SOLVED PAPER – 2016 (COMEDK)

Instructions

- There are 180 questions in all. The number of questions in each section is as given below.

Sections	No. of Questions
Section I : Physics	1–60
Section II : Chemistry	61–120
Section III : Mathematics	121–180

- All the questions are Multiple Choice Questions having four options out of which **ONLY ONE** is correct.
- Candidates will be awarded 1 mark for each correct answer. There will be no negative marking for incorrect answer.
- Time allotted to complete this paper is 3 hrs.

PHYSICS

1. A simple pendulum has a period T inside a lift, when it is stationary. The lift is accelerated upwards with constant acceleration a. The period

a. decreases ***b.*** increases
c. remains same ***d.*** becomes infinite

2. 90 dB sound is x times more intense than 40 dB sound, then x is

a. 5 ***b.*** 50
c. 10^5 ***d.*** 500

3. A star is moving away from the earth with speed v. Change in wavelength $d\lambda$ observed on earth is

a. $\lambda v / c$ ***b.*** $\lambda v / (c + v)$
c. $\lambda c / (c + v)$ ***d.*** $\lambda c / v$

4. An open pipe emits a fundamental frequency n when it emits the 3rd harmonic, then the pipe can accommodate

a. 2 nodes, 2 anti-nodes
b. 3 nodes, 4 anti-nodes
c. 3 nodes, 3 anti-nodes
d. 1 node, 2 anti-nodes

5. In an adiabatic process,

a. temperature remains constant
b. pressure remains constant
c. volume remains constant
d. there is no transfer of heat

6. Carnot heat engine takes 300 J of heat from a source at 627°C and gives some part of it to sink at 27°C. Work done by engine in one cycle is

a. 200 J ***b.*** 300 J ***c.*** 150 J ***d.*** 120 J

7. 15/16th of a radioactive sample disintegrates in 2 h. Mean life of radioactive sample is approximately

a. 30 min ***b.*** 43 min
c. 21 min ***d.*** 15 min

8. Clear images of soft tissues can be well studied using

a. MRI ***b.*** X-rays
c. ultrasonics ***d.*** IR rays

9. Particles which are not composite and hence truly elementary are

a. mesons ***b.*** protons
c. neutrons ***d.*** leptons

10. A logic gate whose output will be in logic 0 state only, when all inputs are in logic 1 state is called
a. AND ***b.*** OR
c. NOR ***d.*** NAND

11. *n*-type and *p*-type semiconductors can be obtained by doping pure silicon, respectively with
a. arsenic, phosphorous
b. indium, aluminium
c. phosphorous, indium
d. aluminium, boron

12. In a CE amplifier, $\beta = 50$, $R_L = 4\,\text{k}\Omega$, $R_i = 500\Omega$. Power gain of the amplifier is
a. 2×10^4 ***b.*** 2×10^2
c. 2×10^3 ***d.*** 2×10^1

13. Electrons are excited from $n = 1$ to $n = 4$ state. During downward transitions, possible number of spectral lines observed in Balmer series is
a. 4 ***b.*** 3
c. 2 ***d.*** 1

14. IR region lies between
a. radiowaves and microwave regions
b. microwaves and visible
c. visible and UV region
d. UV rays and X-ray region

15. A proton and an alpha particle are subjected to same potential difference V. Their de-Broglie wavelengths λ_p, λ_α will be in the ratio
a. 2 : 1 ***b.*** $2\sqrt{2} : 1$
c. 1 : 1 ***d.*** 1 : 2

16. Raman shift depends on
a. incident wavelength
b. incident intensity
c. resolving power of the spectrograph used
d. molecular energy levels of the scatterer

17. ${}_6C^{14}$ and ${}_7N^{15}$ are the examples of
a. isotopes ***b.*** isobars
c. isotones ***d.*** mirror nuclei

18. In an interference experiment, intensity ratio at the bright to dark fringe is 9 : 1. Amplitudes of interfering waves are in the ratio
a. 3 : 1 ***b.*** 9 : 1
c. 2 : 1 ***d.*** 4 : 1

19. In Young's double slit experiment, 1st dark fringe occurs directly opposite to a slit. Wavelength of light used is
a. d^2 / D ***b.*** d / D
c. D^2 / d ***d.*** $2d^2 / D$

20. Newton's ring pattern in reflected system, viewed under white light consists of
a. equally spaced bright and dark bands with central dark spot
b. equally spaced bright and dark bands with central white spot
c. a few coloured rings with central dark spot
d. a few coloured rings with central white spot

21. It is difficult to observe diffraction in case of light waves, because
a. light waves can travel through vacuum
b. speed of light is more
c. light waves are transverse in nature
d. wavelength of light is small

22. A calcite crystal is placed over a dot on a paper sheet and the crystal is rotated. On viewing through the calcite or sees,
a. a single stationary dot
b. two stationary dots
c. two dots rotating about one another
d. one dot rotating about the other stationary dot sometimes coinciding with it

23. Critical angle of the medium is 45°. Polarising angle of incidence at the surface of the medium is
a. 45° ***b.*** 38°
c. 22.5° ***d.*** 54.7°

24. If only 2% of the main current is to be passed through a galvanometer of resistance *G*, then the resistance of shunt should be
a. $G/50$ ***b.*** $G/49$ ***c.*** $50G$ ***d.*** $49G$

25. A small current carrying loop of area A behaves like a tiny magnet of magnetic moment *M*. Current in the loop is
a. MA ***b.*** A / M ***c.*** A^2M ***d.*** M / A

26. Two concentric circular coils, each having 10 turns with radii 0.2 m and 0.4 m carry currents 0.2 A and 0.3 A respectively in opposite directions. Magnetic field at the centre is
a. $(2/3)\mu_0$ ***b.*** $(5/4)\mu_0$
c. $(1/4)\mu_0$ ***d.*** $(1/6)\mu_0$

27. Material of permanent magnet has
a. high retentivity and high coercivity
b. low retentivity and high coercivity
c. low retentivity and low coercivity
d. high retentivity and low coercivity

28. Power factor of a series *L-C-R* circuit is
a. R ***b.*** Z / R
c. R / Z ***d.*** RZ

29. An inductor 1H is connected across 220V, 50 Hz supply. Peak value of current is approximately

a. 0.5 A ***b.*** 0.7 A ***c.*** 1 A ***d.*** 1.4 A

30. Plane polarised light is passed through an analyser and the intensity of emerging light is reduced by 75%. Optical vibrations make an angle θ with the axis of analyser, then θ is

a. 60° ***b.*** 45° ***c.*** 30° ***d.*** 58°

31. A charge 10nC is situated in a medium of relative permittivity 10. The potential due to this charge at a distance of 0.1 m is

a. 900 V ***b.*** 90 V

c. 9 V ***d.*** 0.09 V

32. Dielectric constant of a metal is

a. zero ***b.*** infinite

c. finite ***d.*** unpredictable

33. Distance between the two point charges is increased by 20%. Force of interaction between the charges

a. increases by 10% ***b.*** decreases by 20%

c. decreases by 17% ***d.*** decreases by 31%

34. Potential energy of 2 charge 10 nC each separated by a distance of 0.09 m in air is

a. 10 μJ ***b.*** 1 mJ

c. 10 mJ ***d.*** 10 J

35. A metal plate of thickness d / 2 is introduced in between the plates of a parallel plate air capacitor with plate separation of d, then capacity

a. decreases 2 times ***b.*** increases 2 times

c. remains same ***d.*** becomes zero

36. Specific resistance of a conductor material increases with

a. increase with area of cross-section

b. decrease in length

c. decrease in area of cross-section

d. increases with temperature

37. The resistance of mercury at 4.2 K is

a. infinity

b. greater than at lab temperature

c. same as that of lab temperature

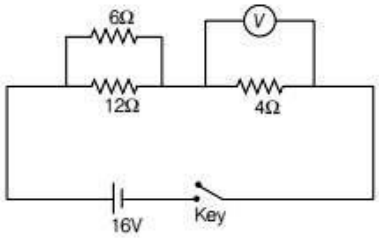
d. almost zero

38. Temperature coefficient of resistance of platinum is 4×10^{-3} / K at 20°C. Temperature at which increase in resistance of platinum is 10% its value at 20°C is

a. 25°C ***b.*** 70°C

c. 45°C ***d.*** 100°C

39. Ideal voltmeter connected as shown reads



a. 16 V ***b.*** 12 V

c. 4 V ***d.*** 8 V

40. When a charged particle moves perpendicular to a uniform magnetic field, then

a. its momentum changes and total energy is same

b. Both momentum and its total energy remain the same

c. Both momentum and its total energy will change

d. total energy changes but momentum remains same

41. 0.04 m of glass contains the same number of waves as 0.05 m of water, when monochromatic light passes through them normally. Refractive index of water is 4/3. Refractive index of glass is

a. 5/3 ***b.*** 5/4 ***c.*** 5/2 ***d.*** 4/5

42. Critical angle will be maximum, when light travels from

a. glass to air ***b.*** glass to water

c. water to air ***d.*** diamond to air

43. A ray of light incident on one face of an equilateral prism at 60° enters and leaves the prism symmetrically. Refractive index of the prism material is

a. 1.5 ***b.*** 1.62 ***c.*** 1.73 ***d.*** 1.8

44. In the spectrum of visible light produced by a prism dispersion is

a. uniform throughout the spectrum

b. maximum in the middle decreases on either sides

c. maximum towards yellow

d. maximum towards violet

45. Convex lens of focal length f made of glass of refractive index 1.5 is immersed in water of refractive index 4/3. Focal length is

a. f ***b.*** greater than f

c. less than f ***d.*** $-f$

46. Two co-axial lenses of power + 4D and – 2D are placed in contact. The focal length of combination is

a. 0.5 m ***b.*** 0.25 m

c. 0.16 m ***d.*** – 0.5 m

47. Eddy currents are produced in a material, when it is

a. heated

b. placed in a time varying magnetic field

c. placed in an electric field

d. placed in a uniform magnetic field

48. Transformer works on 220 V and its efficiency is 80%. If output power is 8 kW, primary current is approximately

a. 35 A ***b.*** 18 A

c. 22 A ***d.*** 45 A

49. Quality factor of a series *L*-*C*-*R* circuit decreases from 3 to 2. Resonant frequency is 600 Hz. Change in bandwidth is

a. zero ***b.*** 100 Hz increase

c. 100 Hz decrease ***d.*** 300 Hz increase

50. A stone dropped from the top of the tower reaches ground in 4 s. Height of the tower is $(g = 10\ \text{m/s}^2)$

a. 20 m ***b.*** 40 m

c. 60 m ***d.*** 80 m

51. Liquid crystal phase which are more close to the solid than to liquid is

a. Nematic ***b.*** Smectic

c. Lyotropic ***d.*** Cholesteric

52. If the earth shrinks in its size (radius) mass remaining the same, the value of *g* on its surface will

a. increase ***b.*** decrease

c. remains same ***d.*** reduce to zero

53. Two rods of same area of cross-section and lengths and conductivities K_1 and K_2 are connected in series. Then in steady state, conductivity of the combination is

a. $(K_1 + K_2)/(K_1K_2)$

b. $2K_1K_2/(K_1 + K_2)$

c. $(K_1 + K_2)/2$

d. $K_1K_2/(K_1 + K_2)$

54. The square of the resultant of two equal forces acting at a point is equal to three times their product. Angle between them is

a. 30° ***b.*** 45°

c. 60° ***d.*** 90°

55. With the addition of impurities, surface tension of a liquid

a. increases

b. decreases

c. remains constant

d. may increase or decrease depending on impurities

56. Viscosity decreases with increase in temperature is the reason for

(i) hot water moving faster than cold water.

(ii) more viscous oils are used in motorcars during summer than in winter

a. Only (i) is correct ***b.*** Only (ii) correct

c. Both are correct ***d.*** Both are incorrect

57. Moment of momentum of an electron revolving in second Bohr orbit of hydrogen is

a. $2\pi h$ ***b.*** $h/2\pi$

c. h/π ***d.*** $2h/3\pi$

58. The existence of excitation and ionisation energies in an atom is an evidence for

a. stability of an atom

b. electrical neutrality of an atom

c. small size of the atom

d. stationary orbits in an atom

59. Work function of a photosensitive metal is 3 eV. The wavelength of incident radiations which can just eject photoelectrons from the metal is

a. 600 nm ***b.*** 510 nm

c. 414 nm ***d.*** 378 nm

60. Three identical capacitors are first connected in series and then in parallel. The ratio of effective capacitances in the two cases is

a. 9 : 1 ***b.*** 3 : 1

c. 1 : 3 ***d.*** 1 : 9

CHEMISTRY

61. A nitrogen containing organic compound gave an oily liquid on heating with bromine and potassium hydroxide solution. On shaking the product with acetic anhydride, an antipyretic drug was obtained. The reactions indicate that the starting compound is
a. acetamide ***b.*** nitrobenzene
c. aniline ***d.*** benzamide

62. The silver salt of a fatty acid on refluxing with an alkyl halide gives an
a. ether ***b.*** amine
c. acid ***d.*** ester

63. Pick out the one which does not belong to the family
a. ptyalin ***b.*** lipase
c. pepsin ***d.*** cellulose

64. Which of the following is wrongly matched?
a. Decomposition of H_2O_2–first order reaction.
b. Combination of H_2 and Br_2 to give HBr–zero order reaction.
c. Saponification of $CH_3COOC_2H_5$–second order reaction.
d. Hydrolysis of CH_3COOCH_3–pseudo unimolecular reaction.

65. The diameter of colloidal particle ranges from
a. 10^3 m to 10^{-3} m
b. 10^{-3} m to 10^{-6} m
c. 10^{-6} m to 10^{-9} m
d. 10^{-9} m to 10^{-12} m

66. The number of $2p$ electrons having spin quantum number $s = -1/2$ are
a. 2 ***b.*** 3 ***c.*** 6 ***d.*** 0

67. Pick out the alkane which differs from the other members of the group.
a. 2-methyl butane ***b.*** 2, 2-dimethyl butane
c. 2, 2-dimethyl propane ***d.*** Pentane

68. 56 g of nitrogen and 8 g of hydrogen gas are heated in a closed vessel. At equilibrium 34 g of ammonia are present. The equilibrium number of moles of nitrogen, hydrogen and ammonia are respectively
a. 1, 1, 2 ***b.*** 2, 1, 2
c. 1, 2, 2 ***d.*** 2, 2, 1

69. A process is taking place at constant temperature and pressure. Then,
a. $\Delta H = 0$ ***b.*** $\Delta S = 0$
c. $\Delta H = \Delta E$ ***d.*** $\Delta H = T\Delta S$

70. In a galvanic cell, the electrons flow from
a. anode to cathode through the external circuit.
b. cathode to anode through the external circuit.
c. anode to cathode through the solution.
d. cathode to anode through the solution.

71. On treating a mixture of two alkyl halides with sodium metal in dry ether, 2-methyl propane was obtained. The alkyl halides are
a. chloromethane and chloroethane
b. chloromethane and 1-chloropropane
c. 2-chloropropane and chloromethane
d. 2-chloropropane and chloroethane

72. Which of the following statements about benzyl chloride is incorrect?
a. It is a lachrymatory liquid and answers Beilstein's test.
b. It gives a white precipitate with alcoholic silver nitrate.
c. It is less reactive than alkyl halides.
d. It can be oxidised to benzaldehyde by boiling with copper nitrate solution.

73. The main product obtained when a solution of sodium carbonate reacts with mercuric chloride is
a. $HgCO_3$ ***b.*** $HgCO_3$, $Hg(OH)_2$
c. $Hg(OH)_2$ ***d.*** $HgCO_3$, HgO

74. In the electrothermal process, the compound displaced by silica from calcium phosphate is
a. phosphorus ***b.*** phosphorus pentoxide
c. calcium phosphide ***d.*** phosphine

75. The enthalpy of combustion of methane at 25°C is 890 kJ. The heat liberated when 3.2 g of methane is burnt in air is
a. − 890 kJ ***b.*** 178 kJ
c. 445 kJ ***d.*** 278 kJ

76. The pressure and temperature of 4 dm^3 of carbon dioxide gas are doubled. Then, the volume of carbon dioxide gas would be
a. 4 dm^3 ***b.*** 8 dm^3
c. 2 dm^3 ***d.*** 3 dm^3

77. 4 g of copper was dissolved in concentrated nitric acid. The copper nitrate solution on strong heating gave 5g of its oxide. The equivalent weight of copper is
a. 12 ***b.*** 20
c. 23 ***d.*** 32

78. In the manufacture of ammonia by the Haber's process,
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + 92.3$ kJ, which of the following conditions is unfavourable?
a. Reducing the temperature
b. Removing ammonia as it is formed
c. Increasing the temperature
d. Increasing the pressure

79. The chemical equilibrium of a reversible reaction is not influenced by
a. concentration of the reactants
b. temperature
c. pressure
d. catalyst

80. Cumene process is the most important commercial method for the manufacture of phenol. Cumene is
a. vinyl benzene
b. propyl benzene
c. 1-methyl ethyl benzene
d. ethyl benzene

81. A solution contains 1.2046×10^{24} hydrochloric acid molecules in one dm^3 of the solution. The strength of the solution is
a. 4N *b.* 8N *c.* 6N *d.* 2N

82. Nuclear theory of the atom was put forward by
a. Neils Bohr *b.* J.J. Thomson
c. Rutherford *d.* Aston

83. In acetylene molecule, the two carbon atoms are linked by
a. three σ-bonds
b. three π-bonds
c. one σ-bond and two π-bonds
d. two σ-bonds and one π-bond

84. The enthalpy of the reaction,
$H_2(g) + 1/2\,O_2(g) \longrightarrow H_2O(g)$ is ΔH_1 and that of
$H_2(g) + 1/2O_2(g) \longrightarrow H_2O(l)$ is ΔH_2. Then
a. $\Delta H_1 > \Delta H_2$ *b.* $\Delta H_1 = \Delta H_2$
c. $\Delta H_1 < \Delta H_2$ *d.* $\Delta H_1 + \Delta H_2 = 0$

85. A radioactive isotope decays at such a rate that after 192 minutes only $\frac{1}{16}$ of the original amount remains. The half-life of the radioactive isotope is
a. 12 min *b.* 24 min *c.* 32 min *d.* 48 min

86. The reagent which does not give acid chloride on treating with a carboxylic acid is
a. $SOCl_2$ *b.* PCl_3
c. PCl_5 *d.* Cl_2

87. Among the halogens, the one which is oxidised by nitric acid is
a. chlorine *b.* bromine
c. fluorine *d.* iodine

88. The metal which does not form ammonium nitrate by reaction with dilute nitric acid is
a. Pb *b.* Mg *c.* Al *d.* Fe

89. The elements with atomic numbers 9, 17, 35, 53, 85 are all
a. heavy metals *b.* light metals
c. noble gases *d.* halogens

90. In the electrolytic method of obtaining aluminium from purified bauxite, cryolite is added to the charge in order to
a. dissolve bauxite and render it conductor of electricity.
b. lower the melting point of bauxite.
c. minimise the heat loss due to radiation.
d. protect aluminium produced from oxygen.

91. Which of the following is not an amphoteric substance?
a. H_2O *b.* NH_3 *c.* HNO_3 *d.* HCO_3^-

92. When 50 cm^3 of 0.2 N H_2SO_4 is mixed with 50 cm^3 of 1N KOH, the heat liberated is
a. 573 kJ *b.* 573 J
c. 11.46 kJ *d.* 57.3 kJ

93. An artificial radioactive isotope gave $^{14}_{7}N$ after two successive β-particle emissions. The number of neutrons in the parent nucleus must be
a. 5 *b.* 7 *c.* 9 *d.* 14

94. Stainless steel does not rust because
a. nickel present in it, does not rust
b. iron forms a hard chemical compound with chromium present in it
c. chromium and nickel combine with iron
d. chromium forms an oxide layer and protects iron from rusting

95. Which of the following combinations can be used to synthesised ethanol?
a. CH_3MgI and $CH_3COOC_2H_5$
b. CH_3MgI and HCHO
c. CH_3MgI and CH_3COCH_3
d. CH_3MgI and C_2H_5OH

96. The reaction, $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ is carried out in a 1 dm^3 vessel and 2 dm^3 vessel separately. The ratio of the reaction velocities will be
a. 4 : 1 *b.* 8 : 1 *c.* 1 : 8 *d.* 1 : 4

97. In a mixture of acetic acid and sodium acetate the ratio of concentrations of the salt to the acid is increased ten times. Then the pH of the solution

a. decreases ten fold ***b.*** increases ten fold
c. increases by one ***d.*** decreases by one

98. When a mixture of methane and oxygen is passed through heated molybdenum oxide, the main product formed is

a. methanol ***b.*** methanal
c. methanoic acid ***d.*** ethanal

99. Benzene can be obtained by heating either benzoic acid with *X* or phenol with *Y*. *X* and *Y* are respectively

a. zinc dust and sodium hydroxide
b. soda lime and copper
c. zinc dust and soda lime
d. soda lime and zinc dust

100. An organic compound is boiled with alcoholic potash. The product is cooled and acidified with HCl. A white solid separates out. The starting compound may be

a. ethyl acetate ***b.*** methyl acetate
c. ethyl benzoate ***d.*** ethyl formate

101. In qualitative analysis, in order to detect second group basic radical, H_2S gas is passed in the presence of dilute HCl to

a. decrease the dissociation of H_2S
b. increase the dissociation of salt solution
c. increase the dissociation of H_2S
d. decrease the dissociation of salt solution

102. Aluminium displaces hydrogen from dilute HCl, whereas silver does not. The emf of a cell prepared by combining Al / Al^{3+} and Ag / Ag^{+} is 2.46 V. The reduction potential of silver electrode is + 0.80 V. The reduction potential of aluminium electrode is

a. 3.26 V ***b.*** – 1.66V
c. + 1.66 V ***d.*** – 3.26 V

103. The first fraction obtained during the fractionation of petroleum is

a. gasoline ***b.*** diesel oil
c. hydrocarbon gases ***d.*** kerosene oil

104. Which of the following compounds gives trichloromethane on distilling with bleaching powder?

a. Ethanol ***b.*** Methanol
c. Methanal ***d.*** Phenol

105. Benzoin is

a. α-hydroxy aldehyde
b. α-hydroxy ketone
c. compound containing an aldehyde and a ketonic group
d. α, β- unsaturated acid

106. The velocity constant of a reaction at 290 K was found to be $3.2 \times 10^{-3} s^{-1}$. When the temperature is raised to 310 K, it will be about

a. 9.6×10^{-3} ***b.*** 1.28×10^{-2}
c. 6.4×10^{-3} ***d.*** 3.2×10^{-4}

107. Select the pK_a value of the strongest acid from the following.

a. 2.0 ***b.*** 4.5
c. 1.0 ***d.*** 3.0

108. Pick out the unsaturated fatty acid from the following

a. oleic acid ***b.*** palmitic acid
c. stearic acid ***d.*** lauric acid

109. Nylon is not a

a. copolymer ***b.*** homopolymer
c. condensation polymer ***d.*** polyamide

110. The coaltar fraction which contains phenol is

a. heavy oil ***b.*** light oil
c. middle oil ***d.*** green oil

111. The compounds *A* and *B* are mixed in equimolar proportion to form the products, $A + B \rightleftharpoons C + D$. At equilibrium, one-third of *A* and *B* are consumed. The equilibrium constant for the reaction is

a. 2.5 ***b.*** 0.25
c. 0.5 ***d.*** 4.0

112. In froth floatation process for the purification of ores, the particles of ore float because

a. they are insoluble
b. they bear electrostatic charge
c. their surface is not easily wetted by water
d. they are light

113. Which of the following statements about amorphous solids is incorrect?

a. There is no orderly arrangement of particles.
b. They are rigid and incompressible.
c. They melt over a range of temperature.
d. They are anisotropic.

114. Hydrogen diffuses six times faster than gas *A*. The molar mass of gas *A* is

a. 24 ***b.*** 36 ***c.*** 72 ***d.*** 6

115. Dulong and Petit's law is valid only for
a. gaseous elements *b.* solid elements
c. metals *d.* non-metals

116. Identify the gas which is readily adsorbed by activated charcoal
a. H_2 *b.* O_2
c. N_2 *d.* SO_2

117. If the distance between Na^+ and Cl^- ions in sodium chloride crystal is x pm, the length of the edge of the unit cell is
a. $\frac{x}{2}$ pm *b.* $2x$ pm
c. $4x$ pm *d.* $\frac{x}{4}$ pm

118. Which of the following statements is incorrect?
a. In $K_4[Fe(CN)_6]$ the ligand has satisfied both primary and secondary valencies of ferrous ion.
b. In $[Cu(NH_3)_4]SO_4$, the ligand has satisfied only the secondary valency of copper.
c. In $K_3[Fe(CN)_6]$, the ligand has satisfied only the secondary valency of ferric ion.
d. In $K_3[Fe(CN)_6]$, the ligand has satisfied both primary and secondary valencies of ferric ion.

119. 2-acetoxy benzoic acid is used as an
a. antiseptic
b. antipyretic
c. antimalarial
d. antidepressant

120. A nucleoside on hydrolysis gives
a. an aldopentose and a heterocyclic base
b. an aldopentose and orthophosphoric acid
c. a heterocyclic base and orthophosphoric acid
d. an aldopentose, a heterocyclic base and orthophosphoric acid

MATHEMATICS

121. If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, e\}$, then $(A - B) \times (B \cap C) =$
a. $\{(a, c), (a, d), (b, d)\}$
b. $\{(c, a), (d, a)\}$
c. $\{(a, b), (c, d)\}$
d. $\{(a, c), (a, d)\}$

122. The function $f : X \to Y$ defined by $f(x) = \sin x$ is one-one but not onto if X and Y are respectively, is equal to
a. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$ *b.* $\left[0, \frac{\pi}{2}\right]$ and $[-, 1, 1]$
c. $[0, \pi]$ and $[0, 1]$ *d.* R and R

123. If $\log_4 2 + \log_4 4 + \log_4 16 + \log_4 x = 6$, then $x =$
a. 32 *b.* 8
c. 4 *d.* 64

124. If $S_n = \frac{1}{6 \cdot 11} + \frac{1}{11 \cdot 16} + \frac{1}{16 \cdot 21} + \ldots$ to n terms, then $6S_n =$
a. $\frac{1}{(5n + 6)}$ *b.* $\frac{(2n - 1)}{5n + 6}$
c. $\frac{n}{(5n + 6)}$ *d.* $\frac{5n - 4}{5n + 6}$

125. The remainder obtained when $(1!)^2 + (2!)^2 + (3!)^2 + \ldots + (100!)^2$ is divided by 10^2 is
a. 14 *b.* 17 *c.* 28 *d.* 27

126. If $(p \wedge \sim r) \to (\sim p \vee q)$ is false, then the truth values of p, q and r are respectively
a. T, F and T *b.* F, T and T
c. F, F and T *d.* T, F and F

127. If α, β and γ are the roots of the equation $x^3 - 8x + 8 = 0$, then $\Sigma\alpha^2$ and $\Sigma\frac{1}{\alpha\beta}$ are respectively is equal to
a. 16 and 0
b. – 16 and 0
c. 16 and 8
d. 0 and – 16

128. The gcd of 1080 and 675 is
a. 125 *b.* 225
c. 135 *d.* 145

129. If $a \mid (b + c)$ and $a \mid (b - c)$, where $a, b, c \in N$, then
a. $c^2 \equiv a^2 (\text{mod } b^2)$
b. $a^2 \equiv b^2 (\text{mod } c^2)$
c. $a^2 + c^2 = b^2$
d. $b^2 \equiv c^2 (\text{mod } a^2)$

130. If a, b and $c \in N$, which one of the following is not true?
a. $a \mid b$ and $a \mid c \Rightarrow a \mid b + c$
b. $a \mid b + c \Rightarrow a \mid b$ and $a \mid c$
c. $a \mid b$ and $b \mid c \Rightarrow a \mid c$
d. $a \mid b$ and $a \mid c \Rightarrow a \mid 3b + 2c$

131. If $2A+3B=\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix}$ and $A+2B=\begin{bmatrix}5 & 0 & 3\\ 1 & 6 & 2\end{bmatrix}$, then $B=$

a. $\begin{bmatrix}8 & 1 & 2\\ 1 & 10 & 1\end{bmatrix}$ *b.* $\begin{bmatrix}8 & 1 & -2\\ -1 & 10 & -1\end{bmatrix}$

c. $\begin{bmatrix}8 & 1 & 2\\ -1 & 10 & -1\end{bmatrix}$ *d.* $\begin{bmatrix}8 & -1 & 2\\ -1 & 10 & -1\end{bmatrix}$

132. If $O(A)=2\times 3$, $O(B)=3\times 2$, and $O(C)=3\times 3$, which one of the following is not defined?

a. $C(A+B')$ *b.* $C(A+B')'$

c. BAC *d.* $CB+A'$

133. If $A=\begin{bmatrix}1 & -3\\ 2 & k\end{bmatrix}$ and $A^2-4A+10I=A$, then $k=$

a. 1 or 4 *b.* 4 and not 1

c. -4 *d.* 0

134. The value of $\begin{vmatrix}x+y & y+z & z+x\\ x & y & z\\ x-y & y-z & z-x\end{vmatrix}=$

a. 0

b. $(x+y+z)^3$

c. $2(x+y+z)^3$

d. $2(x+y+z)^2$

135. On the set Q of all rational numbers the operation * which is both associative and commutative is given by $a*b=$

a. $2a+3b$ *b.* $ab+1$

c. a^2+b^2 *d.* $a+b+ab$

136. In the group $G=\{1, 5, 7, 11\}$ under multiplication modulo 12, the solution of $7^{-1}\underset{12}{\times}(x\underset{12}{\times}11)=5$ is $x=$

a. 11 *b.* 7 *c.* 1 *d.* 5

137. A subset of the additive group of real numbers which is not a sub group, is

a. $(Q, +)$ *b.* $(N, +)$

c. $(Z, +)$ *d.* $(\{0\}, +)$

138. If $\mathbf{p}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$, $\mathbf{q}=4\hat{\mathbf{k}}-\hat{\mathbf{j}}$ and $\mathbf{r}=\hat{\mathbf{i}}+\hat{\mathbf{k}}$, then the unit vector in tile direction of $3\mathbf{p}+\mathbf{q}-2\mathbf{r}$ is

a. $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

b. $\frac{1}{3}(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

c. $\frac{1}{3}(\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

d. $\frac{1}{3}(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

139. If $\mathbf{a}$ and $\mathbf{b}$ are the two vectors such that $|\mathbf{a}|=3\sqrt{3}$, $|\mathbf{b}|=4$ and $|\mathbf{a}+\mathbf{b}|=\sqrt{7}$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is

a. 150° *b.* 30°

c. 60° *d.* 120°

140. If $\mathbf{a}$ is vector perpendicular to both $\mathbf{b}$ and $\mathbf{c}$, then

a. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=0$ *b.* $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=0$

c. $\mathbf{a}\times(\mathbf{b}+\mathbf{c})=0$ *d.* $\mathbf{a}+(\mathbf{b}+\mathbf{c})=0$

141. If the area of the parallelogram with $\mathbf{a}$ and $\mathbf{b}$ as two adjacent sides is 15 sq units, then the area of the parallelogram having $3\mathbf{a}+2\mathbf{b}$ and $\mathbf{a}+3\mathbf{b}$ as two adjacent sides in sq units is

a. 45 *b.* 75

c. 105 *d.* 120

142. The locus of the point which moves such that the ratio of its distances from two fixed points in the plane is always a constant $k(<1)$ is

a. circle *b.* straight line

c. ellipse *d.* hyperbola

143. If the lines $x+3y-9=0$, $4x+by-2=0$ and $2x-y-4=0$ are concurrent, then $b=$

a. 0 *b.* 1 *c.* 5 *d.* -5

144. The lines represented by $ax^2+2hxy+by^2=0$ are perpendicular to each other if

a. $h=0$ *b.* $h^2=ab$

c. $a+b=0$ *d.* $h^2=a+b$

145. The equation of the circle having $x-y-2=0$ and $x-y+2=0$ as two tangents and $x+y=0$ as a diameter is

a. $x^2+y^2=1$

b. $x^2+y^2=2$

c. $x^2+y^2-2x+2y-1=0$

d. $x^2+y^2+2x-2y+1=0$

146. If the length of the tangent from any point on the circle $(x-3)^2+(y+2)^2=5r^2$ to the circle $(x-3)^2+(y+2)^2=r^2$ is 16 units, then the area between the two circles in sq units is

a. 16π *b.* 8π

c. 4π *d.* 32π

147. The circles $ax^2+ay^2+2g_1x+2f_1y+c_1=0$ and $bx^2+by^2+2g_2x+2f_2y+c_2=0$ $(a\neq 0$ and $b\neq 0)$ cut orthogonally if

a. $g_1g_2+f_1f_2=c_1+c_2$

b. $b\,g_1g_2+a\,f_1f_2=b\,c_1+ac_2$

c. $g_1g_2+f_1f_2=b\,c_1+a\,c_2$

d. $g_1g_2+f_1f_2=a\,c_1+b\,c_2$

148. The equation of the common tangent of the two touching circles, $y^2 + x^2 - 6x - 12y + 37 = 0$ and $x^2 + y^2 - 6y + 7 = 0$ is

a. $x + y + 5 = 0$ ***b.*** $x + y - 5 = 0$

c. $x - y + 5 = 0$ ***d.*** $x - y - 5 = 0$

149. The equation of the parabola with vertex at $(-1, 1)$ and focus $(2, 1)$ is

a. $y^2 - 2y - 12x + 13 = 0$

b. $y^2 - 2y - 12x + 11 = 0$

c. $x^2 + 2x - 12y + 13 = 0$

d. $y^2 - 2y - 12x - 11 = 0$

150. The equation of the line which is tangent to both the circle $x^2 + y^2 = 5$ and the parabola $y^2 = 40x$ is

a. $2x + y + 5 = 0$ ***b.*** $2x - y - 5 = 0$

c. $2x - y + 5 = 0$ ***d.*** $2x - y \pm 5 = 0$

151. $x = 4(1 + \cos\theta)$ and $y = 3(1 + \sin\theta)$ are the parametric equations of

a. $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$

b. $\frac{(x-4)^2}{16} - \frac{(y-3)^2}{9} = 1$

c. $\frac{(x+4)^2}{16} + \frac{(y+3)^2}{9} = 1$

d. $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1$

152. If the distance between the foci and the distance between the directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3 : 2, then $a : b$ is

a. 2 : 1 ***b.*** 1 : 2

c. $\sqrt{3} : \sqrt{2}$ ***d.*** $\sqrt{2} : 1$

153. The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ have in common

a. centre and vertices only

b. centre, foci and vertices

c. centre, foci and directrices

d. centre only

154. If $\sec\theta = m$ and $\tan\theta = n$, then

$$\frac{1}{m}\left[(m+n) + \frac{1}{(m+n)}\right] =$$

a. mn ***b.*** $2n$ ***c.*** $2m$ ***d.*** 2

155. The value of $\frac{\sin 85^\circ - \sin 15^\circ}{\cos 65^\circ} =$

a. 0 ***b.*** 1 ***c.*** -1 ***d.*** 2

156. From an aeroplane flying, vertically above a horizontal road, the angles of depression of two consecutive stones on the same side of the aeroplane are observed to be 30° and 60° respectively. The height at which the aeroplane is flying in km, is

a. 2 ***b.*** $\frac{2}{\sqrt{3}}$ ***c.*** $\frac{\sqrt{3}}{2}$ ***d.*** $\frac{4}{\sqrt{3}}$

157. If the angles of a triangle are in the ratio 3 : 4 : 5, then the sides are in the ratio

a. 3 : 4 : 5 ***b.*** $2 : \sqrt{3} : \sqrt{3} + 1$

c. $\sqrt{2} : \sqrt{6} : \sqrt{3} + 1$ ***d.*** $2 : \sqrt{6} : \sqrt{3} + 1$

158. If $\cos^{-1} x = \alpha, (0 < x < 1)$ and $\sin^{-1}(2x\sqrt{1-x^2}) + \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \frac{2\pi}{3}$, then $\tan^{-1}(2x) =$

a. $\frac{\pi}{2}$ ***b.*** $\frac{\pi}{3}$ ***c.*** $\frac{\pi}{4}$ ***d.*** $\frac{\pi}{6}$

159. If $a > b > 0$, then the value of $\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{a+b}{a-b}\right)$ depends on

a. neither a nor b ***b.*** a and not b

c. b and not a ***d.*** both a and b

160. Which one of the following equations has no solution?

a. $\sqrt{3}\sin\theta - \cos\theta = 2$

b. $\cos\theta + \sin\theta = \sqrt{2}$

c. $\operatorname{cosec}\theta \cdot \sec\theta = 1$

d. $\operatorname{cosec}\theta - \sec\theta = \operatorname{cosec}\theta \cdot \sec\theta$

161. The complex number $\frac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$, when represented in the Argand diagram lies

a. on the X-axis (Real axis)

b. on the Y-axis (Imaginary axis)

c. in the first quadrant

d. in the second quadrant

162. If $2x = -1 + \sqrt{3}i$, then the value of $(1 - x^2 + x)^6 - (1 - x + x^2)^6 =$

a. 0 ***b.*** 64 ***c.*** -64 ***d.*** 32

163. The modulus and amplitude of $(1 + i\sqrt{3})^8$ are respectively

a. 256 and $\frac{8\pi}{3}$ ***b.*** 2 and $\frac{2\pi}{3}$

c. 256 and $\frac{2\pi}{3}$ ***d.*** 256 and $\frac{\pi}{3}$

164. The value of $\lim_{x \to 0} \frac{5^x - 5^{-x}}{2x} =$

a. $2\log 5$ *b.* 1

c. 0 *d.* $\log 5$

165. Which one of the following is not true always?

a. If a function $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x)$ exists.

b. If $f(x)$ and $g(x)$ are differentiable at $x = a$, then $f(x) + g(x)$ is also differentiable at $x = a$.

c. If $f(x)$ is continuous at $x = a$, then it is differentiable at $x = a$.

d. If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$.

166. If $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots \infty$ with $|x| > 1$, then $\frac{dy}{dx} =$

a. $\frac{-y^2}{x^2}$ *b.* $\frac{y^2}{x^2}$ *c.* x^2y^2 *d.* $\frac{x^2}{y^2}$

167. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then $f'(x) =$

a. $3x^2 + \frac{3}{x^4}$ *b.* $1 + \frac{1}{x^2}$

c. $x^2 - \frac{1}{x^2}$ *d.* $3x^2 - 3$

168. The derivative of $a^{\sec x}$ w.r.t. $a^{\tan x} (a > 0)$ is

a. $a^{\sec x - \tan x}$ *b.* $\sin x a^{\sec x - \tan x}$

c. $\sin x a^{\tan x - \sec x}$ *d.* $\sec x\, a^{\sec x - \tan x}$

169. If $\sin(x + y) + \cos(x + y) = \log(x + y)$, then $\frac{d^2y}{dx^2} =$

a. 1 *b.* -1 *c.* 0 *d.* $\frac{-y}{x}$

170. If $f(x)$ is a function such that $f''(x) + f(x) = 0$ and $g(x) = [f(x)]^2 + [f'(x)]^2$ and $g(3) + 8$, then $g(8) =$

a. 8 *b.* 3

c. 0 *d.* 5

171. If the curve $y = 2x^3 + ax^2 + bx + c$ passes through the origin and the tangents drawn to it at $x = -1$ and $x = 2$ are parallel to the X-axis, then the values of a, b and c are respectively,

a. $3, -12$ and 0 *b.* $-3, 12$ and 0

c. $-3, -12$ and 0 *d.* $12, -3$ and 0

172. A circular sector of perimeter 60 m with maximum area is to be constructed, The radius of the circular arc in metre must be

a. 10 *b.* 15

c. 5 *d.* 20

173. The tangent and the normal drawn to the curve $y = x^2 - x + 4$ at $P(1, 4)$ cut the X-axis at A and B respectively. If the length of the subtangent drawn to the curve at P is equal to the length of the subnormal, then the area of the ΔPAB in square units is,

a. 16 *b.* 8 *c.* 32 *d.* 4

174. $\int \frac{(x^3 + 3x^2 + 3x + 1)}{(x + 1)^5} dx =$

a. $\tan^{-1} x + c$ *b.* $\log(x + 1) + c$

c. $\frac{1}{5}\log(x + 1) + c$ *d.* $-\frac{1}{(x + 1)} + c$

175. $\int \frac{\operatorname{cosec} x}{\cos^2\left(1 + \log \tan \frac{x}{2}\right)} dx =$

a. $-\tan[1 + \log \tan x/2] + c$

b. $\sec^2[1 + \log \tan x/2] + c$

c. $\tan[1 + \log \tan x/2] + c$

d. $\sin^2[1 + \log \tan x/2] + c$

176. $\int \frac{dx}{x\sqrt{x^6 - 16}}$

a. $\sec^{-1}\left(\frac{x^3}{4}\right) + c$ *b.* $\frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right) + c$

c. $\cosh^{-1}\left(\frac{x^3}{4}\right) + c$ *d.* $\frac{1}{3}\sec^{-1}\left(\frac{x^3}{4}\right) + c$

177. If $I_1 = \int_0^{\pi/2} x \sin x\, dx$ and $I_2 = \int_0^{\pi/2} x \cos x\, dx$, then which one of the following is true?

a. $I_1 = I_2$ *b.* $I_1 + I_2 = 0$

c. $I_1 = \frac{\pi}{2} I_2$ *d.* $I_1 + I_2 = \frac{\pi}{2}$

178. If $f(x)$ is defined in $[-2, 2]$ by $f(x) = 4x^2 - 3x + 1$ and $g(x) = \frac{f(-x) - f(x)}{(x^2 + 3)}$, then $\int_{-2}^{2} g(x)\, dx =$

a. 24 *b.* 0 *c.* -48 *d.* 64

179. The area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$ in sq units =

a. $\frac{4}{3}$ *b.* $\frac{2}{3}$ *c.* $\frac{1}{3}$ *d.* $\frac{8}{3}$

180. The solution of the differential equation $e^{-x}(y + 1)\, dy + (\cos^2 x - \sin 2x) y\, (dx) = 0$ subjected to the condition that $y = 1$, when $x = 0$ is

a. $(y + 1) + e^x \cos^2 x = 2$

b. $y + \log y = e^x \cos^2 x$

c. $\log(y + 1) + e^x \cos^2 x = 1$

d. $y + \log y + e^x \cos^2 x = 2$

ANSWERS

Physics

1. (a)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (d)
11. (c)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (a)	**20.** (a)
21. (d)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (c)
31. (b)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (b)	**36.** (d)	**37.** (d)	**38.** (c)	**39.** (d)	**40.** (a)
41. (a)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (b)	**46.** (a)	**47.** (b)	**48.** (d)	**49.** (b)	**50.** (d)
51. (b)	**52.** (a)	**53.** (b)	**54.** (c)	**55.** (d)	**56.** (c)	**57.** (c)	**58.** (a)	**59.** (c)	**60.** (d)

Chemistry

61. (d)	**62.** (d)	**63.** (d)	**64.** (b)	**65.** (c)	**66.** (b)	**67.** (b)	**68.** (a)	**69.** (a)	**70.** (a)
71. (c)	**72.** (c)	**73.** (d)	**74.** (b)	**75.** (b)	**76.** (a)	**77.** (d)	**78.** (c)	**79.** (d)	**80.** (c)
81. (d)	**82.** (c)	**83.** (c)	**84.** (c)	**85.** (d)	**86.** (d)	**87.** (d)	**88.** (a)	**89.** (d)	**90.** (b)
91. (c)	**92.** (b)	**93.** (c)	**94.** (d)	**95.** (b)	**96.** (b)	**97.** (c)	**98.** (b)	**99.** (d)	**100.** (c)
101. (a)	**102.** (b)	**103.** (c)	**104.** (a)	**105.** (b)	**106.** (b)	**107.** (c)	**108.** (a)	**109.** (b)	**110.** (c)
111. (b)	**112.** (c)	**113.** (d)	**114.** (c)	**115.** (b)	**116.** (d)	**117.** (b)	**118.** (d)	**119.** (b)	**120.** (a)

Mathematics

121. (d)	**122.** (b)	**123.** (a)	**124.** (c)	**125.** (b)	**126.** (d)	**127.** (a)	**128.** (c)	**129.** (d)	**130.** (c)
131. (c)	**132.** (a)	**133.** (a)	**134.** (a)	**135.** (d)	**136.** (c)	**137.** (b)	**138.** (d)	**139.** (a)	**140.** (b)
141. (c)	**142.** (c)	**143.** (d)	**144.** (c)	**145.** (b)	**146.** (*)	**147.** (*)	**148.** (d)	**149.** (d)	**150.** (a)
151. (a)	**152.** (d)	**153.** (a)	**154.** (d)	**155.** (b)	**156.** (c)	**157.** (d)	**158.** (b)	**159.** (a)	**160.** (c)
161. (b)	**162.** (a)	**163.** (c)	**164.** (d)	**165.** (c)	**166.** (a)	**167.** (*)	**168.** (b)	**169.** (c)	**170.** (a)
171. (c)	**172.** (b)	**173.** (a)	**174.** (d)	**175.** (c)	**176.** (b)	**177.** (d)	**178.** (b)	**179.** (a)	**180.** (d)

Note (*) None of the option is correct.

HINTS & SOLUTIONS

Physics

1. *(a)* The time period of a pendulum is given by

$$T = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$$

When a lift containing a simple pendulum is accelerated upwards with acceleration a, then its time period becomes

$$T' = \frac{1}{2\pi}\sqrt{\frac{l}{(g+a)}}$$

Clearly, $T' \angle T$

Hence, its time period decreases.

2. *(c)* The level of sound is given by

$$L = 10\log\left(\frac{I}{I_0}\right)\text{dB}$$

Let for 40 dB sound intensity be I and for 90dB sound intensity be I_x, then

$$40 = 10\log\left(\frac{I}{I_0}\right)$$

$$\Rightarrow \quad \log\left(\frac{I}{I_0}\right) = 4 \qquad \ldots(i)$$

and $$90 = 10\log\left(\frac{I_x}{I_0}\right)$$

$$\Rightarrow \quad \log\left(\frac{I_x}{I_0}\right) = 9$$

Given, $I_x = xI$

$$\Rightarrow \quad \log\left(\frac{xI}{I_0}\right) = 9$$

or $$\log x + \log\left(\frac{I}{I_0}\right) = 9$$

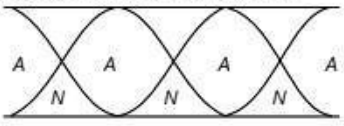
$$\Rightarrow \quad \log x + 4 = 9 \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad \log x = 5 \quad \Rightarrow \quad x = 10^5$$

3. *(a)* The Doppler shift in wavelength of light from star is given by

$$d\lambda = \frac{\lambda v}{c}$$

4. *(b)* For third harmonic, $n = 3$. So, for an open pipe, the propagation of waves can be shown as



$\therefore$ Nodes = 3 and anti-nodes = 4

5. *(d)* In an adiabatic process, there is no transfer of heat takes place between a thermodynamic system and its surroundings.

6. *(a)* The efficiency of a Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Given, $T_1 = 627°\text{C} = 627 + 273 = 900\ \text{K}$

and $T_2 = 27°\text{C} = 27 + 273 = 300\ \text{K}$

$$\Rightarrow \quad \eta = 1 - \frac{300}{900} = \frac{600}{900} = \frac{2}{3}$$

Also, $$\eta = \frac{\text{Work done}}{\text{Heat taken}}$$

$$\Rightarrow \text{ Work done} = \eta \times \text{Heat taken}$$

$$= \frac{2}{3} \times 300 = 200\ \text{J}$$

7. *(b)* The undecayed nuclei in a radioactive sample are given by

$$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

Given, $N = N_0 - \frac{15}{16}N_0 = \frac{1}{16}N_0,\ t = 2\,\text{h}$

$$\Rightarrow \quad \frac{N_0}{16} = N_0\left(\frac{1}{2}\right)^{\frac{2}{t_{1/2}}}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^{\frac{2}{t_{1/2}}} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$$

$$\Rightarrow \quad \frac{2}{t_{1/2}} = 4 \Rightarrow t_{1/2} = \frac{1}{2}\,\text{h} = 30\ \text{min}$$

$\therefore$ Mean life, $\tau = 1.44 t_{1/2}$

$$= 1.44 \times 30 = 43.2\ \text{min} \simeq 43\ \text{min}$$

8. *(a)* MRI (magnetic resonance imaging) gives a detailed image of soft tissues like in brain.

9. *(d)* Of these leptons are truly elementary particles. While rest are composed of quarks, so they are composite particles.

10. *(d)* The output of NAND gate is as shown in the table below

Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Thus, it will give logic 0 only when both inputs are in logic 1 state.

11. *(c)* n - type semiconductor has electrons as majority charge carriers. So, they are obtained by doping with pentavalent atoms like phosphorous, arsenic, etc.

p-type semiconductor has holes as majority charge carriers. So, they are obtained by doping with trivalent atoms like aluminium, boron, indium, etc.

12. *(a)* Given, current gain, $\beta = 50$

Input resistance, $R_i = 500\,\Omega$

Output resistance, $R_L = 4\,\text{k}\Omega = 4000\,\Omega$

$$\text{Voltage gain} = \text{Current gain} \times \frac{R_L}{R_i}$$

$$= 50 \times \frac{4000}{500} = 400$$

$$\text{Power gain} = \text{Current gain} \times \text{Voltage gain}$$

$$= 50 \times 400 = 2 \times 10^4$$

13. *(c)* In Balmer series, the electrons moves to $n = 2$ state

So, possible transitions are $4 \rightarrow 2$, $3 \rightarrow 2$.

$\therefore$ Number of spectral lines are 2.

14. *(b)* The IR region lies between microwave and visible region of electromagnetic spectrum.

15. *(b)* The de-Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

The momentum of proton,

$$p_p = \sqrt{2mE} = \sqrt{2meV}$$

Similarly, for α - particle,

$$p_\alpha = \sqrt{2 \times 4m \times 2e \times V}$$

$$= \sqrt{16meV}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{16meV}}{\sqrt{2meV}} = \sqrt{8}$$

$$= 2\sqrt{2} : 1$$

16. *(a)* Raman scattering is an inelastic scattering of a photon by molecules which are excited to higher vibrational or rotational energy level. Raman shift depends on the wavelength of incident radiation.

17. *(c)* In ${}_6C^{14}$, number of neutrons = 14 − 6 = 8

In ${}_7N^{15}$, number of neutrons = 15 − 7 = 8

These are the examples of isotones.

18. *(c)* Given, $\frac{I_{max}}{I_{min}} = \frac{9}{1}$

As we know,

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{9}{1}$$

$$\Rightarrow \quad \frac{a_1 + a_2}{a_1 - a_2} = \frac{3}{1}$$

$$\Rightarrow \quad a_1 + a_2 = 3a_1 - 3a_2$$

$$\Rightarrow \quad a_1 = 2a_2$$

or $$\frac{a_1}{a_2} = 2 : 1$$

19. *(a)* For dark fringe, path difference,

$$\Delta x = \left(n + \frac{1}{2}\right)\lambda$$

Also, $$\Delta x = \frac{dy}{D} \Rightarrow \frac{dy}{D} = \left(n + \frac{1}{2}\right)\lambda$$

For 1st dark fringe at directly opposite point of slit $n = 0$, $y = \frac{d}{2}$

$$\Rightarrow \quad \frac{d\left(\frac{d}{2}\right)}{D} = \frac{\lambda}{2} \Rightarrow \lambda = \frac{d^2}{D}$$

20. *(a)* Newton's rings are fringes of equal thickness. These circular fringes were discovered by Newton, hence called Newton's rings. These rings are formed with equally spaced bright and dark band with central spot as dark.

21. *(d)* Diffraction pattern using light is difficult to observe in daily life because wavelength of light is very small.

22. *(d)* When a calcite crystal is placed over a dot on a piece of paper, one will observe two dots from above. If the crystal is rotated, then the dot produced by emergent ray will rotate around the stationary dot, which sometimes coincide with it.

23. *(d)* According to Brewster's law,

$$\Rightarrow \quad \mu = \tan i_p$$

At critical angle, $\mu = \frac{1}{\sin i_C} = \frac{1}{\sin 45°}$

$$\Rightarrow \quad \mu = \sqrt{2}$$

$$\therefore \quad \sqrt{2} = \tan i_p$$

$$\Rightarrow \quad i_p = \tan^{-1}\sqrt{2} = 54.7°$$

24. *(b)* For shunt resistance connected to galvanometer, current is given by

$$I_g = \left(\frac{S}{S+G}\right)I$$

$$\frac{2}{100}I = \left(\frac{S}{S+G}\right)I$$

$$\Rightarrow \quad S + G = 50S$$

or $$S = \frac{G}{49}$$

25. *(d)* Magnetic moment of a coil of area A carrying current I is given by

$$M = IA$$

$$\Rightarrow \text{Current, } I = \frac{M}{A}$$

26. *(b)* The magnetic field at the centre of a circular coil carrying current I is given by

$$B = \frac{\mu_0 nI}{2R}$$

where, n = number of turns.

For coil 1, $B_1 = \frac{\mu_0 \times 10 \times 0.2}{2 \times 0.2} = 5\mu_0$

For coil 2, $B_2 = \frac{\mu_0 \times 10 \times 0.3}{2 \times 0.4} = \frac{15}{4}\mu_0$

$\therefore$ Net magnetic field,

$$B = B_1 - B_2 = 5\mu_0 - \frac{15}{4}\mu_0 = \frac{5}{4}\mu_0$$

27. *(a)* Permanent magnet should have high retentivity to persist magnetism on removal of magnetic field. It should also have high coercivity, so that the magnetism is not lost by external magnetic field.

28. *(c)* The power factor is the cosine of phase angle between voltage and current. For *L*-*C*-*R* circuit, it is given by

$$\cos\phi = \frac{R}{Z}$$

where, Z = impedance of circuit.

29. *(c)* Given, $L = 1$ H, $V = 220$ V, $f = 50$ Hz

Peak value of current $I_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega L}$

$$\Rightarrow\ I_0 = \frac{\sqrt{2}V}{2\pi fL} = \frac{\sqrt{2} \times 220}{2 \times \pi \times 50 \times 1} \simeq 1\text{ A}$$

30. *(c)* Let I_0 be the intensity of light incident on polariser.

On passing through polariser, it gets reduced to $\frac{I_0}{2}$.

So, according to law of Malus,

$$I = \frac{I_0}{2}\cos^2\theta$$

Given, $$I = \frac{75}{100} \times \frac{I_0}{2} = \frac{3I_0}{8}$$

$\therefore$ $$\frac{3I_0}{8} = \frac{I_0}{2}\cos^2\theta \ \Rightarrow\ \cos\theta = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $$\theta = 30°$$

31. *(b)* Potential due to a point charge at a distance r is given by

$$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \times \frac{q}{r}$$

Given, $q = 10\text{ nC} = 10 \times 10^{-9}$ C,

$\varepsilon_r = 10$, $r = 0.1$ m

$$\Rightarrow\ V = \frac{9 \times 10^9 \times 10 \times 10^{-9}}{10 \times 0.1} = 90\text{ V}$$

32. *(b)* Permittivity of metal is very high, so its dielectric constant is infinite.

33. *(d)* Force between two point charges is given by

$$F = \frac{kq_1q_2}{r^2}$$

When $r' = r + 20\%$ of $r = 1.2r$, then

$$F' = \frac{kq_1q_2}{(1.2r)^2} = \frac{kq_1q_2}{1.44r^2} = \frac{F}{1.44}$$

Percentage decrease in force $= \frac{F - F'}{F} \times 100$

$$= \frac{F - \frac{F}{1.44}}{F} \times 100 = \frac{0.44}{1.44} \times 100 = 31\%$$

34. *(a)* Given $q_1 = q_2 = 10\text{ nC} = 10 \times 10^{-9}$C

$r = 0.09$ m

Potential energy of two point charges is given by

$$U = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1q_2}{r}$$

$$= 9 \times 10^9 \times \frac{10 \times 10^{-9} \times 10 \times 10^{-9}}{0.09} = 10^{-5}\text{ J}$$

$$= 10\mu\text{J}$$

35. *(b)* Given, $t = \frac{d}{2}$

The capacitance is given by

$$C = \frac{\varepsilon_0 A}{d}$$

For dielectric medium between plates,

$$C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

For metal, $K =$ infinity

$$\Rightarrow\ C' = \frac{\varepsilon_0 A}{d - \frac{d}{2}} = \frac{2\varepsilon_0 A}{d} = 2C$$

Hence, capacitance increases 2 times.

36. *(d)* The specific resistance is the property of material which does not depend on its geometry. It increases with increase in temperature.

37. *(d)* At 4.2 K, the resistance of mercury suddenly drops to zero and behaves as a super conductor at this temperature (called critical temperature).

38. *(c)* Given, $\alpha_r = 4 \times 10^{-3}$ / K, $T_1 = 20°$C

Final resistance, $R_2 = R + 10\%R = 1.1R$

We know that, $\Delta R = \alpha_r R \Delta T$

$$\Rightarrow\ 1.1R - R = 4 \times 10^{-3} \times R \times (T_2 - 20)$$

$$\Rightarrow\ 0.1 = 4 \times 10^{-3} \times (T_2 - 20)$$

$$\Rightarrow\ T_2 = 25 + 20 = 45\,°\text{C}$$

39. *(d)* Here, 6Ω and 12Ω resistors are in parallel combination, so their equivalent resistance,

$$R_{eq} = \frac{6 \times 12}{6 + 12} = \frac{72}{18} = 4\,\Omega$$

Now, R_{eq} and 4 Ω are in series, so total resistance,

$$R_t = 4 + 4 = 8\,\Omega$$

Emf of battery, $E = 16$ V

$\therefore$ Reading of voltmeter, $V' = \frac{4}{R_t} \times E = \frac{4 \times 16}{8} = 8\text{V}$

40. *(a)* When a charged particle moves in a uniform magnetic field normally, then a magnetic force acts on it. Due to this force, it will move in a circular path.

At every point of circle, its velocity changes direction. So, its momentum also changes but the total energy will remain same.

41. *(a)* If n be the number of waves, then wavelength of light in glass, $\lambda_g = \frac{0.04}{n}$

and wavelength of light in water, $\lambda_w = \frac{0.05}{n}$

Speed of light in glass, $v_g = \lambda_g \times f = \frac{0.04f}{n}$

where, f = frequency of light.

Similarly, speed of light in water, $v_w = \lambda_w \times f = \frac{0.05f}{n}$

Relative refractive index

$$= \frac{\mu_w}{\mu_g} = \frac{v_g}{v_w} = \frac{0.04f}{n} \times \frac{n}{0.05f} = \frac{4}{5}$$

$$\Rightarrow \mu_g = \frac{5}{4} \times \mu_w = \frac{5}{4} \times \frac{4}{3} = \frac{5}{3}$$

42. *(b)* The critical angle between two media is given by

$$i_c = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$$

As we know that, $\mu_{air} = 1, \mu_{water} = 1.33, \mu_{glass} = 1.5$ and $\mu_{diamond} = 2.4$.

From given values, $\frac{\mu_1}{\mu_2}$ is maximum for water-glass interface. So, critical angle will also be maximum, when light travels from glass to water.

43. *(c)* Given, angle of prism, $A = 60°$

As, the ray of emergence is symmetrical to incident ray. So, the angle of minimum deviation, $D_m = 60°$.

$$\therefore \text{Refractive index of prism, } \mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\left(\frac{60° + 60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 60°}{\sin 30°}$$

$$= \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3} = 1.73$$

44. *(d)* In the spectrum of visible light produced by a prism, the violet light have minimum wavelength. So, the dispersion will be maximum towards violet.

45. *(b)* Given, $\mu_g = 1.5, \mu_w = \frac{4}{3}$

From lens Maker's formula,

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f(\mu_g - 1)} \qquad \ldots(i)$$

When lens is immersed in water, then

$$\frac{1}{f'} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{\mu_g}{\mu_w} - 1\right)\frac{1}{f(\mu_g - 1)} \qquad \text{[using Eq. (i)]}$$

$$\frac{f}{f'} = \left(\frac{1.5}{4/3} - 1\right) \times \frac{1}{(1.5 - 1)}$$

$$= \left(\frac{9}{8} - 1\right) \times 2 = \frac{1}{4}$$

$$\Rightarrow \quad f' = 4f \text{ or } f' > f$$

$\therefore$ The focal length of lens is greater than f in water.

46. *(a)* The focal length of combination of two lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{(1/4)} + \frac{1}{(-1/2)} \qquad \left(\because f = \frac{1}{P}\right)$$

$$= 4 - 2 = 2$$

$$\Rightarrow \quad f = 0.5 \text{ m}$$

47. *(b)* Eddy currents are produced in a material, when it is placed in a time varying magnetic field. It is based on the Faraday's law of etectromagnetic induction.

48. *(d)* Given, $V_p = 220$ V, $\eta = 80\% = 0.8$

$$P_0 = 8 \text{ kW} = 8000 \text{ W}$$

$$\text{Efficiency of transformer, } \eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\Rightarrow \quad \eta = \frac{P_o}{P_i} \quad \Rightarrow \quad 0.8 = \frac{8000}{P_i}$$

$$\Rightarrow \quad P_i = \frac{8000}{0.8} = 10000$$

$$\Rightarrow \quad V_p I_p = 10000$$

$$\text{or} \quad I_p = \frac{10000}{220} = 45.5 \text{ A} \simeq 45 \text{ A}$$

49. *(b)* Given, $f_0 = 600$ Hz, $Q_1 = 3, Q_2 = 2$

The bandwidth in *L-C-R* circuit,

$$\beta = \frac{f_0}{Q}$$

As, quality factor decreases, bandwidth increases. This increase in bandwidth is given by

$$\Delta\beta = \beta_2 - \beta_1 = \frac{f_0}{Q_2} - \frac{f_0}{Q_1} = f_0\left(\frac{1}{Q_2} - \frac{1}{Q_1}\right)$$

$$= 600\left(\frac{1}{2} - \frac{1}{3}\right) = 100 \text{ Hz}$$

50. *(d)* Given, $t = 4$ s, $u = 0$, $g = 10 \text{ ms}^{-2}$

Using equation of motion,

$$h = ut + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (4)^2 = 80 \text{ m}$$

51. *(b)* In liquid crystal phases, there are 4 types of phases as Nematic, Smectic, Cholesteric and Discotic. Of these, Smectic phase is more close to solid than to liquid.

52. *(a)* The acceleration due to gravity at earth's surface is given by

$$g = \frac{GM}{R^2}$$

As, mass remains constant.

$$\Rightarrow \quad g \propto \frac{1}{R^2}$$

so, as the radius of earth decreases or the earth shrinks in size, the value of g increases.

53. *(b)* The thermal resistance of a rod is given by

$$R = \frac{l}{KA}$$

As, length and area of two rods are same, then in series combination,

$$R = R_1 + R_2$$

$$\Rightarrow \quad \frac{2l}{K_s A} = \frac{l}{K_1 A} + \frac{l}{K_2 A}$$

$$\Rightarrow \quad \frac{2}{K_s} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{K_1 + K_2}{K_1 K_2}$$

$$\Rightarrow \quad K_s = \frac{2K_1 K_2}{K_1 + K_2}$$

54. *(c)* Let F_1 and F_2 be the two forces acting at a point, then their resultant,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

Given, $F_1 = F_2 = F$

$$\Rightarrow \quad F_R = \sqrt{2F^2 + 2F^2 \cos\theta}$$

$$\Rightarrow \quad F_R^2 = 2F^2(1 + \cos\theta) \qquad \ldots(i)$$

As per question,

$$F_R^2 = 3F_1 F_2 = 3F^2$$

$$\Rightarrow \quad 2F^2(1 + \cos\theta) = 3F^2 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad 1 + \cos\theta = \frac{3}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \text{ or } \theta = 60°$$

55. *(d)* The addition of impurities in a liquid may increase or decrease the surface tension of liquid depending on the nature of impurities.

If the impurities are highly soluble in liquid, then it will increase the surface tension. But if it is less soluble in liquid, then it will decrease the surface tension of liquid.

56. *(c)* The viscosity decreases with increase in temperature. Due to this reason, the hot water moves faster than cold water. Since in summer temperature is high, so more viscous oils are used in motorcars.

57. *(c)* Moment of momentum of an electron in nth orbit is given by

$$L = \frac{nh}{2\pi}$$

For an electron in second Bohr orbit, $n = 2$

$$\therefore \quad L = \frac{2h}{2\pi} = \frac{h}{\pi}$$

58. *(a)* The excitation energy is the minimum energy required to excite an electron from ground state of atom to any of its excited state and ionisation energy is the amount energy needed to remove an electron from an atom.

So, lesser the excitation energy and more the ionisation energy, more the stability of atom.

59. *(c)* Given, work function, $\phi = 3$ eV

The wavelength of incident radiation is given by

$$\lambda = \frac{hc}{\phi} = \frac{1242}{\phi} = \frac{1242}{3} = 414 \text{ nm}$$

60. *(d)* The equivalent capacitance of three capacitors connected in series is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Given, $C_1 = C_2 = C_3 = C$

$$\frac{1}{C_s} = \frac{3}{C} \quad \text{or} \quad C_s = \frac{C}{3}$$

Similarly, for parallel combination,

$$C_p = C + C + C = 3C$$

$\therefore$ Ratio of equivalent capacitance,

$$\frac{C_s}{C_p} = \frac{C/3}{3C} = \frac{1}{9} \text{ or } 1 : 9$$

61. *(d)* An organic compound gives oily liquid on heating with bromine and potassium hydroxide solution. On shaking with acetic anhydride, an antipyretic drug acetanilide obtained is benzamide.

$$\underset{\text{Benzamide}}{C_6H_5—CONH_2} \xrightarrow{Br_2/KOH} C_6H_5NH_2 \xrightarrow{(CH_3CO)_2O} \underset{\text{Acetanilide}}{C_6H_5NHCOCH_3}$$

62. *(d)* Silver salt of fatty acid on refluxing with an alkyl halide gives an ester.

$$\underset{\text{Silver salt of fatty acid}}{CH_3COOAg} + \underset{\text{Alkyl halide}}{C_2H_5Br} \longrightarrow \underset{\text{Ester}}{CH_3COOC_2H_5} + AgBr$$

63. *(d)* Pepsin, ptyalin, lipase are all enzymes used for the breaking down of nutrients, while cellulose is a polysaccharide.

64. *(b)* Combination of H_2 and Br_2 to give HBr, is second order reaction.

$$H_2 + Br_2 \longrightarrow 2HBr$$

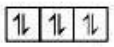
Rate law expression, $r = k[H_2][Br_2]$

$\therefore$ Order w.r.t. H_2 and Br_2 is one.

$\therefore$ Overall order = 1 + 1 = 2

65. *(c)* The diameter of colloidal particle ranges from 10^{-6} m to 10^{-9} m.

66. *(b)* The 2*p*-orbital has electronic configuration as follows



$\therefore$ Three electrons are with $-1/2$ spin quantum number and other three electrons are with $+1/2$ spin quantum number.

67. *(b)* The molecular formula of 2-methyl butane, 2,2-dimethyl propane and pentane is same i.e. (C_5H_{12}). They are structural isomers.

68. *(a)* $\underset{\substack{1\text{ mole}\\(28\text{ g})}}{N_2} + \underset{\substack{3\text{ moles}\\(6\text{ g})}}{3H_2} \longrightarrow \underset{\substack{2\text{ moles}\\(34\text{ g})}}{2NH_3}$

$\therefore$ Mass of N_2 left = 56 − 28 = 28 g

$\therefore$ Moles of N_2 left $= \frac{28}{28} = 1$ mole

Mass of H_2 left = 8 − 6 = 2 g

$\therefore$ Moles of H_2 left $= \frac{2}{2} = 1$ mole

Moles of ammonia $= \frac{34}{17} = 2$ moles

69. *(a)* For constant pressure and temperature process, ΔH and $\Delta U = 0$

70. *(a)* In a galvanic cell, the electron flow from anode to cathode through the external circuit.

71. *(c)* The mixture of 2-chloropropane and chloromethane on treating with sodium metal in dry ether gives 2-methyl propane i.e.,

$$\underset{\text{2-chloropropane}}{CH_3—\underset{\displaystyle CH_3}{\underset{|}{CH}}—Cl} + \underset{\text{Chloromethane}}{Cl—CH_3} \xrightarrow[\text{Dry ether}]{2Na} \underset{\text{2-methylpropane}}{CH_3—\underset{\displaystyle CH_3}{\underset{|}{CH}}—CH_3} + 2NaCl$$

72. *(c)* Benzyl chloride is more reactive than alkyl halides as benzyl carbocation is stabilised by resonance. Hence, it easily gives nucleophilic substitution reaction.

73. *(d)* The main product obtained when sodium carbonate reacts with mercuric chloride is mercuric oxide.

$$HgCl_2 + Na_2CO_3 \longrightarrow HgO + CO_2 + 2NaCl \text{ or } HgCO_3 + 2NaCl$$

74. *(b)* In a electrothermal process, the compound displayed by silica from calcium phosphite is phosphorus pentoxide as follows:

$$2\,Ca_3(PO_4)_2 + 6\,SiO_2 \longrightarrow \underset{\text{(Slag)}}{6CaSiO_3} + P_4O_{10}$$

75. *(b)* $\underset{16\text{ g}}{CH_4} + \underset{64\text{ g}}{2O_2} \longrightarrow \underset{44\text{ g}}{CO_2} + \underset{36\text{ g}}{2H_2O}$; $\Delta H = 890$ kJ/mol

Heat liberated on combustion of 16 g CH_4 is 890 kJ.

$\therefore$ Heat liberation from 3.2 g methane

$$= \frac{3.2}{16} \times 890 = \frac{890}{5}$$

$$= 178\text{ kJ}$$

76. *(a)* According to gas equation,

$$pV = nRT$$

$$\therefore \quad V = \frac{nRT}{p}$$

As both p and T are double, V remains constant i.e., 4 dm^3.

77. *(d)* Copper on reacting with conc. HNO_3 gives copper nitrate.

$$Cu + HNO_3\text{ (conc.)} \longrightarrow \underset{\text{Copper nitrate}}{Cu(NO_3)_2} + 2H_2O + 2NO_2$$

On strongly heating with copper nitrate, copper oxide is obtained.

$$2Cu(NO_3)_2 \longrightarrow \underset{\substack{\text{Copper oxide}\\(5\text{ g})}}{2CuO} + 4NO_2 + O_2$$

$\therefore$ Equivalent weight of Cu

= Weight of copper × Equivalent weight of oxygen

= 4 × 8 = 32

78. *(c)* For the following reaction,

$$N_2 + 3H_2 \rightleftharpoons 2NH_3 + 92.3\ kJ$$

(i) Reducing temperature is favourable as it is an exothermic process and shift equilibrium towards right.

(ii) Also on removing NH_3 equilibrium shift towards right according to Le-Chatelier's principle.

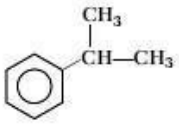
(iii) Δn_g is negative for the process. Increase in pressure, equilibrium towards right.

For an exothermic reaction, increasing temperature shifts the reaction to reactant side.

79. *(d)* The chemical equilibrium of a reversible reaction is not influenced by catalyst as it only increase the rate of reaction to achieve the equilibrium faster. The increase in rate of forward direction is equal to increase in rate of backward direction at equilibrium.

$\therefore$ Equilibrium point is not shifted.

80. *(c)* The structure of cumene is



The IUPAC name is 1-methyl ethyl benzene.

81. *(d)* Moles of HCl $= \dfrac{1.2046 \times 10^{24}}{6.023 \times 10^{23}} = 2$

$$\therefore \text{Normality} = \frac{\text{moles of HCl}}{\text{volume of solution (L or dm}^3)}$$

$$= \frac{2}{1} = 2\ N$$

82. *(c)* Nuclear theory of atom was put forward by Rutherford. He discovered α and β-rays and projected the law of radioactive decay.

83. *(c)* In acetylene or ethyne molecule, the two carbon atoms are linked by one σ and two π-bonds.

84. *(c)* $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g);\ \Delta H_1 = +ve$

$H_2O(g) \longrightarrow H_2O(l)$ $\quad ;\ \Delta H_3 = +ve$

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l)$ $\quad ;\ \Delta H_2 = \Delta H_1 + \Delta H_3$

$\therefore \Delta H_2 > \Delta H_1$

85. *(d)* For radioactive decay

$$t_{1/2} = \frac{0.693}{k}$$

For concentration to reduce to $\frac{1}{16}$ of original amount

$$t = t_{1/2} \times 4$$

$$\therefore \quad t_{1/2} = \frac{192}{4} = 48\ \text{min}$$

86. *(d)* The reagent, Cl_2 does not give acid chloride on heating with carboxylic acids, while other reagents give acid chloride.

$$R\text{—}COOH \xrightarrow{PCl_5} R\text{—}COCl + POCl_3 + HCl$$

$$R\text{—}COOH \xrightarrow{PCl_3} 3R\text{—}COCl + H_3PO_3$$

$$R\text{—}COOH \xrightarrow{SOCl_2} R\text{—}COCl + SO_2\uparrow + HCl\uparrow$$

87. *(d)* Reducing property of halogens increases from F to I.

$\therefore$ I get oxidised by nitric acid.

$$\overset{0}{I_2} + 10\ HNO_3 \longrightarrow \overset{+5}{2HIO_3} + 10\ NO_2 + 4H_2O$$

Iodine — Nitric acid — Iodic acid

Oxidation

88. *(a)* Lead form nitric oxide with dil. HNO_3. It does not form ammonium nitrate by reaction with dil. nitric acid.

$$3Pb + 8HNO_3 \longrightarrow 3Pb(NO_3)_2 + \underset{\text{Nitric oxide}}{2NO} + 4H_2O$$

89. *(d)* The elements with atomic number 9, 17, 35, 53 and 85 are all halogens, they have electronic configuration of ns^2np^5.

90. *(b)* In the electrolytic method of obtaining aluminium from purified bauxite, cryolite is added to the charge in order to lower the melting point of bauxite and brings conductivity.

91. *(c)* HNO_3 is not an amphoteric substance as, it behave only as acid. It can only donate H^+ ions. While, substance like HCO_3^-, H_2O, and NH_3 can act as both donor and acceptor to H^+ ions, so they are amphoteric in nature.

92. *(b)* Equivalent moles of H_2SO_4 and KOH undergoing neutralisation

$$= 50 \times 0.2 \times 10^{-3} = 0.01$$

$\therefore$ Heat of neutralisation $= 0.01 \times 57.3\ kJ = 573\ J$

93. *(c)*

$$\underset{\text{Parent nucleus}}{{}^{14}X_5} \longrightarrow {}^{14}X_6 + {}^{0}_{-1}e$$

$$\downarrow$$

$${}^{14}N_7 + {}^{0}_{-1}e$$

$\therefore$ The number of neutrons in parent nucleus $= 14 - 5$

$= 9$

94. *(d)* Stainless steel does not rust because chromium forms an oxide layer and protects iron from rusting.

95. *(b)* Ethanol can be synthesised as follows:

$$CH_3MgI + \underset{\text{Formaldehyde}}{HCHO} \xrightarrow{H_2O}$$

$$\underset{\text{Ethanol}}{CH_3\text{—}CH_2OH} + Mg(OH)I$$

96. *(b)* The rate law expression is, $r = k[SO_2]^2[O_2]$

As, volume increase 2 times, concentration reduces by $\frac{1}{2}$ times i.e., $[SO_2]_1 = [2SO_2]$ and $[O_2]_1 = [2O_2]$

$\therefore$ New rate, $r_1 = k[2SO_2]^2[2O_2]$

$\therefore$ $r_1 = 8r$

$\therefore$ $\frac{r_1}{r} = \frac{8}{1}$

97. *(c)* The pH of buffer solution is given as,

$$pH_1 = pK_a + \log\frac{[CH_3COONa]}{[CH_3COOH]} \quad ...(i)$$

Given, $[CH_3COONa]_2/[CH_3COOH]_2 = 10[CH_3COONa]/[CH_3COOH]$

$pH_2 = pK_a + \log 10[CH_3COONa]/[CH_3COOH]$

$pH_2 = pK_a + \log[CH_3COONa]/[CH_3COOH] + \log 10$

$\therefore$ $pH_2 = pH_1 + 1$ [From Eq. (i)]

98. *(b)* When mixture of methane and oxygen is passed through heated molybdenum oxide, the main product formed is methanal.

$$\underset{\text{Methane}}{CH_4} + O_2 \xrightarrow{\text{Mo-oxide}} \underset{\text{Methanal}}{HCHO} + H_2O$$

99. *(d)* Benzene can be obtained by heating benzoic acid with soda lime and phenol with zinc dust as follows:

$$\underset{\text{Benzoic acid}}{C_6H_5COOH} + \underset{\text{Soda lime (X)}}{2NaOH} \xrightarrow{\Delta} \underset{\text{Benzene}}{C_6H_6} + Na_2CO_3$$

$$\underset{\text{Phenol}}{C_6H_5—OH} + \underset{\substack{\text{Zinc dust}\\(Y)}}{Zn} \xrightarrow{\Delta} C_6H_6 + ZnO$$

100. *(c)* Ethyl benzoate is boiled with alc. potash to give ethyl alcohol and potassium benzoate.

$$\underset{\text{Ethyl benzoate}}{C_6H_5COOCH_2CH_3} \xrightarrow[H_2O]{\text{Alc. potash}} C_6H_5COOK + CH_3CH_2OH$$

Potassium benzoate is converted into benzoic acid, a white solid which separates out.

$$C_6H_5COOK + HCl \longrightarrow C_6H_5COOH + KCl$$

101. *(a)* On passing H_2S gas in presence of dil. HCl for qualitative analysis of second group due to common ion effect, lower concentration of sulphide ions is obtained. It is sufficient for the precipitation of second group cations in forms of their sulphides and hence their dissociation decreases.

102. *(b)* $E^\circ = E^\circ_{(Ag^+/Ag)} - E^\circ_{(Al^{3+}/Al)}$

$2.46 = 0.80 - E^\circ_{(Al^{3+}/Al)}$

$\therefore$ $E^\circ_{(Al^{3+}/Al)} = -1.66\ V$

$\therefore$ Reduction potential $= -1.66$ V.

103. *(c)* The first fraction obtained during the fractionation of petroleum is hydrocarbon gases.

104. *(a)* Ethanol gives trichloromethane on distilling with bleaching powder.

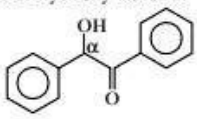
(I) $Cl_2 + H_2O \longrightarrow 2HCl + [O]$

$\underset{\text{Ethanol}}{CH_3CH_2OH} + [O] \longrightarrow CH_3CHO + H_2O$

(II) $CH_3CHO + 3Cl_2 \longrightarrow CCl_3CHO + 3HCl$

(III) $CCl_3CHO + Ca(OH)_2 \longrightarrow \underset{\text{Trichloro methane}}{2CHCl_3} + (HCOO)_2Ca$

105. *(b)* Benzoin is a α-hydroxy ketone.



106. *(b)* For every 10K rise in temperature, rate of reaction double.

$\therefore$ From 298 K to 310 K, the reaction rate became $(2)^2$ times i.e., 4 times.

$\therefore$ $k = 3.2 \times 10^{-3}\ s^{-1} \times 4 = 1.28 \times 10^{-2} s^{-1}$

107. *(c)* The acidic strength of an acid increase with ionisation.

The value of pK_a is inversely proportional to ionisation constant (K_a).

$\therefore$ The acid with pK_a value 1.0 is strongest.

108. *(a)* Unsaturated fatty acids contains one or more double bond in their carbon chains.

Their general formula is $C_nH_{2n-2x}O_2$.

$\therefore$ Oleic acid $(C_{18}H_{34}O_2)$ is unsaturated fatty acid with one double bond.

109. *(b)* Nylon is not a homopolymer. It is a polymer of hexamethylenediamine and adipic acid.

110. *(c)* The coaltar fraction which contains phenol is called middle oil.

111. *(b)*

	A	+	B	$\rightleftharpoons$	C	+	D
Initially	1		1		0		0
at equilibrium	$\frac{2}{3}$		$\frac{2}{3}$		$\frac{1}{3}$		$\frac{1}{3}$

$\therefore$ Equilibrium constant $(K) = \frac{[C][D]}{[A][B]}$

$$= \frac{\frac{1}{3}\cdot\frac{1}{3}}{\frac{2}{3}\cdot\frac{2}{3}} = \frac{1}{4} = 0.25$$

112. *(c)* The particles of ore float because their surface is not easily wetted by water.

113. *(d)* Amorphous solids are isotropic in nature i.e., they shows same properties in all the directions.

114. *(c)* Rate of diffusion $\propto \frac{1}{\sqrt{\text{Molar mass}}}$

$\therefore$ $\frac{r_H}{r_A} = \sqrt{\frac{M_A}{M_H}}$

$\because$ $r_H = 6r_A$

$\therefore$ $6 = \sqrt{\frac{M_A}{2}}$

$\therefore$ $M_A = 72$

115. *(b)* Dulong and Petit's law is valid only for solid elements. According to this law, the product of atomic mass and specific heat of a solid element is approximately equal to 6.4 cal/mol.

116. *(d)* The gas which has greater critical temperature is readily adsorbed by activated charcoal.

The order of critical temperature is

$$H_2 < N_2 < O_2 < SO_2$$

$\therefore$ SO_2 gas is readily adsorbed by chorcoal.

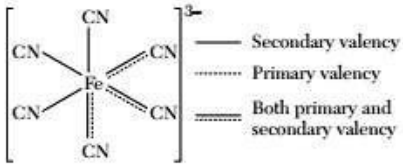
117. *(b)* In fcc unit cell of NaCl, Na^+ ion is present at edge centre and Cl^- ion is present at corner.

$\therefore$ Edge length, $a = 2r^+ + 2r^-$

$$\therefore \quad a = 2(r^+ + r^-)$$

$$= 2x \qquad [\because x = r^+ + r^-]$$

118. *(d)* In $K_3[Fe(CN)_6]$, the ligand has satisfied both primary and secondary valencies of ferric ion. The + 3 charge of ferric ion is satisfied by negative charge on cyanide ion. The coordination sphere's secondary valency is satisfied by six cyanide ligands.



119. *(b)* 2-acetoxy benzoic acid is aspirin. It is used as an antipyretic to reduce body temperature.

120. *(a)* Nucleoside on hydrolysis gives an aldopentose and a heterocyclic base i.e. purine and pyrimidine.

Mathematics

121. *(d)* Given that, $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $c = \{a, d, c\}$

Now, $A - B = \{a, b, c\} - \{b, c, d\} = \{a\}$

and $B \cap C = \{b, c, d\} \cap \{a, d, c\} = \{c, d\}$

$(A - B) \times (B \cap C) = \{a\} \times \{c, d\} = \{(a, c), (a, d)\}$

122. *(b)* We have,

$f : X \to Y$

X : domain and Y : codomain

In one-one function,

$$x_1 \neq x_2$$
$$f(x_1) \neq f(x_2)$$

In onto function,

Range = domain

$f(x) = \sin x$

So, if $X = \left[0, \frac{\pi}{2}\right]$ and $Y = [-1, 1]$

For each value of $x \in X$ and range = [0, 1], there exists a unique value for $y \in Y$

Hence, function is one-one but not onto.

123. *(a)* We have,

$\log_4 2 + \log_4 4 + \log_4 16 + \log_4 x = 6$

$$\log_4 (2 \times 4 \times x \times 16) = 6$$
$$\log_4 (128x) = 6$$
$$\Rightarrow \quad 128x = 4^6$$
$$\Rightarrow \quad 128x = 64 \times 4^3$$
$$\Rightarrow \quad 2x = 4^3$$
$$\Rightarrow \quad x = \frac{64}{2} = 32$$

124. *(c)* We have,

$$S_n = \frac{1}{6 \cdot 11} + \frac{1}{11 \cdot 16} + \frac{1}{16 \cdot 21} + \ldots + n \text{ terms}$$

$$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \ldots \frac{1}{5n-1} - \frac{1}{5n+6}\right)$$

$$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{5n+6}\right)$$

$$= \frac{1}{5}\left[\frac{5n+6-6}{6(5n+6)}\right]$$

$$= \frac{1}{5}\left[\frac{5n}{6(5n+6)}\right]$$

So, $6S_n = \frac{n}{(5n+6)}$

125. *(b)* Here, terms greater than 5!, i.e.

$(5!)^2, (6!)^2, \ldots\ldots, (100!)^2$ is divisible by 100.

$\therefore$ For terms

$(5!)^2, (6!)^2, \ldots (100!)^2$ remainder is 0.

Now, consider $(1!)^2 + (2!)^2 + (3!)^2 + (4!)^2$

$$= 1 + 4 + 36 + 576$$
$$= 617$$

When, 617 is divided by 100, its remainder is 17.

So, required remainder is 17.

126. *(d)* Given that, $(p \wedge \sim r) \to (\sim p \vee q)$ is false

$\Rightarrow (p \wedge \sim r)$ is true and $(\sim p \vee q)$ is false

$\Rightarrow$ (p is true and $\sim r$ is true)

and ($\sim p$ is false and q is false)

$\Rightarrow p$ is true, r is false and q is false.

127. *(a)* Since, α, β and γ are the roots of the equation $x^3 - 8x + 8 = 0$, then

$$\alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \gamma\alpha = -8,$$
$$\alpha\beta\gamma = -8 \qquad \ldots (i)$$

Therefore, $(\alpha + \beta + \gamma)^2 = 0$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = 0$

$\Rightarrow \alpha^2+\beta^2+\gamma^2=-2(-8)=16$

$\Sigma\alpha^2=16$

and $\dfrac{1}{\alpha\beta}+\dfrac{1}{\beta\gamma}+\dfrac{1}{\gamma\alpha}=\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}=\dfrac{0}{-8}=0$

$\Sigma\dfrac{1}{\alpha\beta}=0$

128. *(c)* $1080=2^3\times3^3\times5$ (prime factorization)

$675=3^3\times5^2$

So, Greatest Common divisor $=3^3\times5=27\times5=135$

129. *(d)* $b\equiv c \pmod a$

So, $\dfrac{b+c}{a}$ and $\dfrac{b-c}{a}=\dfrac{(b+c)(b-c)}{a^2}$

$=\dfrac{b^2-c^2}{a^2}$ or $\dfrac{a^2}{b^2-c^2}$

Here, $b^2\equiv c^2 \pmod{a^2}$

130. *(c)* By taking option (c),

$a\mid(b+c)\not\Rightarrow a\mid b$ and $a\mid c$

e.g. $6\mid18\not\Rightarrow 6\mid1$ and $6\mid17$

131. *(c)* We have, $2A+3B=\begin{bmatrix}2&-1&4\\3&2&5\end{bmatrix}$ ... (i)

and $A+2B=\begin{bmatrix}5&0&3\\1&6&2\end{bmatrix}$ ... (ii)

Multiply Eq. (ii) by 2, we get

$2A+4B=\begin{bmatrix}10&0&6\\2&12&4\end{bmatrix}$ ... (iii)

Now, subtracting Eq. (i) from Eq. (iii), we get

$$B=\begin{bmatrix}10&0&6\\2&12&4\end{bmatrix}-\begin{bmatrix}2&-1&4\\3&2&5\end{bmatrix}=\begin{bmatrix}8&1&2\\-1&10&-1\end{bmatrix}$$

132. *(a)* Given that,

$O(A)=2\times3, O(B)=3\times2,$

$O(C)=3\times3$

$\Rightarrow O(A')=3\times2, O(B')=2\times3$

(a) $C(A+B')$

Now, $O(A+B')=2\times3$

and $O(C)=3\times3$

So, matrix $C(A+B')$ cannot be determined.

(b) $C(A+B')'$

$O(A+B')=2\times3$

$\Rightarrow O(A+B')'=3\times2$ and $O(C)=3\times3$

Therefore, matrix $C(A+B')'$ can be determined.

(c) $O(BA)=3\times3$ and $O(C)=3\times3$

Therefore, matrix BAC can be determined.

(d) $CB+A'$

Now, order of CB = (order of C) (order of B)

= (order of C is 3×3) (order of B is 3×2)

= order of CB is 3×2

Since, $O(A')=3\times2$

Therefore, matrix $CB+A'$ can be determined.

133. *(a)* We have, $A=\begin{bmatrix}1&-3\\2&k\end{bmatrix}$

Now, $A=\begin{bmatrix}1&-3\\2&k\end{bmatrix}\begin{bmatrix}1&-3\\2&k\end{bmatrix}$

$=\begin{bmatrix}1-6&-3-3k\\2+2k&-6+k^2\end{bmatrix}=\begin{bmatrix}-5&-3-3k\\2+2k&k^2-6\end{bmatrix}$

Now, $A^2-4A+10I=A$

$\Rightarrow\begin{bmatrix}-5&-3-3k\\2+2k&k^2-6\end{bmatrix}-4\begin{bmatrix}1&-3\\2&k\end{bmatrix}+10\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}1&-3\\2&k\end{bmatrix}$

$\Rightarrow\begin{bmatrix}-5-4+10&-3-3k+12+0\\2+2k-8+0&k^2-6-4k+10\end{bmatrix}=\begin{bmatrix}1&-3\\2&k\end{bmatrix}$

$\Rightarrow\begin{bmatrix}-1&-3k+9\\2k-6&k^2-4k+4\end{bmatrix}=\begin{bmatrix}1&-3\\2&k\end{bmatrix}$

On comparing both sides, we get

$k^2-4k+4=k$

$\Rightarrow k^2-5k+4=0$

$\Rightarrow k^2-4k-k+4=0$

$\Rightarrow k(k-4)-1(k-4)=0$

$\Rightarrow (k-4)(k-1)=0$

$\Rightarrow k=4,1$

134. *(a)* $\begin{vmatrix}x+y&y+z&z+x\\x&y&z\\x-y&y-z&z-x\end{vmatrix}$

$=\begin{vmatrix}2(x+y+z)&y+z&z+x\\(x+y+z)&y&z\\0&y-z&z-x\end{vmatrix}$

$[\because C_1\to C_1+C_2+C_3]$

$=(x+y+z)\begin{vmatrix}2&y+z&z+x\\1&y&z\\0&y-z&z-x\end{vmatrix}$

$=(x+y+z)\begin{vmatrix}2&y+z&z+x\\0&y-z&z-x\\0&y-z&z-x\end{vmatrix}$

$[\because R_2\to 2R_2-R_1]$

$=(x+y+z)\times0=0$

$[\because R_2$ and R_3 are identical$]$

135. *(d)* If $a*b=b*a$, then the operation is commutative.

If $(a*b)*c=a*(b*c)$, then the operation is associative.

By option (d),

$a*b=a+b+ab$

$b*a=b+a+ba=a+b+ab$

Here, $a*b=b*a$, so the operation is commutative.

$(a * b) * c = (a + b + ab) * c$

$= (a + b + ab) + c + (a + b + ab)c$

$= a + b + ab + c + ac + bc + abc$

$= a + b + c + ab + ac + bc + abc$

$a * (b * c) = a * (b + c + bc)$

$= a + b + c + bc + a(b + c + bc)$

$= a + b + c + bc + ab + ac + abc$

Here, $(a * b) * c = a * (b * c)$.

So, the operation is associative.

136. *(c)* Given that, $G = \{1, 5, 7, 11\}$ is a group under multiplication module 12.

$\because \quad 7^{-1} = 7 \qquad [\because 7^{-1} \otimes_{12} 7 = 1]$

Now, $\quad 7^{-1} \otimes_{12} (x \otimes_{12} 11) = 5$

$\Rightarrow \quad 7 \otimes_{12} (11 \otimes_{12} x) = 5$

$\Rightarrow \quad (7 \otimes_{12} 11) \otimes_{12} x = 5$

$\{7 \otimes_{12} 11 =$ remainder after dividing 77 from 12$\}$

$\Rightarrow \quad 5 \otimes_{12} x = 5$

$\Rightarrow \quad x = 1$

137. *(b)* In additive subgroup 'o' is the identity.

So, required set $(N, +)$ which is not a subgroup.

138. *(d)* We have, $\mathrm{p} = \hat{\mathrm{i}} + \hat{\mathrm{j}}$, $\mathrm{q} = 4\hat{\mathrm{k}} - \hat{\mathrm{j}}$ and $\mathrm{r} = \hat{\mathrm{i}} + \hat{\mathrm{k}}$

So, $3\mathrm{p} + \mathrm{q} - 2\mathrm{r} = 3\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 4\hat{\mathrm{k}} - \hat{\mathrm{j}} - 2\hat{\mathrm{i}} - 2\hat{\mathrm{k}}$

$= \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$

Now, required unit vector $= \dfrac{\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}}}{\sqrt{1 + 4 + 4}}$

$= \dfrac{1}{3}(\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}})$

139. *(a)* We have, $|\mathrm{a}| = 3\sqrt{3}$, $|\mathrm{b}| = 4$ and $|\mathrm{a} + \mathrm{b}| = \sqrt{7}$

Now, $|\mathrm{a} + \mathrm{b}|^2 = |\mathrm{a}|^2 + |\mathrm{b}|^2 + 2|\mathrm{a}||\mathrm{b}| \cos\theta$

$\Rightarrow \quad (\sqrt{7})^2 = (3\sqrt{3})^2 + 16 + 2(3\sqrt{3})(4)\cos\theta$

$\Rightarrow \quad 7 = 27 + 16 + 24\sqrt{3}\cos\theta$

$\Rightarrow \quad 24\sqrt{3}\cos\theta = -36$

$\Rightarrow \quad \cos\theta = -\dfrac{36}{24\sqrt{3}}$

$\Rightarrow \quad \cos\theta = -\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta = 150°$

140. *(b)* Given, $\mathbf{a}$ is perpendicular to $\mathbf{b}$ and $\mathbf{c}$.

Thus, $\mathbf{a}$ is perpendicular to the plane of $\mathbf{b}$ and $\mathbf{c}$.

Now, cross product of $\mathbf{b}$ and $\mathbf{c}$ will give a vector perpendicular to plane of $\mathbf{b}$ and $\mathbf{c}$. This vector will be parallel to $\mathbf{a}$.

Now, cross product of two parallel is zero vector.

Thus, $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = 0$

141. *(c)* We know that, if $\mathbf{a}$ and $\mathbf{b}$ are two adjacent sides of a parallelogram, then

Area $= |\mathbf{a} \times \mathbf{b}| = 15$ (given) ... (i)

If the sides are $(3\mathbf{a} + 2\mathbf{b})$ and $(\mathbf{a} + 3\mathbf{b})$

Then,area of parallelogram

$= 3|(3\mathbf{a} + 2\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b})|$

$= |3\,\mathbf{a} \times \mathbf{a} + 9\mathbf{a} \times \mathbf{b} + 2\mathbf{b} \times \mathbf{a} + 6\mathbf{b} \times \mathbf{a}|$

$= |0 + 9\mathbf{a} \times \mathbf{b} - 2\mathbf{a} \times \mathbf{b} + 0|$

$= |7(\mathbf{a} \times \mathbf{b})|$

$= 7\,|\mathbf{a} \times \mathbf{b}|$

$= 7 \times 15 = 105$ sq units

142. *(c)* The locus of the point which such that the ratio of its distance from two fixed point in the plane is always a constant $k(k < 1)$ is an ellipse.

143. *(d)* Given lines are concurrent, then there coefficient determinant is zero.

So, $\begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$

$\Rightarrow \; 1(-4b - 2) - 3(-16 + 4) - 9(-4 - 2b) = 0$

$\Rightarrow -4b - 2 + 36 + 36 + 18b = 0$

$\Rightarrow \; 14b + 70 = 0$

$\Rightarrow \; b = -5$

144. *(c)* We know that the angle between the two lines $ax^2 + 2hxy + by^2 = 0$ is given by

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

$\because \quad \theta = 90°$

$\therefore \quad a + b = 0$

145. *(b)* Since, the equation of tangents $x - y - 2 = 0$ and $x - y + 2 = 0$ are parallel.

Therefore, distance between them = Diameter of the circle

$$= \left|\frac{2 - (-2)}{\sqrt{1^2 + 1^2}}\right|$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Radius $= \dfrac{1}{2}(2\sqrt{2}) = \sqrt{2}$

Now, required equation of circle is

$(x - 0)^2 + (y - 0)^2 = (\sqrt{2})^2$

$x^2 + y^2 = 2$

146. *(*)* Let point $P(x_1, y_1)$ be any point on the circle, therefore it satisfy the circle

$(x_1 - 3)^2 + (y_1 + 2)^2 = 5r^2$ ... (i)

The length of the tangent drawn from point $p(x_1, y_1)$ to the circle $(x - 3)^2 + (y + 2)^2 = r^2$ is

$\sqrt{(x_1 - 3)^2 + (y_1 + 2)^2 - r^2} = \sqrt{5r^2 - r^2}$

$= \sqrt{4r^2} = 2r$

$\Rightarrow \quad 2r = 16$

$\Rightarrow \quad r = 8$

So, area between two circles

$$= \pi(5r^2) - \pi r^2$$
$$= 5\pi r^2 - \pi r^2$$
$$= 4\pi r^2 = 4\pi \times 64$$
$$= 256\ \pi \text{ sq units}$$

147. *(*)* Given equation of circles are

$$ax^2 + ay^2 + 2g_1x + 2f_1y + c_1 = 0$$

and $bx^2 + by^2 + 2g_2x + 2f_2y + c_2 = 0$

It can be rewritten as,

$$x^2 + y^2 + \frac{2g_1}{a}x + \frac{2f_1}{a}y + \frac{c_1}{a} = 0$$

and $x^2 + y^2 + \frac{2g_2}{b}x + \frac{2f_2}{b}y + \frac{c_2}{b} = 0$

So, centres of circle $\left(-\frac{g_1}{a}, -\frac{f_1}{a}\right)$

and $\left(-\frac{g_2}{b}, -\frac{f_2}{b}\right)$ respectively.

We know that, if two circles cut orthogonally, then

$$2(g_1g_2 + f_1f_2) = c_1 + c_2$$
$$\Rightarrow 2\left(\frac{g_1g_2}{ab} + \frac{f_1f_2}{ab}\right) = \frac{c_1}{a} + \frac{c_2}{b}$$
$$2(g_1g_2 + f_1f_2) = bc_1 + ac_2$$

148. *(d)* Let

$$S_1 \equiv x^2 + y^2 - 6x - 12y + 37 = 0$$

and $S_2 \equiv x^2 + y^2 - 6y + 7 = 0$

The equation of common tangent of the two circles is

$S_1 - S_2 = 0$

$\Rightarrow x^2 + y^2 - 6x - 12y + 37 - (x^2 + y^2 - 6y + 7) = 0$

$\Rightarrow -6x + 6y + 30 = 0$

$\Rightarrow x - y - 5 = 0$

149. *(d)* Given, vertex of the parabola $(h, k) = (-1, 1)$ and its focus is

$(a + h, k) = (2, 1)$ or $a + h = 2 \Rightarrow a = 3$

We know that as the y-coordinate of vertex and focus are same, therefore axis of parabola is parallel to X-axis.

Thus, equation of parabola is

$$(y - k)^2 = 4a(x - h)$$
$$(y - 1)^2 = 4 \times 3(x + 1)$$
$$(y - 1)^2 = 12x + 12$$
$$y^2 + 1 - 2y = 12x + 12$$
$$y^2 - 12x - 2y - 11 = 0$$

150. *(a)* Let the equation of line by $y = mx + C$

Since, this is the tangent to the circle $x^2 + y^2 = 5$

So, $C = \pm a\sqrt{1 + m^2}$

$$= \pm\sqrt{5}\sqrt{1 + m^2} \quad \ldots (i)$$

Also, the above line is tangent to the parabola

$$y^2 = 40x$$

So, $c = \frac{a}{m} = \frac{10}{m}$ ... (ii)

By Eqs. (i) and (ii), we get

$$\frac{10}{m} = \pm\sqrt{5}\sqrt{1 + m^2}$$
$$m^4 + m^2 - 20 = 0$$
$$(m^2 + 5)(m^2 - 4) = 0$$
$$m^2 = 4\ (m^2 \neq -5)$$
$$m = \pm 2 \Rightarrow c = \pm 5$$

Here, $y = \pm 2x \pm 5$

151. *(a)* Given, $x = 4(1 + \cos\theta)$

and $y = 3(1 + \sin\theta)$

$\Rightarrow \cos\theta = \frac{x}{4} - 1 = \frac{x - 4}{4}$

and $\sin\theta = \frac{y}{3} - 1 = \frac{y - 3}{3}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \frac{(x - 4)^2}{16} + \frac{(y - 3)^2}{9} = 1$

152. *(d)* Equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between foci = $2ae$

and distance between directrices = $\frac{2a}{e}$

According to question, we have

$$\frac{2ae}{2a/e} = \frac{3}{2}$$
$$\Rightarrow e^2 = \frac{3}{2}$$
$$\because b^2 = a^2(e^2 - 1)$$
$$\Rightarrow \frac{b^2}{a^2} = \frac{3}{2} - 1 = \frac{1}{2}$$
$$\Rightarrow \frac{b}{a} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow a : b = \sqrt{2} : 1$$

153. *(a)* Equation of ellipse is

$$\frac{x^2}{25} + \frac{y^2}{16} = 1,\ a > b$$

and equation of hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{16} = 1,\ a > b$$

Let e and e' be the eccentricities of the ellipse and hyperbola.

So, $e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25 - 16}{25}} = \frac{3}{5}$

and $e' = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{25+16}{25}} = \frac{\sqrt{41}}{5}$

(i) Centre of ellipse (0, 0) and centre of hyperbola is (0, 0)

(ii) Foci of ellipse are $(\pm ae, 0)$ or $(\pm 3, 0)$. Foci of hyperbola are $(\pm ae', o)$ or $(\pm\sqrt{41}, 0)$.

(iii) Direction of ellipse are

$$x = \pm\frac{a}{e} \Rightarrow x = \pm\frac{25}{3}$$

and directrices of hyperbola are $x = \pm\frac{a}{e}$

$$\Rightarrow \quad x = \pm\frac{25}{\sqrt{41}}$$

(iv) Vertices of ellipse are $(\pm a, 0)$ or $(\pm 5, 0)$. Vertices of hyperbola are $(\pm a, 0)$ or $(\pm 5, 0)$.
From the above discussions, their are common is centre and vertices.

154. *(d)* We have, $\sec\theta = m$ and $\tan\theta = n$

$$\text{Now, } \frac{1}{m}\left[(m+n)+\frac{1}{(m+n)}\right]$$

$$= \frac{1}{\sec\theta}\left[(\sec\theta+\tan\theta)+\frac{1}{(\sec\theta+\tan\theta)}\right]$$

$$= \frac{1}{\sec\theta}\left[\frac{(\sec\theta+\tan\theta)^2+1}{(\sec\theta+\tan\theta)}\right]$$

$$= \frac{1}{\sec\theta}\left[\frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}{(\sec\theta+\tan\theta)}\right]$$

$$= \frac{1}{\sec\theta}\left[\frac{\sec^2\theta+\sec^2\theta-1+2\sec\theta+\tan\theta+1}{\sec\theta+\tan\theta}\right]$$

$$= \frac{1}{\sec\theta}\left[\frac{2\sec^2\theta+2\sec\theta\tan\theta}{\sec\theta+\tan\theta}\right]$$

$$= \frac{1}{\sec\theta}\left[\frac{2\sec\theta(\sec\theta+\tan\theta)}{(\sec\theta+\tan\theta)}\right]$$

$$= \frac{1}{\sec\theta}(2\sec\theta) = 2$$

155. *(b)* $\dfrac{\sin 85^\circ - \sin 35^\circ}{\cos 65^\circ}$

$$= \frac{2\cos\left(\frac{85^\circ+35^\circ}{2}\right)\sin\left(\frac{85^\circ-35^\circ}{2}\right)}{\cos 65^\circ}$$

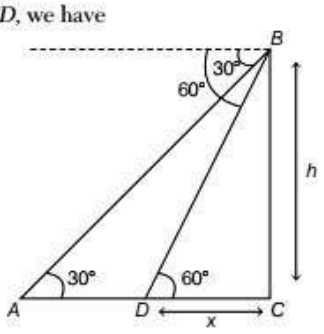
$$= \frac{2\cos 60^\circ \sin 25^\circ}{\cos 65^\circ}$$

$$= \frac{2\times\frac{1}{2}\times\sin 25^\circ}{\cos(90^\circ-25^\circ)}$$

$$= \frac{\sin 25^\circ}{\sin 25^\circ} = 1$$

156. *(c)* Let the distance of two consecutive stones are $(x, x+1)$.

In ΔBCD, we have



$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots \text{(i)}$$

In ΔABC, we have

$$\tan 30^\circ = \frac{h}{x+1}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+1}$$

$$\Rightarrow \quad \frac{h}{\sqrt{3}} + 1 = \sqrt{3}h \qquad \text{[by Eq. (i)]}$$

$$\Rightarrow \quad h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 1$$

$$\Rightarrow \quad h\cdot\left(\frac{2}{\sqrt{3}}\right) = 1 \quad \Rightarrow \quad h = \frac{\sqrt{3}}{2}$$

157. *(d)* Let the angles of triangle are $3\theta, 4\theta, 5\theta$.

As we know that,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \quad 3\theta + 4\theta + 5\theta = 180^\circ$$

$$\Rightarrow \quad 12\theta = 180^\circ \Rightarrow \theta = 15^\circ$$

So, angle are $45^\circ, 60^\circ, 75^\circ$.

$$\text{Now, } \sin A = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin B = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin C = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

So, $a : b : c = \sin A : \sin B : \sin C$

$$= \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$= 2 : \sqrt{6} : \sqrt{3}+1$$

158. *(b)* Given that, $\cos^{-1}x = \alpha, (0 < x < 1)$ $\qquad \ldots$ (i)

$$\Rightarrow \quad x = \cos\alpha$$

$$\text{Thus, } \sin^{-1}(2x\sqrt{1-x^2}) + \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1}(2\cos\alpha\sqrt{1-\cos^2\alpha}) + \sec^{-1}\left(\frac{1}{2\cos^2\alpha-1}\right)=\frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1}(\sin 2\alpha)+\sec^{-1}(\sec 2\alpha)=\frac{2\pi}{3}$$

$$\Rightarrow 2\alpha+2\alpha=\frac{2\pi}{3}\Rightarrow\alpha=\frac{\pi}{6}$$

Now, $x=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2} \Rightarrow 2x=\sqrt{3}$

Therefore, $\tan^{-1}(2x)=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$

159. *(a)* We have,

$$\tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}\left(\frac{a+b}{a-b}\right)$$

$$=\tan^{-1}\left[\frac{\frac{a}{b}+\frac{a+b}{a-b}}{1-\left(\frac{a}{b}\right)\left(\frac{a+b}{a-b}\right)}\right]$$

$$=\tan^{-1}\left[\frac{\frac{a^2-ab+ab+b^2}{b(a-b)}}{\frac{b(a-b)-a(a+b)}{b(a-b)}}\right]$$

$$=\tan^{-1}\left[\frac{a^2+b^2}{ab-b^2-a^2-ab}\right]$$

$$=\tan^{-1}\left[\frac{a^2+b^2}{-(a^2+b^2)}\right]=\tan^{-1}(-1)$$

It does not depends neither a nor b.

160. *(c)* By option (c),

$\operatorname{cosec}\theta\cdot\sec\theta=1$

$\sin\theta\cos\theta=1$

$2\sin\theta\cos\theta=2$

$\sin 2\theta=2$

As we know that range of $\sin x$ is $[-1, 1]$.

Hence, this equation has no solution.

161. *(b)* Let $z=\frac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$

$$=\frac{\sqrt{3}(-1+\sqrt{3}i)(1-i)}{(\sqrt{3})^2(\sqrt{3}+i)(1+i)(i)}$$

$$=\frac{(-1+\sqrt{3}i)(1-i)}{\sqrt{3}(\sqrt{3}i+i^2)(1+i)}=\frac{(-1+\sqrt{3}i)(1-i)}{\sqrt{3}(-1+\sqrt{3}i)(1+i)}$$

$$=\frac{1-i}{(\sqrt{3})1+i}\times\frac{(1-i)}{(1-i)}=\frac{(1-i)^2}{(\sqrt{3})(1+1)}$$

$$=\frac{1+i^2-2i}{2\sqrt{3}}=\frac{-2i}{2\sqrt{3}}=-\frac{i}{\sqrt{3}}$$

= Purely imaginary

162. *(a)* We have,

$$2x=-1+\sqrt{3}i \Rightarrow x=\frac{-1+\sqrt{3}i}{2}=\omega$$

So, $(1-x^2+x)^6-(1-x+x^2)^6$

$$=(1-\omega^2+\omega)^6-(1-\omega-\omega^2)^6$$

$$=(-2\omega^2)^6-(-\omega-\omega)^6=2^6\omega^{12}-2^6\omega^6$$

$$=2^6-2^6=0 \qquad [\because \omega^3=1]$$

163. *(c)* Let $z=(1+i\sqrt{3})^8$

$$=\left(2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right)^8$$

$$=[2(\cos 60^\circ+i\sin 60^\circ)]^8=(2^8e^{i\pi/3})^8$$

$$=2^8.e^{\frac{8\pi}{3}i}=2^8.e^{\left(2\pi+\frac{2\pi}{3}\right)i}=2^8.e^{\frac{2\pi}{3}i}$$

So, modulus $=2^8=256$

and amplitude $=\frac{2\pi}{3}$

164. *(d)* $\lim_{x\to 0}\frac{5^x-5^{-x}}{2x}=\lim_{x\to 0}\frac{5^x\log 5+5^{-x}\log 5}{2}$

(Applying L' Hospital rule)

$$=\frac{\log 5+\log 5}{2}=\log 5$$

165. *(c)* If a function $f(x$ is continuous at $x=a)$.

Then, it may or may not be differentiable at $x=a$.

166. *(a)* Given, $y=\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\ldots\infty$

This is an infinite GP with $a=1$

and $r=\frac{1}{x}$

$\because$ $S_\infty=\frac{a}{1-r}$

$$y=\frac{1}{1-\frac{1}{x}}$$

$$y=\frac{x}{x-1}$$

Now, $\frac{dy}{dx}=\frac{(x-1)-x}{(x-1)^2}=\frac{x-1-x}{(x-1)^2}=\frac{-1}{(x-1)^2}$

$$=-\frac{1}{\left(\frac{x^2}{y^2}\right)}=-\frac{y^2}{x^2}$$

167. *(*)* $f\{g(x)\}=x^3-\frac{1}{x^3}$

$$f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$$

$$=\left(x-\frac{1}{x}\right)\left\{\left(x-\frac{1}{x}\right)^2+2+1\right\}$$

$$f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left\{\left(x-\frac{1}{x}\right)^2+3\right\}$$

Replace $x-\frac{1}{x}$ by x,

$$f(x)=(x)(x^2+3)=x^3+3x$$

$$f'(x)=3x^2+3$$

168. *(b)* Let $u=a^{\sec x} \Rightarrow v=a^{\tan x}$

$\log u=\sec x \log a$

$$\frac{du}{dx}=u\log a\sec x\tan x$$

$$=a^{\tan x}\log a\sec x\tan x$$

and $\log v=\tan x\log a$

$$\frac{dv}{dx}=v\log a\sec^2 x$$

$$=a^{\tan x}\log a\sec^2 x$$

$$\frac{du}{dv}=\frac{\left(\frac{du}{dx}\right)}{\frac{dv}{dx}}=\frac{a^{\sec x}\log a\sec x\tan x}{a^{\tan x}\log a\sec^2 x}$$

$$=\frac{a^{\sec x}\sin x}{a^{\tan x}}=\sin x\, a^{\sec x-\tan x}$$

169. *(c)* Given that, $\sin(x+y)+\cos(x+y)=\log(x+y)$

On differentiating w.r.t. x,

$$\cos(x+y)\cdot\left(1+\frac{dy}{dx}\right)-\sin(x+y)\left(1+\frac{dy}{dx}\right)$$

$$=\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\left(1+\frac{dy}{dx}\right)\left\{\cos(x+y)-\sin(x+y)-\frac{1}{(x+y)}\right\}=0$$

$$\Rightarrow \quad 1+\frac{dy}{dx}=0$$

Again differentiating w.r.t. 'x'

$$0+\frac{d^2y}{dx^2}=0 \quad \Rightarrow \quad \frac{d^2y}{dx^2}=0$$

170. *(a)* We have,

$$g(x)=[f(x)]^2+[f'(x)]^2$$

Differentiate the function $g(x)$

$$g'(x)=2f(x)f'(x)+2f'(x)f''(x)$$

Use chain rule,

$$2f'(x)[f(x)+f''(x)]=2f'(x)(0)=0$$

Hence, $g(x)$ is *a* constant function

$$\Rightarrow \quad g(x)=c, \text{ constant}$$

But, $g(3)=8$, so $g(x)=8$

For all real x.

Hence, $g(8)=8$

171. *(c)* $y=2x^3+ax^2+bx+c$ ... (i)

Since, it passes through (0, 0),

$$0=2(0)+a(0)+b(0)+c$$

$$c=0 \quad \text{... (ii)}$$

$$\frac{dy}{dx}=6x^2+2ax+b$$

Since, tangents at $x=-1$ and $x=2$ are parallel to X-axis.

$$\therefore \quad \frac{dy}{dx}=0$$

At $x=-1$

$$6(-1)^2+2a(-1)+b=0$$

$$6-2a+b=0 \quad \text{... (iii)}$$

At $x=-2$

So, $6(2)^2+2a(2)+b=0$

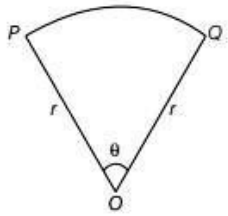
$$24+4a+b=0 \quad \text{... (iv)}$$

By solving Eqs. (iii) and (iv), we get

$$a=-3,\ b=-12$$

Hence, $a=-3,\ b=-12,\ c=0$

172. *(b)*



Perimeter of sector $=2r+r\theta$

$$\Rightarrow \quad 60=2r+r\theta \text{ (given)}$$

$$\Rightarrow \quad \theta=\frac{60-2r}{r}$$

Now, area of sector,

$$(A)=\frac{\pi r^2\theta}{360^\circ}=\frac{\pi r^2(60-2r)}{360\,r}$$

$$=\frac{\pi r}{180}(30-r)$$

$$\frac{dA}{dr}=\frac{\pi}{180}(30-2r)$$

For maximum area $\frac{dA}{dr}=0$

$$\Rightarrow \ 30-2r=0$$

$$\Rightarrow \ 2r=30 \Rightarrow r=15$$

and $\frac{d^2A}{dr^2}=\frac{\pi}{180^\circ}(-2)=-\frac{\pi}{90}<0$

Hence, it is maximum at $r=15$ m

173. *(a)* Given,equation of curve is

$$y=x^2-x+4$$

Slope of tangent at $P(1, 4)$ is

$$\frac{dy}{dx}=2x-1 \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,4)}=2-1=1$$

So, equation of tangent is

$$y-4=1(x-1)$$

$$\Rightarrow \quad y-x=3 \quad \text{... (i)}$$

and equation of normal at point $P(1, 4)$ is

$$y - 4 = -1(x - 1)$$

$$x + y = 5 \quad \ldots \text{(ii)}$$

Since, the tangent cuts X-axis at A.

Therefore, coordinates of A are $(-3, 0)$

and the normal cuts X-axis at B and coordinates of B are $(5, 0)$.

Therefore, area of ΔPAB

$$= \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ -3 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(0) - 4(-3 - 5) + 1(0)]$$

$$= \frac{1}{2}|32| = 16 \text{ sq. units}$$

174. *(d)* $\int \frac{x^3 + 3x^2 + 3x + 1}{(x+1)^5}\,dx$

$$= \int \frac{(x+1)^3}{(x+1)^5}\,dx = \int \frac{dx}{(x+1)^2}$$

$$= -\frac{1}{x+1} + c$$

175. *(c)* Let $I = \int \frac{\text{cosec } x}{\cos^2\left(1 + \log \tan \frac{x}{2}\right)}\,dx$

Put $\quad 1 + \log \tan \frac{x}{2} = t$

$$\Rightarrow \quad \frac{1}{\tan \frac{x}{2}} \cdot \sec^2 \frac{x}{2} \cdot \frac{dx}{2} = dt$$

$$\Rightarrow \quad \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = dt \Rightarrow \frac{dx}{\sin x} = dt$$

$$\Rightarrow \quad \text{cosec } x\,dx = dt$$

So, $I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\,dt = \tan t + c$

$$= \tan\left(1 + \log \tan \frac{x}{2}\right) + c$$

176. *(b)* Let $I = \int \frac{dx}{x\sqrt{(x^3)^2 - 16}}$

Put $\quad x^3 = t \Rightarrow 3x^2 dx = dt$

So, $\quad I = \frac{1}{3}\int \frac{dt}{x^3\sqrt{(x^3)^2 - 16}}$

$$= \frac{1}{3}\int \frac{dt}{t(\sqrt{t^2 - 16}}$$

$$= \frac{1}{3 \times 4}\sec^{-1}\left(\frac{t}{4}\right) + c$$

$$= \frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right) + c$$

177. *(d)* $I_1 = \int_0^{\pi/2} x \sin x\,dx$

$$= [-x \cos x + \int \cos x]_0^{\pi/2}$$

$$= [-x \cos x + \sin x]_0^{\pi/2}$$

$$= 0 + \sin \frac{\pi}{2} - 0 = 1$$

Similarly, $I_2 = \int_0^{\pi/2} x \cos x\,dx$

$$= [x \sin x - \int \sin x]_0^{\pi/2}$$

$$= [x \sin x + \cos x]_0^{\pi/2}$$

$$= \frac{\pi}{2}\sin \frac{\pi}{2} - 1 = \frac{\pi}{2} - 1$$

Hence, $I_1 + I_2 = 1 + \frac{\pi}{2} - 1 = \frac{\pi}{2}$

178. *(b)* Given that, $f(x) = 4x^2 - 3x + 1$

$$g(x) = \frac{f(-x) - f(x)}{x^2 + 3}$$

Therefore,

$$g(x) = \frac{(4x^2 + 3x + 1) - (4x^2 - 3x + 1)}{x^2 + 3} = \frac{6x}{x^2 + 3}$$

Now, $g(-x) = -\frac{6x}{x^2 + 3} = g(x)$

which is an odd function.

Thus, $\int_{-2}^{2} g(x)\,dx = 0$

179. *(a)* Required area

$$= \int_0^2 [(x + 2) - (x^2 - x + 2)]\,dx$$

$$= \int_0^2 [x + 2 - x^2 + x - 2]\,dx = \int_0^2 (-x^2 + 2x)\,dx$$

$$= \left[-\frac{x^3}{3} + x^2\right]_0^2$$

$$= -\frac{8}{3} + 4 - (0) = \frac{-8 + 12}{3} = \frac{4}{3}$$

180. *(d)* Given, differential equation is

$$e^{-x}(y + 1)\,dy + (\cos^2 x - \sin 2x)y\,dx = 0$$

$$\Rightarrow \left(\frac{y+1}{y}\right)dy + e^x(\cos^2 x - \sin 2x)dx = 0$$

$$\Rightarrow \left(1 + \frac{1}{y}\right)dy + (e^x \cos^2 x - e^x \sin 2x)\,dx = 0$$

$$\Rightarrow d(y + \log y) + d(e^x \cos^2 x) = 0$$

On integrating above equation, we get

$$y + \log y + e^x \cos^2 x = c$$

At $y(0) = 1$

$$\Rightarrow 1 + 0 + e^0 \cdot 1 = c \quad \Rightarrow c = 2$$

Hence, required solution is

$$y + \log y + e^x \cos^2 x = 2$$