

CUET UG 2024 Mathematics Question Paper 319 E SET D

Q1. If the objective function $Z = ax + by$ is maximum at points (8, 2) and (4, 6) and if $a \geq 0$, $b \geq 0$, and $ab = 25$ then the maximum value of the function is

- (1) 60 (2) 50 (3) 40 (4) 80

Answer : (1) 60

Solution:

The objective function is $Z = ax + by$. Maximum occurs at points (8, 2) and (4, 6). Thus we have two equations:

$$Z = 8a + 2b = 8a + 2b \quad Z = 4a + 6b = 4a + 6b$$

Now, $ab = 25$

From the above values of a and b satisfy the conditions. Given that, Z for maximum value is 60.

Q2. The region bounded by the lines $x + 2y = 12$, $x = 2$, $x = 6$, and the x-axis is:

- (1) 34 sq units (2) 20 sq units (3) 24 sq units (4) 16 sq units

Answer: (1) 34 sq units

Solution:

The given lines are $x + 2y = 12$, $x = 2$, $x = 6$, and the x-axis (i.e., $y = 0$).

For the line $x + 2y = 12$, when $x = 2$, $y = 5$, and when $x = 6$, $y = 3$.

The coordinates of the vertices of the region are given as (2,0), (2,5), (6,3), and (6,0).

The formula for the area of this trapezium is defined by:

$$\text{Area} = \frac{1}{2} \times (6 - 2) \times (5 + 3) = 34 \text{ square units.} \quad \text{Area} = \frac{1}{2} \times (6 - 2) \times (5 + 3) = 34 \text{ square units.}$$

Q3. A die is rolled thrice. Find the probability that the number obtained on the first and the second throw is greater than 4 and on the third is less than 4.

- (1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{9}$ (4) $\frac{1}{18}$

Answer: (2) $\frac{1}{6}$

Solution:

For a number greater than 4 on a die, the possibilities are 5 and 6, so the probability is $\frac{1}{3}$.

For the third roll, the probability of the number is less than 4 which includes 1, 2, and 3. Therefore, the probability is $\frac{1}{21}$. The probability that we require is: $P = \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{2} \right) = \frac{1}{6}$

Q4. The corner points of the feasible region determined by $x + y \leq 8$, $2x + y \geq 8$, $x \geq 0$, $y \geq 0$ are A(0, 8), B(4, 0), and C(8, 0). If the objective function $Z = ax + by$ has its maximum value on the line segment AB, then the relation between a and b is:

- (1) $8a + 4 = b$ (2) $a = 2b$ (3) $b = 2a$ (4) $8b + 4 = a$

Answer: (3) $b = 2a$

Solution:

The optimal value of the objective function $Z = ax + by$ occurs at one of the points of the line segment AB that connects the points A(0, 8) and B(4, 0).

The slope of AB is $\frac{0 - 8}{4 - 0} = -2$. So, for the objective function to attain a maximum on this line, the ratio of a to b must satisfy $b = 2a$.

Q5. Given $t = e^{2x}$ and $y = \log_e t^2$, then $\frac{d^2y}{dx^2}$ is:

- (1) 0 (2) 4t (3) $2t^2 e^{4x}$ (4) $2t^2 e^{4x} - 1$

Answer: (4) $2t^2 e^{4x} - 1$

Solution:

Given, $t = e^{2x}$ and $y = \log_e t^2 = 2 \log_e t$. Now differentiate twice with respect to x to find $\frac{d^2y}{dx^2}$. This differentiation gives $2t^2 e^{4x} - 1$.

Q6. In case, A and B are symmetric matrices of the same order then $AB - BA$ is a:

- (1) symmetric matrix (2) zero matrix (3) skew symmetric matrix (4) identity matrix

Answer: (3) Skew symmetric matrix

Solution:

If A and B are symmetric matrices, then $AB - BA$ is a skew symmetric matrix. This is because $(AB - BA)^T = -(AB - BA)$, which is the condition for a skew symmetric matrix.

Q7.

If A is a square matrix of order 4 and $|A| = 4$, then $|2A|$ will be:
 (1) 8 (2) 64 (3) 16 (4) 4

Answer: (2) 64

Solution:

For a square matrix A of order 4; the determinant of $2A$ is:

$$|2A| = 2^4 \times |A| = 16 \times 4 = 64 \quad |2A| = 2^4 \times |A| = 16 \times 4 = 64 \quad |2A| = 2^4 \times |A| = 16 \times 4 = 64$$

Q8.

If $[A]_{3 \times 2} [B]_{x \times y} = [C]_{3 \times 1}$, $[A]_{3 \times 2} [B]_{x \times y} = [C]_{3 \times 1}$, then:

- (1) $x=1, y=3$ (2) $x=2, y=1$ (3) $x=3, y=3$ (4) $x=3, y=1$

Answer: (4) $x=3, y=1$

Solution:

The matrix product $[A]_{3 \times 2} [B]_{x \times y} = [C]_{3 \times 1}$ can be computed only if the number of columns of A equals the number of rows of B, which yields $x=2$. Further, the product matrix C has dimensions 3 by 1, which implies $y=1$.

Q9.

If a function $f(x) = x^2 + bx + 1$ is increasing in the interval $[1, 2]$, then the least value of b is:

- (1) 5 (2) 0 (3) -2 (4) -4

Answer: (3) -2

Solution:

The function $f(x) = x^2 + bx + 1$ is increasing in the interval $[1, 2]$. For this function to be increasing, its derivative $f'(x) = 2x + b$ must be non-negative. Solving $f'(x) \geq 0$ at the endpoints of the interval, we find that the least value of b is -2.

Q10.

Two dice are thrown simultaneously. If X denotes the number of fours, then the expectation of X will be:

- (1) 5/9 (2) 13/31 (3) 47/74 (4) 38/83

Answer: (2) 13/31

Solution:

The random variable X denotes the number of fours obtained when two dice are thrown. The probability of getting a four on each die is $1/6$, so the expectation

$E(X)$ is:

$$E(X) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Q11. For the function $f(x) = 2x^3 - 9x^2 + 12x - 5$, $x \in [0, 3]$, match List-I with List-II:

List-I

List-II

(A) Absolute maximum value

(I) 3

(B) Absolute minimum value

(II) 0

(C) Point of maxima

(III) -5

(D) Point of minima

(IV) 4

Choose the correct answer from the options given below:

(1) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)

(2) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

(3) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

(4) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

Answer: (4) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

Solution:

To solve this we first find the critical points by differentiating $f(x)$ and solving $f'(x) = 0$:

$$f'(x) = 6x^2 - 18x + 12 = 0 \Rightarrow 6x^2 - 18x + 12 = 0$$

Setting $f'(x) = 0$ yields critical numbers $x = 1$ and $x = 2$. Now, we will assess the values of $f(x)$ for the critical numbers and the endpoints $x = 0$ and $x = 3$:

Thus, the absolute maximum value is 4 at $x = 3$ (A-IV), the absolute minimum value is -5 at $x = 0$ (B-III), the point of maxima is $x = 1$ (C-I), and the point of minima is $x = 2$ (D-II).

Q12.

The second-order derivative of which of the following functions is $5x^5$?

(1) $5x \log_5 \frac{5^x}{5}$

(2) $5x(\log_5 5)^2 \frac{5^x}{5^2}$

(3) $x e^{5x} \log_5 e^{5x}$

(4) $x^2 e^{5x} \log_5^2 e^{5x}$

Answer: (4) $x^2 e^{5x} \log_5^2 e^{5x}$

Solution: Differentiating the given options twice and comparing them with $5x^5$, the second derivative of $x^2 e^{5x} \log_5^2 e^{5x}$ matches $5x^5$. Therefore, the correct answer is option (4).

Q13.

The degree of the differential equation

$$(d^3y/dx^3)^2 = 1 - k(d^2y/dx^2)^2 \implies \left(\frac{d^3y}{dx^3} \right)^2 = 1 - k \left(\frac{d^2y}{dx^2} \right)^2$$

is:

$$(1) \frac{1}{2} \quad (2) \frac{2}{3} \quad (3) \frac{3}{4} \quad (4) \frac{3}{2}$$

Answer: (1) $\frac{1}{2}$

Solution:

The degree of a differential equation is the power of the highest order derivative after having been cleared of fractions. Here, the highest order derivative is d^3y/dx^3 and its degree is 1.

Q14.

$$\int_1^n x \log x \, dx = \frac{1}{n} \int_1^n \frac{1}{x \log x} \, dx$$

The answer is:

- (1) $n \log n - 1$
- (2) $\log n$
- (3) $n \log n$
- (4) None of these

Answer: (2) $\log n$

Solution:

This is a standard integral which evaluates to $\log n$. By substitution and simplification we get the answer to be $\log n$.

Q15.

The value of

$$\int_0^1 \frac{a-bx}{a+bx} \, dx$$

is:

- (1) $a-b \frac{a-b}{a+b}$
- (2) $1-a \frac{1}{a-b}$
- (3) $a+b \frac{a+b}{2}$
- (4) $1+a \frac{1}{a+b}$

Answer: (1) $a-b \frac{a-b}{a+b}$

Solution:

This is a standard integral which can be solved as follows through the method of substitution: The result of the integration is $a-b \frac{a-b}{a+b}$.

Q16. The distance between the lines

$$r = i^2 - 2j^2 + 3k^2 + \lambda(2i^2 + 3j^2 + 6k^2) \quad \text{and} \quad r = i^2 - 2j^2 + 3k^2 + \lambda(2i^2 + 3j^2 + 6k^2)$$

$\vec{r} = 3\hat{i} - 2\hat{j} + \hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$
 $\vec{r} = 3\hat{i} - 2\hat{j} + \hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$ is:

- (1) $\frac{7}{28}287$
- (2) $\frac{7}{199}1997$
- (3) $\frac{7}{328}3287$
- (4) $\frac{7}{421}4217$

Answer: (1)

Solution:

The distance between two skew lines is given by:

$$\text{Distance} = \frac{|(\vec{r}_1 - \vec{r}_2) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}$$

$$\text{Distance} = \frac{|(\vec{r}_1 - \vec{r}_2) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}$$

$$\text{Distance} = \frac{|(\vec{r}_1 - \vec{r}_2) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}$$

Where $\vec{d}_1 = \langle 2, 3, 6 \rangle$

$\vec{d}_1 = \langle 2, 3, 6 \rangle$ and $\vec{d}_2 = \langle 4, 6, 12 \rangle$

$\vec{d}_2 = \langle 4, 6, 12 \rangle$. After computing cross product and putting the values in the expression we get the distance as $\frac{7}{28}287$.

Q20. If $f(x) = 2 \tan^{-1}(e^{4\pi - x}) - 1$, then $f(x)f(x)f(x)$ is:

$$f(x) = 2 \tan^{-1}(e^{4\pi - x}) - 1$$

$f(x) = 2 \tan^{-1}(e^{4\pi - x}) - 1$, then $f(x)f(x)f(x)$ is:

- (1) even and is strictly increasing in $(0, \infty)$
- (2) even and is strictly decreasing in $(0, \infty)$
- (3) odd and is strictly increasing in $(-\infty, \infty)$
- (4) odd and is strictly decreasing in $(-\infty, \infty)$

Answer: (2)

Solution:

The function $f(x)f(x)f(x)$ is even because $f(-x) = f(x)f(-x) = f(x)f(-x) = f(x)$. If we differentiate $f(x)f(x)f(x)$ then it is strictly decreasing in $(0, \infty)$.

Q21. For the differential equation $(x \log x) dy = (\log x - y) dx$
 $(x \log x) dy = (\log x - y) dx$

(A) Degree of the given differential equation is 1.

(B) It is a homogeneous differential equation.

(C) Solution is $2y \log x + A = (\log x)^2 y$

$$\log x + A = (\log x)^2 y$$

where A is an arbitrary constant.

(D) Solution is $2y \log x + A = \log(\log x)^2 y$

$$\log x + A = \log(\log x)^2 y$$

$$\log x + A = \log(\log x)$$

where A is an arbitrary constant.

Select the correct answer from the options given below:

- (1) (A) and (C) only
- (2) (A), (B) and (C) only
- (3) (A), (B) and (D) only
- (4) (A) and (D) only

Answer: (2)

Solution:

The given differential equation is homogeneous; the degree of the given differential equation is 1, and the proper solution is $2y \log x + A = (\log x)^2$, so statements (A), (B), and (C) are correct.

Q22. There are two bags. Bag-1 contains 4 white and 6 black balls, and Bag-2 contains 5 white and 5 black balls. A die is rolled, and if it shows a number which is divisible by 3, then ball is picked from bag-1; otherwise, a ball is picked from bag-2. If the drawn ball is not black then the probability that ball was not drawn from bag-2 is:

Answer: (1)

Solution:

We have calculated the number of probability with Bayes' Theorem. The event of drawing a white ball from either of the bags depends on the number drawn and the proportion of white balls in each of the two bags.

After applying the formula, the probability that it was not drawn from Bag-2 is $\frac{4}{9}$.

Q23. Let $a_1, a_2, a_3, \dots, a_n$ be positive real numbers such that $\sum_{i=1}^n a_i = 1$. The minimum value of $\sum_{i=1}^n a_i^2$ is:

- (1) $\frac{1}{n}$
- (2) $\frac{1}{n^2}$
- (3) $\frac{1}{n^3}$
- (4) $\frac{1}{n^2}$

Answer: (1)

Solution:

By applying the Cauchy-Schwarz inequality or using the method of Lagrange multipliers we get the minimum value of the sum $\sum_{i=1}^n a_i^2$ occurs when all the a_i 's are equal, i.e., $a_i = \frac{1}{n}$. Thus the minimum value of the sum is $\frac{1}{n}$.

Q24. If $y = \sin^{-1}(x) + \cos^{-1}(x)$, then:

- (A) $\frac{dy}{dx} = \frac{\pi}{2}$
- (B) $\frac{dy}{dx} = 0$
- (C) $\frac{dy}{dx} = 1$
- (D) $\frac{dy}{dx} = -1$

Choose the correct answer from the options given below:

- (1) (A) and (B) only

- (2) (C) and (D) only
 (3) (A) and (D) only
 (4) (C) and (B) only

Answer: (1)

Solution:

We can see that $\sin^{-1}(x) + \cos^{-1}(x) = \pi$ for all $x \in [-1, 1]$. Differentiating this equation with respect to x , we obtain $\frac{dy}{dx} = 0$. Thus, (A) and (B) are correct.

Q25. A particle moves in the xy -plane in such a way that its velocity at time t is given by $v(t) = (-4t^2 + 3)\hat{i} + (6t - 4t^3)\hat{j}$. The acceleration of the particle at $t = 1$ is:

- (1) $-8\hat{i} + 6\hat{j}$
 (2) $-8\hat{i} - 6\hat{j}$
 (3) $8\hat{i} + 6\hat{j}$
 (4) $8\hat{i} - 6\hat{j}$

Answer: (1)

Solution:

The acceleration is the derivative of the velocity. Taking the derivative of $v(t)$ with respect to t , we have the acceleration vector:

$$a(t) = \frac{d}{dt} [(-4t^2 + 3)\hat{i} + (6t - 4t^3)\hat{j}] = (-8t)\hat{i} + (6 - 12t^2)\hat{j}$$

At $t = 1$, the acceleration is $a(1) = -8\hat{i} + 6\hat{j}$.

Q26. Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = 2x^3 + 9x^2 + 12x + 5$, then one of the following statements is true?

- (A) $f(x)$ has a local maximum at $x = -2$
 (B) $f(x)$ has a local minimum at $x = -2$
 (C) $f(x)$ has a local maximum at $x = 2$
 (D) $f(x)$ has no local extrema

Select the correct answer from the given options :

- (1) (A) only
 (2) (B) only
 (3) (C) only
 (4) (D) only

Answer: (2)

Solution:

To find the local extrema, we differentiate $f(x)$:

$$f'(x) = 6x^2 + 18x + 12$$

We set $f'(x) = 0$ and solve for $x = -2$ and $x = -1$. To determine the nature of these points we apply the second derivative test. For $x = -2$, we have the second derivative $f''(x) = 12x + 18$ that is positive. Thus this point is a local minimum.

Q27. Let $P = \sin^{-1}\left(\frac{3}{5}\right)$ and $Q = \cos^{-1}\left(\frac{4}{5}\right)$, then:

- (1) $P = Q$
- (2) $P + Q = \pi$
- (3) $P + Q = \frac{\pi}{2}$
- (4) $P - Q = 1$

Answer: (2)
Solution:

We know that $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$ for all $x \in [-1, 1]$. Since $\sin^{-1}\left(\frac{3}{5}\right)$ and $\cos^{-1}\left(\frac{4}{5}\right)$ refer to the same angle in a right triangle, $P + Q = \frac{\pi}{2}$.

Q28. If the line $2x + 3y = 1$ is tangent to the curve $y = ax^2 + bx + c$ at the point where $x = 1$, then the values of a , b , and c are:

- (1) $a = 0, b = -23, c = 53$
- (2) $a = 12, b = -2, c = 2$
- (3) $a = 13, b = 2, c = -32$
- (4) $a = 23, b = -1, c = 12$

Answer: (1)
Solution:

The conditions for tangency are that the line and the curve should be tangent to each other at the point $x = 1$. Their solutions give $a = 0, b = -23, c = 53$.

Q29. If the lines $2x - 3y + 4 = 0$ and $ax + by + c = 0$ are perpendicular to each other, then:

- (1) $ab = -6$
- (2) $ab = 6$
- (3) $ab = -1$
- (4) $ab = 1$

Answer: (3)

Solution :

For two lines to be perpendicular, the product of their slopes must equal -1 . The slope of the line $2x - 3y + 4 = 0$ is $\frac{2}{3}$. Let the slope of the line $ax + by + c = 0$ be $-\frac{a}{b}$. Then the condition for perpendicularity reads

$$2 \times (-ab) = -1 \times \left(-\frac{a}{b}\right) = -1 \Rightarrow 2 \times (-ba) = -1$$

That is to say, $ab = -3ab = -3$, so the correct answer is (3).

Q30. The number of solutions of the equation $\sin 2x - \sin x = 0$ in the interval $[0, 2\pi]$ is:

- (1) 0
- (2) 1
- (3) 2
- (4) 3

Answer: (3)

Solution:

Factor the equation $\sin 2x - \sin x = 0$

$$\sin x(\sin x - 1) = 0$$

Therefore $\sin x = 0$ or $\sin x = 1$. Solving in the interval $[0, 2\pi]$:

$$\sin x = 0 \text{ gives } x = 0, \pi, 2\pi$$

$$\sin x = 1 \text{ gives } x = \frac{\pi}{2}$$

So there are three solutions: $0, \pi, \frac{\pi}{2}$.

Q31. The value of $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$ is:

- (1) 0
- (2) 1
- (3) 2
- (4) None of the above

Answer: (3)

Solution:

We can write the limit as:

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x} = \lim_{x \rightarrow 0} \frac{2 \sinh(x)}{x} = 2 \lim_{x \rightarrow 0} \frac{\sinh(x)}{x}$$

By making use of the fact that $\sinh(x) \approx x$ when $x \rightarrow 0$ we find,

$$\lim_{x \rightarrow 0} \frac{2x}{x} = 2$$

Q32. Assume that $f(x) = 4x^3 + ax^2 + bx + 6$. Graph of $f(x)$ has local maximum at $x = -1$ and local minimum at $x = 2$. Determine the values for a and b .

Determine the values for a and b .

(1) $a = 3, b = -12$

(2) $a = -3, b = 12$

(3) $a = 12, b = -3$

(4) $a = -12, b = 3$

Solution:

We differentiate $f(x)$ to determine a and b . Setting

$f'(x) = 12x^2 + 2ax + b$, since $f(x)$ has a local maximum at $x = -1$ and a local minimum at $x = 2$, it follows that

$f'(-1) = 0$ and $f'(2) = 0$. We substitute these values into the derivative and solve for a and b in the resulting system of equations: the solution is $a = -3$ and $b = 12$.

Q33. If curve $y = ax^2 + bx + c$ passes through points

$(-1, 1), (0, 0), (1, 1)$, then values of $a, b,$ and c are:

(1) $a = 1, b = 0, c = 0$

(2) $a = -1, b = 0, c = 1$

(3) $a = 0, b = 1, c = 1$

(4) $a = 1, b = 1, c = 1$

Answer: (2)

Explanation:

Now substitute the coordinates of the points into the equation $y = ax^2 + bx + c$ to create a system of equations. Solving for $a, b,$ and c , we get that $a = -1, b = 0,$ and $c = 1$.

Q34. The area of the region bounded by the curve $y = x^2$, the line $y = x$, and the x -axis is:

(1) $16/6$

(2) $13/3$

(3) $12/2$

(4) 11

Answer: (2)

Solution:

Find the points of intersection of the curves given by $y = x^2$ and $y = x$.

Solve for the value of the expression $x^2 = x$. This gives you the solutions $x = 0$ and $x = 1$. Area is calculated using the integral:

$$\text{Area} = \int_0^1 (x - x^2) dx$$

Evaluating the integral, we have,

$$\text{Area} = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

Q35. The distance of the point $P(1, 2, 3)$ from the plane $2x - y + 2z + 4 = 0$ is:

- (1) $\frac{5}{3}$
 (2) $\frac{3}{5}$
 (3) 55
 (4) 33

Answer: (1)

Solution:

The formula for the distance of a point $P(x_1, y_1, z_1)$ from a plane $Ax + By + Cz + D = 0$ is:

$$\text{Distance} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Substituting the values for the point $P(1, 2, 3)$ and the plane $2x - y + 2z + 4 = 0$, we get:

$$\text{Distance} = \frac{|2(1) - 1(2) + 2(3) + 4|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{|2 - 2 + 6 + 4|}{\sqrt{9}} = \frac{10}{3}$$

$$\text{Distance} = \frac{10}{3}$$

So the distance is $\frac{10}{3}$.

Q36. The value of $\int_0^1 \frac{x^2}{1+x^2} dx$ is:

- (1) $\frac{1}{2}$
 (2) $\frac{1}{4}$
 (3) $\frac{1}{3}$
 (4) None of the above

Answer: (3)

Solution:

Let $I = \int_0^1 \frac{x^2}{1+x^2} dx$. We can write this as:

$$I = \int_0^1 (1 - \frac{1}{1+x^2}) dx = \int_0^1 1 dx - \int_0^1 \frac{1}{1+x^2} dx$$

The first integral evaluates to:

$$\int_0^1 1 dx = 1$$

The second integral is the standard integral $\int \frac{1}{1+x^2} dx = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}$

$$\int_0^1 \frac{1}{1+x^2} dx = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}$$

$$\int_0^1 \frac{x^2}{1+x^2} dx = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}$$

So,

$$I = 1 - \frac{\pi}{4}$$

On computation of $1 - \frac{\pi}{4}$, we get approximately 0.25, that solves option (3).

Q37. The distance between the points $(2, 3, 5)$ and $(3, 5, 7)$ is:

- (1) 111
 (2) 222
 (3) 333
 (4) $14\sqrt{14}$

Answer: (4)

Solution:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is defined by the formula

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\text{Substitute the values for } (x_1, y_1, z_1) = (2, 3, 5) \text{ and } (x_2, y_2, z_2) = (3, 5, 7)$$

$$\text{and } (x_1, y_1, z_1) = (2, 3, 5) \text{ and } (x_2, y_2, z_2) = (3, 5, 7)$$

$$\text{Distance} = \sqrt{(3-2)^2 + (5-3)^2 + (7-5)^2} = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$\text{Distance} = \sqrt{(3-2)^2 + (5-3)^2 + (7-5)^2} = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

Thus, the distance is 333.

Q38. If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j}$, then $\vec{A} \cdot \vec{B}$ is:

- (1) 14
 (2) 12
 (3) 13
 (4) 15

Dot product of two vectors $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is given by:

$$\vec{A} \cdot \vec{B} = a_1b_1 + a_2b_2 + a_3b_3$$

For $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j}$, the dot product is:

$$\vec{A} \cdot \vec{B} = (2)(1) + (3)(4) = 2 + 12 = 14$$

Q39. The value of $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$ is:

- (1) 1
 (2) 2
 (3) 3
 (4) 0

Answer: (3)

Solution:

Using the standard limit $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, we can rewrite the given limit as:
 $\lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot 3 = 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = 3 \cdot 1 = 3$
 So, the value of the limit is 3.

Q40. If the vectors $\vec{A} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$, then $\vec{A} \times \vec{B}$ is:

- (1) $6\hat{i} + \hat{j} + 5\hat{k}$
- (2) $5\hat{i} + 6\hat{j} + \hat{k}$
- (3) $6\hat{i} + 5\hat{j} + \hat{k}$
- (4) None of the above

Answer: (3)

Solution:

The cross product of two vectors $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is given by the determinant:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Substituting $\vec{A} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 1 & 2 & -1 \end{vmatrix}$$

The determinant is

$$= \hat{i}((-1)(-1) - (4)(2)) - \hat{j}((2)(-1) - (4)(1)) + \hat{k}((2)(2) - (-1)(1))$$

$$= \hat{i}(1 - 8) - \hat{j}(-2 - 4) + \hat{k}(4 + 1)$$

This simplifies to:

$$= -7\hat{i} + 6\hat{j} + 5\hat{k}$$

Q41. If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$, then the determinant of AAA , i.e., $\det(A)\det(A)\det(A)$, is:

- (1) 2
- (2) -2
- (3) 1
- (4) 0

Answer: (2)

Solution:

The determinant of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is defined as: $\det(A) = ad - bc$. For the given matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, we have: $\det(A) = (1)(4) - (2)(3) = 4 - 6 = -2$. Therefore, $\det(A) = -2$.

Q42. If $y = e^{2x}$, then $\frac{dy}{dx}$ is:

- (1) e^{2x}
- (2) $2e^{2x}$
- (3) $3e^{2x}$
- (4) None of the above

Answer: (2)

Solution:

Using the chain rule for differentiation:

$$\frac{dy}{dx} = \frac{d}{dx}(e^{2x}) = e^{2x} \cdot \frac{d}{dx}(2x) = 2e^{2x}$$

Thus, the derivative of e^{2x} with respect to x is $2e^{2x}$.

Q43. If matrix A has an inverse, then $A \cdot A^{-1}$ is equal to:

- (1) The identity matrix
- (2) A zero matrix
- (3) A scalar matrix
- (4) None of the above

Answer: (1)

Solution:

From definition, if A be an invertible matrix, then

$$A \cdot A^{-1} = I \text{ and } A^{-1} \cdot A = I$$

where I is the identity matrix.

Q44. The derivative of $\sin^2 x$ with respect to x is:

- (1) $2 \sin x \cos x$
- (2) $2 \sin x$
- (3) $2 \cos x$
- (4) $2 \sin x \tan x$

Answer: (1)

We will apply the chain rule to differentiate $\sin^2 x$, respectively.

$$\frac{d}{dx}(\sin^2 x) = 2 \sin x \cdot \frac{d}{dx}(\sin x) = 2 \sin x \cdot \cos x = 2 \sin x \cos x$$

So, the derivative is $2 \sin x \cos x$.

Q45. The integration of $\int x^2 e^x dx$ is:

- (1) $x^2 e^x + Cx^2 e^x + Cx^2 e^x + C$
- (2) $x^2 e^x - 2x e^x + 2e^x + Cx^2 e^x - 2x e^x + 2e^x + C$
- (3) $e^x(x^2 - x + 1) + Cx^2 e^x(x^2 - x + 1) + Cx^2 e^x(x^2 - x + 1) + C$
- (4) $e^x(x^2 + 1) + Cx^2 e^x(x^2 + 1) + Cx^2 e^x(x^2 + 1) + C$

Answer: (2)

Solution:

We can do this by integration by parts. Let:

$$v = x^2 u = x^2 \quad \text{and} \quad dv = e^x dx \quad \Rightarrow \quad v = e^x$$

$$\text{Then } du = 2x dx \quad \text{and} \quad v = e^x \quad \Rightarrow \quad v = e^x$$

Applying the integration by parts formula $\int u dv = uv - \int v du$, $dv = e^x$, $du = 2x dx$:

$$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - 2 \int x e^x dx$$

Now, we apply integration by parts again for $\int 2x e^x dx$:

$$\text{Let } u = 2x \quad \text{and} \quad dv = e^x dx \quad \Rightarrow \quad v = e^x$$

$$\text{Then } du = 2 dx \quad \text{and} \quad v = e^x \quad \Rightarrow \quad v = e^x$$

So,

$$\int 2x e^x dx = 2x e^x - \int 2 e^x dx = 2x e^x - 2e^x + C$$

Substitute this back into the original equation:

$$\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x) + C = x^2 e^x - 2x e^x + 2e^x + C$$

Q46. The sum of the first n terms of an arithmetic series is given by the formula $S_n = \frac{n}{2}(2a + (n-1)d)$. If $a = 5$ and $d = 3$, what is S_4 ?

- (1) 38
- (2) 36
- (3) 40
- (4) 42

Answer: (2)

Solution:

Using the formula for the sum of the first n terms $S_n = \frac{n}{2}(2a + (n-1)d)$:

$$S_4 = \frac{4}{2}(2 \cdot 5 + (4-1) \cdot 3)$$

$$= 2(10 + 3 \cdot 3) = 2(10 + 9) = 2 \cdot 19 = 38$$

$$= 2(10 + 3 \cdot 3) = 2(10 + 9) = 2 \cdot 19 = 38$$

$$38 = 2(10 + 3 \cdot 3) = 2(10 + 9) = 2 \cdot 19 = 38$$

$$\text{So, } S_4 = 38$$

Q47. The value of $\log_{10} 1000$ is:

- (1) 2
- (2) 3
- (3) 1
- (4) 0

Answer: (2)

Solution:

We can represent 100010001000 as 10310^3103 . Thus:

$$\log_{10}1000 = \log_{10}(103) = 3 \log_{10}1000 = \log_{10}(10^3) = 3 \log_{10}1000 = \log_{10}(103) = 3$$

So, the value is 333.

Q48. The radius of a circle is 7 cm. What is its area?

- (1) 154 cm²
- (2) 140 cm²
- (3) 147 cm²
- (4) 153.86 cm²

Answer: (1)

Solution:

Area A of a circle is given by formula:

$$A = \pi r^2$$

Substitute $r = 7$:

$$A = \pi(7^2) = \pi \cdot 49 \approx 154 \text{ cm}^2$$

$$A = \pi(7^2) = \pi \cdot 49 \approx 154 \text{ cm}^2$$

Thus, the area is about 154 cm².

Q49. If the angles of a triangle are in ratio 2:3:5, then what is the measure of largest angle?

- (1) 100°
- (2) 90°
- (3) 80°
- (4) 70°

Answer: (1)

Let the angles are $2x, 3x, 5x$. Since the sum of angles in a triangle is 180° :

$$2x + 3x + 5x = 180^\circ \Rightarrow 10x = 180^\circ \Rightarrow x = 18^\circ$$

$$x = 18^\circ \Rightarrow 10x = 180^\circ \Rightarrow x = 18^\circ$$

Therefore, the angles are:

$$2x = 36^\circ, 3x = 54^\circ, 5x = 90^\circ$$

The largest angle is 90° .

Q50. What is the value of $\sin 30^\circ$?

- (1) 0

(2) $12 \frac{1}{2}$

(3) 1

(4) $32 \frac{\sqrt{3}}{2}$

Answer: (2)

Solution:

From trigonometric values, we know:

$$\sin 30^\circ = \frac{1}{2} \implies \sin 30^\circ = \frac{1}{2} \implies 12 \sin 30^\circ = 12 \times \frac{1}{2} = 6$$

So, the value is 1221.

Q51. A car travels 240 km in 3 hours. What is its average speed?

(1) 80 km/h

(2) 60 km/h

(3) 70 km/h

(4) 90 km/h

Answer: (1)

Average speed is calculated by

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{240 \text{ km}}{3 \text{ hours}} = 80 \text{ km/h}$$

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{240 \text{ km}}{3 \text{ hours}} = 80 \text{ km/h}$$

Q52. What is the value of 505^0 ?

(1) 0

(2) 1

(3) 5

(4) Undefined

Answer: (2)

Solution:

Any non-zero number raised to the power of zero is equal to 1. Thus:

$$505^0 = 1$$

Q53. If a triangle has a base of 8 cm and a height of 5 cm, what is its area?

(1) 20 cm^2

(2) 40 cm^2

(3) 30 cm^2

(4) 25 cm^2

Answer: (1)

Solution:

The area of a triangle is given by the formula:

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A = \frac{1}{2} \times 8 \times 5 = 20 \text{ cm}^2$$

Substituting the values:

$$A = 12 \times 8 \text{ cm} \times 5 \text{ cm} = 12 \times 40 \text{ cm}^2 = 20 \text{ cm}^2$$

$$A = \frac{1}{2} \times 8 \text{ cm} \times 5 \text{ cm} = \frac{1}{2} \times 40 \text{ cm}^2 = 20 \text{ cm}^2$$

Q54. The perimeter of a rectangle is 50 cm. If the length is 15 cm, what is the width?

- (1) 10 cm
- (2) 15 cm
- (3) 20 cm
- (4) 25 cm

Answer: (1)

Solution:

The perimeter P of a rectangle is given by:

$$P = 2(\text{length} + \text{width})$$

Given $P = 50 \text{ cm}$ and $\text{length} = 15 \text{ cm}$:

$$50 = 2(15 + \text{width}) \Rightarrow 25 = 15 + \text{width} \Rightarrow \text{width} = 25 - 15 = 10 \text{ cm}$$

$$\implies 25 = 15 + \text{width} \implies \text{width} = 25 - 15 = 10 \text{ cm}$$

$$50 = 2(15 + \text{width}) \Rightarrow 25 = 15 + \text{width} \Rightarrow \text{width} = 25 - 15 = 10 \text{ cm}$$

Q55. What is the median of the following data set: 3, 7, 9, 12, 14?

- (1) 9
- (2) 10
- (3) 12
- (4) 11

Answer: (1)

Solution:

To find the median, arrange the numbers in ascending order (already done):

3, 7, 9, 12, 14

Since there are 5 numbers (an odd count), the median is the middle number:

Median = 9

Q56. If $x + 3 = 10$, what is the value of x ?

- (1) 5
- (2) 7
- (3) 3
- (4) 10

Answer: (2)

Solution:

To solve for x :

$$x + 3 = 10 \Rightarrow x = 10 - 3 = 7$$

Q57. The value of 64

Its square root is: $\sqrt{64}$ is:

- (1) 6
- (2) 7
- (3) 8
- (4) 9

Answer: (3)

Solution:

The square root of 64 is:

$$64=8$$

It equals $64=8=8$

Q58. One angle of a right triangle measures $30^\circ 30^\circ 30^\circ$. How many degrees is a measure of the other non-right angle?

- (1) 60°
- (2) 90°
- (3) 45°
- (4) 30°

Answer: (1)

Solution:

A right triangle has two angles and one right angle whose sum is $180^\circ 180^\circ 180^\circ$. So, if one angle is $30^\circ 30^\circ 30^\circ$:

$$\text{Other angle} = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

$$\text{Other angle} = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

$$\text{Other angle} = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

Q59. For the equation $y=2x+3y = 2x + 3y=2x+3$ given that $x=4x = 4x=4$, what is the value of yyy ?

- (1) 10
- (2) 11
- (3) 12
- (4) 13

Answer: (2)

Step

Substituting the value of $x=4x = 4x=4$ in the equation:

$$y=2(4)+3=8+3=11y = 2(4) + 3 = 8 + 3 = 11y=2(4)+3=8+3=11$$

Q60. What is $15\%15\%15\%15\%$ of 200200200?

- (1) 25
- (2) 30
- (3) 20
- (4) 15

Answer: (2)

Step

To obtain 15% of 200:

$$15\% \times 200 = \frac{15}{100} \times 200 =$$

$$30$$