CUET PG Geophysics Question Paper with Solutions

Time Allowed : 1 hour 45 minutes	Maximum Marks : 300	Total questions : 75

General Instructions

Read the following instructions very carefully and strictly follow them:

(i) This question paper comprises 75 questions. All questions are compulsory.

(ii) Each question carries 04 (four) marks.

(iii) For each correct response, candidate will get 04 (four) marks.

(iv) For each incorrect response, 01 (one) mark will be deducted from the total score.

(v) Un-answered/un-attempted response will be given no marks.

(vi) To answer a question, the candidate needs to choose one option as correct option.

(vii) However, after the process of Challenges of the Answer Key, in case there are multiple correct options or change in key, only those candidates who have attempted it correctly as per the revised Final Answer Key will be awarded marks.

(viii) In case a Question is dropped due to some technical error, full marks shall be given to all the candidates irrespective of the fact who have attempted it or not

1. The shape of air film formed between the plano-convex lens and the glass slab in Newton's ring experiment is:

1. Rectangular shape
2. Uniform thickness
3. Wedge shape
4. No air film is formed

Correct Answer: 3. Wedge shape.

Solution:

When a plano-convex lens is placed on a flat glass slab, an air film is created between the two surfaces. The air film has a wedge shape because the curvature of the plano-convex lens causes the thickness of the air film to increase radially outward from the point of contact. This variation in thickness results in the formation of Newton's rings due to constructive and destructive interference of light.

Quick Tip

Newton's rings are an interference pattern observed due to the wedge-shaped air film. These patterns provide valuable information about the wavelength of light and lens properties.

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2. Match List I with List II:

List I	**List II**
A. The linear momentum of the system remains constant	(IV) The net external force acting on a system of particles is zero
B. The angular momentum of the system remains constant	(III) The external torque acting on a system of particles is zero
C. Inertial frame	(I) The frames relative to which an unaccelerated body appears unaccelerated
D. Non-inertial frame	(II) The frames relative to which an unaccelerated body appears accelerated

Choose the correct answer from the options given below:

1. A-III, B-IV, C-I, D-II

2. A-IV, B-III, C-I, D-II

3. A-IV, B-III, C-II, D-I

4. A-III, B-I, C-IV, D-II

Correct Answer: 2. A-IV, B-III, C-I, D-II.

Solution:

The matching relies on fundamental principles of physics:

1. A. Linear momentum conservation: No external force implies no net change in momentum (IV).

2. B. Angular momentum conservation: No external torque implies angular momentum remains constant (III).

3. C. Inertial frame: These frames have no acceleration; unaccelerated bodies appear unaccelerated (I).

4. D. Non-inertial frame: These are accelerating frames; unaccelerated bodies appear to accelerate due to fictitious forces (II).

Quick Tip

Understanding inertial and non-inertial frames is crucial for analyzing dynamics in different reference systems.

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3. Match List I with List II:

Choose the correct answer from the options given below:

. A-I, B-IV, C-II, D-III

2. A-III, B-II, C-IV, D-I

3. A-I, B-II, C-III, D-IV

4. A-III, B-IV, C-II, D-I

Correct Answer: 4. A-III, B-IV, C-II, D-I.

List I (Maxwell's Equations)	List II (Corresponding Laws)
A. Maxwell's First Equation	(III) Gauss Law in Electrostatics
B. Maxwell's Second Equation	(IV) Gauss Law in Magnetostatics
C. Maxwell's Third Equation	(II) Faraday's Laws of Electromagnetic Induction
D. Maxwell's Fourth Equation	(I) Ampère-Maxwell Law

Solution:

Maxwell's equations describe the fundamental laws of electromagnetism:

- Maxwell's First Equation: Gauss's Law for electricity (III).
- Maxwell's Second Equation: Gauss's Law for magnetism (no magnetic monopoles exist) (IV).
- Maxwell's Third Equation: Faraday's Law of Induction (II).
- Maxwell's Fourth Equation: Ampère-Maxwell Law (I).

Quick Tip

Maxwell's equations are the cornerstone of classical electromagnetism and unify electric and magnetic fields.

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4. The efficiency of Carnot's engine working between the steam point and the ice point is:

1. 26.8%
2. 2.68%
3. 62.8%
4. 82.6%

Correct Answer: 1. 26.8%.

Solution:

Using Carnot's efficiency formula:

$$\eta = \frac{T_H - T_L}{T_H},$$

convert temperatures to Kelvin: $T_H = 373\,\text{K}, T_L = 273\,\text{K}$. Substituting:

$$\eta = \frac{373 - 273}{373} = \frac{100}{373} \approx 26.8\%.$$

Quick Tip

Efficiency calculations always require absolute temperatures (Kelvin scale).

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5. Match List I with List II:

List I (Thermodynamic Derivatives)	List II (Maxwell Relations)
A. $(\partial S/\partial P)_T$	(III) $-(\partial V/\partial T)_P$
B. $(\partial T/\partial V)_S$	(IV) $-(\partial P/\partial S)_V$
C. $(\partial T/\partial P)_S$	(II) $(\partial V/\partial S)_P$
D. $(\partial S/\partial V)_T$	(I) $(\partial P/\partial T)_V$

Choose the correct answer from the options given below:

1. A-III, B-IV, C-II, D-I
2. A-II, B-IV, C-III, D-I
3. A-I, B-II, C-III, D-IV
4. A-Iv, B-III, C-II, D-I

Correct Answer: 1. A-III, B-IV, C-II, D-I.

Solution:

The Maxwell relations connect thermodynamic derivatives via the properties of potentials:

- $(\partial S/\partial P)_T = -(\partial V/\partial T)_P$.
- $(\partial S/\partial V)_T = (\partial P/\partial T)_V$.
- $(\partial T/\partial V)_S = -(\partial P/\partial S)_V$.
- $(\partial T/\partial P)_S = (\partial V/\partial S)_P$.

Quick Tip

Maxwell relations simplify the computation of partial derivatives in thermodynamic systems.

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6. Which of the following is true for a reversible process?

1. The pressure and temperature of the working substance and the surroundings should differ at any stage.
2. The process should take place very slowly during operation.
3. The process should take place very fast during operation.
4. Energy should be lost due to conduction or radiation during operation.

Correct Answer: 2. The process should take place very slowly during operation.

Solution:

A reversible process is one where the system and its surroundings are in a state of near-equilibrium throughout the process. To maintain this equilibrium, changes occur extremely slowly, ensuring that there are no abrupt differences in pressure, temperature, or other state variables. Any rapid process would lead to irreversibilities such as turbulence or friction, while energy losses (e.g., conduction or radiation) would make the process irreversible.

Quick Tip

In thermodynamics, reversible processes are idealized processes with no entropy generation. They serve as a benchmark for maximum efficiency.

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7. If the radiation emitted by a star has a maximum intensity at a wavelength of 446 nm, its surface temperature is approximately:

1. 650 K
2. 65 K
3. 6500 K
4. 65000 K

Correct Answer: 3. 6500 K.

Solution:

Using Wien's Displacement Law:

$$\lambda_{\text{max}} \cdot T = 2.9 \times 10^{-3} \text{ m} \cdot \text{K}.$$

Given $\lambda_{\text{max}} = 446 \, \text{nm} = 446 \times 10^{-9} \, \text{m}$:

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{\text{max}}} = \frac{2.9 \times 10^{-3}}{446 \times 10^{-9}} \approx 6500 \, \text{K}.$$

Quick Tip

Wien's law relates the peak wavelength of a blackbody to its temperature. Use λ_{max} in meters and convert the result to Kelvin.

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8. Ultraviolet light of wavelength 350 nm and intensity $1.00 \, \text{W/m}^2$ falls on a potassium surface. The maximum kinetic energy of the photoelectron is:

1. 3.3 eV
2. 1.9 eV
3. 3.2 eV
4. 1.3 eV

Correct Answer: 4. 1.3 eV.

Solution:

Using the energy of a photon:

$$E_\gamma = \frac{hc}{\lambda},$$

where $h = 6.626 \times 10^{-34}$ Js, $c = 3.0 \times 10^8 \, \text{m/s}$, $\lambda = 350 \times 10^{-9} \, \text{m}$:

$$E_\gamma = \frac{6.626 \times 10^{-34} \cdot 3.0 \times 10^8}{350 \times 10^{-9}} \approx 5.68 \times 10^{-19} \, \text{J}.$$

Converting to eV ($1 \, \text{eV} = 1.602 \times 10^{-19} \, \text{J}$):

$$E_\gamma \approx \frac{5.68 \times 10^{-19}}{1.602 \times 10^{-19}} \approx 3.55 \, \text{eV}.$$

The work function of potassium $\phi = 2.25 \, \text{eV}$, so:

$$K_{\text{max}} = E_\gamma - \phi = 3.55 \, \text{eV} - 2.25 \, \text{eV} = 1.30 \, \text{eV}.$$

Quick Tip

For photoelectric effect calculations, remember to convert the photon's energy into eV before subtracting the work function.

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9. Which of the following statements are true for Compton effect?

1. The wavelength of the scattered X-rays is larger than that of the incident X-rays.
2. The change in wavelength depends on the intensity of the incident X-rays.
3. The change in wavelength depends on the scattering angle.
4. The wavelength of the scattered X-rays is smaller than that of the incident X-rays.

Choose the correct answer from the options given below:

1. A and B are correct
2. B and C are correct
3. A and D are correct
4. A and C are correct

Correct Answer: 4. A and C are correct.

Solution:

The Compton effect demonstrates that the wavelength of scattered X-rays is longer than that of incident X-rays. This wavelength shift depends on the scattering angle θ and is given by:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta),$$

where h is Planck's constant, m_e is the electron mass, and c is the speed of light. Intensity has no role in determining the wavelength shift.

Quick Tip

The Compton shift equation shows dependence on angle θ but not on the intensity of the incident beam.

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10. Clausius-Clapeyron's latent heat equation is represented as:

1. $\left(\frac{\partial P}{\partial T}\right)_s = \frac{L}{T(V_2 - V_1)}$

2. $\left(\frac{\partial T}{\partial P}\right)_s = \frac{L}{T(V_2 - V_1)}$
3. $\left(\frac{\partial V}{\partial T}\right)_s = \frac{L}{T(V_2 - V_1)}$
4. $\left(\frac{\partial P}{\partial T}\right)_s = \frac{L}{V(V_2 - V_1)}$

Correct Answer: 1. $\left(\frac{\partial P}{\partial T}\right)_s = \frac{L}{T(V_2 - V_1)}$.

Solution:

The Clausius-Clapeyron equation relates the slope of a phase boundary in a P-T diagram to the latent heat of the phase transition:

$$\frac{dP}{dT} = \frac{L}{T\Delta V},$$

where L is the latent heat, T is the absolute temperature, and $\Delta V = V_2 - V_1$ is the change in specific volume during the phase transition.

Quick Tip

This equation is critical for understanding phase transitions and calculating equilibrium curves.

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11. X-ray of wavelength 10.0 pm are scattered from a target in a Compton experiment. If the X-rays are scattered through $45°$, the scattered wavelength is:

1. 1.07 pm
2. 70.1 pm
3. 10.7 pm
4. 107 pm

Correct Answer: 3. 10.7 pm.

Solution:

The Compton shift formula is:

$$\Delta\lambda = \frac{h}{m_e c}\big(1 - \cos\theta\big).$$

For $\theta = 45°$, and using $\frac{h}{m_e c} \approx 2.43\,\text{pm}$,

$$\Delta\lambda = 2.43\,\text{pm} \times \big(1 - \cos 45°\big) \approx 2.43\,\text{pm} \times 0.2929 \approx 0.71\,\text{pm}.$$

Hence,

$$\lambda_{\text{scattered}} = \lambda_{\text{initial}} + \Delta\lambda = 10.0\,\text{pm} + 0.71\,\text{pm} = 10.71\,\text{pm} \approx 10.7\,\text{pm}.$$

Quick Tip

Keep in mind that $\frac{h}{m_e c} \approx 2.43 \times 10^{-12}\,\text{m}$. For $\theta < 90^\circ$, the Compton shift is relatively small.

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12. Which of the following statements are correct for the Second Law of Thermodynamics?

1. It is impossible to derive a continuous supply of energy from a body by cooling it below the temperature of its surroundings.

2. It is possible to derive a continuous supply of energy from a body by cooling it below the temperature of its surroundings.

3. It is impossible for a self-acting machine, unaided by any external agency, to convey heat from a body at lower temperature to another body at higher temperature.

4. It is possible for a self-acting machine, unaided by any external agency, to convey heat from a body at lower temperature to another body at higher temperature.

Choose the correct answer from the options given below:

1. A. and B. only
2. B. and C. only
3. B. and D. only
4. A. and C. only

Correct Answer: 4. A. and C. only.

Solution:

Statements A. and C. summarize two well-known impossibility statements of the second law: 1. No heat engine can continuously produce work by just cooling a single reservoir below its surroundings.

2. No spontaneous (self-acting) device can transfer heat from cold to hot without external energy.

Quick Tip

The Second Law has several equivalent formulations (Kelvin–Planck and Clausius statements). They all forbid "something for nothing" from a single reservoir or spontaneous cold-to-hot flow.

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13. 10 g of water at $100°C$ is converted into steam at the same temperature. If the latent heat of steam is 540 cal/g, the change in entropy is:

1. 14.47 cal/K
2. 144.7 cal/K
3. 74.47 cal/K
4. 47.47 cal/K

Correct Answer: 1. 14.47 cal/K

Solution:

$$\text{Heat absorbed, } Q = m\,L = 10\,\text{g} \times 540\,\text{cal/g} = 5400\,\text{cal}.$$

At $100°C$, the temperature in Kelvin is $T = 373\,\text{K}$. Change in entropy:

$$\Delta S = \frac{Q}{T} = \frac{5400\,\text{cal}}{373\,\text{K}} \approx 14.47\,\text{cal/K}.$$

Quick Tip

For phase changes, entropy change is given by $\Delta S = Q/T = mL/T$, where L is latent heat.

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14. Higher-energy photons are scattered from electrons initially at rest. Assume the photons are backscattered ($\theta = 180°$) and their energies are much higher than the electron's rest mass energy. The wavelength shift will be $\frac{h}{m_e c} = 2.43 \times 10^{-12}\,\text{m}$:

1. $4.86 \times 10^{-10}\,\text{m}$
2. $48.6 \times 10^{-12}\,\text{m}$
3. $4.86 \times 10^{-12}\,\text{m}$

4. 0.486×10^{-12} m

Correct Answer: 3. 4.86×10^{-12} m.

Solution:

Compton shift formula: $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$. For backscatter, $\theta = 180^\circ \implies \cos 180^\circ = -1$.

Thus,

$$\Delta\lambda = \frac{h}{m_e c}\left[1 - (-1)\right] = 2\,\frac{h}{m_e c} = 2 \times 2.43 \times 10^{-12}\,\text{m} = 4.86 \times 10^{-12}\,\text{m}.$$

Quick Tip

Backscattering ($\theta = 180^\circ$) doubles the maximum Compton shift compared to $\theta = 0^\circ$.

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15. Which statements are correct for a P–N junction diode?

A. For forward bias, P-side is connected to the positive terminal and N-side to the negative terminal of the battery.

B. For forward bias, P-side is connected to the negative terminal and N-side to the positive terminal.

C. For reverse bias, P-side is connected to the negative terminal and N-side to the positive terminal.

D. For reverse bias, P-side is connected to the positive terminal and N-side to the negative terminal.

Choose the correct answer from the options given below:

1. A and B only
2. A and D only
3. A and C only
4. B, C, and D only

Correct Answer: 3. A and C only

Solution:

- In forward bias: P-side is connected to the positive terminal, and N-side to the negative terminal of the battery (statement A is correct; B is incorrect).

- In reverse bias: P-side is connected to the negative terminal, and N-side to the positive terminal (statement C is correct; D is incorrect).

The correct biasing ensures the junction operates as intended: forward bias reduces the depletion region, allowing current flow, while reverse bias increases it, preventing significant conduction.

Quick Tip

Forward bias lowers the junction barrier, enabling current flow; reverse bias increases the barrier, blocking current except for leakage.

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16. In an LCR circuit, the current and emf differ in phase by Φ. The value of Φ is:

1. $\Phi = \tan^{-1}\left[(X_L - X_C)/R\right]$
2. $\Phi = \tan^{-1}\left[(X_L + X_C)/R\right]$
3. $\Phi = \tan^{-1}\left[R/(X_L - X_C)\right]$
4. $\Phi = \tan^{-1}\left[R/(X_L + X_C)\right]$

Correct Answer: 1. $\Phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$.

Solution:

For a series LCR circuit, the total impedance is:

$$Z = \sqrt{R^2 + (X_L - X_C)^2},$$

where $X_L = \omega L$ is the inductive reactance, $X_C = 1/(\omega C)$ is the capacitive reactance, and R is the resistance. The phase angle Φ (between voltage and current) satisfies:

$$\tan\Phi = \frac{X_L - X_C}{R}.$$

If $X_L > X_C$, the circuit behaves inductively, and the voltage leads the current. If $X_C > X_L$, the circuit is capacitive, and the current leads the voltage.

Quick Tip

The relative sizes of X_L and X_C determine whether the circuit is inductive or capacitive. Resonance occurs when $X_L = X_C$.

17. If the resonance frequency of an acoustic system is 300 Hz and the half-power frequencies are 150 Hz and 450 Hz, the quality factor is:

1. 1.25

2. 1.50

3. 1.0

4. 1.75

Correct Answer: 3. 1.0.

Solution:

The quality factor Q around a resonance frequency f_0 is defined as:

$$Q = \frac{f_0}{\Delta f},$$

where $\Delta f = f_2 - f_1$ is the bandwidth between the half-power frequencies. Substituting the given values:

$$f_0 = 300\,\text{Hz}, \quad f_1 = 150\,\text{Hz}, \quad f_2 = 450\,\text{Hz},$$

$$\Delta f = f_2 - f_1 = 450\,\text{Hz} - 150\,\text{Hz} = 300\,\text{Hz}.$$

Thus,

$$Q = \frac{f_0}{\Delta f} = \frac{300}{300} = 1.0.$$

Quick Tip

The quality factor Q indicates the sharpness of the resonance. A higher Q corresponds to a narrower bandwidth.

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18. A Fraunhofer diffraction pattern is produced by a circular aperture of radius 0.05 cm at the focal plane of a convex lens of focal length 20 cm. If the wavelength $\lambda = 5 \times 10^{-5}\,\text{cm}$, the radius of the first dark ring is:

1. $1.22 \times 10^{-3}\,\text{cm}$

2. $12.20 \times 10^{-3}\,\text{cm}$

3. $12.20 \times 10^{-2}\,\text{cm}$

4. $12.20\,\text{cm}$

Correct Answer: 2. $12.20 \times 10^{-3}\,\text{cm}$.

Solution:

The radius of the first dark ring in Fraunhofer diffraction for a circular aperture is given by:

$$r_1 = 1.22\,\frac{\lambda f}{D},$$

where D is the diameter of the aperture. Given:

$$\text{Radius } a = 0.05\,\text{cm}, \quad D = 2a = 0.1\,\text{cm}, \quad f = 20\,\text{cm}, \quad \lambda = 5 \times 10^{-5}\,\text{cm}.$$

Substitute the values:

$$r_1 = 1.22 \times \frac{5 \times 10^{-5} \times 20}{0.1} = 1.22 \times 10^{-2}\,\text{cm} = 12.20 \times 10^{-3}\,\text{cm}.$$

Quick Tip

Fraunhofer diffraction assumes the light source and observation screen are effectively at infinity, realized using lenses.

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19. Brewster's law can be expressed as:

1. $\mu = \tan r$, where μ is refractive index and r is the angle of refraction

2. $\tan i = \frac{1}{\mu}$, where μ is refractive index and i is the polarizing angle

3. $\cos r = \mu$, where μ is refractive index and r is the angle of refraction

4. $\mu = \tan i$, where μ is refractive index and i is the polarizing angle

Correct Answer: 4. $\mu = \tan i$.

Solution:

According to Brewster's law, the polarizing angle i_p satisfies:

$$\tan i_p = \mu,$$

where μ is the refractive index of the medium. At this angle, the reflected light is completely plane-polarized.

Quick Tip

At Brewster's angle, the reflected and refracted rays are perpendicular to each other.

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20. In a Fraunhofer N-slit diffraction experiment, a grating has 5000 lines/cm, and monochromatic light of wavelength $5 \times 10^{-5}\,\text{cm}$ is used. What is the highest order spectrum that may be observed?

1. 1
2. 2
3. 3
4. 4

Correct Answer: 4. 4.

Solution:

The grating equation is:

$$d\,\sin\theta = n\,\lambda,$$

where $d = 1/\text{number of lines per cm} = 1/5000 = 2 \times 10^{-4}\,\text{cm}$. For maximum diffraction ($\sin\theta \leq 1$):

$$n\,\lambda \leq d \quad \Rightarrow \quad n \leq \frac{d}{\lambda} = \frac{2 \times 10^{-4}}{5 \times 10^{-5}} = 4.$$

Thus, the highest observable integer order is $n = 4$.

Quick Tip

For diffraction gratings, the maximum order is determined by the ratio d/λ.

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21. The function $f(x)$ is said to be piecewise continuous, if it satisfies the following conditions (Dirichlet conditions):

(A) The function must have a finite number of maxima and minima.

(B) The function must have a finite number of infinite discontinuities, in a period of one oscillation.

(C) The function must have an infinite number of maxima and minima.

(D) The function must have a finite number of finite discontinuities, in a period of one oscillation.

Choose the correct answer from the options given below:

1. A. and D. only.
2. A. and C. only.
3. B and C. only.
4. B and D. only.

Correct Answer: 1. A and D only.

Solution:

A piecewise continuous function satisfies specific conditions, such as having a finite number of maxima, minima, and finite discontinuities within any given interval. Infinite discontinuities are excluded, as they make the function non-piecewise continuous. These conditions allow the function to be represented and analyzed using mathematical tools like Fourier series.

Quick Tip

Piecewise continuity ensures that functions are manageable for integration and analysis in various mathematical and physical contexts.

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22. Match List I with List II:

List I   List II

A. Fraunhofer Diffraction   I. Interaction of the light waves from two different wave fronts.

B. Fresnel Diffraction   II. The distance between the source and the screen are effectively at infinite distance.

C. Interference of Light   III. It's a phenomenon in which the wave vibrations are restricted to a particular direction in a plane.

D. Polarization of Light   IV. The source and screen or both are at finite distances from the aperture or obstacle.

Choose the correct answer from the options given below:

1. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

2. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

3. (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

4. (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

Correct Answer: 4. (A) - (II), (B) - (IV), (C) - (I), (D) - (III).

Solution:

Fraunhofer diffraction (A) occurs when the source and the observation screen are at large distances, creating parallel light waves (II). Fresnel diffraction (B) involves finite distances between the source, aperture, and screen, requiring the consideration of wavefront curvature (IV). Interference (C) results from the interaction of waves from different sources or paths (I). Polarization (D) restricts wave vibrations to specific directions in a plane (III).

Quick Tip

Understanding these optical phenomena is essential for applications in microscopy, spectroscopy, and communication systems.

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23. The inter-molecular distance between two atoms of a hydrogen molecule is 0.77 Å and the mass of a proton is 1.67×10^{-27} Kg. The moment of inertia of a molecule is:

1. 4.95×10^{-47} Kg-m^2

(b) 0.495×10^{-47} Kg-m^2

3. 4.95×10^{-47} Kg-m^2

4. 45.9×10^{-47} Kg-m^2

Correct Answer: (b) 0.495×10^{-47} Kg-m^2.

Solution:

The moment of inertia I for a diatomic molecule is given by:

$$I = \mu r^2,$$

where μ is the reduced mass and r is the inter-nuclear distance. For H_2:

$$\mu = \frac{m \cdot m}{2m} = \frac{m}{2} = 0.835 \times 10^{-27}\,\text{kg}, \quad r = 0.77 \times 10^{-10}\,\text{m}.$$

Substituting:

$$I = 0.835 \times 10^{-27} \cdot (0.77 \times 10^{-10})^2 = 0.495 \times 10^{-47}\,\text{kg} \cdot \text{m}^2.$$

Quick Tip

Moment of inertia calculations are vital for determining rotational spectra in molecular physics.

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24. For Fresnel half period zones:

(A) The amplitude of disturbance is directly proportional to the area of the zone.

(B) The radii of half period zones are directly proportional to the square root of the natural number.

(C) The area of each zone is directly proportional to the wavelength of the light.

(D) The radii of half period zones are directly proportional to the square root of the wavelength of the light.

Choose the correct answer from the options given below:

1. (A), (B), and (C) only.
2. (B), (C), and (D) only.
3. (A), (B), (C), and (D) only.
4. (A), (C), and (D) only.

Correct Answer: 4. (A), (C), and (D) only

Solution:

Fresnel half-period zones divide a wavefront into concentric regions, each contributing a specific phase difference to the resultant amplitude at a point. The properties of these zones include: (A) True: Amplitude is proportional to the area of the zone.

(C) True: Zone area depends on the wavelength.

(D) True: Radii scale with the square root of the wavelength.

Quick Tip

Fresnel zones explain the intensity variations observed in diffraction patterns.

25. In order to introduce a phase difference of $\pi/2$ in a Quarter wave plate, the thickness of the crystal should have a value of... (Given: wavelength = 5893 $\times 10^{-10}$ m, refractive index for O-ray is 1.65836 and refractive index for E-ray is 1.48641):

1. 8.57 $\times 10^{-4}$ mm.
2. 8.57 $\times 10^{-4}$ cm.
3. 8.57 $\times 10^{-4}$ m.
4. 8.57 $\times 10^{-4}$ nm.

Correct Answer: 1. 8.57 $\times 10^{-4}$ mm.

Solution:

The thickness d required to introduce a phase difference of $\pi/2$ is given by:

$$d = \frac{\lambda}{4 \cdot |n_o - n_e|}.$$

Substitute:

$$\lambda = 5893 \times 10^{-10}\,\text{m}, \quad n_o = 1.65836, \quad n_e = 1.48641.$$

$$d = \frac{5893 \times 10^{-10}}{4 \cdot |1.65836 - 1.48641|} \approx 8.57 \times 10^{-6}\,\text{m} = 8.57 \times 10^{-4}\,\text{mm}.$$

Quick Tip

Quarter-wave plates are vital in optics for converting linear polarization to circular polarization.

26. In the case of a rolling solid sphere on an inclined plane making an angle of 30° with the horizontal plane, the acceleration of the sphere rolling down the plane is (where acceleration due to gravity is 9.8 m/s^2):

1. 3.5 m/s^2
2. 35 m/s^2
3. 5.3 m/s^2
4. 0.53 m/s^2

Correct Answer: 1. 3.5 m/s^2.

Solution:

For a solid sphere rolling without slipping, the acceleration a down an inclined plane is:

$$a = \frac{5}{7} g \sin(\theta).$$

Here, $\theta = 30^\circ$ and $g = 9.8\,\mathrm{m/s^2}$. Substituting:

$$a = \frac{5}{7} \cdot 9.8 \cdot \sin(30^\circ) = \frac{5}{7} \cdot 9.8 \cdot 0.5 = 3.5\,\mathrm{m/s^2}.$$

This accounts for both translational and rotational motion, reducing the acceleration compared to an object sliding without friction.

Quick Tip

Rolling motion involves both rotational and translational energy. The factor $5/7$ results from the moment of inertia of a solid sphere.

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27. In an LCR circuit with $L = 2$ mH, $C = 2$ μF, and $R = 0.2$, the quality factor will be:

1. 100
2. 10
3. 1000
4. 1

Correct Answer: 1. 100.

Solution:

The quality factor Q of an LCR circuit is given by:

$$Q = \frac{\omega_0 L}{R},$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$ is the resonant angular frequency. First, calculate ω_0:

$$\omega_0 = \frac{1}{\sqrt{(2 \times 10^{-3}) \cdot (2 \times 10^{-6})}} \approx 500\,\mathrm{rad/s}.$$

Substitute into Q:

$$Q = \frac{\omega_0 L}{R} = \frac{500 \cdot 2 \cdot 10^{-3}}{0.2} = 5 \times 10 = 100.$$

Quick Tip

The quality factor measures the sharpness of resonance. A higher Q indicates lower energy loss.

---

28. The four satellites are lying close to the earth at distances $h_1, h_2, h_3,$ and h_4 meters respectively, away from the center of the earth. If the values of h_i's are given in terms of R (radius of the earth), write the time period of the satellite in increasing order:

1. $h_1 = R/3$
2. $h_2 = R/4$
3. $h_3 = R/5$
4. $h_4 = R/2$

Choose the correct answer from the options given below:

1. (C), (B), (A), (D)
2. (B), (A), (C), (D)
3. (B), (A), (D), (C)
4. (C), (B), (D), (A)

Correct Answer: 1. (C), (B), (A), (D)

Solution:

The orbital period T is related to the distance $r = R + h$ from the Earth's center by Kepler's third law:

$$T \propto \sqrt{r^3}.$$

The smaller the altitude h, the shorter the orbital period. Given:

$$h_3 = R/5 < h_2 = R/4 < h_1 = R/3 < h_4 = R/2,$$

the corresponding periods are in the order: $T_3 < T_2 < T_1 < T_4$.

Quick Tip

Kepler's third law explains the relationship between orbital distance and period, crucial for satellite deployment.

29. The position vector of a point in the frame S moving with constant velocity 10 cm/s along the X-axis is given by (11,9.8) cm. The position with respect to S if the two frames were coincident only 1/2 second earlier:

1. (11, 9.8)
2. (16, 9.8)
3. (16, 13.10)
4. (11, 13.10)

Correct Answer: 2. (16, 9.8).

Solution:

The relative motion is along the x-axis. The displacement of frame S in 0.5 seconds is:

$$\Delta x = 10\,\text{cm/s} \cdot 0.5\,\text{s} = 5\,\text{cm}.$$

Adding this displacement to the x-coordinate:

$$x_{\text{new}} = 11 + 5 = 16\,\text{cm}.$$

The y-coordinate remains unchanged ($y = 9.8\,\text{cm}$).

Quick Tip

Relative motion calculations require identifying the direction of velocity and any relative displacements.

---

30. If the radius of Earth becomes half of its present value, with its mass remaining the same, the duration of one day will become:

1. 8h
2. 6h
3. 12h
4. 18h

Correct Answer: 2. 6h.

Solution:

The Earth's rotational period depends on its moment of inertia $I = kMR^2$. Halving the radius R changes the moment of inertia:

$$I_{\text{new}} = \frac{1}{4} I_{\text{old}}.$$

With angular momentum conserved ($L = I\omega$):

$$\omega_{\text{new}} = \frac{\omega_{\text{old}}}{0.25} = 4\omega_{\text{old}},$$

where $\omega = \frac{2\pi}{T}$. Thus:

$$T_{\text{new}} = \frac{T_{\text{old}}}{4} = \frac{24}{4} = 6\,\text{h}.$$

Quick Tip

Changes in Earth's size alter rotational periods due to moment of inertia's dependence on radius squared.

---

31. For Newton's Ring Experiment:

(A) The condition to get constructive interference is $2nt = (m + 1/2)$, $m = 0, 1, 2, \ldots$, where n is the refractive index of the film and t is the thickness of the film.

(B) The condition to get destructive interference is $2nt = m\lambda$, $m = 0, 1, 2, \ldots$, where n is the refractive index of the film and t is the thickness of the film.

(C) The condition to observe the colored ring is that the source of light should be polychromatic.

(D) The condition to observe the colored ring is that the source of light should be monochromatic.

Choose the correct answer from the options given below:

(a) (A), (B), and (D) only

(b) (A), (B), and (C) only

(c) (A), (B), (C), and (D)

(d) (B), (C), and (D) only

Correct Answer: (b) (A), (B), and (C) only.

Solution:

In Newton's rings: - Constructive interference: $2nt = (m + 1/2)\lambda$, where $m = 0, 1, 2, \ldots$,

- Destructive interference: $2nt = m\lambda$, where $m = 0, 1, 2, \ldots$,

- Colored rings appear only with polychromatic light because the interference condition varies for each wavelength. Monochromatic light will produce a uniform fringe pattern.

Quick Tip

Polychromatic light produces the beautiful colored rings in Newton's ring experiments due to different wavelengths interfering constructively at different points.

---

32. In Michelson Interferometer, the distance traversed by the mirror between two successive disappearances is 0.289 mm. The difference between the wavelengths of two lines is (Assume the wavelength of one line is 5890 Å):

(a) 6 Å

(b) 12 Å

(c) 120 Å

(d) 60 Å

Correct Answer: (b) 12 Å.

Solution:

In a Michelson interferometer, the fringe spacing or wavelength difference is related to the distance traveled by the movable mirror:

$$\Delta\lambda = \frac{\lambda^2}{2\Delta x}.$$

Here:

$$\Delta x = 0.289\,\text{mm} = 0.289 \times 10^{-3}\,\text{m}, \quad \lambda = 5890\,\text{Å} = 5890 \times 10^{-10}\,\text{m}.$$

Substitute into the formula:

$$\Delta\lambda = \frac{(5890 \times 10^{-10})^2}{2 \cdot (0.289 \times 10^{-3})} \approx 12\,\text{Å}.$$

Quick Tip

Michelson interferometers are highly sensitive to small wavelength differences, making them ideal for measuring fine spectral lines.

---

33. Consider a silicon pn junction at $T = 300\,K$ with doping concentrations of acceptor $N_A = 10^{16}$ cm^{-3} and donor $N_D = 10^{15}$ cm^{-3}. Assume that intrinsic concentration $n_i = 1.5 \times 10^{10}$ cm^{-3}, relative permittivity = 11.7 and $V_{bi} = 0.635$ V. The width of the space charge region in the p-n junction is:

(a) 9.51 m

(b) 0.951 m

(c) 95.1 m

(d) 5.91 m

Correct Answer: (a) 9.51 m.

Solution:

The width of the space charge region W is:

$$W = \sqrt{\frac{2\epsilon V_{bi}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}.$$

Given:

$$\epsilon = \epsilon_r \epsilon_0 = (11.7)(8.854\times10^{-14})\,\text{F/cm}, \quad V_{bi} = 0.635\,\text{V}, \quad N_A = 10^{16}\,\text{cm}^{-3}, \quad N_D = 10^{15}\,\text{cm}^{-3}.$$

Substitute these into the formula:

$$W \approx 9.51\,\mu\text{m}.$$

Quick Tip

The width of the depletion region depends on doping levels and built-in voltage. Lower doping widens the depletion region.

---

34. Match List I with List II

LIST I	LIST II
A. Bipolar npn transistor operate in the cut-off mode.	I. Both junctions are reverse biased.
B. Bipolar npn transistor operate in the saturation mode.	II. Both junctions are forward biased.
C. Bipolar npn transistor operate in the inverse active mode.	III. Base-emitter forward biased; base-collector reverse biased.
D. Bipolar npn transistor operate in the forward active mode.	IV. Base-emitter reverse biased; base-collector forward biased.

Choose the correct answer from the options given below:

(a) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

(b) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

(c) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(d) (A) - (IV), (B) - (I), (C) - (II), (D) - (III)

Correct Answer: (c) (A) - (I), (B) - (II), (C) - (III), (D) - (IV).

Solution:

The operating modes of BJTs are defined by the biasing of the base-emitter and base-collector junctions:

- Cut-off mode: Both junctions reverse biased.
- Saturation mode: Both junctions forward biased.
- Forward active mode: Base-emitter forward biased; base-collector reverse biased.
- Inverse active mode: Base-emitter reverse biased; base-collector forward biased.

Quick Tip

The correct mode of operation for a transistor is essential for its application as an amplifier, switch, or oscillator.

---

35. Match List I with List II

LIST I	LIST II
A. $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	I. Gauss's Law in magnetostatics
B. $\nabla \cdot \mathbf{B} = 0$	II. Faraday's Law of electromagnetic induction
C. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	III. Gauss's Law in electrostatics
D. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	IV. Modified Ampere's Law

Choose the correct answer from the options given below:

(a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(b) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

(c) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

(d) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (b) (A) - (III), (B) - (I), (C) - (II), (D) - (IV).

Solution:

- $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$: Gauss's Law in electrostatics relates the electric field divergence to charge density.

- $\nabla \cdot \mathbf{B} = 0$: Gauss's Law in magnetostatics states there are no magnetic monopoles.

- $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$: Faraday's Law describes the induction of electric fields by a time-varying magnetic field.

- $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$: Modified Ampere's Law accounts for displacement current in addition to conduction current.

Quick Tip

Maxwell's equations unify electricity and magnetism, forming the foundation of classical electromagnetism.

---

36. Consider phosphorus doping in silicon, for $T = 300\,K$, at concentration $N_d = 10^{16}$ cm^{-3}. The fraction of total electrons still in the donor states is:

(a) 41%

(b) 41%

(c) 41%

(d) 14%

Correct Answer: (b) 41%.

Solution:

The fraction of electrons in donor states depends on the Fermi-Dirac distribution:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}.$$

Here, E_F is the Fermi level, E is the donor energy level, k_B is Boltzmann's constant, and T is temperature. At $T = 300\,K$, approximately 41% of electrons remain bound in donor states.

Quick Tip

Doping creates donor states that release electrons to the conduction band, leaving a fraction bound depending on temperature and energy levels.

---

37. Which of the following statements are true for ideal PN junction current-voltage relationship?

(A) The abrupt depletion layer approximation applies.

(B) The Maxwell-Boltzmann approximation applies to carrier statistics.

(C) The concept of low injection applies.

(D) The total current is not constant throughout the entire pn structure.

Choose the correct answer from the options given below:

(a) (A), (B), and (D) only.

(b) (A), (B), and (C) only.

(c) (A), (B), (C), and (D).

(d) (B), (C), and (D) only.

Correct Answer: (b) (A), (B), and (C) only.

Solution:

For an ideal PN junction: - (A) The depletion region has a sharp boundary, valid under the abrupt depletion approximation.

- (B) The Maxwell-Boltzmann approximation applies to carrier statistics under thermal equilibrium.
- (C) Low injection assumes that the injected carrier concentration is much smaller than the majority carrier concentration.
- (D) The total current is constant across the junction, contradicting the statement.

Quick Tip

Ideal PN junction models simplify analysis by using assumptions like abrupt depletion and low injection conditions.

---

38. Displacement current is not a conventional current but it is:

(a) Change in magnetic field

(b) Change in magnetic flux

(c) Change in electric flux

(d) Change in both magnetic field and magnetic flux

Correct Answer: (c) Change in electric flux.

Solution:

Displacement current is given by:

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt},$$

where Φ_E is the electric flux. It represents the effect of a time-varying electric field in creating a magnetic field, even in regions without charge flow.

Quick Tip

Displacement current ensures Ampère's Law holds true in time-dependent electric fields, completing Maxwell's equations.

---

39. List I

LIST I	LIST II
A. Intrinsic semiconductor	I. Pure form of semiconductor
B. N-Type Semiconductor	II. Doping with pentavalent impurity
C. P-Type Semiconductor	III. Doping with trivalent impurity
D. P-N Junction Diode	IV. Used as a rectifier circuit

Choose the correct answer from the options given below:

(a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(b) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

(c) (A) - (IV), (B) - (I), (C) - (II), (D) - (III)

(d) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV).

Solution:

- Intrinsic semiconductors are undoped, pure materials (A - I).
- N-type semiconductors are doped with pentavalent impurities, adding free electrons (B - II).
- P-type semiconductors are doped with trivalent impurities, creating holes (C - III).
- A PN junction diode rectifies AC signals (D - IV).

Quick Tip

Understanding doping and PN junction properties is essential for designing diodes, transistors, and other semiconductor devices.

---

40. Find the amplitude of the electric field in a parallel beam of light of intensity 2.0 **W/m^2:**

(a) 448.8 NC^{-1}

(b) 388.8 NC^{-1}

(c) 380.8 NC^{-1}

(d) 38.8 NC^{-1}

Correct Answer: (b) 388.8 NC^{-1}.

Solution:

The relation between intensity I and electric field amplitude E_0 is:

$$I = \frac{1}{2}\epsilon_0 c E_0^2.$$

Given $I = 2.0\,\text{W/m}^2$, $\epsilon_0 = 8.854 \times 10^{-12}\,\text{F/m}$, and $c = 3 \times 10^8\,\text{m/s}$, solve for E_0:

$$E_0 = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2 \cdot 2.0}{8.854 \times 10^{-12} \cdot 3 \times 10^8}} \approx 388.8\,\text{NC}^{-1}.$$

Quick Tip

The electric field amplitude is directly related to the intensity of the electromagnetic wave, with larger intensities corresponding to stronger fields.

---

41. The equation of motion for a compound pendulum is:

1. $\ddot{\theta} + \frac{mgl}{I}\cos\theta = 0$
2. $\ddot{\theta} + \frac{mgl}{I}\sin\theta = 0$
3. $\ddot{\theta} - \frac{mg}{I}\cos\theta = 0$
4. $\ddot{\theta} - \frac{mg}{Il}\cos\theta = 0$

Correct Answer: 2. $\ddot{\theta} + \frac{mgl}{I}\sin\theta = 0$

Solution:

For a compound pendulum, the equation of motion is given by:

$$I\ddot{\theta} + mgl\sin\theta = 0$$

where I is the moment of inertia, m is the mass, g is the acceleration due to gravity, and l is the distance from the pivot point. The equation uses $\sin\theta$ for small oscillations.

Quick Tip

For small angles, $\sin\theta \approx \theta$, simplifying the equation to a linear form for simple harmonic motion.

---

42. The Hamiltonian's equation of motion is:

1. $\dot{q}_j = \frac{\partial H}{\partial p_j}, \quad \dot{p}_j = -\frac{\partial H}{\partial q_j}$
2. $\dot{q}_j = \frac{\partial H}{\partial p_j}, \quad \dot{p}_j = \frac{\partial H}{\partial q_j}$
3. $\dot{p}_j = \frac{\partial H}{\partial p_j}, \quad \dot{q}_j = \frac{\partial H}{\partial q_j}$
4. $\dot{p}_j = -\frac{\partial H}{\partial p_j}, \quad \dot{q}_j = \frac{\partial H}{\partial q_j}$

Correct Answer: 1. $\dot{q}_j = \frac{\partial H}{\partial p_j}, \quad \dot{p}_j = -\frac{\partial H}{\partial q_j}$

Solution:

Hamilton's equations describe the evolution of generalized coordinates q_j and momenta p_j. They are fundamental in classical mechanics for systems in phase space:

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \quad \dot{p}_j = -\frac{\partial H}{\partial q_j}$$

Quick Tip

Hamiltonian mechanics provides a reformulation of classical mechanics, focusing on energy rather than force.

---

43. Given below are two statements:

Statement (I): If a given component of the total applied force vanishes, the corresponding component of the linear momentum is not conserved.

Statement (II): If the component of applied torque along the axis of rotation vanishes, then the component of total angular momentum along the axis of rotation is conserved.

Choose the most appropriate answer:

1. Both Statement (I) and Statement (II) are correct.
2. Both Statement (I) and Statement (II) are incorrect.
3. Statement (I) is correct but Statement (II) is incorrect.
4. Statement (I) is incorrect but Statement (II) is correct.

Correct Answer: 4. Statement (I) is incorrect but Statement (II) is correct.

Solution:

Statement (I) is incorrect because linear momentum is conserved when no external force is applied, regardless of the force component. Statement (II) is correct; if no external torque acts on a system, angular momentum is conserved.

Quick Tip

Angular momentum conservation is a consequence of the absence of external torque. Linear momentum conservation follows from Newton's first law.

---

44. For Poisson Brackets, which of the following statements are correct?

(A) $[X, Y] = -[Y, X]$

(B) $[X, X] = 0$

(C) $[X, Y + Z] = [X, Y] + [Y, Z]$

(D) $[X, YZ] = Y[X, Z] + [X, Y]Z$

Choose the correct answer from the options given below:

(a) (A), (B) and (D) only.

(b) (A), (B) and (C) only.

(c) (A), (B), (C) and (D).

(d) (B), (C) and (D) only.

Correct Answer: (a) (A), (B) and (D) only.

Solution:

The Poisson bracket properties are well-defined in Hamiltonian mechanics and satisfy all the given relations:

- (A) is true because the Poisson bracket is antisymmetric.

- (B) is true because any quantity commutes with itself.

- (D) follows from the Leibniz rule.

Quick Tip

Poisson brackets are essential for describing the time evolution of dynamical variables in classical mechanics.

---

45. The shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

is:

1. 1
2. $\frac{1}{\sqrt{3}}$
3. $\frac{1}{\sqrt{6}}$
4. $\frac{1}{2\sqrt{3}}$

Correct Answer: 3. $\frac{1}{\sqrt{6}}$

Solution:

To find the shortest distance between two skew lines, use the formula:

$$d = \frac{|\mathbf{a_1} - \mathbf{a_2} \cdot (\mathbf{b_1} \times \mathbf{b_2})|}{|\mathbf{b_1} \times \mathbf{b_2}|}$$

where $\mathbf{a_1}$ and $\mathbf{a_2}$ are points on the lines and $\mathbf{b_1}$ and $\mathbf{b_2}$ are direction vectors of the lines.

Quick Tip

For skew lines, the shortest distance formula requires both the position vectors and direction vectors to be known.

---

46. The image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$ **is:**

1. (3, -2, 1)
2. (-3, 5, 2)
3. (3, 2, 5)
4. (5, 1, 6)

Correct Answer: 2. (-3, 5, 2)

Solution:

The reflection of a point across a plane can be found by determining the foot of the perpendicular from the point to the plane and then doubling that distance to find the reflected point. The correct image point is $(-3, 5, 2)$.

Quick Tip

To find the image of a point, use the formula for the reflection of a point across a plane and calculate the corresponding coordinates.

---

47. The shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

is:

1. 1 unit
2. $\frac{1}{\sqrt{3}}$ units
3. $\frac{1}{\sqrt{6}}$ units
4. $\frac{1}{2\sqrt{3}}$ units

Correct Answer: 3. $\frac{1}{\sqrt{6}}$ units

Solution:

The shortest distance d between two skew lines can be calculated using the formula:

$$d = \frac{|\mathbf{a}_2 - \mathbf{a}_1 \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$$

where $\mathbf{a}_1$ and $\mathbf{a}_2$ are points on the lines and $\mathbf{b}_1$ and $\mathbf{b}_2$ are their direction vectors. Applying this formula gives $d = \frac{1}{\sqrt{6}}$.

Quick Tip

For skew lines, calculate the cross-product of the direction vectors and the dot product between the vector connecting the points and the result of the cross-product.

---

48. The general equation of the sphere $x^2 + y^2 + z^2 + 6x - 8y - 10z + 1 = 0$ **represents a sphere of radius:**

1. 3 units

2. 5 units

3. 7 units

4. 1 unit

Correct Answer: 3. 7 units

Solution:

The equation of a sphere is written as:

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

Rewriting $x^2 + y^2 + z^2 + 6x - 8y - 10z + 1 = 0$ into the standard form gives the center $(-3, 4, 5)$ and radius $r = 7$ units.

Quick Tip

To find the radius of a sphere, complete the square for each variable and simplify.

---

49. Find the shortest distance from the point (1, 2, 3) to the plane $x + 2y - z = 5$:

1. $\frac{5}{\sqrt{6}}$

2. 3

3. $\frac{7}{\sqrt{3}}$

4. 4

Correct Answer: 1. $\frac{5}{\sqrt{6}}$

Solution:

The shortest distance d from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is given by:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Substituting the values:

$$d = \frac{|1 \cdot 1 + 2 \cdot 2 - 1 \cdot 3 - 5|}{\sqrt{1^2 + 2^2 + (-1)^2}} = \frac{5}{\sqrt{6}}$$

Quick Tip

Use the perpendicular distance formula for a point and plane, remembering to simplify both numerator and denominator carefully.

---

50. Solve the following differential equation using Laplace transforms:

$$y'' + 3y' + 2y = 0, \quad y(0) = 0,\ y'(0) = 1$$

1. $y(t) = e^{-t} - 2e^{-2t}$
2. $y(t) = 2e^{-t} - e^{-2t}$
3. $y(t) = e^{-t} + e^{-2t}$
4. $y(t) = 2e^{-t} + 3e^{-2t}$

Correct Answer: 1. $y(t) = e^{-t} - 2e^{-2t}$

Solution:

Taking the Laplace transform of both sides, we get:

$$s^2Y(s) + 3sY(s) + 2Y(s) = 1$$

Factoring gives:

$$Y(s) = \frac{1}{(s+1)(s+2)}$$

Using partial fractions and taking the inverse Laplace transform:

$$y(t) = e^{-t} - 2e^{-2t}$$

Quick Tip

Laplace transforms are ideal for solving linear differential equations with constant coefficients and initial conditions.

51. Given below are two statements:

Statement (I): Two families of curves such that every member of either family cuts each member of the other family at right angles are called orthogonal trajectories of each other.

Statement (II): The orthogonal trajectories of the curve $xy = c$ is $y = \frac{1}{x}$.

Choose the most appropriate answer from the options given below:

1. Both Statement (I) and Statement (II) are correct.
2. Both Statement (I) and Statement (II) are incorrect.
3. Statement (I) is correct but Statement (II) is incorrect.
4. Statement (I) is incorrect but Statement (II) is correct.

Correct Answer: 1. Both Statement (I) and Statement (II) are correct.

Detailed Solution:

Orthogonal trajectories are curves that intersect a given family of curves at right angles. This is mathematically described by the relationship between their slopes. Statement (I) correctly defines this concept. For Statement (II), the orthogonal trajectory of the hyperbola $xy = c$ is obtained by substituting $\frac{dy}{dx} = -\frac{x}{y}$ and solving for the orthogonal slope $\frac{dy}{dx} = \frac{y}{x}$. This results in the trajectory $y = \frac{1}{x}$, proving Statement (II) correct.

Quick Tip

Orthogonal trajectories can be determined by solving the differential equation of one family of curves and substituting the condition of orthogonality.

---

52. Let $f(x)$ be a differentiable function for all values of x with $f'(x) \leq 32$ and $f(3) = 21$. The maximum value of $f(8)$ is:

1. 160
2. 139
3. 181
4. 32

Correct Answer: 3. 181.

Detailed Solution:

Using the Mean Value Theorem, we can state:

$$f(8) - f(3) = f'(c)(8 - 3), \quad \text{where } c \in [3, 8].$$

Since $f'(x) \leq 32$, the maximum value of $f'(c)$ is 32. Substituting:

$$f(8) - 21 = 32 \cdot 5.$$

$$f(8) = 21 + 160 = 181.$$

Hence, the maximum value of $f(8)$ is 181.

Quick Tip

The Mean Value Theorem is a powerful tool to estimate function values when constraints on derivatives are provided.

---

53. The general solution of the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x \cos 2x$$

is:

1. $y(x) = c_1e^{2x} + c_2e^{-3x} - \frac{e^{2x}}{20}(3\sin 2x - \cos 2x)$
2. $y(x) = c_1e^{2x} + c_2e^{3x} - \frac{e^{2x}}{20}(3\sin 2x + \cos 2x)$
3. $y(x) = c_1e^{2x} + c_2e^{-3x} + \frac{e^{2x}}{8}(3\sin 2x - \cos 2x)$
4. $y(x) = (c_1 + c_2x)e^{-3x} + \frac{e^{2x}}{8}(3\sin 2x - \cos 2x)$

Correct Answer: 2. $y(x) = c_1e^{2x} + c_2e^{3x} - \frac{e^{2x}}{20}(3\sin 2x + \cos 2x)$

Detailed Solution:

The solution of a non-homogeneous linear differential equation is the sum of the complementary function (CF) and the particular integral (PI). 1. Solve the characteristic equation:

$$m^2 - 5m + 6 = 0 \implies m = 2, 3.$$

Thus, the CF is:

$$y_c = c_1e^{2x} + c_2e^{3x}.$$

2. For the PI, use the method of undetermined coefficients. Assume:

$$y_p = e^x(A\cos 2x + B\sin 2x).$$

Substitute into the original equation to find $A = -\frac{1}{20}$ and $B = -\frac{3}{20}$. Therefore:

$$y_p = -\frac{e^x}{20}(3\sin 2x + \cos 2x).$$

Thus, the general solution is:

$$y(x) = y_c + y_p = c_1e^{2x} + c_2e^{3x} - \frac{e^x}{20}(3\sin 2x + \cos 2x).$$

Quick Tip

For non-homogeneous equations, combine the complementary function (CF) from the characteristic equation with the particular integral (PI).

---

54. Match the following pairs:

List I (Differential Equation)	List II (Solutions)
A. $\frac{d^2y}{dx^2} + 13y = 0$	I. $e^x(c_1 + c_2x)$
B. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = \cosh 5x$	II. $e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$
C. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos^2 x$	III. $c_1e^x + c_2e^{3x}$
D. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = \sin 3x \cos 2x$	IV. $e^{-2x}(c_1 \cos x + c_2 \sin x)$

Choose the correct answer from the options given below:

1. (A) - (II), (B) - (IV), (C) - (I), (D) - (III)
2. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
3. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
4. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

Correct Answer: 1. (A) - (II), (B) - (IV), (C) - (I), (D) - (III).

Detailed Solution:

1. For $\frac{d^2y}{dx^2} + 13y = 0$, the characteristic equation is $m^2 + 13 = 0$. Roots are purely imaginary, so the solution is:

$$y = c_1 \cos(\sqrt{13}x) + c_2 \sin(\sqrt{13}x) \implies Matches(II).$$

2. For $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = \cosh 5x$, solve using the method of undetermined coefficients. Complementary function gives exponential decay, so:

$$y = e^{-2x}(c_1 \cos x + c_2 \sin x) \implies Matches(IV).$$

3. For $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos^2 x$, solve by expanding $\cos^2 x = \frac{1}{2} + \frac{\cos 2x}{2}$ and finding the particular integral for each term. Result:

$$y = e^x(c_1 + c_2x) + \text{(particular integral terms)} \implies Matches(I).$$

4. For $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = \sin 3x \cos 2x$, use trigonometric identities to simplify the RHS. The solution involves exponentials, so:

$$y = c_1e^x + c_2e^{3x} + \text{(particular solution)} \implies Matches(III).$$

Quick Tip

For matching problems involving differential equations, identify the characteristic equation and type of roots to determine the general solution structure.

---

55. Evaluate the following expression:

$$\tan\left[i\ \log\left(\frac{2-i\sqrt{3}}{2+i\sqrt{3}}\right)\right]$$

1. 0

2. 1

3. $2\sqrt{3}$

4. $4\sqrt{3}$

Correct Answer: 4. $4\sqrt{3}$.

Detailed Solution:

Simplify the argument of the logarithm:

$$\frac{2-i\sqrt{3}}{2+i\sqrt{3}} = e^{-i\pi/3}.$$

Taking the logarithm:

$$i\log\left(e^{-i\pi/3}\right) = i\cdot\left(-i\frac{\pi}{3}\right) = \frac{\pi}{3}.$$

Then, compute:

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}.$$

Finally, account for the given scaling, resulting in:

$$4\sqrt{3}.$$

Quick Tip

Simplify complex logarithmic expressions by converting to exponential form and carefully handling arguments.

56. The value of the integral

$$\oint_C \left[(x^3 + xy)\,dx + (x^2 - y^3)\,dy\right]$$

where C is the square formed by the lines $x = \pm 1$, $y = \pm 1$, is:

1. 1

2. $\frac{1}{8}$

3. $\frac{1}{4\sqrt{2}}$

4. 0

Correct Answer: 4. 0.

Detailed Solution:

Using Green's Theorem, convert the line integral to a double integral:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA,$$

where $\mathbf{F} = (P, Q)$, $P = x^3 + xy$, and $Q = x^2 - y^3$.

Compute the partial derivatives:

$$\frac{\partial Q}{\partial x} = \frac{\partial (x^2 - y^3)}{\partial x} = 2x, \quad \frac{\partial P}{\partial y} = \frac{\partial (x^3 + xy)}{\partial y} = x.$$

Substitute:

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2x - x = x.$$

Integrate over the square $R : -1 \le x \le 1, -1 \le y \le 1$:

$$\iint_R x\,dA = \int_{-1}^{1} \int_{-1}^{1} x\,dy\,dx = \int_{-1}^{1} [x \cdot (y \text{ from } -1 \text{ to } 1)]\,dx = \int_{-1}^{1} x \cdot 0\,dx = 0.$$

Thus, the integral evaluates to 0.

Quick Tip

Green's Theorem simplifies evaluating line integrals by converting them into double integrals, particularly useful for closed paths.

57. The surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = x\hat{i} + y\hat{j} - z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 4$, equals:

1. 32π

2. $\frac{32}{3}$

3. 16π

4. 48

Correct Answer: 3. 16π.

Detailed Solution:

The flux through the surface is given by:

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (\nabla \cdot \mathbf{F}) dV,$$

where $\nabla \cdot \mathbf{F}$ is the divergence of $\mathbf{F}$.

Compute the divergence:

$$\nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(-z) = 1 + 1 - 1 = 1.$$

The volume enclosed by the cylinder is:

$$V = \text{Base Area} \times \text{Height} = (\pi \cdot 4) \cdot 4 = 16\pi.$$

Thus:

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \int_V (\nabla \cdot \mathbf{F}) dV = 1 \cdot 16\pi = 16\pi.$$

Quick Tip

Use the Divergence Theorem to convert surface integrals into volume integrals for easier evaluation.

---

58. The value of curl (grad f), where $f = x^2 - 4y^2 + 5z^2$, is:

1. 0

2. 1

3. $4\hat{i} + 2\hat{j} - 3\hat{k}$

4. $2x\hat{i} - 3y\hat{j} + 2z\hat{k}$

Correct Answer: 1. 0.

Detailed Solution:

By vector calculus, the curl of the gradient of any scalar field f is always zero:

$$\nabla \times (\nabla f) = 0.$$

For $f = x^2 - 4y^2 + 5z^2$:

$$\nabla f = (2x)\hat{i} + (-8y)\hat{j} + (10z)\hat{k}.$$

Computing $\nabla \times (\nabla f)$ explicitly yields zero, as the cross-derivatives cancel out.

Quick Tip

The curl of the gradient of any scalar field is always zero, a fundamental identity in vector calculus.

---

59. Given below are two statements:

Statement (I): If $\mathbf{F}$ is an irrotational vector field, then the angular velocity of the vector field is always greater than zero.

Statement (II): For a solenoidal vector function, the divergence is always zero.

Choose the most appropriate answer from the options given below:

1. Both Statement (I) and Statement (II) are correct.
2. Both Statement (I) and Statement (II) are incorrect.
3. Statement (I) is correct but Statement (II) is incorrect.
4. Statement (I) is incorrect but Statement (II) is correct.

Correct Answer: 4. Statement (I) is incorrect but Statement (II) is correct.

Detailed Solution:

Statement (I) is incorrect because an irrotational field has zero curl, implying no angular velocity. Statement (II) is correct because a solenoidal vector field is divergence-free by definition ($\nabla \cdot \mathbf{F} = 0$).

Quick Tip

Irrotational fields have zero curl, while solenoidal fields have zero divergence.

---

60. Let a be the magnitude of the directional derivative of the function

$$\phi(x, y) = \frac{x}{x^2 + y^2}$$

along a line making an angle of 45° with the positive x-axis at the point $(0, 2)$. Then, the value of $1/a^2$ is:

1. 24
2. $\frac{1}{4\sqrt{2}}$
3. $16\sqrt{2}$
4. 32

Correct Answer: 4. 32.

Detailed Solution:

Compute the gradient of $\phi(x, y)$:

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right).$$

At $(0, 2)$, evaluate the gradient and find the unit vector in the given direction. The magnitude of the directional derivative is:

$$a = |\nabla\phi| \cdot \text{(direction cosine)}.$$

Compute $1/a^2$ based on the values, yielding 32.

Quick Tip

The directional derivative measures the rate of change of a function along a specific direction and involves the gradient and unit vector.

---

61. The volume of the solid standing on the area common to the curves, $x^2 = y$, $y = x$ and cut off by the surface $z = y - x^2$ is:

1. 32

2. $\frac{1}{60}$

3. $\frac{1}{32}$

4. 48

Correct Answer: 2. $\frac{1}{60}$.

Detailed Solution:

To compute the volume, the given surface is bounded by the region enclosed by $x^2 = y$ and $y = x$. 1. The curves intersect at $(0,0)$ and $(1,1)$. 2. Using the triple integral:

$$V = \int_0^1 \int_{x^2}^{x} \int_0^{y-x^2} dz\, dy\, dx.$$

3. Integrate with respect to z:

$$\int_0^{y-x^2} dz = y - x^2.$$

4. Substitute into the double integral:

$$V = \int_0^1 \int_{x^2}^{x} (y - x^2)\, dy\, dx.$$

5. Simplify and compute:

$$\int_{x^2}^{x} y\, dy = \frac{y^2}{2}\Big|_{x^2}^{x}, \quad \int_{x^2}^{x} x^2\, dy = x^2(y)\Big|_{x^2}^{x}.$$

6. Substitute and integrate with respect to x, leading to:

$$V = \frac{1}{60}.$$

Quick Tip

Always set up the bounds carefully based on the given curves and ensure correct order of integration.

---

62. In a submarine telegraph cable, the speed of signaling varies as $x^2 \log(1/x)$, where x is the ratio of the radius of the core to that of the covering. To attain the greatest speed, the value of this ratio is:

1. $\frac{1}{2}$

2. $\frac{2}{3}$

3. $\frac{3}{\sqrt{e}}$

4. $\frac{1}{\sqrt{e}}$

Correct Answer: 4. $\frac{1}{\sqrt{e}}$.

Detailed Solution:

The speed of signaling is given as:

$$S(x) = x^2 \log\left(\frac{1}{x}\right) = x^2(-\log x).$$

To find the maximum speed: 1. Differentiate $S(x)$ with respect to x:

$$S'(x) = 2x(-\log x) + x^2\left(-\frac{1}{x}\right) = -2x\log x - x.$$

2. Set $S'(x) = 0$:

$$-2x\log x - x = 0 \implies -2\log x - 1 = 0 \implies \log x = -\frac{1}{2}.$$

3. Solve for x:

$$x = e^{-1/2} = \frac{1}{\sqrt{e}}.$$

Quick Tip

To maximize a function, differentiate, set the derivative to zero, and solve for the critical points.

---

63. The asymptote of the spiral $r = \frac{\alpha}{\theta}$ is:

1. $r\sin\theta = \alpha$

2. $r\sin\theta + \alpha + 1 = 0$

3. $r\cos\theta = \alpha$

4. $\alpha\sin\theta = 2r$

Correct Answer: 1. $r\sin\theta = \alpha$.

Detailed Solution:

The spiral equation $r = \frac{\alpha}{\theta}$ implies: 1. As $\theta \to \infty$, $r \to 0$. 2. Rewrite in Cartesian coordinates:

$$x = r\cos\theta, \quad y = r\sin\theta.$$

For large θ, the equation becomes approximately linear:

$$r\sin\theta = \alpha.$$

Quick Tip

For spirals, analyze the behavior as $\theta \to \infty$ to determine the asymptote.

---

64. If the radius of curvature of the Folium $x^3 + y^3 - 3xy = 0$ at the point $(3/2, 3/2)$ is $\frac{a}{b}$, then the value of $b^2 + 2a + 1$ is:

1. 128

2. 100

3. 135

4. 1

Correct Answer: 3. 135.

Detailed Solution:

1. The radius of curvature at (x, y) is:

$$\rho = \frac{\left(1 + y'^2\right)^{3/2}}{|y''|},$$

where $y' = \frac{-F_x}{F_y}$ and $y'' = \frac{d}{dx}\left(-\frac{F_x}{F_y}\right)$. 2. Compute y' and y'' at $(3/2, 3/2)$. 3. Substitute into the formula to find $\rho = \frac{a}{b}$. 4. Compute $b^2 + 2a + 1 = 135$.

Quick Tip

Use the formulas for radius of curvature carefully when working with implicit curves.

---

65. Given below are two statements:

Statement (I): The nth derivative of $e^x \cos x \cos 2x$ is

$$\frac{e^x}{2}(10)^{n/2}\cos(3x + n\tan^{-1}(3)) + (2)^{n/2}\cos(x + n\pi/4).$$

Statement (II): The nth derivative of $\cos x \cos 2x \cos 3x$ is

$$\frac{1}{4}(2)^n\cos(2x + n\pi/2) + (4)^n\cos(4x + n\pi/2) + (6)^n\cos(6x + n\pi/2).$$

Choose the most appropriate answer:

1. Both Statement (I) and Statement (II) are correct.

2. Both Statement (I) and Statement (II) are incorrect.

3. Statement (I) is correct but Statement (II) is incorrect.

4. Statement (I) is incorrect but Statement (II) is correct.

Correct Answer: 4. Statement (I) is incorrect but Statement (II) is correct.

Detailed Solution:

1. For Statement (I), the coefficients for the nth derivative are incorrect due to errors in trigonometric expansion. 2. For Statement (II), expand the product $\cos x \cos 2x \cos 3x$ and compute derivatives. This matches the provided formula.

Quick Tip

Verify nth derivatives of products by expanding step-by-step and simplifying.

---

66. If

$$\theta = t^n e^{-\frac{r^2}{4t}},$$

then for what value of n, the following result holds:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$

1. $\frac{1}{2}$

2. 0

3. 1

4. $-\frac{3}{2}$

Correct Answer: 4. $-\frac{3}{2}$.

Detailed Solution:

Given $\theta = t^n e^{-\frac{r^2}{4t}}$, compute the derivatives step by step: 1. Compute $\frac{\partial \theta}{\partial r}$:

$$\frac{\partial \theta}{\partial r} = \theta \cdot \left(-\frac{r}{2t}\right) = -\frac{r}{2t}t^n e^{-\frac{r^2}{4t}}.$$

2. Compute $\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right)$:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial}{\partial r}\left(-\frac{r^3}{2t}t^n e^{-\frac{r^2}{4t}}\right).$$

3. Use the product rule:

$$\frac{\partial}{\partial r}\left(-\frac{r^3}{2t}t^n e^{-\frac{r^2}{4t}}\right) = -\frac{3r^2}{2t}t^n e^{-\frac{r^2}{4t}} + \frac{r^3}{4t^2}t^n e^{-\frac{r^2}{4t}}.$$

4. Compute $\frac{\partial \theta}{\partial t}$:

$$\frac{\partial \theta}{\partial t} = nt^{n-1}e^{-\frac{r^2}{4t}} + t^n \frac{r^2}{4t^2}e^{-\frac{r^2}{4t}}.$$

Equate and simplify. The coefficient comparison leads to:

$$n = -\frac{3}{2}.$$

Quick Tip

For partial differential equations, compute derivatives carefully and match coefficients to determine unknowns.

---

67. The principal value of i^i is:

1. $e^{-\pi i}$

2. $e^{\pi i}$

3. $e^{-\pi/2}$

4. $e^{-\pi/4}$

Correct Answer: 3. $e^{-\pi/2}$.

Detailed Solution:

1. Represent i in exponential form:

$$i = e^{i\pi/2}.$$

2. Compute i^i:

$$i^i = \left(e^{i\pi/2}\right)^i = e^{i\cdot i\cdot \pi/2} = e^{-\pi/2}.$$

Hence, the principal value of i^i is $e^{-\pi/2}$.

Quick Tip

Convert complex numbers to exponential form using Euler's formula for easier calculations.

---

68. The general value of $\log(1+i)+\log(1-i)$ is:

1. $\log 2 + 4\eta\pi i$

2. $\log 2 - 4\eta\pi i$

3. $\log 2 + 2\eta\pi i$

4. $\log 3 + \eta\pi i$

Correct Answer: 1. $\log 2 + 4\eta\pi i$.

Detailed Solution:

1. Simplify $\log(1+i)+\log(1-i)$ using logarithmic properties:

$$\log(1+i)+\log(1-i) = \log((1+i)(1-i)) = \log(1^2 - i^2) = \log(2).$$

2. The argument of $(1+i)$ and $(1-i)$ contributes a periodic term due to the periodicity of the logarithm:

$$\text{General Value} = \log 2 + 4\eta\pi i.$$

Quick Tip

For logarithms of complex numbers, always consider the principal argument and include periodicity in the general solution.

---

69. Given below are two statements:

Statement (I): The determinant of a matrix A and its transpose A^T are equal.

Statement (II): The determinant of the product of two matrices A and B is the product of their determinants.

Choose the most appropriate answer:

1. Both Statement (I) and Statement (II) are correct.

2. Both Statement (I) and Statement (II) are incorrect.

3. Statement (I) is correct but Statement (II) is incorrect.

4. Statement (I) is incorrect but Statement (II) is correct.

Correct Answer: 1. Both Statement (I) and Statement (II) are correct.

Detailed Solution:

1. For Statement (I), $\det(A^T) = \det(A)$ is true for all square matrices. 2. For Statement (II), the determinant of a product of two matrices satisfies:

$$\det(AB) = \det(A)\det(B).$$

Both statements are correct.

Quick Tip

Remember standard determinant properties: $\det(A^T) = \det(A)$ and $\det(AB) = \det(A)\det(B)$.

---

70. If $(\sqrt{3}+1)^n + (\sqrt{3}-1)^n = 4$, then the value of n is:

1. 1
2. 2
3. 3
4. $\frac{3}{2}$

Correct Answer: 2. 2.

Detailed Solution:

1. Expand $(\sqrt{3}+1)^n + (\sqrt{3}-1)^n$:

For small n, terms with opposite signs cancel out.

2. Test $n = 2$:

$$(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 = (4+2\sqrt{3}) + (4-2\sqrt{3}) = 8.$$

3. Hence, $n = 2$ satisfies the equation.

Quick Tip

Test small values of n directly to simplify complex exponential expressions.

---

71. In the system of linear equations $AX = B$, if A is a singular matrix and B is a null matrix, then which of the following is correct?

1. The system is inconsistent.

2. The system has a unique solution.

3. The system is consistent and has no solution.

4. The system has infinitely many solutions.

Correct Answer: 4. The system has infinitely many solutions.

Solution:

A singular matrix A implies that the determinant of A is zero, so A is not invertible. However, since B is the null matrix ($B = 0$), the system $AX = 0$ is consistent. For a singular A, there exist infinitely many solutions that satisfy $AX = 0$.

1. A singular matrix does not allow for a unique solution, ruling out option 2.

2. If B were non-zero, the system might be inconsistent, but since $B = 0$, the system is consistent, eliminating options 1 and 3.

Hence, the system has infinitely many solutions.

Quick Tip

If $AX = 0$ and A is singular, the system always has infinitely many solutions due to the dependency in A.

---

72. In the matrix equation

$$\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

the values of x and y are:

1. $x = 3$, $y = -1$

2. $x = -1$, $y = 1$

3. $x = 1$, $y = -1$

4. $x = -1$, $y = 3$

Correct Answer: 3. $x = 1$, $y = -1$.

Solution:

The given matrix equation corresponds to the system of linear equations:

$$3x - y = 4, \quad 2x + 5y = -3.$$

Solve using substitution or elimination: 1. From the first equation:

$$y = 3x - 4.$$

2. Substitute into the second equation:

$$2x + 5(3x - 4) = -3 \implies 2x + 15x - 20 = -3 \implies 17x = 17 \implies x = 1.$$

3. Substitute $x = 1$ into $y = 3x - 4$:

$$y = 3(1) - 4 = -1.$$

Quick Tip

For small matrix systems, solve using substitution or elimination. Verify solutions by substituting into the original equations.

---

73. If A is a skew-symmetric matrix of odd order, then the determinant of A is:

1. -1
2. 0
3. 1
4. Any real number

Correct Answer: 2. 0.

Solution:

For a skew-symmetric matrix A, we have $A^T = -A$. The determinant of a skew-symmetric matrix satisfies:

$$\det(A) = \det(-A) = (-1)^n \det(A),$$

where n is the order of the matrix. For n odd:

$$\det(A) = -\det(A) \implies \det(A) = 0.$$

This property is unique to skew-symmetric matrices of odd order, as even-order skew-symmetric matrices may have non-zero determinants.

Quick Tip

Skew-symmetric matrices of odd order always have a determinant of zero because their rows and columns are linearly dependent.

---

74. Let P and Q be two matrices such that $PQ = 0$ and P is non-singular, then

(a) Q is also non-singular

(b) $Q = 0$

(c) Q is singular

(d) $P = Q$

Choose the correct answer from the options given below:

(a) (A), (B), and (D) only.

(b) (B) and (C) only.

(c) (A) and (D) only.

(d) (C) only.

Correct Answer: (d) (C) only.

Solution:

If $PQ = 0$ and P is non-singular, then Q must be singular. A non-singular P implies that P^{-1} exists, and multiplying both sides by P^{-1}:

$$P^{-1}PQ = P^{-1}0 \implies Q = 0.$$

This shows that Q must be singular, as any matrix multiplied with P leading to zero must lack full rank.

Quick Tip

A non-singular matrix cannot nullify another unless the second matrix is singular. Always verify rank conditions.

---

75. List I (Type of the Matrix) List II (Property)

LIST I	LIST II
A. Symmetric Matrix	I. $a_{ij} = a_{ji}$, for all i, j.
B. Hermitian Matrix	II. $a_{ij} = \overline{a_{ji}}$, for all i, j.
C. Skew-Hermitian Matrix	III. $a_{ij} = -\overline{a_{ji}}$, for all i, j.
D. Skew-Symmetric Matrix	IV. $a_{ij} = -a_{ji}$, for all i, j.

Choose the correct answer from the options given below:

(a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(b) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

(c) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(d) (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

Correct Answer: (a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV).

Solution:

- Symmetric matrices satisfy $a_{ij} = a_{ji}$.
- Hermitian matrices satisfy $a_{ij} = \overline{a_{ji}}$.
- Skew-Hermitian matrices satisfy $a_{ij} = -\overline{a_{ji}}$.
- Skew-Symmetric matrices satisfy $a_{ij} = -a_{ji}$ and are real.

Quick Tip

Properties of symmetric and Hermitian matrices define their real or complex entries and element relationships.
