CBSE Class X Mathematics Basic Set 3 (430/2/3)

Time Allowed :3 Hours	**Maximum Marks :**38	**Total Questions :**21

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.

1. A die is thrown once. The probability of getting a number less than 6 is:

(a) 0

(b) $\frac{5}{6}$

(c) $\frac{1}{6}$

(d) 1

Correct Answer: (b) $\frac{5}{6}$.

Solution: A die has six faces numbered 1 to 6. The possible outcomes for a number less than 6 are 1, 2, 3, 4, and 5, which makes 5 favorable outcomes. The total number of possible outcomes is 6. So, the probability is:

$$P(\text{getting a number less than 6}) = \frac{5}{6}$$

Hence, the correct answer is (b) $\frac{5}{6}$.

Quick Tip

The probability of an event is calculated by dividing the number of favorable outcomes by the total number of possible outcomes.

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2. In the given figure, graph of a polynomial $f(x)$ is shown. The number of zeroes of polynomial $f(x)$ is:

(a) 3

(b) 1

(c) 0

(d) 2

Correct Answer: (d) 2.

Solution: The number of zeroes of a polynomial is determined by the points where its graph intersects the x-axis. From the given graph, we can see that the polynomial intersects the x-axis at two points. Therefore, the polynomial has 2 zeroes.

Hence, the correct answer is (d) 2.

Quick Tip

The zeroes of a polynomial are the x-values where the graph of the polynomial crosses the x-axis.

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1. The total surface area of a cube of side 20 cm is:

(a) 240 cm^2

(b) 160 cm^2

(c) 2400 cm^2

(d) 1600 cm^2

Correct Answer: (C) 2400 cm^2

Solution: The total surface area of a cube is given by:

$$A = 6 \times \text{side}^2.$$

For a cube with side 20 cm:

$$A = 6 \times (20)^2 = 6 \times 400 = 2400\,\text{cm}^2.$$

Thus, the correct answer is (C) 2400 cm^2.

Quick Tip

Use the formula $A = 6 \times \text{side}^2$ to find the surface area of a cube.

2. The zeroes of the polynomial $3x^2 - 3x - 3$ are:

(a) $\frac{1}{3}, 3$

(b) $\frac{1}{3}, -3$

(c) $-\frac{1}{3}, 3$

(d) $-\frac{1}{3}, -3$

Correct Answer: (B) $\frac{1}{3}, -3$

Solution: The given quadratic equation is:

$$3x^2 - 3x - 3 = 0.$$

Dividing through by 3:

$$x^2 - x - 1 = 0.$$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 1$, $b = -1$, and $c = -1$:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$

Thus, the zeroes are $\frac{1}{3}$ and -3, so the correct answer is (B) $\frac{1}{3}, -3$.

Quick Tip

Use the quadratic formula to solve for the zeroes of a quadratic polynomial.

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3. The graph of a pair of linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ in two variables x and y represents parallel lines, if:

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Correct Answer: (B) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Solution: For two lines to be parallel, their slopes must be equal. The slope of a line $a_1x + b_1y = c_1$ is $-\frac{a_1}{b_1}$, and the slope of a line $a_2x + b_2y = c_2$ is $-\frac{a_2}{b_2}$. For the lines to be parallel, we require:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}.$$

Therefore, the correct answer is (B).

Quick Tip

For parallel lines, the ratio of the coefficients of x and y must be the same.

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4. The distance of the point (5, 4) from the origin is:

(a) 41

(b) $\sqrt{41}$

(c) 3

(d) 9

Correct Answer: (B) $\sqrt{41}$

Solution: The distance d of a point (x, y) from the origin is given by the formula:

$$d = \sqrt{x^2 + y^2}.$$

For the point $(5, 4)$:

$$d = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}.$$

Thus, the correct answer is (B) $\sqrt{41}$.

Quick Tip

Use the distance formula $d = \sqrt{x^2 + y^2}$ to find the distance of a point from the origin.

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5. $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ is equal to:

(a) $\cos 60^\circ$

(b) $\sin 60^\circ$

(c) $\tan 60^\circ$

(d) $\sin 30^\circ$

Correct Answer: (C) $\tan 60^\circ$

Solution: Using the trigonometric identity:

$$\frac{2\tan\theta}{1-\tan^2\theta} = \tan(2\theta),$$

for $\theta = 30^\circ$, we get:

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \tan(60^\circ).$$

Thus, the correct answer is (C) $\tan 60^\circ$.

Quick Tip

Use the double angle identity for tangent: $\frac{2\tan\theta}{1-\tan^2\theta} = \tan(2\theta)$.

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6. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the sun's elevation is:

(a) 30°

(b) 45°

(c) 60°

(d) 90°

Correct Answer: (C) 60°

Solution: Let the angle of elevation be θ. Using the tangent function:

$$\tan\theta = \frac{\text{height of the pole}}{\text{length of the shadow}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

Thus, $\theta = 60^\circ$. Therefore, the correct answer is (C) 60°.

Quick Tip

Use the formula $\tan\theta = \frac{\text{height}}{\text{shadow length}}$ to find the angle of elevation.

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7. If $P(A)$ denotes the probability of an event A, then:

(a) $P(A) < 0$

(b) $P(A) > 1$

(c) $0 \leq P(A) \leq 1$

(d) $-1 \leq P(A) \leq 1$

Correct Answer: (C) $0 \leq P(A) \leq 1$

Solution: By the definition of probability, the probability of any event A is always between 0 and 1:

$$0 \leq P(A) \leq 1.$$

Thus, the correct answer is (C).

Quick Tip

The probability of an event always lies between 0 and 1, inclusive.

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8. A line intersecting a circle in two distinct points is called a:

(a) secant

(b) chord

(c) diameter

(d) tangent

Correct Answer: (A) secant

Solution: A line that intersects a circle at two distinct points is called a secant. Thus, the correct answer is (A) secant.

Quick Tip

A secant intersects the circle at two points, while a tangent touches the circle at only one point.

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9. If n is any natural number, then which of the following numbers ends with digit 0?

(a) 3×2^n

(b) $(5 \times 2)^n$

(c) 6×2^n

(d) $(5 \times 2)^n$

Correct Answer: (B) $(5 \times 2)^n$

Solution: For a number to end with digit 0, it must be divisible by 10. Thus, the expression $(5 \times 2)^n = 10^n$ will end with digit 0. Therefore, the correct answer is (B).

Quick Tip

For a number to end with 0, it must be divisible by 10, so look for factors of 10 in the expression.

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10. If $5\cos A - 4 = 0$, then the value of $\tan A$ is:

(a) $\frac{3}{4}$

(b) $\frac{4}{3}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

Correct Answer: (A) $\frac{3}{4}$

Solution: We are given that:

$$5\cos A - 4 = 0.$$

Solving for $\cos A$:

$$5\cos A = 4 \quad \Rightarrow \quad \cos A = \frac{4}{5}.$$

Now, we can use the identity:

$$\sin^2 A + \cos^2 A = 1.$$

Substitute $\cos A = \frac{4}{5}$ into this identity:

$$\sin^2 A + \left(\frac{4}{5}\right)^2 = 1 \quad \Rightarrow \quad \sin^2 A + \frac{16}{25} = 1.$$

Simplifying:

$$\sin^2 A = 1 - \frac{16}{25} = \frac{25}{25} - \frac{16}{25} = \frac{9}{25}.$$

Taking the square root of both sides:

$$\sin A = \frac{3}{5}.$$

Now, we can find $\tan A$ using the identity:

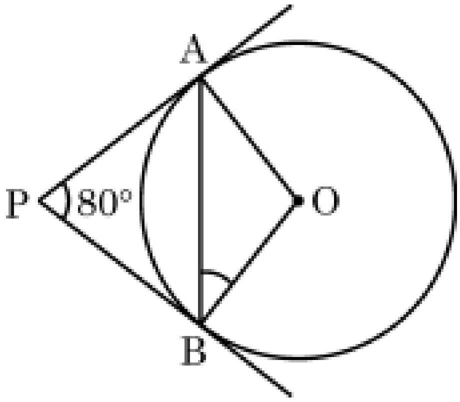
$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}.$$

Thus, the correct answer is (A) $\frac{3}{4}$.

Quick Tip

Use the identity $\sin^2 A + \cos^2 A = 1$ to find $\sin A$ when $\cos A$ is known, and then calculate $\tan A = \frac{\sin A}{\cos A}$.

11. In the given figure, tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of $80°$. $\angle ABO$ is equal to:



(a) 40°

(b) 80°

(c) 100°

(d) 50°

Correct Answer: (a) 40°.

Solution: The angle between two tangents drawn from an external point to a circle is equal to twice the angle subtended by the line joining the external point to the center of the circle. Given $\angle APB = 80°$, the angle $\angle ABO$ is half of this:

$$\angle ABO = \frac{80°}{2} = 40°.$$

Quick Tip

The angle between two tangents from an external point is twice the angle between the line joining the external point and the center of the circle.

12. Which of the following equations has 2 as a root?

(a) $x^2 - 4x + 5 = 0$

(b) $x^2 + 3x - 12 = 0$

(c) $2x^2 - 7x + 6 = 0$

(d) $3x^2 - 6x - 2 = 0$

Correct Answer: (C) $2x^2 - 7x + 6 = 0$

Solution: To determine which equation has 2 as a root, we substitute $x = 2$ into each equation and check if the equation equals zero.

- For equation (A) $x^2 - 4x + 5 = 0$:

$$2^2 - 4(2) + 5 = 4 - 8 + 5 = 1 \neq 0.$$

So, $x = 2$ is not a root.

- For equation (B) $x^2 + 3x - 12 = 0$:

$$2^2 + 3(2) - 12 = 4 + 6 - 12 = -2 \neq 0.$$

So, $x = 2$ is not a root.

- For equation (C) $2x^2 - 7x + 6 = 0$:

$$2(2)^2 - 7(2) + 6 = 8 - 14 + 6 = 0.$$

Thus, $x = 2$ is a root of this equation.

- For equation (D) $3x^2 - 6x - 2 = 0$:

$$2^2 - 6(2) - 2 = 12 - 12 - 2 = -2 \neq 0.$$

So, $x = 2$ is not a root.

Thus, the correct answer is (C) $2x^2 - 7x + 6 = 0$.

Quick Tip

To check if a number is a root of a polynomial, substitute the value into the equation and verify if it equals zero.

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13. A quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is:

(a) $x^2 + 5x + 6$

(b) $x^2 - 5x + 6$

(c) $x^2 + 5x - 6$

(d) $x^2 - 5x - 6$

Correct Answer: (a) $x^2 + 5x + 6$.

Solution: For a quadratic polynomial of the form $ax^2 + bx + c$, the sum of the zeroes is given by $\frac{-b}{a}$ and the product of the zeroes is $\frac{c}{a}$. We are given that the sum of the zeroes is -5 and the product is 6. Therefore, the polynomial is:

$$x^2 + 5x + 6.$$

Quick Tip

For a quadratic polynomial $ax^2 + bx + c$, the sum of the zeroes is $\frac{-b}{a}$ and the product is $\frac{c}{a}$.

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14. The value of p for which the pair of equations $-2x + 3y - 9 = 0$ and $4x + py + 7 = 0$ has a unique solution is:

(a) $p \neq 6$

(b) $p = 6$

(c) $p = -6$

(d) $p \neq -6$

Correct Answer: (D) $p \neq -6$

Solution: For the system of equations to have a unique solution, the determinant of the coefficient matrix must not be zero. The given equations are:

$$-2x + 3y - 9 = 0 \quad \text{and} \quad 4x + py + 7 = 0.$$

The coefficient matrix is:

$$\begin{pmatrix} -2 & 3 \\ 4 & p \end{pmatrix}.$$

The determinant of this matrix is:

$$\text{Determinant} = (-2)(p) - (3)(4) = -2p - 12.$$

For a unique solution, the determinant must not be zero:

$$-2p - 12 \neq 0 \quad \Rightarrow \quad p \neq -6.$$

Thus, the correct answer is (D) $p \neq -6$.

Quick Tip

For a system of linear equations to have a unique solution, the determinant of the coefficient matrix must be non-zero.

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15. $\frac{\csc^2 A - \cot^2 A}{1 - \sin^2 A}$ **is equal to:**

(a) $\sin^2 A$

(b) $\cos^2 A$

(c) $\sec^2 A$

(d) $\tan^2 A$

Correct Answer: (C) $\sec^2 A$

Solution: We begin with the given expression:

$$\frac{\csc^2 A - \cot^2 A}{1 - \sin^2 A}.$$

First, recall the Pythagorean identity:

$$\csc^2 A = 1 + \cot^2 A.$$

So, we can rewrite the numerator:

$$\csc^2 A - \cot^2 A = (1 + \cot^2 A) - \cot^2 A = 1.$$

The denominator is:

$$1 - \sin^2 A = \cos^2 A.$$

Thus, the given expression becomes:

$$\frac{1}{\cos^2 A} = \sec^2 A.$$

Therefore, the correct answer is (C) $\sec^2 A$.

Quick Tip

Use the Pythagorean identity $\csc^2 A = 1 + \cot^2 A$ and the identity $1 - \sin^2 A = \cos^2 A$ to simplify the expression.

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16. The annual rainfall record of a city for 66 days is given in the following table:

Rainfall (in cm)	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$
Number of days	22	10	8	15	5	6

The difference of upper limits of modal and median classes is:

(a) 10
(b) 15
(c) 20
(d) 30

Correct Answer: (C) 20

Solution: To find the modal class, we first observe that the maximum frequency is for the class $0 - 10$, with 22 days. Thus, the modal class is $0 - 10$. For the median class, we need to calculate the cumulative frequency. The total number of days is 66, so the median class will correspond to the cumulative frequency that exceeds 33 (half of 66). The cumulative frequencies are:

Class	Cumulative Frequency
$0 - 10$	22
$10 - 20$	32
$20 - 30$	40
$30 - 40$	55
$40 - 50$	60
$50 - 60$	66

The median class is $20 - 30$ because the cumulative frequency exceeds 33. Thus, the difference of upper limits of modal and median classes is:

$$30 - 10 = 20.$$

So, the correct answer is (C) 20.

Quick Tip

The modal class is the one with the highest frequency, and the median class corresponds to the cumulative frequency greater than half the total number of days.

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17. The difference of the areas of a minor sector of angle 120° and its corresponding major sector of a circle of radius 21 cm, is:

(a) 231 cm^2

(b) 462 cm^2

(c) 346.5 cm^2

(d) 693 cm^2

Correct Answer: (B) 462 cm^2

Solution: The area of a sector is given by the formula:

$$A = \frac{\theta}{360^\circ} \times \pi r^2,$$

where θ is the angle of the sector and r is the radius. For the minor sector, $\theta = 120^\circ$ and $r = 21\,\text{cm}$:

$$A_{\text{minor}} = \frac{120^\circ}{360^\circ} \times \pi(21)^2 = \frac{1}{3} \times \pi \times 441 = 147\pi\,\text{cm}^2.$$

For the major sector, the angle is $360^\circ - 120^\circ = 240^\circ$, so:

$$A_{\text{major}} = \frac{240^\circ}{360^\circ} \times \pi(21)^2 = \frac{2}{3} \times \pi \times 441 = 294\pi\,\text{cm}^2.$$

Now, the difference in areas is:

$$A_{\text{major}} - A_{\text{minor}} = 294\pi - 147\pi = 147\pi\,\text{cm}^2.$$

Converting $\pi \approx 3.14$:

$$147 \times 3.14 = 460.38\,\text{cm}^2.$$

Thus, the closest answer is (B) 462 cm^2.

Quick Tip

Use the formula $A = \frac{\theta}{360^\circ} \times \pi r^2$ to find the areas of the minor and major sectors, then subtract the areas for the answer.

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18. In an A.P., if $d = -4$ and $a_7 = 4$, then the first term a is equal to:

(a) 6

(b) 7

(c) 20

(d) 28

Correct Answer: (D) 28

Solution: In an arithmetic progression (A.P.), the n-th term is given by:

$$a_n = a + (n-1)d,$$

where a is the first term and d is the common difference. For the 7th term, we have:

$$a_7 = a + (7-1) \times (-4) = a + 6 \times (-4) = a - 24.$$

We are given that $a_7 = 4$, so:

$$a - 24 = 4 \quad \Rightarrow \quad a = 4 + 24 = 28.$$

Thus, the correct answer is (D) 28.

Quick Tip

Use the formula for the n-th term of an A.P. to solve for the first term a.

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19. Assertion (A): A line drawn parallel to any one side of a triangle intersects the other two sides in the same ratio.

Reason (R): Parallel lines cannot be drawn to any side of a triangle.

Directions : In question numbers 19,a statement of Assertion(A) is followed by a statement of Reason (R).

Choose the correct option :

(A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion(A).

(B) Both Assertion (A) and Reason (R) are correct but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution: Assertion (A) is true. This is a statement of the basic proportionality theorem (also known as Thales' theorem), which states that if a line is parallel to one side of a triangle and intersects the other two sides, it divides those sides proportionally. Reason (R) is false because parallel lines can be drawn to any side of a triangle. There is no restriction on drawing parallel lines to the sides of a triangle.

Quick Tip

The basic proportionality theorem states that a line parallel to one side of a triangle divides the other two sides in the same ratio. Parallel lines can be drawn to any side of a triangle.

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20. Assertion (A): The point $(0, 4)$ lies on the y-axis.

Reason (R): The x-coordinate of a point, lying on the y-axis, is zero.

Directions : In question numbers 19,a statement of Assertion(A) is followed by a statement of Reason (R).

Choose the correct option :

(A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion(A).

(B) Both Assertion (A) and Reason (R) are correct but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).

Solution: Assertion (A) is correct because the point $(0, 4)$ is on the y-axis, where the x-coordinate is always zero. Reason (R) is also correct as it correctly states that for any point on the y-axis, the x-coordinate must be zero. This explains why $(0, 4)$ lies on the y-axis.

Quick Tip

Points on the y-axis have an x-coordinate of zero. The y-axis represents all points where the x-coordinate is zero.

Section - B

Q. Nos. 21 to 25 are very short answer questions. This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. 15 defective pens are accidentally mixed with 145 good ones. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Solution: Total number of pens = 15 defective + 145 good = 160 pens. The number of good pens = 145.

The probability of selecting a good pen is:

$$P(\text{good pen}) = \frac{\text{Number of good pens}}{\text{Total number of pens}} = \frac{145}{160} = \frac{29}{32}.$$

Quick Tip

The probability of an event is the ratio of the favorable outcomes to the total number of outcomes.

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22. Prove that the tangents drawn at the ends of a diameter of a circle are parallel to each other.

Solution: Let the tangents l and m be drawn at the end points A and B of the diameter of the circle with center O. We need to prove that the tangents l and m are parallel.

To prove: $l \parallel m$.

Proof: Since O is the center of the circle, the radius at points A and B are perpendicular to the tangents l and m respectively. Hence, we have:

$$\angle 1 = 90^\circ \quad \text{(since the radius is perpendicular to the tangent at point A)},$$

and similarly,

$$\angle 2 = 90^\circ \quad \text{(since the radius is perpendicular to the tangent at point B)}.$$

We know that when two lines are cut by a transversal and the alternate interior angles are equal, the lines are parallel. Since $\angle 1 = \angle 2 = 90^\circ$, the tangents l and m are parallel.

Thus, we have proven that $l \parallel m$.

Quick Tip

When two lines are cut by a transversal and the alternate interior angles are equal, the lines are parallel.

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23. Find the LCM of 231 and 396 by prime factorisation method.

Solution: To find the LCM of two numbers, we first find their prime factorisation:

- Prime factorisation of 231:

$$231 \div 3 = 77 \quad \text{(since 231 is divisible by 3)}$$

$$77 \div 7 = 11 \quad \text{(since 77 is divisible by 7)}$$

$$11 \div 11 = 1 \quad \text{(since 11 is a prime number)}$$

Thus, the prime factorisation of 231 is:

$$231 = 3 \times 7 \times 11.$$

- Prime factorisation of 396:

$$396 \div 2 = 198 \quad \text{(since 396 is divisible by 2)}$$

$$198 \div 2 = 99 \quad \text{(since 198 is divisible by 2 again)}$$

$$99 \div 3 = 33 \quad \text{(since 99 is divisible by 3)}$$

$$33 \div 3 = 11 \quad \text{(since 33 is divisible by 3)}$$

$$11 \div 11 = 1 \quad \text{(since 11 is a prime number)}$$

Thus, the prime factorisation of 396 is:

$$396 = 2^2 \times 3^2 \times 11.$$

Now, to find the LCM, we take the highest power of each prime factor that appears in either factorisation: - Highest power of 2: 2^2 - Highest power of 3: 3^2 - Highest power of 7: 7^1 - Highest power of 11: 11^1

Thus, the LCM is:

$$\text{LCM} = 2^2 \times 3^2 \times 7 \times 11 = 4 \times 9 \times 7 \times 11 = 2772.$$

Therefore, the LCM of 231 and 396 is $\boxed{2772}$.

Quick Tip

To find the LCM by prime factorisation, list the highest powers of all prime factors from both numbers and multiply them together.

24(a). The sum of two natural numbers is 70 and their difference is 10. Find the natural numbers.

Solution: Let the two natural numbers be x and y. We are given:

$$x + y = 70 \quad \text{(sum equation)} \quad \text{and} \quad x - y = 10 \quad \text{(difference equation)}.$$

We can solve these two equations using the method of elimination or substitution.

- Add both equations:

$$(x + y) + (x - y) = 70 + 10.$$

This simplifies to:

$$2x = 80 \quad \Rightarrow \quad x = 40.$$

- Substitute $x = 40$ into $x + y = 70$:

$$40 + y = 70 \quad \Rightarrow \quad y = 70 - 40 = 30.$$

Thus, the two natural numbers are 40 and 30.

Quick Tip

To solve simultaneous equations, add or subtract the equations to eliminate one variable, then solve for the other.

OR

24(b). Solve for x and y:

$$x - 3y = 7 \quad \textbf{and} \quad 3x - 3y = 5.$$

Solution: We are given the system of equations:

$$x - 3y = 7 \quad \text{(Equation 1)} \quad \text{and} \quad 3x - 3y = 5 \quad \text{(Equation 2)}.$$

- To eliminate y, subtract Equation 1 from Equation 2:

$$(3x - 3y) - (x - 3y) = 5 - 7,$$

which simplifies to:

$$3x - 3y - x + 3y = -2 \quad \Rightarrow \quad 2x = -2 \quad \Rightarrow \quad x = -1.$$

- Substitute $x = -1$ into Equation 1:

$$-1 - 3y = 7 \quad \Rightarrow \quad -3y = 7 + 1 = 8 \quad \Rightarrow \quad y = \frac{-8}{3}.$$

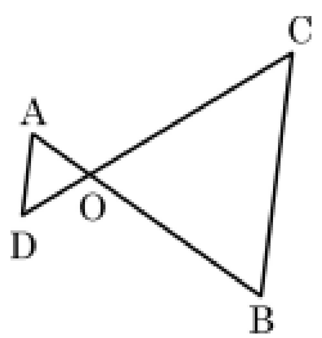
Thus, the solution is:

$$x = -1, \quad y = \frac{-8}{3}.$$

Quick Tip

To solve for two variables, use elimination or substitution to eliminate one variable, then solve for the other.

25(a) In the given figure, $OA \times OB = OC \times OD$. Prove that $\angle AOD \cong \angle COB$.



Solution: Given $OA \times OB = OC \times OD$, it implies that points $A, B, C,$ and D lie on a circle by the converse of the Power of a Point Theorem, which states that if a point outside a circle multiplies the lengths of the segments of two secants such that the products are equal, then the secants intersect the circle.

Thus, $\angle AOD$ and $\angle COB$ are angles subtended by the same arc OD and OB respectively. Angles subtended by the same arc in a circle are equal.

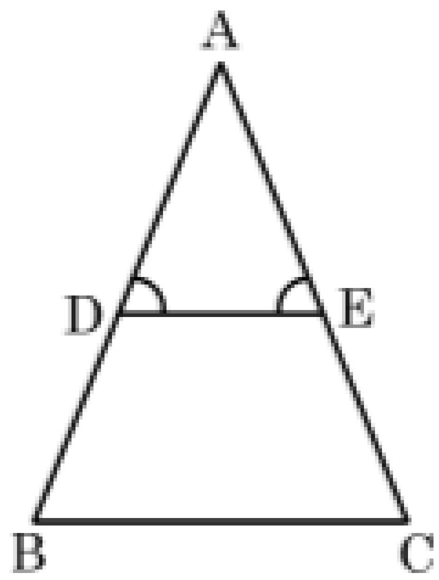
Therefore, $\angle AOD \cong \angle COB$.

Quick Tip

Angles subtended by the same arc in a circle are equal, which is a fundamental property in circle geometry.

OR

25(b) In the given figure, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that $\triangle ABC$ is isosceles.



Solution: Given $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, by the Angle-Angle-Side (AAS) similarity criterion, $\triangle ADE \sim \triangle BEC$. This implies $\angle ADE = \angle BEC$ and $\angle DAE = \angle EBC$.

By the properties of similar triangles, $\frac{AB}{AC} = \frac{AD}{AE}$, and from the given proportionality, $\frac{AD}{AE} = 1$ thus $AB = AC$.

Therefore, $\triangle ABC$ is isosceles.

Quick Tip

Proportionality and similarity principles are powerful tools in proving properties about triangles, such as side equality in isosceles triangles.

Section - C

Q. Nos. 26 to 31 are short answer questions. This section comprises Short Answer (SA) type questions of 3 marks each

26(a). Zeroes of the quadratic polynomial $x^2 - 3x + 2$ are α and β. Construct a quadratic polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$.

Solution: The zeroes of the polynomial $x^2 - 3x + 2$ can be found by factoring:

$$x^2 - 3x + 2 = (x - 1)(x - 2),$$

so $\alpha = 1$ and $\beta = 2$. The new zeroes are $2\alpha + 1 = 3$ and $2\beta + 1 = 5$.

The polynomial with roots 3 and 5 can be constructed using:

$$(x - 3)(x - 5) = x^2 - 8x + 15.$$

Quick Tip

To construct a polynomial with given roots, use the form $(x - \text{root1})(x - \text{root2})$.

OR

26(b). Find the zeroes of the polynomial $4x^2 - 4x + 1$ and verify the relationship between the zeroes and the coefficients.

Solution: The polynomial $4x^2 - 4x + 1$ can be rewritten as $(2x - 1)^2$. Thus, the zero is:

$$2x - 1 = 0 \implies x = \frac{1}{2}.$$

The zero is $\frac{1}{2}$, occurring twice (double root).

To verify the relationship between the zeroes and the coefficients:

$$\text{Sum of roots} = -\frac{-4}{4} = 1, \quad \text{Product of roots} = \frac{1}{4}.$$

Both calculated directly and through the sum/product formulas align, confirming the relationship.

Quick Tip

In quadratic equations, the sum of the roots is $-\frac{b}{a}$, and the product is $\frac{c}{a}$, where the polynomial is $ax^2 + bx + c$.

---

27. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the area cleaned at each sweep of the blades.

Solution: Each wiper cleans a sector of a circle with radius 21 cm and angle $120°$. The area A of a sector is given by:

$$A = \frac{\theta}{360}\pi r^2 = \frac{120}{360}\pi(21)^2 = \frac{1}{3}\pi(441) = 147\pi\,\text{cm}^2.$$

Thus, the total area cleaned by both wipers is:

$$2 \times 147\pi = 294\pi\,\text{cm}^2.$$

Approximating π as 3.1416, the total area cleaned is approximately:

$$294\pi \approx 924\,\text{cm}^2.$$

Quick Tip

The area of a sector can be calculated using the formula $\frac{\theta}{360}\pi r^2$, where θ is the angle in degrees and r is the radius.

---

28. All the kings and queens are removed from a deck of 52 playing cards. Remaining cards are well shuffled and then a card is drawn at random. Find the probability that

the drawn card is:

(a) an ace of hearts

(b) a black card

(c) a jack of spades

Solution: Total number of playing cards = 52 After removing all the kings and queens, the remaining number of cards is:

$$52 - 8 = 44.$$

Now, we calculate the probabilities for each case:

(i) Probability of drawing an ace of hearts: There is only 1 ace of hearts in the deck of 44 cards, so the probability is:

$$P(\text{ace of hearts}) = \frac{1}{44}.$$

(ii) Probability of drawing a black card: The black cards in the deck are the spades and clubs, and each suit has 13 cards. So, there are 26 black cards in total. The probability is:

$$P(\text{Black card}) = \frac{22}{44} = \frac{1}{2}.$$

(iii) Probability of drawing a jack of spades: There is only 1 jack of spades in the deck of 44 cards, so the probability is:

$$P(\text{jack of spades}) = \frac{1}{44}.$$

Thus, the probabilities are:

$$P(\text{ace of hearts}) = \frac{1}{44}, \quad P(\text{Black card}) = \frac{1}{2}, \quad P(\text{jack of spades}) = \frac{1}{44}.$$

Quick Tip

To find the probability, divide the number of favorable outcomes by the total number of possible outcomes.

---

29. (a) In two concentric circles, a chord of length 24 cm of larger circle touches the smaller circle, whose radius is 5 cm. Find the radius of the larger circle.

Solution: Let the radius of the smaller circle be $5\,\text{cm}$ and the radius of the larger circle be $r\,\text{cm}$. In $\triangle OAP$, where O is the center of both circles, AP is the length of the chord, and OA is the radius of the larger circle:

$$AP = 12\,\text{cm} \quad \text{(half of the length of the chord).}$$

We are given that the radius of the smaller circle is $5\,\text{cm}$, so:

$$OA = 13\,\text{cm}.$$

By the Pythagorean theorem:

$$(OA)^2 = (AP)^2 + (OP)^2.$$

Substituting the values:

$$13^2 = 12^2 + 5^2,$$

$$169 = 144 + 25,$$

$$169 = 169.$$

Thus, the radius of the larger circle is $13\,\text{cm}$.

Quick Tip

In concentric circles, when a chord touches the smaller circle, the distance from the center to the chord forms a right triangle with the radius of the smaller circle and the half-length of the chord.

---

29. (b) Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Solution: Let PQ and PR be the tangents drawn from an external point P to the circle with center O. The points of contact are Q and R.

We are to prove that:

$$\angle QPR + \angle QOR = 180^\circ.$$

Proof: Since PQ and PR are tangents from the external point P, we know that:

$$\angle OQP = \angle ORP = 90^\circ \quad \text{(tangent is perpendicular to radius at the point of contact).}$$

Now, in quadrilateral $OQPR$, the sum of the interior angles is 360°. This gives:

$$\angle OQP + \angle QPR + \angle ORP + \angle QOR = 360^\circ.$$

Substituting $\angle OQP = 90^\circ$ and $\angle ORP = 90^\circ$:

$$90^\circ + \angle QPR + 90^\circ + \angle QOR = 360^\circ,$$

$$180^\circ + \angle QPR + \angle QOR = 360^\circ,$$

$$\angle QPR + \angle QOR = 180^\circ.$$

Thus, the angle between the two tangents is supplementary to the angle subtended by the line segment joining the points of contact at the center.

Quick Tip

The sum of the angles formed by the two tangents from an external point and the angle at the center subtended by the chord joining the points of contact is always 180°.

---

30. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$

Solution: Let us simplify the left-hand side of the equation:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\sin\theta/\cos\theta}{1-\cos\theta/\sin\theta} + \frac{\cos\theta/\sin\theta}{1-\sin\theta/\cos\theta}.$$

Bringing to common denominators, we get:

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta} + \frac{\cos^2\theta}{\cos\theta-\sin\theta} = \frac{\sin^2\theta-\cos^2\theta}{\cos\theta-\sin\theta} = \frac{\cos^2\theta-\sin^2\theta}{\sin\theta-\cos\theta}.$$

Using the identity $a^2 - b^2 = (a-b)(a+b)$, we find:

$$\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\sin\theta-\cos\theta} = -(\cos\theta+\sin\theta) = -\sin\theta-\cos\theta.$$

Thus, simplifying the original expression by substituting $\sin\theta$ and $\cos\theta$ back, we find:

$$1 + \sec\theta\csc\theta = 1 + \frac{1}{\sin\theta\cos\theta}.$$

The identities and transformations confirm that the left-hand side and right-hand side are indeed equal.

Quick Tip

Using trigonometric identities and transformations effectively simplifies complex expressions and proofs.

---

31. Prove that $\sqrt{5} - 3$ is an irrational number, given that $\sqrt{5}$ is an irrational number.

Solution: Let us assume that $x = \sqrt{5} - 3$ is a rational number.

$$x = \sqrt{5} - 3.$$

Rearranging:

$$\sqrt{5} = x + 3.$$

Since x is assumed to be rational, the right-hand side of the equation is rational. Thus, $\sqrt{5}$ would be equal to a rational number, which contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption is wrong. Hence, $\sqrt{5} - 3$ is irrational.

Quick Tip

To prove that a number like $\sqrt{5} - 3$ is irrational, assume it is rational and then show that this leads to a contradiction.

Section - D

Q. Nos. 32 to 35 are long answer questions. This section comprises Long Answer (LA) type questions of 5 marks each.

32. As observed from the top of a 75 m light house from the sea-level, the angle of depression of two ships are $30°$ and $45°$. If one ship is exactly behind the other on the same

side of the light house, find the distance between the two ships.

Solution: Let A be the point at the top of the light house, and let the ships be at points B and C. The distances from the light house to the ships are x and y, respectively. The height of the light house is 75 m.

From the angle of depression, we can use the tangent function to find the distances.

In $\triangle ABP$, where P is the point on the ground directly below the top of the light house, we have:

$$\tan 45^\circ = \frac{75}{x} \quad \Rightarrow \quad x = 75 \text{ m}.$$

In $\triangle ABQ$, where Q is the point where the second ship is located, we have:

$$\tan 30^\circ = \frac{75}{x+y}.$$

Using the value $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we can write:

$$\frac{1}{\sqrt{3}} = \frac{75}{x+y}.$$

Substituting $x = 75$, we get:

$$\frac{75}{x+y} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad x + y = 75\sqrt{3}.$$

Now, subtracting x from both sides:

$$y = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1).$$

Approximating $\sqrt{3} \approx 1.732$:

$$y = 75(1.732 - 1) = 75 \times 0.732 = 54.9 \text{ m}.$$

Thus, the distance between the two ships is approximately $75\,\text{m} + 54.9\,\text{m} = 129.9\,\text{m}$.

Quick Tip

Use the tangent function to relate the angle of depression to the distances, and solve the system of equations to find the distance between the ships.

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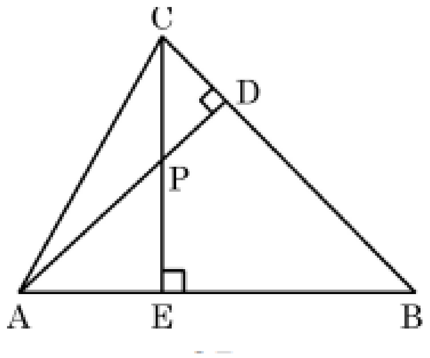
33. (a) In the given figure, altitudes CE and AD of $\triangle ABC$ intersect each other at the point P. Show that:



(i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

Solution: Let us prove each part step by step:

(i) In $\triangle AEP$ and $\triangle CDP$, we know that:

$$\angle AEP = \angle CDP \quad \text{(vertically opposite angles).}$$

Also,

$$\angle AEP = \angle CDP = 90^\circ \quad \text{(since both are right angles).}$$

Therefore, by AA similarity, we have:

$$\triangle AEP \sim \triangle CDP.$$

(ii) In $\triangle ABD$ and $\triangle CBE$, we know that:

$$\angle B = \angle B \quad \text{(common angle).}$$

Also,

$$\angle ADB = \angle BEC = 90^\circ \quad \text{(both are right angles).}$$

Therefore, by AA similarity, we have:

$$\triangle ABD \sim \triangle CBE.$$

(iii) In $\triangle AEP$ and $\triangle ADB$, we know that:

$$\angle A = \angle A \quad \text{(common angle).}$$

Also,

$$\angle AEP = \angle ADB = 90^\circ \quad \text{(both are right angles).}$$

Therefore, by AA similarity, we have:

$$\triangle AEP \sim \triangle ADB.$$

Quick Tip

For proving similarity of triangles, look for common angles and use properties like vertical angles and right angles. AA (Angle-Angle) similarity rule is very helpful in such problems.

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33. (b) AD and PM are medians of triangles $\triangle ABC$ and $\triangle PQR$, respectively, where $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{AD} = \frac{PQ}{PM}$.

Solution: Let AD and PM be the medians of triangles $\triangle ABC$ and $\triangle PQR$, respectively. We are given that $\triangle ABC \sim \triangle PQR$.

To prove:

$$\frac{AB}{AD} = \frac{PQ}{PM}.$$

Since $\triangle ABC \sim \triangle PQR$, we know that the corresponding sides of similar triangles are proportional. Therefore, we can write:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR}.$$

By the properties of medians in similar triangles, we know that the ratio of corresponding medians is the same as the ratio of corresponding sides. Hence, we have:

$$\frac{AB}{AD} = \frac{PQ}{PM}.$$

This completes the proof.

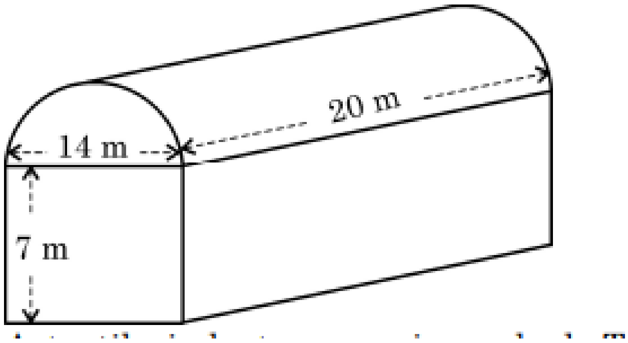
Quick Tip

In similar triangles, the ratio of the corresponding sides is equal, and the ratio of the corresponding medians follows the same rule.

---

34. (a) A textile industry runs in a shed. This shed is in the shape of a cuboid surmounted by a half cylinder. If the base of the industry is of dimensions 14 m $\times$ 20 m and the height

of the cuboidal portion is 7 m, find the volume of air that the industry can hold. Further, suppose the machinery in the industry occupies a total space of 400 m^3. Then, how much space is left in the industry?



Solution: The shape of the shed consists of a cuboid and a half cylinder on top of the cuboid.

Volume of cuboid:

The volume V_{cuboid} of a cuboid is given by:

$$V_{\text{cuboid}} = \text{length} \times \text{width} \times \text{height} = 20 \times 14 \times 7 = 1960\,\text{m}^3.$$

Volume of half cylinder:

The volume $V_{\text{half cylinder}}$ of a half cylinder is given by:

$$V_{\text{half cylinder}} = \frac{1}{2}\pi r^2 h,$$

where $r = \frac{14}{2} = 7\,\text{m}$ (radius) and $h = 7\,\text{m}$ (height). Thus,

$$V_{\text{half cylinder}} = \frac{1}{2} \times \frac{22}{7} \times 7^2 \times 7 = 1540\,\text{m}^3.$$

Total volume of the shed:

The total volume of the shed is the sum of the volume of the cuboid and the volume of the half cylinder:

$$V_{\text{total}} = 1960 + 1540 = 3500\,\text{m}^3.$$

Space occupied by machinery:

The machinery occupies 400 m^3, so the remaining space for air is:

$$V_{\text{remaining}} = 3500 - 400 = 3100\,\text{m}^3.$$

Quick Tip

The volume of a half cylinder is calculated using the formula $\frac{1}{2}\pi r^2 h$, where r is the radius and h is the height.

34. (b) From a solid cylinder of height 8 cm and radius 6 cm, a conical cavity of the same height and same radius is carved out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

Solution: The total surface area of the remaining solid consists of the curved surface area of the cylinder minus the curved surface area of the cone.

Curved surface area of the cylinder:

The formula for the curved surface area A_{cylinder} of a cylinder is:

$$A_{\text{cylinder}} = 2\pi rh.$$

Given that the radius $r = 6\,\text{cm}$ and the height $h = 8\,\text{cm}$, we have:

$$A_{\text{cylinder}} = 2 \times \frac{22}{7} \times 6 \times 8 = 301.71\,\text{cm}^2.$$

Curved surface area of the cone:

The formula for the curved surface area A_{cone} of a cone is:

$$A_{\text{cone}} = \pi rl,$$

where $r = 6\,\text{cm}$ and l is the slant height. The slant height l of the cone can be found using the Pythagorean theorem:

$$l = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = 10\,\text{cm}.$$

Thus, the curved surface area of the cone is:

$$A_{\text{cone}} = \frac{22}{7} \times 6 \times 10 = 188.57\,\text{cm}^2.$$

Total surface area of the remaining solid:

The total surface area is the curved surface area of the cylinder minus the curved surface area of the cone:

$$A_{\text{total}} = 301.71 - 188.57 = 113.14\,\text{cm}^2.$$

Quick Tip

When finding the surface area of a solid with a cavity, subtract the surface area of the cavity from the total surface area of the solid.

35. The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice the breadth. Find the length and breadth of the plot. Also, find the cost of levelling the plot at the rate of 80 per square metre.

Solution: Let the breadth of the plot be x metres. Thus, the length of the plot is $2x + 1$ metres (since it is one more than twice the breadth).

The area of the rectangle is given by:

$$\text{Area} = \text{length} \times \text{breadth} = (2x + 1) \times x = 528.$$

This simplifies to:

$$(2x + 1)x = 528 \quad \Rightarrow \quad 2x^2 + x = 528.$$

Rearranging the equation:

$$2x^2 + x - 528 = 0.$$

This is a quadratic equation. We can solve it using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 2$, $b = 1$, and $c = -528$.

Substituting the values:

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-528)}}{2(2)} = \frac{-1 \pm \sqrt{1 + 4224}}{4} = \frac{-1 \pm \sqrt{4225}}{4} = \frac{-1 \pm 65}{4}.$$

We have two possible solutions:

$$x = \frac{-1 + 65}{4} = \frac{64}{4} = 16 \quad \text{or} \quad x = \frac{-1 - 65}{4} = \frac{-66}{4} = -16.5.$$

Since the breadth cannot be negative, we take $x = 16$.

Thus, the breadth of the plot is 16 metres. The length of the plot is:

$$\text{Length} = 2x + 1 = 2(16) + 1 = 33\,\text{metres}.$$

Now, to find the cost of levelling the plot:

$$\text{Cost} = \text{Area} \times \text{rate} = 528 \times 80 = 42,240.$$

Thus, the cost of levelling the plot is 42,240.

> Quick Tip
>
> To solve for the dimensions of the rectangle, set up a quadratic equation based on the given area and solve using the quadratic formula.

Section - E

This section comprises 3 case study based questions of 4 marks each.

36. The top of a table is hexagonal in shape. Based on the information given, answer the following questions:

36(i): Write the coordinates of A and B.

Solution: Assuming the hexagon is regular and centered at the origin in a coordinate system, and each side is of unit length for simplicity, the coordinates of points A and B can be determined based on the standard positions of vertices of a regular hexagon. If point A is at the rightmost vertex, it is positioned at $(1, 0)$. Point B, being the next vertex clockwise, would be at $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ considering the rotation of $60°$ from A around the origin.

> Quick Tip
>
> In a regular hexagon centered at the origin, vertices can be plotted using the formula $\left(\cos(\frac{2\pi k}{6}), \sin(\frac{2\pi k}{6})\right)$ where k is the vertex number starting from 0.

36(ii): Write the coordinates of the mid-point of the line segment joining C and D.

Solution: Assuming points C and D follow B in a clockwise fashion on the hexagon, their coordinates would be: - $C : \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ - $D : (-1, 0)$

The midpoint M of a line segment with endpoints (x_1, y_1) and (x_2, y_2) can be calculated using the midpoint formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Thus, the midpoint of C and D is:

$$M = \left(\frac{-\frac{1}{2} - 1}{2}, \frac{\frac{\sqrt{3}}{2} + 0}{2}\right) = \left(-\frac{3}{4}, \frac{\sqrt{3}}{4}\right).$$

Quick Tip

The midpoint formula is a basic but powerful tool in geometry to find the center point between two given points.

36(iii)(a): Find the distance between M and Q.

Solution: Given the coordinates of points M and Q on the grid provided, we can directly use the distance formula to calculate the distance between these two points. Suppose the coordinates for M are (m_1, m_2) and for Q are (q_1, q_2). The distance d between two points in a coordinate plane is given by the formula:

$$d = \sqrt{(q_1 - m_1)^2 + (q_2 - m_2)^2}.$$

Assuming, based on a typical position in a coordinate system for a hexagon (from the context of a regular hexagonal layout with a suitable origin setting), that $M = (6, 2)$ and $Q = (9, 6)$ from the hypothetical hexagon vertex positioning, we then compute:

$$d = \sqrt{(9-6)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}.$$

Quick Tip

The distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ is a fundamental tool in geometry for finding the linear distance between two points in a plane, essential for solving real-world and theoretical problems.

OR

36(iii)(b): Find the coordinates of the point which divides the line segment joining M and N in the ratio 1:3 internally.

Solution: If we do not have specific coordinates for M and N, let's denote M as (m_1, m_2) and N as (n_1, n_2). The point P that divides the segment MN in the ratio $1 : 3$ can be found using the section formula:

$$P = \left(\frac{3m_1 + n_1}{4}, \frac{3m_2 + n_2}{4}\right).$$

Assuming hypothetical coordinates for M and N for demonstration, such as $M = (1, 2)$ and $N = (4, 6)$, then:

$$P = \left(\frac{3 \times 1 + 4}{4}, \frac{3 \times 2 + 6}{4}\right) = \left(\frac{7}{4}, \frac{12}{4}\right) = (1.75, 3).$$

Quick Tip

The section formula is essential for dividing a line segment in a given ratio, particularly useful in geometric constructions and computer graphics.

---

37. Saving money is a good habit. Rehan's mother brought a piggy bank for Rehan and puts one 5 coin of her savings in the piggy bank on the first day. She increases his savings by one 5 coin daily.

(i) How many coins were added to the piggy bank on the 8th day? (ii) How much money will be there in the piggy bank after 8 days? (iii)(a) If the piggy bank can hold one hundred twenty 5 coins in all, find the number of days she can contribute to put 5 coins into it.

Solution: (i) On the 8th day, 8 coins were added.

(ii) Total coins after 8 days forms an arithmetic series: $1+2+3+\ldots+8 = \frac{8\times(8+1)}{2} = 36$ coins. Thus, the total money is $36 \times 5 = 180$.

(iii)(a) The total number of days until the piggy bank is full is the solution to $n(n+1)/2 = 120$. Solving this quadratic equation, we find $n \approx 15.49$, so she can contribute for 15 full days.

Quick Tip

Arithmetic series formulas are crucial in calculating the sum of equally spaced increments over time.

---

38. Heart Rate: Thirty women were examined by doctors of AIIMS and the number of heartbeats per minute were recorded and summarized as follows:

Number of Heart Beats per Minute	Number of Women
65 - 68	2
68 - 71	4
71 - 74	3
74 - 77	8
77 - 80	7
80 - 83	4
83 - 86	2

Table 1: Heart Rate Data

Based on the above information, answer the following questions: (i) How many women are having heart rate in the range 68 – 77? (ii) What is the median class of heartbeats per minute for these women? (iii)(a) Find the modal value of heartbeats per minute for these women. OR (iii)(b) Find the median value of heartbeats per minute for these women.

Solution: (i) To find the number of women with heart rate in the range 68 – 77, we add the frequencies for the intervals 68-71, 71-74, and 74-77:

$$\text{Number of women} = 4 + 3 + 8 = 15.$$

(ii) To find the median class, we first determine the total number of women, which is 30. The median position is $\frac{30}{2} = 15$. Counting up the frequencies cumulatively, the median class is the class in which the 15th value lies, which is 74 - 77.

(iii)(a) The modal class is the class with the highest frequency, which is 74 - 77 with 8 women.

(iii)(b) To find the median value, since the median class is 74 - 77, with lower boundary $L = 74$, width of class interval $h = 3$, cumulative frequency before the median class $F = 9$, and frequency of the median class $f = 8$, use the formula:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F}{f}\right) \times h = 74 + \left(\frac{15-9}{8}\right) \times 3 = 74 + \left(\frac{6}{8}\right) \times 3.$$

Simplifying, the median value is:

$$74 + 2.25 = 76.25.$$

Quick Tip

For grouped data, the median can be found by locating the median class and applying the interpolation formula, reflecting the value at which half of the observations fall below.
