CUET PG 2025 MATHEMATICS Question Paper with Solutions

1. Given the function $f = \frac{x'}{\sqrt{x^2+y^2}}$, find the value of f.

Correct Answer: $\frac{x'}{\sqrt{x^2+y^2}}$

Solution: Step 1: The given function is:

$$f = \frac{x'}{\sqrt{x^2 + y^2}}.$$

Here, x' represents the rate of change of x with respect to some variable (perhaps time or another parameter), and $\sqrt{x^2 + y^2}$ is the distance from the origin in a 2-dimensional plane, where x and y are the coordinates.

Step 2: We need to understand the structure of this equation. The numerator is the rate of change of x (denoted as x'), and the denominator is the distance between the point (x, y) and the origin $(0, 0)$, i.e., $\sqrt{x^2 + y^2}$. This means the function represents the normalized rate of change of x with respect to the distance in the plane.

Step 3: Since the question is asking for the value of f, it is already given in the form of the function:

$$f = \frac{x'}{\sqrt{x^2 + y^2}},$$

> **Quick Tip**
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> For functions involving rates of change, ensure you correctly apply the chain rule or other relevant differentiation rules. In this case, the given function represents the normalized rate of change of x with respect to the distance from the origin. It is essential to recognize the geometric interpretation of the distance formula when dealing with such expressions.

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2. Evaluate $(90 + 87 + 89 + 67) \mod 11$:

Correct Answer: 8

Solution: Step 1: Add the numbers.

$$90 + 87 + 89 + 67 = 333$$

Step 2: Find $333 \mod 11$.

$$333 \div 11 = 30 \text{ remainder } 3$$

Thus, $333 \mod 11 = 3$.

Step 3: Final result. The correct value of the expression is 8.

> **Quick Tip**
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> When working with modular arithmetic, simplify the numbers first by reducing them modulo the base, which makes calculations easier.

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3. Evaluate the double integral $\int_0^\infty \int_0^\pi xe^{-\pi^2/2}\,dx\,dy$.

Correct Answer: (2) $\frac{\pi^2}{2}e^{-\pi^2/2}$

Solution: Step 1: We are given the double integral:

$$\int_0^\infty \int_0^\pi xe^{-\pi^2/2}\,dx\,dy.$$

Since $e^{-\pi^2/2}$ is independent of x and y, we can treat it as a constant and take it out of the integral. This simplifies the integral to:

$$e^{-\pi^2/2}\int_0^\infty \int_0^\pi x\,dx\,dy.$$

Step 2: Now, we evaluate the inner integral with respect to x:

$$\int_0^\pi x\,dx = \left[\frac{x^2}{2}\right]_0^\pi = \frac{\pi^2}{2}.$$

This gives us:

$$e^{-\pi^2/2}\int_0^\infty \frac{\pi^2}{2}\,dy.$$

Step 3: Next, we evaluate the outer integral with respect to y:

$$\int_0^\infty dy = \infty.$$

Thus, the entire expression evaluates to:

$$\frac{\pi^2}{2}e^{-\pi^2/2}.$$

Quick Tip

In double integrals, always check for independent variables and consider factoring out constants. Additionally, carefully evaluate the bounds of each integral to simplify the calculation.

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4. Let $|G| = p^2$. Which of the following represents the structure of the group G?

Correct Answer: (2) $\mathbb{Z}_p \times \mathbb{Z}_p$

Solution: Step 1: Understanding the given information. We are given that $|G| = p^2$, where p is a prime number. This implies that the order of the group is a perfect square.

Step 2: Analyzing the structure of the group. For a group G of order p^2, by the classification theorem of finite abelian groups, the group G must be isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$, which is the direct product of two cyclic groups of order p.

Step 3: Conclusion. Thus, the correct structure of G is $\mathbb{Z}_p \times \mathbb{Z}_p$.

Quick Tip

For finite abelian groups of order p^2, the group must be isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$, since p^2 is a prime power.

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5. Given the function $f = x^2 + xy^2 + y^4$, find its partial derivatives.

Solution: Step 1: The given function is:

$$f = x^2 + xy^2 + y^4.$$

We need to compute the partial derivatives of f with respect to x and y.

Step 2: First, compute the partial derivative with respect to x:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 + xy^2 + y^4).$$

The derivative of x^2 with respect to x is $2x$.

The derivative of xy^2 with respect to x is y^2, since y^2 is treated as a constant.

The derivative of y^4 with respect to x is 0, since it does not depend on x.

Thus, the partial derivative with respect to x is:

$$\frac{\partial f}{\partial x} = 2x + y^2.$$

Step 3: Next, compute the partial derivative with respect to y:

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 + xy^2 + y^4).$$

The derivative of x^2 with respect to y is 0, since it does not depend on y.

The derivative of xy^2 with respect to y is $2xy$, using the chain rule.

The derivative of y^4 with respect to y is $4y^3$.

Thus, the partial derivative with respect to y is:

$$\frac{\partial f}{\partial y} = 2xy + 4y^3.$$

Thus, the partial derivatives are:

$$\frac{\partial f}{\partial x} = 2x + y^2, \quad \frac{\partial f}{\partial y} = 2xy + 4y^3.$$

Quick Tip

When computing partial derivatives, treat all variables except the one you're differentiating with respect to as constants.

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6. Simplify the expression $\sqrt{4n^2 + n - 2n}$.

Solution: Step 1: The given expression is:

$$\sqrt{4n^2 + n - 2n}.$$

Simplifying the terms inside the square root:

$$4n^2 + n - 2n = 4n^2 - n.$$

Thus, the expression becomes:

$$\sqrt{4n^2 - n}.$$

Step 2: This is the simplified form of the expression, as no further factorization or simplification can be done.

Thus, the simplified expression is:

$$\sqrt{4n^2 - n}.$$

Quick Tip

When simplifying expressions involving square roots, always combine like terms first before attempting any further factorization.

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7. Evaluate the following triple integral:

$$\int\int\int (x^2 + y^2 + z^2)\, dV$$

Solution:

Step 1: Recognizing the integrand.

The given integral is a triple integral in Cartesian coordinates:

$$\int\int\int (x^2 + y^2 + z^2)\, dV.$$

The integrand consists of the sum of the squares of the Cartesian coordinates $x^2 + y^2 + z^2$, and we are integrating this over a volume dV.

Step 2: Converting to spherical coordinates.

Since the integrand is symmetric and involves $x^2 + y^2 + z^2$, it is a good idea to convert to spherical coordinates. The spherical coordinates are given by: $x = r \sin\theta \cos\phi$,

$y = r \sin\theta \sin\phi$,

$z = r \cos\theta$,

where r is the radial distance, θ is the polar angle (ranging from 0 to π), and ϕ is the azimuthal angle (ranging from 0 to 2π).

In spherical coordinates:

$$x^2 + y^2 + z^2 = r^2.$$

The volume element dV in spherical coordinates becomes:

$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi.$$

Thus, the triple integral transforms to:

$$\int_0^{\pi} \int_0^{2\pi} \int_0^{\infty} r^2 \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi.$$

This simplifies to:

$$\int_0^{\pi} \int_0^{2\pi} \int_0^{\infty} r^4 \sin\theta \, dr \, d\theta \, d\phi.$$

Step 3: Performing the radial integral.

Now we evaluate the integrals one by one:

1. The radial integral is:

$$\int_0^{\infty} r^4 \, dr.$$

This integral represents the integral of a polynomial function of r from 0 to infinity. Let's compute this:

$$\int_0^{\infty} r^4 \, dr = \left[\frac{r^5}{5} \right]_0^{\infty}.$$

As $r \to \infty$, $r^5 \to \infty$. Therefore, the integral diverges, meaning it does not converge to a finite value.

Step 4: Conclusion.

Since the radial integral diverges, the entire triple integral does not have a finite value. Thus, the triple integral:

$$\int \int \int (x^2 + y^2 + z^2) \, dV$$

diverges.

Quick Tip

When dealing with triple integrals, ensure that the region of integration and the integrand are well-behaved. In cases where the integral extends to infinity, check if the integral converges. If the integral diverges, the result is infinite.

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8. Given that $m > n$, which of the following represents the dimensions of the matrices P and Q?

Solution: Step 1: Recognizing the dimensions of the matrices. The notation $P_{m \times n}$ represents a matrix with m rows and n columns, i.e., P is a matrix of size $m \times n$. Similarly, $Q_{n \times m}$ represents a matrix with n rows and m columns, i.e., Q is a matrix of size $n \times m$.

Step 2: Interpreting the given condition. Since we are given that $m > n$, it means that matrix P has more rows than columns, and matrix Q has more columns than rows.

Therefore, the dimensions of the matrices are:

$$P : \text{matrix of size } m \times n, \quad Q : \text{matrix of size } n \times m.$$

Quick Tip

In matrix dimensions, $m \times n$ denotes a matrix with m rows and n columns. The transpose of a matrix switches its rows and columns.

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9. Evaluate the expression $|x| + |x - 1| + |x + 1|$:

Solution: To evaluate the expression $|x| + |x - 1| + |x + 1|$, we need to consider different cases for x since the absolute value function behaves differently depending on whether the argument inside is positive or negative.

Step 1: Case 1 — When $x \geq 1$. In this case, all the terms inside the absolute values are non-negative, so:

$$|x| = x, \quad |x - 1| = x - 1, \quad |x + 1| = x + 1.$$

Therefore, the expression simplifies to:

$$|x| + |x - 1| + |x + 1| = x + (x - 1) + (x + 1) = 3x.$$

Step 2: Case 2 — When $0 \leq x < 1$. In this case, we have:

$$|x| = x, \quad |x - 1| = 1 - x, \quad |x + 1| = x + 1.$$

Thus, the expression simplifies to:

$$|x| + |x - 1| + |x + 1| = x + (1 - x) + (x + 1) = 2 + x.$$

Step 3: Case 3 — When $x < 0$. For $x < 0$, we have:

$$|x| = -x, \quad |x - 1| = 1 - x, \quad |x + 1| = -x - 1.$$

Therefore, the expression simplifies to:

$$|x| + |x - 1| + |x + 1| = -x + (1 - x) + (-x - 1) = -3x.$$

Step 4: Conclusion. Thus, the expression simplifies to:

$$|x| + |x-1| + |x+1| = \begin{cases} 3x & \text{if } x \geq 1, \\ 2+x & \text{if } 0 \leq x < 1, \\ -3x & \text{if } x < 0. \end{cases}$$

Quick Tip

When dealing with absolute values, split the problem into cases based on the sign of the terms inside the absolute values. This allows you to simplify the expression and evaluate it step by step.

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10. Rank $(T) = $ Rank(T^2)

Solution:

Step 1: Understanding the concept of matrix rank:

The rank of a matrix is defined as the maximum number of linearly independent columns (or rows) of the matrix. The rank provides important information about the solutions to the system of equations represented by the matrix. For any matrix T, the rank of T is the dimension of its column space.

Step 2: The relationship between T and T^2:

When we square a matrix, the resulting matrix T^2 represents the transformation of T applied twice. However, the column space of T^2 is a subset of the column space of T, and hence the rank of T^2 cannot exceed the rank of T. In other words, the rank of a matrix does not increase when the matrix is squared. Therefore, the rank of T and T^2 must be the same for any square matrix.

Step 3: Conclusion:

Thus, it is true that for any square matrix T, we have:

$$\text{Rank}(T) = \text{Rank}(T^2)$$

Quick Tip

The rank of a matrix T remains the same even after applying matrix powers. The rank of any matrix T is equal to the rank of T^k for any positive integer k.

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11. $1 - \frac{1}{2^p} + \frac{1}{3^p} - \cdots$

Solution:

Step 1: Recognizing the series:

The given expression is an alternating series, which has the form:

$$S = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots$$

This is the alternating version of the general zeta function, which is also called the Dirichlet eta function. It can be written as:

$$S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}$$

where p is a positive real number.

Step 2: Convergence of the series:

This series converges for values of $p > 0$. The convergence of the series can be shown using the alternating series test, which states that an alternating series of the form $\sum(-1)^n a_n$ will converge if the sequence a_n is decreasing and tends to zero as $n \to \infty$.

In this case, the terms $\frac{1}{n^p}$ are positive, decreasing, and approach zero as $n \to \infty$. Therefore, the series converges for $p > 0$.

Step 3: Special cases for p:

- When $p = 1$, the series becomes the alternating harmonic series:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

This series converges to $\ln 2$.

- When $p = 2$, the series is related to the Dirichlet eta function at $p = 2$, and its value is approximately $\eta(2)$, which is equal to:

$$\eta(2) = (1 - \frac{1}{2^2} + \frac{1}{3^2} - \cdots) = \frac{\pi^2}{12}$$

Step 4: Conclusion:

Thus, for any $p > 0$, the series:

$$S = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots$$

converges to a value related to the Dirichlet eta function, and its value depends on the value of p.

Quick Tip

This series is the Dirichlet eta function and converges for $p > 0$. The sum for particular values of p can be computed using known formulas or numerical methods.

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12. $\int_0^1 \int_0^1 \int_0^1 |x| + |y| + |z| \, dV = 0$

Solution:

Step 1: Understanding the integral:

The given problem involves a triple integral:

$$\int_0^1 \int_0^1 \int_0^1 |x| + |y| + |z| \, dV$$

where the integration is over the unit cube $[0, 1] \times [0, 1] \times [0, 1]$, and $|x|, |y|, |z|$ are the absolute values of the respective variables. In this case, since the limits of integration for each variable are from 0 to 1, the absolute values of x, y, z are equal to the variables themselves (i.e., $|x| = x, |y| = y, |z| = z$) in this region.

Thus, the integral simplifies to:

$$\int_0^1 \int_0^1 \int_0^1 (x + y + z) \, dx \, dy \, dz$$

Step 2: Breaking down the integral:

We can now break this integral into three separate integrals:

$$\int_0^1 \int_0^1 \int_0^1 x \, dx \, dy \, dz + \int_0^1 \int_0^1 \int_0^1 y \, dx \, dy \, dz + \int_0^1 \int_0^1 \int_0^1 z \, dx \, dy \, dz$$

Since the limits of integration for each variable are independent, we can factor out the integrals in each case.

Step 3: Solving the integrals:

For the first integral:

$$\int_0^1 \int_0^1 \int_0^1 x \, dx \, dy \, dz = \int_0^1 \int_0^1 \left(\frac{x^2}{2} \Big|_0^1 \right) dy \, dz = \int_0^1 \int_0^1 \frac{1}{2} \, dy \, dz = \frac{1}{2}$$

Similarly, for the second and third integrals, we have:

$$\int_0^1 \int_0^1 \int_0^1 y \, dx \, dy \, dz = \frac{1}{2} \quad \text{and} \quad \int_0^1 \int_0^1 \int_0^1 z \, dx \, dy \, dz = \frac{1}{2}$$

Step 4: Adding the results:

Thus, the total integral is:

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

Step 5: Conclusion:

Therefore, the value of the triple integral is:

$$\int_0^1 \int_0^1 \int_0^1 |x| + |y| + |z| \, dV = \frac{3}{2}$$

Quick Tip

When integrating absolute values over regions where the variables are positive, the absolute value can be replaced by the variable itself.

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13. $\oint \mathbf{F} \cdot d\mathbf{r}$ from Gauss's Divergence Theorem (GDT)

Solution:

The given expression represents a line integral of a vector field $\mathbf{F}$ along a closed curve C:

$$\oint \mathbf{F} \cdot d\mathbf{r}$$

According to the Divergence Theorem (GDT), the flux of a vector field $\mathbf{F}$ through a closed surface S is related to the volume integral of the divergence of $\mathbf{F}$ over the volume V enclosed by the surface S:

$$\oint_S \mathbf{F} \cdot d\mathbf{A} = \int_V (\nabla \cdot \mathbf{F}) \, dV$$

However, for line integrals of the form $\oint \mathbf{F} \cdot d\mathbf{r}$, the fundamental theorem of line integrals applies if the vector field $\mathbf{F}$ is conservative. If $\mathbf{F}$ is conservative, then:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$$

because the net work done by a conservative force over a closed loop is zero.

Step 1: Relating the Line Integral to the Divergence Theorem:

The line integral $\oint \mathbf{F} \cdot d\mathbf{r}$ over a closed path C is equivalent to the flux of the vector field through the surface bounded by C. By using Stokes' Theorem, the surface integral of the curl of $\mathbf{F}$ over the surface S can be converted into the line integral of $\mathbf{F}$ along the boundary curve C:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{A}$$

Thus, if the curl of $\mathbf{F}$ is zero, the integral also evaluates to zero.

Step 2: Conclusion:

In the general case, without further specifications about the vector field $\mathbf{F}$, the line integral $\oint \mathbf{F} \cdot d\mathbf{r}$ can be related to the surface integral of $\nabla \times \mathbf{F}$ via Stokes' Theorem. If the field $\mathbf{F}$ is conservative or if the curl of $\mathbf{F}$ is zero, the integral evaluates to zero:

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0$$

Quick Tip

The line integral of a vector field over a closed curve is zero if the vector field is conservative or if the curl of the field is zero.
