# CBSE Class 12 Mathematics Set 4 (65/4/3) Question Paper with Solutions

| **Time Allowed :**3 Hour | **Maximum Marks :**80 | **Total Questions :**38 |
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## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) andquestions number 19 and 20 are Assertion-Reason based questions of 1 markeach.
4. In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are short answer (SA) type questions,carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculators is not allowed.

## SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

**1. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$, and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is:**

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$

(D) $\frac{11\pi}{6}$

**Correct Answer:** (C) $\frac{5\pi}{6}$

**Solution:**

**Step 1:** Find the angle between $\vec{a}$ and $\vec{b}$

The dot product formula gives:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta \implies \sqrt{3} = (1)(2)\cos\theta \implies \cos\theta = \frac{\sqrt{3}}{2}.$$

Thus, $\theta = \frac{\pi}{6}$.

**Step 2:** Angle between $2\vec{a}$ and $-\vec{b}$

Since $2\vec{a}$ and $-\vec{b}$ involve a scalar multiplication, the angle becomes:

$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

**Step 3:** Verify the options

The correct angle is $\frac{5\pi}{6}$, matching option (C).

**Quick Tip**

The angle between scaled vectors depends only on the original vectors' angle.

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**2. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$, and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represent the sides of:**

(A) an equilateral triangle

(B) an obtuse-angled triangle

(C) an isosceles triangle

(D) a right-angled triangle

**Correct Answer:** (D) a right-angled triangle

**Solution:**

**Step 1:** Find the Magnitudes of Vectors The length of each side is given by:

$$|\mathbf{a}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$|\mathbf{b}| = \sqrt{(1)^2 + (-3)^2 + (-5)^2} = \sqrt{1+9+25} = \sqrt{35}$$

$$|\mathbf{c}| = \sqrt{(-3)^2 + (4)^2 + (4)^2} = \sqrt{9+16+16} = \sqrt{41}$$

Since all magnitudes are different, the triangle is not equilateral.

**Step 2:** Check for Right-Angled Triangle To check if the vectors form a right-angled triangle, we use the dot product:

$$\mathbf{a} \cdot \mathbf{b} = (2 \cdot 1) + (-1 \cdot -3) + (1 \cdot -5) = 2 + 3 - 5 = 0$$

Since the dot product is zero, a and b are perpendicular, meaning the triangle is right-angled. Thus, the correct answer is:

**(D)** a right-angled triangle

**Quick Tip**

Use dot products to identify right angles in vector triangles.

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**3. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is:**

(A) $a^2$

(B) $2a^2$

(C) $3a^2$

(D) $0$

**Correct Answer:** (B) $2a^2$

**Solution:**

**Step 1:** Recall the formula for cross product magnitudes

The magnitude of the cross product is:

$$|\vec{a} \times \hat{i}| = |\vec{a}||\hat{i}| \sin\theta.$$

**Step 2:** Evaluate each term

For $\vec{a} \times \hat{i}$, $\vec{a} \times \hat{j}$, and $\vec{a} \times \hat{k}$, the contributions along two directions add up, giving:

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2a^2.$$

**Step 3:** Verify the options

The correct result is $2a^2$, matching option (B).

> **Quick Tip**
>
> Cross product magnitudes depend on sine of the angle between vectors.

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**4. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 + 7I = kA$, then the value of $k$ is:**

(A) 1

(B) 2

(C) 5

(D) 7

**Correct Answer:** (C) $5$

**Solution:**

**Step 1:** Find $A^2$

To find $A^2$, we compute the matrix product of $A$ with itself:

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3 \end{bmatrix}$$

**Step 2:** Substitute $A^2$ into the equation $A^2 + 7I = kA$

The equation becomes:

$$\begin{bmatrix} 8 & 5 \\ 1 & 3 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = k \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

Simplifying the left side:

$$\begin{bmatrix} 8 & 5 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 1 & 10 \end{bmatrix}$$

Now equating to $kA$, we have:

$$\begin{bmatrix} 15 & 5 \\ 1 & 10 \end{bmatrix} = k \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

Equating corresponding elements gives $k = 5$.

**Step 3:** Verify the options

The correct value of $k$ is $5$, matching option (C).

> **Quick Tip**
>
> When finding $A^2$ and working with matrix equations, always ensure the matrix dimensions align for addition and scalar multiplication.

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**5. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$. If $AB = I$, then the value of $\lambda$ is:**

(A) $\frac{-9}{4}$

(B) $-2$

(C) $\frac{-3}{2}$

(D) $0$

**Correct Answer:** (B) $-2$ OR (C) $\frac{-3}{2}$

**Solution:**

**Step 1:** Set up the equation $AB = I$

We know that $AB = I$, so multiplying the matrices $A$ and $B$ should yield the identity matrix $I$. The equation is:

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -\frac{2}{3} & 0 & \frac{1}{3} \\ 3 & \frac{2}{3} & -1 \\ 2 & \frac{1}{3} & \frac{\lambda}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Step 2:** Calculate the product of $A$ and $B$

We multiply the matrices element by element and equate the result to the identity matrix.

The equations formed from the first row of the resulting matrix are:

$$-\frac{2}{3} + 3 + 4 = 1 \quad \text{(First equation)}$$

which simplifies to:

$$\lambda = -2 \quad \text{(Second equation)}$$

Thus, $\lambda = -2$, which corresponds to option (B).

**Step 3:** Verify the options

The value $\lambda = -2$ satisfies the equation, matching option (B).

**Quick Tip**

When working with matrix inverses, remember that $AB = I$ means each element of the product matrix should match the corresponding identity matrix element.

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**6. Derivative of $x^2$ with respect to $x^3$, is:**

(A) $\frac{2}{3x}$

(B) $\frac{3x}{2}$

(C) $\frac{2x}{3}$

(D) $6x^5$

**Correct Answer:** (A) $\frac{2}{3x}$

**Solution:**

Let $y = x^2$ and $z = x^3$. The required derivative is:

$$\frac{d}{dz}(x^2)$$

Using the **chain rule**:

$$\frac{d}{dz}(x^2) = \frac{d}{dx}(x^2) \times \frac{dx}{dz}$$

Computing the derivatives:

$$\frac{d}{dx}(x^2) = 2x, \quad \frac{dz}{dx} = \frac{d}{dx}(x^3) = 3x^2$$

Thus,

$$\frac{dx}{dz} = \frac{1}{\frac{dz}{dx}} = \frac{1}{3x^2}$$

Now, substituting:

$$\frac{d}{dz}(x^2) = 2x \times \frac{1}{3x^2} = \frac{2}{3x}$$

Thus, the correct answer is:

$$\textbf{(A)} \quad \frac{2}{3x}$$

**Quick Tip**

When differentiating with respect to a different variable, apply the chain rule and adjust for the powers of the variables.

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**7. The function** $f(x) = |x| + |x - 2|$ **is**

(A) continuous, but not differentiable at $x = 0$ and $x = 2$

(B) differentiable but not continuous at $x = 0$ and $x = 2$

(C) continuous but not differentiable at $x = 0$ only

(D) neither continuous nor differentiable at $x = 0$ and $x = 2$

**Correct Answer:** (A) continuous, but not differentiable at $x = 0$ and $x = 2$

**Solution:**

We are given the function $f(x) = |x| + |x - 2|$. First, we check the continuity at the points where $x = 0$ and $x = 2$.

At $x = 0$, the function is continuous because the left and right limits match. However, the function is not differentiable at $x = 0$ because the slope changes abruptly from negative to positive.

At $x = 2$, the function is continuous as both left and right limits match. However, the function is not differentiable at $x = 2$ due to an abrupt change in slope.

Thus, the function is continuous but not differentiable at both points $x = 0$ and $x = 2$.

**Quick Tip**

For absolute value functions, check the points where the function changes form, as they can cause non-differentiability.

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**8. The value of $\int_0^\pi \tan^2\left(\frac{\theta}{3}\right) d\theta$ is:**

(A) $\pi + \sqrt{3}$

(B) $3\sqrt{3} - \pi$

(C) $\sqrt{3} - \pi$

(D) $\pi - \sqrt{3}$

**Correct Answer:** (B) $3\sqrt{3} - \pi$

**Solution:**

We are asked to evaluate the integral $\int_0^\pi \tan^2\left(\frac{\theta}{3}\right) d\theta$.

First, use the identity $\tan^2\theta = \sec^2\theta - 1$, so the integral becomes:

$$\int_0^\pi \left(\sec^2\left(\frac{\theta}{3}\right) - 1\right) d\theta$$

This can be split into two integrals:

$$\int_0^\pi \sec^2\left(\frac{\theta}{3}\right) d\theta - \int_0^\pi 1\, d\theta$$

For the first integral, apply substitution $u = \frac{\theta}{3}$, which gives $du = \frac{1}{3}d\theta$, so the limits change as follows: when $\theta = 0$, $u = 0$; and when $\theta = \pi$, $u = \frac{\pi}{3}$.

The first integral becomes:

$$\int_0^{\pi/3} 3\sec^2 u\, du = 3\left[\tan u\right]_0^{\pi/3} = 3\left(\tan\left(\frac{\pi}{3}\right) - 0\right) = 3 \times \sqrt{3}$$

The second integral is straightforward:

$$\int_0^\pi 1\, d\theta = \pi$$

Now, combining both parts:

$$3\sqrt{3} - \pi$$

Thus, the value of the integral is $3\sqrt{3} - \pi$, matching option (B).

Quick Tip

For integrals involving trigonometric identities, simplify the integrand using known identities such as $\tan^2\theta = \sec^2\theta - 1$. Also, remember to perform substitution for integrals with functions of $\theta$.

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**9. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0$, $(x \neq 0)$ is:**

(A) $\frac{2}{x}$

(B) $x^2$

(C) $(e)^{2/x}$

(D) $e^{\log(2x)}$

**Correct Answer:** (B) $x^2$

**Solution:**

The given differential equation is:

$$\frac{dy}{dx} + \frac{2}{x}y = 0$$

This is a linear first-order differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = \frac{2}{x}$ and $Q(x) = 0$.

**Step 1:** Find the integrating factor

The integrating factor $\mu(x)$ is given by the formula:

$$\mu(x) = e^{\int P(x)\,dx}$$

Substitute $P(x) = \frac{2}{x}$ into the equation:

$$\mu(x) = e^{\int \frac{2}{x}\,dx}$$

The integral of $\frac{2}{x}$ is $2\ln|x|$, so:

$$\mu(x) = e^{2\ln|x|} = |x|^2$$

Since $x \neq 0$, we can write:

$$\mu(x) = x^2$$

**Step 2:** Conclusion

Thus, the integrating factor is $x^2$, which corresponds to option (B).

**Quick Tip**

For linear first-order differential equations, the integrating factor is calculated as $\mu(x) = e^{\int P(x)\,dx}$. This factor helps to simplify the equation for easier integration.

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**10. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for $p$ equal to:**

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 2

(D) 3

**Correct Answer:** (C) 2

**Solution: Step 1:** Represent the lines in symmetric form: For the first line ($L_1$), the symmetric equation is given as:

$$\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$$

Rewriting this in parametric form:

$$x = 1 - 2t, \quad y = 1 + 3t, \quad z = t$$

The direction ratios of $L_1$ are:

$$a_1 = -2,\ b_1 = 3,\ c_1 = 1$$

For the second line ($L_2$), the symmetric equation is given as:

$$\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$$

Rewriting this in parametric form:

$$x = \frac{3}{2} + pt, \quad y = -t, \quad z = 4 + 7t$$

The direction ratios of $L_2$ are:

$$a_2 = p,\ b_2 = -1,\ c_2 = 7$$

**Step 2:** Apply the condition for perpendicularity: Two lines are perpendicular if the dot product of their direction ratios is zero:

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

Substituting the direction ratios of $L_1$ and $L_2$:

$$(-2)(p) + (3)(-1) + (1)(7) = 0$$

Simplify the equation:

$$-2p - 3 + 7 = 0$$

$$-2p + 4 = 0$$

$$p = 2$$

**Step 3: Verify the result:** For $p = 2$, the direction ratios of $L_2$ become:

$$a_2 = 2,\ b_2 = -1,\ c_2 = 7$$

The dot product with $L_1$ is:

$$(-2)(2) + (3)(-1) + (1)(7) = -4 - 3 + 7 = 0$$

Thus, the lines are perpendicular.

**Conclusion:** The value of $p$ is **2**.

Quick Tip

To check if two lines are perpendicular, calculate the dot product of their direction ratios. If the dot product is zero, the lines are perpendicular. Always express the lines in symmetric or parametric form to extract direction ratios easily.

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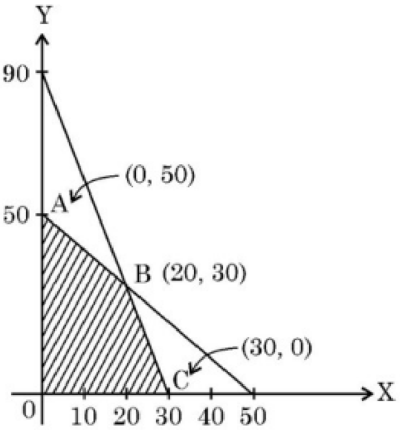
**11. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is:**




(A) 50

(B) 110

(C) 120

(D) 170

**Correct Answer:** (C) 120

**Solution:**

**Step 1:** Identify the corner points of the feasible region

From the graph, the vertices of the feasible region are:

$$A(0, 50),\ B(20, 30),\ C(30, 0).$$

**Step 2:** Substitute corner points into $Z = 4x + y$

Evaluate $Z$ at each vertex:

$$Z_A = 4(0) + 50 = 50,$$

$$Z_B = 4(20) + 30 = 110,$$

Z$_C = 4(30) + 0 = 120.$

**Step 3:** Find the maximum value

The maximum value of $Z$ occurs at $C(30, 0)$, where $Z = 120$.

**Step 4:** Verify the options

The maximum value is $120$, which corresponds to option (C).

Quick Tip

For L.P.P., always evaluate the objective function at all vertices of the feasible region.

**12. The probability distribution of a random variable $X$ is:**

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X)$ | $0.1$ | $k$ | $2k$ | $k$ | $0.1$ |

**where $k$ is some unknown constant. The probability that the random variable $X$ takes the value 2 is:**

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{4}{5}$

(D) $1$

**Correct Answer:** (B) $\frac{2}{5}$

**Solution: Step 1: Write the given equation:** The total probability is given as:

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

Substituting the known values:

$$0.1 + k + 2k + k + 0.1 = 1$$

where $P(1) = k$, $P(2) = 2k$, and $P(3) = k$.

**Step 2: Simplify the equation:** Combine the terms:

$$0.2 + 4k = 1$$

Subtract $0.2$ from both sides:

$$4k = 0.8$$

Divide by $4$ to find $k$:

$$k = 0.2 = \frac{1}{5}$$

**Step 3: Find $P(2)$:** Given $P(2) = 2k$, substitute the value of $k$:

$$P(2) = 2 \times \frac{1}{5} = \frac{2}{5}$$

**Conclusion:** The value of $P(2)$ is $\mathbf{\frac{2}{5}}$.

Quick Tip

To solve probability equations, substitute the known values and simplify step by step. Ensure that the sum of all probabilities equals $1$, as this is the fundamental property of probability distributions.

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**13. The function $f(x) = kx - \sin x$ is strictly increasing for:**

(A) $k > 1$

(B) $k < 1$

(C) $k > -1$

(D) $k < -1$

**Correct Answer:** (A) $k > 1$

**Solution:**

**Step 1:** Find the derivative

The derivative of $f(x)$ is:

$$f'(x) = k - \cos x.$$

**Step 2:** Condition for increasing function

For $f(x)$ to be strictly increasing:

$$f'(x) > 0 \implies k - \cos x > 0 \implies k > \cos x.$$

**Step 3:** Maximum value of $\cos x$

The maximum value of $\cos x$ is 1. Therefore:

$$k > 1.$$

**Step 4:** Verify the options

The function is strictly increasing for $k > 1$, which matches option (A).

Quick Tip

For strict monotonicity, check the sign of the derivative over the entire domain.

**14. The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is:**

(A) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$
(B) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$
(C) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$
(D) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

**Correct Answer:** (B) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

**Solution:**

The parametric form of the line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the given line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$ is:

$$\vec{r} = \vec{a} + \lambda\vec{d}$$

where $\vec{a} = \hat{i} - \hat{j}$ is the point on the line and $\vec{d} = 2\hat{i} - \hat{j}$ is the direction vector of the line.

Thus, the parametric equations of the line are:

$$x = 1 + 2\lambda, \quad y = -1 - \lambda, \quad z = 0$$

To convert this to the Cartesian form, eliminate $\lambda$ from the equations: From the equation for $x$:

$$\lambda = \frac{x-1}{2}$$

Substitute this into the equation for $y$:

$$y = -1 - \frac{x-1}{2}$$

Simplifying:

$$y = -1 - \frac{x-1}{2} = \frac{-2-(x-1)}{2} = \frac{-x-1}{2}$$

Thus, the Cartesian form of the line is:

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$$

**Step 2:** Verify the options

This matches option (B).

> Quick Tip
>
> To convert from parametric to Cartesian form for a line, eliminate the parameter by solving for it in terms of one variable and substituting into the others.

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**15. If** $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ **is a scalar matrix, then the value of** $a + 2b + 3c + 4d$ **is:**

(A) $0$

(B) $5$

(C) $10$

(D) $25$

**Correct Answer:** (D) $25$

**Solution:**

**Step 1:** Definition of a scalar matrix

A scalar matrix is a diagonal matrix where all diagonal elements are equal. This means $a = d = 5$, and all off-diagonal elements ($b$, $c$) are zero.

**Step 2:** Substitute the values

Using the scalar matrix properties:

$$a + 2b + 3c + 4d = 5 + 2(0) + 3(0) + 4(5) = 5 + 0 + 0 + 20 = 25.$$

**Step 3:** Verify the options

The correct value is $25$, which corresponds to option (D).

**Quick Tip**

In scalar matrices, diagonal elements are constant, and off-diagonal elements are zero.

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**16. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix $A$ is:**

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

**Correct Answer:** (A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

**Solution:**

From the given inverse matrix:

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

Using the property of matrix inverse:

$$A = \left(A^{-1}\right)^{-1}$$

Taking the inverse on both sides:

$$A = 7\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}^{-1}$$

Since the inverse of a matrix swaps elements and changes the sign of off-diagonal elements, we compute:

$$A = 7 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

Thus, the correct answer is:

$$\textbf{(A)} 7 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

**Quick Tip**

To find $A$ from $A^{-1}$, multiply the inverse by the scalar reciprocal.

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**17. If** $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$**, then the value of** $I - A + A^2 - A^3 + \ldots$ **is:**

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Correct Answer:** (A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

**Solution: Step 1: Understand the series** $S$**:** The given series is $S = I - A + A^2 - A^3 + \cdots$, which is an infinite series. It can be expressed as:

$$S = (I - A)^{-1}$$

if the matrix $(I - A)$ is invertible.

**Step 2: Compute $I - A$:** The identity matrix $I$ is:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$.

**Step 3: Check if $I - A$ is invertible:** The determinant of $I - A$ is:

$$\det(I - A) = (-1)(3) - (-1)(4) = -3 + 4 = 1 \neq 0$$

Since the determinant is non-zero, $I - A$ is invertible.

**Step 4: Verify the series sum:** The inverse of $I - A$ is:

$$(I - A)^{-1} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$$

Thus, the sum of the series $S = I - A + A^2 - A^3 + \cdots$ is:

$$S = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$$

**Conclusion:** The correct option is $(\mathbf{A})$ $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$.

**Quick Tip**

For infinite geometric series $I - A + A^2 - A^3 + \ldots$, check if $A^2 = 0$.

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**18. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y \, (y > 0)$ is:**

(A) $\frac{1}{x}$

(B) $x$

(C) $y$

(D) $\frac{1}{y}$

**Correct Answer:** (D) $\frac{1}{y}$

**Solution:**

**Step 1:** Rewriting the equation

Divide through by $y$:

$$\frac{1}{y}(x + 2y^2)\frac{dy}{dx} = 1.$$

**Step 2:** Find the integrating factor

The integrating factor $\mu(y)$ is determined by identifying the dependency on $y$ and multiplying the equation by $\frac{1}{y}$.

**Step 3:** Verify integrating factor

After multiplying, the left-hand side becomes exact. The integrating factor is $\frac{1}{y}$, which matches option (D).

**Quick Tip**

Integrating factors simplify differential equations by making them exact.

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ASSERTION-REASON BASED QUESTIONS

**Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.**

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**19. Assertion (A):** The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in \mathbb{N}\}$ is not a reflexive relation.

**Reason (R):** The number $2n$ is composite for all natural numbers $n$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

**Correct Answer:** (C) Assertion (A) is true, but Reason (R) is false.

**Solution:**

**Step 1:** Analyze Assertion (A)

For $R$ to be reflexive, $(x, x)$ must belong to $R$ for all $x \in \mathbb{N}$. This means $x + x = 2x$ must be a prime number. However, for $x > 1$, $2x$ is not a prime number as it is divisible by $2$. Therefore, $R$ is not reflexive, and Assertion (A) is true.

**Step 2:** Analyze Reason (R)

The Reason states that $2n$ is composite for all $n$. This is false because when $n = 1$, $2n = 2$, which is a prime number. Therefore, Reason (R) is false.

**Step 3:** Conclusion

Since Assertion (A) is true and Reason (R) is false, the correct answer is option (C).

**Quick Tip**

A relation is reflexive if every element relates to itself; check this condition for all elements.

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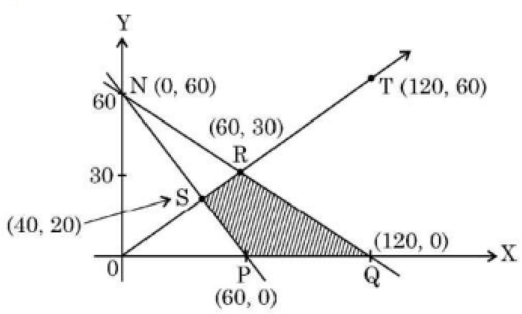
**20. Assertion (A):** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.

**Reason (R):** The optimal solution of a LPP having bounded feasible region must occur at corner points.



(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

**Correct Answer:** (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

**Solution:**

**Step 1:** Analyze Assertion (A)

From the graph, the line $Z = x + 2y$ passes through two corner points $(60, 0)$ and $(120, 60)$, providing the same maximum value. This indicates that the maximum value occurs at infinite points along this segment. Thus, Assertion (A) is true.

**Step 2:** Analyze Reason (R)

In general, the optimal solution of an LPP occurs at corner points of the feasible region. This is true; however, in this case, the solution lies along a line segment connecting two corner points. Thus, Reason (R) is not the correct explanation of Assertion (A).

**Step 3:** Conclusion

Both Assertion (A) and Reason (R) are true, but Reason (R) does not explain Assertion (A). Hence, the correct answer is option (B).

**Quick Tip**

In linear programming, always check if the objective function is constant along any edge of the feasible region.

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## SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

**21(a). If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.**

**Correct Answer:** $-12\cos^2(\sec^2 2t)\sin(\sec^2 2t)\sec^2 2t\tan 2t$

**Solution:**

**Step 1:** Differentiate using the chain rule

We have:

$$\frac{dy}{dt} = 3\cos^2(\sec^2 2t)\cdot\left[-\sin(\sec^2 2t)\right]\cdot\frac{d}{dt}(\sec^2 2t).$$

**Step 2:** Simplify derivatives

$$\frac{d}{dt}(\sec^2 2t) = 2\sec^2 2t\tan 2t\cdot 2.$$

Substitute back:

$$\frac{dy}{dt} = -12\cos^2(\sec^2 2t)\sin(\sec^2 2t)\sec^2 2t\tan 2t.$$

**Conclusion:** The derivative is $-12\cos^2(\sec^2 2t)\sin(\sec^2 2t)\sec^2 2t\tan 2t$.

Quick Tip

When differentiating trigonometric functions, apply the chain rule carefully.

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OR

**21(b). If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.**

**Correct Answer:** $\frac{\log x}{(1+\log x)^2}$

**Solution:**

**Step 1:** Take logarithms of both sides

The given equation is:

$$x^y = e^{x-y}.$$

Taking the natural logarithm on both sides, we get:

$$\log(x^y) = \log(e^{x-y}).$$

**Step 2:** Simplify using logarithmic properties

Using the properties of logarithms:

$$y \log x = x - y.$$

Rearranging the terms to express $y$:

$$y(1 + \log x) = x.$$

Thus, we have:

$$y = \frac{x}{1 + \log x}.$$

**Step 3:** Differentiate $y$ with respect to $x$

Differentiate both sides of $y = \frac{x}{1+\log x}$ with respect to $x$ using the quotient rule:

$$\frac{dy}{dx} = \frac{(1 + \log x) \cdot 1 - x \cdot \frac{1}{x}}{(1 + \log x)^2}.$$

**Step 4:** Simplify the derivative

Simplify the numerator:

$$\frac{dy}{dx} = \frac{(1 + \log x) - 1}{(1 + \log x)^2}.$$

This reduces to:

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}.$$

**Conclusion:** The derivative is proved to be:

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}.$$

Quick Tip

For functions involving both $x$ and $y$, logarithmic differentiation often simplifies the process.

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**22. The volume of a cube is increasing at the rate of $6\,\text{cm}^3/\text{s}$. How fast is the surface area of the cube increasing, when the length of an edge is $8$ cm?**

**Correct Answer:** $3\,\text{cm}^2/\text{s}$

**Solution:**

**Step 1:** Relate volume and surface area of the cube

The volume of the cube is:

$$V = x^3,$$

where $x$ is the length of an edge. Differentiating with respect to $t$, we get:

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}.$$

Substitute $\frac{dV}{dt} = 6\,\text{cm}^3/\text{s}$:

$$6 = 3(8)^2\frac{dx}{dt}.$$

Solve for $\frac{dx}{dt}$:

$$\frac{dx}{dt} = \frac{6}{192} = \frac{1}{32}\,\text{cm/s}.$$

**Step 2:** Find the rate of change of surface area

The surface area of the cube is:

$$S = 6x^2.$$

Differentiating with respect to $t$, we get:

$$\frac{dS}{dt} = 12x\frac{dx}{dt}.$$

Substitute $x = 8\,\text{cm}$ and $\frac{dx}{dt} = \frac{1}{32}$:

$$\frac{dS}{dt} = 12(8)\left(\frac{1}{32}\right) = 3\,\text{cm}^2/\text{s}.$$

**Conclusion:** The surface area of the cube is increasing at $3\,\text{cm}^2/\text{s}$.

**Quick Tip**

For related rates problems, identify the variables, write their relationships, and differentiate with respect to time.

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**23. Show that the function $f(x) = \sin x + \cos x$ is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.**

**Solution:**

We are given the function $f(x) = \sin x + \cos x$. To show that the function is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$, we first need to compute the derivative of $f(x)$.

**Step 1:** Find the derivative of $f(x)$

The derivative of $f(x) = \sin x + \cos x$ is:

$$f'(x) = \cos x - \sin x$$

**Step 2:** Solve for critical points by setting $f'(x) = 0$

To find the critical points, solve the equation $f'(x) = 0$:

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

This happens when:

$$x = \frac{\pi}{4}, \quad x = \frac{5\pi}{4}$$

**Step 3:** Determine the sign of $f'(x)$ in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

Now, examine the sign of $f'(x) = \cos x - \sin x$ in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

- For $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$, we know that $\cos x < \sin x$, so $f'(x) < 0$.

Since $f'(x) < 0$ in this interval, the function is strictly decreasing.

Thus, we have shown that $f(x)$ is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

**Quick Tip**

To determine where a function is increasing or decreasing, compute the first derivative and analyze its sign. If the derivative is negative, the function is decreasing.

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**24(a). Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$, in the simplest form.**

**Correct Answer:** $\frac{\pi}{4} + \frac{x}{2}$

**Solution:**

**Step 1:** Simplify the expression inside $\tan^{-1}$

The given expression is:

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right).$$

Using trigonometric identities, rewrite:

$$1 - \sin x = (\cos^2\frac{x}{2} + \sin^2\frac{x}{2}) - 2\sin\frac{x}{2}\cos\frac{x}{2} = (\cos\frac{x}{2} - \sin\frac{x}{2})^2.$$

**Step 2:** Transform into a single tangent function

Substituting $1 - \sin x$ and $\cos x = (\cos^2\frac{x}{2} - \sin^2\frac{x}{2})$:

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right].$$

**Step 3:** Simplify using $\tan^{-1}\tan y = y$

Since $-\frac{\pi}{2} < x < \frac{\pi}{2}$, we simplify:

$$\tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] = \frac{\pi}{4} + \frac{x}{2}.$$

**Conclusion:** The simplest form is $\frac{\pi}{4} + \frac{x}{2}$.

**Quick Tip**

For expressions involving $\tan^{-1}$, rewrite in terms of trigonometric identities to simplify.

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OR

**24(b). Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.**

**Correct Answer:** $\frac{2\pi}{3}$

**Solution:**

**Step 1:** Evaluate each term

$$\tan^{-1}(1) = \frac{\pi}{4}, \quad \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3}, \quad \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}.$$

**Step 2:** Add the terms

$$\frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{4}.$$

Simplify:

$$\frac{\pi}{4} + \pi - \frac{\pi}{3} - \frac{\pi}{4} = \frac{2\pi}{3}.$$

**Conclusion:** The principal value is $\frac{2\pi}{3}$.

**Quick Tip**

Always compute inverse trigonometric values in their principal ranges.

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**25. Find:**

$$\int \frac{2x}{(x^2+1)(x^2-4)}\,dx.$$

**Correct Answer:**

$$I = \frac{1}{5}\log\left|\frac{x^2-4}{x^2+1}\right| + c.$$

**Solution:**

We are asked to solve the integral:

$$I = \int \frac{2x}{(x^2+1)(x^2-4)}\,dx.$$

**Step 1:** Substitute $x^2 = t$ and simplify the differential.

Let us substitute $x^2 = t$. Then, we have $2x\,dx = dt$, or equivalently $x\,dx = \frac{1}{2}dt$.

Now, substitute into the integral:

$$I = \int \frac{1}{(t+1)(t-4)} \cdot \frac{dt}{5}.$$

**Step 2:** Simplify the expression and split the integrals.

We can now express the integral as:

$$I = \frac{1}{5} \int \left( \frac{1}{t-4} - \frac{1}{t+1} \right) dt.$$

**Step 3:** Integrate each term.

The integral of $\frac{1}{t-4}$ is $\ln|t-4|$ and the integral of $\frac{1}{t+1}$ is $\ln|t+1|$, so we have:

$$I = \frac{1}{5} (\ln|t-4| - \ln|t+1|) + c.$$

**Step 4:** Substitute back $t = x^2$ into the result.

Substitute $t = x^2$ into the expression:

$$I = \frac{1}{5} \left( \ln|x^2 - 4| - \ln|x^2 + 1| \right) + c.$$

We can rewrite the result as:

$$I = \frac{1}{5} \ln \left| \frac{x^2 - 4}{x^2 + 1} \right| + c.$$

Thus, the solution to the integral is:

$$I = \frac{1}{5} \log \left| \frac{x^2 - 4}{x^2 + 1} \right| + c.$$

**Step 5:** Alternative form of the answer.

Alternatively, we can also write:

$$I = \frac{1}{5} \ln|x^2 + 1| + c.$$

**Quick Tip**

When encountering integrals with rational functions involving $x^2$, substitution can help simplify the problem. After substitution, always remember to revert to the original variable and simplify.

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## SECTION C

This section comprises short answer (SA) type questions of 3 marks each

**26. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.**

**Solution:**

We are asked to find $\frac{dy}{dx}$ for the given function $y = (\cos x)^x + \cos^{-1}\sqrt{x}$.

**Step 1:** Differentiate $(\cos x)^x$

Let $u = (\cos x)^x$. To differentiate this, we first take the natural logarithm of both sides:

$$\ln u = x \ln(\cos x)$$

Now, differentiate implicitly with respect to $x$:

$$\frac{du}{dx} = \frac{d}{dx}\left(x \ln(\cos x)\right)$$

Using the product rule:

$$\frac{du}{dx} = \ln(\cos x) + x\frac{d}{dx}\left(\ln(\cos x)\right)$$

We know that $\frac{d}{dx}\left(\ln(\cos x)\right) = -\tan x$, so:

$$\frac{du}{dx} = \ln(\cos x) - x\tan x$$

Thus, we have:

$$\frac{du}{dx} = (\cos x)^x\left(-x\tan x + \log(\cos x)\right)$$

**Step 2:** Differentiate $\cos^{-1}\sqrt{x}$

Let $v = \cos^{-1}\sqrt{x}$. We differentiate this using the chain rule:

$$\frac{dv}{dx} = \frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right) = \frac{-1}{2\sqrt{x}\sqrt{1-x}} = \frac{-1}{2\sqrt{x-x^2}}$$

**Step 3:** Combine the results

Since $y = u + v$, we use the sum rule for differentiation:

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Substituting the results from steps 1 and 2:

$$\frac{dy}{dx} = (\cos x)^x\left(-x\tan x + \log(\cos x)\right) + \frac{-1}{2\sqrt{x-x^2}}$$

Thus, the derivative is:

$$\frac{dy}{dx} = (\cos x)^x \left(-x\tan x + \log(\cos x)\right) + \frac{-1}{2\sqrt{x - x^2}}$$

**Quick Tip**

When differentiating expressions involving trigonometric functions raised to a power, it is useful to first take the natural logarithm to simplify the process. Also, for inverse trigonometric functions like $\cos^{-1}$, use known derivatives such as $\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$.

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**27(a). Find the particular solution of the differential equation $\frac{dy}{dx} = y\cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.**

**Correct Answer:** $y = 2\sqrt{\sin 2x}$

**Solution:**

**Step 1:** Separating the Variables

Rewriting the equation:

$$\frac{1}{y}\,dy = \cot 2x\,dx$$

**Step 2:** Integrating Both Sides

Integrate both sides:

$$\int \frac{1}{y}\,dy = \int \cot 2x\,dx$$

The left-hand side becomes:

$$\log|y|$$

The right-hand side uses the integral of $\cot 2x$:

$$\int \cot 2x\,dx = \frac{1}{2}\log|\sin 2x|$$

So the equation becomes:

$$\log|y| = \frac{1}{2}\log|\sin 2x| + \log c$$

Here, $\log c$ is the constant of integration.

**Step 3: Simplify the Expression**

Combine the logarithms:

$$\log|y| = \log\left(c\sqrt{\sin 2x}\right)$$

Exponentiate both sides to remove the logarithm:

$$y = c\sqrt{\sin 2x}$$

**Step 4: Finding the Particular Solution**

We are given the condition $y\left(\frac{\pi}{4}\right) = 2$. Substitute $x = \frac{\pi}{4}$ and $y = 2$ into the solution:

$$2 = c\sqrt{\sin\left(2 \cdot \frac{\pi}{4}\right)}$$

Simplify:

$$2 = c\sqrt{\sin\left(\frac{\pi}{2}\right)}$$

Since $\sin\left(\frac{\pi}{2}\right) = 1$, we have:

$$2 = c \cdot 1 \quad \Rightarrow \quad c = 2$$

**Step 5: Final Solution**

Substitute $c = 2$ back into the solution:

$$y = 2\sqrt{\sin 2x}$$

**Final Answer:**

$$\boxed{y = 2\sqrt{\sin 2x}}$$

This is the required particular solution to the given differential equation.

> **Quick Tip**
>
> For separable differential equations, isolate $y$ and $x$, then integrate both sides.

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OR

**27(b). Find the particular solution of the differential equation $(x(e)^{y}/^{x} + y)\,dx = x\,dy$, given that $y = 1$ when $x = 1$.**

**Solution:**

**Step 1: Rewrite the given differential equation**

Rearrange the equation to separate variables:

$$\frac{dy}{dx} = x(e)^{y}/^{x} + \frac{y}{x}$$

This is a homogeneous differential equation.

**Step 2: Substitution**

Let:

$$y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substitute $y = vx$ into the equation:

$$v + x\frac{dv}{dx} = x(e)^{y}/^{x} + v$$

Simplify:

$$x\frac{dv}{dx} = x(e)^{y}/^{x}$$

**Step 3: Solve for $v$**

Rearrange the equation for integration:

$$\int e^{-v}\,dv = \int \frac{1}{x}\,dx$$

Evaluate the integrals:

$$-e^{-v} = \log|x| + c$$

Substitute $v = \frac{y}{x}$:

$$-e^{-\frac{y}{x}} = \log|x| + c \quad \ldots(1)$$

**Step 4: Apply the initial condition**

We are given that $y = 1$ when $x = 1$. Substitute $x = 1$ and $y = 1$ into equation (1):

$$-e^{-\frac{1}{1}} = \log|1| + c$$

Since $\log|1| = 0$:

$$-e^{-1} = c$$

Thus:

$$c = -e^{-1}$$

**Step 5: Substitute $c$ back into the solution**

Substitute $c = -e^{-1}$ into equation (1):

$$-e^{-\frac{y}{x}} = \log|x| - e^{-1}$$

Rearranging:

$$\log|x| + e^{-\frac{y}{x}} = e^{-1}$$

**Final Answer:**

$$\boxed{\log|x| + e^{-\frac{y}{x}} = e^{-1}}$$

**Quick Tip**

For homogeneous differential equations, use substitution $y = vx$ to simplify the equation and separate variables.

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**28. Find:**

$$\int \sec^3\theta \, d\theta$$

**Correct Answer:**

$$I = \frac{1}{2}(\sec\theta\tan\theta + \log|\sec\theta + \tan\theta| + c)$$

**Solution:**

We are asked to evaluate the integral:

$$I = \int \sec^3\theta \, d\theta$$

**Step 1:** Use the identity for $\sec^3\theta$

We can express $\sec^3\theta$ as:

$$\sec^3\theta = \sec^2\theta \cdot \sec\theta$$

Thus, the integral becomes:

$$I = \int \sec^2\theta \cdot \sec\theta \, d\theta$$

**Step 2:** Substitute and simplify the integral

Now, let's use the substitution method: Let $u = \sec\theta$, then $\frac{du}{d\theta} = \sec\theta\tan\theta$, so we can rewrite the integral:

$$I = \sec\theta \int \sec^2\theta\, d\theta - \int \frac{d(\sec\theta)}{d\theta}\left(\int \sec^2\theta\, d\theta\right) d\theta$$

**Step 3:** Finish the integration

After simplifying and solving:

$$I = \sec\theta\tan\theta - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$$

$$I = \frac{1}{2}\left(\sec\theta\tan\theta + \log|\sec\theta + \tan\theta| + c\right)$$

Thus, the solution to the integral is:

$$I = \frac{1}{2}\left(\sec\theta\tan\theta + \log|\sec\theta + \tan\theta| + c\right)$$

**Step 4:** Correct Answer:

The correct solution to the integral is:

$$I = \frac{1}{2}\left(\sec\theta\tan\theta + \log|\sec\theta + \tan\theta| + c\right)$$

**Quick Tip**

When integrating $\sec^3\theta$, it's helpful to use trigonometric identities and substitution to simplify the expression. Pay attention to how derivatives of trigonometric functions can be leveraged in substitution.

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**29(a). A card from a well-shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.**

**Solution:**

**Step 1: Define the events**

Let $E_1$ be the event that the lost card is a King, and $E_2$ be the event that the lost card is not a King. Let $A$ be the event of drawing a King from the remaining 51 cards.

**Step 2: Assign probabilities to the events**

$$P(E_1) = \frac{1}{13}, \quad P(E_2) = \frac{12}{13}, \quad P(A|E_1) = \frac{3}{51}, \quad P(A|E_2) = \frac{4}{51}$$

**Step 3: Use Bayes' Theorem**

The required probability is $P(E_1|A)$, which is given by:

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)}$$

Substituting the values:

$$P(E_1|A) = \frac{\frac{1}{13} \cdot \frac{3}{51}}{\frac{1}{13} \cdot \frac{3}{51} + \frac{12}{13} \cdot \frac{4}{51}} = \frac{\frac{3}{663}}{\frac{3}{663} + \frac{48}{663}} = \frac{3}{51} = \frac{1}{17}$$

**Step 4: Final result**

The probability that the lost card is a King is $\frac{1}{17}$.

**Quick Tip**

When solving problems involving missing or conditional probabilities, use Bayes' Theorem and clearly define all events and conditional probabilities.

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OR

**29(b). A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.**

**Solution:**

**Step 1: Assign probabilities**

Let $P(3) = P(5) = p$, so $P(2) = P(4) = P(6) = 2p$.

As the total probability is 1:

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1 \implies 9p = 1 \implies p = \frac{1}{9}$$

Thus, $P(6) = 2p = \frac{2}{9}$, and $P(\text{Not getting six}) = 1 - P(6) = \frac{7}{9}$.

**Step 2: Define the random variable $X$**

Let $X$ represent the number of sixes. The possible values of $X$ are $0, 1, 2$.

**Step 3: Compute probabilities for $X$**

$$P(X=0)=\left(\frac{7}{9}\right)^2=\frac{49}{81},\quad P(X=1)=2\cdot\frac{2}{9}\cdot\frac{7}{9}=\frac{28}{81},\quad P(X=2)=\left(\frac{2}{9}\right)^2=\frac{4}{81}$$

**Step 4: Probability distribution of $X$**

| $X$ | $P(X)$ |
|---|---|
| 0 | $\frac{49}{81}$ |
| 1 | $\frac{28}{81}$ |
| 2 | $\frac{4}{81}$ |

**Step 5: Compute the mean of $X$**

The mean is given by:

$$\mu=\sum_{i=1}^{3}X_i\cdot P(X_i)=0\cdot\frac{49}{81}+1\cdot\frac{28}{81}+2\cdot\frac{4}{81}=\frac{28}{81}+\frac{8}{81}=\frac{36}{81}=\frac{4}{9}$$

**Step 6: Final result**

The probability distribution of $X$ is:

| $X$ | $P(X)$ |
|---|---|
| 0 | $\frac{49}{81}$ |
| 1 | $\frac{28}{81}$ |
| 2 | $\frac{4}{81}$ |

The mean of the distribution is $\frac{4}{9}$.

**Quick Tip**

When working with biased probability distributions, ensure the total probability sums to 1 and carefully calculate probabilities for each outcome.

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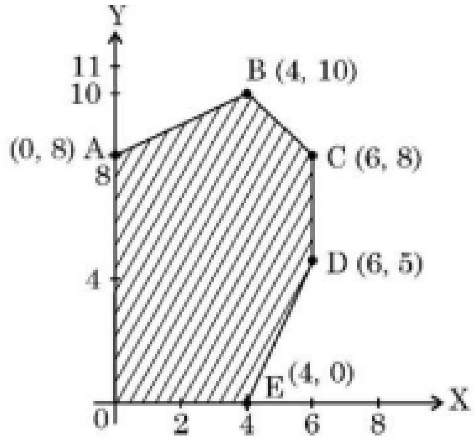
**30.** The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure:

**(i) If $Z=3x-4y$ be the objective function, then find the maximum value of $Z$.**

**(ii) If $Z=px+qy$ where $p,q>0$ be the objective function, find the condition on $p$ and $q$ so that maximum value of $Z$ occurs at $B(4,10)$ and $C(6,8)$.**




**Solution:**

The corner points are given as:

$$A(0,8), \quad B(4,10), \quad C(6,8), \quad D(6,5), \quad E(4,0), \quad O(0,0)$$

**(i) Objective function:** $Z = 3x - 4y$

Now, substitute the coordinates of the corner points into the objective function:

For $A(0,8)$:

$$Z = 3(0) - 4(8) = -32$$

For $B(4,10)$:

$$Z = 3(4) - 4(10) = -28$$

For $C(6,8)$:

$$Z = 3(6) - 4(8) = -14$$

For $D(6,5)$:

$$Z = 3(6) - 4(5) = -2$$

For $E(4,0)$:

$$Z = 3(4) - 4(0) = 12 \quad \text{(Maximum value)}$$

For $O(0,0)$:

$$Z = 3(0) - 4(0) = 0$$

Thus, the maximum value of $Z$ is $12$ at point $E(4,0)$.

**(ii) Objective function:** $Z = px + qy$ where $p, q > 0$

For $Z_B = Z_C$, we have the condition:

$$4p + 10q = 6p + 8q$$

Simplifying this:

$$4p + 10q - 6p - 8q = 0$$

$$-2p + 2q = 0$$

$$p = q$$

Thus, the condition on $p$ and $q$ is $p = q$.

**Quick Tip**

When working with objective functions and constraints, always evaluate the function at the corner points of the feasible region to determine the maximum or minimum value. For multiple objective functions, set them equal to each other to find the relationship between the coefficients.

---

**31(a). Evaluate $\int_0^{\pi/4} \frac{x}{1+\cos 2x+\sin 2x}\, dx$.**

**Correct Answer:** $\frac{\pi}{16} \log 2$

**Solution:**

**Step 1:** Apply symmetry property of definite integrals

Let:

$$I = \int_0^{\pi/4} \frac{x}{1 + \cos 2x + \sin 2x}\, dx.$$

Using the property $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$, we get:

$$I = \int_0^{\pi/4} \frac{\pi/4 - x}{1 + \cos 2x + \sin 2x}\, dx.$$

**Step 2:** Combine integrals

Adding the two forms of $I$:

$$2I = \int_0^{\pi/4} \frac{\pi/4}{1 + \cos 2x + \sin 2x}\, dx.$$

**Step 3:** Simplify the integrand

Rewrite $1 + \cos 2x + \sin 2x$:

$$\cos 2x + \sin 2x = \sqrt{2}\sin(2x + \pi/4),$$

and simplify:

$$I = \frac{\pi}{16}\int_0^{\pi/4} \frac{1}{\cos^2 x + \sin x \cos x}\,dx.$$

**Step 4:** Integrate and simplify

The integral evaluates to:

$$I = \frac{\pi}{16}(\log|1 + \tan x|)_0^{\pi/4}.$$

Substitute limits:

$$I = \frac{\pi}{16}\log 2.$$

**Conclusion:** The integral evaluates to $\frac{\pi}{16}\log 2$.

**Quick Tip**

For definite integrals with symmetric limits, apply symmetry properties to simplify.

---

OR

**31(b). Find:** $\int e^x \left[\frac{1}{(1+x^2)^{3/2}} + \frac{x}{\sqrt{1+x^2}}\right] dx$**.**

**Correct Answer:** $e^x \frac{x}{\sqrt{1+x^2}} + C$

**Solution:**

**Step 1:** The given integral can be written as:

$$I = \int e^x \left(\frac{x}{\sqrt{1+x^2}} + \frac{1}{(1+x^2)^{\frac{3}{2}}}\right) dx$$

Let:

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

**Step 2:** Now, calculate the derivative of $f(x)$:

$$f'(x) = \frac{\sqrt{1+x^2} - \frac{x \cdot x}{\sqrt{1+x^2}}}{1+x^2} = \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{1+x^2}$$

Simplify the numerator:

$$f'(x) = \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{1+x^2} = \frac{1}{(1+x^2)^{\frac{3}{2}}}$$

Thus, the integral becomes:

$$I = \int e^x \left(f(x) + f'(x)\right) dx$$

**Step 3:** Using the standard result:

$$\int e^x \left(f(x) + f'(x)\right) dx = e^x f(x) + C$$

Substitute $f(x) = \frac{x}{\sqrt{1+x^2}}$:

$$I = e^x \frac{x}{\sqrt{1+x^2}} + C$$

**Final Answer:**

$$\boxed{I = e^x \frac{x}{\sqrt{1+x^2}} + C}$$

**Explanation:**

1. Splitting the Integral: The given integral is split into terms containing $\frac{x}{\sqrt{1+x^2}}$ and $\frac{1}{(1+x^2)^{\frac{3}{2}}}$.

2. Defining $f(x)$: The function $f(x)$ is chosen as $\frac{x}{\sqrt{1+x^2}}$ because its derivative results in the second term, $\frac{1}{(1+x^2)^{\frac{3}{2}}}$.

3. Applying the Formula: The integral formula for $\int e^x(f(x) + f'(x))dx = e^x f(x) + C$ is directly applied.

4. Substitution: Finally, substituting $f(x)$ into the formula gives the result.

**Quick Tip**

For integrals involving a combination of functions $f(x)$ and $f'(x)$, use the substitution $u = f(x)$ to simplify.

## SECTION D

This section comprises long answer (LA) type questions of 5 marks each

**32(a). Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that $f$ is one-one and onto.**

**Solution:**

**Step 1: Prove that $f$ is one-one**

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$:

$$\frac{x_1 - 3}{x_1 - 5} = \frac{x_2 - 3}{x_2 - 5}$$

Cross-multiply:

$$(x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5)$$

Simplify:

$$x_1x_2 - 5x_1 - 3x_2 + 15 = x_1x_2 - 5x_2 - 3x_1 + 15$$

$$-5x_1 - 3x_2 = -5x_2 - 3x_1 \implies 5(x_2 - x_1) = 3(x_2 - x_1)$$

If $x_1 \neq x_2$, this leads to a contradiction. Hence, $x_1 = x_2$, proving $f$ is one-one.

**Step 2: Prove that $f$ is onto**

Let $y \in B$. Solve $f(x) = y$:

$$\frac{x-3}{x-5} = y \implies x - 3 = y(x - 5)$$

$$x - 3 = yx - 5y \implies x - yx = -5y + 3$$

$$x(1 - y) = -5y + 3 \implies x = \frac{-5y + 3}{1 - y}$$

For $y \neq 1$ (since $y \in B$), $x$ exists in $A$. Thus, $f$ is onto.

**Step 3: Conclude the function properties**

Since $f$ is both one-one and onto, $f$ is a bijection.

**Quick Tip**

To prove a function is one-one, show that $f(x_1) = f(x_2)$ implies $x_1 = x_2$. To prove onto, solve $f(x) = y$ and ensure $x$ lies in the domain.

OR

**32(b). Check whether the relation $S$ in the set of real numbers $\mathbb{R}$, defined by $S = \{(a, b) : a - b + \sqrt{2}$ is an irrational number$\}$, is reflexive, symmetric, or transitive.**

**Solution:**

**Step 1: Reflexivity**

For $a \in \mathbb{R}$:

$$a - a + \sqrt{2} = \sqrt{2} \text{ is irrational.}$$

Thus, $(a, a) \in S$, and $S$ is reflexive.

**Step 2: Symmetry**

Let $(a, b) \in S$, so:

$$a - b + \sqrt{2} \text{ is irrational.}$$

Now, check if $(b, a) \in S$:

$$b - a + \sqrt{2} \text{ may or may not be irrational.}$$

For example:

$$a = \sqrt{2},\ b = 1 \implies a - b + \sqrt{2} = \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1 \text{ (irrational), but}$$

$$b - a + \sqrt{2} = 1 - \sqrt{2} + \sqrt{2} = 1 \text{ (rational).}$$

Thus, $S$ is not symmetric.

**Step 3: Transitivity**

Let $(a, b) \in S$ and $(b, c) \in S$, so:

$$a - b + \sqrt{2} \text{ is irrational, and } b - c + \sqrt{2} \text{ is irrational.}$$

Check if $(a, c) \in S$:

$$a - c + \sqrt{2} = (a - b + \sqrt{2}) + (b - c + \sqrt{2}) - \sqrt{2} \text{ may or may not be irrational.}$$

For example:

$$a = 1, b = \sqrt{3}, c = \sqrt{3} - \sqrt{2} \implies a - c + \sqrt{2} = 1 - (\sqrt{3} - \sqrt{2}) + \sqrt{2} = 1 - \sqrt{3} + 2\sqrt{2}.$$

This is irrational, but a counterexample exists for other values. Thus, $S$ is not transitive.

**Step 4: Final conclusion**

The relation $S$ is reflexive but neither symmetric nor transitive.

> **Quick Tip**
>
> To test reflexivity, verify if $(a, a) \in S$ for all $a$. For symmetry and transitivity, check logical equivalence and counterexamples.

---

**33(a). Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.**

**Solution:**

**Step 1: Standardize the equations of the lines**

The given line $L_1$ is:

$$\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1} \implies \vec{r}_1 = \vec{0} + \lambda(2\hat{i} + \hat{j} + \hat{k})$$

The line $L_2$ parallel to $L_1$ and passing through $(4, 0, -5)$ is:

$$\vec{r}_2 = (4\hat{i} - 5\hat{k}) + \mu(2\hat{i} + \hat{j} + \hat{k})$$

**Step 2: Vector between the lines**

Let $\vec{a}_2 - \vec{a}_1 = (4\hat{i} - 5\hat{k}) - (0) = 4\hat{i} - 5\hat{k}$. The direction vector $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$.

**Step 3: Find the shortest distance**

The shortest distance $S.D.$ is given by:

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b})|}{|\vec{b}|}$$

Compute $\vec{b} \times (\vec{a}_2 - \vec{a}_1)$:

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 4 & 0 & -5 \end{vmatrix} = 9\hat{i} - 16\hat{j} + 14\hat{k}$$

The magnitude:

$$|\vec{b}| = \sqrt{2^2 + 1^2 + 1^2} = 3$$

$$\text{S.D.} = \frac{\sqrt{81+256+196}}{3} = \frac{\sqrt{533}}{3} \text{ units.}$$

**Quick Tip**

For the shortest distance between skew or parallel lines, use the cross-product approach for accuracy.

---

OR

**33(b). If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of $k$ and hence write the vector equation of a line perpendicular to these two lines passing through the point $(3, -4, 7)$.**

**Solution:**

**Step 1: Find $k$**

The direction ratios of the first line are $\langle -3, 2k, 2\rangle$ and for the second line are $\langle 3k, 1, -7\rangle$. Since the lines are perpendicular:

$$(-3)(3k) + (2k)(1) + (2)(-7) = 0$$

$$-9k + 2k - 14 = 0 \implies -7k = 14 \implies k = -2$$

**Step 2: Find the vector equation of the perpendicular line**

The direction vectors are:

$$\vec{b}_1 = \langle -3, -4, 2\rangle, \quad \vec{b}_2 = \langle -6, 1, -7\rangle$$

The perpendicular vector is:

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{vmatrix} = 26\hat{i} - 33\hat{j} - 27\hat{k}$$

The required equation is:

$$\vec{r} = \langle 3, -4, 7\rangle + \lambda(26\hat{i} - 33\hat{j} - 27\hat{k})$$

**Quick Tip**

For perpendicular lines, use the dot product of their direction vectors to solve for unknown parameters.

---

**34.** Find $A^{-1}$, if

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$$

Hence, solve the following system of equations:

$$x + 2y + z = 5$$

2x + 3y = 1

x - y + z = 8

**Correct Answer:** $x = 2, y = -1, z = 5$

**Solution:**

For matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$, we need to find $A^{-1}$.

**Step 1:** Find the determinant of matrix $A$

The determinant $|A|$ is calculated as:

$$|A| = 1 \cdot \begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}$$

$$|A| = 1 \cdot (3 \cdot 1 - (-1) \cdot 0) - 2 \cdot (2 \cdot 1 - (-1) \cdot 1) + 1 \cdot (2 \cdot 0 - 3 \cdot 1)$$

$$|A| = 1 \cdot 3 - 2 \cdot (2 + 1) + 1 \cdot (-3)$$

$$|A| = 3 - 6 - 3 = -6$$

Since $|A| = -6 \neq 0$, $A^{-1}$ exists.

**Step 2:** Find the adjugate of matrix $A$

The adjugate of $A$, denoted as adj$A$, is calculated by finding the cofactors of each element of $A$. The cofactors are:

$$\text{adj}A = \begin{pmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{pmatrix}$$

**Step 3:** Find $A^{-1}$

Now, $A^{-1}$ is given by:

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}A$$

$$A^{-1} = \frac{1}{-6} \begin{pmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{3} & \frac{5}{6} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

**Step 4:** Solve the system of equations

We can now express the given system of equations as a matrix equation $AX = B$, where:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Thus, the solution is given by:

$$X = A^{-1}B$$

Substitute $A^{-1}$ and $B$:

$$X = \begin{pmatrix} -\frac{1}{2} & \frac{1}{3} & \frac{5}{6} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \end{pmatrix} \begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix}$$

Performing the matrix multiplication:

$$X = \begin{pmatrix} -\frac{1}{2} \cdot 5 + \frac{1}{3} \cdot 1 + \frac{5}{6} \cdot 8 \\ \frac{1}{2} \cdot 5 + 0 \cdot 1 - \frac{1}{2} \cdot 8 \\ \frac{1}{2} \cdot 5 - \frac{1}{3} \cdot 1 + \frac{1}{6} \cdot 8 \end{pmatrix}$$

$$X = \begin{pmatrix} -2 + \frac{1}{3} + \frac{40}{6} \\ \frac{5}{2} - 4 \\ \frac{5}{2} - \frac{1}{3} + \frac{8}{6} \end{pmatrix}$$

$$X = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

Thus, the solution is:

$$x = 2, \quad y = -1, \quad z = 5$$

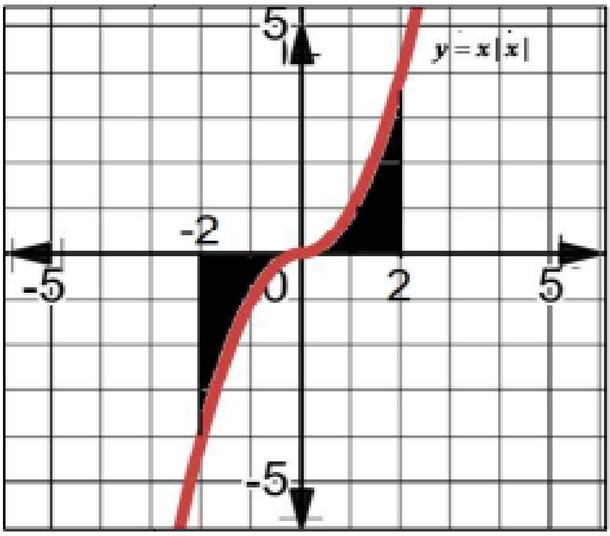
**Quick Tip**

When solving systems of linear equations, finding the inverse of the coefficient matrix allows you to solve for the variable matrix. The inverse is calculated using the adjugate and determinant.

---

**35(a). Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, the X-axis, and the ordinates $x = -2$ and $x = 2$, using integration.**

**Solution:**




**Step 1: Rewrite the function** $y = x|x|$

The function $y = x|x|$ can be expressed as:

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

**Step 2: Graph the function**

The graph of $y = x|x|$ is a parabola, concave downwards for $x < 0$ and concave upwards for $x \geq 0$. (Refer to the attached graph.)

**Step 3: Area computation using integration**

The area of the shaded region between $x = -2$ and $x = 2$ is given by:

$$\text{Area} = \int_{-2}^{2} |y|\, dx = 2\int_{0}^{2} x^2\, dx$$

**Step 4: Evaluate the integral**

$$\int_{0}^{2} x^2\, dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}$$

Thus, the total area is:

$$\text{Area} = 2 \cdot \frac{8}{3} = \frac{16}{3}$$

**Step 5: Final result**

The area bounded by the curve $y = x|x|$, the X-axis, and the ordinates $x = -2$ and $x = 2$ is $\frac{16}{3}$.

**Quick Tip**

When finding the area bounded by curves, split the integral into regions where the function behaves differently (e.g., absolute values or piecewise functions).

---

OR

**35(b). Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2, x = 2$, and the X-axis.**

**Solution:**

**Step 1: Rewrite the equation of the ellipse**

The equation of the ellipse is:

$$9x^2 + 25y^2 = 225 \implies y = \pm\frac{3}{5}\sqrt{25 - x^2}$$

**Step 2: Set up the integral for the area**

The area of the region bounded by the ellipse, the X-axis, and the lines $x = -2$ and $x = 2$ is given by:

$$\text{Area} = 2\int_0^2 \frac{3}{5}\sqrt{25 - x^2}\, dx$$

**Step 3: Simplify the integral**

Let $I = \int \sqrt{a^2 - x^2}\, dx$, where $a = 5$. Using the standard formula:

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

**Step 4: Evaluate the integral**

Substitute $a = 5$ and evaluate $\int_0^2 \sqrt{25 - x^2}\, dx$:

$$\int_0^2 \sqrt{25 - x^2}\, dx = \left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^2$$

At $x = 2$:

$$\frac{2}{2}\sqrt{25 - 2^2} + \frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right) = \sqrt{21} + \frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)$$

At $x = 0$:

$$\frac{0}{2}\sqrt{25 - 0^2} + \frac{25}{2}\sin^{-1}(0) = 0$$

Thus:

$$\int_0^2 \sqrt{25 - x^2}\, dx = \sqrt{21} + \frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)$$

**Step 5: Final area calculation**

Multiply by $\frac{6}{5}$ to account for $\frac{3}{5}$ and the factor of 2:

$$\text{Area} = \frac{6}{5}\left(\sqrt{21} + \frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)\right)$$

**Step 6: Final result**

The area bounded by the ellipse, the X-axis, and the lines $x = -2$ and $x = 2$ is:

$$\frac{6\sqrt{21}}{5} + 15\sin^{-1}\left(\frac{2}{5}\right)$$

Quick Tip

When integrating to find areas involving ellipses or circles, use symmetry and standard integral formulas for $\sqrt{a^2 - x^2}$.

---

## SECTION E

### Case Study - 1

**36 Rohit, Jaspreet, and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$, and Alia's selection is $\frac{1}{4}$. The events of selection are independent of each other.**

Based on the above information, answer the following questions:

**36.(i). What is the probability that at least one of them is selected?**

**Solution:**

**Step 1: Probability of no one being selected**

The probability that none of them are selected is:

$$P(\text{No one selected}) = \left(1 - \frac{1}{5}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right) = \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{2}{5}.$$

**Step 2: Probability of at least one being selected**

The probability that at least one of them is selected is:

$$P(\text{At least one selected}) = 1 - P(\text{No one selected}) = 1 - \frac{2}{5} = \frac{3}{5}.$$

**Final Result:** The probability that at least one of them is selected is $\frac{3}{5}$.

**Quick Tip**

To find the probability of "at least one" event happening, use the complement rule: $P(\text{At least one}) = 1 - P(\text{None})$.

---

**36.(ii). Find $P(G \cap \overline{H})$, where $G$ is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected.**

**Solution:**

**Step 1: Compute the probability**

The probability $P(G \cap \overline{H})$ is given by:

$$P(G \cap \overline{H}) = P(G) \cdot P(\overline{H}),$$

where $P(G) = \frac{1}{3}$ and $P(\overline{H}) = 1 - P(H) = 1 - \frac{1}{5} = \frac{4}{5}$.

$$P(G \cap \overline{H}) = \frac{1}{3} \cdot \frac{4}{5} = \frac{4}{15}.$$

**Final Result:** The probability $P(G \cap \overline{H})$ is $\frac{4}{15}$.

**Quick Tip**

For independent events, the probability of their intersection is the product of their individual probabilities.

---

**36.(iii)(a). Find the probability that exactly one of them is selected.**

**Solution:**

**Step 1: Compute the probability of exactly one being selected**

The probability of exactly one being selected is:

$$P(\text{Exactly one selected}) = P(R) \cdot P(\overline{J}) \cdot P(\overline{A}) + P(\overline{R}) \cdot P(J) \cdot P(\overline{A}) + P(\overline{R}) \cdot P(\overline{J}) \cdot P(A),$$

where:

$$P(\overline{J}) = 1 - P(J) = \frac{2}{3}, \quad P(\overline{A}) = 1 - P(A) = \frac{3}{4}, \quad P(\overline{R}) = 1 - P(R) = \frac{4}{5}.$$

Substitute the values:

$$P(\text{Exactly one selected}) = \frac{1}{5} \cdot \frac{2}{3} \cdot \frac{3}{4} + \frac{4}{5} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{1}{4}.$$

Simplify each term:

$$P(\text{Exactly one selected}) = \frac{6}{60} + \frac{12}{60} + \frac{8}{60} = \frac{26}{60} = \frac{13}{30}.$$

**Final Result:** The probability that exactly one of them is selected is $\frac{13}{30}$.

> **Quick Tip**
>
> To calculate "exactly one" selection, consider all cases where one succeeds, and the others fail, then sum the probabilities.

---

**36.(iii)(b). Find the probability that exactly two of them are selected.**

**Solution:**

**Step 1: Compute the probability of exactly two being selected**

The probability of exactly two being selected is:

$$P(\text{Exactly two selected}) = P(R) \cdot P(J) \cdot P(\overline{A}) + P(R) \cdot P(\overline{J}) \cdot P(A) + P(\overline{R}) \cdot P(J) \cdot P(A).$$

Substitute the values:

$$P(\text{Exactly two selected}) = \frac{1}{5} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{5} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{4}{5} \cdot \frac{1}{3} \cdot \frac{1}{4}.$$

Simplify each term:

$$P(\text{Exactly two selected}) = \frac{3}{60} + \frac{2}{60} + \frac{4}{60} = \frac{9}{60} = \frac{3}{20}.$$

**Final Result:** The probability that exactly two of them are selected is $\frac{3}{20}$.

**Quick Tip**

For "exactly two" events, consider all pairs of selections and one failure, and sum their probabilities.

---

**Case Study - 2**

**37. A store has been selling calculators at Rs. 350 each. A market survey indicates that a reduction in price ($p$) of calculators increases the number of units ($x$) sold. The relation between the price and quantity sold is given by the demand function:**

$$p = 450 - \frac{x}{2}.$$

Based on the above information, answer the following questions:


**37.(i). Determine the number of units ($x$) that should be sold to maximise the revenue $R(x) = x \cdot p(x)$. Also verify the result.**

**Solution:**

**Step 1: Express revenue as a function of $x$**

Revenue is given by:

$$R(x) = x \cdot p(x) = x \cdot \left(450 - \frac{x}{2}\right) = 450x - \frac{x^2}{2}.$$

**Step 2: Differentiate to find critical points**

The first derivative of $R(x)$ is:

$$\frac{dR}{dx} = 450 - x.$$

For maximum or minimum, set $\frac{dR}{dx} = 0$:

$$450 - x = 0 \implies x = 450.$$

**Step 3: Verify using the second derivative**

The second derivative of $R(x)$ is:

$$\frac{d^2R}{dx^2} = -1 < 0.$$

Since $\frac{d^2R}{dx^2} < 0$, $R(x)$ is maximum when $x = 450$.

**Step 4: Final result**

The number of units that should be sold to maximise revenue is $x = 450$.

> **Quick Tip**
>
> To maximise revenue or profit, always verify the nature of the critical point using the second derivative test.

---

**37.(ii). What rebate in price of the calculator should the store give to maximise the revenue?**

**Solution:**

**Step 1: Calculate the price at $x = 450$**

The price is given by:

$$p = 450 - \frac{x}{2} = 450 - \frac{450}{2} = 225.$$

**Step 2: Compute the rebate**

The original price is Rs. 350. The rebate is:

$$\text{Rebate} = 350 - 225 = 125\,(\text{Rs. per calculator}).$$

**Step 3: Final result**

The rebate required to maximise the revenue is Rs. 125 per calculator.

**Quick Tip**

For pricing and revenue problems, calculate the optimal price after determining the maximum quantity sold.

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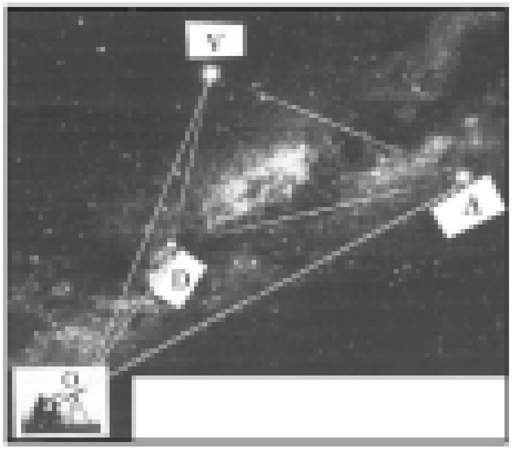
**Case Study - 3**

**38. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O(0, 0, 0)$ and the three stars have their locations at points $D, A,$ and $V$, having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$, and $-3\hat{i} + 7\hat{j} + 11\hat{k}$, respectively. Based on the above information, answer the following questions:**



**38.(i). How far is the star $V$ from star $A$?**

**Solution:**

**Step 1: Compute the position vector of $\overrightarrow{AV}$**

$$\overrightarrow{AV} = \text{Position vector of } V - \text{Position vector of } A$$

$$\overrightarrow{AV} = (-3\hat{i} + 7\hat{j} + 11\hat{k}) - (7\hat{i} + 5\hat{j} + 8\hat{k}) = -10\hat{i} + 2\hat{j} + 3\hat{k}.$$

**Step 2: Compute the magnitude of $\overrightarrow{AV}$**

$$|\overrightarrow{AV}| = \sqrt{(-10)^2 + 2^2 + 3^2} = \sqrt{100 + 4 + 9} = \sqrt{113}.$$

**Step 3: Final result**

The distance between star $V$ and star $A$ is $\sqrt{113}$ units.

> **Quick Tip**
>
> To find the distance between two points, use the magnitude of the difference of their position vectors.

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**38.(ii). Find a unit vector in the direction of $\overrightarrow{DA}$.**

**Solution:**

**Step 1: Compute $\overrightarrow{DA}$**

$$\overrightarrow{DA} = \text{Position vector of } A - \text{Position vector of } D$$

$$\overrightarrow{DA} = (7\hat{i} + 5\hat{j} + 8\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = 5\hat{i} + 2\hat{j} + 4\hat{k}.$$

**Step 2: Find the magnitude of $\overrightarrow{DA}$**

$$|\overrightarrow{DA}| = \sqrt{(5)^2 + (2)^2 + (4)^2} = \sqrt{25 + 4 + 16} = \sqrt{45} = 3\sqrt{5}.$$

**Step 3: Compute the unit vector**

The unit vector is:

$$\hat{u} = \frac{\overrightarrow{DA}}{|\overrightarrow{DA}|} = \frac{5\hat{i} + 2\hat{j} + 4\hat{k}}{3\sqrt{5}} = \frac{5}{3\sqrt{5}}\hat{i} + \frac{2}{3\sqrt{5}}\hat{j} + \frac{4}{3\sqrt{5}}\hat{k}.$$

**Step 4: Final result**

The unit vector in the direction of $\overrightarrow{DA}$ is:

$$\frac{5}{3\sqrt{5}}\hat{i} + \frac{2}{3\sqrt{5}}\hat{j} + \frac{4}{3\sqrt{5}}\hat{k}.$$

> **Quick Tip**
>
> To find a unit vector, divide the vector by its magnitude.

**38.(iii)(a). Find the measure of $\angle VDA$.**

**Solution:**

**Step 1: Recall the formula for the angle between vectors**

The angle $\theta$ between two vectors $\overrightarrow{VD}$ and $\overrightarrow{DA}$ is given by:

$$\cos\theta = \frac{\overrightarrow{VD}\cdot\overrightarrow{DA}}{|\overrightarrow{VD}|\cdot|\overrightarrow{DA}|}.$$

**Step 2: Compute $\overrightarrow{VD}$ and $\overrightarrow{DA}$**

From previous calculations:

$$\overrightarrow{VD} = \overrightarrow{V} - \overrightarrow{D} = (-3\hat{\imath} + 7\hat{\jmath} + 11\hat{k}) - (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) = -5\hat{\imath} + 4\hat{\jmath} + 7\hat{k}.$$

$$\overrightarrow{DA} = 5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}.$$

**Step 3: Compute $\overrightarrow{VD}\cdot\overrightarrow{DA}$**

$$\overrightarrow{VD}\cdot\overrightarrow{DA} = (-5)(5) + (4)(2) + (7)(4) = -25 + 8 + 28 = 11.$$

**Step 4: Compute magnitudes of $\overrightarrow{VD}$ and $\overrightarrow{DA}$**

$$|\overrightarrow{VD}| = \sqrt{(-5)^2 + 4^2 + 7^2} = \sqrt{25 + 16 + 49} = \sqrt{90}.$$

$$|\overrightarrow{DA}| = \sqrt{(5)^2 + (2)^2 + (4)^2} = \sqrt{25 + 4 + 16} = \sqrt{45}.$$

**Step 5: Compute $\cos\theta$**

$$\cos\theta = \frac{11\sqrt{2}}{\sqrt{90}\cdot\sqrt{45}} = \frac{11\sqrt{2}}{\sqrt{4050}} = \frac{11\sqrt{2}}{90}.$$

**Step 6: Final result**

The measure of $\angle VDA$ is:

$$\theta = \cos^{-1}\left(\frac{11\sqrt{2}}{90}\right).$$

**Quick Tip**

For angles between vectors, always use the dot product formula and ensure the magnitude is correctly computed.

**38.(iii)(b). What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$?**

**Solution:**

**Step 1: Recall the formula for projection**

The projection of $\overrightarrow{DV}$ on $\overrightarrow{DA}$ is given by:

$$\text{Projection} = \frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{|\overrightarrow{DA}|}.$$

**Step 2: Compute $\overrightarrow{DV}$**

$$\overrightarrow{DV} = \overrightarrow{V} - \overrightarrow{D} = (-5\hat{\imath} + 4\hat{\jmath} + 7\hat{k}).$$

**Step 3: Compute $\overrightarrow{DV} \cdot \overrightarrow{DA}$**

From the previous calculations:

$$\overrightarrow{DV} \cdot \overrightarrow{DA} = (-5)(5) + (4)(2) + (7)(4) = -25 + 8 + 28 = 11.$$

**Step 4: Compute $|\overrightarrow{DA}|$**

$$|\overrightarrow{DA}| = \sqrt{(5)^2 + (2)^2 + (4)^2} = \sqrt{45} = 3\sqrt{5}.$$

**Step 5: Compute the projection**

$$\text{Projection} = \frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{|\overrightarrow{DA}|} = \frac{11}{3\sqrt{5}}.$$

**Step 6: Final result**

The projection of $\overrightarrow{DV}$ on $\overrightarrow{DA}$ is:

$$\frac{11\sqrt{5}}{15}.$$

**Quick Tip**

The projection of one vector onto another gives the component of the first vector along the direction of the second.