Differentiation JEE Main PYQ -1

Total Time: 20 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Differentiation

1. If $f(x)=x^2+g'(1)x+g''(2)$ and $g(x)=f(1)x^2+xf'(x)+f''(x)$, then the value of $f(4)-g(4)$ is equal to ______ **(+4)**

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2. Let $f:R\to R$ be a differentiable function such that $f'(x)+f(x)=\int_0^2 f(t)dt$If $f(0)=e^{-2}$, then $2f(0)-f(2)$ is equal to_____ **(+4)**

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3. Let f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right]$ such that $f(x)>0$; and $f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}dt=e,\forall x\in\left[0,\frac{\pi}{2}\right]$ Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to _______ **(+4)**

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4. If the solution of the equation $\log_{\cos x}\cot x+4\log_{\sin x}\tan x=1, x\in\left(0,\frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha+\beta$ is equal to : **(+4)**

a. 6

b. 4

c. 5

d. 3

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5. If f and g are differentiable functions in $(0,1)$ satisfying $f(0)=2=g(1), g(0)=0$ and $f(1)=6$, then for some $c\in]0,1[$ **(+4)**

a. $2f'(c)=g'(c)$

b. $2f'(c)=3g'(c)$

c. $f'(c)=g'(c)$

d. $f'(c)=2g'(c)$

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6. For $x\epsilon R, f(x)=|\log 2-\sin x|$ and $g(x)=f(f(x))$, then : **(+4)**

a. g is not differentiable at $x=0$

b. $g'(0) = cos(log2)$

c. $g'(0) = -cos(log2)$

d. g is differentiable at $x = 0$ and $g'(0) = -sin(log2)$

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7. If for $x \epsilon \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$, then $g(x)$ equals : **(+4)**

a. $\frac{3x\sqrt{x}}{1-9x^3}$

b. $\frac{3x}{1-9x^3}$

c. $\frac{3}{1+9x^3}$

d. $\frac{9}{1+9x^3}$

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8. For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dy}{dx}$ is equal to : **(+4)**

a. $\log_e 2x$

b. $\frac{x \log_e 2x + \log_e 2}{x}$

c. $x \log_e 2x$

d. $\frac{x \log_e 2x - \log_e 2}{x}$

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9. If $f(x) = sin\,(sin\,x)$ and $f''(x) + tan\,x\, f'(x) + g(x) = 0$, then $g(x)$ is : **(+4)**

a. $cos^2\,x\,cos\,(sin\,x)$

b. $sin^2\,x\,cos\,(cos\,x)$

c. $sin^2\,x\,sin\,(cos\,x)$

d. $cos^2\,x\,sin\,(sin\,x)$

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10. If $x = \sqrt{2^{cosec^{-1}}t}$ and $y = \sqrt{2^{\sec^{-1}}t}(|t| \geq 1)$, then $\frac{dy}{dx}$ is equal to : **(+4)**

a. $\frac{y}{x}$

b. $\frac{x}{y}$

c. $-\frac{y}{x}$

d. $-\frac{x}{y}$

Answers

1. **Answer: 14 - 14**

Explanation:

The correct answer is 14.

$f(x) = x^2 + g'(1)x + g''(2)$

$f'(x) = 2x + g'(1)$

$f''(x) = 2$

$g(x) = f(1)x^2 + x\,[2x + g'(1)] + 2$

$g'(x) = 2f(1)x + 4x + g'(1)$

$g''(x) = 2f(1) + 4$

$g''(x) = 0$

$2f(1) + 4 = 0$

$f(1) = -2$

$-2 = 1 + g'(1) = g'(1) = -3$

So $f'(x) = 2x - 3$

$f(x) = x^2 - 3x + c$

$c = 0$

$f(x) = x^2 - 3x$

$g(x) = -3x + 2$

$f(4) - g(4) = 14$

Concepts:

1. **Continuity & Differentiability:**

Definition of Differentiability

f(*x*) is said to be differentiable at the point *x* = *a,* if the derivative f '(a) be at every point in its domain. It is given by

The maxima lie between the minima and the width of the central maximum is simply the distance between the 1st order minima from the centre of the screen on both sides of the centre.

The position of the minima given by y (measured from the centre of the screen) is:

tanθ≈θ≈y/D

For small θ,

sin θ≈θ

⇒ λ = a sin θ≈aθ

⇒ θ = y/D = λa

⇒ y = λDa

The width of the central maximum is simply twice this value

⇒ **Width of central maximum** = 2λDa

⇒ **Angular width of central maximum** = 2θ = 2λa

Definition of Continuity

Mathematically, a function is said to be continuous at a point *x* = *a,* if

It is implicit that if the left-hand limit (L.H.L), right-hand limit (R.H.L), and the value of the function at *x*=*a* exist and these parameters are equal to each other, then the function *f* is said to be continuous at *x*=*a*.

$\lim_{x\to a}$
f(x) Exists, and

$\lim_{x\to a}$
f(x) = *f*(*a*)

If the function is unspecified or does not exist, then we say that the function is discontinuous.

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2. Answer: 1 - 1

Explanation:

The correct answer is 1.

$\frac{dy}{dx} + y = k$

$y \cdot e^x = k \cdot e^x + c$

$f(0) = e^{-2}$

$\Rightarrow c = e^{-2} - k$

$\therefore y = k + \left(e^{-2} - k\right) e^{-x}$

$$\text{now } k = \int_0^2 \left(k + \left(e^{-2} - k\right) e^{-x}\right) dx$$
$$\Rightarrow k = e^{-2} - 1$$
$$\therefore y = \left(e^{-2} - 1\right) + e^{-x}$$
$$f(2) = 2e^{-2} - 1, f(0) = e^{-2}$$
$$2f(0) - f(2) = 1$$

Concepts:

1. Continuity & Differentiability:

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The maxima lie between the minima and the width of the central maximum is simply the distance between the 1st order minima from the centre of the screen on both sides of the centre.

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It is implicit that if the left-hand limit (L.H.L), right-hand limit (R.H.L), and the value of the function at *x*=*a* exist and these parameters are equal to each other, then the function *f* is said to be continuous at *x*=*a*.

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f(x) Exists, and

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If the function is unspecified or does not exist, then we say that the function is discontinuous.

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3. **Answer: 27 - 27**

Explanation:

The correct answer is 27

$f(x)+\int\limits_0^x f(t)\sqrt{1-(\log_e f(t))^2}dt=e$

$\Rightarrow f(0)=e$

$f'(x)+f(x)\sqrt{1-(\ln f(x))^2}=0$

$f(x)=y$

$\frac{dy}{dx}=-y\sqrt{1-(\ln y)^2}$

$\int \frac{dy}{y\sqrt{1-(\ln y)^2}}=-\int dx$

Put $\ln y=t$

$\int \frac{dt}{\sqrt{1-t^2}}=-x+C$

$\sin^{-1}t=-x+C\Rightarrow \sin^{-1}(\ln y)=-x+C$

$\sin^{-1}(\ln f(x))=-x+C$

$f(0)=e$

$\Rightarrow \frac{\pi}{2}=C$

$\Rightarrow \sin^{-1}(\ln f(x))=-x+\frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right)=\frac{-\pi}{6}+\frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right)=\frac{\pi}{3}$

$\Rightarrow \ln f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$, we need $\left(6\times\frac{\sqrt{3}}{2}\right)^2=27$

Concepts:

1. Logarithmic Differentiation:

Logarithmic differentiation is a method to find the derivatives of some complicated functions, using logarithms. There are cases in which differentiating the logarithm of a given function is simpler as compared to differentiating the function itself. By the proper usage of properties of logarithms and chain rule finding, the derivatives become easy. This concept is applicable to nearly all the non-zero functions which are differentiable in nature.

Therefore, in calculus, the differentiation of some complex functions is done by taking logarithms and then the logarithmic derivative is utilized to solve such a function.

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4. **Answer: b**

Explanation:

The correct answer is (B) : 4

$\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x = 1$

$\Rightarrow \frac{\ln \cos x - \ln \sin x}{\ln \cos x} + 4 \frac{\ln \sin x - \ln \cos x}{\ln \sin x} = 1$

$\Rightarrow (\ln \sin x)^2 - 4(\ln \sin x)(\ln \cos x) + 4(\ln \cos x)^2 = 1$

$\Rightarrow \ln \sin x = 2 \ln \cos x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$

$\therefore \alpha + \beta = 4$

Concepts:

1. Logarithmic Differentiation:

Logarithmic differentiation is a method to find the derivatives of some complicated functions, using logarithms. There are cases in which differentiating the logarithm of a given function is simpler as compared to differentiating the function itself. By the proper usage of properties of logarithms and chain rule finding, the derivatives become easy. This concept is applicable to nearly all the non-zero functions which are differentiable in nature.

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5. **Answer: d**

Explanation:

Using, mean value theorem

$f'(c) = \frac{f(1)-f(0)}{1-0} = 4$

$g'(c) = \frac{g(1)-g(0)}{1-0} = 2$

so, $f'(c) = 2g'(c)$

Concepts:

1. Continuity:

A function is said to be continuous at a point $x = a$, if

$\lim_{x \to a}$

f(x) Exists, and

$\lim_{x \to a}$

$f(x) = f(a)$

It implies that if the left hand limit (L.H.L), right hand limit (R.H.L) and the value of the function at $x=a$ exists and these parameters are equal to each other, then the function f is said to be continuous at $x=a$.

If the function is undefined or does not exist, then we say that the function is discontinuous.

Conditions for continuity of a function: For any function to be continuous, it must meet the following conditions:

- The function f(x) specified at x = a, is continuous only if f(a) belongs to real number.
- The limit of the function as x approaches a, exists.
- The limit of the function as x approaches a, must be equal to the function value at x = a.

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6. **Answer: b**

Explanation:

$g(x) = |\log_e 2 - \sin(|\log_e 2 - \sin x|)|$
At $x = 0, g(x) = \log_e(2) - \sin(\log_e 2 - \sin x)$
$\therefore g'(x) = \cos(\log_e(2) - \sin x) \times \cos(x)$
$\Rightarrow g'(0) = \cos(\log_e(2))$

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7. Answer: d

Explanation:

$f(x) = 2\tan^{-1}(3x\sqrt{x})$

For x $\in \left(0, \frac{1}{4}\right)$

$f'(x) = \frac{9\sqrt{x}}{1+9x^3}$

$g(x) = \frac{9}{1+9x^3}$

Concepts:

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8. Answer: d

Explanation:

$$(2x)^{2y} = 4e^{2x-2y}$$
$$2y\ell n2x = \ell n4 + 2x - 2y$$
$$y = \frac{x+\ell n2}{1+\ell n2x}$$
$$y' = \frac{(1+\ell n2x)-(x+\ell n2)\frac{1}{x}}{(1+\ell n2x)^2}$$
$$y' = 1 + \ell n2x)^2 = [\frac{x\ell n2x-\ell n2}{x}]$$

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9. Answer: d

Explanation:

$f(x) = sin\ (sin\,x)$
$\Rightarrow f'(x) = cos\,(sin\,x)\,.cos\,x$
$\Rightarrow f''(x) = -sin\ (sin\,x)\,.cos^2\,cos\,(sin\,x)\,.\,(-sin\,x)$
$= -cos^2\,x.sin\,(sin\,x) - sin\,x.cos\,(sin\,x)$
Now $f''(x) + tanx.f'(x) + g(x) = 0$
$\Rightarrow g(x) = cos^2\,x.sin\,(sin\,x) + sin\,x.cos\,(sin\,x) - tan\,x.cos\,x.cos\,(sin\,x)$
$\Rightarrow g(x) = cos^2\,x.sin\,(sin\,x)\,.$

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10. Answer: c

Explanation:

Given: $x=\sqrt{2^{\text{cose }^{-1} t}}$ and $y=\sqrt{2^{\sec ^{-1} t}}(|t| \geq 1)$

Now, $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$

$$=\frac{\frac{1}{2\sqrt{2 \sec^{-1} t}} 2^{\sec^{-1} t} \ln\left(\frac{1}{t\sqrt{t^2-1}}\right)}{-\frac{1}{2\sqrt{2 cosec^{-1} t}} 2^{\text{cosec }^{-1}} \ln\left(\frac{1}{t\sqrt{t^2-1}}\right)}$$

$$=-\frac{\sqrt{2^{ec^{-1} t}}}{\sqrt{2^{\text{omee }^{-1} y}}}=\frac{-y}{x}$$

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