

SOLVED PAPER – 2018 (COMEDK)

Instructions

- There are 180 questions in all. The number of questions in each section is as given below.

Sections	No. of Questions
Section I : Physics	1-60
Section II : Chemistry	61-120
Section III : Mathematics	121-180

- All the questions are Multiple Choice Questions having four options out of which **ONLY ONE** is correct.
- Candidates will be awarded 1 mark for each correct answer. There will be no negative marking for incorrect answer.
- Time allotted to complete this paper is 3 hrs.

PHYSICS

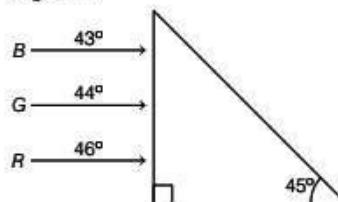
1. A ray of light enters from a rarer to a denser medium. The angle of incidence is i . Then, the reflected and refracted rays are mutually perpendicular to each other. The critical angle for the pair of media is

- $\sin^{-1}(\tan i)$
- $\tan^{-1}(\sin i)$
- $\sin^{-1}(\cot i)$
- $\cos^{-1}(\tan i)$

2. A fish in water (refractive index n) looks at a bird vertically above in the air. If y is the height of the bird and x is the depth of the fish from the surface, then the distance of the bird as estimated by the fish is

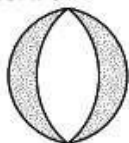
- $x + y \left(1 - \frac{1}{n}\right)$
- $x + ny$
- $x + y \left(1 + \frac{1}{n}\right)$
- $y + x \left(1 - \frac{1}{n}\right)$

3. Figure shows a mixture of blue, green and red coloured rays incident normally on a right angled prism. The critical angles of the material of the prism for red, green and blue are 46° , 44° and 43° , respectively. The arrangement will separate



- red colour from blue and green
 - blue colour from red and green
 - green colour from red and blue
 - All the three colours
4. A convex and a concave lens separated by distance d are then put in contact. The focal length of the combination
- decreases
 - increases
 - becomes zero
 - remains the same

5. A convex lens is made of 3 layers of glass of 3 different materials as in the figure. A point object is placed on its axis. The number of images of the object are



- a. 1
b. 2
c. 3
d. 4
6. If μ_0 is permeability of free space and ϵ_0 is permittivity of free space, then the speed of light in vacuum is given by
- a. $\sqrt{\mu_0 \epsilon_0}$
b. $\sqrt{\mu_0 / \epsilon_0}$
c. $\sqrt{\frac{1}{\mu_0 \epsilon_0}}$
d. $\sqrt{\epsilon_0 / \mu_0}$
7. In Young's double slit experiment, a third slit is made in between the double slits. Then,
- a. intensity of fringes totally disappears
b. only bright light is observed on the screen
c. fringes of unequal width are formed
d. contrast between bright and dark fringes is reduced
8. The maximum number of possible interference maxima when slit separation is equal to 4 times the wavelength of light used in a double slit experiment is
- a. ∞
b. 9
c. 8
d. 4
9. In a Fraunhofer diffraction experiment at a single slit using a light of wavelength 400 nm, the first minimum is formed at an angle of 30° . The direction θ of the first secondary maximum is given by
- a. $\sin^{-1}(2/3)$
b. $\sin^{-1}(3/4)$
c. $\sin^{-1}(1/4)$
d. $\tan^{-1}(2/3)$
10. Maximum diffraction takes place in a given slit for
- a. γ -rays
b. ultraviolet light
c. infrared light
d. radio waves
11. An unpolarised beam of intensity I_0 falls on a polaroid. The intensity of the emergent light is
- a. $I_0 / 2$
b. I_0
c. $I_0 / 4$
d. Zero
12. Which of the following is a dichroic crystal?
- a. Quartz
b. Tourmaline
c. Mica
d. Selenite

13. Two identical metal spheres charged with $+12\mu\text{F}$ and $-8\mu\text{F}$ are kept at certain distance in air. They are brought into contact and then kept at the same distance. The ratio of the magnitudes of electrostatic forces between them before and after contact is

a. 12 : 1
b. 8 : 1
c. 24 : 1
d. 4 : 1

14. A small conducting sphere of radius r is lying concentrically inside a bigger hollow conducting sphere of radius R . The bigger and smaller spheres are charged with Q and q ($Q > q$) and are insulated from each other. The potential difference between the spheres will be

a. $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{q}{R} \right)$
b. $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{R} - \frac{Q}{r} \right)$
c. $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{Q}{R} \right)$
d. $\frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} + \frac{q}{r} \right)$

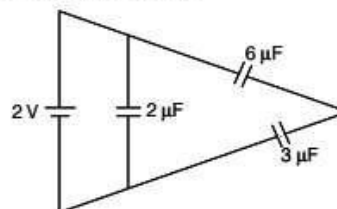
15. The charges Q , $+q$ and $+q$ are placed at the vertices of an equilateral triangle of side l . If the net electrostatic potential energy of the system is zero, then Q is equal to

a. $-q/2$
b. $-q$
c. $+q/2$
d. Zero

16. How many $6\mu\text{F}$, 200 V condensers are needed to make a condenser of $18\mu\text{F}$, 600 V?

a. 9
b. 18
c. 3
d. 27

17. The total energy stored in the condenser system shown in the figure will be

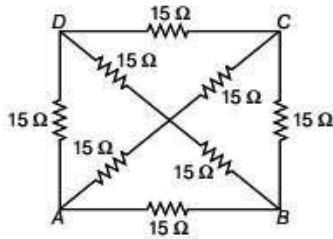


a. $2\mu\text{J}$
b. $4\mu\text{J}$
c. $8\mu\text{J}$
d. $16\mu\text{J}$

18. A metal wire is subjected to a constant potential difference. When the temperature of the metal wire increases, the drift velocity of the electron in it

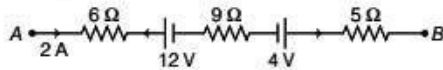
a. increases, thermal velocity of the electron decreases
b. decreases, thermal velocity of the electron decreases
c. increases, thermal velocity of the electron increases
d. decreases, thermal velocity of the electron increases

19. The equivalent resistance between the points A and B will be (each resistance is $15\ \Omega$)

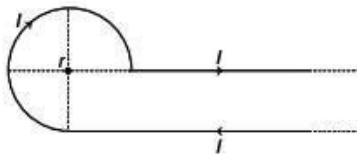


- a. $30\ \Omega$ b. $8\ \Omega$
c. $10\ \Omega$ d. $40\ \Omega$
20. The terminals of a 18 V battery with an internal resistance of $1.5\ \Omega$ are connected to a circular wire of resistance $24\ \Omega$ at two points distant at one quarter of the circumference of a circular wire. The current through the bigger arc of the circle will be
- a. 0.75 A b. 1.5 A
c. 2.25 A d. 3 A

21. The potential difference between A and B in the following figure is



- a. 32 V b. 48 V
c. 24 V d. 14 V
22. The magnetic field at the centre of a circular current carrying conductor of radius r is B_c . The magnetic field on its axis at a distance r from the centre is B_a . The value of $B_c : B_a$ will be
- a. $1 : \sqrt{2}$ b. $1 : 2\sqrt{2}$
c. $2\sqrt{2} : 1$ d. $\sqrt{2} : 1$
23. Current I is flowing in a conductor shaped as shown in the figure. The radius of the curved part is r and the length of straight portion is very large. The value of the magnetic field at the centre O will be

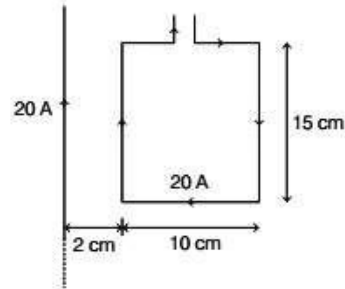


- a. $\frac{\mu_0 I}{4\pi r} \left(\frac{3\pi}{2} + 1 \right)$ b. $\frac{\mu_0 I}{4\pi r} \left(\frac{3\pi}{2} - 1 \right)$
c. $\frac{\mu_0 I}{4\pi r} \left(\frac{\pi}{2} + 1 \right)$ d. $\frac{\mu_0 I}{4\pi r} \left(\frac{\pi}{2} - 1 \right)$

24. Two tangent galvanometers A and B are identical except in their number of turns. They are connected in series. On passing a current through them, deflections of 60° and 30° are produced. The ratio of the number of turns in A and B is

- a. $1 : 3$ b. $3 : 1$
c. $1 : 2$ d. $2 : 1$

25. The resultant force on the current loop PQRS due to a long current carrying conductor will be



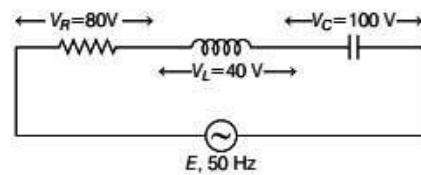
- a. 10^{-4} N
b. $3.6 \times 10^{-4}\text{ N}$
c. $1.8 \times 10^{-4}\text{ N}$
d. $5 \times 10^{-4}\text{ N}$

26. A certain current on passing through a galvanometer produces a deflection of 100 divisions. When a shunt of one ohm is connected, then the deflection reduces to 1 division. The galvanometer resistance is
- a. $100\ \Omega$ b. $99\ \Omega$ c. $10\ \Omega$ d. $9.9\ \Omega$

27. Two similar circular loops carry equal currents in the same direction. On moving the coils further apart, the electric current will

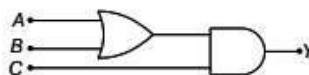
- a. increase in both
b. decrease in both
c. remain unaltered
d. increases in one and decreases in the second

28. The value of alternating emf E in the given circuit will be



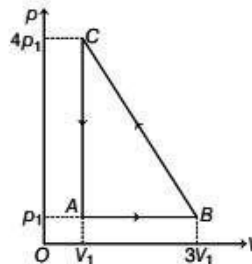
- a. 220 V b. 140 V
c. 100 V d. 20 V

29. A current of 5 A is flowing at 220 V in the primary coil of a transformer. If the voltage produced in the secondary coil is 2200 V and 50% of power is lost, then the current in the secondary will be
 a. 2.5 A b. 5 A
 c. 0.25 A d. 0.5 A
30. For a series L - C - R circuit at resonance, the statement which is not true ?
 a. Peak energy stored by a capacitor = peak energy stored by an inductor
 b. Average power = Apparent power
 c. Wattless current is zero.
 d. Power factor is zero.
31. Solar spectrum is an example for
 a. line emission spectrum
 b. continuous emission spectrum
 c. band absorption spectrum
 d. line absorption spectrum
32. When a piece of metal is illuminated by a monochromatic light of wavelength λ , then stopping potential is 3V-s. When same surface is illuminated by light of wavelength 2λ , then stopping potential becomes V_s . The value of threshold wavelength for photoelectric emission will be
 a. 4λ b. 8λ
 c. $4/3\lambda$ d. 6λ
33. The maximum kinetic energy of emitted electrons in a photoelectric effect does not depend upon
 a. wavelength b. frequency
 c. intensity d. work-function
34. The ratio of minimum wavelength of Lyman and Balmer series will be
 a. 1.25 b. 0.25
 c. 5 d. 10
35. Hydrogen atom does not emit X-rays because
 a. it contains only a single electron
 b. energy levels in it are far apart
 c. its size is very small
 d. energy levels in it are very close to each other
36. If an electron and a proton have the same de-Broglie wavelength, then the kinetic energy of the electron is
 a. Zero
 b. less than that of a proton
 c. more than that of a proton
 d. equal to that of a proton
37. Two protons are kept at a separation of $40 \text{ \AA}$. F_n is the nuclear force and F_e is the electrostatic force between them. Then,
 a. $F_n \gg F_e$ b. $F_n = F_e$
 c. $F_n < F_e$ d. $F_n \approx F_e$
38. Blue colour of sea water is due to
 a. interference of sunlight reflected from the water surface
 b. scattering of sunlight by the water molecules
 c. image of sky in water
 d. refraction of sunlight
39. The ratio of the nuclear radii of elements with mass numbers 216 and 125 is
 a. 216 : 125 b. $\sqrt{216} : \sqrt{125}$
 c. 6 : 5 d. None of these
40. On bombarding U^{235} by slow neutron, 200 MeV energy is released. If the power output of atomic reactor is 1.6 MW, then the rate of fission will be
 a. $5 \times 10^{22} / s$ b. $5 \times 10^{16} / s$
 c. $8 \times 10^{16} / s$ d. $20 \times 10^{16} / s$
41. The masses of two radioactive substances are same and their half-lives are 1 year and 2 years, respectively. The ratio of their activities after 6 years will be
 a. 1 : 4 b. 1 : 2
 c. 1 : 3 d. 1 : 6
42. ${}_{92}U^{235}$ undergoes successive disintegrations with the end product of ${}_{82}Pb^{203}$. The number of α and β particles emitted are
 a. $\alpha = 6, \beta = 4$ b. $\alpha = 6, \beta = 0$
 c. $\alpha = 8, \beta = 6$ d. $\alpha = 3, \beta = 3$
43. The most stable particle in Baryon group is
 a. neutron b. omega-particle
 c. proton d. lamda-particle
44. In an unbiased p - n junction
 a. potential at p is more than that at n
 b. potential at p is less than that at n
 c. potential at p is equal to that at n
 d. potential at p is +ve and that at n is -ve
45. To get an output $Y = 1$ from the circuit shown, the inputs A , B and C must be respectively



- a. 0, 1, 0 b. 1, 0, 0
 c. 1, 0, 1 d. 1, 1, 0

46. Dimensional formula for the universal gravitational constant G is
 a. $[M^{-1}L^2T^{-2}]$ b. $[M^0L^0T^0]$
 c. $[M^{-1}L^3T^{-2}]$ d. $[M^{-1}L^3T^{-1}]$
47. A body is projected vertically upwards. The times corresponding to height h , while ascending and descending are t_1 and t_2 , respectively. Then, the velocity of projection is (g is acceleration due to gravity)
 a. $g\sqrt{t_1t_2}$ b. $\frac{gt_1t_2}{t_1+t_2}$
 c. $\frac{g\sqrt{t_1t_2}}{2}$ d. $\frac{g(t_1+t_2)}{2}$
48. A mass of 10 kg is suspended from a spring balance. It is pulled a side by a horizontal string, so that it makes an angle of 60° with the vertical. The new reading of the balance is
 a. 20 kg-wt b. 10 kg-wt
 c. $10\sqrt{3}$ kg-wt d. $20\sqrt{3}$ kg-wt
49. A body weight 50 g in air and 40 g in water. How much would it weight in a liquid of specific gravity 1.5?
 a. 30 g b. 35 g
 c. 65 g d. 45 g
50. A body of mass 4 kg is accelerated upon by a constant force, travels a distance of 5 m in the 1st second and a distance of 2 m in the 3rd second. The force acting on the body is
 a. 2 N b. 4 N
 c. 6 N d. 8 N
51. A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest its time period is T . With what acceleration should the lift be accelerated upwards in order to reduce its period to $T/2$? (g is acceleration due to gravity)
 a. $2g$ b. $3g$
 c. $4g$ d. g
52. If γ is the ratio of specific heats and R is the universal gas constant, then the molar specific heat at constant volume C_V is given by
 a. γR b. $\frac{(\gamma-1)R}{\gamma}$
 c. $\frac{R}{\gamma-1}$ d. $\frac{\gamma R}{\gamma-1}$
53. An ideal gas is taken via path ABCA as shown in figure.



The network done in the whole cycle is

- a. $3p_1V_1$ b. $-3p_1V_1$ c. $6p_1V_1$ d. Zero
54. In which of the processes, does the internal energy of the system remain constant?
 a. Adiabatic b. Isochoric
 c. Isobaric d. Isothermal
55. The coefficient of thermal conductivity of copper is 9 times that of steel. In the composite cylindrical bar shown in the figure, what will be the temperature at the junction of copper and steel?
-
- a. 75°C b. 67°C c. 25°C d. 33°C
56. The equation of a simple harmonic wave is given by $y = 6\sin 2\pi(2t - 0.1x)$, where x and y are in mm and t is in seconds. The phase difference between two particles 2 mm apart at any instant is
 a. 18° b. 36° c. 54° d. 72°
57. With what velocity should an observer approach a stationary sound source, so that the apparent frequency of sound should appear double the actual frequency? (v is velocity of sound).
 a. $v/2$ b. $3v$ c. $2v$ d. v
58. If a black body emits 0.5 J of energy per second when it is at 27°C , then the amount of energy emitted by it when it is at 627°C will be
 a. 40.5 J b. 162 J c. 13.5 J d. 135 J
59. A string vibrates with a frequency of 200 Hz. When its length is doubled and tension is altered, it begins to vibrate with a frequency of 300 Hz. The ratio of the new tension to the original tension is
 a. 9 : 1 b. 1 : 9 c. 3 : 1 d. 1 : 3
60. How many times more intense is a 60 dB sound than a 30 dB sound?
 a. 1000 b. 2 c. 100 d. 4

CHEMISTRY

- 61.** Which of the following is not an ore of magnesium?
 a. Carnallite b. Dolomite
 c. Calamine d. Sea water
- 62.** The atomic numbers of Ni and Cu are 28 and 29 respectively. The electron configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10}$ represents
 a. Cu^+ b. Cu^{2+}
 c. Ni^{2+} d. Ni
- 63.** In the following, the element with the highest ionisation energy is
 a. $[\text{Ne}] 3s^2, 3p^1$ b. $[\text{Ne}] 3s^2, 3p^3$
 c. $[\text{Ne}] 3s^2, 3p^2$ d. $[\text{Ne}] 3s^2, 3p^4$
- 64.** In the conversion of Br_2 to BrO_3^- , the oxidation number of Br changes from
 a. zero to +5 b. +1 to +5
 c. zero to -3 d. +2 to +5
- 65.** Among the alkali metals caesium is the most reactive because
 a. its incomplete shell is nearest to the nucleus
 b. it has a single electron in the valence shell
 c. it is the heaviest alkali metal
 d. the outermost electron is more loosely bound than the outermost electron of the other alkali metals.
- 66.** Which of the following represents the Lewis structure of N_2 molecule?
 a. $\text{N} \equiv \text{N}$ b. $\text{N} \equiv \text{N}$
 c. $\text{N} - \text{N}$ d. $\text{N} = \text{N}$
- 67.** Hydrogen bond is strongest in
 a. $\text{S}-\text{H} \cdots \text{O}$ b. $\text{O}-\text{H} \cdots \text{S}$
 c. $\text{F}-\text{H} \cdots \text{F}$ d. $\text{O}-\text{H} \cdots \text{N}$
- 68.** The decomposition of a certain mass of CaCO_3 gave 11.2 dm^3 of CO_2 gas at STP. The mass of KOH required to completely neutralise the gas is
 a. 56 g b. 28 g
 c. 42 g d. 20 g
- 69.** The density of a gas is 1.964 g dm^{-3} at 273 K and 76 cm Hg. The gas is
 a. CH_4 b. C_2H_6
 c. CO_2 d. Xe
- 70.** 0.06 mole of KNO_3 solid is added to 100 cm^3 of water at 298 K. The enthalpy of $\text{KNO}_3(\text{aq})$ solution is 35.8 kJ mol^{-1} . After the solute is dissolved the temperature of the solution will be
 a. 293 K b. 298 K c. 301 K d. 304 K
- 71.** 4 moles each of SO_2 and O_2 gases are allowed to react to form SO_3 in a closed vessel. At equilibrium 25 % of O_2 is used up. The total number of moles of all the gases present at equilibrium is
 a. 6.5 b. 7.0 c. 8.0 d. 2.0
- 72.** An example for auto-catalysis is
 a. oxidation of NO to NO_2
 b. oxidation of SO_2 to SO_3
 c. decomposition of KClO_3 to KCl and O_2
 d. oxidation of oxalic acid by acidified KMnO_4
- 73.** During the fusion of an organic compound with sodium metal, nitrogen of the compound is converted into
 a. NaNO_2 b. NaNH_2
 c. NaCN d. NaNC
- 74.** Identify the product Y in the following reaction sequence

$$\begin{array}{c} \text{CH}_2-\text{CH}_2-\text{COO} \\ | \\ \text{CH}_2-\text{CH}_2-\text{COO} \end{array} \text{Ca} \xrightarrow{\text{Heat}} \text{X} \xrightarrow[\text{HCl, heat}]{\text{Zn-Hg}} \text{Y}$$

 a. pentane b. cyclobutane
 c. cyclopentane d. cyclopentanone
- 75.** The reaction $\text{C}_2\text{H}_5\text{ONa} + \text{C}_2\text{H}_5\text{I} \longrightarrow \text{C}_2\text{H}_5\text{OC}_2\text{H}_5 + \text{NaI}$ is known as
 a. Kolbe's synthesis
 b. Wurtz's synthesis
 c. Williamson's synthesis
 d. Grignard's synthesis
- 76.** ΔG° vs T plot in the Ellingham's diagram, slopes downwards for the reaction
 a. $\text{Mg} + \frac{1}{2} \text{O}_2 \longrightarrow \text{MgO}$
 b. $2\text{Ag} + \frac{1}{2} \text{O}_2 \longrightarrow \text{Ag}_2\text{O}$
 c. $\text{C} + \frac{1}{2} \text{O}_2 \longrightarrow \text{CO}$
 d. $\text{CO} + \frac{1}{2} \text{O}_2 \longrightarrow \text{CO}_2$

77. Which of the following reaction taking place in the blast furnace is endothermic ?
 a. $\text{CaCO}_3 \longrightarrow \text{CaO} + \text{CO}_2$
 b. $2\text{C} + \text{O}_2 \longrightarrow 2\text{CO}$
 c. $\text{C} + \text{O}_2 \longrightarrow \text{CO}_2$
 d. $\text{Fe}_2\text{O}_3 + 3\text{CO} \longrightarrow 2\text{Fe} + 3\text{CO}_2$
78. Liquor ammonia bottles are opened only after cooling. This is because
 a. it is a mild explosive
 b. it is a corrosive liquid
 c. it is a lachrymatory
 d. it generates high vapour pressure
79. The formation of $\text{O}_2^+[\text{PtF}_6]^-$ is basis for the formation of xenon fluorides. This is because
 a. O_2 and Xe have comparable sizes
 b. Both O_2 and Xe are gases
 c. O_2 and Xe have comparable ionisation energies
 d. O_2 and Xe have comparable electronegativities
80. The highest magnetic moment is shown by the transition metal ion with the configuration
 a. $3d^2$ b. $3d^5$ c. $3d^7$ d. $3d^9$
81. A transition metal ion exists in its highest oxidation state. It is expected to behave as
 a. a chelating agent
 b. a central metal, in a coordination compound
 c. an oxidising agent
 d. a reducing agent
82. In which of the following complex ion, the central metal ion is in a state of sp^3d^2 hybridisation?
 a. $[\text{CoF}_6]^{3-}$ b. $[\text{Co}(\text{NH}_3)_6]^{3+}$
 c. $[\text{Fe}(\text{CN})_6]^{3-}$ d. $[\text{Cr}(\text{NH}_3)_6]^{3+}$
83. Which of the following can participate in linkage isomerism?
 a. NO_2 b. $\text{H}_2\ddot{\text{N}}\text{CH}_2\text{CH}_2\ddot{\text{N}}\text{H}_2$
 c. H_2O d. $\ddot{\text{N}}\text{H}_3$
84. Which of the following has the highest bond order?
 a. N_2 b. O_2 c. He_2 d. H_2
85. Which of the following is diamagnetic?
 a. H_2^+ b. O_2 c. Li_2 d. He_2^+
86. The concentration of a reactant X decreases from 0.1 M to 0.025 M in 40 min. If the reaction follows 1 order kinetics, the rate of the reaction when the concentration of X is 0.01 M will be
 a. $1.73 \times 10^{-4} \text{ M min}^{-1}$ b. $3.47 \times 10^{-4} \text{ M min}^{-1}$
 c. $3.47 \times 10^{-5} \text{ M min}^{-1}$ d. $1.73 \times 10^{-5} \text{ M min}^{-1}$
87. Chemical reactions with very high E_a values are generally
 a. very fast b. very slow
 c. moderately fast d. spontaneous
88. Which of the following does not conduct electricity?
 a. Fused NaCl b. Solid NaCl
 c. Brine solution d. Copper
89. When a quantity of electricity is passed through CuSO_4 solution, 0.16 g of copper gets deposited. If the same quantity of electricity is passed through acidulated water, then the volume of H_2 liberated at STP will be [Given, atomic weight of Cu = 64]
 a. 4.0 cm^3 b. 56 cm^3 c. 604 cm^3 d. 8.0 cm^3
90. Solubility product of a salt AB is $1 \times 10^{-8} \text{ M}^2$ in a solution in which the concentration of A^+ ions is 10^{-3} M . The salt will precipitate when the concentration of B^- ions is kept
 a. between 10^{-8} M to 10^{-7} M
 b. between 10^{-7} M to 10^{-6} M
 c. $> 10^{-5} \text{ M}$
 d. $< 10^{-8} \text{ M}$
91. Which one of the following condition will increase the voltage of the cell represented by the equation.
 $\text{Cu}(s) + 2\text{Ag}^+(aq) \rightleftharpoons \text{Cu}^{2+}(aq) + 2\text{Ag}(s)$
 a. increase in the dimensions of Cu electrode
 b. increase in the dimensions of Ag electrode
 c. increase in the concentration of Cu^{2+} ions
 d. increase in the concentration of Ag^+ ions
92. The pH of 10^{-8} M HCl solution is
 a. 8 b. more than 8
 c. between 6 and 7 d. slightly more than 7
93. The mass of glucose that should be dissolved in 50 g of water in order to produce the same lowering of vapour pressure as is produced by dissolving 1g of urea in the same quantity of water is
 a. 1 g b. 3 g c. 6 g d. 18 g
94. Osmotic pressure observed when benzoic acid is dissolved in benzene is less than that expected from theoretical considerations. This is because
 a. benzoic acid is an organic solute
 b. benzoic acid has higher molar mass than benzene
 c. benzoic acid gets associated in benzene
 d. benzoic acid gets dissociated in benzene

95. For a reaction to be spontaneous at all temperatures
 a. ΔG and ΔH should be negative
 b. ΔG and ΔH should be positive
 c. $\Delta G = \Delta S = 0$
 d. $\Delta H < \Delta G$
96. Which of the following electrolyte will have maximum flocculation value for $\text{Fe}(\text{OH})_3$ sol.?
 a. NaCl b. Na_2S
 c. $(\text{NH}_4)_3\text{PO}_4$ d. K_2SO_4
97. For a reversible reaction,

$$\text{X}(\text{g}) + 3\text{Y}(\text{g}) \rightleftharpoons 2\text{Z}(\text{g})$$

 $\Delta H = -40 \text{ kJ}$ the standard entropies of X, Y and Z are 60, 40 and $50 \text{ JK}^{-1} \text{ mol}^{-1}$ respectively.
 The temperature at which the above reaction attains equilibrium is about
 a. 400 K b. 500 K
 c. 273 K d. 373 K
98. The radii of Na^+ and Cl^- ions are 95 pm and 181 pm respectively. The edge length of NaCl unit cell is
 a. 276 pm b. 138 pm
 c. 552 pm d. 415 pm
99. Inductive effect involves
 a. displacement of σ -electrons
 b. delocalisation of π -electrons
 c. delocalisation of σ -electrons
 d. displacement of π -electrons
100. The basicity of aniline is less than that of cyclohexylamine. This is due to
 a. + R-effect of $-\text{NH}_2$ group
 b. - I-effect of $-\text{NH}_2$ group
 c. - R-effect of $-\text{NH}_2$ group
 d. hyperconjugation effect
101. Methyl bromide is converted into ethane by heating it in ether medium with
 a. Al b. Zn
 c. Na d. Cu
102. Which of the following compound is expected to be optically active?
 a. $(\text{CH}_3)_2\text{CHCHO}$
 b. $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHO}$
 c. $\text{CH}_3\text{CH}_2\text{CHBrCHO}$
 d. $\text{CH}_3\text{CH}_2\text{CBr}_2\text{CHO}$
103. Which cycloalkane has the lowest heat of combustion per CH_2 group?
 a. cyclopropane b. cyclobutane
 c. cyclopentane d. cyclohexane
104. The catalyst used in the preparation of an alkyl chloride by the action of dry HCl on an alcohol is
 a. anhydrous AlCl_3 b. FeCl_3
 c. anhydrous ZnCl_2 d. Cu
105. In the reaction,

$$\text{R}-\text{X} \xrightarrow[\text{KCN}]{\text{Alcoholic}} \text{A} \xrightarrow[\text{HCl}]{\text{Dilute}} \text{B}, \text{ the product B is}$$

 a. alkyl chloride b. aldehyde
 c. carboxylic acid d. ketone
106. Which of the following compound would not evolve CO_2 when treated with NaHCO_3 solution?
 a. Salicylic acid b. Phenol
 c. Benzoic acid d. 4-nitro benzoic acid
107. By heating phenol, with chloroform in alkali, it is converted into
 a. salicylic acid b. salicylaldehyde
 c. anisole d. phenyl benzoate
108. When a mixture of calcium benzoate and calcium acetate is dry distilled, the resulting compound is
 a. acetophenone b. benzaldehyde
 c. benzophenone d. acetaldehyde
109. Which of the following does not give benzoic acid on hydrolysis?
 a. Phenyl cyanide b. Benzoyl chloride
 c. Benzyl chloride d. Methyl benzoate
110. Which of the following would undergo Hofmann reaction to give a primary amine?

$$\begin{array}{c} \text{O} \\ || \\ \text{a. R}-\text{C}-\text{Cl} \end{array}$$

 a. $\text{R}-\text{C}(=\text{O})-\text{Cl}$ b. RCO NH CH_3
 c. RCONH_2 d. RCOOR
111. Glucose contains in addition to aldehyde group
 a. one secondary OH and four primary OH groups
 b. one primary OH and four secondary OH groups
 c. two primary OH and three secondary OH groups
 d. three primary OH and two secondary OH groups
112. A distinctive and characteristic functional group of fats is
 a. a peptide group b. an ester group
 c. an alcoholic group d. a ketonic group
113. At pH = 4 glycine exists as

$$\begin{array}{c} + \\ \text{a. H}_3\text{N}-\text{CH}_2-\text{COO}^- \end{array}$$

 a. $\text{H}_3\text{N}^+-\text{CH}_2-\text{COO}^-$ b. $\text{H}_3\text{N}^+-\text{CH}_2-\text{COOH}$
 c. $\text{H}_2\text{N}-\text{CH}_2-\text{COOH}$ d. $\text{H}_2\text{N}-\text{CH}_2-\text{COO}^-$

- 114.** Insulin regulates the metabolism of
 a. minerals b. amino acids
 c. glucose d. vitamins
- 115.** The formula mass of Mohr's salt is 392 g. The iron present in it is oxidised by KMnO_4 in acid medium. The equivalent mass of Mohr's salt is
 a. 392 b. 31.6
 c. 278 d. 156
- 116.** The brown ring test for nitrates depends on
 a. the reduction of nitrate to nitric oxide
 b. oxidation of nitric oxide to nitrogen dioxide
 c. reduction of ferrous sulphate to iron
 d. oxidising action of sulphuric acid
- 117.** Acrolein test is positive for
 a. polysaccharides b. proteins
 c. oils and fats d. reducing sugars
- 118.** An organic compound which produces a bluish green coloured flame on heating in presence of copper is
 a. chlorobenzene b. benzaldehyde
 c. aniline d. benzoic acid
- 119.** For a reaction, $A + B \longrightarrow C + D$,
 If the concentration of A is doubled without altering the concentration of B, the rate gets doubled. If the concentration of B is increased by nine times without altering the concentration of A, the rate gets tripled. The order of the reaction is
 a. 2 b. 1 c. $\frac{3}{2}$ d. $\frac{4}{3}$
- 120.** Which of the following solutions will exhibit highest boiling point?
 a. 0.01 M $\text{Na}_2\text{SO}_4(aq)$ b. 0.01 M $\text{KNO}_3(aq)$
 c. 0.015 M urea(aq) d. 0.015 M glucose(aq)

MATHEMATICS

- 121.** If $u = a - b$, $v = a + b$, and $|a| = |b| = 2$, then $|u \times v|$ is
 a. $2\sqrt{16 - (a \cdot b)^2}$ b. $2\sqrt{4 - (a \cdot b)^2}$
 c. $\sqrt{16 - (a \cdot b)^2}$ d. $\sqrt{4 - (a \cdot b)^2}$
- 122.** The volume of the tetrahedron formed by the points (1, 1, 1), (2, 1, 3) (3, 2, 2) and (3, 3, 4) in cubic units is
 a. $\frac{5}{6}$ b. $\frac{6}{5}$
 c. 5 d. $\frac{2}{3}$
- 123.** Unit vector perpendicular to $\hat{i} - 2\hat{j} + 2\hat{k}$ and lying in the plane containing $\hat{i} + \hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{j} + \hat{k}$ is
 a. $8\hat{i} - 7\hat{j} + 11\hat{k}$ b. $8\hat{i} + 7\hat{j} - 11\hat{k}$
 c. $8\hat{i} - 7\hat{j} - 11\hat{k}$ d. $\frac{1}{\sqrt{234}}(8\hat{i} - 7\hat{j} - 11\hat{k})$
- 124.** In the group $Q - \{1\}$ under the binary operation $*$ defined by $a * b = a + b + ab$ the inverse of 10 is
 a. $\frac{1}{10}$ b. $\frac{11}{10}$ c. $-\frac{11}{10}$ d. $-\frac{10}{11}$
- 125.** In the group $\{1, 2, 3, 4, 5, 6\}$ under multiplication mod 7, $2^{-1} X_7^4 =$
 a. 1 b. 4
 c. 2 d. 3
- 126.** The group $(Z, +)$ has
 a. exactly one sub-group
 b. only two sub-groups
 c. no sub-groups
 d. infinitely many subgroups
- 127.** If $3x \equiv 5 \pmod{7}$, then
 a. $x \equiv 2 \pmod{7}$ b. $x \equiv 3 \pmod{7}$
 c. $x \equiv 4 \pmod{7}$ d. None of these
- 128.** The argument of the complex number $\sin\left(\frac{6\pi}{5}\right) + i\left(1 + \cos\frac{6\pi}{5}\right)$ is
 a. $\frac{\pi}{10}$ b. $\frac{5\pi}{6}$ c. $-\frac{\pi}{10}$ d. $\frac{2\pi}{5}$
- 129.** The maximum value of $n < 101$, such that $1 + \sum_{k=1}^n i^k = 0$ is
 a. 96 b. 97 c. 99 d. 100
- 130.** The value of $(-1 + \sqrt{-3})^{62} + (-1 - \sqrt{-3})^{62}$ is
 a. 2^{62} b. 2^{+3}
 c. -2^{62} d. 0
- 131.** All complex number z which satisfy the equation $\left|\frac{z - 6i}{z + 6i}\right| = 1$ lie on the
 a. imaginary axis b. real axis
 c. neither of the axes d. None of the above

132. The value of $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$ is

- a. $\left(\sqrt{\frac{1+x^2}{2+x^2}}\right)$ b. $\left(\sqrt{\frac{2+x^2}{1+x^2}}\right)$
 c. $\left(\sqrt{\frac{x^2-2}{x^2-1}}\right)$ d. $\left(\sqrt{\frac{x^2-1}{x^2-2}}\right)$

133. The value of $\alpha (\neq 0)$ for which the function $f(x) = 1 + \alpha x$ is the inverse of itself is

- a. -2 b. 2
 c. -1 d. 1

134. If x occurs in the expansion of $\left(x + \frac{1}{x}\right)^n$, then its coefficient is

- a. $\frac{n!}{(r!)^2}$ b. $\frac{n!}{(r+1)!(r-1)!}$
 c. $\frac{n!}{\left(\frac{n+r}{2}\right)!\left(\frac{n-r}{2}\right)!}$ d. $\frac{n!}{\left[\left(\frac{n}{2}\right)!\right]^2}$

135. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then $\cot(A-B) =$

- a. $\frac{1}{y} - \frac{1}{x}$ b. $\frac{1}{x} - \frac{1}{y}$
 c. $\frac{1}{x} + \frac{1}{y}$ d. None of these

136. $\cos^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{4} + \cos^2 \frac{5\pi}{12} =$

- a. $\frac{3}{2}$ b. $\frac{3-\sqrt{3}}{2}$
 c. $\frac{2}{3}$ d. $\frac{2}{3+\sqrt{3}}$

137. If $\sin \theta$, $\cos \theta$ and $\tan \theta$ are in GP, then $\cot^6 \theta - \cot^2 \theta$ is

- a. 1 b. 1/2
 c. 2 d. 3

138. If $\frac{3x^2 - 2x + 4}{(x-1)^6} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} + \frac{A_3}{(x+1)^3} + \frac{A_4}{(x+1)^4} + \frac{A_5}{(x+1)^5} + \frac{A_6}{(x+1)^6}$, then

- $(A_1 + A_2 + A_3 - A_4 - A_5 - A_6) =$
 a. (0, 0) b. (-8, -12)
 c. (8, -12) d. (-8, 12)

139. If $\log_2(2^{x-1} + 6) + \log_2(4^{x-1}) = 5$, then $x =$

- a. 4 b. 1
 c. 3 d. 2

140. If a, b, c , and d are the roots of the equation $x^4 + 2x^3 + 3x^2 + 4x + 5 = 0$, then

$1 + a^2 + b^2 + c^2 + d^2$ is equal to

- a. -2 b. -1
 c. 2 d. 1

141. If $c_0, c_1, c_2, \dots, c_n$ are binomial coefficients of order n , then the value of $\frac{c_1}{2} + \frac{c_2}{4} + \frac{c_3}{6} + \dots$

- a. $\frac{2^n + 1}{n-1}$ b. $\frac{2^n - 1}{n+1}$
 c. $\frac{2^n + 1}{n-1}$ d. $\frac{2^n}{n+1}$

142. The value of $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right)}$ is

- a. 4 b. $\frac{1}{4}$
 c. 2 d. $\frac{1}{2}$

143. If $n(A) = n(B) = m$, then the number of possible bijection from A to B is

- a. m b. m^2
 c. $m!$ d. $2m$

144. $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] =$

- a. $\sin^{-1}x - \sin^{-1}\sqrt{1-x^2}$
 b. $\sin^{-1}x + \sin^{-1}\sqrt{1-x}$
 c. $\sin^{-1}x - \sin^{-1}\sqrt{x}$
 d. $\sin^{-1}x + \sin^{-1}\sqrt{x}$

145. If $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \tan 4\theta \tan 7\theta$, then the general solution is

- a. $\theta = \frac{n\pi}{4}$ b. $\theta = \frac{n\pi}{12}$
 c. $\theta = \frac{n\pi}{6}$ d. None of these

146. If a circle with the point $(-1, 1)$ as its center touches the straight line $x + 2y + 9 = 0$, then the coordinates of the points of contact is

- a. $(-3, 3)$ b. $(-3, -3)$
 c. $(0, 0)$ d. $\left(\frac{7}{3}, -\frac{17}{3}\right)$

147. If the circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other, then

- a. $fg = f'g'$ b. $f'g = fg'$
 c. $ff' = gg'$ d. None of these

- 148.** The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 4x + 2y - 4 = 0$ is
 a. 1 b. 2 c. 3 d. 4

- 149.** The length of the tangent drawn from any point on the circle $x^2 + y^2 - 4x + 6y - 4 = 0$ to the circle $x^2 + y^2 - 4x + 6y = 0$ is
 a. 8 b. 4
 c. 2 d. None of these

- 150.** If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is
 a. 25 b. 9 c. 16 d. 4

- 151.** The latusrectum of the ellipse is half the minor axis. Then, its eccentricity is
 a. $\frac{1}{\sqrt{2}}$ b. $\frac{1}{\sqrt{3}}$
 c. $\frac{\sqrt{3}}{2}$ d. None of these

- 152.** The ends of the latusrectum of the parabola $x^2 + 10x - 16y + 25 = 0$ are
 a. (3, 4), (-13, 4) b. (5, -8), (-5, 8)
 c. (3, -4), (13, 4) d. (-3, 4), (13, -4)

- 153.** Which of the following functions is differentiable at $x = 0$?
 a. $\cos(|x|) + |x|$ b. $\cos(|x|) - |x|$
 c. $\sin(|x|) + |x|$ d. $\sin(|x|) - |x|$

- 154.** If $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$, then $\frac{dy}{dx} =$
 a. $\tan t$ b. $\cot t$
 c. $-\cot t$ d. $\tan t$

- 155.** If $\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then
 a. $a = 1 = b$
 b. $a = \cos 2\theta$, $b = \sin 2\theta$
 c. $a = \sin 2\theta$, $b = \cos 2\theta$
 d. $a = \cos \theta$, $b = \sin \theta$

- 156.** If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is

- a. $\begin{bmatrix} 1 & 2^n - 2 \\ 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$
 c. $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & n^2 \\ 1 & 1 \end{bmatrix}$

- 157.** If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then the value of the

$$\text{determinant} \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} \text{ is}$$

- a. q b. 0 c. p d. $p^2 - 2q$

- 158.** The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} \text{ in the interval } \left[-\frac{\pi}{4}, \frac{\pi}{4} \right] \text{ is}$$

- a. 0 b. 1 c. 2 d. 3

- 159.** The sum of non-prime positive divisors of 450 is
 a. 1209 b. 1299 c. 1199 d. 1099

- 160.** The last digit of $\sum_{\substack{1 < p < 100 \\ p \rightarrow \text{prime}}} p! - \sum_{n=1}^{50} (2n)!$ is

- a. 2 b. 4 c. 6 d. 8

- 161.** The interval I such that $\int_0^1 \frac{dx}{\sqrt{1+x^4}} \in I$ is given by

- a. $\left[0, \frac{1}{\sqrt{2}} \right]$ b. $\left[\frac{1}{\sqrt{2}}, 1 \right]$
 c. $[\sqrt{2}, 2]$ d. $\left[\sqrt{2}, \frac{7}{4} \right]$

- 162.** $\int_0^{\pi/2} \log(\tan x) dx =$
 a. $\frac{\pi}{2}$ b. 0 c. 1 d. $\frac{\pi}{4}$

- 163.** The value of $\int_{-2}^2 (ax^3 + bx + c) dx$ depends on the

- a. value of b b. value of c
 c. value of a d. values of a and b

- 164.** The area of the region bound by the curves $y = x^2$ and $y = 4x - x^2$ is

- a. $\frac{16}{3}$ sq. units b. $\frac{8}{3}$ sq. units
 c. $\frac{4}{3}$ sq. units d. $\frac{2}{3}$ sq. unit

- 165.** The particular solution of $\frac{y}{x} \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, when

$$x = 1, y = 2 \text{ is}$$

- a. $5(1+y^2) = 2(1+x^2)$ b. $2(1+y^2) = 5(1+x^2)$
 c. $5(1+y^2) = (1+x^2)$ d. $(1+y^2) = 2(1+x^2)$

166. The solution of the differential equation

$$\frac{dy}{dx} = (x+y)^2 \text{ is}$$

- a. $\frac{1}{x+y} = c$ b. $\sin^{-1}(x+y) = x+c$
c. $\tan^{-1}(x+y) = c$ d. $\tan^{-1}(x+y) = x+c$

167. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

- a. $e^{1/e}$ b. $\sqrt{e}$
c. 1 d. e^2

168. Let x be a number which exceeds its square by the greatest possible quantity, then $x =$

- a. $1/2$ b. $1/4$ c. $3/4$ d. $1/3$

169. The subtangent at $x = \pi/2$ on the curve $y = x \sin x$ is

- a. 0 b. 1
c. $\pi/2$ d. None of these

170. The value of $\int \frac{10^{x/2}}{\sqrt{10^{-x} - 10^x}} dx$ is

- a. $\frac{1}{\log_e 10} \sin^{-1}(10^x) + c$
b. $2\sqrt{10^{-x} + 10^x} + c$
c. $\frac{1}{\log_e 10} \sinh^{-1}(10^x) + c$
d. $\frac{-1}{\log_e 10} \sinh^{-1}(10^x) + c$

171. $\int e^x \left\{ \frac{1 + \sin x \cos x}{\cos^2 x} \right\} dx =$

- a. $e^x \cos x + c$ b. $e^x \sec x \tan x + c$
c. $e^x \tan x + c$ d. $e^x \cos^2 x - 1 + c$

172. $\int \frac{x^2 + 1}{x^4 + 1} dx$

- a. $\frac{1}{\sqrt{2}} \log_e(x^2 + 1) + c$ b. $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 + 1}{x\sqrt{2}}\right) + c$
c. $\frac{1}{\sqrt{2}} \tan^{-1}(x^2 - 1) + c$ d. $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 1}{x\sqrt{2}}\right) + c$

173. The locus of the mid-point of the intercept of the line $x \cos \alpha + y \sin \alpha = p$ between the coordinate axes is

- a. $x^{-2} + y^{-2} = 4p^{-2}$ b. $x^{-2} + y^{-2} = p^{-2}$
c. $x^2 + y^2 = 4p^{-2}$ d. $x^2 + y^2 = p^2$

174. If the line through $A = (4, -5)$ is inclined at an angle 45° with the positive direction of the X-axis, then the coordinates of the two points on opposite sides of A at a distance of $3\sqrt{2}$ units are

- a. (7, 2), (1, 8) b. (7, 2), (1, -8)
c. (7, -2), (1, -8) d. (7, 2), (-1, 8)

175. If the line $px + qy = 0$ coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$, then

- a. $ap^2 + 2hpq + bq^2 = 0$ b. $aq^2 + 2hpq + bp^2 = 0$
c. $aq^2 - 2hpq + bp^2 = 0$ d. None of these

176. The function

$$f(x) = \left(\frac{\log_e(1+ax) - \log_e(1-bx)}{x} \right) \text{ is}$$

undefined at $x = 0$. The value which should be assigned to f at $x = 0$ so that it is continuous at $x = 0$ is

- a. $\frac{a+b}{2}$ b. $a+b$ c. $\log_e(ab)$ d. $a-b$

177. $\lim_{n \rightarrow \infty} \frac{(1^2 + 2^2 + \dots + n^2) \sqrt[n]{n}}{(n+1)(n+10)(n+100)} =$

- a. 3 b. $\frac{1}{3}$ c. $\frac{2}{3}$ d. ∞

178. The number of triangles in a complete graph with 10 non-collinear vertices is

- a. 360 b. 240 c. 120 d. 60

179. The angle between hands of a clock when the time is 4.25 AM is signed to

- a. $17 \frac{1}{2}^\circ$ b. $14 \frac{1}{2}^\circ$
c. $13 \frac{1}{2}^\circ$ d. $12 \frac{1}{2}^\circ$

180. The point (5, -7) lies outside the circle

- a. $x^2 + y^2 - 8x = 0$
b. $x^2 + y^2 - 5x + 7y = 0$
c. $x^2 + y^2 - 5x + 7y - 1 = 0$
d. $x^2 + y^2 - 8x + 7y - 2 = 0$

ANSWERS

Physics

1. (c)	2. (b)	3. (a)	4. (a)	5. (a)	6. (c)	7. (d)	8. (*)	9. (c)	10. (d)
11. (a)	12. (b)	13. (c)	14. (a)	15. (a)	16. (d)	17. (c)	18. (d)	19. (b)	20. (a)
21. (b)	22. (c)	23. (a)	24. (b)	25. (d)	26. (b)	27. (a)	28. (c)	29. (c)	30. (d)
31. (d)	32. (a)	33. (c)	34. (b)	35. (d)	36. (c)	37. (a)	38. (b)	39. (c)	40. (b)
41. (a)	42. (c)	43. (a)	44. (b)	45. (c)	46. (c)	47. (d)	48. (a)	49. (b)	50. (c)
51. (b)	52. (c)	53. (b)	54. (d)	55. (a)	56. (d)	57. (d)	58. (a)	59. (a)	60. (a)

Chemistry

61. (c)	62. (a)	63. (b)	64. (a)	65. (d)	66. (a)	67. (c)	68. (a)	69. (c)	70. (a)
71. (b)	72. (d)	73. (c)	74. (c)	75. (c)	76. (c)	77. (a)	78. (a)	79. (c)	80. (b)
81. (c)	82. (a)	83. (a)	84. (a)	85. (c)	86. (b)	87. (b)	88. (b)	89. (b)	90. (c)
91. (d)	92. (c)	93. (b)	94. (c)	95. (a)	96. (a)	97. (b)	98. (c)	99. (a)	100. (a)
101. (c)	102. (c)	103. (d)	104. (c)	105. (c)	106. (b)	107. (b)	108. (a)	109. (c)	110. (c)
111. (b)	112. (b)	113. (b)	114. (c)	115. (a)	116. (a)	117. (c)	118. (a)	119. (c)	120. (a)

Mathematics

121. (a)	122. (a)	123. (d)	124. (d)	125. (c)	126. (d)	127. (c)	128. (c)	129. (c)	130. (c)
131. (b)	132. (a)	133. (c)	134. (c)	135. (c)	136. (a)	137. (a)	138. (d)	139. (d)	140. (b)
141. (b)	142. (a)	143. (c)	144. (c)	145. (b)	146. (b)	147. (b)	148. (b)	149. (c)	150. (c)
151. (c)	152. (a)	153. (d)	154. (d)	155. (b)	156. (c)	157. (b)	158. (b)	159. (c)	160. (a)
161. (b)	162. (b)	163. (b)	164. (b)	165. (b)	166. (d)	167. (a)	168. (a)	169. (c)	170. (a)
171. (c)	172. (d)	173. (a)	174. (c)	175. (c)	176. (b)	177. (b)	178. (c)	179. (a)	180. (a)

Note (*) None of the option is correct.

HINTS & SOLUTIONS

Physics

1. (c) The angle of incidence = i

Let r be the angle of refraction.

According to given situation, reflected rays and refracted rays are perpendicular to each other.

i.e. $i + r = 90^\circ$... (i)

$\therefore$ According to Snell's law,

$$\begin{aligned}\mu &= \frac{\sin i}{\sin r} \\ &= \frac{\sin i}{\sin(90^\circ - i)} \quad [\because \text{From Eq. (i)}] \\ &= \frac{\sin i}{\cos i}\end{aligned}$$

$$\mu = \tan i \quad \dots (ii)$$

If i_c be the critical angle for the pair of media, then

$$\mu = \frac{1}{\sin i_c} \quad \dots (iii)$$

From Eqs. (ii) and (iii), we get

$$\tan i = \frac{1}{\sin i_c}$$

$$\Rightarrow \sin i_c = \cot i$$

$$\Rightarrow i_c = \sin^{-1}(\cot i)$$

2. (b) Refractive index of water = n

Since, fish looks the bird from water (denser medium) to air (rarer medium).

Hence, normal shift $d = (n - 1)h$

But according to question, real depth $h = y$

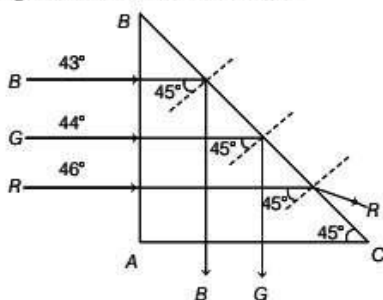
$$\therefore d = (n - 1)y \quad (\because \text{height of bird} = y) \dots (i)$$

The bird's height from the surface of the water is equal to the summation of the normal shift and height of the bird from the surface.

$$\begin{aligned}\text{i.e. } d' &= y + d = y + (n - 1)y \quad [\text{From Eq. (i)}] \\ d' &= ny \quad \dots (ii)\end{aligned}$$

Total actual distance between bird and fish
 $= x + d' = x + ny$

3. (a) The given situation is shown below



Since, critical angles for blue colour and green colour are less than angle of incidence, hence total internal

reflection occurs for blue and green colours of light. Again, since critical angle for red colour is greater than angle of incidence hence red colour transmitted outside from surface BC.

4. (a) Let f_1 and f_2 are focal lengths of convex and concave lens respectively, when they are kept in contact co-axially, then focal length F of the combination is given as

$$\begin{aligned}\frac{1}{F} &= \frac{1}{f_1} + \frac{1}{(-f_2)} \quad \left[\begin{array}{l} \text{Focal length of concave} \\ \text{lens is negative.} \end{array} \right] \\ \Rightarrow \frac{1}{F} &= \frac{f_2 - f_1}{f_1 f_2} \\ \Rightarrow F &= \frac{f_1 f_2}{f_2 - f_1} \quad \dots (i)\end{aligned}$$

When both lenses are kept at a distance d , then their combined focal length F' is given as

$$\begin{aligned}\frac{1}{F'} &= \frac{1}{f_1} + \frac{1}{(-f_2)} - \frac{d}{f_1(-f_2)} \\ \Rightarrow \frac{1}{F'} &= \frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2} \\ \Rightarrow \frac{1}{F'} &= \frac{f_2 - f_1 + d}{f_1 f_2} \\ \Rightarrow F' &= \frac{f_1 f_2}{f_2 - f_1 + d} \quad \dots (ii)\end{aligned}$$

From Eqs. (i) and (ii), it is clear that $F' < F$
 i.e. focal length of combination will decrease.

5. (a) Since, convex lens made of 3 layers of different materials have only one focal length. Thus, for a given object, there is only one image.
 6. (c) Speed of light in terms of permittivity of free space ϵ_0 and permeability of free space μ_0 is given as

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

7. (d) When a third slit is made in between the double slits in YDSE, then contrast between bright and dark fringes is reduced. It is because in the case of double slits, light passes only two slits. Now, the same light is passing through the three slits which results in reducing the contrast of bright fringes as well as dark fringes.

8. (*) In double slit experiment, for possible interference maxima on the screen is given as

$$d \sin \theta = n\lambda$$

Given, slit width, $d = 4\lambda$

$$\therefore 4\lambda \sin \theta = n\lambda$$

$$\Rightarrow 4 \sin \theta = n$$

Since, maximum value of $\sin \theta = 1$

∴ For maximum number of possible interference maxima,

$$\begin{aligned} 4 \cdot 1 &= n \\ \Rightarrow n &= 4 \\ \text{i.e., } n &= 0, 1, 2, 3 \end{aligned}$$

∴ Maximum number of possible maxima will be 7
i.e. $0, \pm 1, \pm 2, \pm 3$

Hence, no option is correct.

9. (c) Given, wavelength of used light,

$$\lambda = 400 \text{ nm}$$

For minima in Fraunhofer diffraction,

$$d \sin \theta = n\lambda$$

For first minima $n = 1$, where $\theta = 30^\circ$

$$\therefore d \sin 30^\circ = 1 \times 400$$

$$\Rightarrow d = 800 \text{ nm}$$

For maxima in diffraction,

$$d \sin \theta = (2n + 1) \frac{\lambda}{2}$$

For first maxima $n = 0$

$$\therefore d \sin \theta = (0 + 1) \frac{\lambda}{2}$$

$$\Rightarrow d \sin \theta = \frac{\lambda}{2}$$

$$\Rightarrow 800 \sin \theta = \frac{400}{2}$$

$$\Rightarrow \sin \theta = \frac{1}{4}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{1}{4} \right)$$

10. (d) Greater is the wavelength of wave higher will be its degree of diffraction, since radiowave has maximum wavelength, hence maximum diffraction takes place in a given slit for radio waves.

11. (a) When an unpolarised beam of intensity I_0 falls on a polaroid, then after polarisation, the intensity of the emergent beam of light becomes $\frac{I_0}{2}$.

12. (b) Materials which have different absorption for perpendicular incident plane for light are called dichroic material crystal. Tourmaline is one example of such type of crystal.

13. (c) Given, $Q_1 = 12 \mu\text{F} = 12 \times 10^{-6} \text{ F}$

$$\begin{aligned} Q_2 &= -8 \mu\text{F} \\ &= -8 \times 10^{-6} \text{ F} \end{aligned}$$

Initial value of electrostatic force of attraction between both spheres,

$$\begin{aligned} F_1 &= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{12 \times 10^{-6} \times 8 \times 10^{-6}}{r^2} \\ F_1 &= \frac{1}{4\pi\epsilon_0} \frac{96 \times 10^{-12}}{r^2} \end{aligned} \quad \dots(i)$$

When both charged spheres are kept in contact, then new charges on the spheres will be

$$\begin{aligned} Q'_1 = Q'_2 &= \frac{Q_1 + Q_2}{2} \\ &= \frac{12 + (-8)}{2} = 2 \mu\text{F} = 2 \times 10^{-6} \text{ F} \end{aligned}$$

∴ Final value of electrostatic force between the spheres,

$$\begin{aligned} F_2 &= \frac{1}{4\pi\epsilon_0} \frac{Q'_1 \times Q'_2}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2 \times 10^{-6} \times 2 \times 10^{-6}}{r^2} \\ F_2 &= \frac{1}{4\pi\epsilon_0} \frac{4 \times 10^{-12}}{r^2} \end{aligned} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \frac{F_1}{F_2} = \frac{\frac{1}{4\pi\epsilon_0} \frac{96 \times 10^{-12}}{r^2}}{\frac{1}{4\pi\epsilon_0} \frac{4 \times 10^{-12}}{r^2}}$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{24}{1}$$

$$\Rightarrow F_1 : F_2 = 24 : 1$$

14. (a) According to given situation, potential at the surface of small conducting sphere of radius r ,

$$V_r = \frac{kq}{r} + \frac{kQ}{R}$$

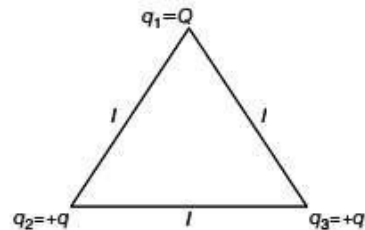
Potential at the surface of bigger conducting sphere of radius R ,

$$V_R = \frac{kq}{R} + \frac{kQ}{R}$$

∴ Potential difference between the spheres is given as

$$\begin{aligned} V_r - V_R &= \frac{kq}{r} + \frac{kQ}{R} - \frac{kq}{R} - \frac{kQ}{R} \\ &= \frac{kq}{r} - \frac{kq}{R} = k \left(\frac{q}{r} - \frac{q}{R} \right) \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r} - \frac{q}{R} \right] \end{aligned}$$

15. (a) The given situation is shown below



Net potential energy of the system = 0

$$\begin{aligned} \text{i.e. } U_{\text{net}} &= 0 \\ \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{l} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{l} + \frac{1}{4\pi\epsilon_0} \frac{q_3 q_1}{l} &= 0 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow q_1 q_2 + q_2 q_3 + q_3 q_1 = 0 \\
 &\Rightarrow Q \cdot q + q \cdot q + q \cdot Q = 0 \\
 &\Rightarrow 2Qq + q^2 = 0 \\
 &\Rightarrow q(2Q + q) = 0 \\
 &\text{Since, } q \neq 0 \\
 &\therefore 2Q + q = 0 \\
 &\Rightarrow Q = \frac{-q}{2}
 \end{aligned}$$

16. (d) We know that, in series combination of capacitors, net potential difference is equal to sum of the individual potential difference of each capacitor
 i.e., $V = V_1 + V_2 + V_3 \dots$ [$V_1 = V_2 = V_3 = 200 \text{ V}$]
 $\Rightarrow 600 = n \times 200$
 $\Rightarrow n = 3$

So, there should be 3 capacitors in series to obtain the required potential difference. The equivalent capacitance of 3 capacitors in series is given as

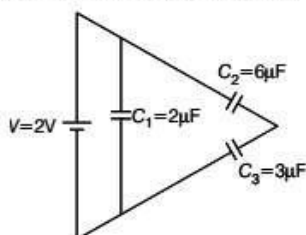
$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow C_{eq} = 2 \mu\text{F}$$

Now, we require $18 \mu\text{F}$ capacitance for this, we need 9 number of such capacitors in parallel.

$$\therefore \text{Total number of capacitors (condensers)} = 9 \times 3 = 27$$

17. (c) The given circuit diagram is redrawn as



Equivalent capacitance is given as

$$C_{eq} = \frac{C_2 C_3}{C_2 + C_3} + C_1 = \frac{6 \times 3}{6 + 3} + 2 = 4 \mu\text{F}$$

$\therefore$ Total energy stored in the condenser system,

$$\begin{aligned}
 U &= \frac{1}{2} C_{eq} \cdot V^2 = \frac{1}{2} \times 4 \times 10^{-6} \times 2^2 \\
 &= 8 \times 10^{-6} \text{ J} = 8 \mu\text{J}
 \end{aligned}$$

18. (d) Drift velocity of the electron is given as

$$v_d = \frac{eE\tau}{m}$$

where, e = charge on electron,

E = electric field,

τ = relaxation time

and m = mass of the electron.

$$\text{Since, } E = \frac{V}{l}$$

$$\therefore v_d = \frac{eV\tau}{ml}$$

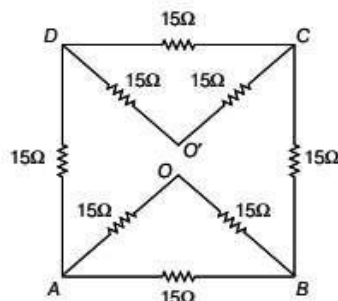
Since, e , V , m and l are constant.

$$\therefore v_d \propto \tau$$

On increasing the temperature of metal wire, relaxation time decreases, therefore drift velocity of electrons also decreases.

But on increasing temperature of metal wire, thermal velocity of electron increases.

19. (b) Redrawing the given circuit network as

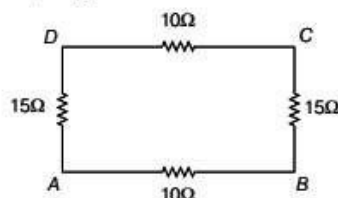


Equivalent resistance between points D and C,

$$R_{DC} = \frac{(15 + 15) \times 15}{(15 + 15) + 15} = \frac{30 \times 15}{45} = 10 \Omega$$

$$\text{Similarly, } R_{AB} = R_{DC} = 10 \Omega$$

Now, circuit, diagram reduces as



$\therefore$ Equivalent resistance between points A and B is given as

$$\begin{aligned}
 R_{AB} &= (15 + 10 + 15) \parallel 10 = 40 \parallel 10 = \frac{40 \times 10}{40 + 10} \\
 &= \frac{400}{50} = 8 \Omega
 \end{aligned}$$

20. (a) Since, the terminals of battery is connected at one quarter distance. Hence, ratio of lengths between two parts will be

$$\frac{l_1}{l_2} = \frac{1}{3}$$

$$\Rightarrow l_1 : l_2 = 1 : 3$$

Since, $R \propto l$

$$\therefore R_1 : R_2 = 1 : 3$$

$$R_1 = \frac{24}{1 + 3} = 6 \Omega$$

$$R_2 = 24 \times \frac{3}{1 + 3} = 18 \Omega$$

Since, potential difference across R_1 and R_2 is same, so they are connected in parallel.

$\therefore$ Equivalent resistance of R_1 and R_2 ,

$$R' = \frac{R_1 R_2}{R_1 + R_2} \\ = \frac{6 \times 18}{6 + 18} = 4.5 \Omega$$

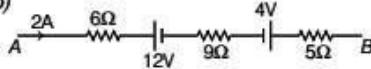
$$\therefore \text{Total resistance, } R_T = R' + r \\ = 4.5 + 1.5 \quad [\because r = 1.5 \Omega] \\ = 6 \Omega$$

$$\text{Current, } I = \frac{V}{R_T} = \frac{18}{6} = 3 \text{ A}$$

According to current division rule, current through bigger arc (i.e. through R_2)

$$I_{R_2} = I \cdot \frac{R_1}{R_1 + R_2} = 3 \times \frac{6}{6 + 18} = 0.75 \text{ A}$$

21. (b)



Applying Kirchhoff's voltage law, we get

$$-V_{AB} + 6 \times 2 + 12 + 9 \times 2 - 4 + 5 \times 2 = 0$$

$$\Rightarrow -V_{AB} + 48 = 0$$

$$\Rightarrow V_{AB} = 48 \text{ V}$$

22. (c) Magnetic field at the centre of circular carrying conductor,

$$B_c = \frac{\mu_0 I}{2r} \quad \dots(i)$$

Magnetic on the axis at a distance x from the centre is given as

$$B_{\text{axis}} = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$

Here, $x = r$

$$B_{\text{axis}} = \frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}} \\ = \frac{\mu_0 I r^2}{2(2r^2)^{3/2}} = \frac{\mu_0 I r^2}{2 \cdot 2\sqrt{2} \cdot r^3}$$

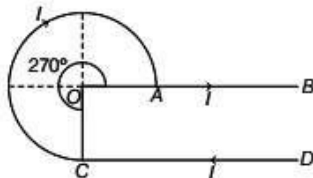
$$B_{\text{axis}} = \frac{\mu_0 I}{4\sqrt{2}r}$$

$$B_{\text{axis}} = B_a = \frac{\mu_0 I}{4\sqrt{2}r}$$

$$\therefore \frac{B_c}{B_a} = \frac{\mu_0 I / 2r}{\frac{\mu_0 I}{4\sqrt{2}r}} = \frac{4\sqrt{2}r}{2r} = 2\sqrt{2}$$

$$B_c : B_a = 2\sqrt{2} : 1$$

23. (a) The given current carrying conductor is shown below



Magnetic field at the centre due to circular wire,

$$B_1 = \frac{\mu_0 I}{2r} \times \frac{270}{360} = \frac{\mu_0 I}{2r} \times \frac{3}{4}$$

$$B_1 = \frac{3\mu_0 I}{8r}$$

[in downward to the plane of paper]

Magnetic field at centre O due current carrying wire AB will be zero because point O lies along AB

$$\therefore B_2 = 0$$

Magnetic field at O due to current carrying wire CD

$$B_3 = \frac{\mu_0 I}{4\pi r} \text{ (in downward to the plane of paper)}$$

$\therefore$ Net magnetic field at the centre,

$$B_{\text{net}} = B_1 + B_2 + B_3 = \frac{3\mu_0 I}{8r} + 0 + \frac{\mu_0 I}{4\pi r} \\ = \frac{\mu_0 I}{r} \left[\frac{3}{8} + \frac{1}{4\pi} \right] = \frac{\mu_0 I}{4\pi r} \left[\frac{3\pi}{2} + 1 \right]$$

24. (b) Given, $\theta_A = 60^\circ$, $\theta_B = 30^\circ$

We know that, deflection in galvanometer θ and current I are related as

$$\tan \theta = KNI$$

where, K = constant,

N = number of turns in the coil of galvanometer

and I = current.

$$\therefore \frac{\tan \theta_A}{\tan \theta_B} = \frac{N_A}{N_B}$$

$$\Rightarrow \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{N_A}{N_B}$$

$$\Rightarrow \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{N_A}{N_B}$$

$$\Rightarrow \frac{3}{1} = \frac{N_A}{N_B}$$

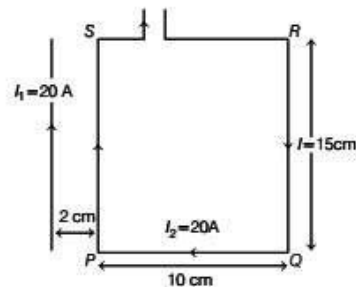
$$\Rightarrow N_A : N_B = 3 : 1$$

25. (d) $I_1 = 20 \text{ A}$, $I_2 = 20 \text{ A}$

$$r_1 = 2 \text{ cm}$$

$$r_2 = (10 + 2) \text{ cm} = 12 \text{ cm}$$

The given situation is shown in the figure.



The forces on arm SR and PQ are equal in magnitude and opposite in direction and also having same line of action, hence they cancel to each other.

∴ Resultant force on the current loop PQRS,

$$\begin{aligned}
 F &= F_{PS} - F_{QR} \\
 &= \frac{\mu_0}{2\pi} \frac{I_1 I_2 \times l}{r_1} - \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{r_2} \\
 &= \frac{\mu_0}{2\pi} I_1 I_2 l \left[\frac{1}{r_1} - \frac{1}{r_2} \right] \\
 &= 2 \times 10^{-7} \times 20 \times 20 \times 15 \times 10^{-2} \left[\frac{1}{2 \times 10^{-2}} - \frac{1}{12 \times 10^{-2}} \right] \\
 &= 5 \times 10^{-4} \text{ N}
 \end{aligned}$$

26. (b) Given, shunt resistance, $R_S = 1 \Omega$

Total current, $I = 100 \text{ A}$

Galvanometer current, $I_g = 1 \text{ A}$

We know that,

$$\begin{aligned}
 I_g &= \frac{IR_S}{R_S + G} \\
 \Rightarrow 1 &= \frac{100 \times 1}{1 + G} \\
 \Rightarrow 1 + G &= 100 \\
 \Rightarrow G &= 99 \Omega
 \end{aligned}$$

27. (a) On moving the circular loops further apart, flux linked with loops will be reduced. Therefore, according to Lenz's law, current will increase in both circular loops to increase the linked flux.

28. (c) The given circuit diagram is L - C - R series circuit, where

$$V_R = 80 \text{ V}, V_C = 100 \text{ V}$$

$$V_L = 40 \text{ V}$$

Since, $V_C > V_L$

Value of emf E is given as

$$\begin{aligned}
 E &= \sqrt{V_R^2 + (V_C - V_L)^2} \\
 &= \sqrt{80^2 + (100 - 40)^2} \\
 &= \sqrt{6400 + 3600} = 100 \text{ V}
 \end{aligned}$$

29. (c) For a transformer,

$$I_P = 5 \text{ A}, V_P = 220 \text{ V}$$

$$V_S = 2200 \text{ V}$$

Power at primary side, $P_i = V_P I_P$

$$= 220 \times 5 = 1100 \text{ W}$$

Since, 50% power is lost, hence power at secondary side,

$$P_0 = 50\% \text{ of } P_i = \frac{50}{100} \times 1100$$

$$\Rightarrow P_0 = 550 \text{ W}$$

$$\Rightarrow V_S I_S = 550$$

$$\Rightarrow 2200 \times I_S = 550$$

$$\Rightarrow I_S = \frac{550}{2200} = \frac{1}{4} = 0.25 \text{ A}$$

30. (d) In series L - C - R circuit,

$$X_L = X_C$$

$$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + 0^2}$$

$$Z = R$$

∴ Power factor = $\cos \phi$

$$= \frac{R}{Z} = \frac{R}{R} = 1$$

Hence, power factor is not zero.

Therefore option (d) is not correct.

In L - C - R series resonance circuit, peak energy stored by a capacitor is equal to peak energy stored by an inductor.

Average power, $P_{avg} = V_{rms} \times I_{rms} \times \cos \phi$

$$= V_{rms} \times I_{rms}$$

$$= \text{apparent power}$$

L - C - R series resonance circuit is purely resistive because $X_L = X_C$, hence no wattless current exists in this circuit therefore wattless current is zero.

31. (d) Solar spectrum is a line absorption spectrum which is also called as Fraunhofer line of missing wavelengths.

32. (a) According to first situation wavelength of incident radiation,

$$\lambda_1 = \lambda$$

Stopping potential, $V_{01} = 3V_s$

According to Einstein's photoelectric equation,

$$K_{max} = h\nu_1 - \phi$$

$$\Rightarrow eV_{01} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_0}$$

[where, λ_0 = threshold wavelength.]

$$\Rightarrow e \cdot 3V_s = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

$$\Rightarrow 3V_s e = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(i)$$

According to second situation, wavelength of incident radiation,

$$\lambda_2 = 2\lambda$$

Stopping potential, $V_{02} = V_s$

Again, Einstein's photoelectric equation,

$$K_{max} = h\nu_2 - \phi$$

$$\Rightarrow eV_{02} = hc \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_0} \right)$$

$$\Rightarrow eV_s = hc \left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(ii)$$

Dividing Eq. (i) by Eq (ii), we get

$$3 = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{2\lambda} - \frac{1}{\lambda_0}}$$

$$\Rightarrow \frac{3}{2\lambda} - \frac{3}{\lambda_0} = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

$$\Rightarrow \lambda_0 = 4\lambda$$

33. (c) According to Einstein's photoelectric equation maximum kinetic energy,

$$K_{\max} = eV_0 = h\nu - \phi_0 \\ = h(\nu - \nu_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Hence, in photoelectric effect, maximum kinetic energy of emitted photoelectrons depends on work function of metal surface, frequency and wavelength of incident radiation but does not depend on intensity of incident radiation.

34. (b) We know that, wavelength of hydrogen spectrum is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For minimum wavelength of Lyman series,

$$n_1 = 1 \text{ and } n_2 = \infty \\ \therefore \frac{1}{(\lambda_L)_{\min}} = R\left(\frac{1}{(1)^2} - \frac{1}{\infty}\right) = R \\ \Rightarrow (\lambda_L)_{\min} = \frac{1}{R}$$

For minimum wavelength of Balmer series,

$$n_1 = 2 \text{ and } n_2 = \infty \\ \therefore \frac{1}{(\lambda_B)_{\min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) \\ = R\left(\frac{1}{4} - 0\right) = \frac{R}{4} \\ \Rightarrow (\lambda_B)_{\min} = \frac{4}{R} \\ \therefore \frac{(\lambda_L)_{\min}}{(\lambda_B)_{\min}} = \frac{\frac{1}{R}}{\frac{4}{R}} = \frac{1}{4} = 0.25$$

35. (d) Hydrogen atom does not emit X-rays because its energy level are very close to each other. Hence, the transition of electron emits high energy radiations having short wavelength.

36. (c) Given, $\lambda_e = \lambda_p$... (i)

Since, de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \\ \Rightarrow \lambda = \frac{h}{\sqrt{2mK}}$$

where, K is kinetic energy.

$$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} \cdot \sqrt{\frac{K_p}{K_e}} \\ \Rightarrow \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} \cdot \sqrt{\frac{K_p}{K_e}} \quad [\text{From Eq. (i)}] \\ \Rightarrow 1 = \sqrt{\frac{m_p}{m_e}} \cdot \sqrt{\frac{K_p}{K_e}}$$

$$\Rightarrow \sqrt{\frac{K_e}{K_p}} = \sqrt{\frac{m_p}{m_e}}$$

Squaring both sides, we get

$$\frac{K_e}{K_p} = \frac{m_p}{m_e}$$

since $m_p > m_e$

$$\therefore K_e > K_p$$

37. (a) Nuclear force between two protons at a separation of $40\text{\AA}$ is much greater than electrostatic force between them.

$$\text{i.e. } F_n \gg F_e$$

38. (b) Blue colour of water in the sea is due to scattering of light by water molecules. As blue colour of light has smaller wavelength, therefore scattering of blue colour of light is exceptionally large.

Therefore, blue colour of sea water is due to scattering of sunlight by the water molecules.

39. (c) Nuclear radius and mass number A are related with following relation,

$$R = R_0 A^{1/3}$$

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

Given, $A_1 = 216$ and $A_2 = 125$

$$\therefore \frac{R_1}{R_2} = \left(\frac{216}{125}\right)^{1/3} \\ = \left[\left(\frac{6}{5}\right)^3\right]^{1/3} \\ = \frac{6}{5}$$

$$\therefore R_1 : R_2 = 6 : 5$$

40. (b) Given, output power, $P_o = 1.6 \text{ MW}$
 $= 1.6 \times 10^6 \text{ W}$

Energy released per fission,

$$E_1 = 200 \text{ MeV} \\ = 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \\ = 320 \times 10^{-13} \text{ J} \\ = 3.2 \times 10^{-11} \text{ J}$$

The rate of fission R is given as

$$R = \frac{P_o}{E_1} \\ = \frac{1.6 \times 10^6}{3.2 \times 10^{-11}} \\ = 0.5 \times 10^{17} = 5 \times 10^{16} / \text{s}$$

41. (a) Let initial masses of two radioactive substances be N_0 .

If T_1 and T_2 be the half-lives of two radioactive substances,

Then, $T_1 = 1 \text{ yr}$

$T_2 = 2$ yrs
 Total time, $t = 6$ yrs
 Hence, remaining mass of the first substances after
 $t = 6$ yrs

$$N_1 = N_0 \left(\frac{1}{2} \right)^{\frac{t}{T_1}} = N_0 \left(\frac{1}{2} \right)^{6/1}$$

$$= \frac{N_0}{2^6}$$

$$\Rightarrow N_1 = \frac{N_0}{64}$$

Similarly, remaining mass of the second substance after
 $t = 6$ yrs

$$N_2 = N_0 \left(\frac{1}{2} \right)^{\frac{t}{T_2}} = N_0 \left(\frac{1}{2} \right)^{6/2} = \frac{N_0}{8}$$

$$\Rightarrow N_2 = \frac{N_0}{8}$$

$$\therefore \frac{N_1}{N_2} = \frac{\frac{N_0}{64}}{\frac{N_0}{8}} = \frac{1}{8} \Rightarrow \frac{N_1}{N_2} = \frac{1}{8} \quad \dots(i)$$

Since, activity, $A = \lambda N = \frac{0.693}{T} N$

$$\Rightarrow A \propto \frac{N}{T}$$

$$\therefore \frac{A_1}{A_2} = \frac{N_1 T_2}{N_2 T_1} = \frac{1}{8} \times \frac{2}{1} = \frac{1}{4}$$

$$\Rightarrow A_1 : A_2 = 1 : 4$$

42. (c) When an α - particle emits from a radioactive nuclei, then its mass number is decreased by 4 unit and atomic number is decreased by 2 unit.

When a β -particle emits from a radioactive nuclei, then its mass number is unaffected, whereas atomic number is increased by one unit.

Here, initial nuclei = ${}_{92}\text{U}^{235}$

Final nuclei = ${}_{82}\text{P}^{203}$

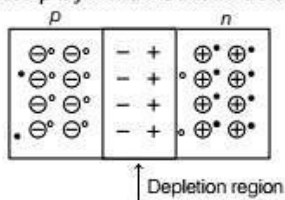
Change in mass number = $235 - 203 = 32$

Number of α -particle emitted = $\frac{32}{4} = 8$

$\therefore$ Number of emitted β -particles
 $= 8 \times 2 - (92 - 82) = 16 - 10 = 6$

43. (a) Neutron is the most stable particle in the baryon group.

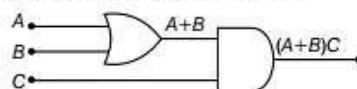
44. (b) An unbiased p - n junction is shown below.



From the diagram, it is clear that in depletion region, n -side has immobile positive charge, whereas p -side has immobile negative charge.

Therefore, potential at p is less than that of n .

45. (c) The given logic diagram is shown as



Output of logic gate is given as

$$Y = (A + B)C$$

In option (a), $A = 0, B = 1, C = 0$

$$\therefore Y = (0 + 1) \cdot 0 = 0$$

In option (b), $A = 1, B = 0, C = 0$

$$\therefore Y = (1 + 0) \cdot 0 = 0$$

In option (c), $A = 1, B = 0, C = 1$

$$\therefore Y = (1 + 0) \cdot 1 = 1$$

In option (d), $A = 1, B = 1, C = 0$

$$\therefore Y = (1 + 1) \cdot 0 = 0$$

Hence, to get an output $Y = 1$

$$A = 1, B = 0 \text{ and } C = 1$$

46. (c) According to Newton's law of gravitation,

$$F = \frac{G m_1 m_2}{r^2}$$

where, F = gravitational force between two bodies of masses m_1 and m_2 ,

r = distance between the two masses

and G = universal gravitational constant.

$$\therefore G = \frac{F r^2}{m_1 m_2}$$

$$\therefore [G] = \frac{[F][r^2]}{[m_1][m_2]}$$

$$= \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

47. (d) Let u be the initial vertical velocity of the body by which it is projected vertically upward.

If t be the time taken by the body to reach a height h from the point of projection, then using equation of vertical motion,

$$h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow gt^2 - 2ut + 2h = 0$$

$$\Rightarrow t = \frac{2u \pm \sqrt{4u^2 - 4g \times 2h}}{2g}$$

$$= \frac{u \pm \sqrt{u^2 - 2gh}}{g}$$

It means t has two values which is already given in question,

$$t_1 = \frac{u + \sqrt{u^2 - 2gh}}{g}$$

and
$$t_2 = \frac{u - \sqrt{u^2 - 2gh}}{g}$$

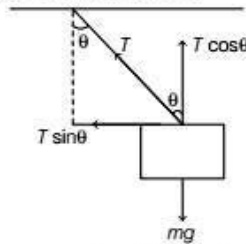
$$\therefore t_1 + t_2 = \frac{2u}{g}$$

$$\Rightarrow u = \frac{g(t_1 + t_2)}{2}$$

48. (a) Given, mass suspended from spring balance,
 $m = 10 \text{ kg}$

$\theta = 60^\circ$

The given situation is shown below



From the diagram, reading of spring balance,

$$F_s = T \quad \dots(i)$$

Since, $T \cos \theta = mg$

$$T = \frac{mg}{\cos \theta} \quad \dots(ii)$$

From Eqs. (i) and (ii) we get

$$\begin{aligned} F_s &= \frac{mg}{\cos \theta} = \frac{mg}{\cos 60^\circ} \\ &= 2mg \\ &= 2 \times 10 \times g \\ &= 20 \text{ kg-wt} \end{aligned}$$

49. (b) Weight of body in air,
 $m_1 = 50 \text{ g} = 5 \times 10^{-2} \text{ kg}$

Weight of body in water,

$m_2 = 40 \text{ g} = 4 \times 10^{-2} \text{ kg}$

Buoyant force on the body,

$$\begin{aligned} F' &= m_1 g - m_2 g \\ &= g(m_1 - m_2) \\ &= g(5 \times 10^{-2} - 4 \times 10^{-2}) \\ F' &= 10^{-2} \text{ g N} \quad \dots(i) \end{aligned}$$

But buoyant force is given as

$$F' = V \rho g \quad [\text{From Eq. (i)}]$$

$$\Rightarrow 10^{-2} \text{ g} = V \rho g$$

$$\Rightarrow 10^{-2} = \rho V \quad [\because \rho = 10^3 \text{ kg/m}^3]$$

$$\Rightarrow V = 10^{-5} \text{ m}^3$$

$\therefore$ Loss of weight in liquid $= \rho_l V g$

$$\begin{aligned} W' &= 1.5 \rho \times 10^{-5} \times 10 \quad [\because \rho_l = 1.5 \rho] \\ &= 1.5 \times 10^3 \times 10^{-5} \times 10 \\ &= 0.15 \text{ N} \end{aligned}$$

$$\therefore \text{Loss of weight (in kg)} = \frac{w'}{g} = \frac{0.15}{10} = 0.015 \text{ kg} = 15 \text{ g}$$

$$\therefore \text{Weight of body in water} = 50 - 15 = 35 \text{ g}$$

50. (c) We know that, displacement travelled by a moving body with constant acceleration a in n th second,

$$s_n = u + \frac{1}{2} a (2n - 1)$$

For first second, $n = 1$

$$\therefore s_1 = u + \frac{1}{2} a (2 \times 1 - 1)$$

$$5 = u + \frac{a}{2} \quad \dots(i) [\because s_1 = 5 \text{ m}]$$

For third second, $n = 3$

$$\therefore s_3 = u + \frac{a}{2} (2 \times 3 - 1)$$

$$2 = u + \frac{5a}{2} \quad \dots(ii) [\because s_3 = 2 \text{ m}]$$

Subtracting Eq. (i) from Eq. (ii), we get

$$-3 = 2a$$

$$\Rightarrow a = \frac{-3}{2} \text{ m/s}^2$$

Force acting on the body,

$$F = ma = 4 \times \left(\frac{-3}{2} \right) = -6 \text{ N}$$

Hence, force acting on the body in opposite direction of its motion will be 6N.

51. (b) Time-period of simple pendulum is given as

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \dots(i)$$

If a be the acceleration of the lift, so that time-period of simple pendulum is reduced to $T/2$.

i.e. $T' = T/2$

$$\Rightarrow 2\pi \sqrt{\frac{l}{g+a}} = \frac{1}{2} \times 2\pi \sqrt{\frac{l}{g}}$$

$$\Rightarrow \frac{l}{g+a} = \frac{1}{4} \cdot \frac{l}{g}$$

$$\Rightarrow g+a = 4g$$

$$\Rightarrow a = 3g$$

52. (c) According to Mayer's formula,

$$C_p - C_v = R$$

where, C_p = specific heat at constant pressure,

C_v = specific heat at constant volume

R = gas constant.

$$\Rightarrow \frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

$$\Rightarrow \gamma - 1 = \frac{R}{C_v}$$

$$\Rightarrow C_v = \frac{R}{\gamma - 1}$$

53. (b) Net work done in cyclic process ABCA,

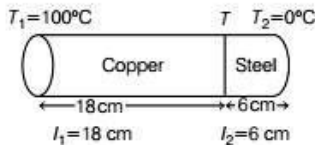
$$\begin{aligned} W_{\text{net}} &= \text{Area enclosed by path ABCA} \\ &= \frac{1}{2} \times (3V_1 - V_1)(4p_1 - p_1) \\ &= \frac{1}{2} \times 2V_1 \times 3p_1 = 3p_1V_1 \end{aligned}$$

Since, cyclic process is taken in anti-clockwise direction, hence net work done will be negative.

$$W_{\text{net}} = -3p_1V_1$$

54. (d) In isothermal process, temperature remains constant, hence internal energy of the system remains constant in isothermal process.

55. (a) The given situation is shown below



Thermal conductivity of copper
 $= 9 \times$ thermal conductivity of steel
 $K_c = 9 K_s$

Since, both bar are connected in series, hence heat current along the bar will be same.

$$\begin{aligned} \text{i. e. } \frac{dQ}{dt} &= \frac{K_c A (T_1 - T)}{l_1} \\ &= \frac{K_s A (T - T_2)}{l_2} \\ \Rightarrow \frac{9K_s A (100 - T)}{18} &= \frac{K_s A (T - 0)}{6} \\ \Rightarrow 3(100 - T) &= T \\ 300 - 3T &= T \\ \Rightarrow 4T &= 300 \\ \Rightarrow T &= 75^\circ\text{C} \end{aligned}$$

56. (d) Equation of simple harmonic wave is given as

$$y = 6 \sin 2\pi(2t - 0.1x)$$

Path difference, $\Delta x = 2 \text{ mm}$

Comparing with standard equation of wave,

$$\begin{aligned} y &= A \sin(\omega t - kx) \\ &= A \sin 2\pi \left(vt - \frac{k}{2\pi} \cdot x \right) \end{aligned}$$

$$\frac{k}{2\pi} = 0.1$$

$$k = 0.2\pi$$

$$\text{Wavelength, } \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10 \text{ mm}$$

$\therefore$ We know that, phase difference,

$$\begin{aligned} \phi &= \frac{2\pi}{\lambda} \times \text{path difference } (\Delta x) \\ &= \frac{2\pi}{10} \times 2 = \frac{2\pi}{5} = \frac{2 \times 180}{5} = 72^\circ \end{aligned}$$

57. (d) Given, sound source is stationary, $v_s = 0$

Apparent frequency, $f' = 2f$

where, f is actual frequency of sound.

By using Doppler's effect,

$$f' = f \left(\frac{v + v_0}{v} \right)$$

[where, v is velocity of sound A.]

$$\Rightarrow 2f = f \left(\frac{v + v_0}{v} \right)$$

$$\Rightarrow 2v = v + v_0$$

$$\Rightarrow v = v_0$$

$$\Rightarrow v_0 = v$$

58. (a) Given, $T_1 = 27^\circ\text{C} = (273 + 27)\text{K} = 300\text{K}$

$$\begin{aligned} T_2 &= 627^\circ\text{C} \\ &= (627 + 273)\text{K} \\ &= 900\text{K} \end{aligned}$$

Energy emitted per second

$$E_1 = 0.5 \text{ J}$$

$$E_2 = ?$$

According to Stefan's Boltzmann law,

Energy radiated per unit area per second is given by

$$E = \sigma T^4$$

$$\Rightarrow \frac{E_2}{E_1} = \left(\frac{T_2}{T_1} \right)^4$$

$$= \left(\frac{900}{300} \right)^4 = 81$$

$$\Rightarrow E_2 = 81E_1 = 81 \times 0.5 = 40.5 \text{ J}$$

59. (a) Given, $L_2 = 2L_1$, $f_1 = 200 \text{ Hz}$, $f_2 = 300 \text{ Hz}$

We know that, frequency of vibrating string is given as

$$f = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

where, μ = mass per unit length of string

and L = length of the string.

$$\Rightarrow f \propto \frac{\sqrt{T}}{L}$$

$$\Rightarrow \frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \cdot \frac{L_1}{L_2}$$

$$\Rightarrow \sqrt{\frac{T_2}{T_1}} = \frac{f_2 L_2}{f_1 L_1} = \frac{300}{200} \times \frac{2L_1}{L_1}$$

$$\sqrt{\frac{T_2}{T_1}} = 3$$

$$\Rightarrow \frac{T_2}{T_1} = 9$$

$$\therefore T_2 : T_1 = 9 : 1$$

60. (a) Loudness of sound is given as

$$L = 10 \log_{10} \frac{I}{I_0}$$

So, we get $L_2 - L_1 = 10 \log_{10} \frac{I_2}{I_1}$

here $L_1 = 30 \text{ dB}$, $L_2 = 60 \text{ dB}$

$$\therefore 60 - 30 = 10 \log_{10} \frac{I_2}{I_1}$$

$$\Rightarrow 3 = \log_{10} \frac{I_2}{I_1}$$

$$\Rightarrow \frac{I_2}{I_1} = 10^3 \Rightarrow I_2 = 1000 I_1$$

Chemistry

61. (c) Carnallite : $\text{KCl} \cdot \text{MgCl}_2 \cdot 6\text{H}_2\text{O}$

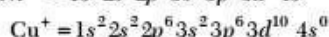
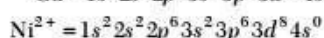
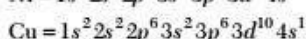
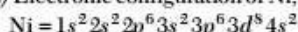
Dolomite : $\text{MgCO}_3 \cdot \text{CaCO}_3$

Calamine : ZnCO_3

Sea water : ore of magnesium

$\therefore$ Calamine is not an ore of magnesium.

62. (a) Electronic configuration of Ni, Cu and their ions are



Electronic configuration represents Cu^+ ions.

63. (b) The element $[\text{Ne}]3s^2 3p^3$ have half-filled p -orbital.

As a result, it is more stable.

$\therefore$ The ionisation energy is highest for this element.

64. (a) The oxidation number of Br in Br_2 is zero.

The oxidation number of Br is BrO_3^-

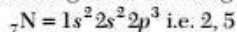
$$x + 3 \times (-2) = -1$$

$$x = -1 + 6$$

$$x = +5$$

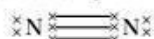
65. (d) Among the alkali metals, caesium is most reactive because the outermost electron is more loosely bound than the outermost electron of other alkali metals.

66. (a) Electronic configuration of nitrogen is



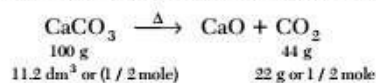
In outer most shell, it has five electrons.

$\therefore$ It share 3 electrons to complete its octet as follows :

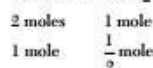
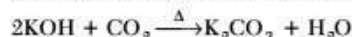


67. (c) Hydrogen bond is strongest in $\text{F}-\text{H} \cdots \text{F}$, as the electronegativity of fluorine is highest.

68. (a) Decomposition of CaCO_3 as follows:



Neutralisation reaction of KOH and CO_2 as follows:



1 mole of CO_2 reacts with 2 moles of KOH

1 / 2 mole of CO_2 reacts with 1 mole of KOH

$\therefore$ Mass of $\text{KOH} = 1 \times 56.1 = 56.1 \text{ g} \approx 56 \text{ g}$

$$69. (c) M = \frac{dRT}{p} = \frac{1.964 \times 0.0821 \times 273}{1}$$

$$\left[\begin{array}{l} \therefore p = 76 \text{ cm Hg} = 1 \text{ atm,} \\ d = 1.964 \text{ g dm}^{-3} \\ R = 0.0821 \text{ dm}^3 \text{ atm K}^{-1} \text{ mol}^{-1} \end{array} \right]$$

$M = 44 \text{ g / mol}$

$\therefore$ So, gas is CO_2 .

70. (a) 0.06 mole KNO_3 added to 100 cm^3 of water at 298 K.

$$\Delta H = 35.8 \text{ kJ/mol}$$

$$\Delta H = m \times C \times \Delta T$$

$$\left[\begin{array}{l} \Delta H \text{ for } 0.06 \text{ mole} = 35.8 \times 0.06 = 2.148 \text{ kJ} \\ \text{Mass} = 100 \text{ cm}^3 = 100 \text{ g} \end{array} \right]$$

$$\therefore 2.148 = 100 \times 0.004184 \text{ kJ} \times \Delta T$$

$$\Delta T = 5.133 \text{ K}$$

It is an endothermic process as heat absorbed.

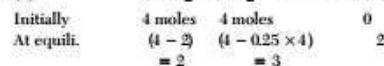
So, temperature will decrease.

$\therefore$ Final temperature

$$T = 298 - 5.133$$

$$T = 292.867 \text{ K} \approx 293 \text{ K}$$

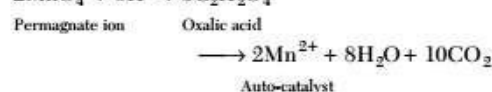
71. (b) $2\text{SO}_2 + \text{O}_2 \rightleftharpoons 2\text{SO}_3$



$\therefore$ Total moles at equilibrium = $2 + 3 + 2 = 7$ moles.

72. (d) An example of auto-catalysis is oxidation of oxalic acid by acidified KMnO_4 solution.

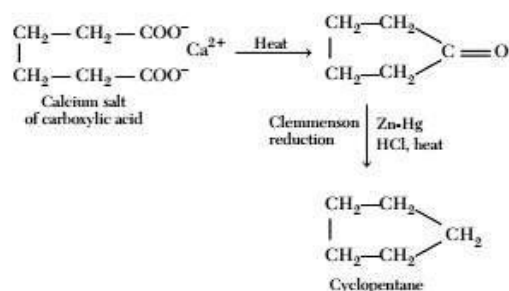
In this reaction, one of the product (Mn^{2+}) formed act as a catalyst and speed up the reaction.



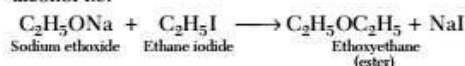
73. (c) When a nitrogenous organic compound is fused with sodium, the nitrogen present in the compound is converted into sodium cyanide.

This reaction is also called Lassaigne's test.

74. (c) The complete reaction is as follows :



75. (c) Williamson ether synthesis is an organic reaction, forming an ether from alkyl halide and deprotonated alcohol i.e.

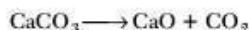


76. (c) For the reaction, $\text{C} + \frac{1}{2}\text{O}_2 \longrightarrow \text{CO}$,

ΔG° vs T plot in the Ellingham's diagram slopes downwards.

As for this reaction ΔG° become more negative with increase in temperature.

77. (a) Decomposition of calcium carbonate to form calcium oxide and carbondioxide is endothermic reaction.



78. (a) Liquor ammonia bottle is opened only after cooling because it is a mild explosive.

Note The concentrated ammonia solution in water results in liquor ammonia.

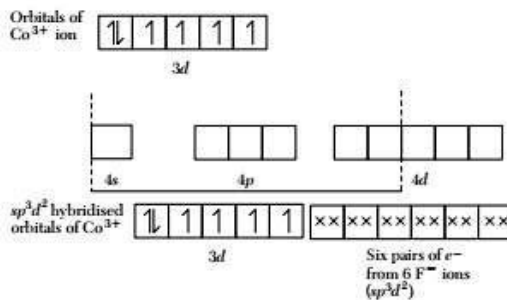
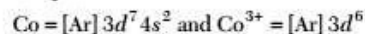
79. (c) The formation of $\text{O}_2^+[\text{PtF}_6]^-$ is the basis for the formation of xenon fluorides because O_2 and Xe have comparable ionisation energies.

80. (b) Magnetic moment of the element / ion is highest for configuration $3d^5$, $\uparrow\uparrow\uparrow\uparrow\uparrow$ because number of unpaired electron is maximum i.e. five.

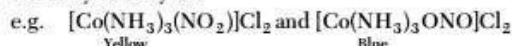
81. (c) If a transition metal ion exists in its highest oxidation state, then it behave as oxidising agent. The oxidation number of metal can only decrease from its highest oxidation number i.e. it will undergoes reduction and oxidise other element.

82. (a) In $[\text{CoF}_6]^{3-}$, F^- is a weak ligand. It cannot pair up d -electrons and form outer orbital octahedral complex while NH_3 and CN^- are strong ligand.

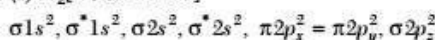
In $[\text{CoF}_6]^{3-}$, outer orbital $4d$ is used in hybridisation. Thus, it is called outer orbital or high spin or spin free complex.



83. (a) Linkage isomerism can be shown by compound containing ambidentate ligand. In NO_2 , NO_2 can bind either by N or by O.



84. (a) N_2 [14 electrons] =



$$\begin{aligned}
 \text{Bond order} &= \frac{N_b - N_a}{2} \left[\begin{array}{l} N_b \text{ and } N_a \text{ are bonding and} \\ \text{anti bonding molecular} \\ \text{orbital } s\text{-electron} \end{array} \right] \\
 &= \frac{10 - 4}{2} = 3
 \end{aligned}$$

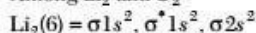
$\therefore \text{N}_2$ has highest bond order i.e. 3.

While O_2 , He_2 and H_2 have 2, 0 and 1/2 bond order respectively.

85. (c) H_2^+ and He_2^+ contains odd number of electrons

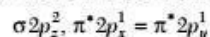
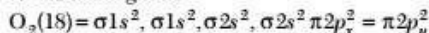
$\therefore$ They are paramagnetic.

Among Li_2 and O_2



Li have no unpaired electron.

$\therefore$ It is diamagnetic



It has two unpaired electrons. So, it is paramagnetic.

86. (b) For 1st order reaction

$$\begin{aligned}
 k &= \frac{2.303}{t} \log \frac{[R_0]}{[R]} = \frac{2.303}{40} \log \left[\frac{0.1}{0.025} \right] \\
 &= \frac{2.303}{40} \times \log 4 = 0.0347 \text{ min}^{-1}
 \end{aligned}$$

Rate expression,

$$r = k[A] = 0.0347 \times 0.01 = 3.47 \times 10^{-4} \text{ M min}^{-1}$$

87. (b) According to Arrhenius equation

$$k = A e^{\frac{-E_a}{RT}}$$

Here, k decreases with increase in E_a (activation energy).

Chemical reaction with very high E_a values are generally slow.

88. (b) Solid NaCl does not conduct electricity because the ions (Na^+ and Cl^-) are well arranged in the crystal structure and not free to move to conduct electricity.

89. (b) According to second law of Faraday,

$$\frac{w_1}{E_1} = \frac{w_2}{E_2}$$

$$\frac{0.16}{64} = \frac{w_2}{2}$$

$$w_2 = \frac{0.16 \times 2}{64} = \frac{0.01}{2} = 0.005 \text{ g}$$

$$\text{Volume of } \text{H}_2 = \frac{0.005}{2} \times 22.400 \text{ cm}^3 = \frac{22.4}{4} = 56 \text{ cm}^3$$

90. (c) $K_{sp} = 10^{-8} \text{ M}^2$

$$K_{sp} = [A^+][B^-]$$

$$10^{-8} = (10^{-3})[B^-]$$

$$[B^-] = 10^{-5} \text{ M}$$

$\therefore$ Salt will precipitate, when concentration of $[B^-] > 10^{-5} \text{ M}$.

91. (d) According to Nernst equation,

$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{0.0591}{n} \log \frac{[\text{Cu}^{+2}]}{[\text{Ag}^+]^2}$$

$\therefore$ The voltage of cell will increase with increase in concentration of Ag^+ ions.

92. (c) $\text{pH} = -\log[\text{H}^+]$

$$[\text{H}^+] = 10^{-8} + 10^{-7} = 1.1 \times 10^{-7}$$

$$\text{pH} = -\log(1.1 \times 10^{-7})$$

$$\text{pH} = 7 - \log(1.1)$$

$\therefore$ It is between 6 and 7.

93. (b) Lowering in vapour pressure is directly proportional to moles of solute particles

$$(\Delta p)_{\text{glucose}} = (\Delta p)_{\text{urea}}$$

$$(\chi_B)_{\text{glucose}} = (\chi_B)_{\text{urea}}$$

$$\text{i.e. } \frac{W_B}{50} \times \frac{18}{180} = \frac{1 \times 18}{50 \times 60}$$

$$W_B = 3 \text{ g}$$

94. (c) Osmotic pressure is a colligative property. It depends upon the number of solute particles. Benzoic acid undergoes dimerisation.

$\therefore$ Number of particles reduces ($i < 1$) Hence, osmotic pressure observed is less than the theoretical consideration.

95. (a) For a reaction to be spontaneous at all temperature ΔG should be negative. As we know $\Delta G = \Delta H - T\Delta S$

$$= -ve - T(+ve) = -ve$$

$\therefore \Delta G$ and ΔH should be negative.

96. (a) Flocculation value $\propto \frac{1}{\text{Coagulating power}}$

$\text{Fe}(\text{OH})_3$ is a positively charged sol.

For coagulating $\text{Fe}(\text{OH})_3$ negatively charged electrolyte is used and greater the value of $-ve$ charge, coagulating power will be strong. Among given electrolyte, NaCl has lowest coagulating power. Hence, its flocculation value will be maximum.

97. (b) $\Delta H = -40 \text{ kJ} = -40 \times 10^3 \text{ J}$

$$\Delta S = \Delta S_{\text{Product}} - \Delta S_{\text{Reactant}}$$

$$= 2(50) - 3(40) - 60$$

$$= 100 - 120 - 60 = -80$$

$$\Delta G = \Delta H - T\Delta S$$

$$0 = -40 \times 10^3 - T(-80)$$

$$T = \frac{40 \times 10^3}{80} = 500 \text{ K.}$$

98. (c) NaCl forms fcc structure.

$$\therefore (\text{Diagonal length})^2 = (\sqrt{2}a)^2 = 2(r^+)^2 + 2(r^-)^2$$

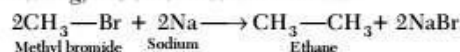
Where, a = edge length, r^+ and r^- are radii of Na^+ and Cl^- ions.

$$a = 2[(r^+)^2 + (r^-)^2] = 2(95 + 181) = 2 \times 276 = 552 \text{ pm}$$

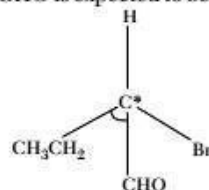
99. (a) Inductive effect involves the displacement of σ -electrons from less electronegative element to more electronegative element in a compound.

100. (a) The basicity of aniline is less than that of cyclohexylamine because of +R-effect of NH_2 group in aniline.

101. (c) Methyl bromide is converted into ethane by heating, it is ether medium with Na.

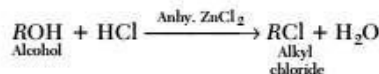


102. (c) The compound having chiral centre, $\text{CH}_3\text{CH}_2\text{CHBrCHO}$ is expected to be optically active.

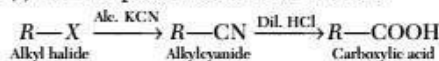


103. (d) Cyclohexane has the lowest heat of combustion per CH_2 group. As, stability increase upto 6 membered ring due to decrease in angular strain. Also heat of combustion is measure for chemical stability.

104. (c) The catalyst used in preparation of an alkyl chloride by action of dry HCl on alcohol is anhy. ZnCl_2 . This process is also called Grove's process.



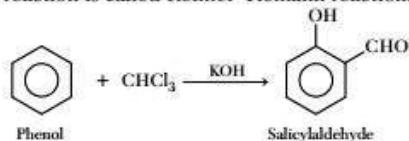
105. (c) The complete reaction is as follows :



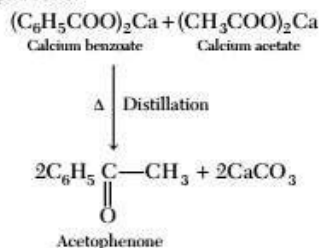
106. (b) Phenol does not decompose sodium bicarbonate (NaHCO_3) as it is a weaker acid than carbonic acid. $\therefore \text{CO}_2$ gas is not evolved by phenol.

107. (b) Phenol on heating with chloroform in alkali get converted into salicylaldehyde.

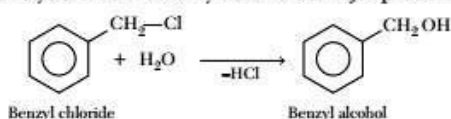
This reaction is called Reimer-Tiemann reaction.



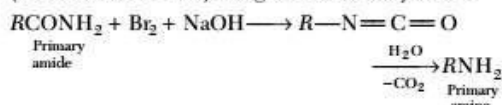
108. (a) When a mixture of calcium benzoate and calcium acetate is dry distilled, the resulting compound is acetophenone.



109. (c) Benzylchloride does not give benzoic acid on hydrolysis. It forms benzyl alcohol as major product.

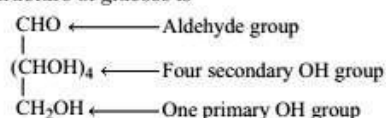


110. (c) In Hofmann bromamide reaction, primary amide reacts with bromine and sodium hydroxide to form isocyanate, which is hydrolysed to give primary amine (with one carbon less) along with decarboxylation.



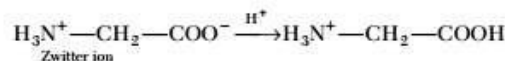
111. (b) Glucose contains in addition to aldehyde group on primary OH and four secondary OH groups.

The structure of glucose is



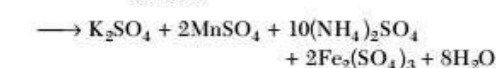
112. (b) Fats are triglycerides which are triesters of fatty acid with glycerol. So, the characteristic feature of fats is an ester group.

113. (b) At $\text{pH} = 4$, glycine reacts with H^+ ion as



114. (c) Insulin regulate the metabolism of glucose (carbohydrates).

115. (a) The reaction between Mohr's salt and KMnO_4 is



Oxidation state of Fe changes from +2 to +3.

$\therefore$ Equivalent weight of Mohr's salt

$$= \frac{\text{molecular weight}}{n\text{-factor}} = \frac{392}{1} = 392 \text{ g}$$

116. (a) The brown ring is appeared due to formation of a brown complex between nitric oxide and Fe^{2+} ion.

This nitric oxide is formed as a result of reduction of nitrate. Brown ring test for nitrates depends on reduction of nitrate to nitric oxide.

117. (c) Acrolein test is positive for oils and fats when fat is heated in presence of KHSO_4 , the glycerol portion of molecule is dehydrated and form unsaturated aldehyde. $\text{CH}_2=\text{CH}-\text{CHO}$ (acrolein) a bad smelling compound.

118. (a) An organic compound which produces a bluish green flame, when heated in presence of copper is chlorobenzene. This is called Beilstein's test used to detect presence of halogen in an organic compound.

119. (c) Let rate expression, $r = k[A]^x[B]^y$... (i)

If, $[A_1] = [2A]$, then $r_1 = 2r$

$\therefore x = 1$

If, $[B_2] = [9B]$, then $r_2 = 3r$

$\therefore y = \frac{1}{2}$

Rate expression, $r = k[A][B]^{1/2}$ [By Eq. (i)]

$\therefore \text{Order} = 1 + \frac{1}{2} = \frac{3}{2}$

120. (a) For, assume complete dissociation

For Na_2SO_4 ; $i = 3$, $m = 0.01$

$$\Delta T_b = iK_b m$$

$$\Delta T_b = 3 \times K_b \times 0.01 = 0.03 K_b$$

For, 0.01 M KNO_3 ;

$$i = 2, m = 0.01$$

$$\Delta T_b = 2 \times K_b \times 0.01 K_b = 0.02 K_b$$

While, urea and glucose are non-electrolyte

They do not dissociate into ions highest boiling point is shown by 0.01 M Na_2SO_4 (aq) solution.

Mathematics

121. (a) Given that, $\mathbf{u} = \mathbf{a} - \mathbf{b}$, $\mathbf{v} = \mathbf{a} + \mathbf{b}$

$$\text{and } |\mathbf{a}| = |\mathbf{b}| = 2$$

$$\text{Now, } |\mathbf{u} \times \mathbf{v}| = |\mathbf{a} - \mathbf{b} \times \mathbf{a} + \mathbf{b}|$$

$$= 2(\mathbf{a} \times \mathbf{b}) = 2|\mathbf{a}||\mathbf{b}|\sin\theta$$

$$= 2 \times 2 \times 2\sin\theta = 8\sin\theta = 8\sqrt{1 - \cos^2\theta}$$

$$= 8\sqrt{1 - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)^2}$$

$$= 8\sqrt{1 - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{4}\right)^2} = 8\sqrt{1 - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{16}\right)^2}$$

$$= \frac{8}{4}\sqrt{16 - (\mathbf{a} \cdot \mathbf{b})^2} = 2\sqrt{16 - (\mathbf{a} \cdot \mathbf{b})^2}$$

122. (a) Let $A(1, 1, 1)$, $B(2, 1, 3)$, $C(3, 2, 2)$ and $D(3, 3, 4)$.

$$\text{So, } \mathbf{AB} = \hat{i} + 2\hat{k}, \mathbf{AC} = 2\hat{i} + \hat{j} + \hat{k}$$

$$\text{and } \mathbf{AD} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\text{Volume of tetrahedron} = \frac{1}{6} \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix}$$

$$= \frac{1}{6} |1(3-2) + 0(6-2) + 2(4-2)|$$

$$= \frac{1}{6}(1+0+4) = \frac{5}{6} \text{ cubic units}$$

123. (d) Let $\mathbf{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\mathbf{c} = -\hat{i} + 2\hat{j} + \hat{k}$

$$\text{Now, } \mathbf{b} \times \mathbf{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ -1 & 2 & 1 \end{vmatrix} = 5\hat{i} + \hat{j} + 3\hat{k}$$

$$\text{Now, } \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 5 & 1 & 3 \end{vmatrix} = -8\hat{i} + 7\hat{j} + 11\hat{k}$$

Now, any vector perpendicular to $\mathbf{a}$ and lying in the plane containing $\hat{i} + \hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{j} + \hat{k}$ is $\pm(\mathbf{a} \times (\mathbf{b} \times \mathbf{c}))$.

$$\therefore \text{Required unit vector} = \pm \frac{(-8\hat{i} + 7\hat{j} + 11\hat{k})}{\sqrt{(-8)^2 + (7)^2 + (11)^2}}$$

$$= \pm \frac{(-8\hat{i} + 7\hat{j} + 11\hat{k})}{\sqrt{234}}$$

124. (d) Let the inverse element of 10 be e , then

$$a * e = 0$$

$$\text{Therefore, } 10 * e = 10 + e + 10e = 0$$

$$\Rightarrow 10 + 11e = 0$$

$$\Rightarrow 11e = -10$$

$$\Rightarrow e = \frac{-10}{11}$$

125. (c) Under Multiplication Modulo 7, inverse of 2 is

$$2 \times 4 = 1 \pmod{7}$$

$$\text{Hence, } 2^{-1} = 4$$

$$\text{Now, } 2^{-1} \times 4 = 4 \times 4 = 16$$

$$\text{Now, } 16 = 2 \times 7 + 2$$

Hence dividing 16 by 7 gives a remainder 2,

Therefore, value of above multiplication mod is 2.

126. (d) The group $(\mathbb{Z}, +)$ has infinitely many subgroups.

127. (c) Given, $3x \equiv 5 \pmod{7}$

$$\Rightarrow 3x - 5 \equiv 0 \pmod{7} = \frac{7}{3x-5}$$

$$\Rightarrow 3x - 5 = 7P, \text{ where } P \in \mathbb{Z}$$

$$\Rightarrow x \equiv 4 \pmod{7}$$

128. (c) Let $z = \sin\left(\frac{6\pi}{5}\right) + i\cos\left(1 + \cos\frac{6\pi}{5}\right)$

$$\text{Putting } \sin\frac{6\pi}{5} = r \cos\alpha \quad \dots(i)$$

$$\text{and } 1 + \cos\frac{6\pi}{5} = r \sin\alpha \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\tan\alpha = \frac{1 + \cos\frac{6\pi}{5}}{\sin\left(\frac{6\pi}{5}\right)} = \frac{2\cos^2\frac{3\pi}{5}}{2\sin\frac{3\pi}{5}\cos\frac{3\pi}{5}}$$

$$= \cot\frac{3\pi}{5} = \tan\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)$$

$$\tan\alpha = \tan\left(-\frac{\pi}{10}\right)$$

$$\therefore \text{Argument to } z \text{ is } \alpha = -\frac{\pi}{10}$$

129. (c) We have,

$$\sum_{n=1}^n i^k = i + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + \dots$$

$$\text{Now as, } i^4 = i^8 = i^{12} = \dots i^{4n} = 1$$

$$\text{and } i + i^2 + i^3 + i^4 = 0$$

$$\text{So, } \sum_{n=1}^n i^k = i + i^2 + i^3 + i^4 + i^5 + i^6 + \dots i^7 + i^8 + i^9 + \dots$$

$$= i + i^2 + i^3 + i^4 + (i + i^2 + i^3 + i^4) + (i + i^2 + i^3 + i^4) + \dots$$

$$= 0 + 0 + 0 + \dots$$

When we take $n = 99$ after 96th term, we will be left with $0 + i + i^2 + i^3 = -1$

So, for $n = 99$

$$\Rightarrow 1 + \sum_{k=1}^n i^k = 0$$

130. (c) We have,

$$\begin{aligned} & (-1 + \sqrt{-3})^{62} + (-1 - \sqrt{-3})^{62} \\ &= 2^{62} \left[\left(\frac{-1 + \sqrt{3}i}{2} \right)^{62} + \left(\frac{-1 - \sqrt{3}i}{2} \right)^{62} \right] \\ &= 2^{62} [\omega^{62} + \omega^{124}] \\ &= 2^{62} [\omega + \omega^2] \\ &= 2^{62}(-1) = -2^{62} \end{aligned}$$

131. (b) We have,

$$\left| \frac{z - 6i}{z + 6i} \right| = 1$$

$$\text{or } |z - 6i| = |z + 6i|$$

$$\text{Let } z = x + iy$$

$$\text{or } |x + iy - 6i| = |x + iy + 6i|$$

$$|x + (y - 6)i| = |x + (y + 6)i|$$

$$\Rightarrow \sqrt{x^2 + (y - 6)^2} = \sqrt{x^2 + (y + 6)^2}$$

$$\Rightarrow x^2 + (y - 6)^2 = x^2 + (y + 6)^2$$

$$\Rightarrow y^2 + 36 - 12y = y^2 + 36 + 12y$$

$$\Rightarrow 24y = 0$$

$$\Rightarrow y = 0$$

Hence, it lies on the real axis.

132. (a) We have,

$$\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$$

$$\text{Now, } \cos(\tan^{-1}x) = \cos\left[\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right]$$

$$= \frac{1}{\sqrt{x^2+1}}$$

$$[\because \cos(\cos^{-1}x) = x, \forall x \in [-1, 1]]$$

$$\text{So, } \cot^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) = \sin^{-1}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}\right)$$

$$\text{Thus, } \sin\left[\cot^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)\right] = \sin\left[\sin^{-1}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}\right)\right]$$

$$= \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}} \quad [\because \sin(\sin^{-1}x) = x, \forall x \in [-1, 1]]$$

133. (c) Let $y = 1 + \alpha x$

$$[f(x) = y]$$

$$\Rightarrow x = \frac{y-1}{\alpha}$$

$$\Rightarrow f^{-1}(x) = \frac{x-1}{\alpha}$$

It is given that, $f(x) = f^{-1}(x)$

$$\Rightarrow 1 + \alpha x = \frac{x-1}{\alpha}$$

$$\Rightarrow \alpha + \alpha^2 x = x - 1$$

On compare the coefficients, we get

$$\alpha^2 = 1 \text{ and } \alpha = -1$$

$$\Rightarrow \alpha = -1$$

134. (c) Let x^r occurs in the expansion of $\left(x + \frac{1}{x}\right)^n$

$$T_{r+1} = {}^nC_p x^{n-p} \left(\frac{1}{x}\right)^p$$

$$= {}^nC_p x^{n-p} x^{-p} = {}^nC_p x^{n-2p}$$

$$\text{let } n - 2p = r$$

$$\Rightarrow n - r = 2p \Rightarrow p = \frac{n-r}{2}$$

$$\text{So, coefficient of } x^r = {}^nC_p = \frac{n!}{p!(n-p)!}$$

$$= \frac{n!}{\left(\frac{n-r}{2}\right)! \left(n - \frac{n-r}{2}\right)!}$$

$$= \frac{n!}{\left(\frac{n-r}{2}\right)! \left(\frac{n+r}{2}\right)!}$$

135. (c) We have,

$$\tan A - \tan B = x$$

$$\text{and } \cot B - \cot A = y$$

$$\Rightarrow \frac{1}{\tan B} - \frac{1}{\tan A} = y$$

$$\Rightarrow \frac{\tan A - \tan B}{\tan A \tan B} = y$$

$$\Rightarrow \tan A \tan B = \frac{x}{y}$$

$$\Rightarrow \cot A \cot B = \frac{y}{x}$$

$$\text{Now, } \cot(A-B) = \frac{1 + \cot A \cot B}{\cot B - \cot A}$$

$$= \frac{1 + \frac{y}{x}}{\frac{y}{x}} = \frac{x+y}{xy} = \frac{1}{y} + \frac{1}{x}$$

$$\begin{aligned}
 136. (a) \cos^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{4} + \cos^2 \frac{5\pi}{12} \\
 = \cos^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{4} + \cos^2 \left(\frac{\pi}{2} - \frac{\pi}{12} \right) \\
 = \cos^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{12} \\
 = 1 + \cos^2 \frac{\pi}{4} = 1 + \left(\frac{1}{\sqrt{2}} \right)^2 = 1 + \frac{1}{2} = \frac{3}{2}
 \end{aligned}$$

137. (a) $\therefore \sin \theta, \cos \theta$ and $\tan \theta$ are in GP.

$$\begin{aligned}
 \text{So, } \cos^2 \theta &= \sin \theta \cdot \tan \theta \\
 \Rightarrow \cos^3 \theta &= \sin^2 \theta \\
 \Rightarrow \cot^3 \theta &= \frac{1}{\sin \theta} = \operatorname{cosec} \theta
 \end{aligned}$$

Squaring both sides, we get

$$\begin{aligned}
 \cot^6 \theta &= \operatorname{cosec}^2 \theta \\
 \Rightarrow \cot^6 \theta &= 1 + \cot^2 \theta \\
 \Rightarrow \cot^6 \theta - \cot^2 \theta &= 1
 \end{aligned}$$

138. (d) We have,

$$\begin{aligned}
 \frac{3x^2 - 2x + 4}{(x-1)^6} &= \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} + \frac{A_3}{(x+1)^3} \\
 &\quad + \frac{A_4}{(x+1)^4} + \frac{A_5}{(x+1)^5} + \frac{A_6}{(x+1)^6} \dots (i)
 \end{aligned}$$

Now, put $x = 0$ in Eq. (i), we get

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 4 \quad \dots (ii)$$

If $A_1 + A_3 + A_5 = -8$

and $A_2 + A_4 + A_6 = 12$

satisfies the equation.

Hence, only option (i.e. $-8, 12$) satisfies the solution.

139. (d) We have,

$$\log_2(2^{x-1} + 6) + \log_2(4^{x-1}) = 5$$

$$\Rightarrow 2^{x-1} + 6)(2^{2x-2}) = 2^5$$

$$\text{Let } y = 2^{x-1}$$

$$\Rightarrow (y+6)y^2 = 2^5$$

$$\Rightarrow y^3 + 6y^2 - 32 = 0$$

$$\Rightarrow (y-2)(y^2 + 8y + 16) = 0$$

$$\text{So, } \begin{aligned} y &= 2 \\ 2^{x-1} &= 2^1 \end{aligned}$$

On comparing both sides, we get

$$\begin{aligned}
 x-1 &= 1 \\
 x &= 2
 \end{aligned}$$

140. (b) As,

$$\begin{aligned}
 (a+b+c+d)^2 &= a^2 + b^2 + c^2 + d^2 \\
 &\quad + 2(ab+bc+ad+bc+bd+cd) \\
 \Rightarrow a^2 + b^2 + c^2 + d^2 &= (a+b+c+d)^2 \\
 &\quad - 2[ab+bc+ac+ad+bd+cd]
 \end{aligned}$$

Now, as a, b, c, d are the roots of the equation.

We have,

$$a+b+c+d = \frac{-2}{1} = -2$$

$$\text{and } ab+ac+ad+bc+bd+cd = \frac{3}{1} = 3$$

We have, $1+a^2+b^2+c^2+d^2$

$$= 1 + (-2)^2 - 2(3)$$

$$= 1 + 4 - 6 = -1$$

141. (b) We have,

$$c_1x + c_3x^3 + c_5x^5 + \dots = \frac{1}{2} \{ (1+x)^n - (1-x)^n \}$$

$$\Rightarrow \int_0^1 (c_1x + c_3x^3 + c_5x^5 + \dots) dx$$

$$= \frac{1}{2} \int_0^1 (1+x)^n - (1-x)^n dx$$

$$\Rightarrow \frac{c_1}{2} + \frac{c_3}{4} + \frac{c_5}{6} + \dots = \frac{2^n - 1}{n+1}$$

$$142. (a) (0.2)^{\log_{\sqrt{5}} \left(\frac{1}{4} + \frac{1}{4.2} + \frac{1}{4} \left(\frac{1}{2} \right)^2 + \dots \infty \right)}$$

$$= (0.2)^{\log_{\sqrt{5}} \frac{1/4}{1 - \frac{1}{2}}}$$

$$= (0.2)^{\log_{\sqrt{5}} \left(\frac{1}{2} \right)}$$

$$= \left(\frac{1}{5} \right)^{\log_{\sqrt{5}} \left(\frac{1}{2} \right)} = \left(\frac{1}{2} \right)^{2 \log_5 \frac{1}{5}}$$

$$= \frac{1}{2}^{-2 \log_5 5} = \left(\frac{1}{2} \right)^{-2}$$

$$= (2^{-1})^{-2} = 2^2 = 4$$

143. (c) If set A and B has m elements, then the number of bijections from A to B is the total number of arrangements of n distinct items taken all at a time, i.e. $m!$.

$$144. (c) \text{ Let, } y = \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$$

$$= \sin^{-1} [x\sqrt{1-(\sqrt{x})^2} - \sqrt{x}\sqrt{1-x^2}]$$

$$= \sin^{-1} x - \sin^{-1} \sqrt{x}$$

$$[\text{Using } \sin^{-1} x - \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}]]$$

145. (b) We have,

$$\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \tan 4\theta \tan 7\theta$$

$$\tan \theta + \tan 4\theta = \tan \theta \tan 4\theta \tan 7\theta - \tan 7\theta$$

$$\tan \theta + \tan 4\theta = -\tan 7\theta(1 - \tan \theta \tan 4\theta)$$

$$\frac{\tan \theta + \tan 4\theta}{1 - \tan \theta \tan 4\theta} = -\tan 7\theta$$

$$\tan(\theta + 4\theta) = -\tan 7\theta$$

$$\tan 5\theta = \tan(-7\theta)$$

$$\therefore 5\theta = -7\theta$$

$$12\theta = 0 \text{ or } 12\theta = n\pi$$

$$\text{So, } \theta = \frac{n\pi}{12}, \forall n \in I$$

146. (b) The normal to the tangent will pass through the centre. Also, the intersection of the normal and tangent will give the point of contact.

Hence, consider the equation of tangent

$$y = -\frac{n}{2} - 4.5$$

$$\text{Here, } m = \frac{-1}{2}$$

Therefore, slope of the normal is 2.

Since, slope of normal and tangent are mutually perpendicular.

Therefore, equation of the normal at $(1, -1)$ is

$$\frac{y-1}{x+1} = 2 \text{ or } y-1 = 2x+2$$

$$\Rightarrow 2x - y = -3$$

Therefore, we have two equations $2x - y = -3$

and $x + 2y + 9 = 0$

Solving these two equations give us $(x, y) = (-3, -3)$

Hence, point of contact is $(-3, -3)$.

147. (b) For given condition,

$$\frac{2g}{2g'} = \frac{2f}{2f'}$$

$$\Rightarrow gf' = fg'$$

148. (b) Given, circles are $x^2 + y^2 = 4$ and

$$x^2 + y^2 - 4x + 2y - 4 = 0$$

$$\text{i.e., } x^2 + y^2 = 4 \quad \dots(i)$$

$$\text{and } (x-2)^2 - 4 + (y+1)^2 - 1 - 4 = 0$$

$$\Rightarrow x^2 + y^2 = 4 \text{ and } (x-2)^2 + (y+1)^2 = 9 \quad \dots(ii)$$

Center and radius of circles (i) is

$$c_1 = (0, 0), r_1 = 2$$

and center and radii of circle (ii) is

$$c_2 = (2, -1), r_2 = 3$$

So, difference between c_1 and $c_2 = \sqrt{5}$

Now here,

$$1 = r_2 - r_1 < \sqrt{5} = c_1 c_2 < 5 = r_1 + r_2$$

$$r_2 - r_1 < c_1 c_2 < r_1 + r_2$$

Hence, there will be two common tangents.

149. (c) Length of tangent from $x^2 + y^2 + 2gx + 2fy + c_1 = 0$

To the circle $x^2 + y^2 + 2gx + 2fy + c_2 = 0$

$$\text{is } \sqrt{c_2 - c_1}$$

Here, $c_2 = 4$ and $c_1 = 0$

So, required length $= \sqrt{4 - 0} = \sqrt{4} = 2$

150. (c) Eccentricity for $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $b^2 = a^2(1 - e^2)$ and eccentricity for

$$\frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1 \text{ is}$$

$$e_1 = \frac{a_1^2 + b_1^2}{a_1^2}$$

$$\text{So, } e_1 = \sqrt{1 + \frac{81}{144}} = \frac{15}{12}$$

$$\text{Again, foci} = a_1 e_1 = \frac{12}{5} \times \frac{15}{12} = 3$$

So, focus of hyperbola is $(3, 0) = (ae, 0)$

and focus of ellipse is $(ae, 0) = (5a, 0)$

As, this foci are same, so

$$5a = 3$$

$$\therefore e = \frac{3}{5}$$

$$\text{So, } e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{b^2}{25}$$

$$\Rightarrow \frac{b^2}{25} = 1 - e^2 = 1 - \frac{9}{25}$$

$$\Rightarrow \frac{b^2}{25} = \frac{16}{25}$$

$$\Rightarrow b^2 = 16$$

151. (c) Given that, $\frac{2b^2}{a} = \frac{2b}{2}$

$$a = 2b$$

Now, $b^2 = a^2(1 - e^2)$

$$b^2 = 4b^2(1 - e^2)$$

$$\frac{1}{4} = 1 - e^2$$

$$\Rightarrow e^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow e = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

152. (a) Given, equation of parabola is
 $x^2 + 10x - 16y + 25 = 0$

It can be rewritten as

$$(x+5)^2 = 16y \quad \dots(i)$$

Here $a = 4$ and vertex $\equiv (-5, 0)$

and focus $\equiv (-5, 4)$

So, y -coordinate of ends of latusrectum will be 4.

Putting in equation, we have

$$(x+5)^2 = 16 \times 4 = 64$$

$$\Rightarrow x+5 = \pm 8$$

$$\text{So, } x = 3, -13$$

Substituting x in Eq. (i), we get $y = 4, 4$

Hence, ends of latusrectum will be $(3, 4)$ and $(-13, 4)$.

153. (d) (a) We have,

$$\begin{aligned} f(x) &= \cos |x| + |x| \\ &= \begin{cases} \cos(-x) + (-x), & x < 0 \\ \cos(x) + x, & x \geq 0 \end{cases} \\ &= \begin{cases} \cos x - x, & x < 0 \\ \cos x + x, & x \geq 0 \end{cases} \end{aligned}$$

$$\therefore f'(x) = \begin{cases} -\sin x - 1, & x < 0 \\ -\sin x + 1, & x \geq 0 \end{cases}$$

$$\therefore f'(0^-) = -1 \text{ and } f'(0^+) = 1$$

$$\Rightarrow f'(0^-) \neq f'(0^+)$$

So, it is not differentiable at $x = 0$.

(b) We have,

$$\begin{aligned} f(x) &= \cos |x| - |x| \\ &= \begin{cases} \cos(-x) - (-x), & x < 0 \\ \cos(x) - x, & x \geq 0 \end{cases} \\ &= \begin{cases} \cos x + x, & x < 0 \\ \cos x - x, & x \geq 0 \end{cases} \end{aligned}$$

$$\therefore f'(x) = \begin{cases} -\sin x + 1, & x < 0 \\ -\sin x - 1, & x \geq 0 \end{cases}$$

$$\therefore f'(0^-) = 1 \text{ and } f'(0^+) = -1$$

$$\Rightarrow f'(0^-) \neq f'(0^+)$$

So, it is not differentiable at $x = 0$.

(c) We have,

$$\begin{aligned} f(x) &= \sin |x| + |x| \\ &= \begin{cases} \sin(-x) + (-x), & x < 0 \\ \sin(+x) + (+x), & x \geq 0 \end{cases} \\ &= \begin{cases} -\sin x - x, & x < 0 \\ \sin x + x, & x \geq 0 \end{cases} \end{aligned}$$

$$\therefore f'(x) = \begin{cases} -\cos x - 1, & x < 0 \\ \cos x + 1, & x \geq 0 \end{cases}$$

$$\therefore f'(0^-) = -1 - 1 = -2$$

$$\text{and } f'(0^+) = 1 + 1 = 2 \Rightarrow f'(0^-) \neq f'(0^+)$$

So, it is not differentiable at $x = 0$.

(d) We have,

$$\begin{aligned} f(x) &= \sin |x| - |x| \\ &= \begin{cases} \sin(-x) - (-x), & x < 0 \\ \sin x - x, & x \geq 0 \end{cases} \\ &= \begin{cases} -\sin x + x, & x < 0 \\ \sin x - x, & x \geq 0 \end{cases} \end{aligned}$$

$$\therefore f'(x) = \begin{cases} -\cos x + 1, & x < 0 \\ \cos x - 1, & x \geq 0 \end{cases}$$

$$\therefore f'(0^-) = -1 + 1 = 0$$

$$\text{and } f'(0^+) = 1 - 1 = 0$$

$$\Rightarrow f'(0^-) = f'(0^+)$$

So, it is differentiable at $x = 0$.

154. (d) We have,

$$x = a \left(\cos t + \log \tan \frac{t}{2} \right)$$

$$\text{Now, } \frac{dx}{dt} = a \left\{ -\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2} \cdot \frac{1}{2} \right\}$$

$$= a \left\{ -\sin t + \frac{1}{\sin t} \right\}$$

$$= a \left(\frac{1 - \sin^2 t}{\sin t} \right) = \frac{a \cos^2 t}{\sin t}$$

and $y = a \sin t$

$$\frac{dy}{dt} = a \cos t$$

$$\text{So, } \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{a(\cos t \sin t)}{a \cos^2 t} = \tan t$$

155. (b) We have,

$$\begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{Now, } \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \left(\frac{1}{1 + \tan^2 \theta} \right)$$

$$\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 - \tan^2 \theta & -2 \tan \theta \\ 2 \tan \theta & 1 - \tan^2 \theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\therefore a = \cos 2\theta, b = \sin 2\theta$$

156. (c) We have, $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$\begin{aligned} A^2 &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & 2+2 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A^3 &= A^2 \times A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & 2+4 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 1 & 2 \times 3 \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$$

157. (b) We have, α, β and γ are roots of equation

$$x^3 + px + q = 0 \quad \dots(i)$$

$$\text{Here, } \alpha + \beta + \gamma = 0$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = p$$

$$\alpha\beta\gamma = -q$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} \alpha + \beta + \lambda & \beta & \gamma \\ \alpha + \beta + \lambda & \gamma & \alpha \\ \alpha + \beta + \lambda & \alpha & \beta \end{vmatrix} = (\alpha + \beta + \lambda) \begin{vmatrix} 1 & \beta & \gamma \\ 1 & \gamma & \alpha \\ 1 & \alpha & \beta \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$, we get

$$\begin{aligned} (\alpha + \beta + \lambda) \begin{vmatrix} 1 & \beta & \gamma \\ 0 & \gamma - \beta & \alpha - \gamma \\ 0 & \alpha - \beta & \beta - \gamma \end{vmatrix} \\ = (\alpha + \beta + \lambda)[(\gamma - \beta)(\beta - \gamma) - (\alpha - \gamma)(\alpha - \beta)] \\ = 0 \times [(\gamma - \beta)(\beta - \gamma) - (\alpha - \gamma)(\alpha - \beta)] = 0 \end{aligned}$$

$$158. (b) \begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} 2\cos x + \sin x & \cos x & \cos x \\ 2\cos x + \sin x & \sin x & \cos x \\ 2\cos x + \sin x & \cos x & \sin x \end{vmatrix} = 0$$

$$\Rightarrow (2\cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

[Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$(2\cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$$\Rightarrow (2\cos x + \sin x)[1 \cdot (\sin x - \cos x)^2] = 0$$

$$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$$

$$\Rightarrow 2\cos x = -\sin x \text{ or } \sin x = \cos x$$

$$\Rightarrow \tan x = -2 \text{ which is not possible as for}$$

$$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \text{ we get } -1 \leq \tan x \leq 1$$

$$\text{or } \tan x = 1$$

$$\text{So, } x = \frac{\pi}{4}. \text{ Hence, only one real root exist.}$$

159. (c) Prime factors of 450 is $2 \times 3 \times 3 \times 5 \times 5$.

Now, non-prime divisors will be combinations of these.

When two are multiplied,

$$2 \times 3 = 6, 2 \times 5 = 10$$

$$3 \times 3 = 9, 3 \times 5 = 15, 5 \times 5 = 25$$

When three are multiplied,

$$2 \times 3 \times 3 = 18, 2 \times 3 \times 5 = 30, 2 \times 5 \times 5 = 50,$$

$$3 \times 3 \times 5 = 45, 3 \times 5 \times 5 = 75$$

When four are multiplied,

$$2 \times 3 \times 3 \times 5 = 90, 2 \times 3 \times 5 \times 5 = 150,$$

$$3 \times 3 \times 5 \times 5 = 225$$

When five are multiplied,

$$2 \times 3 \times 3 \times 5 \times 5 = 450$$

Also, 1 is also a non-prime divisor.

So, sum of divisors

$$= 9 + 15 + 25 + 30 + 50 + 45 + 75 + 90 \\ + 150 + 225 + 450 + 1 = 1199$$

$$160. (a) \sum_{\substack{1 < p < 100 \\ p \rightarrow \text{prime}}} p! - \sum_{n=1}^{50} (2n!) = (2! + 3! + 5! + 7! + \dots) \\ - (2! + 4! + 6! + \dots)$$

After 5! the number will have 0 at the one's place.

So, they will no effect the last digit of the total sum.

$$\text{Now last digit of } (2! + 3!) = 2 + 6 = 8$$

$$\text{Last digit of } (2! + 4!) = 2 + 4 = 6$$

$$\text{Last digit of } \sum_{\substack{1 < p < 100 \\ p \rightarrow \text{prime}}} p! - \sum_{n=1}^{50} (2n!) = 8 - 6 = 2$$

$$161. (b) \text{ Let } I = \int_0^1 \frac{1}{\sqrt{1+x^4}} dx$$

We have, $0 \leq x \leq 1$

$$0 \leq x^4 \leq 1$$

$$\Rightarrow 1 \leq 1 + x^4 \leq 2$$

$$\Rightarrow 1 \leq \sqrt{1+x^4} \leq \sqrt{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \leq \frac{1}{\sqrt{1+x^4}} \leq 1$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{2}} dx \leq \int_0^1 \frac{1}{\sqrt{1+x^4}} dx \leq \int_0^1 1 dx$$

$$\Rightarrow \frac{1}{\sqrt{2}} \leq \int_0^1 \frac{1}{\sqrt{1+x^4}} dx \leq 1$$

$$\therefore I \in \left[\frac{1}{\sqrt{2}}, 1 \right]$$

$$162. (b) \text{ Let } I = \int_0^{\pi/2} \log(\tan x) dx$$

$$\Rightarrow I = \int_0^{\pi/2} \log \tan \left(\frac{\pi}{2} - x \right) dx$$

$$\Rightarrow I = \int_0^{\pi/2} \log \cot x dx$$

$$\Rightarrow I = \int_0^{\pi/2} \log \frac{1}{\tan x} dx$$

$$\Rightarrow I = - \int_0^{\pi/2} \log \tan x dx$$

$$\Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

$$163. (b) \text{ Let } I = \int_{-2}^2 (ax^3 + bx + c) dx \quad \dots(i)$$

$$= \int_{-2}^2 (a(-2+2-x)^3 + b(-2+2)x + c) dx$$

$$\left[\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$\Rightarrow I = \int_{-2}^2 (-ax^3 - bx + c) dx \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-2}^2 2c dx$$

$$I = 4c$$

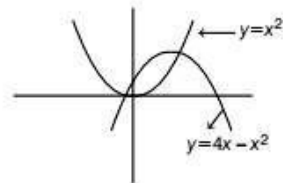
So, value of I depends on the value of c .

$$164. (b) \text{ Given equation of curves are } y = x^2 \text{ and}$$

$$y = 4x - x^2$$

$$y - 4 = -4 + 4x - x^2$$

$$y - 4 = -(x^2 - 4x + 4) = -(x-2)^2$$



$$\text{Now, required area} = \int_0^2 \{(4x - x^2) - x^2\} dx$$

$$= \int_0^2 \{(4x - 2x^2)\} dx$$

$$= \left[2x^2 - \frac{2}{3}x^3 \right]_0^2$$

$$= \left[\left(8 - \frac{16}{3} \right) - 0 \right]$$

$$= \frac{24 - 16}{3} = \frac{8}{3} \text{ sq. units}$$

$$165. (b) \text{ We have, } \frac{y}{x} \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

$$\frac{y dy}{1+y^2} = \frac{x dx}{1+x^2}$$

$$\frac{2y dy}{1+y^2} = \frac{2x dx}{1+x^2}$$

Integrating both sides, we get

$$\int \frac{2y dy}{1+y^2} = \int \frac{2x dx}{1+x^2}$$

$$\ln(1+y^2) = \ln(1+x^2) + \ln c$$

$$1+y^2 = c(1+x^2)$$

Now, $y = 2$ at $x = 1$

$$\text{So, } 5 = c(2)$$

$$2c = 5$$

$$\text{Therefore, } 2(1+y^2) = 5(1+x^2)$$

166. (d) We have,

$$\frac{dy}{dx} = (x+y)^2$$

Let $x+y = z$

$$\Rightarrow \frac{dy}{dx} + 1 = \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$$

Now, given equation becomes

$$\frac{dz}{dx} - 1 = z^2 \Rightarrow \frac{dz}{dx} = 1 + z^2$$

$$dx = \frac{dz}{1+z^2}$$

On integrating both sides, we get

$$\int dx = \int \frac{dz}{1+z^2}$$

$$\Rightarrow c + x = \tan^{-1}(z)$$

$$\Rightarrow \tan^{-1} z = x + c$$

$$\Rightarrow \tan^{-1}(x+y) = x + c$$

$$167. (a) \text{ Let } f(x) = \left(\frac{1}{x}\right)^{2x^2} = x^{-2x^2}$$

$$\therefore \log f(x) = -2x^2 \log x$$

$$\Rightarrow \frac{1}{f(x)} f'(x) = -4x \log x - \frac{2x^2}{x}$$

$$\Rightarrow f'(x) = x^{-2x^2} [-4x \log x - 2x]$$

For maximum,

$$f'(x) = 0$$

$$\Rightarrow x^{-2x^2} [-4x \log x - 2x] = 0$$

$$\Rightarrow -4x \log x - 2x = 0$$

$$\Rightarrow \log x = -\frac{1}{2}$$

$$\Rightarrow x = e^{-\frac{1}{2}}$$

$$\text{Again, } f''(x) = x^{-2x^2} [-4x \log x - 2x]^2 + x^{-2x^2} [-4 \log x - 4 - 2]$$

$$\therefore f''(e^{-\frac{1}{2}}) < 0$$

So, $f(x)$ has maximum value at $x = e^{-\frac{1}{2}}$

$$\therefore f_{\max} = f(e^{-\frac{1}{2}}) = \left(\frac{1}{\sqrt{e}}\right)^{2\left(\frac{1}{\sqrt{e}}\right)^2} = (\sqrt{e})^{\frac{2}{\sqrt{e}}} = e^{1/\sqrt{e}}$$

168. (a) Let the required numbers be x . Then,

$$f(x) = x - x^2$$

$$\Rightarrow f'(x) = 1 - 2x$$

For a local maxima or local minima, we must have

$$f'(x) = 0$$

$$\Rightarrow 1 - 2x = 0$$

$$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

$$\text{Now, } f''(x) = -2 < 0$$

So, $x = \frac{1}{2}$ is point of local maxima.

Hence, required numbers is $\frac{1}{2}$.

169. (c) Given, curve is $y = x \sin x$

...(i)

At $x = \frac{\pi}{2}$, we have $y = \frac{\pi}{2}$

Slope of the tangent at $x = \frac{\pi}{2}$

From Eq. (i), we get

$$\frac{dy}{dx} = \sin x + x \cos x$$

$$\Rightarrow \frac{dy}{dx} = 1 \text{ at } \frac{\pi}{2}$$

Now, equation of tangent will be

$$x - \frac{\pi}{2} = y - \frac{\pi}{2}$$

$$\Rightarrow x = y$$

This will intersect X-axis at $(0, 0)$.

So, the subtangent will be $= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$$\begin{aligned} 170. (a) \text{ Let } I &= \int \frac{10^{x/2}}{\sqrt{10^{-x}} - 10^x} dx \\ &= \int \frac{10^{x/2}}{\sqrt{\frac{1}{10^x}} - 10^x} dx = \int \frac{10^{x/2} \cdot 10^{x/2}}{\sqrt{1 - (10^x)^2}} dx \\ &= \int \frac{10^x}{\sqrt{1 - (10^x)^2}} dx \end{aligned}$$

Put $10^x = t \Rightarrow 10^x \log 10 \, dx = dt$

$$\begin{aligned}\text{So, } I &= \frac{1}{\log 10} \int \frac{dt}{\sqrt{1-t^2}} \\ &= \frac{1}{\log 10} \sin^{-1}(t) + c = \frac{1}{\log 10} \sin^{-1}(10^x) + c\end{aligned}$$

171. (c) We have,

$$\begin{aligned}& \int e^x \left\{ \frac{1 + \sin x \cos x}{\cos^2 x} \right\} dx \\ &= \int e^x \left\{ \frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x} \right\} dx \\ &= \int e^x (\sec^2 x + \tan x) dx = \int e^x \{f'(x) + f(x)\} dx \\ &\quad [\text{where } f(x) = \tan x \text{ and } f'(x) = \sec^2 x] \\ &= e^x f(x) + c = e^x \tan x + c\end{aligned}$$

172. (d) We have, $\int \frac{x^2+1}{x^4+1} dx$

Divide numerator and denominator by x^2 , we get

$$\begin{aligned}\int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx &= \int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 2 + 2} dx \\ &= \int \frac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 2} dx\end{aligned}$$

Let $x - \frac{1}{x} = t$

$$\begin{aligned}\left(1 + \frac{1}{x^2}\right) dx &= dt = \int \frac{dx}{t^2 + 2} \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) + c \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x - \frac{1}{x}}{\sqrt{2}} \right) + c \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + c\end{aligned}$$

173. (a) Given, line segment is $x \cos \alpha + y \sin \alpha = p$

Therefore, it cuts the X-axis and Y-axis at points $(p \sec \alpha, 0)$ and $(0, p \csc \alpha)$.

Hence, the coordinate of the mid point are

$$\left(\frac{p \sec \alpha}{2}, \frac{p \csc \alpha}{2} \right)$$

Therefore, $x = \frac{p \sec \alpha}{2}$ and $y = \frac{p \csc \alpha}{2}$

or $\cos \alpha = \frac{p}{2x}$ and $\sin \alpha = \frac{p}{2y}$

$\therefore \sin^2 \alpha + \cos^2 \alpha = 1$

$$\left(\frac{p}{2x} \right)^2 + \left(\frac{p}{2y} \right)^2 = 1$$

$$\Rightarrow \frac{p^2}{4x^2} + \frac{p^2}{4y^2} = 1 \Rightarrow \frac{1}{4x^2} + \frac{1}{4y^2} = \frac{1}{p^2}$$

$$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2} \Rightarrow x^{-2} + y^{-2} = 4p^{-2}$$

174. (c) If the line is inclined at angle 45° with the positive direction of the X-axis i.e. the points are situated at angle $\theta = 45^\circ$ from X-axis at a distance $r = 3\sqrt{2}$ from point $(4, -5)$.

The two points can be given by

$$(x_0 \pm r \cos \theta, y_0 \pm r \sin \theta)$$

The points are

$$\begin{aligned}& (4 + 3\sqrt{2} \cos 45^\circ, -5 + 3\sqrt{2} \sin 45^\circ) \\ &= \left(4 + 3\sqrt{2} \times \frac{1}{\sqrt{2}}, -5 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} \right) = (7, -2)\end{aligned}$$

and $(4 - 3\sqrt{2} \cos 45^\circ, -5 - 3\sqrt{2} \sin 45^\circ)$

$$= \left(4 - 3\sqrt{2} \times \frac{1}{\sqrt{2}}, -5 - 3\sqrt{2} \times \frac{1}{\sqrt{2}} \right) = (1, -8)$$

Thus, the points are $(1, -8)$ and $(7, -2)$.

175. (c) The equation $ax^2 + 2hxy + by^2 = 0$ can be rewritten as

$$a + \frac{2hy}{x} + b\left(\frac{y}{x}\right)^2 = 0 \quad \dots(i)$$

Consider, $px + qy = 0$

$$px = -qy \Rightarrow \frac{y}{x} = \frac{-p}{q}$$

Substituting in Eq. (i), we get

$$\begin{aligned}& a + 2h\left(\frac{-p}{q}\right) + b\left(\frac{-p}{q}\right)^2 = 0 \Rightarrow a - \frac{2hp}{q} + b\frac{p^2}{q^2} = 0 \\ & \Rightarrow aq^2 - 2hpq + bp^2 = 0\end{aligned}$$

176. (b) We have, $f(x) = \frac{\log_e(1+ax) - \log_e(1-bx)}{x}$

$$= \lim_{x \rightarrow 0} \frac{\log_e(1+ax) - \log_e(1-bx)}{x} \quad \left[\frac{0}{0} \text{ form} \right]$$

Applying L' Hospital rule,

$$\lim_{x \rightarrow 0} \frac{\frac{a}{1+ax} + \frac{b}{1-bx}}{1} = \frac{a}{1} + \frac{b}{1} = a + b$$

177. (b) We have,

$$\begin{aligned}
 & \lim_{n \rightarrow \infty} \frac{(1^2 + 2^2 + \dots + n^2) \sqrt[3]{n}}{(n+1)(n+10)(n+100)} \\
 &= \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+n)}{6(n+1)(n+10)(n+100)} \left(\lim_{n \rightarrow \infty} \sqrt[3]{n} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{2n^2 + n}{6(n+10)(n+100)} \dots(i) \\
 &= \lim_{n \rightarrow \infty} \frac{2 + \frac{1}{n}}{6 \left(1 + \frac{10}{n} \right) \left(1 + \frac{100}{n} \right)} \\
 &= \frac{2+0}{6(1+0)(1+0)} \\
 &= \frac{2}{6} = \frac{1}{3}
 \end{aligned}$$

178. (c) Consider that there are m collinear points out of total n points in a plane. To construct a triangle we require 3 non-collinear points.

Hence, the number of triangles will be ${}^nC_3 - {}^mC_3$.

Since, all the point in the above question are non-collinear, hence $n = 10$ and $m = 0$

Therefore, the total number of triangles are $= {}^{10}C_3$

$$= \frac{10 \times 9 \times 8}{3!} = \frac{90 \times 8}{6} = 15 \times 8 = 120$$

179. (a) Angle traced by the hour hand in 12 h = 360°

Angle traced by it in 4 h 25 min, i.e.

$$\begin{aligned}
 4\frac{5}{12} \text{ h} &= \frac{53}{12} \text{ h} \\
 &= \frac{360}{12} \times \frac{53}{12} = \frac{265}{2} = 132\frac{1}{2}^\circ
 \end{aligned}$$

Angle traced by it in 25 min

$$= \frac{360^\circ}{60} \times 25 = 150^\circ$$

Now, required angle = $\left(150^\circ - 132\frac{1}{2}^\circ \right) = 17\frac{1}{2}^\circ$

180. (a) $x^2 + y^2 - 8x = 0$

$$\Rightarrow 5^2 + (-7)^2 - 8 \times 5$$

$$\Rightarrow 25 + 49 - 40 = 34$$

$$\Rightarrow 34 > 0$$

Here, $34 > 0$, so the points $(5, -7)$ lies outside the circle.