

CUET PG 2024 B.Ed. Mathematics Question Paper with Solutions

Time Allowed :1 Hour 45 Mins	Maximum Marks :300	Total Questions :75
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The examination duration is 105 minutes. Manage your time effectively to attempt all questions within this period.
2. The total marks for this examination are 300. Aim to maximize your score by strategically answering each question.
3. There are 75 mandatory questions to be attempted in the Agro forestry paper. Ensure that all questions are answered.
4. Questions may appear in a shuffled order. Do not assume a fixed sequence and focus on each question as you proceed.
5. The marking of answers will be displayed as you answer. Use this feature to monitor your performance and adjust your strategy as needed.
6. You may mark questions for review and edit your answers later. Make sure to allocate time for reviewing marked questions before final submission.
7. Be aware of the detailed section and sub-section guidelines provided in the exam. Understanding these will aid in effectively navigating the exam.

Question 1: Which of the following is a singleton set?

1. $\{x : |x| = 5, x \in N\}$
2. $\{x : |x| = 7, x \in Z\}$
3. $\{x : x^2 + 5x + 1 = 0, x \in N\}$
4. $\{x : x^2 = 5, x \in N\}$

Correct Answer: 1. $\{x : |x| = 5, x \in N\}$

Solution:

1. Option 1: For the set $\{x : |x| = 5, x \in N\}$, the absolute value condition $|x| = 5$ is satisfied by $x = 5$, which belongs to N . Hence, the set contains exactly one element: $\{5\}$. This is a singleton set.

2. Option 2: For the set $\{x : |x| = 7, x \in Z\}$, the absolute value condition $|x| = 7$ is satisfied by $x = 7$ and $x = -7$. Thus, the set contains two elements: $\{7, -7\}$. This is not a singleton set.

3. Option 3: For the set $\{x : x^2 + 5x + 1 = 0, x \in N\}$, solving the quadratic equation:

$$x = \frac{-5 \pm \sqrt{25 - 4 \cdot 1 \cdot 1}}{2} = \frac{-5 \pm \sqrt{21}}{2}.$$

The roots are irrational and not natural numbers. Thus, the set is empty. It is not a singleton set.

4. Option 4: For the set $\{x : x^2 = 5, x \in N\}$, solving $x^2 = 5$:

$$x = \pm\sqrt{5}.$$

The roots are not natural numbers. Hence, the set is empty and not a singleton set.

Conclusion:

The only singleton set is $\{x : |x| = 5, x \in N\}$, which contains exactly one element: $\{5\}$.

 Quick Tip

A singleton set contains exactly one element. Carefully analyze the given conditions to determine whether they lead to exactly one solution.

Question 2: Let R be a relation defined by $R = \{(a, b) : a \geq b, a, b \in R\}$. Then R is:

1. an equivalence relation
2. reflexive, transitive but not symmetric
3. symmetric, transitive but not reflexive
4. only reflexive

Correct Answer: 2. Reflexive, transitive but not symmetric

Solution:

The relation R is defined as $R = \{(a, b) : a \geq b, a, b \in R\}$. We check the properties of the relation step by step:

- 1. Reflexive:** For any $a \in R$, we have $a \geq a$. Therefore, $(a, a) \in R$. Hence, R is reflexive.
- 2. Symmetric:** If $(a, b) \in R$, then $a \geq b$. However, $b \geq a$ is not guaranteed unless $a = b$. Therefore, R is not symmetric.
- 3. Transitive:** If $(a, b) \in R$ and $(b, c) \in R$, then $a \geq b$ and $b \geq c$. From these, it follows that $a \geq c$. Hence, R is transitive.
- 4. Equivalence Relation:** An equivalence relation requires the properties of reflexivity, symmetry, and transitivity. Since R is not symmetric, it is not an equivalence relation.

Conclusion: The relation R is reflexive, transitive but not symmetric. Thus, The correct answer is Option 2.

 Quick Tip

A relation is reflexive if $a \geq a$, symmetric if $a \geq b$ implies $b \geq a$, and transitive if $a \geq b$ and $b \geq c$ imply $a \geq c$. Check each property systematically.

Question 3: The function $f(x) = \cos x$, $x \in R$ is:

1. an even function
2. an odd function
3. neither even nor odd function
4. a power function

Correct Answer: 1. An even function

Solution:

To determine whether the function $f(x) = \cos x$ is even, odd, or neither, we check the properties of even and odd functions:

1. Even Function: A function $f(x)$ is even if $f(-x) = f(x)$ for all $x \in R$. For $f(x) = \cos x$:

$$f(-x) = \cos(-x).$$

Using the property of the cosine function, $\cos(-x) = \cos x$. Hence, $f(-x) = f(x)$, which shows that $f(x)$ is an even function.

2. Odd Function: A function $f(x)$ is odd if $f(-x) = -f(x)$ for all $x \in R$. For $f(x) = \cos x$:

$$f(-x) = \cos(-x) \neq -\cos x.$$

Thus, $f(x)$ is not an odd function.

3. Neither Even Nor Odd: Since $f(x)$ satisfies the condition for an even function, it cannot be classified as neither even nor odd.

4. Power Function: A power function has the form $f(x) = x^n$, where n is a real number. The function $f(x) = \cos x$ does not fit this form. Hence, it is not a power function.

Conclusion: The function $f(x) = \cos x$ is an even function.

The correct answer is Option 1.

 Quick Tip

For trigonometric functions, check their symmetry:

- $\cos(-x) = \cos x$, so $\cos x$ is even.
- $\sin(-x) = -\sin x$, so $\sin x$ is odd.

Question 4: The number $\frac{(1+i)^n}{(1-i)^{n-2}}$ is equal to:

1. $2i^{n+3}$
2. $2i^{n-3}$
3. $4i^{n+2}$
4. $4i^{n+4}$

Correct Answer: 1. $2i^{n+3}$

Solution:

We are given the expression:

$$\frac{(1+i)^n}{(1-i)^{n-2}}.$$

Step 1: Simplify $1+i$ and $1-i$ in polar form. The modulus of $1+i$ is:

$$|1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

The argument of $1+i$ is:

$$\arg(1+i) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}.$$

Thus,

$$1+i = \sqrt{2}e^{i\pi/4}.$$

Similarly, for $1-i$:

$$|1-i| = \sqrt{2}, \quad \arg(1-i) = -\frac{\pi}{4}.$$

Thus,

$$1-i = \sqrt{2}e^{-i\pi/4}.$$

Step 2: Substitute into the expression. Substitute the polar forms into $\frac{(1+i)^n}{(1-i)^{n-2}}$:

$$\frac{(1+i)^n}{(1-i)^{n-2}} = \frac{(\sqrt{2}e^{i\pi/4})^n}{(\sqrt{2}e^{-i\pi/4})^{n-2}}.$$

Step 3: Simplify the modulus. The modulus is:

$$\frac{(\sqrt{2})^n}{(\sqrt{2})^{n-2}} = \frac{2^{n/2}}{2^{(n-2)/2}} = 2.$$

Step 4: Simplify the argument. The argument is:

$$\frac{e^{in\pi/4}}{e^{-i(n-2)\pi/4}} = e^{in\pi/4} \cdot e^{i(n-2)\pi/4} = e^{i(n+n-2)\pi/4} = e^{i(2n-2)\pi/4}.$$

Step 5: Combine modulus and argument. Thus:

$$\frac{(1+i)^n}{(1-i)^{n-2}} = 2 \cdot e^{i(2n-2)\pi/4}.$$

Step 6: Express in terms of i . Since $e^{i\pi/2} = i$, we have:

$$e^{i(2n-2)\pi/4} = i^{n+3}.$$

Therefore:

$$\frac{(1+i)^n}{(1-i)^{n-2}} = 2i^{n+3}.$$

—
Conclusion: The given expression simplifies to $2i^{n+3}$.
The correct answer is Option 1.

 Quick Tip

To simplify expressions involving complex numbers, always use polar form and properties of exponents for clear calculations.

Question 5: Solution of $|x^2 - 10| \leq 6$ is:

1. $(2, 4)$
2. $(-4, -2)$
3. $(-4, -2) \cup (2, 4)$
4. $[-4, -2] \cup [2, 4]$

Correct Answer: 4. $[-4, -2] \cup [2, 4]$

Solution:

The inequality is given as:

$$|x^2 - 10| \leq 6.$$

Step 1: Break the absolute value inequality. The condition $|x^2 - 10| \leq 6$ can be rewritten as:

$$-6 \leq x^2 - 10 \leq 6.$$

Step 2: Solve for x^2 . Adding 10 to all parts of the inequality:

$$4 \leq x^2 \leq 16.$$

Step 3: Solve for x . Taking the square root on both sides:

$$-4 \leq x \leq -2 \quad \text{or} \quad 2 \leq x \leq 4.$$

Step 4: Write the solution in interval notation. The solution is:

$$x \in [-4, -2] \cup [2, 4].$$

—
Conclusion: The solution of the inequality $|x^2 - 10| \leq 6$ is $[-4, -2] \cup [2, 4]$.
The correct answer is Option 4.

 Quick Tip

When solving absolute value inequalities, split the inequality into two conditions and solve systematically. Don't forget to combine solutions appropriately.

Question 6: If A is an orthogonal matrix, then:

1. $|A| = 0$
2. $|A| = \pm 1$
3. $|A| = \pm 2$
4. $|A| = \pm 3$

Correct Answer: 2. $|A| = \pm 1$

Solution:

An orthogonal matrix A satisfies the condition:

$$A^T A = I,$$

where A^T is the transpose of A , and I is the identity matrix.

Step 1: Determinant property of orthogonal matrices. Taking the determinant on both sides of $A^T A = I$:

$$\det(A^T A) = \det(I).$$

Using the property of determinants, $\det(A^T A) = \det(A^T) \cdot \det(A)$, and since $\det(A^T) = \det(A)$, we have:

$$\det(A)^2 = 1.$$

Step 2: Solve for $|A|$. From $\det(A)^2 = 1$, it follows that:

$$\det(A) = \pm 1.$$

Step 3: Interpret the result. The determinant of an orthogonal matrix can only be $+1$ or -1 . It cannot take any other value.

Conclusion: If A is an orthogonal matrix, $|A| = \pm 1$.

The correct answer is Option 2.

💡 Quick Tip

Orthogonal matrices preserve lengths and angles. Their determinant is always ± 1 , ensuring they represent either a rotation ($+1$) or a reflection (-1).

Question 7: If α, β are roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is:

1. $a^2x^2 + 5acx + 6c^2 + bc = 0$
2. $a^2x^2 + 5bcx + 6b^2 + bc = 0$
3. $a^2x^2 + 6abx + 5b^2 + ac = 0$
4. $a^2x^2 + 5abx + 6b^2 + ac = 0$

Correct Answer: 4. $a^2x^2 + 5abx + 6b^2 + ac = 0$

Solution:

We are tasked with finding the quadratic equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

Step 1: Sum of new roots. The sum of the roots S is:

$$S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5\alpha + 5\beta = 5(\alpha + \beta).$$

For the given quadratic equation $ax^2 + bx + c = 0$, the sum of the roots $(\alpha + \beta)$ is given by:

$$\alpha + \beta = -\frac{b}{a}.$$

Thus:

$$S = 5(\alpha + \beta) = 5\left(-\frac{b}{a}\right) = -\frac{5b}{a}.$$

Step 2: Product of new roots. The product of the roots P is:

$$P = (2\alpha + 3\beta)(3\alpha + 2\beta).$$

Expanding the product:

$$P = 6\alpha^2 + 9\alpha\beta + 6\alpha\beta + 6\beta^2 = 6\alpha^2 + 15\alpha\beta + 6\beta^2.$$

Substitute $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$:

$$P = 6((\alpha + \beta)^2 - 2\alpha\beta) + 15\alpha\beta.$$

Using $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$, we get:

$$P = 6\left(\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}\right) + 15\frac{c}{a}.$$

Simplify:

$$P = 6\left(\frac{b^2}{a^2} - \frac{2c}{a}\right) + \frac{15c}{a}.$$

$$P = \frac{6b^2}{a^2} - \frac{12c}{a} + \frac{15c}{a} = \frac{6b^2}{a^2} + \frac{3c}{a}.$$

Step 3: Quadratic equation. The quadratic equation with roots $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is given by:

$$a^2x^2 - S \cdot ax + P = 0.$$

Substitute $S = -\frac{5b}{a}$ and $P = \frac{6b^2}{a^2} + \frac{3c}{a}$:

$$a^2x^2 + 5abx + (6b^2 + ac) = 0.$$

Conclusion: The required quadratic equation is:

$$a^2x^2 + 5abx + 6b^2 + ac = 0.$$

The correct answer is Option 4.

 Quick Tip

To find the new quadratic equation from transformed roots, use the sum and product of roots formula. Carefully substitute known values from the original quadratic equation.

Question 8: 12 persons are to be arranged on a round table. If two particular persons among them are not to sit side by side, the number of arrangements is:

1. $9 \cdot (10!)$
2. $2 \cdot (10!)$
3. $8 \cdot (45!)$
4. $10!$

Correct Answer: 1. $9 \cdot (10!)$

Solution:

Step 1: Total arrangements without restrictions. For n people seated around a round table, the total number of arrangements is $(n - 1)!$. For 12 people:

$$\text{Total arrangements} = (12 - 1)! = 11!.$$

Step 2: Treat two particular persons as a single unit. If the two particular persons must sit side by side, treat them as one single unit. This reduces the total number of units to 11, and the total arrangements become:

$$(11 - 1)! \cdot 2! = 10! \cdot 2.$$

Step 3: Exclude restricted arrangements. The number of arrangements where the two particular persons are not side by side is the total arrangements minus the restricted arrangements:

$$\text{Valid arrangements} = 11! - (10! \cdot 2).$$

Simplify:

$$\text{Valid arrangements} = 10! \cdot (11 - 2) = 10! \cdot 9.$$

Conclusion: The number of valid arrangements where the two particular persons do not sit side by side is $9 \cdot (10!)$.

The correct answer is Option 1.

 Quick Tip

For circular arrangements, use $(n - 1)!$ for total arrangements and treat specific restrictions using logical groupings or subtractions.

Question 9: The coefficient of x^2 in the expansion of $(1 + 4x + x^2)^{1/2}$ is:

1. -3
2. -2
3. $-\frac{3}{2}$
4. 2

Correct Answer: 3. $-\frac{3}{2}$

Solution:

We are tasked with finding the coefficient of x^2 in the expansion of $(1 + 4x + x^2)^{1/2}$.

Step 1: Let $f(x) = (1 + 4x + x^2)^{1/2}$.

We can expand $f(x)$ using the binomial theorem for $(1 + u)^n$, where $n = \frac{1}{2}$ and $u = 4x + x^2$.

The general term is:

$$T_k = \binom{\frac{1}{2}}{k} (4x + x^2)^k.$$

Step 2: Terms contributing to x^2 .

To find the coefficient of x^2 , we consider the terms of $u^k = (4x + x^2)^k$ that produce x^2 when expanded.

1. For $k = 1$:

$$T_1 = \binom{\frac{1}{2}}{1} (4x + x^2) = \frac{1}{2} (4x + x^2).$$

The x^2 -term here is $\frac{1}{2} \cdot x^2$, with coefficient $\frac{1}{2}$.

2. For $k = 2$:

$$T_2 = \binom{\frac{1}{2}}{2} (4x + x^2)^2.$$

Expanding $(4x + x^2)^2$:

$$(4x + x^2)^2 = 16x^2 + 8x^3 + x^4.$$

The x^2 -term here is $16x^2$.

Now calculate the binomial coefficient:

$$\binom{\frac{1}{2}}{2} = \frac{\frac{1}{2}(\frac{1}{2} - 1)}{2!} = \frac{\frac{1}{2}(-\frac{1}{2})}{2} = -\frac{1}{8}.$$

So, the contribution to x^2 is:

$$T_2 = -\frac{1}{8} \cdot 16x^2 = -2x^2.$$

Step 3: Combine contributions. The total coefficient of x^2 is:

$$\frac{1}{2} - 2 = -\frac{3}{2}.$$

Conclusion: The coefficient of x^2 in the expansion of $(1 + 4x + x^2)^{1/2}$ is $-\frac{3}{2}$.

The correct answer is Option 3.

 Quick Tip

For binomial expansions involving polynomials, carefully expand and select terms that contribute to the required power. Use binomial coefficients to simplify calculations.

Question 10: If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the Arithmetic Mean between a and b , then n is equal to:

1. 0
2. -1
3. 1
4. 2

Correct Answer: 1. 0

Solution:

The Arithmetic Mean (AM) between two numbers a and b is given by:

$$AM = \frac{a+b}{2}.$$

We are given:

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}.$$

Step 1: Cross multiply to simplify the equation. Multiply both sides by $2(a^n+b^n)$:

$$2(a^{n+1}+b^{n+1}) = (a+b)(a^n+b^n).$$

Expand the right-hand side:

$$2a^{n+1}+2b^{n+1} = a^{n+1}+ab^n+b^na^n+b^{n+1}.$$

Step 2: Simplify the terms. Rearranging terms:

$$a^{n+1}+b^{n+1} = ab^n+a^nb.$$

Step 3: Analyze for n . Divide through by a^n (assuming $a \neq 0$):

$$a+b = ab^n+b.$$

Cancel common terms:

$$a = ab^n.$$

Divide through by a (assuming $a \neq 0$):

$$1 = b^n.$$

Step 4: Solve for n . Since $b^n = 1$, and this holds true for all $b > 0$ when $n = 0$, it follows that:

$$n = 0.$$

—
Conclusion: The value of n is 0.
The correct answer is Option 1.

 Quick Tip

For problems involving powers and averages, simplify carefully and use logical reasoning to isolate the variable n .

Question 11: Domain of the function $f(x) = \sqrt{\log_3(\sin x)}$ is:

1. $2n\pi + \frac{\pi}{2}, n \in N$
2. $n\pi + \frac{\pi}{2}, n \in N$
3. $2n\pi - \frac{\pi}{2}, n \in N$
4. $n\pi - \frac{\pi}{2}, n \in N$

Correct Answer: 1. $2n\pi + \frac{\pi}{2}, n \in N$

Solution:

The function $f(x) = \sqrt{\log_3(\sin x)}$ is defined when:

$$\log_3(\sin x) \geq 0 \quad \text{and} \quad \sin x > 0.$$

Step 1: Condition for $\log_3(\sin x) \geq 0$. For $\log_3(\sin x) \geq 0$, we require:

$$\sin x \geq 1.$$

The equality $\sin x = 1$ occurs at:

$$x = \frac{\pi}{2} + 2n\pi, \quad n \in Z.$$

Step 2: Combine conditions. The values of x where $\sin x > 0$ and $\sin x = 1$ align with:

$$x = 2n\pi + \frac{\pi}{2}, \quad n \in N.$$

Step 3: Verify the domain. The domain of $f(x)$ is the set of all x satisfying:

$$x = 2n\pi + \frac{\pi}{2}, \quad n \in N.$$

Conclusion: The domain of the function $f(x) = \sqrt{\log_3(\sin x)}$ is $x = 2n\pi + \frac{\pi}{2}, n \in N$.
The correct answer is Option 1.

 Quick Tip

When solving for domains involving square roots and logarithmic functions, ensure the argument of the logarithm is positive and satisfies all given conditions.

Question 12: Evaluate $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1}$:

1. 1
2. $\frac{1}{2}$
3. $\frac{1}{\sqrt{2}}$
4. $\frac{1}{2\sqrt{2}}$

Correct Answer: 2. $\frac{1}{2}$

Solution:

We need to evaluate:

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1}.$$

Step 1: Simplify the numerator and denominator. The numerator is $\sqrt{2} \cos x - 1$. Substituting $\cos x = \frac{1}{\sqrt{2}}$ when $x = \frac{\pi}{4}$, we have:

$$\sqrt{2} \cos x - 1 = \sqrt{2} \left(\frac{1}{\sqrt{2}} - \frac{x - \frac{\pi}{4}}{\sqrt{2}} \right) - 1.$$

This simplifies to:

$$\sqrt{2} \cos x - 1 = -\frac{x - \frac{\pi}{4}}{\sqrt{2}}.$$

The denominator is $\cot x - 1 = \frac{\cos x}{\sin x} - 1$. Near $x = \frac{\pi}{4}$, we expand:

$$\cot x = 1 - 2\left(x - \frac{\pi}{4}\right).$$

Thus:

$$\cot x - 1 = -2\left(x - \frac{\pi}{4}\right).$$

Step 2: Evaluate the limit. Substitute these simplified expressions into the limit:

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-\frac{x - \frac{\pi}{4}}{\sqrt{2}}}{-2\left(x - \frac{\pi}{4}\right)}.$$

Simplify:

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1} = \frac{\frac{1}{\sqrt{2}}}{2} = \frac{1}{2}.$$

Conclusion: The value of the given limit is $\frac{1}{2}$.

The correct answer is Option 2.

💡 Quick Tip

For evaluating trigonometric limits, simplify using Taylor expansions or small-angle approximations to isolate and cancel terms effectively.

Question 13: Which of the following are true?

- (A) $\frac{d}{dx}(\sin x) = \cos x, \forall x \in R$
 (B) $\frac{d}{dx}(\cos x) = -\sin x, \forall x \in R$
 (C) $\frac{d}{dx}(\tan x) = \sec^2 x, x \in R - \{(2n+1)\pi/2, n \in Z\}$
 (D) $\frac{d}{dx}(\sec x) = \sec x \tan x, x \in R - \{(2n+1)\pi/2, n \in Z\}$

Choose the correct answer from the options given below:

1. (A), (B), (C) only
2. (A), (B), (C), (D)
3. (A), (D), (C) only
4. (C), (B), (D) only

Correct Answer: 2. (A), (B), (C), (D)

Solution:

Step 1: Verify (A) The derivative of $\sin x$ is $\cos x$. This is true for all $x \in R$. Hence, (A) is correct.

Step 2: Verify (B) The derivative of $\cos x$ is $-\sin x$. This is true for all $x \in R$. Hence, (B) is correct.

Step 3: Verify (C) The derivative of $\tan x$ is $\sec^2 x$. However, $\tan x$ is undefined at $x = (2n+1)\pi/2, n \in Z$. Therefore, the domain is $x \in R - \{(2n+1)\pi/2\}$. Hence, (C) is correct.

Step 4: Verify (D) The derivative of $\sec x$ is $\sec x \tan x$. Since $\sec x$ is undefined at $x = (2n+1)\pi/2, n \in Z$, the domain is $x \in R - \{(2n+1)\pi/2\}$. Hence, (D) is correct.

Conclusion: All statements (A), (B), (C), and (D) are correct.

The correct answer is Option 2.

Quick Tip

When dealing with derivatives of trigonometric functions, carefully check for points of discontinuity to determine the domain.

Question 14: Which of the following are true?

- (A) $\int \csc x \cot x \, dx = -\csc x + C$
 (B) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log |x + \sqrt{x^2-a^2}| + C$
 (C) $\int \tan x \, dx = \begin{cases} \log |\sec x| + C, & \text{if } x \neq \frac{\pi}{2} + n\pi \text{ for } n \in Z, \\ -\log |\cos x| + C & \text{(equivalent form)} \end{cases}$
 (D) $\int \frac{dx}{\sqrt{a^2-x^2}} = \begin{cases} \sin^{-1} \frac{x}{a} + C, \\ -\cos^{-1} \frac{x}{a} + C \end{cases}$

Choose the correct answer from the options given below:

1. (A), (C), (D) only
2. (A), (B), (C) only
3. (B), (A), (D) only
4. (C), (B), (D) only

Correct Answer: 1. (A), (C), (D) only

Solution:

Step 1: Verify (A) The integration of $\csc x \cot x$ is:

$$\int \csc x \cot x \, dx = -\csc x + C.$$

This matches the given expression. Hence, (A) is correct.

Step 2: Verify (B) The integration of $\frac{1}{\sqrt{x^2-a^2}}$ is:

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \sinh^{-1}(x) + C,$$

which differs from the given expression. Hence, (B) is incorrect.

Step 3: Verify (C) The integration of $\tan x$ is typically given by:

$$\int \tan x \, dx = \log |\sec x| + C.$$

The equivalent form can be expressed as $-\log |\cos x| + C$, so (C) is correct.

Step 4: Verify (D) The integration of $\frac{1}{\sqrt{a^2-x^2}}$ is:

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C,$$

or equivalently:

$$-\cos^{-1} \frac{x}{a} + C.$$

This matches the given expression. Hence, (D) is correct.

Conclusion: The correct statements are (A), (C), and (D).

The correct answer is Option 1.

Quick Tip

When solving integrals involving trigonometric and inverse trigonometric functions, always verify the correctness of equivalent forms and their domains.

Question 15: Which of the following are true?

- (A) The equation of a circle with centre (h, k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.
 (B) The equation of a parabola with focus at $(a, 0)$, $a > 0$ and directrix $x = -a$ is $y^2 = 4ax$.
 (C) The equation of an ellipse with foci on the x-axis is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
 (D) The length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

Choose the correct answer from the options given below:

1. (A), (B), (C) only
2. (A), (B), (D) only
3. (A), (D), (C) only
4. (C), (B), (D), (A)

Correct Answer: 2. (A), (B), (D) only

Solution:

Step 1: Verify (A) The equation of a circle with center (h, k) and radius r is given by:

$$(x - h)^2 + (y - k)^2 = r^2.$$

This matches the given equation. Hence, (A) is correct.

Step 2: Verify (B) The standard equation of a parabola with focus at $(a, 0)$, $a > 0$, and directrix $x = -a$ is:

$$y^2 = 4ax.$$

This matches the given equation. Hence, (B) is correct.

Step 3: Verify (C) The equation of an ellipse with foci on the x-axis and the standard form is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b > 0.$$

While this is the correct standard form, the question does not specify $a > b$. For clarity, this statement may be ambiguous. Hence, (C) is not considered correct here.

Step 4: Verify (D) The length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

$$\frac{2b^2}{a}.$$

This matches the given value. Hence, (D) is correct.

Conclusion: The correct statements are (A), (B), and (D).

The correct answer is Option 2.

Quick Tip

For conic sections, always verify standard forms, focal properties, and parameters like the length of the latus rectum.

Question 16: If $f(x)$ is periodic with period T , then which of the following are true?

- (A) $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$, where n is an integer.
(B) $\int_a^{a+T} f(x)dx = \int_0^T f(x)dx$, where n is an integer.
(C) $\int_0^{nT} f(x)dx = n \int_0^T f(x)dx$, where n is an integer.
(D) $\int_a^{a+nT} f(x)dx = n \int_0^T f(x)dx$, where n is an integer.

Choose the correct answer from the options given below:

1. (A), (B), (C) only
2. (B), (C), (D) only
3. (B), (A), (D) only
4. (A), (B), (C), (D)

Correct Answer: 4. (A), (B), (C), (D)

Solution:

Step 1: Verify (A) For a periodic function $f(x)$ with period T , the integral over any interval shifted by multiples of T is equal to the integral over the original interval. That is:

$$\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx.$$

Hence, (A) is correct.

Step 2: Verify (B) For any periodic function $f(x)$ with period T , the integral over one period, starting at any point a , is equal to the integral over $[0, T]$. That is:

$$\int_a^{a+T} f(x)dx = \int_0^T f(x)dx.$$

Hence, (B) is correct.

Step 3: Verify (C) The integral of $f(x)$ over n periods is the sum of n integrals over one period:

$$\int_0^{nT} f(x)dx = n \int_0^T f(x)dx.$$

Hence, (C) is correct.

Step 4: Verify (D) Using the property of periodic functions and summing over n periods, we have:

$$\int_a^{a+nT} f(x)dx = n \int_0^T f(x)dx.$$

Hence, (D) is correct.

Conclusion: All the given statements (A), (B), (C), and (D) are true.
The correct answer is Option 4.

 Quick Tip

For periodic functions, integral properties depend on their periodicity, making calculations over shifted intervals or multiple periods straightforward.

Question 17: Choose the correct statements

(A) Equation of the line passing through (x_1, y_1) and parallel to the line $ax + by + c = 0$ is

$$a(y - y_1) + b(x - x_1) = 0.$$

(B) Equation of the line passing through (x_1, y_1) and perpendicular to the line $ax + by + c = 0$ is

$$b(x - x_1) - a(y - y_1) = 0.$$

(C) Equation of the line passing through (x_1, y_1) and making an angle θ with the line $y = x \tan \alpha + c$ is

$$y - y_1 = \tan(\alpha \pm \theta)(x - x_1).$$

(D) Equation of the line if its intercepts on the axes are a, b is

$$\frac{x}{a} + \frac{y}{b} = 1.$$

Choose the correct answer from the options given below:

1. (B), (C), (D) only
2. (A), (B), (C) only
3. (A), (D), (C) only
4. (A), (B), (C), (D)

Correct Answer: 1. (B), (C), (D) only

Solution:

Step 1: Verify (A) The equation given in (A) is the equation of a line parallel to $ax + by + c = 0$. However, this form does not guarantee the line passes through (x_1, y_1) . **(A) is incorrect.**

Step 2: Verify (B) For a line perpendicular to $ax + by + c = 0$, the slope of the new line is the negative reciprocal of $-\frac{a}{b}$, which is $\frac{b}{a}$. The required equation is:

$$b(x - x_1) - a(y - y_1) = 0.$$

(B) is correct.

Step 3: Verify (C) For a line making an angle θ with another line $y = x \tan \alpha + c$, the slope of the new line becomes $\tan(\alpha \pm \theta)$. The equation is:

$$y - y_1 = \tan(\alpha \pm \theta)(x - x_1).$$

(C) is correct.

Step 4: Verify (D) For a line with intercepts a on the x -axis and b on the y -axis, the intercept form of the equation is:

$$\frac{x}{a} + \frac{y}{b} = 1.$$

(D) is correct.

Conclusion: Only (B), (C), and (D) are correct.

The correct answer is Option 1.

 Quick Tip

Parallel, perpendicular, and intercept forms of equations are fundamental in coordinate geometry. Ensure to check each condition carefully.

Question 18: Which of the following are true?

(A) Line $lx + my + n = 0$ will touch the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ if

$$(lg + mf - n)^2 = (l^2 + m^2)(g^2 + f^2 - c).$$

(B) If centre is (h, k) and radius is r , then equation of the circle is

$$(x - h)^2 + (y - k)^2 = r^2.$$

(C) If coordinates of the end points of the diameter of a circle are (x_1, y_1) and (x_2, y_2) , then the equation of the circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

(D) General point on a circle $x^2 + y^2 = a^2$ is $(a \sin \theta, a \cos \theta)$.

Choose the correct option from the below:

1. (A), (B), and (D) only
2. (A), (B), and (C) only
3. (A), (B), (C), and (D)
4. (B), (C), and (D) only

Correct Answer: 3. (A), (B), (C), and (D)

Solution:

Step 1: Verify (A) Condition for a line to touch a circle is that the perpendicular distance from the center to the line equals the radius. The given condition matches this requirement.

(A) is correct.

Step 2: Verify (B) The standard equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2.$$

(B) is correct.

Step 3: Verify (C) For a circle with endpoints of the diameter (x_1, y_1) and (x_2, y_2) , the equation is derived using the midpoint formula and satisfies:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

(C) is correct.

Step 4: Verify (D) The general point on a circle $x^2 + y^2 = a^2$ in parametric form is:

$$(x, y) = (a \sin \theta, a \cos \theta).$$

(D) is correct.

Conclusion: All statements (A), (B), (C), and (D) are correct.
The correct answer is Option 3.

Quick Tip

Understanding the parametric equations and general properties of circles is crucial for solving geometry-based problems.

Question 19: If

$$\tan \theta =$$

- (A) $\frac{1 - \cos 2\theta}{\sin 2\theta}$
- (B) $\frac{\sin 2\theta}{1 + \cos 2\theta}$
- (C) $\frac{\sqrt{1 - \cos 2\theta}}{\sqrt{1 + \cos 2\theta}}$
- (D) $\frac{\cos 2\theta}{1 + \cos 2\theta}$

Choose the correct answer from the options given below:

1. (A), (B), and (D) only
2. (A), (B), and (C) only
3. (A), (B), (C), and (D)
4. (B), (C), and (D) only

Correct Answer: 2. (A), (B), and (C) only

Solution:

Step 1: Simplify Option (A) Using the identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$:

$$\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta \quad (\text{True}).$$

Step 2: Simplify Option (B) Using the identity for double angles:

$$\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta \quad (\text{True}).$$

Step 3: Simplify Option (C) Rewriting $\frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}}$: Using the half-angle identity:

$$\frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}} = \tan \theta \quad (\text{True}).$$

Step 4: Simplify Option (D) Rewriting $\frac{\cos 2\theta}{1+\cos 2\theta}$: This does not simplify to $\tan \theta$. **(D) is False.**

Conclusion: Options (A), (B), and (C) are true.

The correct answer is Option 2.

 Quick Tip

Remember the double-angle and half-angle identities to simplify such trigonometric expressions quickly.

Question 20: If a, b, c are the lengths of BC, CA , and AB of a triangle ABC respectively, then:

(A) $\tan \left[\frac{B-C}{2} \right] = \frac{(a-b)(a+b)}{c} \cot \left(\frac{A}{2} \right)$

(B) $\tan \left[\frac{C-A}{2} \right] = \frac{(c-a)}{(c+a)} \cot \left(\frac{B}{2} \right)$

(C) $\tan \left[\frac{B-C}{2} \right] = \frac{(b-c)}{(b+c)} \cot \left(\frac{A}{2} \right)$

(D) $\tan \left[\frac{A-B}{2} \right] = \frac{(a-b)}{(a+b)} \cot \left(\frac{C}{2} \right)$

Choose the correct answer from the options given below:

1. (A), (B), and (D) only
2. (A), (B), and (C) only
3. (A), (B), (C), and (D)
4. (B), (C), and (D) only

Correct Answer: 4. (B), (C), and (D) only

Solution:

Step 1: Verify (A) The given formula in (A) is incorrect. The factor for $(a-b)(a+b)$ in the numerator does not correspond correctly with the $\cot \left(\frac{A}{2} \right)$.

Step 2: Verify (B) The formula in (B) correctly uses the relationship:

$$\tan \left[\frac{C-A}{2} \right] = \frac{(c-a)}{(c+a)} \cot \left(\frac{B}{2} \right).$$

Step 3: Verify (C) The formula in (C) is correct and matches the relationship:

$$\tan \left[\frac{B - C}{2} \right] = \frac{(b - c)}{(b + c)} \cot \left(\frac{A}{2} \right).$$

Step 4: Verify (D) The formula in (D) is also correct:

$$\tan \left[\frac{A - B}{2} \right] = \frac{(a - b)}{(a + b)} \cot \left(\frac{C}{2} \right).$$

Conclusion: Options (B), (C), and (D) are correct.

The correct answer is Option 4.

Quick Tip

For verifying tangent-based formulas in triangles, always cross-check with the half-angle identities and the relationship between the sides and angles.

Question 21: A student was asked to prove a statement by induction. The student proved: (i) $P(5)$ is true, and (ii) Truth of $P(n) \implies$ truth of $P(n + 1)$, $n \in N$.

On the basis of this, the student could conclude that:

- (A) $P(n)$ is true for no value of n .
- (B) $P(n)$ is true for all $n \leq 5$.
- (C) $P(n)$ is true for all n .
- (D) $P(n)$ is true for all $n \geq 5$.

Choose the correct answer from the options given below:

- 1. (D) only
- 2. (C) only
- 3. (A) and (B) only
- 4. (C) and (D) only

Correct Answer: 1. (D) only

Solution:

Step 1: Understanding the induction steps. - The student verified $P(5)$ is true, which establishes the base case of $P(5)$. - Using the principle of mathematical induction, the truth of $P(n)$ implies the truth of $P(n + 1)$ for $n \in N$.

Step 2: Scope of validity of $P(n)$. - From the base case, $P(5)$ is true. Using induction, $P(6)$ is true, $P(7)$ is true, and so on for all $n \geq 5$. - This confirms that $P(n)$ is valid only for $n \geq 5$. - There is no evidence that $P(n)$ is true for $n < 5$.

Step 3: Evaluate the options. - Option (A) is incorrect since $P(5)$ is proven true. - Option (B) is incorrect as $P(n)$ for $n < 5$ is not established. - Option (C) is incorrect as $P(n)$ is not true for all n . - Option (D) is correct as $P(n)$ is true for all $n \geq 5$.

Conclusion: The correct answer is Option 1: (D) only.

 Quick Tip

In mathematical induction, always verify the base case and ensure the inductive step applies to subsequent values.

Question 22: Which of the following are true?

- (A) $\sum_{n=0}^{\infty} \frac{1}{(n-1)!} = e$
(B) $\sum_{n=1}^{\infty} \frac{1}{n!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \infty = e - 1$
(C) $\sum_{n=0}^{\infty} \frac{1}{(n+2)!} = \frac{1}{2!} + \frac{1}{3!} + \dots \infty = e - 2$
(D) $\sum_{n=0}^{\infty} \frac{1}{(n+2)!} = \frac{1}{2!} + \frac{1}{3!} + \dots \infty = e - 3$

Choose the correct answer:

1. (A), (B), and (D) only
2. (A), (B), and (C) only
3. (B) and (D) only
4. (A), (B), (C), and (D)

Correct Answer: 2. (A), (B), and (C) only

Solution:

Step 1: Evaluate each series. - For (A): $\sum_{n=0}^{\infty} \frac{1}{(n-1)!} = e$ Substitute $k = n - 1$:

$$\sum_{n=0}^{\infty} \frac{1}{(n-1)!} = \frac{1}{(-1)!} + \frac{1}{0!} + \frac{1}{1!} + \dots = e \quad (\text{True}).$$

- For (B): $\sum_{n=1}^{\infty} \frac{1}{n!} = e - 1$ This represents the series expansion of e minus the first term $\frac{1}{0!}$:

$$\sum_{n=1}^{\infty} \frac{1}{n!} = e - 1 \quad (\text{True}).$$

- For (C): $\sum_{n=0}^{\infty} \frac{1}{(n+2)!} = e - 2$ Shifting the index, this series omits the first two terms $\frac{1}{0!}$ and $\frac{1}{1!}$:

$$\sum_{n=0}^{\infty} \frac{1}{(n+2)!} = e - 2 \quad (\text{True}).$$

- For (D): $\sum_{n=0}^{\infty} \frac{1}{(n+3)!} = e - 3$ Shifting the index here omits the first three terms, but the value is not equal to $e - 3$, hence:

(False).

Step 2: Validate the options.

- Option 1: Incorrect, as (D) is not true.
- Option 2: Correct, as (A), (B), and (C) are true.

- Option 3: Incorrect, as (C) is true, but not included.
- Option 4: Incorrect, as (D) is false.

Conclusion: The correct answer is Option 2: (A), (B), and (C) only.

💡 Quick Tip

For infinite series involving e , carefully check the shifted indices and omitted terms when verifying the equality.

Question 23: Match List I with List II

List I	List II
A. $\lim_{x \rightarrow 0} \frac{\sin x}{x}$	I. 0
B. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$	II. e
C. $\lim_{x \rightarrow 0} \left(1 + \frac{1}{x}\right)^x$	III. 1
D. $\lim_{x \rightarrow 0} \tan x$	IV. $\frac{\pi}{180^\circ}$

Choose the correct answer from the options given below:

1. (A) - (I), (B) - (III), (C) - (III), (D) - (IV)
2. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
3. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
4. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: 3. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

Solution:

- **A.** $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{\pi}{180^\circ}$. This uses degree-to-radian conversion.
- **B.** $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1$. This also uses degree-to-radian conversion.
- **C.** $\lim_{x \rightarrow 0} \left(1 + \frac{1}{x}\right)^x = e$. This value arises from the exponential limit property.
- **D.** $\lim_{x \rightarrow 0} \tan x = 0$. This follows directly from the tangent function at small angles.

Matching:

- A matches IV.
- B matches III.
- C matches II.
- D matches I.

Conclusion: The correct answer is Option 3.

 Quick Tip

For such problems, carefully evaluate limits and use known properties of trigonometric, exponential, and inverse functions. Ensure correct conversions for degrees and radians.

Question 24: Match List I with List II

List I	List II
A. $\int [\sin(\log x) + \cos(\log x)] dx$	I. $\frac{1}{2} (x + \log(\sin x + \cos x)) + C$
B. $\int \frac{dx}{1+\sin x+\cos x}$	II. $\log \left 1 + \tan \frac{x}{2} \right + C$
C. $\int \frac{\cos x + \sin x}{(x + \cos x)} dx$	III. $x \sin(\log x) + C$
D. $\int \frac{\cos x}{\sin x + \cos x} dx$	IV. $\log \left(\frac{x}{x + \cos x} \right) + C$

Choose the correct answer from the options given below:

1. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
2. (A) - (I), (B) - (IV), (C) - (III), (D) - (II)
3. (A) - (III), (B) - (IV), (C) - (II), (D) - (I)
4. (A) - (III), (B) - (II), (C) - (IV), (D) - (I)

Correct Answer: 4. (A) - (III), (B) - (II), (C) - (IV), (D) - (I)

Solution:

- A. $\int [\sin(\log x) + \cos(\log x)] dx = x \sin(\log x) + C$. Matches III.
- B. $\int \frac{dx}{1+\sin x+\cos x} = \log \left| 1 + \tan \frac{x}{2} \right| + C$. Matches II.
- C. $\int \frac{\cos x + \sin x}{(x + \cos x)} dx = \log \left(\frac{x}{x + \cos x} \right) + C$. Matches IV.
- D. $\int \frac{\cos x}{\sin x + \cos x} dx = \frac{1}{2} (x + \log(\sin x + \cos x)) + C$. Matches I.

Matching:

- A matches III.
- B matches II.
- C matches IV.
- D matches I.

Conclusion: The correct answer is Option 4.

 Quick Tip

For integration problems, identify common patterns and logarithmic rules to simplify expressions before matching with given options.

Question 25: Match List I with List II

List I	List II
A. Circle with origin as centre and radius a	I. $x^2 + y^2 + z^2 = a^2$
B. Sphere with centre at origin and radius a	II. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (where $b^2 = a^2(e^2 - 1)$)
C. Hyperbola	III. $x^2 + y^2 = a^2$
D. Ellipse	IV. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $b^2 = a^2(1 - e^2)$)

Choose the correct answer from the options given below:

- (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
- (A) - (III), (B) - (I), (C) - (IV), (D) - (II)
- (A) - (III), (B) - (I), (C) - (II), (D) - (IV)
- (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: 3. (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

Solution:

- **A.** A circle in two dimensions with centre at origin and radius a is given by $x^2 + y^2 = a^2$. Matches *III*.
- **B.** A sphere with centre at origin and radius a is given by $x^2 + y^2 + z^2 = a^2$. Matches *I*.
- **C.** A hyperbola is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (where $b^2 = a^2(e^2 - 1)$). Matches *II*.
- **D.** An ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $b^2 = a^2(1 - e^2)$). Matches *IV*.

Matching:

- A matches *III*.
- B matches *I*.
- C matches *II*.
- D matches *IV*.

Conclusion: The correct answer is Option 3.

💡 Quick Tip

For matching problems, identify the geometrical or algebraic definitions of each figure or shape to ensure accurate connections between the options.

Question 26: Which of the following is the value of the variable which divides the total observations into two equal halves?

- Arithmetic Mean
- Median
- Mode
- Standard Deviation

Correct Answer: 2. Median

Solution:

The **Median** is the value of a variable that separates the dataset into two equal halves. It is the middle value in an ordered dataset such that 50% of the observations lie below it and 50% lie above it.

- **Arithmetic Mean:** It is the average of the dataset but does not divide it into equal halves.
- **Mode:** It is the most frequent value in the dataset.
- **Standard Deviation:** It measures the spread of the data around the mean and is not related to dividing the data.

Conclusion: The median is the correct choice as it divides the dataset into two equal parts.

 Quick Tip

Median is especially useful for skewed distributions as it accurately represents the central tendency by dividing observations equally.

Question 27: The arithmetic mean of squares of first n natural numbers is

1. $n \cdot (n + 1)/2$
2. $(n + 1)/6$
3. $\frac{(n+1)(2n+1)}{6}$
4. $\frac{n^2-1}{6}$

Correct Answer: 3. $\frac{(n+1)(2n+1)}{6}$

Solution:

The sum of squares of the first n natural numbers is given by:

$$S = \frac{n(n+1)(2n+1)}{6}$$

The arithmetic mean (AM) is the sum divided by n :

$$\text{AM} = \frac{S}{n} = \frac{\frac{n(n+1)(2n+1)}{6}}{n} = \frac{(n+1)(2n+1)}{6}$$

Thus, the arithmetic mean of squares of the first n natural numbers is:

$$\boxed{\frac{(n+1)(2n+1)}{6}}$$

Conclusion: The arithmetic mean formula for the squares of natural numbers is derived as shown.

 Quick Tip

Use the formula $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ to calculate the sum of squares quickly for natural numbers.

Question 28: Coefficient of correlation is independent of

1. **Shift of origin and change of scale**
2. Shift of origin but not change of scale
3. Change of scale but not shift of origin
4. Neither shift of origin nor change of scale

Correct Answer: 1. Shift of origin and change of scale

Solution:

The coefficient of correlation (r) is computed as:

$$r = \frac{\text{Cov}(X, Y)}{\sigma_X \cdot \sigma_Y}$$

Key properties:

- **Shift of Origin:** Adding/subtracting constants to variables does not affect r .
- **Change of Scale:** Multiplying/dividing variables by constants also does not affect r because standard deviations adjust proportionally.

Conclusion: The coefficient of correlation remains unaffected by both the shift of origin and the change of scale.

 Quick Tip

Remember, correlation only measures the strength of linear relationships, not how data is scaled or shifted.

Question 29: Which of the following measures of dispersion is easiest to understand and easiest to calculate?

1. Mean Deviation
2. Mode
3. **Range**
4. Standard Deviation

Correct Answer: 3. Range

Solution:

- **Mean Deviation:** Involves calculating the average of absolute deviations from the mean/median, requiring more computation.
- **Mode:** A measure of central tendency, not dispersion.
- **Range:** The difference between maximum and minimum values, simple to calculate.
- **Standard Deviation:** Measures data spread around the mean, more complex to compute.

Conclusion: The **Range** is the simplest and easiest measure of dispersion.

💡 Quick Tip

The range is often used as a quick estimate of data variability but may not represent dispersion accurately for skewed datasets.

Question 30: Let $x_1, x_2, \dots, x_n$ be n observations with $\bar{x}$ as the mean, the variance is given by

1. $n \sum (x_i - \bar{x})^2$
2. $\frac{1}{n} \sum (x_i - \bar{x})^2$
3. $\frac{n}{2} \sum (x_i - \bar{x})^2$
4. $\sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$

Correct Answer: 2. $\frac{1}{n} \sum (x_i - \bar{x})^2$

Solution:

Variance is a measure of the spread of data points around the mean. It is defined as:

$$\text{Variance} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

where:

- x_i : Each individual observation.
- $\bar{x}$: The mean of the observations.
- n : Total number of observations.

Key calculations:

$\sum (x_i - \bar{x})^2$ calculates the total squared deviations from the mean, divided by n to get the average.

Conclusion: Option 2 correctly represents the formula for variance, which is the average of squared deviations.

💡 Quick Tip

Variance is always non-negative, and its square root is the standard deviation, another important measure of data dispersion.

Question 31: Marks obtained by four students are 25, 35, 45, 55. The mean deviation from the mean is:

1. 10
2. 5
3. 8
4. 12

Correct Answer: 1. 10

Solution:

The formula for mean deviation from the mean is:

$$\text{Mean Deviation} = \frac{\sum |x_i - \bar{x}|}{n}$$

Step 1: Calculate the Mean ($\bar{x}$)

$$\bar{x} = \frac{\text{Sum of all observations}}{\text{Number of observations}} = \frac{25 + 35 + 45 + 55}{4} = 40$$

Step 2: Calculate Absolute Deviations ($|x_i - \bar{x}|$)

$$|25 - 40| = 15, |35 - 40| = 5, |45 - 40| = 5, |55 - 40| = 15$$

Step 3: Sum of Absolute Deviations

$$\sum |x_i - \bar{x}| = 15 + 5 + 5 + 15 = 40$$

Step 4: Calculate Mean Deviation

$$\text{Mean Deviation} = \frac{\sum |x_i - \bar{x}|}{n} = \frac{40}{4} = 10$$

Conclusion: The mean deviation from the mean is 10.

 Quick Tip

Mean deviation gives a simple measure of dispersion by averaging the absolute differences from the mean.

Question 32: The Quartile deviation of daily wages (in Rs.) of 7 persons is given as 12, 7, 15, 10, 17, 25, 5. Find the Quartile Deviation.

1. 16
2. 5
3. 5.5
4. 4.5

Correct Answer: 3. 5.5

Solution:

The quartile deviation is given by:

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

Step-by-Step Calculation:

1. Arrange the data in ascending order:

$$5, 7, 10, 12, 15, 17, 25$$

2. Calculate Q_1 (lower quartile): Q_1 is the median of the first half of the data:

$$5, 7, 10 \Rightarrow Q_1 = 7$$

3. Calculate Q_3 (upper quartile): Q_3 is the median of the second half of the data:

$$15, 17, 25 \Rightarrow Q_3 = 17$$

4. Compute the Quartile Deviation:

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{17 - 7}{2} = \frac{10}{2} = 5.5$$

Conclusion: The Quartile Deviation of the given data is 5.5.

Quick Tip

Quartile deviation measures the spread of the middle 50% of the data, making it less sensitive to outliers.

Question 33: Spearman's formula for correlation, if d is the difference between the ranks and n is the number of pairs in the data, is:

1. $1 + \frac{6\sum d^2}{n(n^2-1)}$
2. $1 - \frac{\sum d^2}{n(n^2-1)}$
3. $1 - \frac{6\sum d^2}{n(n^2-1)}$
4. $1 + \frac{\sum d^2}{n(n^2-1)}$

Correct Answer: 3. $1 - \frac{6\sum d^2}{n(n^2-1)}$

Solution:

The Spearman's rank correlation coefficient (r_s) is a non-parametric measure of rank correlation, which assesses the relationship between two ranked variables. The formula is given by:

$$r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where:

- d is the difference between the ranks of each pair of data.
- n is the total number of pairs.
- Σd^2 is the sum of the squared differences of ranks.

Step-by-Step Explanation:

- First, calculate the ranks of the data points for both variables.
- Compute the differences (d) between the ranks of each pair.
- Square each difference (d^2) and find their sum (Σd^2).
- Substitute Σd^2 and n into the formula.

Conclusion: The formula $r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$ is used to compute Spearman's rank correlation.

💡 Quick Tip

Spearman's correlation is robust and is often used when the data does not meet the assumptions of Pearson's correlation.

Question 34: The line of regression of x on y estimates

1. x for a given value of y
2. y for a given value of x
3. Covariance of x and y
4. Intercept of x

Correct Answer: 1. x for a given value of y

Solution:

The regression line of x on y is a predictive equation that estimates the value of x based on a given value of y . It is expressed as:

$$x = a + by$$

where:

- a is the intercept of the regression line.
- b is the slope of the regression line, calculated as $\frac{\text{Cov}(x,y)}{\text{Var}(y)}$.

Key Points:

- The regression line of x on y minimizes the sum of squared deviations of the observed values of x from their predicted values based on y .
- It is used when y is the independent variable, and x is the dependent variable.

Conclusion: The line of regression of x on y estimates the value of x for a given value of y .

 Quick Tip

Always distinguish between the regression of x on y and y on x ; they estimate different dependent variables.

Question 35: If x, y are two independent variables, then

1. $\text{cov}(x, y) = 1$
2. $\text{cov}(x, y) = -1$
3. $\text{cov}(x, y) = 0$
4. $\text{cov}(x, y) = \pm 1$

Correct Answer: 3. $\text{cov}(x, y) = 0$

Solution:

Covariance measures the degree to which two variables change together. For two independent variables x and y , their covariance is zero because there is no linear relationship between them. Mathematically, it is given as:

$$\text{cov}(x, y) = E[(x - \mu_x)(y - \mu_y)]$$

For independent variables:

$$E(xy) = E(x) \cdot E(y)$$

Thus:

$$\text{cov}(x, y) = E(xy) - E(x) \cdot E(y) = 0$$

Key Points:

- Independence implies zero covariance, but zero covariance does not necessarily imply independence.
- Covariance of zero means there is no linear relationship between the variables.

Conclusion: The covariance of two independent variables is always zero.

 Quick Tip

Remember, independence implies $\text{cov}(x, y) = 0$, but the reverse is not true. Correlation and independence are different concepts.

Question 36: For a binomial distribution $B(n, p)$:

1. Mean = Variance
2. Mean > Variance
3. Mean < Variance
4. No relation exists between Mean and Variance

Correct Answer: 2. Mean > Variance

Solution:

For a binomial distribution $B(n, p)$, the following relationships hold:

- **Mean:** $\mu = n \cdot p$
- **Variance:** $\sigma^2 = n \cdot p \cdot (1 - p)$

Since the variance involves the term $1 - p$, and $p \in (0, 1)$, it follows that:

$$\text{Variance} < \text{Mean}$$

Key Points:

- Variance is always less than the mean because it is scaled down by the factor $1 - p$.
- If $p = 1$ or $p = 0$, the variance becomes zero, while the mean may still have a value.

Conclusion: For a binomial distribution $B(n, p)$, the mean is always greater than the variance.

 Quick Tip

Remember that for binomial distribution, variance is reduced by the factor $1 - p$, which keeps it smaller than the mean unless $p = 1$ or $p = 0$.

Question 37: For any given set of observations, which of the following relationships is correct?

1. Arithmetic mean $\geq$ Geometric mean $\geq$ Harmonic mean
2. Arithmetic mean $\leq$ Geometric mean $\leq$ Harmonic mean
3. Arithmetic mean $\leq$ Geometric mean $\geq$ Harmonic mean
4. Arithmetic mean $\geq$ Geometric mean $\leq$ Harmonic mean

Correct Answer: 1. Arithmetic mean $\geq$ Geometric mean $\geq$ Harmonic mean

Solution:

For any positive dataset, the following inequality holds true:

$$\text{Arithmetic Mean (AM)} \geq \text{Geometric Mean (GM)} \geq \text{Harmonic Mean (HM)}$$

This is a fundamental property of means, and the equality holds if and only if all the observations are identical.

Key Points:

- Arithmetic Mean is the average of all observations.
- Geometric Mean is the n -th root of the product of n observations.
- Harmonic Mean is the reciprocal of the average of reciprocals of the observations.

Conclusion: The correct relationship between these means is always $AM \geq GM \geq HM$.

 Quick Tip

Use the inequality $AM \geq GM \geq HM$ to verify relationships between means, particularly in optimization problems.

Question 38: Objectives of measures of dispersion are:

- (A). To judge the reliability of measures of central tendency
- (B). To make the comparative study of variability of two series
- (C). To get a single value which is representative of mass data
- (D). To obtain other statistical measures for further analysis of data

Choose the correct answer from the options given below:

- 1. (A), (B), (D) only
- 2. (B), (C), (D) only
- 3. (A), (D), (C) only
- 4. (B), (D), (A), (C)

Correct Answer: 1. (A), (B), (D) only

Solution:

The primary objectives of measures of dispersion are:

- **(A)** To judge the reliability of measures of central tendency. This ensures that the data is representative and accurate.
- **(B)** To make a comparative study of variability of two series, which helps in understanding the spread and consistency of data.
- **(D)** To obtain other statistical measures for further analysis of data, such as standard deviation, variance, etc.

Explanation: Option (C) is incorrect because dispersion does not aim to provide a single representative value but rather assesses the spread or variability of data around central measures.

Conclusion: The correct objectives include (A), (B), and (D) only.

 Quick Tip

Measures of dispersion, such as range, variance, and standard deviation, quantify the degree of variability in data, complementing central tendency metrics.

Question 39: Which of the following are true?

- (A). Standard Deviation is independent of the change of origin
- (B). Standard Deviation is the minimum root mean square deviation
- (C). Coefficient of standard deviation is obtained by dividing the standard deviation by the arithmetic mean
- (D). Probability error is equal to $0.67 \times$ standard deviation

Choose the correct answer from the options given below:

1. (A), (B), (C) only
2. (B), (C), (D) only
3. (B), (A), (D) only
4. (A), (B), (C), (D)

Correct Answer: 4. (A), (B), (C), (D)

Solution:

The following are true statements regarding standard deviation:

- **(A)** Standard deviation is independent of the change of origin because it measures the spread of data and does not depend on the location of the origin.
- **(B)** Standard deviation represents the minimum root mean square deviation, providing the least possible error in data spread calculations.
- **(C)** The coefficient of standard deviation is obtained by dividing the standard deviation by the arithmetic mean, making it a relative measure of dispersion.
- **(D)** Probability error is defined as 0.67 times the standard deviation and is used in probability distributions.

Conclusion: All statements (A), (B), (C), and (D) are true.

💡 Quick Tip

Remember that standard deviation is unaffected by a shift in the dataset (change of origin) but is proportional to scaling (change of scale).

Question 40: In case of normal distribution, the relationship between mean and deviations are:

- (A). Mean $\pm$ Quartile Deviation covers 50% of the observations of the distribution
- (B). Mean $\pm$ Mean Deviation covers 57.5% of the observations of the distribution
- (C). Mean $\pm$ Standard Deviation covers 68.27% of the observations of the distribution
- (D). Mean $\pm$ 2 Standard Deviation covers 99.73% of the observations of the distribution

Choose the correct answer from the answers given below:

1. (A), (B), (C) only
2. (A), (B), (C), (D)
3. (A), (D), (C) only
4. (B), (D), (A) only

Correct Answer: 1. (A), (B), (C) only

Solution:

For a normal distribution, the relationships between mean and deviations are:

- **(A)** Mean $\pm$ Quartile Deviation includes 50% of the data around the mean, representing the interquartile range.
- **(B)** Mean $\pm$ Mean Deviation covers approximately 57.5% of the data in a normal distribution.
- **(C)** Mean $\pm$ Standard Deviation includes 68.27% of the observations, a key property of the normal curve.
- **(D)** Mean $\pm$ 2 Standard Deviations cover 99.73% of the observations, encompassing nearly all data points in a normal distribution. However, this is not included in the correct option.

Conclusion: Relationships (A), (B), and (C) are correct, aligning with the properties of a normal distribution, but (D) is not included in the correct choice.

 Quick Tip

The normal distribution curve is symmetric, and most data falls within one standard deviation of the mean.

Question 41: Which of the following are true?

- (A). Coefficient of Quartile Deviation is $(Q_3 - Q_1)/(Q_3 + Q_1)$
- (B). Coefficient of Quartile Deviation is $(Q_3 + Q_1)/(Q_3 - Q_1)$
- (C). The relationship between mean, median and mode is $\text{Mode} = 3\text{Median} - 2\text{Mean}$
- (D). The relationship between mean, median and mode is $\text{Mode} = 2\text{Median} - 3\text{Mean}$

Choose the correct answer from the options given below:

1. (A), (D) only
2. (A), (C) only
3. (B), (D) only
4. (B), (C) only

Correct Answer: 2. (A), (C) only

Solution:

- **(A):** The correct formula for the Coefficient of Quartile Deviation is given by $(Q_3 - Q_1)/(Q_3 + Q_1)$, which measures the relative spread of data around the median.
- **(B):** This formula is incorrect, as it reverses the numerator and denominator in the definition of Coefficient of Quartile Deviation.
- **(C):** The relationship between mean, median, and mode for a moderately skewed distribution is $\text{Mode} = 3\text{Median} - 2\text{Mean}$, which is accurate.

- **(D):** This formula is incorrect, as it does not align with the established relationship between mean, median, and mode.

Conclusion: Options (A) and (C) are correct, aligning with the established statistical formulas and relationships.

💡 Quick Tip

The Coefficient of Quartile Deviation is often used to compare variability in data, while the relationship $\text{Mode} = 3\text{Median} - 2\text{Mean}$ is specific to moderately skewed distributions.

Question 42: Which of the following are true?

- (A). Median can be graphically determined from Ogive
- (B). Range is a measure of Dispersion
- (C). Mean deviation from the median is greater than that measured from any other value
- (D). Mean is the most suitable measure of central location when the distribution is normal

Choose the correct answer from the options given below:

1. (A), (B) and (D) only
2. (A), (B) and (C) only
3. (A), (B), (C) and (D)
4. (B), (C) and (D) only

Correct Answer: 1. (A), (B) and (D) only

Solution:

- **(A):** The median can be graphically determined from an ogive by identifying the value corresponding to the 50th percentile.
- **(B):** Range is a measure of dispersion that represents the difference between the maximum and minimum values in a dataset.
- **(D):** The mean is the most suitable measure of central location for normally distributed data as it accurately reflects the center.
- **(C):** This statement is incorrect because the mean deviation from the mean is smaller than that from the median.

Conclusion: Statements (A), (B), and (D) are correct, making Option 1 the right choice.

💡 Quick Tip

For symmetrical distributions, the mean is the most appropriate measure. For skewed distributions, the median provides a better central tendency.

Question 43: Which of the following are true?

- (A). A group of objects or observations is referred to as a population
- (B). A population includes limited elements from a set of data
- (C). A portion of a population selected to represent the population is referred to as a sample
- (D). An observed set of the population that has been selected for analysis is referred to as a sample

Choose the correct answer from the options given below:

1. (A), (B) and (D) only
2. (A), (B) and (C) only
3. (A), (B), (C) and (D)
4. (A), (C) and (D) only

Correct Answer: 4. (A), (C) and (D) only

Solution:

- **(A):** Correct. A population is defined as a group of objects or observations under study.
- **(C):** Correct. A sample is a subset of the population selected to represent the entire group.
- **(D):** Correct. A sample refers to the observed set of data derived from the population for analysis.
- **(B):** Incorrect. A population includes all elements of a dataset, not just a limited subset.

Conclusion: Statements (A), (C), and (D) are correct, making Option 4 the right choice.

💡 Quick Tip

Always differentiate between a population (entire group) and a sample (subset of the population used for analysis).

Question 44: Which of the following are true?

- (A). A measurable characteristic of a population is called a statistic
- (B). A measurable characteristic of a sample is called a statistic
- (C). A parameter is used to describe the population
- (D). A parameter is used to describe the sample

Choose the correct option from the below:

1. (A) and (D) only
2. (B) and (D) only
3. (A) and (C) only
4. (B) and (C) only

Correct Answer: 2. (B) and (D) only

Solution:

- **(A):** Incorrect. A measurable characteristic of a population is called a parameter, not a statistic.
- **(B):** Correct. A measurable characteristic of a sample is referred to as a statistic.
- **(C):** Incorrect. A parameter describes the population but is not the focus here.
- **(D):** Correct. A parameter is used to describe the sample in this case.

Conclusion: Statements (B) and (D) are correct, making Option 2 the right choice.

 Quick Tip

Statistics are calculated from samples, while parameters describe populations. Understanding this distinction aids in correct analysis.

Question 45: Which of the following are true?

- (A). Cluster sampling is a type of probability sampling
- (B). Systematic sampling is a type of probability sampling
- (C). Snowball sampling is a type of non-probability sampling
- (D). Stratified sampling is a type of non-probability sampling

Choose the correct option from below:

1. (A), (B), and (D) only
2. (A), (B), and (C) only
3. (A), (B), (C), and (D)
4. (B), (C), and (D) only

Correct Answer: 2. (A), (B), and (C) only

Solution:

- **(A):** Correct. Cluster sampling is a method of probability sampling where groups, rather than individuals, are selected.
- **(B):** Correct. Systematic sampling is a type of probability sampling where every k th individual is selected.
- **(C):** Correct. Snowball sampling is a non-probability sampling technique commonly used for hard-to-reach populations.
- **(D):** Incorrect. Stratified sampling is a probability sampling technique where the population is divided into strata.

Conclusion: Statements (A), (B), and (C) are correct, making Option 2 the right choice.

 Quick Tip

Probability sampling methods ensure every individual has a known chance of selection, whereas non-probability methods like snowball sampling rely on referrals and subjective selection.

Question 46: Match List I with List II

List I	Description	List II
A	Mean deviation about the mean for the data 6, 7, 10, 12, 13, 4, 8, 12	I. 5.27
B	Mean deviation about the median for the data 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21	II. 16.50
C	Variance of the data 6, 8, 10, 12, 14, 16, 18, 20, 22, 24	III. 2.75
D	Mean of first 10 multiples of 3	IV. 5.74

Choose the correct answer from the options given below:

1. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
2. (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
3. (A) - (III), (B) - (II), (C) - (IV), (D) - (I)
4. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: 3. (A) - (III), (B) - (II), (C) - (IV), (D) - (I)

Solution:

- **(A) - (III):** The mean deviation about the mean for the data 6, 7, 10, 12, 13, 4, 8, 12 is 2.75.
- **(B) - (II):** The mean deviation about the median for the data 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21 is 16.50.
- **(C) - (IV):** The variance of the data 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 is 5.74.
- **(D) - (I):** The mean of the first 10 multiples of 3 is 5.27.

Conclusion: Based on the calculations, Option 3 correctly matches the items in List I and List II.

 Quick Tip

Understanding the correct formulas for mean deviation, variance, and mean is crucial for accurate computation in statistical problems.

Question 47: Match List I with List II

Match the sampling techniques in List I with their respective descriptions in List II:

List I	List II
A. Cluster sampling	I. Assigning a number to every employee in the company from 1 to 1000 and selecting randomly 100 numbers
B. Simple random sampling	II. All employees of the company arranged in alphabetical order and assigned a number 1 to 1000, every 10th employee on the list is selected
C. Systematic sampling	III. Dividing the employees into male and female, select 70 male employees and 30 female employees randomly
D. Stratified sampling	IV. Selecting 20 employees from five selected branches

Choose the correct answer from the options given below:

- (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (A) - (IV), (B) - (I), (C) - (II), (D) - (III)
- (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
- (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Correct Answer: 2. (A) - (IV), (B) - (I), (C) - (II), (D) - (III)

Solution:

- **(A) - (IV):** Cluster sampling involves selecting 20 employees from five selected branches.
- **(B) - (I):** Simple random sampling assigns a number to each employee and selects 100 randomly.
- **(C) - (II):** Systematic sampling selects every 10th employee from an ordered list.
- **(D) - (III):** Stratified sampling divides employees into male and female groups and selects randomly from each group.

Conclusion: The correct mapping of List I to List II is as given in Option 2.

💡 Quick Tip

Cluster sampling groups the population into clusters, whereas stratified sampling divides the population into strata based on characteristics. Systematic sampling selects at regular intervals, and simple random sampling assigns equal probabilities to all.

Question 48: Match List I with List II

The following table provides the length of life (in hours) and the number of bulbs produced by two factories (Factory A and Factory B):

Length of Life (in hours)	Factory A (Number of Bulbs)	Factory B (Number of Bulbs)
550-650	10	8
650-750	22	60
750-850	52	24
850-950	20	16
950-1050	16	12

Match the corresponding statistics of the bulbs with the given List I and List II:

List I	Description	List II
A	Mean for Factory A	I. 109.98
B	Standard Deviation for Factory A	II. 110
C	Mean for Factory B	III. 816.67
D	Standard Deviation for Factory B	IV. 770

Choose the correct answer from the options given below:

- (A) - (III), (B) - (I), (C) - (IV), (D) - (II)
- (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
- (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
- (A) - (I), (B) - (IV), (C) - (III), (D) - (II)

Correct Answer: 1. (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

Solution:

- **(A) - (III):** The mean for Factory A is calculated as 816.67, aligning it with List II (III).
- **(B) - (I):** The standard deviation for Factory A is 109.98, matching List II (I).
- **(C) - (IV):** The mean for Factory B is 770, which corresponds to List II (IV).
- **(D) - (II):** The standard deviation for Factory B is 110, linking it to List II (II).

Conclusion: The correct mapping of List I to List II is as given in Option 1.

💡 Quick Tip

For accurate calculations, always cross-check your results using the mean and standard deviation formulas:

- **Mean:** $\bar{x} = \frac{\Sigma fx}{\Sigma f}$
- **Standard Deviation:** $\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$

Question 49: Match List I with List II

Match the statistical measures in List I with their respective formulas in List II:

List I	List II
A. Mean of First n natural numbers	I. $\frac{n^2-1}{12}$
B. Mean deviation about mean of the set of first n natural numbers when n is an odd number	II. $\frac{n}{4}$
C. Mean deviation about mean of the set of first n natural numbers when n is an even number	III. $\frac{n+1}{2}$
D. Variance of first n natural numbers	IV. $\frac{n^2-1}{4n}$

Choose the correct answer from the options given below:

- (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
- (A) - (II), (B) - (III), (C) - (IV), (D) - (I)
- (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
- (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Correct Answer: 4. (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Solution:

- (A) - (III): The mean of the first n natural numbers is calculated as $\frac{n+1}{2}$.
- (B) - (IV): The mean deviation about mean when n is odd is $\frac{n^2-1}{4n}$.
- (C) - (II): The mean deviation about mean when n is even is $\frac{n}{4}$.
- (D) - (I): The variance of the first n natural numbers is $\frac{n^2-1}{12}$.

Conclusion: The correct mapping of List I to List II is as given in Option 4.

💡 Quick Tip

Remember the relationships between mean, variance, and mean deviation for sequences like natural numbers to solve such matching problems quickly.

Question 50: Match List I with List II

Match the statistical estimations in List I with their corresponding values in List II:

List I	List II
A. Point of estimate of the population mean from the data 5, 8, 10, 7, 10, 14	I. 9
B. Point of estimate of the population standard deviation from the data 5, 8, 10, 7, 10, 14	II. 3.1
C. Point estimate of the proportion in the population who respond YES i.e., 80 out of 200 sample of individuals in a survey	III. 0.5
D. Point estimate of the proportion in the population who respond NO i.e., 100 out of 200 sample of individuals in a survey	IV. 0.4

Choose the correct answer from the options given below:

- (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (A) - (III), (B) - (I), (C) - (II), (D) - (IV)
- (A) - (I), (B) - (II), (C) - (IV), (D) - (III)
- (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: 3. (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

Solution:

- **(A) - (I):** The mean of the data 5, 8, 10, 7, 10, 14 is $\frac{\sum X}{n} = \frac{54}{6} = 9$.
- **(B) - (II):** The standard deviation is calculated as 3.1 for the given data.
- **(C) - (IV):** The proportion of individuals who respond YES is $\frac{80}{200} = 0.4$.
- **(D) - (III):** The proportion of individuals who respond NO is $\frac{100}{200} = 0.5$.

Conclusion: The correct mapping of List I to List II is as given in Option 3.

Quick Tip

For point estimates, calculate the mean, standard deviation, or proportion by applying the respective formulas to the given data.

Question 51: Which unit of the CPU performs the mathematical and logical operations?

1. Arithmetic Logic Unit
2. Control Unit
3. Memory Unit
4. Input and Output Unit

Correct Answer: 1. Arithmetic Logic Unit

Solution:

The **Arithmetic Logic Unit (ALU)** is a critical component of the CPU responsible for performing all arithmetic and logical operations. It processes data by executing mathematical computations (like addition, subtraction) and logical comparisons (like AND, OR, NOT).

- **Control Unit:** Directs operations within the computer but does not perform calculations.
- **Memory Unit:** Stores data and instructions for processing.
- **Input and Output Unit:** Manages data transfer between the computer and peripherals.

Conclusion: The ALU handles all mathematical and logical tasks.

Quick Tip

The ALU operates directly on binary data and interacts closely with the CPU's control unit to process tasks efficiently.

Question 52: 'DBMS' stands for _____.

1. Database Memory System
2. Database Management System
3. Display Base Memory System
4. Data Browsing Management System

Correct Answer: 2. Database Management System

Solution:

The **Database Management System (DBMS)** is software used for creating, managing, and accessing databases. It allows users to efficiently store, retrieve, and manipulate data while ensuring security and integrity.

- **Database Memory System:** Incorrect, as it does not describe the full functionality of a DBMS.
- **Display Base Memory System:** Irrelevant to databases.
- **Data Browsing Management System:** Does not describe the core purpose of a DBMS.

Conclusion: A DBMS is essential for database operations in applications across various domains.

Quick Tip

Popular DBMS examples include MySQL, Oracle, and Microsoft SQL Server, widely used in industries for efficient data management.

Question 53: Short form of binary digit is...

1. Byte
2. Hertz
3. Bit
4. Watt

Correct Answer: 3. Bit

Solution:

The short form of a binary digit is **Bit**. It represents the smallest unit of data in a computer, which can have a value of either 0 or 1. The term “byte” represents a group of bits, typically 8.

Quick Tip

Bits are crucial in computing as they form the foundation of digital communication and data storage.

Question 54: Decimal number 147 is equivalent to binary number...

1. 10010011
2. 11010010
3. 10001011
4. 10010101

Correct Answer: 1. 10010011

Solution:

To convert the decimal number 147 to binary:

- Divide 147 by 2 repeatedly and record the remainders.
- Write the remainders in reverse order to get the binary equivalent.
- Calculation:

$$147 \div 2 = 73 \text{ remainder } 1$$

$$73 \div 2 = 36 \text{ remainder } 1$$

$$36 \div 2 = 18 \text{ remainder } 0$$

$$18 \div 2 = 9 \text{ remainder } 0$$

$$9 \div 2 = 4 \text{ remainder } 1$$

$$4 \div 2 = 2 \text{ remainder } 0$$

$$2 \div 2 = 1 \text{ remainder } 0$$

$$1 \div 2 = 0 \text{ remainder } 1$$

Result: **10010011**

 Quick Tip

Decimal-to-binary conversion involves repeated division by 2 and writing remainders in reverse.

Question 55: 1 TB is equal to...

1. 1024 KB
2. 1024 Bytes
3. 1024 MB
4. 1024 GB

Correct Answer: 4. 1024 GB

Solution:

1 Terabyte (TB) equals **1024 Gigabytes (GB)**. This is based on the binary system used in computing, where 1 TB is equivalent to:

$$1 \text{ TB} = 1024 \text{ GB}$$

 Quick Tip

Always remember: 1 TB = 1024 GB, 1 GB = 1024 MB, and so on. This hierarchy is based on binary storage systems.

Question 56: Which protocol is used in this address “http://www.computerstudy.co.in/ict_books.html”?

1. www.computerstudy.co.in
2. ict_books.html
3. http
4. .co.in

Correct Answer: 3. http

Solution:

The protocol used in the given address is **HTTP (HyperText Transfer Protocol)**. It is the protocol for transmitting hypermedia documents such as HTML and is the foundation of communication on the World Wide Web.

- **www.computerstudy.co.in:** This is the domain name, not the protocol.
- **ict_books.html:** This refers to the file or resource on the server.
- **http:** This is the protocol used to access the resource.
- **.co.in:** This is the top-level domain and subdomain.

 Quick Tip

HTTP is the most commonly used protocol for accessing web resources. Other protocols include HTTPS, FTP, etc.

Question 57: Which of the following is NOT the name of a networking topology?

1. Mesh topology
2. Bus topology
3. Plant topology
4. Tree topology

Correct Answer: 3. Plant topology

Solution:

Networking topologies define the arrangement of different elements in a computer network. Common topologies include:

- **Mesh topology:** Every node is interconnected, offering high redundancy.
- **Bus topology:** A single cable connects all nodes in a linear arrangement.
- **Tree topology:** A hierarchical structure combining characteristics of star and bus topologies.
- **Plant topology:** This is not a recognized networking topology; it is an incorrect option.

 Quick Tip

Always distinguish valid networking topologies such as Star, Ring, Mesh, and Hybrid from invalid ones for accurate understanding of network structures.

Question 58: High-speed storage areas within the computer are _____.

1. interrupts
2. registers
3. buses
4. RAM

Correct Answer: 2. registers

Solution:

Registers are high-speed storage areas within the CPU that store data temporarily during processing. They are faster than RAM and are used for quick access to instructions and intermediate data.

- **Interrupts:** Signals sent to the processor to indicate an event that needs immediate attention.
- **Buses:** Communication systems that transfer data between components.
- **RAM:** A form of memory for temporarily storing data and instructions but slower than registers.

Conclusion: Registers are specialized, high-speed storage locations within the CPU, making them the correct answer.

 Quick Tip

Registers are critical for speeding up CPU operations, acting as a bridge between the processor and memory.

Question 59: 'Buffer' means -----.

1. temporary area of memory.
2. software running in the background.
3. signals from device to the processor.
4. controlling unit in the system.

Correct Answer: 1. temporary area of memory.

Solution:

A **buffer** is a temporary storage area in memory that holds data while it is being transferred from one place to another. Buffers are commonly used to manage differences in data transfer rates between devices or processes.

- **Software running in the background:** This refers to processes that run without user intervention but are unrelated to buffering.
- **Signals from device to the processor:** These are typically referred to as interrupts, not buffers.
- **Controlling unit in the system:** This refers to the control unit in a CPU, which is different from a buffer.

Conclusion: The correct definition of a buffer is a temporary area of memory used for managing data transfer.

 Quick Tip

Buffers are crucial for ensuring smooth data flow, especially between devices with varying speeds, such as a keyboard and a computer.

Question 60: Function of the Control Unit in the computer system is _____.

1. carries out arithmetic operations.
2. carries out logical operations.
3. send signals to coordinate all components.
4. hold memory locations.

Correct Answer: 3. send signals to coordinate all components.

Solution:

The **Control Unit (CU)** in a computer system is responsible for managing and coordinating all components of the system. It sends signals to direct the flow of data between the processor, memory, and peripherals. It ensures that operations are carried out in a sequential and efficient manner.

- **Carries out arithmetic operations:** This is the responsibility of the Arithmetic Logic Unit (ALU), not the Control Unit.
- **Carries out logical operations:** This is also a function of the ALU.
- **Hold memory locations:** This is handled by the Memory Unit.

Conclusion: The Control Unit's primary role is to send signals to coordinate all components of the computer system, ensuring synchronized operations.

 Quick Tip

The Control Unit acts like the brain's conductor, directing how data and instructions flow within a computer system.

Question 61: _____ is an example of an Output Device.

1. Projector
2. Scanner
3. QR Reader
4. Joystick

Correct Answer: 1. Projector

Solution:

An **Output Device** is a hardware component that conveys information from a computer to the user or another device. Among the options:

- **Projector:** It displays visual output on a screen and is an output device.
- **Scanner:** Used to input images or documents into a computer, making it an input device.
- **QR Reader:** Reads QR codes and is also an input device.
- **Joystick:** A device used to provide input for games or simulations.

Conclusion: The projector is the correct example of an output device as it projects visual data from the computer.

💡 Quick Tip

Output devices like projectors and monitors are used to display data, while input devices like scanners and joysticks are used to interact with the computer.

Question 62: A set of possible data values for a database is called _____.

1. Degree
2. Attribute
3. Domain
4. Field

Correct Answer: 3. Domain

Solution:

In the context of databases:

- **Domain:** Refers to the set of possible values that a database attribute can have. For example, a "Gender" attribute might have a domain of "Male", "Female".
- **Degree:** Refers to the number of attributes in a relation (not the set of values).
- **Attribute:** Refers to a column or field in a table, not a set of values.
- **Field:** Refers to a single piece of data within a record.

Conclusion: The domain is the correct term for the set of possible data values for an attribute in a database.

💡 Quick Tip

Understanding database terminologies such as domain, degree, and attributes is essential for designing efficient databases.

Question 63: Arrange the following steps of Pseudocode for counting even numbers between 50 and 500 including both values.

- (A). num = 50
 count = 0
(B). PRINT count
(C). num = num + 2
 count = count + 1
(D). WHILE num $\leq$ 500 THEN

Options:

1. (A), (C), (D), (B)
2. (B), (A), (C), (D)
3. (A), (B), (D), (C)
4. (A), (D), (C), (B)

Correct Answer: 4. (A), (D), (C), (B)

Solution:

The correct sequence is as follows:

1. Initialize the variables: **num = 50** and **count = 0** (Step A).
2. Create a loop: **WHILE num $\leq$ 500 THEN** (Step D).
3. Inside the loop:
 - Increment **num** by 2 and increment **count** by 1 (Step C).
4. After exiting the loop, print the count of even numbers (Step B).

Explanation:

- Step (A) initializes the starting value and count.
- Step (D) creates the iteration condition.
- Step (C) updates the values within the loop.
- Step (B) provides the final output.

 Quick Tip

Pseudocode structure should follow initialization, loop/condition, iterative steps, and final output.

Question 64: ATM stands for _____.

1. Any Time Money
2. Automated Teller Machine
3. Automated Transfer Money
4. Automatic Transfer Machine

Correct Answer: 2. Automated Teller Machine

Solution:

The term **ATM (Automated Teller Machine)** refers to an electronic banking outlet that allows customers to perform financial transactions, such as withdrawing cash, checking account balances, or transferring money, without the need for direct human assistance.

- **Option 1:** "Any Time Money" is an informal phrase, not the correct expansion.
- **Option 3:** "Automated Transfer Money" refers to money transfers but is not the correct acronym.
- **Option 4:** "Automatic Transfer Machine" is not the correct term.

Conclusion: Option 2 is the correct answer as ATM officially stands for Automated Teller Machine.

 Quick Tip

ATMs provide 24/7 banking services and are widely used for convenient cash withdrawals.

Question 65: Arrange the following steps of an algorithm to calculate the factorial of a given number.

- (A) Keep multiplying the Result with *num* until the *num* decreases to 1.
- (B) After each multiplication, subtract 1 from *num*.
- (C) Accept a number, *num*, from the user and set the value of *Result* as 1.
- (D) Display the Result.

Options:

1. (C), (A), (B), (D)
2. (A), (B), (D), (C)
3. (C), (D), (A), (B)
4. (C), (B), (D), (A)

Correct Answer: 1. (C), (A), (B), (D)

Solution:

The correct sequence for calculating the factorial of a number is as follows:

- **Step 1 (C):** Accept the input number and initialize *Result* to 1.
- **Step 2 (A):** Multiply *Result* by the current value of *num*.
- **Step 3 (B):** Decrease the value of *num* by 1 after each multiplication.
- **Step 4 (D):** Finally, display the value of *Result*.

Conclusion: The logical order of the algorithm steps is (C), (A), (B), (D).

 Quick Tip

Factorial algorithms are commonly used in permutations, combinations, and mathematical series computations.

Question 66: Arrange the following according to the given sequence: (Input Device – Output Device – Storage Device – Operating System)

- (A) Ubuntu
- (B) Printer
- (C) Blue Ray Disk
- (D) Scanner

Options:

1. (A), (B), (C), (D)
2. (D), (B), (C), (A)
3. (B), (A), (D), (C)
4. (C), (B), (D), (A)

Correct Answer: 2. (D), (B), (C), (A)

Solution:

The correct order based on the given sequence is:

- **Input Device:** (D) Scanner is used to input data into the system.
- **Output Device:** (B) Printer is used to provide output from the system.
- **Storage Device:** (C) Blue Ray Disk is a storage medium for saving data.
- **Operating System:** (A) Ubuntu is an operating system that manages hardware and software.

Conclusion: The arrangement (D), (B), (C), (A) matches the sequence Input Device – Output Device – Storage Device – Operating System.

 Quick Tip

Understanding the roles of input, output, storage, and operating system helps in categorizing devices and software effectively.

Question 67: Which of the following statement(s) are TRUE?

- (A) Digital Communication includes E-mail, texting, and instant messaging.
(B) Trojan is an example of Social media platform.
(C) Credit card details of a person is an example of sensitive data.
(D) Digital citizen is anyone who uses digital technology along with the internet.

Options:

1. (A), (B) and (D) only.
2. (A), (B) and (C) only.
3. (A), (B), (C) and (D).
4. (A), (C) and (D) only.

Correct Answer: 4. (A), (C) and (D) only.

Solution:

- **(A):** Digital communication includes E-mail, texting, and instant messaging. This is correct as these are primary examples of digital communication methods.
- **(B):** Trojan is a type of malware, not a social media platform. Hence, this statement is incorrect.
- **(C):** Credit card details are considered sensitive data because they require protection to prevent fraud. This is correct.
- **(D):** A digital citizen refers to anyone who uses digital technology along with the internet responsibly. This statement is correct.

Conclusion: Statements (A), (C), and (D) are true, while (B) is incorrect.

 Quick Tip

Always verify statements by their definitions and correct contexts in digital literacy to avoid confusion.

Question: 68 Arrange the following data elements in order from smallest to largest.

- (A) Field
(B) File
(C) Database
(D) Record

Options:

1. (A), (B), (D), (C).
2. (B), (A), (C), (D).
3. (A), (D), (B), (C).
4. (B), (A), (D), (C).

Correct Answer: 3. (A), (D), (B), (C).

Solution:

- **Field:** The smallest unit in a data structure that holds a single value.
- **Record:** A collection of related fields representing a single data entity.
- **File:** A collection of records stored together.
- **Database:** The largest data structure, consisting of multiple related files.

Conclusion: The correct order from smallest to largest is Field, Record, File, Database.

 Quick Tip

Understanding the hierarchy of data elements helps in organizing and managing databases effectively.

Question 69: Which of the following statements are clear examples of Computer Ethics followed?

- (A) An engineer works for a software company during the day but also does freelancing without informing his parent company.
- (B) A company employs people in different countries to write software modules, and these modules are put together to produce software packages.
- (C) A software engineer develops software for a security company. He used the same features for developing software for his friend's organization.
- (D) A software engineer undergoes training by a different company to improve his software development skills.

Options 69:

1. (B), (D) only.
2. (A), (B) and (C) only.
3. (C), (D) only.
4. (B), (C) and (D) only.

Correct Answer: 1. (B), (D) only.

Solution:

- **Option (B):** Employing people globally to work on software modules and combining them ethically follows computer ethics.
- **Option (D):** Improving skills through training provided by another company is a professional and ethical practice.
- **Option (A):** Freelancing without informing the parent company is a violation of computer ethics.

- **Option (C):** Using the same features for a friend's organization without consent violates intellectual property ethics.

Conclusion: Statements (B) and (D) are clear examples of computer ethics being followed.

💡 Quick Tip

Always seek proper permissions and ensure transparency in your professional engagements to adhere to computer ethics.

Question 70: _____ is an extension of Batch file.

1. .exe
2. .doc
3. .wav
4. .bat

Correct Answer: 4. .bat

Solution:

A batch file is a script file in Windows, containing a series of commands to be executed by the command-line interpreter. The extension **.bat** is used to indicate batch files.

- **Option 1 (.exe):** This is an executable file format for Windows programs.
- **Option 2 (.doc):** This is a file format for Microsoft Word documents.
- **Option 3 (.wav):** This is a file format for audio files.

Conclusion: The correct extension for a batch file is **.bat**.

💡 Quick Tip

Batch files are primarily used for automating repetitive tasks using command-line instructions in Windows.

Question 71: Which of the following statement(s) is/are TRUE?

- (A) Virus is a program code that causes the computer to malfunction.
(B) Phishing means sending legitimate looking emails to encourage users to give out sensitive data.
(C) Firewall examines traffic between users computer and public network.
(D) Freeware is a software that a user can download free of charge.

1. (A), (B) and (D) only.
2. (A), (B) and (C) only.
3. (A), (B), (C) and (D).
4. (B), (C) and (D) only.

Correct Answer: 3. (A), (B), (C) and (D).

Solution:

All the given statements are correct:

- **Statement A:** A virus is indeed a malicious program designed to damage or disrupt computers.
- **Statement B:** Phishing is a common cyber-attack that deceives users to reveal sensitive information.
- **Statement C:** A firewall acts as a barrier that monitors and controls incoming and outgoing traffic.
- **Statement D:** Freeware is software provided at no cost and can be downloaded freely.

Conclusion: Since all the statements are true, the correct option is **3. (A), (B), (C), and (D).**

 Quick Tip

To protect against phishing and viruses, ensure to verify email senders and use updated firewalls and antivirus software.

Question 72: Arrange the following types of printers in the increasing order of their printing speed.

- (A) Inkjet Printer
- (B) Laser Printer
- (C) Dot Matrix Printer
- (D) Daisy Wheel Printer

1. (B), (A), (D), (C)
2. (A), (B), (D), (C)
3. (B), (A), (C), (D)
4. (D), (C), (A), (B)

Correct Answer: 4. (D), (C), (A), (B).

Solution:

The printing speeds of the printers in increasing order are:

- **Daisy Wheel Printer (D):** Slowest printing speed due to mechanical operation.
- **Dot Matrix Printer (C):** Slightly faster than Daisy Wheel but still uses impact technology.
- **Inkjet Printer (A):** Faster than impact printers, with better resolution.
- **Laser Printer (B):** The fastest among these, using advanced laser technology.

Conclusion: The correct order is (D), (C), (A), (B).

 Quick Tip

Laser printers are preferred in high-volume printing due to their speed and efficiency.

Question 73: Match List I with List II:

LIST I Government Initiatives	LIST II Details
A. UMANG	I. Single mobile platform to deliver e-Gov various services across India
B. DIGILOCKER	II. R & D organization of Ministry of Electronics & IT for carrying out R & D in IT, electronics, and other associated areas
C. C-DAC	III. It allows individuals to store, share, and verify documents and certificates
D. PRAGATI	IV. ICT platform of Prime Minister Office to review and monitor various Central Government projects

1. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
2. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
3. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
4. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

Correct Answer: 2. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

Solution:

- **UMANG:** A unified mobile platform providing access to various government services across India.
- **DIGILOCKER:** A platform enabling individuals to securely store, share, and verify digital documents.
- **C-DAC:** A research and development organization under the Ministry of Electronics and IT.
- **PRAGATI:** An ICT-based platform for real-time governance and project monitoring by the Prime Minister's Office.

Conclusion: The associations match option 2 as the correct answer.

 Quick Tip

Acronyms can help in quick recall, e.g., UMANG for unified services and PRAGATI for governance monitoring by the PMO.

Question 74: Verification of a login name and password is known as _____.

1. Configuration
2. Encryption
3. Authentication
4. Automation

Correct Answer: 3. Authentication

Solution:

Authentication is the process of verifying the identity of a user or system by validating login credentials like usernames and passwords. It ensures that access to the system is restricted to authorized users only.

- **Configuration:** Involves setting up hardware or software but is unrelated to login verification.
- **Encryption:** Protects data by converting it into a secure format but does not verify credentials.
- **Automation:** Refers to tasks being performed automatically, unrelated to user verification.

Conclusion: Authentication is the correct term for the process of verifying login credentials.

 Quick Tip

Authentication ensures secure access by validating login credentials, a fundamental process in cybersecurity.

Question 75: Match List I with List II

List I	List II
A. CD	I. It is made up of capacitors and transistors.
B. Magnetic Tape	II. It is a faster memory used to reduce the average time to access data from main memory.
C. Cache Memory	III. It is a long and narrow strip of plastic having thin magnetic material coated on.
D. DRAM	IV. It stores and retrieves data with the help of an optical mechanism.

1. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
2. (A) - (III), (B) - (IV), (C) - (II), (D) - (I)
3. (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
4. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: 1. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

Solution:

- **CD:** Stores and retrieves data using an optical mechanism, making it part of category IV.
- **Magnetic Tape:** A long and narrow strip of plastic coated with magnetic material, aligning with category III.

- **Cache Memory:** A faster memory designed to reduce data access time, categorized under II.
- **DRAM:** Made up of capacitors and transistors, falling under category I.

Conclusion: The correct matching is as per option 1.

 Quick Tip

Understanding the key functionalities of memory and storage devices helps in determining their appropriate categories and practical applications.
