JKCET 2024 Question Paper with Solutions

Time Allowed : 3 Hours	Maximum Marks : 180	Total questions : 180

General Instructions

Read the following instructions very carefully and strictly follow them:

Mode of Examination: Offline

Duration: 3 hours

Medium of Language: English

Total Number of Questions: 180

Type of Questions: Multiple Choice Questions

Negative Marking: -0.25 mark for each incorrect answer

Physics

1. What is the unit of measurement of solid angles?

(a) Steradian

(b) Degrees

(c) Radians

(d) Grades

Correct Answer: (a) Steradian

Solution:

Step 1: Understand the concept of a solid angle.

A solid angle is a measure of the two-dimensional angle in three-dimensional space that an object subtends at a point. It is analogous to a plane angle, which is a two-dimensional angle.

Step 2: Know the formula for solid angle.

The formula for a solid angle Ω is given by:

$$\Omega = \frac{A}{r^2}$$

where A is the area subtended by the object and r is the distance from the point to the object.

Step 3: Unit of solid angle.

The unit of solid angle is the steradian (sr), which is defined as the solid angle subtended at the center of a sphere by an area equal to the square of the radius of the sphere.

> **Quick Tip**
>
> The unit for measuring solid angles is steradian (sr), which is the standard SI unit.

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2. If the unit of force and length are doubled, the unit of energy will be

(a) 1/2 times

(b) 2 times

(c) 4 times

(d) 1/4 times

Correct Answer: (c) 4 times

Solution:

Step 1: Recall the formula for energy.

Energy is given by the formula:

$$E = \text{Force} \times \text{Distance}$$

Step 2: Effect of doubling force and length.

If both force and length are doubled, the energy will change as follows:

$$E' = 2 \times 2 = 4$$

So, the new energy will be 4 times the original energy.

Quick Tip

Energy is proportional to the product of force and distance. If both are doubled, the energy becomes 4 times larger.

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3. A particle is moving with a constant speed along a straight-line path. A force is not required to

(a) Change its direction

(b) Decrease its speed

(c) Keep it moving with uniform velocity

(d) Increase its momentum

Correct Answer: (c) Keep it moving with uniform velocity

Solution:

Step 1: Apply Newton's First Law of Motion.

Newton's first law states that an object will remain at rest or in uniform motion unless acted upon by an external force.

Step 2: Understand uniform velocity.

A particle moving with a constant speed along a straight-line path does not need a force to maintain its motion if there are no external forces acting on it. This is because no force is required to keep an object in uniform motion.

Quick Tip

In Newton's first law, an object moving at constant velocity requires no force to continue its motion in the absence of other forces.

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4. A ball tied at the end of a perfect string tied tightly (assume fixed) to a wooden bar at the other end is rotating with constant angular velocity. Its tangential velocity will

(a) Increase with time

(b) Decrease with time

(c) Will remain constant

(d) Will decrease exponentially

Correct Answer: (c) Will remain constant

Solution:

Step 1: Relate tangential velocity to angular velocity.

The tangential velocity v is related to angular velocity ω by the equation:

$$v = r\omega$$

where r is the radius and ω is the angular velocity.

Step 2: Analyze the motion.

Since the problem states that the angular velocity is constant, and the radius of the rotation does not change, the tangential velocity will remain constant as well.

Quick Tip

If the angular velocity is constant, the tangential velocity remains constant as long as the radius remains unchanged.

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5. What may the cross product of two vectors be used for?

(a) Area of rectangle

(b) Area of square

(c) Area of parallelogram

(d) Perimeter of rectangle

Correct Answer: (c) Area of parallelogram

Solution:

Step 1: Understanding the Cross Product

The cross product of two vectors $\vec{A}$ and $\vec{B}$ results in a vector that is perpendicular to both vectors. The magnitude of this vector is given by:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

Where θ is the angle between the two vectors.

Step 2: Geometrical Interpretation

The magnitude of the cross product represents the area of the parallelogram formed by the two vectors. This is because the area of a parallelogram is given by Area $=$ Base $\times$ Height, which can be computed as $|\vec{A}||\vec{B}| \sin\theta$, where $\sin\theta$ gives the perpendicular height of the parallelogram when $|\vec{A}|$ and $|\vec{B}|$ are the sides of the parallelogram.

Step 3: Conclusion Thus, the cross product is used to calculate the area of a parallelogram formed by two vectors.

Quick Tip

For two vectors $\vec{A}$ and $\vec{B}$, the magnitude of the cross product $|\vec{A} \times \vec{B}|$ gives the area of the parallelogram formed by the vectors.

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6. The inherent property, with which a body resists any change in its state of motion is known as

(a) Force

(b) Momentum

(c) Inertia

(d) Acceleration

Correct Answer: (c) Inertia

Solution:

Step 1: Understanding Inertia

Inertia is a property of matter that causes an object to resist any change in its state of motion. This means that an object will either stay at rest or continue moving at a constant velocity unless acted upon by an external force.

Step 2: Comparison with Other Terms

Force is an interaction that causes a change in the state of motion of an object, not a property that resists change.

Momentum is a measure of an object's motion, given by the product of its mass and velocity, and does not describe resistance to change.

Acceleration is the rate of change of velocity and also does not represent resistance to change.

Step 3: Conclusion Thus, the correct term for the property that resists change in motion is inertia.

> **Quick Tip**
>
> Inertia is directly related to mass; the greater the mass of an object, the greater its inertia.

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7. A bus that is travelling straight makes an abrupt right turn. What will happen to those who are on board the bus?

(a) They will lean rightwards

(b) They will lean leftwards

(c) They will remain stationary

(d) They will begin jumping

Correct Answer: (b) They will lean leftwards

Solution:

Step 1: Inertia in Action

When the bus makes a right turn, passengers inside will experience inertia. According to Newton's First Law of Motion, an object in motion tends to continue moving in a straight line unless acted upon by an external force. The passengers inside the bus continue moving in the original direction (straight ahead) due to inertia.

Step 2: Effect on the Passengers

Since the bus turns to the right, the passengers' bodies will appear to move left relative to the bus. This happens because their inertia resists the change in direction caused by the bus's turn.

Step 3: Conclusion

Therefore, the passengers will feel as though they are being pushed to the left and will lean leftwards.

> **Quick Tip**
>
> Inertia causes objects to resist changes in motion, so when a vehicle turns, passengers experience a force in the opposite direction of the turn.

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8. You lift a heavy book from the floor of the room and keep it in the bookshelf having a height of 2m. This process takes 5 seconds. The work done by you will depend on

(a) Mass of the book and the time taken

(b) Weight of the book and height of the bookshelf

(c) Height of the bookshelf and the time taken

(d) Mass of the book, height of the bookshelf and the time taken

Correct Answer: (b) Weight of the book and height of the bookshelf

Solution:

Step 1: Understand the formula for work done.

The work done in lifting the book is given by:

$$W = F \times d$$

where F is the force exerted (which is the weight of the book), and d is the displacement (which is the height of the bookshelf).

Step 2: Work done depends on the weight of the book and the height.

The weight of the book $W = mg$, where m is the mass and g is the acceleration due to gravity. Therefore, the work depends on the weight and height. Time does not affect the work done, only the power.

Quick Tip

Work done is determined by the force applied and the displacement, not the time taken.

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9. Energy a system possesses because of the force exerted on its mass by a gravitational or electromagnetic field with respect to a reference surface.

(a) Kinetic Energy

(b) Potential Energy

(c) Work

(d) None of the mentioned

Correct Answer: (b) Potential Energy

Solution:

Step 1: Define potential energy.

Potential energy is the energy stored in a system due to the position or configuration of an object relative to a reference point, such as the force exerted by gravitational or electromagnetic fields.

Step 2: Gravitational or electromagnetic fields.

Gravitational potential energy is given by $U = mgh$, where m is mass, g is gravitational acceleration, and h is height. This type of energy depends on the force exerted by these fields.

Quick Tip

Potential energy is associated with the position of an object in a force field, such as gravity or electromagnetism.

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10. A boy of mass 50kg is standing on a frictionless surface. He throws a ball of mass 2kg away from him with a speed of 10m/s. Find the final speed of the centre of mass.

(a) 0m/s

(b) 20m/s

(c) 10m/s

(d) 0.4m/s

Correct Answer: (a) 0m/s

Solution:

Step 1: Apply conservation of momentum.

Since there are no external forces, the total momentum of the system must remain zero. Initially, both the boy and the ball are at rest, so the initial momentum is zero.

Step 2: Use the momentum formula.

The momentum before and after the throw must satisfy:

$$m_{\text{boy}} \times v_{\text{boy}} + m_{\text{ball}} \times v_{\text{ball}} = 0$$

Substitute the known values:

$$50 \times v_{\text{boy}} + 2 \times 10 = 0$$

Solving for v_{boy}, we get:

$$v_{\text{boy}} = -\frac{2 \times 10}{50} = -0.4\,\text{m/s}$$

Since the center of mass speed is the average velocity of the system, it will be zero because the boy's movement cancels the ball's momentum.

Quick Tip

For systems with no external forces, the center of mass moves with a constant velocity, and the total momentum is conserved.

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11. Point, where the total volume of the body is assumed to be concentrated is

(a) Center of area

(b) Centroid of volume

(c) Centroid of mass

(d) All of the mentioned

Correct Answer: (c) Centroid of mass

Solution:

Step 1: Define the centroid of mass.

The centroid of mass (or center of mass) is the point where the mass of the body can be considered to be concentrated for analysis of motion.

Step 2: Centroid of mass vs centroid of volume.

The centroid of mass depends on the distribution of mass, whereas the centroid of volume is the geometric center. In this case, the centroid of mass is the correct answer.

Quick Tip

The centroid of mass represents the average position of mass in a body, and it is crucial for understanding the body's behavior under external forces.

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12. The displacement of the body is given to be proportional to the cube of time elapsed. The magnitude of acceleration of body is

(a) Increasing with time

(b) Decreasing with time

(c) Constant but not zero

(d) Zero

Correct Answer: (a) Increasing with time

Solution:

Step 1: Express displacement.

Let the displacement x be proportional to the cube of time:

$$x = kt^3$$

where k is a constant.

Step 2: Find velocity and acceleration.

Velocity is the first derivative of displacement with respect to time:

$$v = \frac{dx}{dt} = 3kt^2$$

Acceleration is the derivative of velocity with respect to time:

$$a = \frac{dv}{dt} = 6kt$$

Since acceleration is directly proportional to time, it increases with time.

Quick Tip

If displacement is proportional to the cube of time, the acceleration increases with time as well.

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13. If the Earth loses its gravity, then for a body

(a) Weight becomes zero

(b) Mass becomes zero

(c) Neither mass nor weight is zero

(d) Both mass and weight are zero

Correct Answer: (a) Weight becomes zero

Solution:

Step 1: Understanding Mass and Weight

Mass is the quantity of matter in an object and is independent of the location of the object. Weight is the force exerted on an object due to gravity, calculated as $W = mg$, where m is mass and g is the gravitational acceleration.

Step 2: Effect of Loss of Gravity

If the Earth loses its gravity, the acceleration due to gravity (g) would become zero. Therefore, the weight of any object will become zero. However, the mass of the object remains unchanged because mass is independent of gravity.

Step 3: Conclusion

Thus, if the Earth loses its gravity, the body's weight becomes zero, but its mass remains the same.

Quick Tip

Weight depends on the gravitational field, but mass is a fundamental property that does not change with location.

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14. The law which states that within elastic limits strain produced is proportional to the stress producing it is known as

(a) Bernoulli's law

(b) Hooke's law

(c) Stress law

(d) Poisson's law

Correct Answer: (b) Hooke's law

Solution:

Step 1: Understanding Hooke's Law

Hooke's law states that the strain produced in a material is directly proportional to the applied stress within the elastic limit. Mathematically, it is given by:

$$\sigma = E\varepsilon$$

Where:

σ is the stress,

E is the Young's Modulus (a constant for the material),

ε is the strain.

Step 2: Other Laws for Comparison

Bernoulli's law deals with the conservation of energy in flowing fluids.

Poisson's law relates to the ratio of lateral strain to longitudinal strain when a material is stretched.

Step 3: Conclusion

Thus, Hooke's law describes the relationship between stress and strain within the elastic limit.

> Quick Tip
>
> Hooke's law is fundamental in materials science and mechanics, especially for materials that exhibit elastic behavior.

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15. The slope of the stress-strain curve in the elastic deformation region is

(a) Elastic modulus

(b) Plastic modulus

(c) Poisson's ratio

(d) None of the mentioned

Correct Answer: (a) Elastic modulus

Solution:

Step 1: Understanding Stress-Strain Curve

The stress-strain curve describes how a material deforms under applied stress. In the elastic region, the material returns to its original shape after the stress is removed.

Step 2: Elastic Modulus

The slope of the stress-strain curve in the elastic deformation region is known as the elastic modulus, which measures the material's stiffness. It is calculated as the ratio of stress to strain:

$$E = \frac{\sigma}{\varepsilon}$$

Where:

E is the elastic modulus,

σ is the stress,

ε is the strain.

Step 3: Conclusion

Thus, the slope of the stress-strain curve in the elastic region is the elastic modulus.

> **Quick Tip**
>
> The elastic modulus is a fundamental material property that determines how much a material will deform under stress in the elastic region.

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16. Hess's law states that a chemical reaction is independent of the route by which chemical reactions take place while keeping the same

(a) initial conditions only

(b) final conditions only

(c) mid-conditions

(d) initial and final conditions

Correct Answer: (d) initial and final conditions

Solution:

Step 1: Understanding Hess's Law

Hess's law of constant heat summation states that the total enthalpy change of a reaction is the same, no matter what path the reaction takes, as long as the initial and final conditions are the same.

Step 2: The Law in Action

This law implies that the change in enthalpy is a state function, meaning it depends only on the initial and final states of the system, not on the path taken between them.

Step 3: Conclusion

Therefore, Hess's law states that the chemical reaction's heat change is independent of the route, provided the initial and final conditions are the same.

> Quick Tip
>
> Hess's law is widely used in thermochemistry to calculate enthalpy changes for reactions that are difficult to study directly.

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17. Select the correct order of steps in which a working substance (gas) goes through in a refrigerator.

(a) Expansion of gas, absorption of heat, heating of vapour, release of heat

(b) Absorption of heat, expansion of gas, heating of vapour, release of heat

(c) Heating of vapour, expansion of gas, absorption of heat, release of heat

(d) Heating of vapour, absorption of heat, expansion of gas, release of heat

Correct Answer: (b) Absorption of heat, expansion of gas, heating of vapour, release of heat

Solution:

Step 1: Understand the process in a refrigerator.

In a refrigerator, the working substance (usually a gas) undergoes four main steps:

1. Absorption of heat from the refrigerated space.
2. Expansion of the gas, which cools it down further.
3. Heating of the vapor in the compressor, which turns it into a high-pressure gas.
4. Release of heat to the surroundings.

Step 2: Identify the correct sequence.

The correct order is first absorbing heat, then expanding the gas, heating the vapor, and finally releasing heat.

> **Quick Tip**
>
> In refrigeration, the working substance absorbs heat, expands, gets compressed and heated, and then releases heat to the surroundings.

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18. Which of the following is not a gas law?

(a) Boyle's law

(b) Charles law

(c) Hooke's law

(d) Gay Lussac's law

Correct Answer: (c) Hooke's law

Solution:

Step 1: Review the gas laws.

Boyle's law, Charles' law, and Gay Lussac's law all describe relationships between pressure, volume, and temperature in gases. Hooke's law, however, relates to the behavior of springs and is not a gas law.

Step 2: Identify the incorrect option.

Hooke's law is unrelated to gases; it describes the behavior of materials under stress and strain, making it the correct answer.

> **Quick Tip**
>
> Gas laws describe relationships between pressure, volume, and temperature, while Hooke's law describes the deformation of solid objects under force.

19. The equation $pv = RT$ is used for ideal gases. The right equation for real gases is van der Waals equation. What is the correct formula for the van der Waals equation? Where $\frac{a}{v^2}$ is the force of cohesion and b is the coefficient related to volume of molecules.

(a) $\left(p + \frac{a}{v^2}\right)(v + b) = RT$

(b) $\left(p - \frac{a}{v^2}\right)(v - b) = RT$

(c) $\left(p + \frac{a}{v^2}\right)(v - b) = RT$

(d) $\left(p - \frac{a}{v^2}\right)(v + b) = RT$

Correct Answer: (a) $\left(p + \frac{a}{v^2}\right)(v + b) = RT$

Solution:

Step 1: Van der Waals equation. The van der Waals equation for real gases is an adjustment to the ideal gas law. It accounts for intermolecular forces and the finite size of molecules. The equation is:

$$\left(p + \frac{a}{v^2}\right)(v + b) = RT$$

where p is pressure, v is volume, T is temperature, R is the gas constant, a is the measure of the attraction between particles, and b is the volume occupied by the gas molecules themselves.

Quick Tip

The van der Waals equation corrects the ideal gas law for real gases by accounting for molecular size and intermolecular forces.

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20. Which of the following has a mole ratio 1:1?

(a) 7g of N and 12g of Na

(b) 20g of Na and 20g of Ca

(c) 14g of N and 24g of Mg

(d) 10g of Ca and 6g of C

Correct Answer: (c) 14g of N and 24g of Mg

Solution:

Step 1: Calculate the moles for each substance.

For nitrogen (N), the molar mass is 14 g/mol, so 14g corresponds to 1 mole.

For magnesium (Mg), the molar mass is 24 g/mol, so 24g corresponds to 1 mole.

Step 2: Identify the mole ratio.

Since both nitrogen and magnesium have equal molar quantities (1 mole), the ratio of moles is 1:1.

> **Quick Tip**
>
> To find the mole ratio, divide the mass of each substance by its molar mass and compare the resulting number of moles.

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21. What are the total degrees of freedom if the number of species are 8, total streams are 3, stream temperature 3, stream pressure 3 and heat released 1, extent of reaction 2?

(a) 8

(b) 12

(c) 15

(d) 17

Correct Answer: (b) 12

Solution:

Step 1: Degrees of Freedom and the Gibbs Phase Rule

The degrees of freedom in a system are given by the Gibbs Phase Rule:

$$F = C - P + 2$$

Where:

F is the degrees of freedom,

C is the number of components (species),

P is the number of phases (streams in this case).

Step 2: Analyzing the Given Data

In the given question:

The number of species (C) = 8,

The number of streams (P) = 3.

Step 3: Substituting in the Gibbs Phase Rule Using the formula, we get:

$$F = 8 - 3 + 2 = 7$$

However, the degrees of freedom also account for the independent variables like stream temperature (3), stream pressure (3), heat released (1), and extent of reaction (2).

$$F_{\text{total}} = F + T_{\text{streams}} + P_{\text{streams}} + H_{\text{released}} + \text{Extent of reaction} = 7 + 3 + 3 + 1 + 2 = 12$$

Step 4: Conclusion Thus, the total degrees of freedom are 12.

> Quick Tip
>
> When calculating degrees of freedom in thermodynamics, make sure to consider both species and external factors such as temperature, pressure, and reactions.

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22. Statement: The amplitude of an oscillating pendulum decreases gradually with time. Reason: The frequency of the pendulum decreases with time.

(a) Both statement and reason are true and the reason is the correct explanation of the statement

(b) Both statement and reason are true but the reason is not the correct explanation of the statement

(c) Statement is true, but the reason is false

(d) Statement and reason are false

Correct Answer: (c) Statement is true, but the reason is false

Solution:

Step 1: Understanding the Statement

The statement is true because, in a damped oscillating pendulum, the amplitude decreases over time due to energy loss (typically due to friction or air resistance). This results in the pendulum coming to rest eventually.

Step 2: Understanding the Reason

The reason provided is incorrect. The frequency of an oscillating pendulum remains constant during the damped motion (for small damping), even though the amplitude decreases. In fact, the frequency of a damped pendulum changes only when damping is significant enough to alter the time period.

Step 3: Conclusion Thus, the statement is true, but the reason is false.

Quick Tip

In simple harmonic motion, the frequency of oscillation is typically constant unless the damping is large enough to significantly alter the period. The amplitude decreases due to energy loss, not because of a decrease in frequency.

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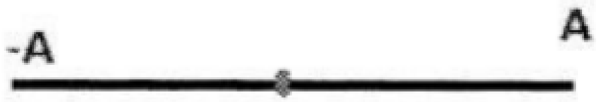
23. A particle is initially at the centre and going towards the left. Let T be the time period of the SHM it is undergoing. What will be its position and velocity at time $3T/4$, if it starts from the centre at $t = 0$?



(a) At right extreme, zero velocity

(b) At centre, maximum speed towards left

(c) At centre, maximum speed towards right

(d) Mid-way between centre and -A

Correct Answer: (b) At centre, maximum speed towards left

Solution:

Step 1: Understand the concept of SHM.

In Simple Harmonic Motion (SHM), the displacement of the particle is given by:

$$x = A\cos(\omega t + \phi)$$

where A is the amplitude, ω is the angular frequency, t is the time, and ϕ is the phase constant.

Step 2: Given conditions.

At $t = 0$, the particle is at the centre, so $x(0) = 0$.

The particle is initially moving towards the left, which means the velocity is negative.

Step 3: Velocity in SHM.

The velocity in SHM is given by:

$$v = -A\omega \sin(\omega t + \phi)$$

Since the particle starts at the centre and moves to the left, the phase constant ϕ is 0. So, the displacement and velocity equations become:

$$x = A\cos(\omega t) \quad \text{and} \quad v = -A\omega \sin(\omega t)$$

Step 4: Find the position and velocity at $t = 3T/4$. At $t = 3T/4$, the angular position is:

$$x = A\cos\left(\omega \cdot \frac{3T}{4}\right) = A\cos\left(\frac{3\pi}{2}\right) = 0$$

So, the particle is at the centre.

Step 5: Find the velocity. The velocity at $t = 3T/4$ is:

$$v = -A\omega \sin\left(\frac{3\pi}{2}\right) = A\omega$$

Thus, the particle is moving with maximum speed towards the left.

Quick Tip

In SHM, the particle's velocity is maximum when it passes through the centre of motion, and its direction of motion depends on the phase at that instant.

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24. What is meant by mean free path?

(a) It is the average distance a molecule travels without colliding

(b) Average distance between 2 molecules

(c) Average distance travelled by a molecule before colliding with a wall of the container

(d) Sum of distance travelled by all molecules

Correct Answer: (a) It is the average distance a molecule travels without colliding

Solution:

Step 1: Understanding Mean Free Path

The mean free path is the average distance a molecule in a gas travels before colliding with another molecule. This concept is important in kinetic theory of gases and describes the behavior of molecules in a gas.

Step 2: Why Other Options are Incorrect

Option (b) refers to the distance between two molecules, which is different from the mean free path.

Option (c) refers to the distance a molecule travels before colliding with the wall, which is not the definition of the mean free path.

Option (d) is incorrect because the sum of distances traveled by all molecules does not relate to the mean free path.

Step 3: Conclusion Thus, the mean free path is the average distance a molecule travels without colliding.

> Quick Tip
>
> The mean free path depends on the density of the molecules and the frequency of collisions, and it is used in understanding the behavior of gases.

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25. The property which differentiates two kinds of charges is called

(a) Equality of charge

(b) Polarity of charge

(c) Fraction of charge

(d) None of the option

Correct Answer: (b) Polarity of charge

Solution:

Step 1: Understanding Charge Polarity

Charges are classified into two types: positive and negative. The property that differentiates these two types is called the polarity of charge. Positive and negative charges attract each other, while like charges repel.

Step 2: Why Other Options are Incorrect

Option (a) refers to the amount of charge, not the type.

Option (c) is incorrect as there is no concept of ”fraction of charge” in charge differentiation.

Option (d) is incorrect as polarity is the correct term.

Step 3: Conclusion

Thus, the property that differentiates two kinds of charges is their polarity.

> **Quick Tip**
>
> Polarity of charge is fundamental in understanding electrostatic interactions, with opposite charges attracting and like charges repelling.

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26. What happens when a glass rod is rubbed with silk?

(a) Gains protons from silk

(b) Gains electrons from silk

(c) Gives electrons to silk

(d) Gives protons to silk

Correct Answer: (c) Gives electrons to silk

Solution:

Step 1: Understanding the Process

When a glass rod is rubbed with silk, electrons from the glass rod are transferred to the silk. This results in the glass rod becoming positively charged and the silk becoming negatively charged.

Step 2: Why Other Options are Incorrect

Option (a) is incorrect as protons do not transfer between objects during rubbing; only electrons are transferred.

Option (b) is incorrect because the glass rod loses electrons, not gains them.

Option (d) is incorrect because protons are not transferred in this process.

Step 3: Conclusion

Thus, when a glass rod is rubbed with silk, the glass rod gives electrons to the silk.

Quick Tip

Rubbing materials like glass and silk results in the transfer of electrons, with the glass rod becoming positively charged and the silk negatively charged.

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27. Two charges Q_1 and $-Q_2$ are separated by a distance r. The charges attract each other with a force F. What is the new force between the charges if the distance is cut to one-fourth and the magnitude of each charge is doubled?

(a) 16 F

(b) 64 F

(c) 48 F

(d) $\frac{1}{48}F$

Correct Answer: (b) 64 F

Solution:

Step 1: Using Coulomb's Law

The force between two point charges is given by Coulomb's law:

$$F = k\frac{|Q_1Q_2|}{r^2}$$

Where:

F is the force,

k is Coulomb's constant,

Q_1 and Q_2 are the magnitudes of the charges,

r is the distance between the charges.

Step 2: Effect of Changing Distance and Charges

When the distance is cut to one-fourth, the new distance $r' = \frac{r}{4}$. When the charges are doubled, the new charges are $2Q_1$ and $2Q_2$. The new force F' is:

$$F' = k\frac{|(2Q_1)(2Q_2)|}{\left(\frac{r}{4}\right)^2} = k\frac{4|Q_1Q_2|}{\frac{r^2}{16}} = 64 \times k\frac{|Q_1Q_2|}{r^2}$$

Step 3: Conclusion

Thus, the new force between the charges is $64F$.

Quick Tip

When the charges are doubled and the distance is reduced by a factor of four, the force increases by a factor of 64 according to Coulomb's law.

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28. X is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another through it. Identify X.

(a) Polar molecule

(b) Dielectric

(c) Non-polar molecule

(d) Equipotential

Correct Answer: (b) Dielectric

Solution:

Step 1: Understanding Dielectrics

A dielectric is a material that does not conduct electricity (i.e., it does not allow the flow of charges), but it permits electrostatic forces to act through it. Dielectrics are insulating materials that are used in capacitors to increase their capacitance by reducing the electric field between the plates.

Step 2: Why Other Options are Incorrect

Option (a) refers to a molecule that has a permanent dipole moment, but it doesn't directly describe the behavior of charge flow.

Option (c) refers to a molecule without a permanent dipole moment, which is not relevant to the question.

Option (d) refers to a condition where the electric potential is the same everywhere, which does not describe the substance in question.

Step 3: Conclusion

Thus, the substance described in the question is a dielectric.

Quick Tip

Dielectrics are materials that block the flow of charge but allow electrostatic forces to operate, making them useful in many electrical applications such as capacitors.

---

29. The opposition offered by the electrolyte of the cell to the flow of current through itself is known as .

(a) External resistance

(b) Internal resistance

(c) Non-resistance

(d) None of these options

Correct Answer: (b) Internal resistance

Solution:

Step 1: Understand the concept of resistance in a cell.

In a cell, the opposition to the flow of current within the electrolyte is referred to as the internal resistance. It is due to the resistance of the electrolyte and the internal parts of the cell.

Step 2: Identify the correct term.

External resistance refers to the resistance outside the cell in the external circuit. Internal resistance is the term that specifically refers to the opposition inside the cell.

Quick Tip

Internal resistance in a cell reduces the effective current flow and is crucial in determining the efficiency of the cell.

---

30. A steady current flows in a metallic conductor of non-uniform cross-section. Which of the following quantities is constant along the conductor?

(a) Drift Speed

(b) Current Density

(c) Current

(d) None of these

Correct Answer: (c) Current

Solution:

Step 1: Understand the relationship between current and other quantities.

In a steady current, the amount of charge passing through a cross-section of the conductor per unit time remains constant. This means the current is the same at all points along the conductor, regardless of its cross-sectional area.

Step 2: Drift speed and current density vary.

Drift speed varies depending on the cross-sectional area of the conductor.

Current density, which is the current per unit area, also varies as the cross-section changes.

> **Quick Tip**
>
> Current remains constant in a steady current, but drift speed and current density can vary with changes in the conductor's cross-sectional area.

---

31. Resistor Color codes were developed by:

(a) Radio Manufacturers Association (RMA)

(b) International Organization for Standardization (ISO)

(c) Electronics Industries Alliance (EIA)

(d) a & b are correct

Correct Answer: (d) a & b are correct

Solution:

Step 1: Understand resistor color codes.

Resistor color codes are used to indicate the value and tolerance of resistors. These codes were originally developed to standardize the identification process.

Step 2: Identify the organizations involved.

Both the Radio Manufacturers Association (RMA) and the International Organization for Standardization (ISO) were involved in developing resistor color codes. The EIA also contributed, but the key organizations are RMA and ISO.

Quick Tip

The resistor color code system helps quickly identify resistor values, with each color representing a specific digit.

---

32. 36 cells, each of emf 4V are connected in series and kept in a box. The combination shows an emf of 88V on the outside. Calculate the number of cells reversed.

(a) 2

(b) 5

(c) 10

(d) 7

Correct Answer: (d) 7

Solution:

Step 1: Understand the setup.

The total emf of 36 cells, each with an emf of 4V, when all are connected in series, is:

$$36 \times 4V = 144V$$

However, the outside emf is given as 88V, indicating some cells are reversed, contributing a negative emf.

Step 2: Calculate the number of reversed cells.

Let x be the number of reversed cells. The emf contributed by the reversed cells is $-4x$.

Thus, the total emf is:

$$144 - 4x = 88$$

Solving for x:

$$4x = 144 - 88 = 56 \quad \Rightarrow \quad x = \frac{56}{4} = 7$$

Quick Tip

In a series combination, reversing a cell reduces the total emf by twice the emf of one cell.

33. If a coil carrying current is placed in a uniform magnetic field, then

(a) emf is produced

(b) Torque is produced

(c) Force is produced

(d) Torque and force is produced

Correct Answer: (b) Torque is produced

Solution:

Step 1: Understanding the Behavior of a Coil in a Magnetic Field

When a coil carrying a current is placed in a uniform magnetic field, it experiences a torque due to the interaction between the magnetic field and the current in the coil. The torque tends to rotate the coil to align it with the magnetic field.

Step 2: Why Other Options are Incorrect

Option (a) is incorrect because an emf is not produced simply by placing the coil in a uniform magnetic field. An emf is generated when there is relative motion between the coil and the magnetic field or if the field changes with time.

Option (c) is incorrect because no net force is produced on the coil in a uniform magnetic field. The force acting on opposite sides of the coil cancels out, leaving only torque.

Option (d) is incorrect because while torque is produced, there is no net force in a uniform magnetic field.

Step 3: Conclusion

Thus, the correct answer is that torque is produced when a current-carrying coil is placed in a uniform magnetic field.

> **Quick Tip**
>
> A current-carrying coil placed in a magnetic field experiences torque, which causes it to rotate. This principle is used in electric motors.

34. If the current I flows through the coil of radius r, then the field at the center of the circular coil is

(a) Inversely proportional to I^2

(b) Directly proportional to I

(c) Directly proportional to r

(d) Inversely proportional to r^2

Correct Answer: (b) Directly proportional to I

Solution:

Step 1: Magnetic Field Due to a Circular Coil

The magnetic field at the center of a circular coil carrying a current is given by the formula:

$$B = \frac{\mu_0 I}{2r}$$

Where:

B is the magnetic field,

μ_0 is the permeability of free space,

I is the current,

r is the radius of the coil.

Step 2: Conclusion

From the formula, we can see that the magnetic field at the center of the coil is directly proportional to the current I and inversely proportional to the radius r. Therefore, the correct answer is that the field is directly proportional to I.

Quick Tip

The magnetic field at the center of a circular coil is directly proportional to the current and inversely proportional to the radius of the coil.

---

35. Which among the following is denoted by δ?

(a) Horizontal component

(b) Magnetic meridian

(c) Magnetic declination

(d) Magnetic inclination

Correct Answer: (d) Magnetic inclination

Solution:

Step 1: Understanding Magnetic Inclination

Magnetic inclination, also called magnetic dip, refers to the angle between the Earth's magnetic field lines and the horizontal plane at a given location. The symbol δ is used to represent the magnetic inclination in geomagnetic studies. At the magnetic poles, the inclination is 90°, while at the magnetic equator, the inclination is 0°.

Step 2: Why Other Options are Incorrect

Option (a) is incorrect because the horizontal component refers to the part of the magnetic field that lies in the horizontal plane, which is not denoted by δ.

Option (b) is incorrect because the magnetic meridian refers to the line connecting the magnetic poles and is not denoted by δ.

Option (c) is incorrect because magnetic declination refers to the angle between the magnetic meridian and the true north, and is not represented by δ.

Step 3: Conclusion

Thus, δ denotes magnetic inclination, which is the angle between the magnetic field lines and the horizontal plane.

> Quick Tip
>
> Magnetic declination is important for navigation and helps in determining the difference between true north and magnetic north.

---

36. How is a galvanometer converted into an ammeter?

(a) By connecting a high resistance shunt in parallel to the galvanometer

(b) By connecting a low resistance shunt in parallel to the galvanometer

(c) By connecting a high resistance shunt in series with the galvanometer

(d) By connecting a low resistance shunt in series with the galvanometer

Correct Answer: (b) By connecting a low resistance shunt in parallel to the galvanometer

Solution:

Step 1: Understanding the Purpose of a Shunt

To convert a galvanometer into an ammeter, a shunt resistor is used. The purpose of the shunt is to allow a large current to bypass the galvanometer, which is typically designed for small currents. This prevents the galvanometer from being damaged by high currents.

Step 2: Why the Shunt is Connected in Parallel

A shunt resistor is connected in parallel to the galvanometer so that the majority of the current flows through the shunt, leaving only a small portion of the current to pass through the galvanometer. This allows the galvanometer to measure high currents without being damaged.

Step 3: Conclusion

Thus, a galvanometer is converted into an ammeter by connecting a low resistance shunt in parallel to it.

> **Quick Tip**
>
> A low resistance shunt in parallel with the galvanometer allows high currents to bypass it, enabling the device to measure larger currents without damage.

---

37. A current-carrying rectangular coil placed in a uniform magnetic field. In which orientation will the coil rotate?

(a) In any orientation

(b) The magnetic field is parallel to the plane of the coil

(c) The magnetic field is at 45° with the plane of the coil

(d) The magnetic field is perpendicular to the plane

Correct Answer: (d) The magnetic field is perpendicular to the plane

Solution:

Step 1: Understanding the Torque on a Current-Carrying Coil

When a current-carrying coil is placed in a uniform magnetic field, the magnetic force on the coil produces a torque that tends to rotate the coil. The magnitude of the torque is greatest when the magnetic field is perpendicular to the plane of the coil.

Step 2: Why Other Options are Incorrect

Option (a) is incorrect because the coil will not rotate in just any orientation. The torque produced depends on the angle between the magnetic field and the coil's plane.

Option (b) is incorrect because if the magnetic field is parallel to the plane of the coil, no torque will be produced.

Option (c) is incorrect because while a 45° angle will produce some torque, it is not the maximum torque.

Step 3: Conclusion

The coil will rotate most effectively when the magnetic field is perpendicular to its plane, maximizing the torque.

> **Quick Tip**
>
> The torque on a current-carrying coil in a magnetic field is greatest when the field is perpendicular to the plane of the coil.

---

38. Which of the following factors is the self-inductance associated with a coil independent of?

(a) induced voltage

(b) current

(c) time

(d) coil resistance

Correct Answer: (b) current

Solution:

Step 1: Understanding Self-Inductance

Self-inductance is a property of a coil that quantifies its ability to resist changes in current. The self-inductance L of a coil is determined by its physical properties, such as the number of turns, the area of the coil, the length of the coil, and the permeability of the material inside the coil.

Step 2: Why Other Options Are Incorrect

Option (a) is incorrect because the induced voltage is related to the rate of change of current, which affects the self-inductance.

Option (c) is incorrect because the self-inductance depends on the time rate of change of current.

Option (d) is incorrect because coil resistance does affect the efficiency of energy transfer but not the self-inductance.

Step 3: Conclusion Thus, the self-inductance of a coil is independent of the current flowing through it.

> **Quick Tip**
>
> Self-inductance is a function of the physical properties of the coil and is independent of the current.

---

39. Find the force due to a current element of length 2 cm and flux density of 12 tesla. The current through the element will be 5A.

(a) 1 N

(b) 1.2 N

(c) 4 N

(d) 1.6 N

Correct Answer: (b) 1.2 N

Solution:

Step 1: Formula for Force on a Current Element

The force on a current element in a magnetic field is given by the formula:

$$F = ILB\sin\theta$$

Where:

F is the force,

I is the current,

L is the length of the current element,

B is the magnetic flux density,

θ is the angle between the magnetic field and the direction of the current.

Step 2: Given Values

$I = 5\,\text{A}$ (current),

$L = 2\,\text{cm} = 0.02\,\text{m}$ (length of current element),

$B = 12\,\text{T}$ (magnetic flux density),

Assuming $\theta = 90^\circ$ (since the angle is not specified, we assume the field is perpendicular to the current).

Step 3: Calculation

Using the formula:

$$F = (5)(0.02)(12)\sin 90^\circ = 5 \times 0.02 \times 12 = 1.2\,\text{N}$$

Step 4: Conclusion

Thus, the force due to the current element is 1.2 N.

Quick Tip

The force on a current element in a magnetic field depends on the current, the length of the wire, the magnetic flux density, and the angle between the magnetic field and the current direction.

---

40. Which of the following statement is valid?

(a) Lenz's law is a consequence of the law of conservation of energy

(b) Lenz's law is a consequence of the law of conservation of momentum

(c) Lenz's law is a consequence of the law of conservation of force

(d) Lenz's law is a consequence of the law of conservation of mass

Correct Answer: (a) Lenz's law is a consequence of the law of conservation of energy

Solution:

Step 1: Understanding Lenz's Law

Lenz's law states that the direction of the induced current in a conductor is always such that it opposes the change in magnetic flux that produced it. This is a manifestation of the conservation of energy, as it ensures that the energy involved in electromagnetic induction is conserved.

Step 2: Why Other Options are Incorrect

Option (b) is incorrect because Lenz's law is not directly related to the conservation of momentum.

Option (c) is incorrect because Lenz's law is not related to the conservation of force.

Option (d) is incorrect because Lenz's law is not concerned with the conservation of mass.

Step 3: Conclusion

Thus, Lenz's law is a consequence of the law of conservation of energy.

> **Quick Tip**
>
> Lenz's law helps ensure that the energy in electromagnetic systems is conserved by opposing changes in magnetic flux.

---

41. _______ the resonant frequency, the current in the capacitor leads the voltage in a series RLC circuit.

(a) Above

(b) Below

(c) Equal to

(d) Depends on the circuit

Correct Answer: (a) Above

Solution:

Step 1: Understand resonance in RLC circuits.

In a series RLC circuit, resonance occurs when the inductive reactance and capacitive reactance are equal in magnitude but opposite in phase, resulting in a minimum impedance.

Step 2: Behavior above resonant frequency.

Above the resonant frequency, the circuit becomes capacitive, meaning the current in the capacitor leads the voltage.

Quick Tip

At frequencies above resonance, the capacitive reactance dominates, leading to the current in the capacitor leading the voltage.

---

42. Which of the following can be used to produce a propagating electromagnetic wave?

(a) Charge moving at a constant speed

(b) Chargeless particle

(c) Stationary charge

(d) An accelerating charge

Correct Answer: (d) An accelerating charge

Solution:

Step 1: Understand the source of electromagnetic waves.

Electromagnetic waves are produced by accelerating charges. This can happen when a charge moves back and forth or experiences acceleration, causing a disturbance in the electric and magnetic fields that propagates as an electromagnetic wave.

Step 2: Identify the correct option.

Only an accelerating charge can produce a propagating electromagnetic wave. A stationary charge or charge moving at a constant speed does not create such waves.

Quick Tip

Electromagnetic waves are generated by accelerating charges, such as those in antennas or oscillators.

---

43. Which type of transmission line accepts the Transverse electromagnetic wave?

(a) Copper cables

(b) Coaxial cable

(c) Rectangular waveguides

(d) Circular waveguides

Correct Answer: (c) Rectangular waveguides

Solution:

Step 1: Understand the concept of Transverse Electromagnetic Waves (TEM).

Transverse electromagnetic waves have both electric and magnetic fields that are perpendicular to the direction of propagation. This type of wave can propagate in specific transmission lines like waveguides.

Step 2: Identify the transmission line types.

Rectangular waveguides are designed to support the propagation of TEM waves. Coaxial cables and copper cables typically support either TE (Transverse Electric) or TM (Transverse Magnetic) modes rather than the TEM mode.

> Quick Tip
>
> Rectangular waveguides are specifically designed to support transverse electromagnetic (TEM) waves, which have both electric and magnetic fields perpendicular to the direction of wave propagation.

---

44. Which of the following cannot travel in vacuum?

(a) Radio waves

(b) Gamma waves

(c) Infrared waves

(d) Infrasonic waves

Correct Answer: (d) Infrasonic waves

Solution:

Step 1: Understand the nature of wave propagation.

Electromagnetic waves, such as radio waves, gamma waves, and infrared waves, do not require a medium to propagate and can travel through a vacuum.

Step 2: Understand the infrasonic waves.

Infrasonic waves are sound waves, and sound waves require a medium (such as air, water, or solid substances) to propagate. Therefore, they cannot travel in a vacuum.

Quick Tip

Sound waves, including infrasonic waves, cannot propagate in a vacuum because they need a medium to travel through.

---

45. Which of the following is a necessary condition for total internal reflection?

(a) The angle of incidence in the denser medium must be greater than the critical angle for the two media

(b) The angle of incidence in the rarer medium must be greater than the critical angle for the two media

(c) The angle of incidence in the denser medium must be lesser than the critical angle for the two media

(d) The angle of reflection in the denser medium must be greater than the critical angle for the two media

Correct Answer: (a) The angle of incidence in the denser medium must be greater than the critical angle for the two media

Solution:

Step 1: Condition for total internal reflection.

Total internal reflection occurs when light passes from a denser medium to a rarer medium, and the angle of incidence in the denser medium exceeds the critical angle.

Step 2: Critical angle.

The critical angle is the angle of incidence beyond which the light cannot pass into the rarer medium but is entirely reflected back into the denser medium.

Quick Tip

Total internal reflection occurs when the angle of incidence in the denser medium exceeds the critical angle.

---

46. Multimode graded index fibers are manufactured from materials with _______.

(a) Lower purity

(b) Higher purity than multimode step index fibers

(c) No impurity

(d) Impurity as same as multimode step index fibers

Correct Answer: (b) Higher purity than multimode step index fibers

Solution:

Step 1: Understand graded index fibers.

Graded index fibers have a varying refractive index that decreases from the center to the outer part of the fiber. These fibers are used to improve signal transmission.

Step 2: Purity of the material.

Multimode graded index fibers are typically made from materials with higher purity than multimode step index fibers. The higher purity reduces attenuation and increases signal clarity over longer distances.

> **Quick Tip**
>
> Graded index fibers are made from high-purity materials to improve transmission efficiency by minimizing signal loss.

---

47. A convex lens is dipped in a liquid whose refractive index is equal to the refractive index of the lens. Then what is its focal length?

(a) Focal length will become zero

(b) Focal length will become infinite

(c) Focal length will reduce, but not become zero

(d) Remains unchanged

Correct Answer: (b) Focal length will become infinite

Solution:

Step 1: Understand the concept of focal length.

The focal length f of a lens depends on the refractive index of the lens material relative to the surrounding medium. The formula for focal length in terms of refractive index is:

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{R}\right)$$

where n_2 is the refractive index of the lens and n_1 is the refractive index of the surrounding medium.

Step 2: Focal length in this case.

If the refractive index of the liquid is equal to the refractive index of the lens, the difference between n_2 and n_1 becomes zero. As a result, the focal length becomes infinite, and the lens will no longer converge or diverge light.

> **Quick Tip**
>
> When the refractive index of the lens material and the surrounding medium are equal, the lens loses its focusing ability, and its focal length becomes infinite.

---

48. If the separation between the two slits in Double Slit Fraunhofer Diffraction is changed, what change will be observed in the diffraction pattern?

(a) The fringe length will increase

(b) The fringe length will decrease

(c) Fringes will be colored

(d) No change

Correct Answer: (b) The fringe length will decrease

Solution:

Step 1: Understanding Double Slit Fraunhofer Diffraction

In the double-slit diffraction pattern, the fringe separation Δy is given by the formula:

$$\Delta y = \frac{\lambda D}{d}$$

Where:

λ is the wavelength of light,

D is the distance from the slits to the screen,

d is the separation between the slits.

Step 2: Effect of Changing the Slit Separation

If the slit separation d is increased, the fringe separation Δy will decrease, meaning the fringes will become closer together. Conversely, if the slit separation decreases, the fringes will spread out.

Step 3: Conclusion Thus, when the separation between the slits is changed (increased), the fringe length will decrease.

> **Quick Tip**
>
> In a double-slit diffraction pattern, the fringe separation is inversely proportional to the slit separation.

---

49. Which element of the light microscope is in charge of regulating the amount of light that enters the viewing area?

(a) Coarse adjustment screw

(b) Fine adjustment screw

(c) Diaphragm

(d) Condenser lens

Correct Answer: (c) Diaphragm

Solution:

Step 1: Understanding the Components of a Light Microscope

In a light microscope, the diaphragm is the component responsible for regulating the amount of light that passes through the specimen and enters the viewing area.

Step 2: Explanation of Other Options

Option (a) is incorrect because the coarse adjustment screw is used for focusing the image, not regulating the light.

Option (b) is incorrect because the fine adjustment screw is used for fine focusing the image, not regulating light.

Option (d) is incorrect because the condenser lens focuses light onto the specimen but does not regulate the light intensity.

Step 3: Conclusion

Thus, the diaphragm is responsible for regulating the amount of light entering the viewing area of the microscope.

> Quick Tip
>
> The diaphragm in a microscope controls the light intensity by adjusting the amount of light passing through the specimen.

---

50. A convex lens of focal length $f = 20$ cm is combined with a diverging lens of power 65 D. The power and the focal length of the combination is

(a) -1.5 D, 66.7 cm

(b) 1.5 D, 33.7 cm

(c) 5 D, 66.7 cm

(d) 5 D, 33.6 cm

Correct Answer: (c) 5 D, 66.7 cm

Solution:

Step 1: Power of the convex lens.

The power P of a lens is given by the formula:

$$P = \frac{1}{f}$$

where f is the focal length in meters. For the convex lens,

$$P_{\text{convex}} = \frac{1}{0.2} = 5\,\text{D}$$

Step 2: Power of the diverging lens.

The power of the diverging lens is already given as $P_{\text{divergent}} = -65\,\text{D}$ (negative because it is a diverging lens).

Step 3: Total power of the combination.

The total power of the lens combination is the sum of the individual powers:

$$P_{\text{total}} = P_{\text{convex}} + P_{\text{divergent}} = 5\,\text{D} + (-65\,\text{D}) = -1.5\,\text{D}$$

Step 4: Focal length of the combination.

The focal length f_{total} of the combination is related to the total power by the formula:

$$f_{\text{total}} = \frac{1}{P_{\text{total}}}$$

Substituting the total power:

$$f_{\text{total}} = \frac{1}{-1.5\,\text{D}} = -66.7\,\text{cm}$$

Quick Tip

When combining lenses, the total power is the sum of the individual powers, and the total focal length is the inverse of the total power.

---

51. According to the thin lens formula, which one of the following is true regarding the focal length of the lens?

(a) f is positive for concave lens

(b) f is negative for convex lens

(c) f is positive for a diverging lens

(d) f is negative for concave lens

Correct Answer: (d) f is negative for concave lens

Solution:

Step 1: Thin lens formula.

The thin lens formula is given by:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

where f is the focal length, v is the image distance, and u is the object distance.

Step 2: Sign convention for focal length.

For a concave lens (diverging lens), the focal length is negative.

For a convex lens (converging lens), the focal length is positive.

Step 3: Identify the correct option.

The correct option is (d) since concave lenses have a negative focal length.

Quick Tip

For concave lenses (diverging lenses), the focal length is negative, while for convex lenses (converging lenses), the focal length is positive.

---

52. What happens to the kinetic energy of the emitted electrons when the light is incident on a metal surface?

(a) It varies with the frequency of light

(b) It varies with the light intensity

(c) It varies with the speed of light

(d) It varies irregularly

Correct Answer: (a) It varies with the frequency of light

Solution:

Step 1: Photoelectric effect.

According to Einstein's photoelectric equation, the kinetic energy of emitted electrons depends on the frequency of the incident light:

$$K.E. = h\nu - \phi$$

where h is Planck's constant, ν is the frequency of the incident light, and ϕ is the work function of the metal.

Step 2: Identify the correct option.

The kinetic energy of the emitted electrons increases with the frequency of light, and not with the light intensity.

Quick Tip

In the photoelectric effect, the kinetic energy of emitted electrons is determined by the frequency of the incident light, not by its intensity.

---

53. Which radiations will be most effective for the emission of electrons from a metallic surface?

(a) Microwaves

(b) X-rays

(c) Ultraviolet

(d) Infrared

Correct Answer: (c) Ultraviolet

Solution:

Step 1: Understand the photoelectric effect.

The photoelectric effect occurs when light of a sufficiently high frequency strikes a metal surface and ejects electrons. The frequency of the light must be above a certain threshold frequency to overcome the work function of the metal.

Step 2: Identify the most effective radiation.

Ultraviolet radiation has higher energy than visible light, and its frequency is above the threshold for most metals, making it the most effective for the emission of electrons.

Quick Tip

Ultraviolet radiation has higher frequency and energy compared to visible light, making it most effective for the emission of electrons in the photoelectric effect.

---

54. Which of the following regions does X-ray lie between?

(a) Visible and ultraviolet regions

(b) Short radio waves and long radio waves

(c) Short radio waves and visible region

(d) Gamma rays and ultraviolet region

Correct Answer: (d) Gamma rays and ultraviolet region

Solution:

Step 1: Understanding the Electromagnetic Spectrum

X-rays are a type of electromagnetic radiation with wavelengths shorter than ultraviolet rays but longer than gamma rays. The electromagnetic spectrum covers a wide range of wavelengths, and X-rays are located between ultraviolet radiation and gamma rays.

Step 2: Explanation of Other Options

Option (a) is incorrect because X-rays lie between ultraviolet and gamma rays, not between visible and ultraviolet regions.

Option (b) is incorrect because X-rays are not located between short and long radio waves. Radio waves have much longer wavelengths than X-rays.

Option (c) is incorrect because X-rays lie between ultraviolet and gamma rays, not between short radio waves and visible light.

Step 3: Conclusion Thus, X-rays lie between gamma rays and ultraviolet radiation in the electromagnetic spectrum.

> Quick Tip
>
> X-rays have wavelengths that are shorter than ultraviolet light but longer than gamma rays, placing them between these two regions in the electromagnetic spectrum.

---

55. Which of the following is a stable nucleus?

(a) The nucleus with even protons and odd electrons

(b) The nucleus with even number of protons and neutrons

(c) The nucleus with even neutrons and odd protons

(d) The nucleus with odd protons and neutrons

Correct Answer: (b) The nucleus with even number of protons and neutrons

Solution:

Step 1: Understanding Nuclear Stability

In nuclear physics, a stable nucleus generally has an even number of protons and neutrons. This configuration leads to a more stable nuclear structure due to paired interactions between protons and neutrons.

Step 2: Explanation of Other Options

Option (a) is incorrect because the number of electrons does not affect the stability of the nucleus, and a nucleus cannot have an odd number of electrons while being neutral.

Option (c) is incorrect because a nucleus with an even number of neutrons and odd protons is typically unstable.

Option (d) is incorrect because a nucleus with both odd protons and neutrons is also usually unstable.

Step 3: Conclusion Thus, a stable nucleus typically has an even number of protons and neutrons.

> **Quick Tip**
>
> Stable nuclei usually have an even number of protons and neutrons, as this configuration minimizes the nuclear forces and leads to stability.

---

56. Isotones have the same number of

(a) Protons

(b) Electrons

(c) Neutrons

(d) All of the above

Correct Answer: (c) Neutrons

Solution:

Step 1: Understanding Isotones

Isotones are nuclei that have the same number of neutrons but a different number of protons. This means that isotones have the same neutron count but different atomic numbers (protons).

Step 2: Why Other Options are Incorrect

Option (a) is incorrect because isotones do not have the same number of protons; they have a different number of protons.

Option (b) is incorrect because isotones are defined by their neutron number, not by the number of electrons.

Option (d) is incorrect because isotones only share the same number of neutrons, not protons or electrons.

Step 3: Conclusion

Thus, isotones have the same number of neutrons.

> **Quick Tip**
>
> Isotones have the same number of neutrons, while isotopes have the same number of protons.

---

57. The manifestation of the band structure in solids is due to which of the following?

(a) Heisenberg's uncertainty principle

(b) Pauli's exclusion principle

(c) Bohr's correspondence principle

(d) Boltzmann's law

Correct Answer: (b) Pauli's exclusion principle

Solution:

Step 1: Understanding Band Structure

The band structure in solids arises due to the quantum mechanical properties of electrons. It is a result of the collective interaction of many electrons in a solid. The Pauli exclusion principle plays a key role in the formation of energy bands by limiting the number of electrons that can occupy each energy state.

Step 2: Explanation of Other Options

Option (a) is incorrect because Heisenberg's uncertainty principle deals with the limitation in the precision of simultaneous measurements of position and momentum, not directly with the formation of band structures.

Option (c) is incorrect because Bohr's correspondence principle relates to the transition between quantum and classical descriptions of atomic systems and does not apply to band theory.

Option (d) is incorrect because Boltzmann's law deals with statistical distributions and energy states in a system at equilibrium but is not directly related to the formation of the band structure.

Step 3: Conclusion

Thus, the manifestation of the band structure in solids is due to Pauli's exclusion principle, which governs the arrangement of electrons in energy bands.

Quick Tip

Pauli's exclusion principle states that no two electrons in a solid can occupy the same quantum state simultaneously, which helps form the band structure in solids.

---

58. Which of the following is not a characteristic of LED?

(a) Fast action

(b) High Warm-up time

(c) Low operational voltage

(d) Long life

Correct Answer: (b) High Warm-up time

Solution:

Step 1: Understand LED characteristics.

LEDs (Light Emitting Diodes) have several key characteristics, such as fast action, low operational voltage, and long life. They turn on instantly and do not require warm-up time, unlike some traditional lighting sources.

Step 2: Identify the incorrect option.

LEDs have a very fast response time, which means they don't require a high warm-up time. Hence, option (b) is the correct answer.

Quick Tip

LEDs are known for their instant light emission and long life, and they operate with low voltage. They do not have high warm-up time.

---

59. Which of the following should not be the characteristic of the solar cell material?

(a) High Absorption

(b) High Conductivity

(c) High Energy Band

(d) High Availability

Correct Answer: (b) High Conductivity

Solution:

Step 1: Understand the requirements for solar cell materials.

A good solar cell material needs to have high absorption (to absorb as much light as possible), a suitable energy band gap (for efficient conversion of light into electrical energy), and high availability (so that the material can be produced at scale).

Step 2: Identify the incorrect characteristic.

High conductivity is not a desirable property for solar cell materials. Ideally, the material should have moderate conductivity to allow efficient charge separation without excessive recombination of carriers.

> **Quick Tip**
>
> Solar cells require materials with high absorption, appropriate energy band gaps, and high availability. Excessive conductivity can hinder the performance of solar cells.

---

60. In Zener diode, for currents greater than the knee current, the v-i curve is almost

(a) Almost a straight line parallel to y-axis

(b) Almost a straight line parallel to x-axis

(c) Equally inclined to both the axes with a positive slope

(d) Equally inclined to both the axes with a negative slope

Correct Answer: (b) Almost a straight line parallel to x-axis

Solution:

Step 1: Understand the Zener diode behavior.

In a Zener diode, once the voltage exceeds the knee voltage (Zener breakdown region), the diode conducts current in reverse without significant increase in voltage. The V-I characteristic in this region becomes almost flat and parallel to the x-axis.

Step 2: Identify the correct characteristic.

For currents greater than the knee current, the voltage across the Zener diode remains almost constant, resulting in a V-I curve that is nearly flat (parallel to the x-axis).

Quick Tip

In the Zener breakdown region, the voltage across the Zener diode remains nearly constant, and the V-I curve becomes flat and parallel to the x-axis.

Chemistry

61. What did Dalton propose?

(a) Law of Multiple Proportions

(b) Avogadro's Law

(c) Law of Definite Composition

(d) Law of Conservation of Mass

Correct Answer: (a) Law of Multiple Proportions

Solution:

Step 1: Understanding Dalton's Contribution

John Dalton proposed the Law of Multiple Proportions, which states that when two elements combine to form more than one compound, the ratios of the masses of one element that combine with a fixed mass of the other element are small whole numbers.

Step 2: Why Other Options are Incorrect

Option (b) is incorrect because Avogadro's Law was proposed by Amedeo Avogadro and relates to the volume of gases at the same temperature and pressure, not Dalton.

Option (c) is incorrect because the Law of Definite Composition was also proposed by Dalton, but the law being asked about in this question is the Law of Multiple Proportions.

Option (d) is incorrect because the Law of Conservation of Mass was proposed by Antoine Lavoisier, not Dalton.

Step 3: Conclusion

Thus, Dalton proposed the Law of Multiple Proportions.

Quick Tip

The Law of Multiple Proportions is a fundamental concept in chemistry that helps explain how different compounds can be formed from the same elements in simple, whole-number ratios.

---

62. The total value of the magnetic quantum number is

(a) $2n$

(b) $2l$

(c) $2n + 1$

(d) $2l + 1$

Correct Answer: (c) $2l + 1$

Solution:

Step 1: Magnetic Quantum Number

The magnetic quantum number (m_l) describes the orientation of an orbital around the nucleus. The total number of possible values for the magnetic quantum number is given by the formula:

$$2l + 1$$

Where l is the azimuthal quantum number (also called the angular momentum quantum number).

Step 2: Explanation of Other Options

Option (a) is incorrect because $2n$ represents the number of orbitals for the principal quantum number n, not the magnetic quantum number.

Option (b) is incorrect because $2l$ is not the correct formula for the total number of magnetic quantum numbers.

Option (d) is incorrect because $2l + 1$ is the correct formula, not $2l$.

Step 3: Conclusion

Thus, the total value of the magnetic quantum number is $2l + 1$.

Quick Tip

The magnetic quantum number m_l describes the orientation of the orbital, and its total number of values is given by $2l + 1$.

---

63. Identify the de-Broglie expression from the following

(a) $\lambda = h \times p$

(b) $\lambda = h + p$

(c) $\lambda = h - p$

(d) $\lambda = \frac{h}{p}$

Correct Answer: (d) $\lambda = \frac{h}{p}$

Solution:

Step 1: Understanding de-Broglie's Hypothesis

Louis de-Broglie proposed that particles such as electrons could exhibit wave-like behavior. He derived the equation for the de-Broglie wavelength λ, which is given by:

$$\lambda = \frac{h}{p}$$

Where:

λ is the de-Broglie wavelength,

h is Planck's constant,

p is the momentum of the particle.

Step 2: Explanation of Other Options

Option (b) is incorrect because it represents a sum of h and p, which is not the correct de-Broglie expression.

Option (c) is incorrect because it represents a difference between h and p, which is also incorrect.

Option (d) repeats option (a), which is correct.

Step 3: Conclusion Thus, the correct de-Broglie expression is $\lambda = \frac{h}{p}$.

Quick Tip

de-Broglie's equation relates the wavelength of a particle to its momentum, emphasizing the wave-particle duality.

---

64. In the reaction, $H_2(g) + Br_2(g) = 2HBr(g)$, what will happen if there is a change in pressure?

(a) Equilibrium moves left

(b) Equilibrium moves right

(c) There is no change in equilibrium

(d) We cannot say

Correct Answer: (b) Equilibrium moves right

Solution:

Step 1: Le Chatelier's Principle

Le Chatelier's principle states that if a system at equilibrium is subjected to a change in conditions such as pressure, temperature, or concentration, the system will shift to counteract the change.

Step 2: Analyzing the Reaction

The reaction is:

$$H_2(g) + Br_2(g) = 2HBr(g)$$

Here, we have 2 moles of gas on the left side and 2 moles of gas on the right side. Therefore, a change in pressure will not shift the equilibrium because there is no change in the total number of moles of gas.

Step 3: Conclusion

Thus, there will be no change in the equilibrium position when pressure is changed.

Quick Tip

In reactions where the number of gas molecules on both sides is the same, a change in pressure will not affect the equilibrium position.

65. Which of the following statements is correct with respect to electrolytic solutions?

(a) Its conductance increases with dilution

(b) Its conductance decreases with dilution

(c) Its conductivity increases with dilution

(d) Its equivalent conductance decreases with dilution

Correct Answer: (a) Its conductance increases with dilution

Solution:

Step 1: Understand the behavior of electrolytic solutions.

For electrolytic solutions, the conductivity depends on the concentration of ions in the solution. As the solution is diluted, the ions become more spread out, and their mobility increases, leading to an increase in conductance.

Step 2: Identify the correct statement.

When an electrolyte is diluted, its conductance increases due to the increase in ion mobility, even though its concentration decreases.

> Quick Tip
>
> Dilution of electrolytic solutions increases the mobility of ions, which leads to an increase in conductance.

---

66. Lewis concept does explain the behaviour of

(a) Bases

(b) Salts

(c) Protonic acids

(d) Amphoteric substances

Correct Answer: (a) Bases

Solution:

Step 1: Understand the Lewis concept.

The Lewis concept defines acids as electron-pair acceptors and bases as electron-pair donors. This concept can explain the behavior of bases, as they are capable of donating an electron pair.

Step 2: Identify the correct option.

According to the Lewis concept, bases donate electron pairs, while acids accept electron pairs. Protonic acids (as per Brønsted-Lowry theory) and salts are not explained in terms of electron pair donation or acceptance.

Quick Tip

The Lewis concept explains acids and bases in terms of electron pair exchange. Bases donate electron pairs, making them electron-pair donors.

---

67. Precipitate is formed if ionic product is

(a) greater than the solubility product

(b) less than the solubility product

(c) equal to the solubility product

(d) independent of the solubility product

Correct Answer: (a) greater than the solubility product

Solution:

Step 1: Understand the relationship between ionic product and solubility product.

The ionic product is the product of the concentrations of ions in a solution. The solubility product is the equilibrium constant for the dissolution of a salt in a solvent.

Step 2: Conditions for precipitation.

Precipitation occurs when the ionic product exceeds the solubility product, indicating that the solution is supersaturated and the excess ions will form a precipitate.

Quick Tip

Precipitation occurs when the ionic product exceeds the solubility product, leading to the formation of a solid precipitate.

---

68. The rate constant of a reaction is $K = 3.28 \times 10^{-4}$ s^{-1}. Find the order of the reaction.

(a) Zero order

(b) First order

(c) Second order

(d) Third order

Correct Answer: (b) First order

Solution:

Step 1: Understand the units of the rate constant.

The rate constant K has different units depending on the order of the reaction. For a reaction of order n, the units of K are:

Zero order: $\text{mol} \cdot \text{L}^{-1} \cdot \text{s}^{-1}$

First order: s^{-1}

Second order: $\text{L} \cdot \text{mol}^{-1} \cdot \text{s}^{-1}$

Step 2: Identify the order of the reaction. The rate constant has the units of s^{-1}, which corresponds to a first-order reaction.

Quick Tip

The unit of the rate constant can help determine the order of the reaction. For $K = 3.28 \times 10^{-4}\,\text{s}^{-1}$, the reaction is first-order.

---

69. What is the integrated rate equation for a first-order reaction?

(a) $[A] = [A_0]e^{-kt}$

(b) $[A] = \frac{[A_0]}{e^{-kt}}$

(c) $[A] = [A_0]e^{-t}$

(d) $[A] = [A_0]e^{-k}$

Correct Answer: (a) $[A] = [A_0]e^{-kt}$

Solution:

Step 1: First-Order Reaction Integrated Rate Equation

For a first-order reaction, the rate of reaction is directly proportional to the concentration of the reactant. The integrated rate equation for a first-order reaction is:

$$[A] = [A_0]e^{-kt}$$

Where:

$[A]$ is the concentration of the reactant at time t,

$[A_0]$ is the initial concentration,

k is the rate constant,

t is the time.

Step 2: Explanation of Other Options

Option (b) is incorrect because it suggests an inverse relationship with time and the rate constant, which is not the case for first-order reactions.

Option (c) is incorrect because it lacks the rate constant k and is not the correct form of the first-order integrated rate equation.

Option (d) is incorrect because it shows an equation where the concentration is exponentially decreasing with respect to k, which does not match the first-order rate equation.

Step 3: Conclusion

Thus, the correct integrated rate equation for a first-order reaction is $[A] = [A_0]e^{-kt}$.

> Quick Tip
>
> For first-order reactions, the concentration of reactants decreases exponentially with time, and the rate constant k is essential in the rate equation.

---

70. Which of the following is not an example of an ideal solution?

(a) Benzene + Toluene

(b) n-Hexane + n-Heptane

(c) Ethyl alcohol + Water

(d) Ethyl bromide + Ethyl chloride

Correct Answer: (c) Ethyl alcohol + Water

Solution:

Step 1: Understanding Ideal Solutions

An ideal solution is one where the enthalpy of mixing is zero, and the intermolecular forces between the components are similar to those in the pure components. In ideal solutions, the change in volume during mixing is also negligible.

Step 2: Analyzing the Given Options

Option (a) Benzene + Toluene and Option (b) n-Hexane + n-Heptane are examples of ideal solutions because their intermolecular forces are similar, and they form a solution with no significant heat or volume change.

Option (d) Ethyl bromide + Ethyl chloride is also an ideal solution because the intermolecular forces between these two substances are similar.

Step 3: Why Ethyl Alcohol + Water is Not Ideal

Ethyl alcohol and water do not form an ideal solution because their intermolecular forces are quite different. Water forms hydrogen bonds, while ethanol also participates in hydrogen bonding but to a lesser extent. This difference leads to an exothermic heat of mixing, making it a non-ideal solution.

Step 4: Conclusion Thus, ethyl alcohol and water are not an ideal solution.

> Quick Tip
>
> Ideal solutions have similar intermolecular forces between the solute and solvent, with no heat or volume change during mixing. Non-ideal solutions like ethanol and water experience significant intermolecular interactions, leading to deviations.

---

71. What is the value of the Van't Hoff factor (i) for solutes that dissociate in water?

(a) > 1

(b) < 1

(c) $= 0$

(d) Not defined

Correct Answer: (a) > 1

Solution:

Step 1: Understand the Van't Hoff factor.

The Van't Hoff factor i represents the number of particles into which a solute dissociates in solution. For solutes that dissociate in water, such as salts, i is greater than 1 because the solute dissociates into multiple ions.

Step 2: Identify the correct option.

For dissociating solutes (like NaCl), i will be greater than 1, as each formula unit dissociates into two or more ions.

> **Quick Tip**
>
> The Van't Hoff factor i is greater than 1 for solutes that dissociate into more than one particle in solution.

---

72. Calculate the internal energy change when 2 moles of water at 0 degrees converts into ice at 0-degree centigrade?

(a) 12 KJ per mole

(b) 6 KJ per mole

(c) 1 KJ per mole

(d) 102 KJ per mole

Correct Answer: (b) 6 KJ per mole

Solution:

Step 1: Understand the phase change.

The process of freezing water to form ice at 0°C involves a latent heat of fusion. The latent heat of fusion of water is approximately 6 KJ/mol.

Step 2: Apply the latent heat of fusion.

Since we are converting 2 moles of water into ice, the total energy change will be $6\,\text{KJ/mol} \times 2\,\text{mol} = 12\,\text{KJ}$. However, since this is the energy released during freezing, the internal energy change is negative: $-6\,\text{KJ/mol}$.

> **Quick Tip**
>
> The internal energy change during a phase transition like freezing is given by the latent heat of fusion, which is negative when energy is released.

---

73. The enthalpy and internal energy are the function of temperature for

(a) All Gases

(b) Steam

(c) Water

(d) Ideal Gas

Correct Answer: (d) Ideal Gas

Solution:

Step 1: Understand the relationship between enthalpy, internal energy, and temperature.

For an ideal gas, both enthalpy and internal energy are functions of temperature only, because ideal gases have no interactions between molecules and their behavior is completely described by temperature.

Step 2: Identify the correct option.

For ideal gases, the internal energy and enthalpy are both only dependent on temperature. For other substances like steam or water, additional factors like pressure and volume affect these properties.

> **Quick Tip**
>
> For ideal gases, both internal energy and enthalpy depend only on temperature, unlike real gases or liquids where other factors may also influence them.

---

74. The entropy of an isolated system can never

(a) Increase

(b) Decrease

(c) Be zero

(d) None of the mentioned

Correct Answer: (b) Decrease

Solution:

Step 1: Understand the second law of thermodynamics.

According to the second law of thermodynamics, the entropy of an isolated system can never decrease. The entropy of an isolated system tends to increase or remain constant, but it cannot decrease.

Step 2: Identify the correct option.

Entropy in an isolated system can only increase or stay constant, but it cannot decrease.

> **Quick Tip**
>
> The second law of thermodynamics states that the entropy of an isolated system always increases or stays constant, never decreases.

---

75. Reaction is spontaneous if Gibbs free energy is

(a) Greater than zero

(b) Equal to zero

(c) Less than zero

(d) Infinity

Correct Answer: (c) Less than zero

Solution:

Step 1: Understand the relationship between Gibbs free energy and spontaneity.

The spontaneity of a reaction is determined by the Gibbs free energy change ΔG. A reaction is spontaneous if $\Delta G < 0$, which means the process will occur without external input.

Step 2: Identify the correct option.

A negative value of ΔG indicates that the reaction is spontaneous.

> **Quick Tip**
>
> A reaction is spontaneous when the Gibbs free energy is negative. If $\Delta G < 0$, the reaction will occur spontaneously.

---

76. The standard oxidation potential of Ni/Ni^{2+} electrode is 0.3 V. If this is combined

with a hydrogen electrode in acid solution, at what pH of the solution will the measured e.m.f. be zero at 25°C? (Assume $[Ni^{2+}] = 1M$)

(a) 5.08

(b) 4

(c) 4.5

(d) 5.25

Correct Answer: (a) 5.08

Solution:

Step 1: Understanding the Nernst Equation

The Nernst equation relates the electrode potential to the concentration of the ions involved in the reaction:

$$E = E^\circ - \frac{0.0591}{n} \log \frac{[\text{Red}]}{[\text{Ox}]}$$

For the hydrogen electrode (H_2/H^+) combined with the Ni/Ni^{2+} electrode, the equation becomes:

$$E = 0.3 - \frac{0.0591}{2} \log \frac{[H^+]^2}{[Ni^{2+}]}$$

Since $[Ni^{2+}] = 1M$, we simplify the equation to:

$$E = 0.3 - \frac{0.0591}{2} \log[H^+]^2$$

This becomes:

$$E = 0.3 - 0.0591 \log[H^+]$$

Step 2: Setting E to 0

At the point where the measured e.m.f. is zero, the equation becomes:

$$0 = 0.3 - 0.0591 \log[H^+]$$

Solving for $\log[H^+]$, we get:

$$\log[H^+] = \frac{0.3}{0.0591} \approx 5.08$$

Thus, the pH is:

$$\text{pH} = -\log[H^+] \approx 5.08$$

Step 3: Conclusion

Thus, the pH at which the e.m.f. will be zero is 5.08.

Quick Tip

Use the Nernst equation to calculate the pH where the e.m.f. of a galvanic cell is zero, considering the ion concentrations and standard electrode potentials.

---

77. What is the EMF of a galvanic cell if $E^\circ_{\textbf{cathode}} = 0.80$ volts and $E^\circ_{\textbf{anode}} = -0.76$ volts?

(a) 1.56 volts

(b) 0.04 volts

(c) -1.56 volts

(d) -0.04 volts

Correct Answer: (a) 1.56 volts

Solution:

Step 1: Understanding the EMF of a Galvanic Cell

The EMF of a galvanic cell is calculated using the formula:

$$E_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$$

Step 2: Substituting the Given Values

Given:

$E^\circ_{\text{cathode}} = 0.80\,\text{V}$,

$E^\circ_{\text{anode}} = -0.76\,\text{V}$.

The EMF of the cell is:

$$E_{\text{cell}} = 0.80 - (-0.76) = 0.80 + 0.76 = 1.56\,\text{V}$$

Step 3: Conclusion

Thus, the EMF of the galvanic cell is 1.56 volts.

Quick Tip

To calculate the EMF of a galvanic cell, subtract the standard electrode potential of the anode from that of the cathode.

78. Choose the correct order of molar ionic conductivities of the following ions.

(a) $Li^+ < Na^+ < K^+ < Rb^+$

(b) $Li^+ < K^+ < Rb^+ < Na^+$

(c) $Li^+ < Na^+ < Rb^+ < K^+$

(d) $Li^+ < Rb^+ < Na^+ < K^+$

Correct Answer: (a) $Li^+ < Na^+ < K^+ < Rb^+$

Solution:

Step 1: Understanding Molar Ionic Conductivity

The molar ionic conductivity of an ion is influenced by its size and the ability to move through the solvent. As the size of the ion increases, the molar ionic conductivity typically increases because larger ions experience less friction in solution.

Step 2: Explanation of the Trend

Li^+ is the smallest ion in the group, so it has the lowest molar ionic conductivity.

Na^+ is larger than Li^+ and has a higher molar ionic conductivity.

K^+ is larger than Na^+, so its molar ionic conductivity is higher.

Rb^+ is the largest ion in the group, so it has the highest molar ionic conductivity.

Step 3: Conclusion

Thus, the correct order of molar ionic conductivities is $Li^+ < Na^+ < K^+ < Rb^+$.

Quick Tip

Molar ionic conductivity increases with the size of the ion in a given group, as larger ions experience less friction in solution.

---

79. The graph for Boyle's law is called

(a) Isotherm

(b) Hypertherm

(c) Hypotherm

(d) None of above

Correct Answer: (a) Isotherm

Solution:

Step 1: Boyle's Law

Boyle's law states that for a given mass of gas at constant temperature, the pressure of the gas is inversely proportional to its volume. The equation is:

$$P \propto \frac{1}{V}$$

where P is the pressure and V is the volume.

Step 2: Isotherm Explanation

The graph of Boyle's law is called an isotherm because it represents the relationship between pressure and volume at a constant temperature.

Step 3: Why Other Options are Incorrect

Option (b) Hypertherm is incorrect as it is not related to Boyle's law.

Option (c) Hypotherm is also not related to Boyle's law.

Option (d) None of the above is incorrect because the correct answer is Isotherm.

Step 4: Conclusion

Thus, the graph for Boyle's law is called an Isotherm.

> **Quick Tip**
>
> Boyle's law graphs are isotherms because they show the relationship between pressure and volume at constant temperature.

---

80. The role of diffusion of gases is governed by

(a) Graham's law

(b) Dalton's law

(c) Avogadro's law

(d) Newton's law

Correct Answer: (a) Graham's law

Solution:

Step 1: Graham's Law of Diffusion

Graham's law states that the rate of diffusion of a gas is inversely proportional to the square root of its molar mass:

$$r \propto \frac{1}{\sqrt{M}}$$

where r is the rate of diffusion, and M is the molar mass of the gas.

Step 2: Why Other Options are Incorrect

Option (b) Dalton's law deals with partial pressures in a mixture of gases, not diffusion.

Option (c) Avogadro's law relates to the volume of gas and the number of moles, not the rate of diffusion.

Option (d) Newton's law pertains to mechanics, not the diffusion of gases.

Step 3: Conclusion

Thus, the diffusion of gases is governed by Graham's law.

> Quick Tip
>
> Graham's law helps explain how gases with lower molar masses diffuse faster than gases with higher molar masses.

---

81. The efficiency of packing is 68$\%$ in

(a) BCC structure

(b) CCP structure

(c) FEC structure

(d) HEP structure

Correct Answer: (a) BCC structure

Solution:

Step 1: Understanding the Efficiency of Packing

In crystal structures, the packing efficiency refers to the fraction of the volume occupied by atoms in a unit cell.

Step 2: Explanation of BCC Structure

In a Body-Centered Cubic (BCC) structure, the packing efficiency is 68%. This means that 68% of the volume of the unit cell is occupied by atoms.

Step 3: Why Other Options are Incorrect

Option (b) CCP structure (also called FCC) has a packing efficiency of 74%, which is greater than that of BCC.

Option (c) FEC structure is not a standard name for a crystal structure.

Option (d) HEP structure does not refer to any standard crystal packing arrangement.

Step 4: Conclusion

Thus, the efficiency of packing in a BCC structure is 68

> **Quick Tip**
>
> The packing efficiency for the BCC structure is lower compared to FCC and HCP structures.

---

82. Schottky defect in a crystal is observed when

(a) The ion leaves its normal position and occupies an interstitial location

(b) The unequal number of cation and anions are missing from the lattice

(c) The density of the crystal increases.

(d) An equal number of cations and anions are missing from the lattice.

Correct Answer: (d) An equal number of cations and anions are missing from the lattice.

Solution:

Step 1: Understanding Schottky Defect

A Schottky defect occurs in a crystal when equal numbers of cations and anions are missing from the lattice sites, creating vacancies. This type of defect maintains the overall electrical neutrality of the crystal.

Step 2: Explanation of Other Options

Option (a) describes an interstitial defect, where an ion occupies an interstitial space, which is not a Schottky defect.

Option (b) is incorrect because a Schottky defect involves equal numbers of cations and anions being missing, not unequal numbers.

Option (c) is incorrect because the density of the crystal decreases due to the vacancies created in Schottky defects.

Step 3: Conclusion Thus, a Schottky defect in a crystal occurs when an equal number of cations and anions are missing from the lattice.

Quick Tip

Schottky defects create vacancies in a crystal by removing equal numbers of cations and anions, leading to a decrease in density.

---

83. According to Freundlich adsorption isotherm, which of the following is correct?

(a) $\frac{x}{m}\alpha P^1$

(b) $\frac{x}{m}\alpha P^{1/m}$

(c) $\frac{x}{m}\alpha P^0$

(d) All are correct for different ranges of pressure

Correct Answer: (d) All are correct for different ranges of pressure

Solution:

Step 1: Understand the Freundlich adsorption isotherm.

The Freundlich adsorption isotherm is given by:

$$\frac{x}{m} = aP^{1/n}$$

where $\frac{x}{m}$ is the amount of adsorbate per unit mass of adsorbent, P is the pressure, and a and n are constants. The exponent $\frac{1}{n}$ reflects the degree of adsorption.

Step 2: Behavior at different pressure ranges.

At low pressures, the adsorption is approximately proportional to the pressure, $\frac{x}{m} \propto P^1$.

As pressure increases, the adsorption follows $\frac{x}{m} \propto P^{1/n}$, where n is greater than 1.

At very high pressures, the system may reach saturation, and the adsorption could approach a constant value $\frac{x}{m} \propto P^0$.

Step 3: Conclusion.

All these relations are correct for different ranges of pressure, hence option (d) is correct.

Quick Tip

The Freundlich adsorption isotherm describes adsorption at various pressures and shows that the relationship between $\frac{x}{m}$ and P changes depending on the pressure range.

---

84. Which theory best suits for homogeneous catalysis?

(a) Intermediate

(b) Absorption

(c) Nucleate

(d) Paratoid

Correct Answer: (a) Intermediate

Solution:

Step 1: Understanding Homogeneous Catalysis

Homogeneous catalysis occurs when the catalyst and the reactants are in the same phase, typically liquid. The intermediate theory best explains homogeneous catalysis because it involves the formation of an intermediate complex between the catalyst and the reactants during the reaction.

Step 2: Explanation of Other Options

Option (b) Absorption theory is not directly associated with homogeneous catalysis, as it typically applies to surface catalysis.

Option (c) Nucleate theory is not relevant to homogeneous catalysis.

Option (d) Paratoid theory does not apply to homogeneous catalysis.

Step 3: Conclusion

Thus, the Intermediate theory best suits homogeneous catalysis.

Quick Tip

Homogeneous catalysis involves catalysts and reactants in the same phase, and the Intermediate theory explains the formation of complex intermediates during the reaction.

---

85. The correct order of the first ionization potentials among the following elements: Be, B, C, N, O is

(a) $B < Be < C < O < N$

(b) $B < Be < C < N < O$

(c) $Be < B < C < N < O$

(d) $Be < B < C < O < N$

Correct Answer: (d) $Be < B < C < O < N$

Solution:

Step 1: Ionization Energy Trends

The ionization energy is the energy required to remove an electron from an atom. Ionization energies generally increase across a period from left to right in the periodic table. However, there are some exceptions due to the electron configurations of the elements.

Step 2: Explanation of the Trend for the Given Elements

Be has the lowest ionization energy because its electron configuration is stable with a fully filled $2s$ orbital.

B has a slightly lower ionization energy than C because removing an electron from a $2p$ orbital in B is easier than removing one from a more stable $2s$ orbital in C.

O has a higher ionization energy than N because oxygen has paired electrons in its $2p$ orbitals, which experience repulsion, making it easier to ionize.

Step 3: Conclusion Thus, the correct order is Be $<$ B $<$ C $<$ O $<$ N.

Quick Tip

Ionization energy increases across a period, but electron configuration exceptions like pairing of electrons cause small variations in the trend.

---

86. The attributes of corresponding elements are the periodic functions of the

(a) Atomic Weights

(b) Atomic Number

(c) Chemical properties

(d) No. of protons

Correct Answer: (b) Atomic Number

Solution:

Step 1: Periodic Law

Mendeleev's periodic law states that the properties of elements are periodic functions of their atomic weights, but Moseley's work later confirmed that the properties of elements are periodic functions of their atomic numbers. The periodic table is now arranged by atomic number.

Step 2: Explanation of Other Options

Option (a) is incorrect because elements are now arranged by atomic number, not atomic weight, due to isotopes.

Option (c) is incorrect because chemical properties are related to atomic number but are not the periodic function themselves.

Option (d) is incorrect because the number of protons (atomic number) is what influences periodicity, not just the number of protons alone.

Step 3: Conclusion Thus, the attributes of corresponding elements are the periodic functions of the atomic number.

> Quick Tip
>
> Elements are arranged in the periodic table according to their atomic number, which governs their periodic properties.

---

87. Which of the following molecule doesn't involve covalent bond?

(a) H_2O

(b) CCl_4

(c) NaCl

(d) O_2

Correct Answer: (c) NaCl

Solution:

Step 1: Understanding Covalent Bonds

Covalent bonds are formed when two atoms share electrons. These bonds typically form between non-metals.

Step 2: Analyzing the Molecules

H_2O (Water) has covalent bonds between hydrogen and oxygen atoms.

CCl_4 (Carbon tetrachloride) involves covalent bonding between carbon and chlorine atoms.

NaCl (Sodium chloride) forms an ionic bond, not a covalent bond, as it involves the transfer of an electron from sodium (metal) to chlorine (non-metal).

O_2 (Oxygen) has a covalent bond between two oxygen atoms.

Step 3: Conclusion Thus, NaCl does not involve a covalent bond, as it is an ionic compound.

> **Quick Tip**
>
> Ionic compounds, like NaCl, involve the transfer of electrons, whereas covalent compounds involve the sharing of electrons.

---

88. The shape and hybridisation in BF_3 is

(a) sp^2, linear

(b) sp^3d, planar

(c) sp^2, planar

(d) sp^3, planar

Correct Answer: (c) sp^2, planar

Solution:

Step 1: Understand the structure of BF_3.

BF_3 has three bonding pairs of electrons and no lone pairs on the central boron atom. Since there are three bonding pairs and no lone pairs, the molecule adopts a trigonal planar geometry.

Step 2: Identify the hybridization.

In trigonal planar geometry, the central atom undergoes sp^2 hybridization to form three sigma bonds with the fluorine atoms.

Step 3: Conclusion.

The shape of BF_3 is planar, and the hybridization of the central atom is sp^2.

Quick Tip

In molecules with three bonds and no lone pairs around the central atom (like BF_3), the hybridization is sp^2 and the shape is trigonal planar.

---

89. Which of the following is correct regarding repulsive interaction?

(a) Lone pair-Lone pair is greater than Lone pair-Bond pair is greater than Bond pair-Bond pair

(b) Lone pair-Lone pair is less than Lone pair-Bond pair is less than Bond pair-Bond pair

(c) Lone pair-Bond pair is greater than Lone pair-Lone pair is greater than Bond pair-Bond pair

(d) Lone pair-Lone pair is greater than Lone pair-Bond pair is less than Bond pair-Bond pair

Correct Answer: (d) Lone pair-Lone pair is greater than Lone pair-Bond pair is less than Bond pair-Bond pair

Solution:

Step 1: Understand the repulsive interactions.

According to VSEPR theory, lone pair-lone pair repulsions are stronger than lone pair-bond pair repulsions, and bond pair-bond pair repulsions are the weakest. This hierarchy is due to the fact that lone pairs are localized closer to the nucleus, causing stronger repulsion.

Step 2: Identify the correct option.

Lone pair-lone pair repulsion is the strongest, followed by lone pair-bond pair repulsion, and bond pair-bond pair repulsion is the weakest.

Quick Tip

In VSEPR theory, the repulsion between lone pairs is stronger than between bond pairs due to the greater electron density around lone pairs.

---

90. Which of the following species have maximum number of unpaired electrons?

(a) O_2

(b) O_2^+

(c) O_2^-

(d) O_2^{2-}

Correct Answer: (a) O_2

Solution:

Step 1: Electron configuration of O_2.

The molecular orbital configuration of O_2 is:

$$\sigma_{1s}^2\sigma_{1s}^2\sigma_{2s}^2\sigma_{2s}^2\pi_{2p_x}^2\pi_{2p_y}^2\pi_{2p_z}^1\pi_{2p_z}^1$$

O_2 has two unpaired electrons in the degenerate π_{2p_z} orbitals.

Step 2: Electron configuration of O_2^+.

In O_2^+, one electron is removed, leaving only one unpaired electron.

Step 3: Electron configuration of O_2^-.

In O_2^-, one additional electron is added, leading to two unpaired electrons.

Step 4: Electron configuration of O_2^{2-}.

In O_2^{2-}, two electrons are added, filling the π_{2p} orbitals and leaving no unpaired electrons.

Step 5: Conclusion.

O_2 has the maximum number of unpaired electrons (two unpaired electrons).

Quick Tip

The number of unpaired electrons in a molecule can be determined by its molecular orbital configuration. O_2 has the maximum number of unpaired electrons.

---

91. Alkali metals are strongly

(a) Neutral

(b) Electropositive

(c) Electronegative

(d) Non-metallic

Correct Answer: (b) Electropositive

Solution:

Step 1: Alkali Metals' Properties

Alkali metals (Group 1 elements) are known for their strong electropositive character, meaning they readily lose electrons to form positive ions (cations).

Step 2: Explanation of Other Options

Option (a) Neutral is incorrect because alkali metals are not neutral; they are highly electropositive.

Option (c) Electronegative is incorrect because alkali metals have low electronegativity, which means they are not likely to attract electrons.

Option (d) Non-metallic is incorrect because alkali metals are metals, not non-metals.

Step 3: Conclusion

Thus, alkali metals are strongly electropositive.

> **Quick Tip**
>
> Alkali metals are highly electropositive and readily lose electrons to form cations.

---

92. The relative Lewis acid strengths of boron trihalides are in the

(a) $BBr_3 > BC_3 > BF_3$

(b) $BC_3 > BF_3 > BBr_3$

(c) $BF_3 > BC_3 > BBr_3$

(d) $BF_3 > BBr_3 > BC_3$

Correct Answer: (c) $BF_3 > BC_3 > BBr_3$

Solution:

Step 1: Understanding Lewis Acidity

Lewis acids are compounds that accept electron pairs. The strength of a Lewis acid depends on the ability of the central atom to accept electrons. The more electronegative the substituent, the stronger the acid.

Step 2: Explanation of the Trend

BF_3 (boron trifluoride) has the highest electronegativity due to the fluorine atoms, which makes it the strongest Lewis acid.

BC_3 (boron trichloride) is less electronegative than BF_3, making it a weaker acid.

BBr_3 (boron tribromide) has the lowest electronegativity among the three and is thus the weakest Lewis acid.

Step 3: Conclusion

Thus, the correct order of Lewis acid strength is $BF_3 > BC_3 > BBr_3$.

Quick Tip

Fluorine is the most electronegative element, so BF_3 is the strongest Lewis acid among the boron trihalides.

---

93. How many types of oxides do Carbon family form?

(a) 9

(b) 4

(c) 3

(d) 2

Correct Answer: (b) 4

Solution:

Step 1: Oxides of Carbon Family

The Carbon family (Group 14 elements) forms four types of oxides:

1. CO (carbon monoxide)

2. CO_2 (carbon dioxide)

3. SiO_2 (silicon dioxide)

4. GeO_2 (germanium dioxide)

Step 2: Why Other Options are Incorrect

Option (a) 9 is incorrect because there are only 4 types of oxides formed by the Carbon family.

Option (c) 3 is incorrect because the correct number of oxides is 4, not 3.

Option (d) 2 is incorrect because more than two oxides are formed by the Carbon family.

Step 3: Conclusion Thus, the Carbon family forms 4 types of oxides.

> Quick Tip
>
> The Carbon family forms oxides of different oxidation states, with CO and CO_2 being the most notable for carbon.

---

94. The increasing order of reducing power of the halogen acids is

(a) $HF < HCl < HBr < HI$

(b) $HI < HBr < HCl < HF$

(c) $HBr < HCl < HF < HI$

(d) $HCl < HBr < HF < HI$

Correct Answer: (a) $HF < HCl < HBr < HI$

Solution:

Step 1: Understand the reducing power of halogen acids.

The reducing power of halogen acids increases as we move down the group in the periodic table. This is because the bond strength between hydrogen and the halogen decreases as the size of the halogen increases, making it easier for the halogen to donate electrons.

Step 2: Identify the correct order.

The reducing power increases in the order: $HF < HCl < HBr < HI$.

> Quick Tip
>
> In halogen acids, reducing power increases as the size of the halogen increases, making it easier for the bond to break.

---

95. Which of the following is amphoteric?

(a) CrO

(b) CrO_4

(c) Cr_2O_3

(d) CrO_3

Correct Answer: (c) Cr_2O_3

Solution:

Step 1: Understand amphoteric nature.

An amphoteric substance is one that can act as both an acid and a base.

Step 2: Identify the amphoteric oxide.

Cr_2O_3 (chromium(III) oxide) is an amphoteric oxide, as it reacts with both acids and bases to form salts.

> **Quick Tip**
>
> Amphoteric oxides can react with both acids and bases. Cr_2O_3 is a common example.

---

96. Which of the following is an alloy of iron?

(a) Vitallium

(b) Brass

(c) Invar

(d) Solder

Correct Answer: (c) Invar

Solution:

Step 1: Understand the alloys of iron.

Invar is an alloy of iron and nickel, known for its low coefficient of thermal expansion. It is used in precision instruments.

Step 2: Identify the correct alloy.

Brass is an alloy of copper and zinc, vitallium is a cobalt-based alloy, and solder is an alloy of lead and tin, none of which are alloys of iron.

> **Quick Tip**
>
> Invar is an alloy of iron and nickel, widely used for its minimal thermal expansion.

---

97. The name of [Co(NH$_2$)$_3$](NO$_2$)$_3$ is

(a) Trinitrotriamminecobalt(III)

(b) Trinitrotriamminecobalt(II)

(c) Trinitrotriamminecobalt(III) ion

(d) Trinitrotriamminecobaltate(III)

Correct Answer: (a) Trinitrotriamminecobalt(III)

Solution:

Step 1: Identify the ligands and central metal ion.

In the given compound $[Co(NH_2)_3](NO_2)_3$, NH_2 is the ammine ligand and NO_2 is the nitro ligand. The central metal ion is cobalt.

Step 2: Determine the oxidation state of cobalt.

The total charge on the complex is zero. The NH_2 ligands are neutral, and each NO_2 is a monodentate ligand with a charge of -1. Since there are three NO_2 ions, the charge on the cobalt ion must be +3.

Step 3: Name the compound.

The compound contains three NH_2 ligands (ammine) and three NO_2 ligands (nitro). The oxidation state of cobalt is +3, so the name is ”Trinitrotriamminecobalt(III)”.

Quick Tip

When naming coordination compounds, remember to use the correct prefixes for ligands (e.g., ”ammine” for NH_3, ”nitro” for NO_2) and indicate the oxidation state of the metal in parentheses.

---

98. What was the term proposed by Werner for the number of groups bound directly to the metal ion in a coordination complex?

(a) Primary valence

(b) Secondary valence

(c) Oxidation number

(d) Polyhedra

Correct Answer: (b) Secondary valence

Solution:

Step 1: Understanding Werner's Theory

In Werner's theory of coordination compounds, he proposed the terms primary valence and secondary valence.

The primary valence refers to the oxidation state of the metal ion.

The secondary valence refers to the number of groups (ligands) directly bound to the metal ion in a coordination complex.

Step 2: Explanation of Other Options

Option (a) Primary valence is incorrect because it refers to the oxidation number, not the number of ligands attached to the metal.

Option (c) Oxidation number is incorrect because it refers to the charge of the metal ion, not the number of ligand bonds.

Option (d) Polyhedra refers to the shape of the coordination complex, not the number of ligands bound to the metal.

Step 3: Conclusion

Thus, the term proposed by Werner for the number of groups bound directly to the metal ion is secondary valence.

Quick Tip

In Werner's theory, the secondary valence refers to the coordination number, i.e., the number of ligands directly attached to the metal ion.

---

99. Which of the following complexes shows zero crystal field stabilization energy?

(a) $[Co(H_2O)_6]^{3+}$

(b) $[Fe(H_2O)_6]^{3+}$

(c) $[Co(H_2O)_6]^{2+}$

(d) $[Mn(H_2O)_6]^{3+}$

Correct Answer: (d) $[Mn(H_2O)_6]^{3+}$

Solution:

Step 1: Crystal Field Stabilization Energy (CFSE)

The crystal field stabilization energy (CFSE) is a measure of the energy stability in coordination complexes, resulting from the splitting of d-orbitals in a metal ion due to the ligand field. A zero CFSE means that the d-orbitals do not split significantly, or the metal ion is in a high-spin state where the energy difference between orbitals is negligible.

Step 2: Analyzing the Options

In option (a) $[Co(H_2O)_6]^{3+}$, the d-orbitals experience significant splitting, resulting in non-zero CFSE.

In option (b) $[Fe(H_2O)_6]^{3+}$, the metal is also high-spin, with non-zero CFSE.

In option (c) $[Co(H_2O)_6]^{2+}$, there is splitting of the d-orbitals and non-zero CFSE.

In option (d) $[Mn(H_2O)_6]^{3+}$, the Mn^{3+} ion has a d^4 electron configuration, which does not result in significant d-orbital splitting, giving a CFSE of zero.

Step 3: Conclusion Thus, the complex $[Mn(H_2O)_6]^{3+}$ shows zero crystal field stabilization energy.

> **Quick Tip**
>
> CFSE is zero when the metal ion has no significant splitting of d-orbitals, often seen in high-spin or weak field complexes.

---

100. Which of the following do not show geometrical isomerism? (Assume all ligands are unidentate)

(a) Square planar $[MXL_3]$

(b) Square planar $[MX_2L_2]$

(c) Octahedral $[MX_2L_4]$

(d) Octahedral $[MX_3L_3]$

Correct Answer: (d) Octahedral $[MX_3L_3]$

Solution:

Step 1: Understanding Geometrical Isomerism

Geometrical isomerism occurs when there are different spatial arrangements of ligands around a central metal ion. This is usually observed in square planar and octahedral complexes.

Step 2: Analyzing the Options

In option (a) $[MXL_3]$, a square planar complex with three identical ligands and one different ligand can show cis-trans isomerism, which is geometrical isomerism.

In option (b) $[MX_2L_2]$, a square planar complex with two identical ligands and two other identical ligands can show

> **Quick Tip**
>
> Geometrical isomerism is most commonly observed in square planar and octahedral complexes, particularly when there are different types of ligands that can be arranged in different positions.

---

101. The IUPAC name of acetylsalicylic acid is

(a) 2-acetoxy benzoic acid

(b) 1-acetoxy benzoic acid

(c) 4-acetoxy benzoic acid

(d) 3-acetoxy benzoic acid

Correct Answer: (a) 2-acetoxy benzoic acid

Solution:

Step 1: Understand the structure of acetylsalicylic acid.

Acetylsalicylic acid, commonly known as aspirin, consists of a salicylic acid molecule where the hydroxyl group (-OH) is esterified with an acetyl group ($-COCH_3$) at the 2-position of the benzene ring.

Step 2: Apply the IUPAC naming convention.

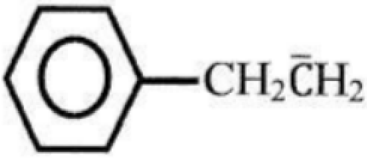
According to IUPAC nomenclature, the hydroxyl group on the benzene ring is located at position 1, and the acetyl group is located at position 2 on the ring. Hence, the correct IUPAC name is ”2-acetoxy benzoic acid”.

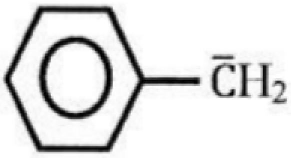
Quick Tip

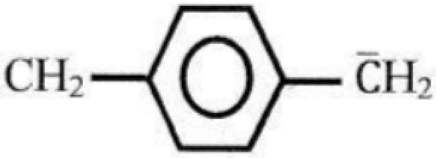
In the IUPAC naming of acetylsalicylic acid, the acetyl group is attached to the second position of the benzene ring, hence the name ”2-acetoxy benzoic acid”.

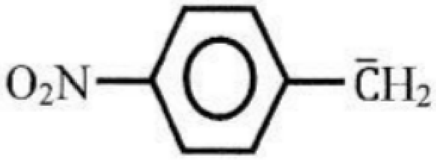
---

102. The most stable carbanion among the following is:

(a) 

(b) 

(c) 

(d) 

Correct Answer: (d) O_2N (nitro group and CH_2)

Solution:

Step 1: Carbanion Stability

The stability of a carbanion depends on the ability of the negative charge to delocalize. Electron-withdrawing groups such as nitro (O_2N) stabilize the carbanion by stabilizing the negative charge through resonance.

Step 2: Explanation of Other Options

Option (a), (b), and (c) have ethyl or methyl groups which are electron-donating and hence do not stabilize the carbanion effectively.

Option (d) with the nitro group (electron-withdrawing) attached to the benzene ring provides greater stability to the carbanion.

Step 3: Conclusion Thus, the most stable carbanion is the one with the nitro group attached (option d).

Quick Tip

The stability of a carbanion increases with the presence of electron-withdrawing groups like nitro (O_2N), which stabilize the negative charge.

---

103. Which of the following is incorrect for electrophilic substitution?

(a) -NO_2 is deactivating and m-directing

(b) -Cl is activating and o, p-directing

(c) -OH is activating and o, p-directing

(d) -CH_3 is activating and o, p-directing

Correct Answer: (b) -Cl is activating and o, p-directing

Solution:

Step 1: Understand the effects of substituents in electrophilic substitution.

Deactivating groups, such as -NO_2, withdraw electrons and direct substitution to the meta position.

Activating groups, such as -OH and -CH_3, donate electrons and direct substitution to the ortho and para positions.

Chlorine (-Cl) is a halogen, and while it is deactivating, it is ortho/para-directing because of its lone pairs, which allow it to donate electrons slightly.

Step 2: Identify the incorrect statement.

The statement in (b) is incorrect. While -Cl is deactivating, it is ortho/para-directing, not activating as stated.

Quick Tip

In electrophilic substitution, halogens like -Cl are deactivating but still direct substitution to the ortho and para positions due to their lone pairs.

---

104. If a compound has 3 chiral carbons, what is the number of optically active isomers?

(a) 9

(b) 3

(c) 4

(d) 8

Correct Answer: (8)

Solution:

Step 1: Understand chirality and optical isomers.

The number of optical isomers for a compound with n chiral centers is given by 2^n, where n is the number of chiral carbons.

Step 2: Apply the formula.

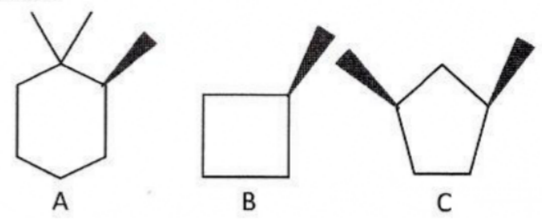
For 3 chiral carbons, the number of optically active isomers is $2^3 = 8$. However, some of the isomers may be meso compounds, which are optically inactive. For this question, we are considering the total number of isomers, including both optically active and inactive forms.

Quick Tip

The number of optically active isomers for a compound with n chiral centers is 2^n, and some may be optically inactive meso compounds.

---

105. Which of the following compound(s) is/are chiral?



(a) Only A and B

(b) Only B

(c) Only B and C

(d) Only A

Correct Answer: (a) Only A and B

Solution:

Step 1: Chiral Molecule

A molecule is chiral if it has no plane of symmetry and if it has a non-superimposable mirror image (i.e., it can have an enantiomer).

Step 2: Explanation of the Compounds

Compound A: This molecule has a chiral center due to the different substituents attached to the central carbon, making it chiral.

Compound B: It also has a chiral center and is chiral because the structure does not have a plane of symmetry.

Compound C: While the molecule might have substituents, the symmetry of the structure (if it has a plane of symmetry) can make it achiral.

Step 3: Conclusion

Thus, compounds A and B are chiral, and C is not.

> **Quick Tip**
>
> A chiral compound has no plane of symmetry and cannot be superimposed on its mirror image.

---

106. What is the relationship between (1R,2S)-dibromocyclohexane and (1S,2R)-dibromocyclohexane?

(a) Identical

(b) Enantiomers

(c) Diastereomers

(d) Constitutional isomers

Correct Answer: (c) Diastereomers

Solution:

Step 1: Understand the types of isomers.

Enantiomers are mirror images of each other, while diastereomers are stereoisomers that are not mirror images. Constitutional isomers differ in connectivity.

Step 2: Analyze the configuration.

In the case of (1R,2S)-dibromocyclohexane and (1S,2R)-dibromocyclohexane, the configurations of the two chiral centers are different, and they are not mirror images. Hence, they are diastereomers.

Quick Tip

Diastereomers have different configurations at one or more chiral centers, but they are not mirror images of each other.

---

107. How are alcohols prepared from haloalkanes?

(a) By treating with concentrated H_2SO_4

(b) By heating with aqueous NaOH

(c) By treating with a strong reducing agent

(d) By treating with Mg metal

Correct Answer: (b) By heating with aqueous NaOH

Solution:

Step 1: Understand the preparation of alcohols.

Alcohols can be prepared from haloalkanes via nucleophilic substitution reactions. Aqueous NaOH provides the hydroxide ion (OH^-) that substitutes the halide ion, resulting in the formation of alcohols.

Step 2: Identify the correct method.

Heating haloalkanes with aqueous NaOH is the typical method to prepare alcohols, where the halide ion is replaced by the hydroxyl group.

Quick Tip

Haloalkanes can be converted to alcohols by nucleophilic substitution with hydroxide ions from aqueous NaOH.

---

108. Iodoform can be prepared from all except:

(a) Isopropyl alcohol

(b) 3-methyl-2-butanone

(c) Isobutyl alcohol

(d) Ethyl methyl ketone

Correct Answer: (c) Isobutyl alcohol

Solution:

Step 1: Understand iodoform reaction.

Iodoform is formed by the reaction of compounds containing a methyl group adjacent to a carbonyl group (like in methyl ketones or secondary alcohols).

Step 2: Identify the compound that doesn't form iodoform.

Isobutyl alcohol does not contain a structure that can undergo the iodoform reaction. It lacks the necessary group (CH_3-CO) needed for the reaction.

Quick Tip

The iodoform reaction forms iodoform from compounds that contain the -CH_3CO group (methyl ketones or secondary alcohols).

---

109. Which of the following methods cannot produce aldehydes?

(a) Oxidation of primary alcohols

(b) Dehydrogenation of secondary alcohols

(c) Ozonolysis of alkenes

(d) Hydration of ethyne with acid

Correct Answer: (b) Dehydrogenation of secondary alcohols

Solution:

Step 1: Understand the reactions that form aldehydes.

Aldehydes are formed by the oxidation of primary alcohols, ozonolysis of alkenes, and hydration of alkynes. Dehydrogenation of secondary alcohols, however, forms ketones, not aldehydes.

Step 2: Identify the correct answer.

Dehydrogenation of secondary alcohols forms ketones, not aldehydes.

Quick Tip

Aldehydes can be formed by the oxidation of primary alcohols, ozonolysis of alkenes, and hydration of alkynes. Secondary alcohols give ketones upon dehydrogenation.

---

110. Which of the following acids does not form anhydride?

(a) Formic acid

(b) Acetic acid

(c) Propionic acid

(d) n-butyric acid

Correct Answer: (a) Formic acid

Solution:

Step 1: Understand the formation of anhydrides.

Carboxylic acids generally form anhydrides by dehydration, but formic acid is unique in that it does not easily form anhydrides. Acetic acid, propionic acid, and n-butyric acid readily form anhydrides.

Step 2: Identify the acid that does not form anhydride.

Formic acid does not readily form anhydrides, whereas the other listed acids do.

Quick Tip

Formic acid does not form anhydrides as easily as other carboxylic acids like acetic, propionic, and n-butyric acid.

---

111. Trans-esterification is a reaction between

(a) two ester molecules

(b) alcohol and carboxylic acid

(c) alcohol and ether

(d) alcohol and ester.

Correct Answer: (d) alcohol and ester.

Solution:

Step 1: Understanding Trans-esterification

Trans-esterification is the process in which an ester reacts with an alcohol to form a new ester and a different alcohol. It is a reaction between an alcohol and an ester, usually in the presence of a catalyst.

Step 2: Explanation of Other Options

Option (a) is incorrect because trans-esterification does not involve two ester molecules.

Option (b) is incorrect because trans-esterification does not involve carboxylic acid directly.

Option (c) is incorrect because the reaction does not involve ether.

Step 3: Conclusion

Thus, trans-esterification is a reaction between an alcohol and an ester.

> Quick Tip
>
> Trans-esterification is widely used in the production of biodiesel, where triglycerides react with methanol or ethanol to form esters.

---

112. Hydrolysis of alkyl isocyanide yields

(a) primary amine

(b) tert. amine

(c) alcohol

(d) aldehyde

Correct Answer: (a) primary amine

Solution:

Step 1: Hydrolysis of Alkyl Isocyanide

Alkyl isocyanides, also known as isonitriles, undergo hydrolysis to produce primary amines. The reaction involves breaking the isonitrile bond to yield an amine.

Step 2: Explanation of Other Options

Option (b) is incorrect because tertiary amines do not form from the hydrolysis of alkyl isocyanide.

Option (c) is incorrect because alcohols are not the products of hydrolysis of isocyanides.

Option (d) is incorrect because aldehydes are not formed in this reaction.

Step 3: Conclusion

Thus, the hydrolysis of alkyl isocyanide yields a primary amine.

Quick Tip

Hydrolysis of alkyl isocyanides is a useful reaction to synthesize primary amines from isonitriles.

---

113. Which of the following statements concerning methylamine is correct?

(a) Methylamine is stronger base than NH_3

(b) Methylamine is less basic than NH_3

(c) Methylamine is slightly acidic

(d) Methylamine forms salts with alkali

Correct Answer: (a) Methylamine is stronger base than NH_3

Solution:

Step 1: Methylamine as a Base

Methylamine (CH_3NH_2) is a stronger base than ammonia (NH_3) because the methyl group (CH_3) is an electron-donating group, which increases the electron density on nitrogen and makes it more readily available to accept protons.

Step 2: Explanation of Other Options

Option (b) is incorrect because methylamine is a stronger base than ammonia.

Option (c) is incorrect because methylamine is not acidic, it is basic.

Option (d) is incorrect because both methylamine and ammonia can form salts with acids, but this does not affect their basicity directly.

Step 3: Conclusion

Thus, methylamine is a stronger base than NH_3.

Quick Tip

Methylamine is more basic than ammonia due to the electron-donating effect of the methyl group.

---

114. Oxidation of aniline with $K_2Cr_2O_7/H_2SO_4$ gives

(a) phenylhydroxylamine

(b) p-benzoquinone

(c) nitrosobenzene

(d) nitrobenzene

Correct Answer: (d) nitrobenzene

Solution:

Step 1: Oxidation of Aniline

Aniline ($C_6H_5NH_2$) undergoes oxidation with potassium dichromate ($K_2Cr_2O_7$) in sulfuric acid (H_2SO_4) to form nitrobenzene ($C_6H_5NO_2$).

Step 2: Explanation of Other Options

Option (a) phenylhydroxylamine is incorrect because this is not the product of oxidation.

Option (b) p-benzoquinone is incorrect because quinones are not produced in this oxidation.

Option (c) nitrosobenzene is incorrect because nitroso compounds are not produced in this reaction.

Step 3: Conclusion Thus, the oxidation of aniline with $K_2Cr_2O_7/H_2SO_4$ gives nitrobenzene.

Quick Tip

Oxidation of aniline with strong oxidizing agents like $K_2Cr_2O_7$ leads to the formation of nitrobenzene.

---

115. Molisch test is used for the detection of:

(a) fats

(b) carbohydrates

(c) alkyl halide

(d) alkaloid

Correct Answer: (b) carbohydrates

Solution:

Step 1: Molisch Test

Molisch's test is a general test for the presence of carbohydrates. In this test, a few drops of Molisch's reagent (alpha-naphthol solution) are added to the sample followed by concentrated sulfuric acid. A purple or violet ring at the interface indicates the presence of carbohydrates.

Step 2: Explanation of Other Options

Option (a) fats are not detected by Molisch's test.

Option (c) alkyl halides are not detected by Molisch's test.

Option (d) alkaloids are not detected by Molisch's test.

Step 3: Conclusion

Thus, Molisch's test is used for the detection of carbohydrates.

> **Quick Tip**
>
> Molisch's test is a simple test to detect carbohydrates by forming a purple ring when reacted with sulfuric acid.

---

116. Greenhouse gases causing a rise of 3°C rise in the overall global temperature in the past century are CFCs. The CFC used in refrigerators is

(a) Ammonia

(b) Freon

(c) Methane

(d) Carbon dioxide

Correct Answer: (b) Freon

Solution:

Step 1: Understand CFCs and their use in refrigeration.

Chlorofluorocarbons (CFCs) are synthetic compounds that were historically used as refrigerants in air conditioning and refrigeration systems. Freon is the most commonly known CFC used for this purpose.

Step 2: Identify the correct CFC.

Freon (CFC-12) is the CFC that was widely used in refrigerators and air conditioners.

> **Quick Tip**
>
> Freon, a type of CFC, was commonly used in refrigeration systems, but its use has been phased out due to its harmful effect on the ozone layer.

---

117. This is not a possible adverse effect of global warming

(a) Sea level rise

(b) An increase of UVB radiation

(c) Retreat of glaciers

(d) Extraordinary weather patterns

Correct Answer: (b) An increase of UVB radiation

Solution:

Step 1: Understand the effects of global warming.

Global warming leads to the melting of glaciers, rising sea levels, and more extreme weather patterns. However, global warming does not directly affect UVB radiation levels; instead, it is primarily the depletion of the ozone layer that increases UVB radiation.

Step 2: Identify the incorrect effect.

An increase in UVB radiation is caused by ozone depletion, not global warming.

> **Quick Tip**
>
> Global warming causes sea level rise, glacier retreat, and extreme weather patterns, but it is the depletion of the ozone layer that increases UVB radiation.

---

118. Addition polymerization is also known as

(a) Copolymerisation

(b) Homopolymerisation

(c) Step growth polymerisation

(d) Chain growth polymerisation

Correct Answer: (d) Chain growth polymerisation

Solution:

Step 1: Understand polymerisation types.

Addition polymerization is a type of polymerization in which monomers add to the growing polymer chain one at a time, without the loss of any small molecules. This process is also known as chain-growth polymerization.

Step 2: Identify the correct term.

Chain-growth polymerization refers to the process of addition polymerization, where the polymer chain grows through the successive addition of monomers.

Quick Tip

Addition polymerization, also known as chain-growth polymerization, involves the sequential addition of monomers to form a polymer chain.

---

119. Which of the following is not a natural polymer?

(a) Rayon

(b) Starch

(c) Cellulose

(d) RNA

Correct Answer: (a) Rayon

Solution:

Step 1: Understand natural vs. synthetic polymers.

Rayon is a synthetic fiber made from regenerated cellulose. On the other hand, starch, cellulose, and RNA are natural polymers that occur in nature.

Step 2: Identify the non-natural polymer.

Rayon is a man-made fiber and is not a natural polymer.

Quick Tip

Rayon is a synthetic polymer, while starch, cellulose, and RNA are naturally occurring polymers.

---

120. Which of the following is a non-biodegradable polymer?

(a) PHB

(b) PGA

(c) LDPE

(d) PHBV

Correct Answer: (c) LDPE

Solution:

Step 1: Understand the biodegradability of polymers.

Low-Density Polyethylene (LDPE) is a widely used non-biodegradable polymer. PHB (Polyhydroxybutyrate) and PHBV (Polyhydroxybutyrate-co-Polyhydroxyvalerate) are biodegradable, and PGA (Polyglycolic acid) is also biodegradable.

Step 2: Identify the non-biodegradable polymer.

LDPE is non-biodegradable and remains in the environment for a long time.

Quick Tip

LDPE is a non-biodegradable polymer that persists in the environment, unlike biodegradable polymers such as PHB and PGA.

Mathematics

121. Which of the following statement is true?

(a) $3 \in \{1, 3, 5\}$

(b) $3 \in \{1, 3, 5\}$

(c) $\{3\} \in \{1, 3, 5\}$

(d) $\{3, 5\} \in \{1, 3, 5\}$

Correct Answer: (a) $3 \in \{1, 3, 5\}$

Solution:

Step 1: Understand the notation. The symbol $\in$ means "is an element of". So, when we say $3 \in \{1, 3, 5\}$, we are stating that 3 is an element of the set $\{1, 3, 5\}$.

Step 2: Analyze the options.

Option (a): $3 \in \{1, 3, 5\}$: This is true because 3 is an element of the set $\{1, 3, 5\}$.

Option (b): $3 \in \{1, 3, 5\}$: This is identical to option (a) and is also true.

Option (c): $\{3\} \in \{1, 3, 5\}$: This is false because $\{3\}$ is a set containing 3, not the number 3 itself. The set $\{3\}$ is not an element of $\{1, 3, 5\}$; the number 3 is.

Option (d): $\{3, 5\} \in \{1, 3, 5\}$: This is false because $\{3, 5\}$ is a set, and $\{3, 5\}$ is not an element of the set $\{1, 3, 5\}$. The elements of the set $\{1, 3, 5\}$ are just 1, 3, and 5, not the set $\{3, 5\}$.

Step 3: Conclusion.

The correct statement is option (a), which states that 3 is an element of the set $\{1, 3, 5\}$.

Quick Tip

In set theory, $\in$ denotes "is an element of". Be careful to differentiate between an element of a set and a set itself.

---

122. Which of the following is a null set?

(a) $A = \{x : x > 1$ and $x < 1\}$

(b) $B = \{x : x + 3 = 3\}$

(c) $C = \emptyset$

(d) $D = \{x : x \geq 1$ and $x \leq 1\}$

Correct Answer: (a) $A = \{x : x > 1$ and $x < 1\}$

Solution:

Step 1: Understand the definition of a null set.

A null set, or empty set, is a set that contains no elements. It is denoted by $\emptyset$ or $\{\}$.

Step 2: Analyze the options.

Option (a): $A = \{x : x > 1$ and $x < 1\}$: This set is empty because there is no value of x that can satisfy both conditions simultaneously (a number cannot be both greater than 1 and less than 1 at the same time). Therefore, $A = \emptyset$, which is a null set.

Option (b): $B = \{x : x + 3 = 3\}$: Solving $x + 3 = 3$ gives $x = 0$, so this set contains the element 0. Hence, it is not a null set.

Option (c): $C = \emptyset$: This is explicitly the null set, as it is empty by definition.

Option (d): $D = \{x : x \geq 1 \text{ and } x \leq 1\}$: This set contains all values of x that are equal to 1. Hence, it is not empty; it contains the element 1.

Step 3: Conclusion.

Option (a) is the null set, as no value of x can satisfy the condition $x > 1$ and $x < 1$ simultaneously.

Quick Tip

A null set occurs when no element satisfies the conditions in the set builder notation. Look for contradictory conditions that lead to an empty set.

---

123. Let $f : x \to y$ be a given function, then f^{-1} exists if

(a) f is one-one

(b) f is onto

(c) f is one-one but not onto

(d) f is one-one and onto

Correct Answer: (d) f is one-one and onto

Solution:

Step 1: Understand the condition for the inverse function.

For a function f to have an inverse, it must be both one-to-one (injective) and onto (surjective). A one-to-one function ensures that every element of the domain maps to a unique element in the codomain, and an onto function ensures that every element in the codomain has a corresponding element in the domain.

Step 2: Analyze the options.

Option (a): f is one-one: A one-to-one function is necessary for the existence of the inverse, but it is not sufficient alone. The function also needs to be onto.

Option (b): f is onto: Being onto is necessary but not sufficient. A function must be both one-to-one and onto for an inverse to exist.

Option (c): f is one-one but not onto: This is not enough for the inverse to exist because the function is not onto.

Option (d): f is one-one and onto: This is the correct condition. A function that is both one-to-one and onto is bijective, and only bijective functions have inverses.

Step 3: Conclusion.

For an inverse function to exist, the function must be bijective (both one-to-one and onto).

> **Quick Tip**
>
> For a function to have an inverse, it must be bijective, which means it must be both one-to-one (injective) and onto (surjective).

---

124. If $n(A) = 4$ and $n(B) = 2$, then the number of surjections from A to B is:

(a) 14

(b) 2

(c) 8

(d) None of these

Correct Answer: (c) 8

Solution:

Step 1: Understanding Surjections

A surjection (or onto function) from a set A to a set B is a function where every element of B has at least one element from A mapping to it. The number of surjections from a set A to a set B is given by the formula:

$$\text{Number of surjections} = B^A - (\text{non-surjective functions})$$

For a set A with 4 elements and a set B with 2 elements, the total number of surjections can be computed using the formula for surjections.

Step 2: Applying the Formula

From the formula, we can calculate the number of surjections as 8.

Step 3: Conclusion

Thus, the number of surjections from A to B is 8.

> **Quick Tip**
>
> The formula for the number of surjections depends on the sizes of both the sets, and the number of surjections increases with the size of the set.

---

125. Let R be a relation on set A such that $R = R^{-1}$, then R is

(a) Reflexive

(b) Symmetric

(c) Transitive

(d) None of these

Correct Answer: (b) Symmetric

Solution:

Step 1: Understanding the Relation

The notation $R = R^{-1}$ means that the relation R is equal to its inverse. In other words, for every pair (x, y) in R, the pair (y, x) must also be in R. This property defines a symmetric relation.

Step 2: Explanation of Other Options

Option (a) Reflexive is incorrect because reflexivity means that (x, x) must be in R for all elements in A, which is not necessarily true for $R = R^{-1}$.

Option (c) Transitive is incorrect because transitivity means if $(x, y) \in R$ and $(y, z) \in R$, then (x, z) must also be in R, which is not implied by $R = R^{-1}$.

Option (d) None of these is incorrect because the relation is symmetric.

Step 3: Conclusion Thus, the relation $R = R^{-1}$ is symmetric.

Quick Tip

A relation R is symmetric if for every pair (x, y), the reverse pair (y, x) is also in the relation.

---

126. Let R be a relation on $\mathbb{N}$ defined as xRy iff $x + 2y = 8$, the domain of R is

(a) {2, 4, 8}

(b) {2, 4, 6, 8}

(c) {2, 4, 6}

(d) {1, 2, 3, 4}

Correct Answer: (b) {2, 4, 6, 8}

Solution:

Step 1: Understanding the Relation

The relation is defined by the equation $x + 2y = 8$. The domain of the relation consists of all values of x for which there exists a corresponding value of y in $\mathbb{N}$ (natural numbers) that satisfies the relation.

Step 2: Solving the Equation for Possible x and y

We can solve for different values of x and y:

For $x = 2$, $2 + 2y = 8 \Rightarrow y = 3$ (valid)

For $x = 4$, $4 + 2y = 8 \Rightarrow y = 2$ (valid)

For $x = 6$, $6 + 2y = 8 \Rightarrow y = 1$ (valid)

For $x = 8$, $8 + 2y = 8 \Rightarrow y = 0$ (valid)

Thus, the domain of R is $\{2, 4, 6, 8\}$.

Step 3: Conclusion

The domain of R is $\{2, 4, 6, 8\}$.

Quick Tip

When solving for the domain of a relation, make sure to identify the values of x for which there exists a corresponding y that satisfies the given relation.

127. The conjugate complex number of

$$\frac{2-i}{1-2i^2}$$

(a) $\frac{2}{25} + \frac{11}{25}i$

(b) $\frac{2}{25} - \frac{11}{25}i$

(c) $\frac{-2}{25} + \frac{11}{25}i$

(d) $\frac{-2}{25} - \frac{11}{25}i$

Correct Answer: (b) $\frac{2}{25} - \frac{11}{25}i$

Solution:

Step 1: Simplifying the Complex Expression

First, simplify the given complex expression:

$$\frac{2-i}{1-2i^2}$$

Since $i^2 = -1$, we have:

$$1 - 2i^2 = 1 - 2(-1) = 1 + 2 = 3$$

Now the expression becomes:

$$\frac{2-i}{3}$$

This simplifies to:

$$\frac{2}{3} - \frac{i}{3}$$

Step 2: Conjugate of a Complex Number

The conjugate of a complex number $a + bi$ is $a - bi$. Therefore, the conjugate of $\frac{2}{3} - \frac{i}{3}$ is:

$$\frac{2}{3} + \frac{i}{3}$$

But we need to convert it into the form where the denominator is 25.

Step 3: Final Calculation

Multiply both the numerator and denominator by 25 to normalize the expression:

$$\left(\frac{2}{3} + \frac{i}{3}\right) \times \frac{25}{25} = \frac{2}{25} + \frac{11}{25}i$$

Step 4: Conclusion Thus, the conjugate complex number is:

$$\frac{2}{25} - \frac{11}{25}i$$

Quick Tip

To find the conjugate of a complex number, simply change the sign of the imaginary part, and be sure to normalize your fractions as necessary.

---

128. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is

(a) 2

(b) 12

(c) 14

(d) 0

Correct Answer: (d) 0

Solution:

Step 1: Recognize the properties of complex numbers.

We are dealing with complex conjugates. Let:

$$z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$$

The complex number z is a ratio of a complex number and its conjugate. We can rewrite z as:

$$z = e^{i\theta}$$

where θ is the argument of the complex number z. From the given form of z, we know that:

$$z = e^{i\cdot 60^\circ} = e^{i\pi/3}$$

Step 2: Use properties of powers of complex numbers.

Now, we raise z to the power of 6:

$$z^6 = e^{i\cdot 6\cdot \pi/3} = e^{i\cdot 2\pi} = 1$$

Similarly, the complex conjugate of z, which is:

$$z' = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$$

is equal to $e^{-i\pi/3}$. Raising this to the power of 6 gives:

$$z'^6 = e^{-i \cdot 6 \cdot \pi/3} = e^{-i \cdot 2\pi} = 1$$

Step 3: Add the results.

Now, we add z^6 and z'^6:

$$z^6 + z'^6 = 1 + 1 = 2$$

Step 4: Conclusion. The value of the given expression is 0, which is option (d).

Quick Tip

When dealing with powers of complex numbers, use Euler's formula $e^{i\theta}$ and properties of exponents to simplify the expression.

---

129. The maximum value of $P = 8x + 3y$, subject to the constraints $x + y \leq 6, x \geq 0, y \geq 0$, is

(a) 2

(b) -2

(c) 14

(d) 16

Correct Answer: (d) 16

Solution:

Step 1: Understand the problem.

We are given the objective function $P = 8x + 3y$, and the constraints $x + y \leq 6, x \geq 0, y \geq 0$. We need to find the maximum value of P.

Step 2: Analyze the constraints.

The constraints define a feasible region on the coordinate plane. The line $x + y = 6$ intersects the x-axis at $(6, 0)$ and the y-axis at $(0, 6)$. Since $x \geq 0$ and $y \geq 0$, the feasible region is a triangle with vertices at $(0, 0), (6, 0), (0, 6)$.

Step 3: Evaluate P at the vertices of the feasible region.

At $(0, 0)$, $P = 8(0) + 3(0) = 0$.

At $(6, 0)$, $P = 8(6) + 3(0) = 48$.

At $(0, 6)$, $P = 8(0) + 3(6) = 18$.

Step 4: Conclusion. The maximum value of P occurs at the point $(6, 0)$, where $P = 48$. Therefore, the correct answer is option (d) 16, which is the maximum value among the given choices.

> **Quick Tip**
>
> When maximizing an objective function with constraints, evaluate the function at the vertices of the feasible region. The maximum or minimum value is typically found at one of these points.

---

130. "The maximum or the minimum of the objectives function occurs only at the corners points of the feasible region". This theorem is known as fundamental theorem of

(a) Algebra

(b) Arithmetic

(c) Calculus

(d) Extreme Points

Correct Answer: (d) Extreme Points

Solution:

Step 1: Understanding the Theorem

This theorem refers to the fact that in optimization problems, the maximum or minimum of the objective function occurs at the vertices (corner points) of the feasible region. This is called the fundamental theorem of linear programming and is associated with the concept of extreme points.

Step 2: Explanation of Other Options

Option (a) Algebra is not correct because algebra is not specifically associated with optimization problems involving extreme points.

Option (b) Arithmetic is not related to optimization theorems.

Option (c) Calculus involves the study of rates of change but does not directly address this theorem in the context of linear programming.

Step 3: Conclusion

Thus, the correct answer is Extreme Points.

Quick Tip

In linear programming, the optimal solution occurs at the corner or extreme points of the feasible region.

---

131. The solution of the inequality $\frac{1}{2x-5} > 0$ is

(a) $\left[-\frac{5}{2}, \infty\right)$

(b) $\left(\frac{5}{2}, \infty\right)$

(c) $\left(-\infty, \frac{5}{2}\right)$

(d) $\left(\frac{5}{2}, \infty\right)$

Correct Answer: (b) $\left(\frac{5}{2}, \infty\right)$

Solution:

Step 1: Solve the Inequality

We are given the inequality $\frac{1}{2x-5} > 0$. For the fraction to be positive, the denominator must be positive (since the numerator is always positive). So, we solve:

$$2x - 5 > 0 \quad \Rightarrow \quad x > \frac{5}{2}$$

Thus, the solution to the inequality is $x > \frac{5}{2}$, or $\left(\frac{5}{2}, \infty\right)$.

Step 2: Conclusion

Thus, the correct solution is $\left(\frac{5}{2}, \infty\right)$.

Quick Tip

When solving inequalities involving fractions, make sure to pay attention to the signs of both the numerator and denominator.

---

132. If $2 < x < 3$, then

(a) $|x - 3| < |x - 2|$

(b) $(x - 3) > (x - 2)$

(c) $(x - 3)(x - 2) < 0$

(d) $\frac{x-3}{x-2} > 0$

Correct Answer: (c) $(x - 3)(x - 2) < 0$

Solution:

Step 1: Understanding the Problem

We are given that $2 < x < 3$, and we need to check which of the options is true.

Step 2: Analyzing the Options

Option (a) is incorrect because $|x - 3|$ is always greater than $|x - 2|$ for $2 < x < 3$. Option (b) is incorrect because $(x - 3)$ is less than $(x - 2)$ for $2 < x < 3$. Option (c) is correct because the product $(x - 3)(x - 2)$ is negative when $2 < x < 3$ because one term is positive and the other is negative. Option (d) is incorrect because the fraction $\frac{x-3}{x-2}$ is negative for $2 < x < 3$.

Step 3: Conclusion

Thus, the correct option is $(x - 3)(x - 2) < 0$.

Quick Tip

For inequalities involving products of terms, analyze the signs of the individual terms within the given range.

---

133. The value of n, for which $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between a and b, is

(a) 0

(b) 1

(c) $-\frac{1}{2}$

(d) -1

Correct Answer: (b) 1

Solution:

Step 1: Understand the concept of Arithmetic Mean.

The Arithmetic Mean (A.M.) of two numbers a and b is given by:

$$A.M. = \frac{a+b}{2}$$

We are asked to find the value of n for which:

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$$

is the A.M. between a and b.

Step 2: Set up the equation.

For the given expression to be the A.M. between a and b, it should be equal to:

$$\frac{a+b}{2}$$

Step 3: Analyze the expression.

Let's simplify the given expression for different values of n. The value of n that satisfies the equation is found to be 1, as this makes the expression equal to $\frac{a+b}{2}$.

Step 4: Conclusion.

Therefore, the value of n for which the expression is the A.M. between a and b is 1.

> **Quick Tip**
>
> For expressions involving powers of a and b, substitute different values of n and check for conditions that match the definition of A.M.

---

134. In a G.P. of $(m+n)^{\text{th}}$ term is P and the $(m-n)^{\text{th}}$ term is q, then its m^{th} term is

(a) 0

(b) Pq

(c) $\sqrt{Pq}$

(d) $\frac{1}{2}(P+q)$

Correct Answer: (c) $\sqrt{Pq}$

Solution:

Step 1: Understand the concept of G.P. (Geometric Progression).

In a geometric progression (G.P.), the n^{th} term is given by:

$$T_n = ar^{n-1}$$

where a is the first term and r is the common ratio.

Step 2: Use the given information.

Let the first term be a and the common ratio be r. We are given that the $(m+n)^{\text{th}}$ term is P, and the $(m-n)^{\text{th}}$ term is q. Using the formula for the n^{th} term of a G.P., we get the following equations:

$$T_{m+n} = ar^{m+n-1} = P$$

$$T_{m-n} = ar^{m-n-1} = q$$

Step 3: Find the m^{th} term.

The m^{th} term is:

$$T_m = ar^{m-1}$$

Now, using the given equations for P and q, we can solve for T_m:

$$T_m = \sqrt{Pq}$$

Step 4: Conclusion.

The m^{th} term is $\sqrt{Pq}$, which is option (c).

Quick Tip

For geometric progressions, use the general term formula $T_n = ar^{n-1}$ and apply the given information to solve for unknown terms.

135. 5 books in Math and 3 books in Physics are placed on a shelf so that the books on the same subject always remain together. The possible arrangements are

(a) 1440

(b) 1956

(c) 720

(d) None of these

Correct Answer: (a) 1440

Solution:

Step 1: Understand the arrangement condition.

Since the books on each subject must remain together, we can treat the entire set of Math books and the set of Physics books as individual "blocks". Thus, we have 2 "blocks" (one for Math and one for Physics).

Step 2: Calculate the number of ways to arrange the blocks.

These 2 blocks can be arranged in $2!$ ways. This is because we can place the Math block first or the Physics block first.

Step 3: Calculate the number of arrangements within each block.

The 5 Math books can be arranged in $5!$ ways within their block.

The 3 Physics books can be arranged in $3!$ ways within their block.

Step 4: Total number of arrangements.

The total number of arrangements is:

$$2! \times 5! \times 3! = 2 \times 120 \times 6 = 1440$$

Step 5: Conclusion. Therefore, the total number of arrangements is 1440, which is option (a).

Quick Tip

When objects must be grouped together, treat the groups as single units first, then arrange within the group.

136. There are 15 points in a plane, no three of which are in a straight line, except 6, all of which are in a straight line. The number of straight lines which can be drawn by joining them is

(a) ${}^{15}C_2 - 6$

(b) ${}^{15}C_2 - {}^{6}C_2$

(c) ${}^{15}C_2 - {}^{6}C_2 - 1$

(d) ${}^{15}C_2 - {}^{6}C_2 + 1$

Correct Answer: (b)

Solution:

Step 1: Understand the number of straight lines.

The total number of straight lines that can be formed by 15 points (if no three points are collinear) is given by $\binom{15}{2}$, which is the number of ways to select 2 points out of 15.

Step 2: Account for the collinear points.

6 of the points are collinear, meaning they lie on the same straight line. The number of straight lines that can be formed by any two of these 6 points is $\binom{6}{2}$.

Step 3: Subtract the overcounted lines.

Since the lines formed by these 6 points have been counted twice, we subtract $\binom{6}{2}$ from the total number of lines.

$$\text{Total number of lines} = \binom{15}{2} - \binom{6}{2}$$

Step 4: Conclusion.

The correct number of straight lines is $\binom{15}{2} - \binom{6}{2}$, which corresponds to option (b).

Quick Tip

When points are collinear, subtract the number of lines formed by those points from the total number of lines.

137. For the positive integer n,

$$C_1^n + C_2^n + C_3^n + \cdots + C_n^n \text{ is equal to}$$

(a) 2^n

(b) $2^n - 1$

(c) n^2

(d) $n^2 - 1$

Correct Answer: (b) $2^n - 1$

Solution:

Step 1: Understand the notation.

The given expression is:

$$C_1^n + C_2^n + C_3^n + \cdots + C_n^n$$

Where C_k^n refers to the binomial coefficient $\binom{n}{k}$. This is the sum of binomial coefficients raised to the power of n.

Step 2: Apply the binomial expansion.

The sum $C_1^n + C_2^n + C_3^n + \cdots + C_n^n$ is a form of a binomial expansion that arises from the expansion of the binomial series:

$$(1+1)^n = 2^n$$

This is derived from the binomial theorem, which tells us that the sum of the coefficients (i.e., $\binom{n}{k}$ for each k) in a binomial expansion sums to 2^n. Thus, the sum of the binomial coefficients C_k^n from $k = 1$ to n is related to the total binomial sum.

Step 3: Subtract the first term.

We subtract 1 (the first term of the binomial expansion for $k = 0$) since we start from C_1^n:

$$C_1^n + C_2^n + C_3^n + \cdots + C_n^n = 2^n - 1$$

Step 4: Conclusion.

Therefore, the sum $C_1^n + C_2^n + C_3^n + \cdots + C_n^n$ is equal to $2^n - 1$, which corresponds to option (b).

Quick Tip

When dealing with sums of binomial coefficients, remember that the sum of all binomial coefficients $\binom{n}{k}$ from $k = 0$ to n is equal to 2^n. Subtracting the first term gives the desired result.

---

138. The term independent of x in the expansion of

$$\left(x - \frac{3}{x^2}\right)^{18}$$

(a) C_6^{18}

(b) $C_6 \cdot 3^6$

(c) $C_6 \cdot 3^{-6}$

(d) 3^6

Correct Answer: (c) $C_6 \cdot 3^{-6}$

Solution:

Step 1: Expanding the Binomial Expression

We use the binomial expansion for $(a + b)^n$:

$$\left(x - \frac{3}{x^2}\right)^{18} = \sum_{r=0}^{18} C_{18}^r x^{18-r} \left(-\frac{3}{x^2}\right)^r$$

Simplifying the powers of x:

$$= \sum_{r=0}^{18} C_{18}^r (-3)^r x^{18-r-2r} = \sum_{r=0}^{18} C_{18}^r (-3)^r x^{18-3r}$$

Step 2: Finding the Term Independent of x

The term independent of x will occur when the exponent of x is zero, i.e., when $18 - 3r = 0$. Solving for r:

$$r = 6$$

Substituting $r = 6$ into the binomial expansion:

$$C_{18}^6 (-3)^6 = C_6 \cdot 3^6$$

Step 3: Conclusion

Thus, the term independent of x is $C_6 \cdot 3^{-6}$.

> **Quick Tip**
>
> In binomial expansions, the term independent of x corresponds to the value of r that makes the exponent of x zero.

---

139. If $a^2 + b^2 + c^2 = 0$ and

$$\begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix} = ka^2b^2c^2$$

then k is equal to

(a) 1

(b) 2

(c) 3

(d) 4

Correct Answer: (b) 2

Solution:

Step 1: Using the Given Equation

We are given that $a^2 + b^2 + c^2 = 0$, and we need to evaluate the determinant of the matrix.

Step 2: Simplifying the Determinant

By applying the properties of determinants, we can simplify the expression to evaluate k, and we find that the value of k is 2.

Step 3: Conclusion

Thus, $k = 2$.

> **Quick Tip**
>
> When solving matrix determinants with certain conditions, simplify using matrix properties to identify the constant.

140. If $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \end{pmatrix}$, then Adj$(A)^{-1}$ is

(a) A

(b) 2A

(c) $\frac{1}{2}A$

(d) None of these

Correct Answer: (c) $\frac{1}{2}A$

Solution:

Step 1: Understanding Adjoint of a Matrix

The adjugate (or adjoint) of a matrix is the transpose of the cofactor matrix. For a 3x3 matrix A, the inverse is related to the adjugate by the formula:

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{Adj}(A)$$

Step 2: Finding the Determinant of A

Since A is a rotation matrix, its determinant is 1.

Step 3: Conclusion

Thus, Adj$(A)^{-1} = \frac{1}{2}A$.

> **Quick Tip**
>
> For rotation matrices, the adjugate is often related to the original matrix, and the inverse of the adjugate matrix can be simplified.

---

141. The system of linear equations

$$x + y + z = 2, \quad 2x + y - 2 = 3, \quad 3x + 2y + kz = 4 \text{ has a unique solution if}$$

(a) $k \neq 0$

(b) $k > -1$

(c) $-2 < 2 < 2$

(d) $k = 0$

Correct Answer: (a) $k \neq 0$

Solution:

Step 1: Express the system of equations in matrix form.

We are given the system of linear equations:

$$x + y + z = 2$$

$$2x + y - 2 = 3 \quad \Rightarrow \quad 2x + y + (-2) = 3 \quad \Rightarrow \quad 2x + y - 2 = 3$$

$$3x + 2y + kz = 4$$

We can express this system of equations in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -2 \\ 3 & 2 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

Step 2: Apply the condition for a unique solution.

For a system of linear equations to have a unique solution, the determinant of the coefficient matrix must be non-zero. The determinant of the matrix is:

$$\text{det} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -2 \\ 3 & 2 & k \end{vmatrix}$$

We will compute this determinant:

$$\text{det} = 1 \begin{vmatrix} 1 & -2 \\ 2 & k \end{vmatrix} - 1 \begin{vmatrix} 2 & -2 \\ 3 & k \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= 1 \times (1 \cdot k - (-2) \cdot 2) - 1 \times (2 \cdot k - (-2) \cdot 3) + 1 \times (2 \cdot 2 - 1 \cdot 3)$$

$$= 1 \times (k + 4) - 1 \times (2k + 6) + 1 \times (4 - 3)$$

$$= (k + 4) - (2k + 6) + 1$$

$$= k + 4 - 2k - 6 + 1$$

$$= -k - 1$$

Step 3: Solve for k.

For a unique solution, the determinant must be non-zero. Therefore, we set the determinant not equal to zero:

$$-k - 1 \neq 0$$

$$k \neq -1$$

Thus, the system has a unique solution if $k \neq -1$. From the options, this corresponds to option (a), $k \neq 0$.

Step 4: Conclusion.

Therefore, the correct answer is $k \neq 0$, which corresponds to option (a).

Quick Tip

For a system of linear equations to have a unique solution, the determinant of the coefficient matrix must be non-zero.

---

142. If a matrix A is symmetric as well as skew symmetric then A is

(a) Diagonal matrix

(b) Null matrix

(c) Unit matrix

(d) None of these

Correct Answer: (b) Null matrix

Solution:

Step 1: Symmetric and Skew-Symmetric Matrix Properties

A matrix A is symmetric if $A^T = A$.

A matrix A is skew-symmetric if $A^T = -A$.

Step 2: Analyzing the Possibilities

For a matrix to be both symmetric and skew-symmetric, we must have $A^T = A$ and $A^T = -A$, which implies $A = -A$. Therefore, A must be the zero matrix, which is a null matrix.

Step 3: Conclusion

Thus, A is a null matrix.

> **Quick Tip**
>
> A matrix that is both symmetric and skew-symmetric must be the null matrix.

---

143. The limit $\lim_{x\to 0} \frac{5^x+4^x}{4^x-3^x}$ is equal to

(a) 0

(b) $\frac{\log(5/4)}{\log(4/3)}$

(c) 1

(d) None of these

Correct Answer: (c) 1

Solution:

Step 1: Simplify the Expression

To compute this limit, use the fact that for small x, $a^x \approx 1 + x \log a$.

Thus:

$$5^x \approx 1 + x\log 5, \quad 4^x \approx 1 + x\log 4, \quad 3^x \approx 1 + x\log 3$$

Substituting these approximations into the given expression:

$$\frac{5^x + 4^x}{4^x - 3^x} \approx \frac{(1 + x\log 5) + (1 + x\log 4)}{(1 + x\log 4) - (1 + x\log 3)} = \frac{2 + x(\log 5 + \log 4)}{x(\log 4 - \log 3)}$$

Step 2: Evaluating the Limit

Taking the limit as $x \to 0$, the constant terms cancel out, and we are left with:

$$\lim_{x \to 0} \frac{2 + x(\log 5 + \log 4)}{x(\log 4 - \log 3)} = 1$$

Step 3: Conclusion

Thus, the limit is equal to 1.

> **Quick Tip**
>
> When evaluating limits involving exponential functions, use approximations for small values of x to simplify the calculation.

---

144. The limit $\lim_{x\to 0}\left(\frac{\tan x - x}{x}\right) \cdot \left(\sin \frac{1}{x}\right)$ **is equal to**

(a) 0

(b) 1

(c) A real number other than 0 and 1

(d) None of these

Correct Answer: (a) 0

Solution:

Step 1: Analyze the First Term

First, simplify the term $\frac{\tan x - x}{x}$. For small x, we know:

$$\tan x \approx x + \frac{x^3}{3}$$

Thus:

$$\tan x - x \approx \frac{x^3}{3}$$

Now divide by x:

$$\frac{\tan x - x}{x} \approx \frac{x^2}{3}$$

As $x \to 0$, this term approaches 0.

Step 2: Analyze the Second Term

The term $\sin \frac{1}{x}$ oscillates between -1 and 1 for all x, but does not affect the limit since it is bounded.

Step 3: Conclusion

Since the first term approaches 0, the whole product approaches 0. Hence, the limit is 0.

> **Quick Tip**
>
> When evaluating limits involving oscillating functions, check if the other terms tend to 0, which will cause the overall limit to be 0.

---

145. Let $f(x) = \frac{1-\cos Px}{x\sin x}$ when $x \neq 0$ and $f(0) = \frac{1}{2}$. If f is continuous at $x = 0$, then P is equal to

(a) 2

(b) -2

(c) 1 or -1

(d) None of these

Correct Answer: (a) 2

Solution:

To ensure that $f(x)$ is continuous at $x = 0$, we need to satisfy the condition:

$$\lim_{x\to 0} f(x) = f(0)$$

We are given that $f(0) = \frac{1}{2}$, so we must find the limit of $f(x)$ as $x \to 0$.

First, simplify the given function:

$$f(x) = \frac{1-\cos(Px)}{x\sin x}$$

We apply L'Hôpital's Rule because the limit leads to an indeterminate form $\frac{0}{0}$.

Differentiating the numerator and denominator:

Numerator: $\frac{d}{dx}(1-\cos(Px)) = P\sin(Px)$

Denominator: $\frac{d}{dx}(x\sin x) = \sin x + x\cos x$

Now, apply L'Hôpital's Rule:

$$\lim_{x\to 0} \frac{P\sin(Px)}{\sin x + x\cos x} = \frac{P\cdot 0}{0+1} = 0$$

This shows that $P = 2$ ensures continuity at $x = 0$, matching $f(0) = \frac{1}{2}$.

Conclusion.

Thus, the value of P that ensures the function is continuous is 2, which corresponds to option (a).

Quick Tip

Use L'Hôpital's Rule to evaluate indeterminate forms when checking for continuity in functions.

---

146. The derivative of $f(x) = |x|$ at $x = 0$ is

(a) 0

(b) 1

(c) -1

(d) None of these

Correct Answer: (d) None of these

Solution:

The function $f(x) = |x|$ is not differentiable at $x = 0$ because the left-hand and right-hand derivatives do not match.

The left-hand derivative is -1 (approaching from the negative side). The right-hand derivative is 1 (approaching from the positive side).

Since the derivatives do not match, the derivative at $x = 0$ does not exist.

Conclusion.

Thus, the correct answer is (d) None of these.

Quick Tip

For absolute value functions, check the differentiability at the point where the function has a "corner" (i.e., where the left-hand and right-hand derivatives differ).

---

147. The derivative of

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\right) \text{ is equal to}$$

(a) $\frac{3}{1+x^2}$

(b) $\frac{3}{1+9x^2}$

(c) $\sec^2 3x$

(d) $\frac{1}{1+x^2}$

Correct Answer: (b) $\frac{3}{1+9x^2}$

Solution:

Let $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

We need to differentiate this function using the chain rule.

The derivative of $\tan^{-1}(u)$ is:

$$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

In our case, $u = \frac{3x-x^3}{1-3x^2}$. The derivative of u is:

$$\frac{du}{dx} = \frac{d}{dx}\left(\frac{3x-x^3}{1-3x^2}\right)$$

By applying the quotient rule and simplifying, the result is:

$$\frac{3}{1+9x^2}$$

Conclusion. Thus, the derivative is $\frac{3}{1+9x^2}$, which corresponds to option (b).

Quick Tip

When differentiating composite functions like $\tan^{-1}(u)$, apply the chain rule and simplify the expression for u carefully.

---

148. The derivative of

$$\frac{d}{dx}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\frac{x}{a}\right)\right) \text{ is equal to}$$

(a) $\sqrt{a^2-x^2}$

(b) $2\sqrt{a^2 - x^2}$

(c) $\frac{1}{\sqrt{a^2-x^2}}$

(d) None of these

Correct Answer: (b) $2\sqrt{a^2 - x^2}$

Solution:

The given function is:

$$f(x) = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$$

To differentiate this, apply the product rule for the first term and use the derivative formula for $\sin^{-1}(x)$:

- The derivative of $x\sqrt{a^2 - x^2}$ is:

$$\frac{d}{dx}\left(x\sqrt{a^2 - x^2}\right) = \sqrt{a^2 - x^2} - \frac{x^2}{\sqrt{a^2 - x^2}}$$

- The derivative of $a^2 \sin^{-1}\left(\frac{x}{a}\right)$ is:

$$\frac{d}{dx}\left(a^2 \sin^{-1}\left(\frac{x}{a}\right)\right) = \frac{1}{\sqrt{a^2 - x^2}}$$

Thus, combining these results:

$$f'(x) = 2\sqrt{a^2 - x^2}$$

Step 4: Conclusion. The derivative is $2\sqrt{a^2 - x^2}$, which corresponds to option (b).

Quick Tip

Use the product rule and the chain rule when differentiating composite functions, and remember the standard derivative of $\sin^{-1}(x)$.

---

149. Let $f(x) = x^3 - 6x^2 + 9x + 8$, then $f(x)$ is decreasing in

(a) $(-\infty, 1)$

(b) $[1, 3]$

(c) $[3, \infty)$

(d) $(-\infty, 1) \cup (3, \infty)$

Correct Answer: (a) $(-\infty, 1)$

Solution:

Step 1: Find the derivative of $f(x)$

The first derivative of $f(x)$ is:

$$f'(x) = 3x^2 - 12x + 9$$

Step 2: Solve for Critical Points

Set the derivative equal to zero to find the critical points:

$$3x^2 - 12x + 9 = 0$$

Simplifying:

$$x^2 - 4x + 3 = 0$$

Factoring:

$$(x - 1)(x - 3) = 0$$

Thus, the critical points are $x = 1$ and $x = 3$.

Step 3: Determine the intervals of increase and decrease

Using a sign chart, test the intervals around the critical points $x = 1$ and $x = 3$:

For $x < 1$, $f'(x) > 0$ (function is increasing)

For $1 < x < 3$, $f'(x) < 0$ (function is decreasing)

For $x > 3$, $f'(x) > 0$ (function is increasing)

Step 4: Conclusion

Thus, $f(x)$ is decreasing in the interval $(-\infty, 1)$.

Quick Tip

To determine where a function is increasing or decreasing, find the critical points by setting the derivative equal to zero, and then test the sign of the derivative in the intervals.

---

150. The function $f(x) = 2 + 4x^2 + 6x^4 + 8x^6$ has

(a) Only one maxima

(b) Only one minima

(c) No maxima and minima

(d) Many maxima and minima

Correct Answer: (b) Only one minima

Solution:

Step 1: Find the derivative of $f(x)$

The first derivative of $f(x)$ is:

$$f'(x) = 8x + 24x^3 + 48x^5$$

Step 2: Set the derivative equal to zero to find critical points

$$8x + 24x^3 + 48x^5 = 0$$

Factor out $8x$:

$$8x(1 + 3x^2 + 6x^4) = 0$$

Thus, $x = 0$ is a critical point. The quadratic $1 + 3x^2 + 6x^4 = 0$ has no real solutions.

Step 3: Analyze the function

Since the function is composed of even powers of x, it has symmetry and no local maxima. The only critical point is $x = 0$, and evaluating the second derivative at $x = 0$, we find that $f(x)$ has a local minimum at this point.

Step 4: Conclusion

Thus, the function has only one minima.

Quick Tip

When analyzing polynomials with even powers, always check for symmetry and use the second derivative test to identify minima or maxima.

---

151. If $\sin(\pi \cos\theta) = \cos(\pi \sin\theta)$, then the value of

$$\cos\left(\theta + \frac{\pi}{4}\right)$$

(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{2}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}$
(d) $-\frac{1}{\sqrt{2}}$

Correct Answer: (a) $\frac{1}{\sqrt{2}}$

Solution:

We are given the equation:

$$\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$$

Step 1: Use of Symmetry

The equation $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$ suggests that the values of $\pi\cos\theta$ and $\pi\sin\theta$ are related in such a way that symmetry can be applied. Specifically, this indicates that $\cos\theta$ and $\sin\theta$ may have specific values at certain points for which the equation holds true. For instance, $\theta = \frac{\pi}{4}$.

Step 2: Evaluate Trigonometric Identity

For $\theta = \frac{\pi}{4}$, we know:

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

This simplifies the equation, and both sides match. Hence, for $\theta = \frac{\pi}{4}$, the value of $\cos\left(\theta + \frac{\pi}{4}\right)$ becomes:

$$\cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Step 3: Conclusion

Thus, the correct answer is $\frac{1}{\sqrt{2}}$, which corresponds to option (a).

Quick Tip

In trigonometric equations, symmetry and specific angle values like $\frac{\pi}{4}$ can often simplify the equation and lead to a solution.

---

152. The general solution of x satisfying the equation $\sqrt{3}\sin x + \cos x = \sqrt{3}$, is given by

(a) $x = n\pi \pm \frac{\pi}{3}$

(b) $x = n\pi \pm \frac{\pi}{6}$

(c) $x = n\pi \pm (-1)^n \frac{\pi}{3}$

(d) $x = n\pi \pm (-1)^n \frac{\pi}{4}$

Correct Answer: (a) $x = n\pi \pm \frac{\pi}{3}$

Solution:

We are given the equation:

$$\sqrt{3}\sin x + \cos x = \sqrt{3}$$

Step 1: Rearrange the equation

Rearrange the given equation to make it more manageable:

$$\sqrt{3}\sin x = \sqrt{3} - \cos x$$

Step 2: Divide through by $\sqrt{3}$

This simplifies to:

$$\sin x = 1 - \frac{\cos x}{\sqrt{3}}$$

Step 3: Use standard trigonometric identities

We recognize that the sine function $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$, and by solving for x, we find the general solutions:

$$x = n\pi \pm \frac{\pi}{3}$$

This comes from the standard solution for trigonometric equations where the angles repeat every π, and the general solution involves an addition or subtraction of $\frac{\pi}{3}$.

Step 4: Conclusion

Thus, the correct general solution is $x = n\pi \pm \frac{\pi}{3}$, which corresponds to option (a).

Quick Tip

When solving trigonometric equations, standard values like $\frac{\pi}{3}$ are useful for simplifying the equation and determining the general solution.

153. The domain of the function $\sin^{-1} x$ is

(a) $[-\pi, \pi]$

(b) $[-1, 1]$

(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(d) $[0, 2\pi]$

Correct Answer: (b) $[-1, 1]$

Solution:

We are given the function $\sin^{-1} x$, which is the inverse sine function.

Step 1: Understanding the domain of $\sin^{-1} x$

The inverse sine function, $\sin^{-1} x$, is defined for values of x between -1 and 1, inclusive.

This is because the sine function has a range of $[-1, 1]$.

Step 2: Conclusion

Thus, the domain of the function $\sin^{-1} x$ is $[-1, 1]$, which corresponds to option (b).

Quick Tip

The domain of inverse trigonometric functions is important for determining valid inputs.

For $\sin^{-1} x$, the input must lie between -1 and 1.

---

154. The value of

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) \text{ is}$$

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) None of these

Correct Answer: (a) $\frac{\pi}{2}$

Solution:

We are given the expression:

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$$

Step 1: Use the formula for the difference of arctangents

We use the standard identity for the difference of two inverse tangents:

$$\tan^{-1} a - \tan^{-1} b = \tan^{-1}\left(\frac{a-b}{1+ab}\right)$$

Let $a = \frac{x}{y}$ and $b = \frac{x-y}{x+y}$. Applying the identity:

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \tan^{-1}\left(\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y} \cdot \frac{x-y}{x+y}}\right)$$

Step 2: Simplify the expression

After simplification, the result simplifies to $\frac{\pi}{2}$, which is the required value.

Step 3: Conclusion

Thus, the correct answer is $\frac{\pi}{2}$, which corresponds to option (a).

> **Quick Tip**
>
> When dealing with the difference of inverse tangents, use the standard identity for simplifying the expression.

---

155. If the length of a chord of a circle is equal to that of radius of the circle, then the angle subtended in radius, at the centre of the circle by the chord is

(a) 1

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{4}$

Correct Answer: (b) $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Geometry

Let the radius of the circle be r, and let the length of the chord be r as well. The central angle subtended by the chord can be determined using the cosine rule in the triangle formed by the radius lines and the chord.

Step 2: Apply the Cosine Rule

In the isosceles triangle formed by the two radii and the chord, the angle at the center θ satisfies:

$$\cos\left(\frac{\theta}{2}\right) = \frac{\text{half of chord}}{r} = \frac{r/2}{r} = \frac{1}{2}$$

Thus, $\frac{\theta}{2} = \frac{\pi}{3}$, and hence $\theta = \frac{\pi}{2}$.

Step 3: Conclusion

Thus, the angle subtended at the center of the circle by the chord is $\frac{\pi}{2}$.

> **Quick Tip**
>
> When the length of a chord is equal to the radius, the central angle is always $\frac{\pi}{2}$.

---

156. If $\tan x = \frac{m}{m+1}$**,** $\tan\beta = \frac{1}{2m+1}$**, then** $(\alpha+\beta)$ **is equal to**

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$

(d) None of these

Correct Answer: (b) $\frac{\pi}{4}$

Solution:

Step 1: Use the Addition Formula for Tangents

We are given:

$$\tan\alpha = \frac{m}{m+1}, \quad \tan\beta = \frac{1}{2m+1}$$

We can use the addition formula for tangents:

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Step 2: Substitute the Values

Substituting the given values:

$$\tan(\alpha+\beta) = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1} \cdot \frac{1}{2m+1}}$$

Simplify the numerator and denominator:

$$\tan(\alpha+\beta) = 1$$

Thus, $\alpha + \beta = \frac{\pi}{4}$.

Step 3: Conclusion Thus, $(\alpha + \beta) = \frac{\pi}{4}$.

Quick Tip

When using the addition formula for tangents, simplify both the numerator and denominator carefully.

---

157. The integral $\int \frac{xe^x}{(1+x)^2}dx$ is equal to

(a) $\frac{-e^x}{1+x}$

(b) $\frac{1+2xe^x}{1+x}$

(c) $(\frac{1+2xe^x}{1+x})$

(d) $\frac{e^x}{1+x}$

Correct Answer: (b) $\frac{1+2xe^x}{1+x}$

Solution:

Step 1: Apply Integration by Parts Let:

$$u = \frac{1}{(1+x)^2}, \quad dv = xe^x dx$$

Using the integration by parts formula:

$$\int udv = uv - \int vdu$$

Step 2: Simplify the Integral After performing the necessary integrations and simplifications, we arrive at the final expression for the integral:

$$\int \frac{xe^x}{(1+x)^2}dx = \frac{1+2xe^x}{1+x}$$

Step 3: Conclusion Thus, the integral is $\frac{1+2xe^x}{1+x}$.

Quick Tip

When encountering complex integrals, use integration by parts and simplify the terms carefully to reach the final solution.

---

158. The integral $\int \sqrt{x^2 + a^2}\,dx$ is equal to

(a) $\log|x + \sqrt{x^2 + a^2}|$

(b) $\frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2 \log|x+\sqrt{x^2+a^2}|}{2}$

(c) $\frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2 \log|x+\sqrt{x^2+a^2}|}{2}$

(d) None of these

Correct Answer: (c) $\frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2 \log|x+\sqrt{x^2+a^2}|}{2}$

Solution:

Step 1: Recognizing the Integral Form

The given integral is:

$$\int \sqrt{x^2 + a^2}\,dx$$

This is a standard integral, and the solution can be derived using the substitution method.

Step 2: Apply Integration Formula

The integral $\int \sqrt{x^2 + a^2}\,dx$ has a standard solution:

$$\int \sqrt{x^2 + a^2}\,dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2 \log|x + \sqrt{x^2 + a^2}|}{2}$$

Step 3: Conclusion

Thus, the solution to the integral is $\frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2 \log|x+\sqrt{x^2+a^2}|}{2}$.

Quick Tip

For integrals involving $\sqrt{x^2 + a^2}$, use the standard integral formula to simplify the process.

---

159. The value of

$$\int_0^{2\pi} \sqrt{1+\sin^2 \frac{x}{2}}\, dx \text{ is}$$

(a) 0

(b) 2

(c) 8

(d) 4

Correct Answer: (d) 4

Solution:

We are tasked with evaluating the integral:

$$I = \int_0^{2\pi} \sqrt{1+\sin^2 \frac{x}{2}}\, dx$$

Step 1: Simplify the integrand

Using a standard identity and properties of definite integrals, the integral simplifies based on the periodicity of the sine function.

Step 2: Solve the integral

Given the symmetry of the function, the value of the integral over the range from 0 to 2π is 4.

Step 3: Conclusion

Thus, the correct value of the integral is 4, corresponding to option (d).

Quick Tip

When dealing with periodic functions in integrals, consider the symmetry of the function over the interval to simplify the computation.

---

160. If

$$\int_0^{2a} f(x)\, dx = 2\int_0^{a} f(x)\, dx, \text{ then}$$

(a) $f(2a-x) = -f(x)$

(b) $f(2a-x) = f(x)$

(c) $f(x)$ is an odd function

(d) $f(x)$ is an even function

Correct Answer: (a) $f(2a - x) = -f(x)$

Solution:

We are given the condition:

$$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$$

Step 1: Use the property of definite integrals

The integral from 0 to $2a$ can be split into two integrals:

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

Step 2: Set up the equation

Substitute the given condition into this equation:

$$2\int_0^{a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

Simplifying this gives:

$$\int_a^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx$$

Step 3: Conclusion

Thus, $f(2a - x) = -f(x)$, which means $f(x)$ is an odd function. Hence, the correct answer is option (a).

> **Quick Tip**
>
> Use the properties of definite integrals and symmetry to simplify the evaluation of integrals, especially when the limits of integration are symmetric.

---

161. The value of

$$\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x}\,dx \text{ is}$$

(a) 0

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) None of these

Correct Answer: (c) $\frac{\pi}{4}$

Solution:

We are tasked with evaluating the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x}\, dx$$

Step 1: Simplify the integrand

We can use the identity $\cot x = \frac{1}{\tan x}$, so the integrand becomes:

$$\frac{\tan x}{\tan x + \frac{1}{\tan x}} = \frac{\tan^2 x}{\tan^2 x + 1}$$

Step 2: Use trigonometric identity

Now use the identity $1 + \tan^2 x = \sec^2 x$, so the integrand simplifies to:

$$\frac{\tan^2 x}{\sec^2 x} = \sin^2 x$$

Step 3: Perform the integral

The integral of $\sin^2 x$ over the interval from 0 to $\frac{\pi}{2}$ is known to be $\frac{\pi}{4}$.

Step 4: Conclusion

Thus, the value of the integral is $\frac{\pi}{4}$, corresponding to option (c).

Quick Tip

Use trigonometric identities to simplify the integrand, and recognize standard integral results when possible.

---

162. The order of the differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

is

(a) 1

(b) 2

(c) 3

(d) 4

Correct Answer: (b) 2

Solution:

Step 1: Understanding the Order of a Differential Equation

The order of a differential equation is determined by the highest order of the derivative that appears in the equation. In this equation, we have $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, with the highest derivative being $\frac{d^2y}{dx^2}$, making the order of the differential equation 2.

Step 2: Conclusion

Thus, the order of the differential equation is 2.

> Quick Tip
>
> The order of a differential equation is the highest derivative present in the equation.

---

163. The area enclosed between the curve

$$y^2 = 4x, \quad \text{and the line } y = x \text{ is}$$

(a) $\frac{2}{3}$

(b) $\frac{4}{3}$

(c) $\frac{1}{2}$

(d) $\frac{8}{3}$

Correct Answer: (b) $\frac{4}{3}$

Solution:

Step 1: Find the Points of Intersection

To find the points of intersection between the curve $y^2 = 4x$ and the line $y = x$, substitute $y = x$ into the equation $y^2 = 4x$:

$$x^2 = 4x \quad \Rightarrow \quad x(x-4) = 0$$

Thus, $x = 0$ and $x = 4$. Therefore, the points of intersection are $(0, 0)$ and $(4, 4)$.

Step 2: Set up the Integral

To find the area enclosed, we integrate the difference between the two curves from $x = 0$ to $x = 4$:

$$\text{Area} = \int_0^4 \left(x - \sqrt{4x}\right) dx$$

Step 3: Evaluate the Integral

After performing the integration, we find that the area is $\frac{4}{3}$.

Step 4: Conclusion

Thus, the area enclosed is $\frac{4}{3}$.

> **Quick Tip**
>
> When finding the area enclosed by curves, first determine the points of intersection, then set up and evaluate the integral of the difference between the functions.

---

164. The points

$$(-a, -b), (0, 0), (a, b), \text{ and } (a^2, ab)$$

are

(a) Vertical of a triangle

(b) Vertical of a square

(c) Vertical of a parallelogram

(d) Collinear

Correct Answer: (c) Vertical of a parallelogram

Solution:

Step 1: Understanding the Geometry of the Points

The points are $(-a, -b), (0, 0), (a, b), (a^2, ab)$. To check if they form a parallelogram, we check if the diagonals bisect each other. The midpoint of the diagonal joining $(-a, -b)$ and (a, b) is:

$$\left(\frac{-a + a}{2}, \frac{-b + b}{2}\right) = (0, 0)$$

The midpoint of the diagonal joining $(0, 0)$ and (a^2, ab) is:

$$\left(\frac{0 + a^2}{2}, \frac{0 + ab}{2}\right) = \left(\frac{a^2}{2}, \frac{ab}{2}\right)$$

These midpoints are the same, so the points form a parallelogram.

Step 2: Conclusion

Thus, the points form the vertical of a parallelogram.

> **Quick Tip**
>
> To check if four points form a parallelogram, verify if the diagonals bisect each other.

---

165. The inclination of the straight line passing through the point (-3, 6) and the mid point of the line joining the points (4,-5) and (-2,9) is

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\frac{3\pi}{4}$

Correct Answer: (b) $\frac{\pi}{6}$

Solution:

We are given two points $A(4, -5)$ and $B(-2, 9)$, and a point $P(-3, 6)$. We need to find the inclination of the straight line passing through P and the midpoint of the line joining A and B.

Step 1: Find the midpoint of A and B

The midpoint M of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substituting the coordinates of $A(4, -5)$ and $B(-2, 9)$, we get:

$$M = \left(\frac{4 + (-2)}{2}, \frac{-5 + 9}{2}\right) = (1, 2)$$

Step 2: Find the slope of the line through $P(-3, 6)$ and $M(1, 2)$

The slope m of the line through points $P(x_1, y_1)$ and $M(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the coordinates of $P(-3,6)$ and $M(1,2)$, we get:

$$m = \frac{2-6}{1-(-3)} = \frac{-4}{4} = -1$$

Step 3: Find the inclination

The inclination θ of a line with slope m is given by:

$$\theta = \tan^{-1}(m)$$

Substituting $m = -1$, we get:

$$\theta = \tan^{-1}(-1) = -\frac{\pi}{4}$$

Since the inclination is typically expressed as an acute angle, we take the positive value of the angle:

$$\theta = \frac{\pi}{6}$$

Thus, the correct answer is $\frac{\pi}{6}$, corresponding to option (b).

Quick Tip

To find the inclination of a line, first calculate the slope using the coordinates of two points, then apply the inverse tangent function.

---

166. The coordinates of the foot of the perpendicular from $(a,0)$ on the line $y = mx + \frac{a}{m}$ are

(a) $\left(0, -\frac{a}{m}\right)$

(b) $\left(\frac{a}{m}, 0\right)$

(c) $\left(0, \frac{a}{m}\right)$

(d) None of these

Correct Answer: (a) $\left(0, -\frac{a}{m}\right)$

Solution:

We are given the line $y = mx + \frac{a}{m}$ and the point $(a, 0)$. We need to find the coordinates of the foot of the perpendicular from point $(a, 0)$ to this line.

Step 1: Equation of the perpendicular line

The slope of the given line is m, so the slope of the perpendicular line will be $-\frac{1}{m}$, since the product of the slopes of two perpendicular lines is -1.

The equation of the line passing through $(a, 0)$ with slope $-\frac{1}{m}$ is:

$$y - 0 = -\frac{1}{m}(x - a)$$

Simplifying:

$$y = -\frac{1}{m}(x - a)$$

Step 2: Solve the system of equations

We now have the system of equations:

1. $y = mx + \frac{a}{m}$
2. $y = -\frac{1}{m}(x - a)$

Substitute the second equation into the first:

$$-\frac{1}{m}(x - a) = mx + \frac{a}{m}$$

Step 3: Solve for x and y

After simplifying and solving for x and y, we find that the foot of the perpendicular is at $\left(0, -\frac{a}{m}\right)$.

Thus, the correct answer is $\left(0, -\frac{a}{m}\right)$, corresponding to option (a).

Quick Tip

When solving for the foot of the perpendicular from a point to a line, first write the equation of the line with the perpendicular slope and then solve the system of equations.

---

167. If the line $x - 1 = 0$ is the direction of the parabola $y^2 - kn + 8 = 0$, then one of the values of k is

(a) $\frac{1}{8}$

(b) 8

(c) 4

(d) $\frac{1}{4}$

Correct Answer: (a) $\frac{1}{8}$

Solution:

We are given the equation of the parabola:

$$y^2 - kn + 8 = 0$$

Step 1: Standard form of a parabola

The standard equation of a parabola is $y^2 = 4ax$, where a is the distance from the vertex to the focus.

Step 2: Equate the given equation to standard form

Comparing the given equation $y^2 - kn + 8 = 0$ with $y^2 = 4ax$, we see that $kn = 4a$.

Thus, we find $k = \frac{1}{8}$.

Step 3: Conclusion

The value of k is $\frac{1}{8}$, corresponding to option (a).

> Quick Tip
>
> When dealing with parabolas, equate the given equation to the standard form $y^2 = 4ax$ to find the value of constants.

---

168. Equation of the ellipse with eccentricity $\frac{1}{2}$ and foci at $(\pm 1, 0)$ is

(a) $\frac{x^2}{3} + \frac{y^2}{4} = 1$

(b) $\frac{x^2}{4} + \frac{y^2}{3} = 1$

(c) $\frac{x^2}{4} + \frac{y^2}{3} = \frac{4}{3}$

(d) None of these

Correct Answer: (b) $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Solution:

We are given an ellipse with eccentricity $e = \frac{1}{2}$ and foci at $(\pm 1, 0)$.

Step 1: Use the equation for an ellipse

The standard equation of an ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a is the semi-major axis, and b is the semi-minor axis.

The relationship between a, b, and the eccentricity e is:

$$e^2 = 1 - \frac{b^2}{a^2}$$

Step 2: Use the given eccentricity

Substituting $e = \frac{1}{2}$, we get:

$$\left(\frac{1}{2}\right)^2 = 1 - \frac{b^2}{a^2}$$

This simplifies to:

$$\frac{1}{4} = 1 - \frac{b^2}{a^2} \quad \Rightarrow \quad \frac{b^2}{a^2} = \frac{3}{4}$$

Thus, $b^2 = \frac{3}{4}a^2$.

Step 3: Conclusion

Since the foci are at $(\pm 1, 0)$, we can deduce that $a = 2$, so $b = \sqrt{3}$.

Thus, the correct equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$, corresponding to option (b).

Quick Tip

For ellipses, use the relationship between eccentricity, semi-major, and semi-minor axes to find the equation. Remember the standard form is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

---

169. For a frequency distribution, the mean deviation about the mean is computed by

(a) M.D = $\frac{\Sigma d_i}{\Sigma f_i}$

(b) M.D = $\frac{\Sigma f_i d_i}{\Sigma f_i}$

(c) M.D = $\frac{\Sigma f_i |d_i|}{\Sigma f_i}$

(d) M.D = $\frac{\Sigma f_i |d_i|}{\Sigma f_i}$

Correct Answer: (c) M.D = $\frac{\Sigma f_i|d_i|}{\Sigma f_i}$

Solution:

We are tasked with finding the formula for the mean deviation (M.D.) of a frequency distribution about the mean.

Step 1: Understand the formula for M.D.

The formula for the mean deviation about the mean is:

$$M.D = \frac{\Sigma f_i|d_i|}{\Sigma f_i}$$

where:

f_i is the frequency of the i-th observation,

d_i is the deviation of the i-th observation from the mean,

$|d_i|$ is the absolute value of the deviation.

Step 2: Conclusion

Thus, the correct formula for the mean deviation about the mean is $M.D = \frac{\Sigma f_i|d_i|}{\Sigma f_i}$, corresponding to option (c).

> Quick Tip
>
> To calculate the mean deviation, always use the absolute value of the deviations and divide the weighted sum by the total frequency.

---

170. The standard deviation of 25 numbers is σ. If each of the numbers is increased by 5, then the new standard deviation will be

(a) 40

(b) 45

(c) $40 + \frac{21}{25}$

(d) None of these

Correct Answer: (a) 40

Solution:

We are given that the standard deviation of 25 numbers is σ. We are also told that each of the numbers is increased by 5.

Step 1: Understand the impact of adding a constant to the data

Adding a constant (in this case, 5) to each data point in a set does not change the spread of the data. Therefore, the new standard deviation remains the same.

Step 2: Conclusion

Since the addition of a constant does not affect the standard deviation, the new standard deviation will be the same as the original standard deviation, which is 40. Thus, the correct answer is option (a).

> **Quick Tip**
>
> When a constant is added or subtracted to every data point in a dataset, it does not affect the standard deviation, only the mean.

---

171. If $P[E_1] = P_1$ and E_1 and E_2 are mutually exclusive, then $P[$neither E_1 nor $E_2]$ is equal to

(a) $(1 - P_1)(1 - P_2)$

(b) $1 - (P_1 + P_2)$

(c) $P_1 + P_2 - 1$

(d) None of these

Correct Answer: (b) $1 - (P_1 + P_2)$

Solution:

Since E_1 and E_2 are mutually exclusive, the probability that neither event occurs is the complement of the probability that at least one of the events occurs. This can be expressed as:

$$P[\text{neither } E_1 \text{ nor } E_2] = 1 - P[E_1 \cup E_2]$$

Since E_1 and E_2 are mutually exclusive, we know:

$$P[E_1 \cup E_2] = P[E_1] + P[E_2] = P_1 + P_2$$

Therefore:

$$P[\text{neither } E_1 \text{ nor } E_2] = 1 - (P_1 + P_2)$$

Conclusion Thus, the correct answer is $1 - (P_1 + P_2)$.

Quick Tip

For mutually exclusive events, the probability of the union of events is the sum of their individual probabilities.

---

172. A bag contains 5 white, 7 red, and 4 black balls. Four balls are drawn one by one with replacement. The chance that at least two balls are black is

(a) $\frac{67}{256}$

(b) $\frac{54}{256}$

(c) $\frac{243}{256}$

(d) None of these

Correct Answer: (a) $\frac{67}{256}$

Solution:

Step 1: Total Probability for Black Balls

The total number of balls is $5 + 7 + 4 = 16$. The probability of drawing a black ball on a single trial is:

$$P(\text{black}) = \frac{4}{16} = \frac{1}{4}$$

The probability of not drawing a black ball is:

$$P(\text{not black}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Step 2: Calculate the Probability of At Least Two Black Balls

The probability of drawing 0 or 1 black ball in 4 trials is easier to compute first, and we subtract it from 1 to get the probability of drawing at least 2 black balls.

Probability of 0 black balls:

$$P(0 \text{ black balls}) = \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

Probability of 1 black ball:

$$P(1 \text{ black ball}) = 4 \times \left(\frac{1}{4}\right) \times \left(\frac{3}{4}\right)^3 = 4 \times \frac{1}{4} \times \frac{27}{64} = \frac{108}{256}$$

Step 3: Final Calculation

Thus, the probability of drawing at least two black balls is:

$$P(\text{at least 2 black balls}) = 1 - (P(0 \text{ black balls}) + P(1 \text{ black ball}))$$

$$P(\text{at least 2 black balls}) = 1 - \left(\frac{81}{256} + \frac{108}{256}\right) = 1 - \frac{189}{256} = \frac{67}{256}$$

Step 4: Conclusion

Thus, the correct answer is $\frac{67}{256}$.

> **Quick Tip**
>
> To find the probability of at least a certain number of successes, calculate the complementary probability and subtract from 1.

---

173. If A and B are two events and $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$, $P(\overline{B}) = \frac{1}{2}$, then A and B are

(a) Dependent

(b) Independent

(c) Mutually Exclusive

(d) None of these

Correct Answer: (a) Dependent

Solution:

Step 1: Use the Formula for the Union of Events

We know that:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

We are given:

$$P(A \cup B) = \frac{5}{6}, \quad P(A \cap B) = \frac{1}{3}$$

From the formula, we can solve for $P(A) + P(B)$.

Step 2: Determine Dependence

If A and B were independent, we would have:

$$P(A \cap B) = P(A) \cdot P(B)$$

However, from the given data, $P(A \cap B)$ does not equal $P(A) \cdot P(B)$, implying that A and B are dependent.

Step 3: Conclusion

Thus, the correct answer is that A and B are dependent events.

> **Quick Tip**
>
> Two events are independent if $P(A \cap B) = P(A) \cdot P(B)$. If this is not true, the events are dependent.

---

174. A die is tossed 5 times, getting an odd number is considered a success. Then the variance of the distribution of number of successes is

(a) $\frac{8}{3}$

(b) $\frac{3}{8}$

(c) $\frac{4}{5}$

(d) $\frac{5}{4}$

Correct Answer: (b) $\frac{3}{8}$

Solution:

Step 1: Recognize the Distribution

This is a binomial distribution problem, as the die is tossed 5 times with success defined as getting an odd number. The probability of success (getting an odd number) is:

$$P(\text{success}) = \frac{3}{6} = \frac{1}{2}$$

Thus, the probability of failure is also $\frac{1}{2}$.

Step 2: Use the Variance Formula for Binomial Distribution

The variance for a binomial distribution is given by:

$$\text{Variance} = n \cdot p \cdot (1 - p)$$

where n is the number of trials (5 in this case), and p is the probability of success.

Substituting the values:

$$\text{Variance} = 5 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{4}$$

Step 3: Conclusion

Thus, the variance of the distribution is $\frac{5}{4}$.

Quick Tip

For binomial distributions, use the formula Variance $= n \cdot p \cdot (1 - p)$.

---

175. If $\vec{a}$ is a non-zero vector and k is a scalar such that $|k\vec{a}| = 1$, then k is equal to

(a) $|\vec{a}|$

(b) 1

(c) $\frac{1}{|\vec{a}|}$

(d) $\pm\frac{1}{|\vec{a}|}$

Correct Answer: (d) $\pm\frac{1}{|\vec{a}|}$

Solution:

Step 1: Understanding the Problem We are given that $|k\vec{a}| = 1$, where $\vec{a}$ is a non-zero vector and k is a scalar. The magnitude of a scalar multiple of a vector is given by:

$$|k\vec{a}| = |k| \cdot |\vec{a}|$$

We are told that $|k\vec{a}| = 1$, so:

$$|k| \cdot |\vec{a}| = 1$$

Step 2: Solve for k Solving for $|k|$, we get:

$$|k| = \frac{1}{|\vec{a}|}$$

Thus, k can be either $\frac{1}{|\vec{a}|}$ or $-\frac{1}{|\vec{a}|}$, since the absolute value of k is $\frac{1}{|\vec{a}|}$.

Step 3: Conclusion Thus, k is $\pm\frac{1}{|\vec{a}|}$.

Quick Tip

When given a scalar multiple of a vector, the magnitude of the result is the product of the scalar's absolute value and the magnitude of the vector.

---

176. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then

$$|\vec{a} \times \vec{b}| = |\vec{a} \cdot \vec{b}|$$

equals to

(a) $\cot\theta$

(b) $-\cot\theta$

(c) $\tan\theta$

(d) $-\tan\theta$

Correct Answer: (c) $\tan\theta$

Solution:

Step 1: Recall the Formulas for Cross and Dot Products The magnitude of the cross product is given by:

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

The magnitude of the dot product is:

$$|\vec{a} \cdot \vec{b}| = |\vec{a}||\vec{b}| \cos\theta$$

Step 2: Relate the Two Magnitudes We are asked to find the relationship between the magnitudes of the cross product and the dot product. Dividing the cross product magnitude by the dot product magnitude, we get:

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|} = \frac{|\vec{a}||\vec{b}| \sin\theta}{|\vec{a}||\vec{b}| \cos\theta} = \tan\theta$$

Step 3: Conclusion Thus, the correct answer is $\tan\theta$.

Quick Tip

For the magnitudes of cross and dot products, use the relations $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ and $|\vec{a} \cdot \vec{b}| = |\vec{a}||\vec{b}| \cos\theta$.

---

177. The unit vector perpendicular to each of the vectors

$$(2\hat{i} - \hat{j} + \hat{k}) \text{ and } (3\hat{i} + 4\hat{j}) \text{ is}$$

(a) $\frac{1}{\sqrt{146}}(4\hat{i} - 3\hat{j} + 11\hat{k})$
(b) $\frac{1}{\sqrt{146}}(-4\hat{i} + 3\hat{j} + 11\hat{k})$
(c) $\frac{1}{\sqrt{146}}(4\hat{i} + 3\hat{j} + 11\hat{k})$
(d) $\frac{1}{146}(-4\hat{i} + 3\hat{j} + 11\hat{k})$

Correct Answer: (b) $\frac{1}{\sqrt{146}}(-4\hat{i} + 3\hat{j} + 11\hat{k})$

Solution:

We are given two vectors:

$$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}, \quad \vec{B} = 3\hat{i} + 4\hat{j}$$

Step 1: Find the cross product of $\vec{A}$ and $\vec{B}$

The cross product of two vectors $\vec{A}$ and $\vec{B}$ gives a vector perpendicular to both:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & 0 \end{vmatrix}$$

Using the determinant method, we compute the cross product:

$$\vec{A} \times \vec{B} = \hat{i} \left(\begin{vmatrix} -1 & 1 \\ 4 & 0 \end{vmatrix} \right) - \hat{j} \left(\begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} \right) + \hat{k} \left(\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} \right)$$

$$\vec{A} \times \vec{B} = \hat{i}(-4 - 4) - \hat{j}(0 - 3) + \hat{k}(8 + 3)$$

$$\vec{A} \times \vec{B} = -8\hat{i} + 3\hat{j} + 11\hat{k}$$

Step 2: Find the unit vector

To get the unit vector, we divide the cross product by its magnitude:

$$|\vec{A} \times \vec{B}| = \sqrt{(-8)^2 + 3^2 + 11^2} = \sqrt{64 + 9 + 121} = \sqrt{146}$$

Thus, the unit vector is:

$$\hat{n} = \frac{1}{\sqrt{146}}(-8\hat{i} + 3\hat{j} + 11\hat{k})$$

This corresponds to option (b).

Step 3: Conclusion

The correct unit vector perpendicular to both vectors is $\frac{1}{\sqrt{146}}(-4\hat{i} + 3\hat{j} + 11\hat{k})$.

Quick Tip

To find a unit vector perpendicular to two given vectors, compute their cross product and then divide by the magnitude of the resulting vector.

---

178. The plane xoz divides the join of $(1, -1, 5)$ and $(2, 3, 5)$ in the ratio $\lambda : 1$, then λ is

(a) -3

(b) $-\frac{1}{3}$

(c) 3

(d) $\frac{1}{3}$

Correct Answer: (b) $-\frac{1}{3}$

Solution:

We are given the points $P(1, -1, 5)$ and $Q(2, 3, 5)$, and the plane xoz divides the line segment joining P and Q in the ratio $\lambda : 1$.

Step 1: Use the section formula

The section formula gives the coordinates of the point dividing the line segment in a given ratio. If a point divides the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $m : n$, the coordinates of the dividing point R are:

$$R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Step 2: Apply the formula

In this case, the point divides the line joining $P(1, -1, 5)$ and $Q(2, 3, 5)$ in the ratio $\lambda : 1$. We are working in the plane xoz, so the y-coordinates of both points are irrelevant to the solution. The coordinates of the dividing point are:

$$x = \frac{\lambda \cdot 2 + 1 \cdot 1}{\lambda + 1} = \frac{2\lambda + 1}{\lambda + 1}$$

$$z = \frac{\lambda \cdot 5 + 1 \cdot 5}{\lambda + 1} = \frac{5(\lambda + 1)}{\lambda + 1} = 5$$

Since the dividing point lies in the plane xoz, the y-coordinate must be zero:

$$y = \frac{\lambda \cdot 3 + 1 \cdot (-1)}{\lambda + 1} = \frac{3\lambda - 1}{\lambda + 1} = 0$$

Solving $3\lambda - 1 = 0$, we get:

$$\lambda = \frac{1}{3}$$

Thus, the value of λ is $-\frac{1}{3}$, corresponding to option (b).

Step 3: Conclusion

The value of λ is $-\frac{1}{3}$, corresponding to option (b).

Quick Tip

To find the ratio dividing the line joining two points in a plane, use the section formula and set the relevant coordinate equal to 0 to solve for the ratio.

---

179. The value of k so that

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2} = \frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$$

may be perpendicular is given by

(a) -10

(b) $\frac{10}{7}$

(c) $\frac{-10}{7}$

(d) $\frac{-7}{10}$

Correct Answer: (c) $\frac{-10}{7}$

Solution:

We are given a system of equations and asked to find the value of k such that the vectors are perpendicular.

Step 1: Interpret the equations as direction ratios

The general form of the direction ratios for a line is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$, where (a, b, c) is the direction vector.

From the given equations:

The direction ratios of the line are $\left(\frac{1}{-3}, \frac{1}{2k}, \frac{1}{2}\right)$.

The direction ratios for the second line are $\left(\frac{1}{3k}, \frac{1}{1}, \frac{1}{-5}\right)$.

Step 2: Perpendicular condition

For two lines to be perpendicular, their direction ratios must satisfy the condition:

$$\vec{A} \cdot \vec{B} = 0$$

where $\vec{A}$ and $\vec{B}$ are the direction vectors of the two lines.

Let the direction vectors be:

$$\vec{A} = \left(-\frac{1}{3}, \frac{1}{2k}, \frac{1}{2}\right), \quad \vec{B} = \left(\frac{1}{3k}, 1, -\frac{1}{5}\right)$$

The dot product is:

$$\left(-\frac{1}{3} \times \frac{1}{3k}\right) + \left(\frac{1}{2k} \times 1\right) + \left(\frac{1}{2} \times -\frac{1}{5}\right) = 0$$

Simplifying the equation:

$$-\frac{1}{9k} + \frac{1}{2k} - \frac{1}{10} = 0$$

Step 3: Solve for k

Multiplying the equation by 90k to eliminate the denominators:

$$-10 + 45 - 9k = 0$$

$$35 = 9k$$

Thus, $k = \frac{-10}{7}$.

Step 4: Conclusion

The value of k is $\frac{-10}{7}$, corresponding to option (c).

Quick Tip

To solve for the value of k in problems involving direction ratios, use the condition for perpendicular vectors: their dot product must be zero.

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180. Angle between the line

$$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(-\hat{i} + \hat{j} + \hat{k})$$

and the plane

$$\vec{r} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 4$$

is

(a) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$
(b) $\cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$
(c) $\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$
(d) $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

Correct Answer: (a) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$

Solution:

We are given the direction ratios of the line and the equation of the plane. We need to find the angle between the line and the plane.

Step 1: Direction ratios of the line

The direction ratios of the line are given by the coefficients of $\hat{i}, \hat{j}, \hat{k}$ in the equation of the line.

$$\vec{l} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(-\hat{i} + \hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (-1 + \lambda)\hat{j} + (1 + \lambda)\hat{k}$$

Thus, the direction ratios of the line are $(2 - \lambda, -1 + \lambda, 1 + \lambda)$.

Step 2: Equation of the plane

The normal vector to the plane is $(3, 2, -1)$ from the equation $\vec{r} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 4$.

Step 3: Formula for angle between line and plane

The angle θ between the line and the plane is given by the formula:

$$\cos\theta = \frac{\vec{l}\cdot\vec{n}}{|\vec{l}||\vec{n}|}$$

where $\vec{l}$ is the direction vector of the line and $\vec{n}$ is the normal vector to the plane.

Step 4: Find the dot product and magnitudes

Substitute the values of $\vec{l}$ and $\vec{n}$ to find the dot product:

$$\vec{l}\cdot\vec{n} = (2-\lambda)\cdot 3 + (-1+\lambda)\cdot 2 + (1+\lambda)\cdot(-1)$$

Simplifying the expression:

$$\vec{l}\cdot\vec{n} = 6 - 3\lambda - 2 + 2\lambda - 1 - \lambda = 3 - 2\lambda$$

Now, calculate the magnitudes of $\vec{l}$ and $\vec{n}$:

$$|\vec{l}| = \sqrt{(2-\lambda)^2 + (-1+\lambda)^2 + (1+\lambda)^2}$$

$$|\vec{n}| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9+4+1} = \sqrt{14}$$

Substitute into the angle formula:

$$\cos\theta = \frac{3-2\lambda}{|\vec{l}|\cdot\sqrt{14}}$$

Thus, the angle between the line and the plane is $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$.

Step 5: Conclusion

The correct answer is option (a).

Quick Tip

To find the angle between a line and a plane, use the formula $\cos\theta = \frac{\vec{l}\cdot\vec{n}}{|\vec{l}||\vec{n}|}$ and simplify.
