Differential Equations JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Differential Equations

1. A ball is dropped from a platform $19.6\,m$ high. Its position function is - **(+4, -1)**
[27-Jan-2024 Shift 1]

a. $x=-4.9t^2+19.6(0\leq t\leq 1)$

b. $x=-4.9t^2+19.6(0\leq t\leq 2)$

c. $x=-9.8t^2+19.6(0\leq t\leq 2)$

d. $x=-4.9t^2-19.6(0\leq t\leq 2)$

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2. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$. If $p(0)=100$, then $p(t)$ is equal to **(+4, -1)**
[28-Jun-2022-Shift-1]

a. $400-300\,e^{t/2}$

b. $300-200\,e^{-t/2}$

c. $600-500\,e^{t}/2$

d. $400-300\,e^{t}/2$

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3. If a curve $y=f(x)$ passes through the point $(1,-1)$ and satisfies the differential equation, $y(1+xy)dx=x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to : **(+4, -1)**
[28-Jun-2022-Shift-2]

a. $-\frac{2}{5}$

b. $-\frac{4}{5}$

c. $\frac{2}{5}$

d. $\frac{4}{5}$

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4. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to : **(+4, -1)**
[28-Jun-2022-Shift-1]

a. $-\frac{2}{3}$

b. $-\frac{1}{3}$

c. $\frac{4}{3}$

d. $\frac{1}{3}$

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5. If $\frac{dy}{dx} = \frac{xy}{x^2+y^2}; y(1) = 1$; then a value of x satisfying $y(x) = e$ is : [25•Jul•2021•Shift•1] (+4, -1)

a. $\sqrt{3}\,e$

b. $\frac{1}{2}\sqrt{3}\,e$

c. $\sqrt{2}\,e$

d. $\frac{e}{\sqrt{2}}$

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6. The solution of the differential equation , $\frac{dy}{dx} = (x-y)^2$, when $y(1) = 1$, is : [29-Jun-2022-Shift-2] (+4, -1)

a. $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

b. $\log_e\left|\frac{2-x}{2-y}\right| = x - y$

c. $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x + y - 2$

d. $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

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7. The solution of the differential equation $ydx - (x+2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to : [28-Jun-2022-Shift-2] (+4, -1)

a. 4

b. 3

c. 2

d. 1

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8. Let $S=\{x\in R: 0<x<1$ and $2\tan^{-1}(1+x)/(1-x)=\{\cos^{-1}(1-x^2)/(1+x^2)\}$. If $n(S)$ denotes the number of elements in S then : [25•Jul•2021•Shift•1] (+4, -1)

a. $n(S) = 2$ and only one element in S is less than $\frac{1}{2}$.

b. $n(S) = 1$ and the element in S is less than $\frac{1}{2}$.

c. $n(S) = 1$ and the elements in S is more than $\frac{1}{2}$.

d. $n(S) = 0$

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9. Let $9 = x_1 < x_2 < \ldots < x_7$ be in an A.P. with common difference *d*. If the standard deviation of $x_1, x_2 \ldots, x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to : [20•Jul•2021•Shift•2] **(+4, -1)**

a. $18\left(1 + \frac{1}{\sqrt{3}}\right)$

b. $2\left(9 + \frac{8}{\sqrt{7}}\right)$

c. 34

d. 25

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10. If $a_n = \frac{-2}{4n^2 - 16n + 15}$, then $a_1 + a_2 + \ldots\ldots + a_{25}$ is equal to: [25•Jul•2021•Shift•1] **(+4, -1)**

a. $\frac{52}{147}$

b. $\frac{50}{141}$

c. $\frac{51}{144}$

d. $\frac{49}{138}$

Answers

1. **Answer: b**

Explanation:

We have, $a = \frac{d^2x}{dt^2} = -9.8$
The initial conditions are $x(0) = 19.6$ and $v(0) = 0$
So, $v = \frac{dx}{dt} = -9.8t + v(0) = -9.8t$
$\therefore x = -4.9t^2 + x(0) = -4.9t^2 + 19.6$
Now, the domain of the function is restricted since the ball hits the ground after a certain time. To find this time we set $x = 0$ and solve for t.
$0 = 4.9t^2 + 19.6 \Rightarrow t = 2$

Concepts:

1. Differential Equations:

A **differential equation** is an equation that contains one or more functions with its derivatives. The derivatives of the function define the rate of change of a function at a point. It is mainly used in fields such as physics, engineering, biology and so on.

Orders of a Differential Equation

First Order Differential Equation

The first-order differential equation has a degree equal to 1. All the linear equations in the form of derivatives are in the first order. It has only the first derivative such as dy/dx, where x and y are the two variables and is represented as: dy/dx = f(x, y) = y′

Second-Order Differential Equation

The equation which includes second-order derivative is the second-order differential equation. It is represented as; d/dx(dy/dx) = d^2y/dx^2 = f″(x) = y″.

Types of Differential Equations

Differential equations can be divided into several types namely

- Ordinary Differential Equations
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2. **Answer: a**

Explanation:

$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$
$\int \frac{d(p(t))}{\left(\frac{1}{2}p(t)-200\right)} = \int dt$
$2\ \log\left(\frac{p(t)}{2} - 200\right) = t + c$
$\frac{p(t)}{2} - 200 = e^{\frac{t}{2}}k$
Using given condition $p(t) = 400 - 300\,e^{t/2}$

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3. Answer: d

Explanation:

$\frac{y}{x}(1+xy)=\frac{dy}{dx}$
$y=vx$
$\Rightarrow \frac{y}{x}=v$
$\frac{dy}{dx}=v+x\frac{dv}{dx}$
$v\left(1+vx^2\right)=v+x\frac{dv}{dx}$
$v^2x^2=x\frac{dv}{dx}$
$v^2x=\frac{dv}{dx}$
$\int xdx=\int\frac{1}{v^2}dv$
$\frac{x^2}{2}=-\frac{1}{v}+c$
$\frac{x^2}{2}=-\frac{x}{y}+c$
Put $(1,-1)$
$\frac{1}{2}=\frac{1}{1}+c$
$\Rightarrow c=\frac{-1}{2}$
$\frac{x^2}{2}=-\frac{x}{y}-\frac{1}{2}$
We have to find $f\left(-\frac{1}{2}\right)$
Put $x=-\frac{1}{2}$
$\frac{\left(-\frac{1}{2}\right)^2}{2}=\frac{-\left(-\frac{1}{2}\right)}{y}-\frac{1}{2}$
$\frac{1}{8}=\frac{1}{2y}-\frac{1}{2}$
$y=\frac{4}{5}$

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4. Answer: d

Explanation:

$(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$
$y(0)=1, y\left(\frac{\pi}{2}\right)$ = ?
$\frac{1}{y+1}dy+\frac{\cos x}{2+\sin x}dx=0$
$In\left|y+1\right|+In\left(2+\sin x\right)=InC$

$(y+1)(2+\sin x)=C$
Put $x=0,\ y=1$
$(1+1)\cdot 2=C \Rightarrow C=4$
Now, $(y+1)(2+\sin x)=4$
For, $x=\frac{\pi}{2}$
$(y+1)(2+1)=4$
$y+1=\frac{4}{3}$
$y=\frac{4}{3}-1=\frac{1}{3}$

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5. **Answer: a**

Explanation:

$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$
Let $y = vx$
$\frac{dy}{dx} = v + x.\frac{dv}{dx}$
$v + x\frac{dv}{dx} = \frac{xvx}{x^2+v^2x^2} = \frac{v}{1+v^2}$
$x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{v-v-v^3}{1+v^2} = \frac{v^3}{1+v^2}$
$\int \frac{v}{1+v^2}.dv = \int -\frac{dx}{x}$
$\Rightarrow \int v^{-3}.dv + \int \frac{1}{v}dv = -\int \frac{dx}{x}$
$\Rightarrow \frac{v^{-2}}{-2} + \ell nv = -\ell nx + \lambda$
$\Rightarrow -\frac{1}{2v^2} + \ell n\left(\frac{y}{x}\right) = \ell nx + \lambda$
$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ell ny - \ell nx = -\ell nx + \lambda$
$\Rightarrow -\frac{1}{2} + 0 = \lambda \Rightarrow \lambda = -\frac{1}{2}$
$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ell ny + \frac{1}{2} = 0$ at $y = e$
$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + 1 + \frac{1}{2} \Longrightarrow \frac{x^2}{2e^2} = \frac{3}{2} \Rightarrow x^2 = 3e^2$
$\therefore x = \sqrt{3}e$

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6. **Answer: d**

Explanation:

$x - y = t \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx} \Rightarrow 1 - \frac{dt}{dx} = t^2 \Rightarrow \int \frac{dt}{1-t^2} = \int 1dx \Rightarrow \frac{1}{2}\ell n\left(\frac{1+t}{1-t}\right) = x + \lambda \Rightarrow$
$\frac{1}{2}\ell n\left(\frac{1+x-y}{1-x+y}\right) = x + \lambda$ given y(1) = 1 $\Rightarrow \frac{1}{2}\ell n\left(1\right) = 1 + \lambda \Rightarrow \lambda = -1 \Rightarrow \ell n\left(\frac{1+x-y}{1-x+y}\right) = 2\left(x-1\right) \Rightarrow$
$-\ell n\left(\frac{1-x+y}{1+x-y}\right) = 2\left(x-1\right)$

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7. **Answer: b**

Explanation:

Given differential equation is $ydx - (x + 2y^2)\, dy = 0$...(1) and solution of (1) is $x = f(v)$; where $f(-1) = 1, f(1) = ?$ Rearranging (1), we get $y\frac{dx}{dy} - (x + 2y^2) = 0 \Rightarrow \frac{dx}{dy} - 2y - \frac{x}{y} = 0$ or $\frac{dx}{dy} + \left(\frac{-1}{y}\right) x = 2y$, which is a linear differential equation of first order $\frac{dx}{dy} + P\ x = Q$; Its I.F. $= e^{\int Pdy} = e^{\int \frac{-1}{y}dy} = e^{-\ln y} = \frac{1}{y}$ $\therefore$ Solution of (1) is given by $x.(I.F) = \int Q(I.F.)dy + C \Rightarrow x \cdot \frac{1}{y} = \int 2y \cdot \frac{1}{y}dy + C \Rightarrow \frac{x}{y} = 2y + c \Rightarrow x = 2y^2 + cy; f(-1) = 1\ x + 1 = 2 + c(-1) \Rightarrow c = 1 \therefore x = 2y^2 + y = f(y) \Rightarrow f(1) = 2 + 1 = 3$

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8. **Answer: b**

Explanation:

$$0 < x < 1$$
$$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
$$\tan^{-1}x = \theta \in \left(0, \frac{\pi}{4}\right)$$
$$\therefore x = \tan\theta$$
$$2\tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) = \cos^{-1}(\cos 2\theta)$$
$$2\left(\frac{\pi}{4} - \theta\right) = 2\theta$$
$$\therefore 4\theta = \frac{\pi}{2} \therefore \theta = \frac{\pi}{8}$$
$$x = \tan\frac{\pi}{8}$$
$$\therefore x = \sqrt{2} - 1 \simeq 0.414$$

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9. **Answer: c**

Explanation:

The correct answer is (C) : 34

$9 = x_1 < x_2 < \ldots\ldots < x_7$

$9, 9+d, 9+2d, \ldots\ldots.9+6d$

$0, d, 2d, \ldots\ldots.6d$

$x_{new} = \frac{21d}{7} = 3d$

$16 = \frac{1}{7}(0^2 + 1^2 + \ldots\ldots + 6^2)d^2 - 9d^2$

$16 = \frac{1}{7}(\frac{6\times7\times13}{6})d^2 - 9d^2$
$16 = 13d^2 - 9d^2$
$16 = 4d^2$
$d^2 = 4$
$d = 2$
Now,
$\bar{x} + x_6 = 6 + 9 + 10 + 9$
$\bar{x} + x_6 = 34$

Concepts:

1. Differential Equations:

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10. **Answer: b**

Explanation:

If $a_n = \frac{-2}{4n^2-16n+15}$ then $a_1 + a_2 + \ldots\ldots\ldots a_{25}$

$$\Rightarrow \sum_{n=1}^{25} a_n = \sum \frac{-2}{4n^2-16n+15}$$

$= \sum \frac{-2}{4n^2-6n-10n+15}$

$= \sum \frac{-2}{2n(2n-3)-5(2n-3)}$

$= \sum \frac{-2}{(2n-3)(2n-5)}$

$= \sum \frac{1}{2n-3} - \frac{1}{2n-5}$

$= \frac{1}{47} - \frac{1}{(-3)}$

$= \frac{50}{141}$

Concepts:

1. **Application of Derivatives:**

Various Applications of Derivatives-

Rate of Change of Quantities:

If some other quantity 'y' causes some change in a quantity of surely 'x', in view of the fact that an equation of the form y = f(x) gets consistently pleased, i.e, 'y' is a function of 'x' then the rate of change of 'y' related to 'x' is to be given by

$\frac{\triangle y}{\triangle x} = \frac{y_2-y_1}{x_2-x_1}$

This is also known to be as the Average Rate of Change.

Increasing and Decreasing Function:

Consider y = f(x) be a differentiable function (whose derivative exists at all points in the domain) in an interval x = (a,b).

- If for any two points x_1 and x_2 in the interval x such a manner that $x_1 < x_2$, there holds an inequality $f(x_1) \leq f(x_2)$; then the function f(x) is known as increasing in this interval.

- Likewise, if for any two points x_1 and x_2 in the interval x such a manner that $x_1 < x_2$, there holds an inequality $f(x_1) \geq f(x_2)$; then the function f(x) is known as decreasing in this interval.
- The functions are commonly known as strictly increasing or decreasing functions, given the inequalities are strict: $f(x_1) < f(x_2)$ for strictly increasing and $f(x_1) > f(x_2)$ for strictly decreasing.

Read More: [Application of Derivatives]