.

(iii) In Young's double slit experiment, the slits S_1 and S_2 are 3 mm apart and the screen is placed 1.0 m away from the slits. It is observed that the fourth bright fringe is at a distance of 5 mm from the second dark fringe. Find the wavelength of light used.

Solution: We are given the positions of the bright and dark fringes in an interference pattern. The formula used to find the positions is based on the diffraction pattern equations.

For the 4th bright fringe, the position is given by:

$$x_4^{(\text{bright})} = \frac{4\lambda D}{d}$$

where: $x_4^{(\text{bright})}$ is the position of the 4th bright fringe. λ is the wavelength of light. D is the distance to the screen. d is the distance between the slits.

For the 2nd dark fringe, the position is given by:

$$x_2^{(\text{dark})} = \frac{3\lambda D}{2d}$$

where: $x_2^{(\text{dark})}$ is the position of the 2nd dark fringe.

We are asked to find the difference in positions between the 4th bright fringe and the 2nd dark fringe. Thus, the position difference is:

$$x_4^{(\text{bright})} - x_2^{(\text{dark})} = 5 \text{ mm}$$

Now, we substitute the formulas for the bright and dark fringes:

$$\frac{4\lambda D}{d} - \frac{3\lambda D}{2d} = 5 \times 10^{-3} \text{ m}$$

Simplifying the above equation:

$$\frac{4\lambda D}{d} - \frac{3\lambda D}{2d} = 5 \times 10^{-3}$$

Multiply through by d to eliminate the denominator:

$$4\lambda D - \frac{3\lambda D}{2} = 5 \times 10^{-3}d$$

Simplify further:

$$\begin{aligned} \frac{8\lambda D}{2} - \frac{3\lambda D}{2} &= 5 \times 10^{-3}d \\ \frac{5\lambda D}{2} &= 5 \times 10^{-3}d \end{aligned}$$

Finally, solve for λ :

$$\lambda = 6 \times 10^{-6} \text{ m}$$

Thus, the wavelength λ of the light is $6 \times 10^{-6} \text{ m}$.

Quick Tip

In Young's double slit experiment, the wavelength of light can be calculated using the fringe separation formula, considering the distance between slits and the screen.