AP EAPCET (AP EAMCET) May 15 2023 Shift 2 Question Paper with Solution

Time Allowed :180 minutes	**Maximum Marks :**160	**Total Questions :**160

General Instructions

Read the following instructions carefully and follow them strictly:

1. The test duration is **180 minutes**, and the paper contains **160 questions**.
2. It is divided into three sections: **Mathematics (80 questions)**, **Physics (40 questions)**, and **Chemistry (40 questions)**.
3. Each subject includes two parts:
   (i) **Section-A:** Multiple choice questions (MCQs) with one correct answer. Each correct answer earns 4 marks, and each wrong answer deducts 1 mark.
   (ii) **Section-B:** Numerical value questions. Each carries 4 marks for a correct answer and –1 for a wrong one. Answers should be rounded to the nearest integer.
4. The use of calculators, magnifying glasses, rulers, erasers, scratch pads, rough sheets, protractors, or highlighters is **not permitted**.
5. The console displays a **watermark** and has **auto-save** enabled.
6. Changes to font, background color, or themes are **not allowed**. There is no help button.
7. The answer key is shared with the delivery engine; examiner permissions are **restricted (Can't View)**.
8. Marks, reports, and progress bar will **not be displayed** during the test.

MATHEMATICS

1. If $f(a) = \log\left|\frac{1-a}{1+a}\right|$ for $a \neq \{-1, 1\}$, then the set of values of all a, for which $f\left(\frac{2a}{1+a^2}\right) > 0$ is:

(1) $(0, \infty) - \{1\}$

(2) $(-\infty, 0) - \{-1\}$

(3) $(-\infty, \infty) - \{-1, 1\}$

(4) $(-1, 1)$

Correct Answer: (2) $(-\infty, 0) - \{-1\}$

Solution: We are given:

$$f(a) = \log\left|\frac{1-a}{1+a}\right|, \quad a \neq -1, 1$$

We are required to find values of a such that:

$$f\left(\frac{2a}{1+a^2}\right) > 0$$

Let $x = \frac{2a}{1+a^2}$. Then:

$$f(x) = \log\left|\frac{1-x}{1+x}\right| > 0 \Rightarrow \left|\frac{1-x}{1+x}\right| > 1$$

This implies:

$$\left|\frac{1-x}{1+x}\right| > 1 \Rightarrow \left|\frac{1-x}{1+x}\right|^2 > 1 \Rightarrow \left(\frac{1-x}{1+x}\right)^2 > 1 \Rightarrow \frac{(1-x)^2}{(1+x)^2} > 1$$

$$\Rightarrow (1-x)^2 > (1+x)^2 \Rightarrow 1 - 2x + x^2 > 1 + 2x + x^2 \Rightarrow -2x > 2x \Rightarrow -4x > 0 \Rightarrow x < 0$$

So, we want:

$$\frac{2a}{1+a^2} < 0$$

Now, observe the sign of $\frac{2a}{1+a^2}$: - The denominator $1 + a^2 > 0$ for all real a - Hence, $\frac{2a}{1+a^2} < 0$ when $2a < 0 \Rightarrow a < 0$

Also, we are given $a \neq \pm 1$

$$\Rightarrow \boxed{a \in (-\infty, 0) \setminus \{-1\}}$$

Quick Tip

When dealing with composite functions inside logarithms, simplify step by step and carefully examine the domain and sign of the inner expressions.

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2. If a real valued function f is defined by $f(x) = \frac{ax+\sqrt{a^2-x^2}}{bx}$, then f is:

(1) only one-one

(2) only onto

(3) both one-one and onto

(4) neither one-one nor onto

Correct Answer: (2) only onto

Solution:

We are given the function:

$$f(x) = \frac{ax + \sqrt{a^2 - x^2}}{bx}$$

where a and b are constants.

Step 1: Determine the domain of $f(x)$ - The square root expression $\sqrt{a^2 - x^2}$ implies: $a^2 - x^2 \geq 0 \Rightarrow x \in [-a, a]$ - The denominator $bx \neq 0 \Rightarrow x \neq 0$

So, the domain of $f(x)$ is:

$$x \in [-a, a] \setminus \{0\}$$

Step 2: Test for injectivity (One-One) Suppose $f(x_1) = f(x_2)$. Then:

$$\frac{ax_1 + \sqrt{a^2 - x_1^2}}{bx_1} = \frac{ax_2 + \sqrt{a^2 - x_2^2}}{bx_2}$$

This equation does not imply $x_1 = x_2$, and is not generally solvable to prove injectivity. Hence, f is not one-one.

Example: Take $a = 1, b = 1$, then:

$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2} + \sqrt{1 - \left(\frac{1}{2}\right)^2}}{\frac{1}{2}} = \frac{\frac{1}{2} + \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 1 + \sqrt{3}$$

$$f\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2} + \sqrt{1 - \left(\frac{1}{2}\right)^2}}{-\frac{1}{2}} = \frac{\frac{\sqrt{3}-1}{2}}{-\frac{1}{2}} = -(\sqrt{3} - 1) = 1 - \sqrt{3}$$

These are not equal, but no general injectivity is evident. Try plotting or checking symmetry — ultimately, f fails the horizontal line test across its domain.

Conclusion: Not one-one.

Step 3: Test for surjectivity (Onto) Let us observe the function behavior:

$$f(x) = \frac{a}{b} + \frac{\sqrt{a^2 - x^2}}{bx}$$

As $x \to 0^+$, $f(x) \to +\infty$; As $x \to 0^-$, $f(x) \to -\infty$; As $x \to \pm a$, the square root $\to 0$, and the value tends to $\frac{a}{b}$

Hence, $f(x)$ sweeps all values in $\mathbb{R}$, depending on x's sign and proximity to 0, making it **onto**.

Quick Tip

Always examine domain restrictions carefully when roots and denominators are involved. Use boundary analysis and symmetry to test injectivity and surjectivity.

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3. If the cofactors of the elements 3, 7, and 6 of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 7 \\ 2 & 4 & 6 \end{bmatrix}$ **are** a, b, c **respectively, then evaluate:**

$$[a\ b\ c]\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + [a\ b\ c]\begin{bmatrix} 3 \\ 7 \\ 6 \end{bmatrix}$$

(1) -1

(2) 1

(3) 0

(4) 3

Correct Answer: (3) 0

Solution: We are given matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 7 \\ 2 & 4 & 6 \end{bmatrix}$$

and the cofactors of entries 3, 7, and 6, which are the elements A_{13}, A_{23}, A_{33} respectively. We define:

$$[a\ b\ c] = \text{cofactors of } (3, 7, 6)$$

We need to evaluate:

$$[a\ b\ c] \cdot \left(\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 7 \\ 6 \end{bmatrix} \right) = [a\ b\ c] \cdot \begin{bmatrix} 4 \\ 11 \\ 8 \end{bmatrix}$$

Now calculate each cofactor:

- $a = \text{Cofactor of } 3 = (-1)^{1+3} \cdot \begin{vmatrix} 4 & -1 \\ 2 & 4 \end{vmatrix} = 1 \cdot (16 + 2) = 18$ -

$b = \text{Cofactor of } 7 = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = -1 \cdot (4 - 4) = 0$ -

$c = \text{Cofactor of } 6 = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix} = 1 \cdot (-1 - 8) = -9$

So, $[a\ b\ c] = [18\ 0\ -9]$

Now compute:

$$18 \cdot 4 + 0 \cdot 11 + (-9) \cdot 8 = 72 + 0 - 72 = 0$$

$$\boxed{0}$$

Quick Tip

When summing dot products, it's efficient to combine vectors first, then apply cofactors. Pay attention to signs in cofactor expansion.

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4. Let $B = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}$**,** $C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$**, and if a matrix** A **is such that** $BAC = I$**, then** $A^{-1} =$

(1) $\begin{bmatrix} -3 & -5 & 5 \\ 0 & 9 & 14 \\ 2 & 2 & 6 \end{bmatrix}$

(2) $\begin{bmatrix} -3 & -5 & 5 \\ 0 & 0 & 9 \\ 2 & 14 & 16 \end{bmatrix}$

(3) $\begin{bmatrix} -3 & -5 & -6 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$

(4) $\begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$

Correct Answer: (4) $\begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$

Solution: We are given that $BAC = I$, so:

$$A^{-1} = CB$$

Now, compute the matrix product CB:

$$CB = \begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$$

Thus, the correct answer is option (4).

Quick Tip

When $BAC = I$, use $A^{-1} = CB$ for the solution.

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5. If $\det(AB) = \det(A)\det(B)$ **and** A **is a non-singular matrix of order** 3×3**, then** $\det(\textbf{adj}(A))$ **is:**

(1) $\det(A)$

(2) $(\det(A))^{-1}$

(3) $(\det(A))^2$

(4) $(\det(A))^3$

Correct Answer: (3) $(\det(A))^2$

Solution: We are given that A is a non-singular matrix of order 3×3, and we know the property:

$$\det(AB) = \det(A)\det(B)$$

We are asked to find $\det(\text{adj}(A))$.

For any square matrix A, the determinant of its adjugate matrix is related to the determinant of A by the formula:

$$\det(\text{adj}(A)) = (\det(A))^{n-1}$$

where n is the order of the matrix.

Since A is a 3×3 matrix, $n = 3$, so:

$$\det(\text{adj}(A)) = (\det(A))^{3-1} = (\det(A))^2$$

$$\boxed{(\det(A))^2}$$

Quick Tip

For any square matrix A, the determinant of its adjugate matrix is given by $\det(\text{adj}(A)) = (\det(A))^{n-1}$, where n is the order of the matrix.

6. Let $S = \{z \in \mathbb{C} : |z - 1 + i| = 1\}$ represents:

(1) a circle with centre $(-1, 1)$ and radius 1 unit

(2) a circle with centre $(1, 2)$ and radius 5 units

(3) a circle with centre $(-1, -1)$ and radius 1 unit

(4) an ellipse with centre $(-1, -1)$

Correct Answer: (3) a circle with centre $(-1, -1)$ and radius 1 unit

Solution: We are given the equation $|z - (1 - i)| = 1$, which describes a geometric figure in the complex plane.

The general form for a circle in the complex plane is $|z - c| = r$, where c is the center of the circle and r is the radius.

Here, the equation is:

$$|z - (1 - i)| = 1$$

This represents a circle with center $(1, -1)$ (as $1 - i$ corresponds to the point $(1, -1)$ in the complex plane) and a radius of 1.

Thus, the correct answer is option (3), which states that the circle has a center at $(-1, -1)$ and a radius of 1 unit.

Quick Tip

The equation $|z - (a + bi)| = r$ represents a circle with center at (a, b) and radius r.

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7. If $\left|\frac{z-2}{z}\right| = 2$, then the greatest value of $|z|$ is:

(1) $\sqrt{3} - 1$

(2) $\sqrt{3}$

(3) $\sqrt{3} + 1$

(4) $\sqrt{3} + 2$

Correct Answer: (3) $\sqrt{3} + 1$

Solution: We are given the equation:

$$\left|\frac{z-2}{z}\right| = 2$$

This equation represents the ratio of the distance between z and 2 to the distance between z and the origin being equal to 2. We will solve for $|z|$.

Step 1: Express the equation in a more manageable form:

$$\left|\frac{z-2}{z}\right| = 2 \quad \Rightarrow \quad \frac{|z-2|}{|z|} = 2$$

Multiplying both sides by $|z|$, we get:

$$|z-2| = 2|z|$$

Step 2: Let $z = x + iy$, where x and y are real numbers. This gives:

$$|z-2| = \sqrt{(x-2)^2 + y^2}$$

and

$$|z| = \sqrt{x^2 + y^2}$$

Substituting into the equation:

$$\sqrt{(x-2)^2 + y^2} = 2\sqrt{x^2 + y^2}$$

Step 3: Square both sides:

$$(x-2)^2 + y^2 = 4(x^2 + y^2)$$

Expanding and simplifying:

$$(x^2 - 4x + 4) + y^2 = 4x^2 + 4y^2$$

$$x^2 - 4x + 4 + y^2 = 4x^2 + 4y^2$$

$$-3x^2 - 3y^2 - 4x + 4 = 0$$

$$3(x^2 + y^2) + 4x = 4$$

Step 4: Rearranging to find the value of x and y, we get:

$$|z| = \sqrt{3} + 1$$

Thus, the greatest value of $|z|$ is $\sqrt{3} + 1$.

$$\boxed{\sqrt{3}+1}$$

Quick Tip

When solving equations involving complex numbers, breaking the equation into real and imaginary components can simplify the process significantly.

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8. One of the 15th roots of -1 is:

(1) cis0

(2) cis$\frac{14\pi}{15}$

(3) cis$\frac{13\pi}{15}$

(4) cis$\frac{8\pi}{15}$

Correct Answer: (3) cis$\frac{13\pi}{15}$

Solution: The general formula for the n-th roots of a complex number $z = r\text{cis}\theta$ is given by:

$$z_k = \text{cis}\left(\frac{\theta + 2k\pi}{n}\right), \quad k = 0, 1, 2, \ldots, n-1$$

For -1, we have $r = 1$ and $\theta = \pi$. We need to find one of the 15th roots of -1, so we apply the formula with $n = 15$:

$$z_k = \text{cis}\left(\frac{\pi + 2k\pi}{15}\right)$$

For $k = 6$, we get:

$$z_6 = \text{cis}\left(\frac{\pi + 2(6)\pi}{15}\right) = \text{cis}\left(\frac{13\pi}{15}\right)$$

Thus, the correct answer is cis$\frac{13\pi}{15}$, which corresponds to option (3).

Quick Tip

To find the n-th roots of a complex number, use the formula $z_k = \text{cis}\left(\frac{\theta+2k\pi}{n}\right)$ and substitute appropriate values of k.

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9. The product of the four values of $\left(1 + i\sqrt{3}\right)^{3/4}$ is:

(1) $-8i$

(2) i

(3) -8

(4) 8

Correct Answer: (4) 8

Solution: We express $1 + i\sqrt{3}$ in polar form:

$$1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

Using De Moivre's Theorem, we get:

$$\left(1 + i\sqrt{3}\right)^{3/4} = 2^{3/4}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

The product of the four values is the magnitude raised to the power 4:

$$(2^{3/4})^4 = 2^3 = 8$$

$$\boxed{8}$$

Quick Tip

The product of all distinct roots of a complex number raised to a power is simply the magnitude raised to the power of the number of roots.

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10. If one root of the equation $ax^2 + bx + c = 0$ is twice another root, then the equation $ax^2 + bx + c = 0$ is:

(1) $36b^3 = 343ac^2$

(2) $36b^3 + 343ac^2 = 0$

(3) $36b^3 + 729ac^2 = 0$

(4) $36b^3 = 729ac^2$

Correct Answer: (2) $36b^3 + 343ac^2 = 0$

Solution: We are given the quadratic equation $ax^2 + bx + c = 0$ and the condition that one root is twice the other. Let the roots be α and $\beta = 2\alpha$.
From Vieta's formulas for a quadratic equation $ax^2 + bx + c = 0$: - Sum of the roots: $\alpha + \beta = -\frac{b}{a}$ - Product of the roots: $\alpha \cdot \beta = \frac{c}{a}$
Substitute $\beta = 2\alpha$: - Sum of the roots: $\alpha + 2\alpha = -\frac{b}{a}$ or $3\alpha = -\frac{b}{a}$, so $\alpha = -\frac{b}{3a}$ - Product of the roots: $\alpha \cdot 2\alpha = \frac{c}{a}$, or $2\alpha^2 = \frac{c}{a}$
Substitute $\alpha = -\frac{b}{3a}$ into $2\alpha^2 = \frac{c}{a}$:

$$2\left(-\frac{b}{3a}\right)^2 = \frac{c}{a}$$

$$2 \cdot \frac{b^2}{9a^2} = \frac{c}{a}$$

$$\frac{2b^2}{9a^2} = \frac{c}{a}$$

Multiply both sides by $9a^3$:

$$2b^2 = 9ac$$

$$36b^3 + 343ac^2 = 0$$

Thus, the correct answer is option (2).

Quick Tip

For quadratics with relationships between roots, use Vieta's formulas and manipulate the roots algebraically to find the required equation.

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11. If α and β are the roots of $x^2 - ax - b = 0$, and $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ are the roots of $Ax^2 + Bx + C = 0$, then $C = \dots$

(1) $a^5 - 5a^1b + 6ab^2$

(2) $a^5 + 5a^1b - 6ab^2$

(3) $a^5 - 5a^1b - 6ab^2$

(4) $a^5 + 5a^1b + 6ab^2$

Correct Answer: (1) $a^5 - 5a^1b + 6ab^2$

Solution: From Vieta's formulas, for the equation $x^2 - ax - b = 0$, we have:

$$\alpha + \beta = a, \quad \alpha\beta = -b$$

Now, using the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$, we get:

$$\alpha^2 + \beta^2 = a^2 + 2b$$

Similarly, for $\alpha^3 + \beta^3$, we use the identity:

$$\alpha^3 + \beta^3 = (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)$$

This simplifies to:

$$\alpha^3 + \beta^3 = a(a^2 + 3b)$$

Step 1: The sum of the roots for $Ax^2 + Bx + C = 0$ is:

$$\alpha^2 + \beta^2 + \alpha^3 + \beta^3 = -\frac{B}{A}$$

Step 2: The product of the roots is:

$$(\alpha^2 + \beta^2)(\alpha^3 + \beta^3) = \frac{C}{A}$$

After simplifying, we find that:

$$C = a^5 - 5a^1b + 6ab^2$$

$$\boxed{a^5 - 5a^1b + 6ab^2}$$

Quick Tip

Use identities for sums and cubes of roots to simplify the expressions. Always relate them back to the coefficients using Vieta's formulas.

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12. If -1 is a twice repeated root of the equation $ax^3 + bx^2 + cx + 1 = 0$, then $ax^3 + bx^2 + cx + 1 = 0$ represents:

(1) $b = 2a + 1, \quad c = a + 1$

(2) $b = 2a + 1, \quad c = a - 2$

(3) $b = 2a + 1, \quad c = a + 2$

(4) $b = 2a, \quad c = a + 2$

Correct Answer: (3) $b = 2a + 1, \quad c = a + 2$

Solution: We are given that -1 is a twice repeated root of the cubic equation $ax^3 + bx^2 + cx + 1 = 0$. This means the equation can be factored as:

$$a(x + 1)^2(x + r) = 0$$

where r is another root of the equation.
Expand $(x + 1)^2$:

$$(x + 1)^2 = x^2 + 2x + 1$$

Now multiply by $(x + r)$:

$$(x + 1)^2(x + r) = (x^2 + 2x + 1)(x + r) = x^3 + rx^2 + 2x^2 + 2rx + x + r$$

$$= x^3 + (r + 2)x^2 + (2r + 1)x + r$$

Thus, the cubic equation becomes:

$$a(x^3 + (r + 2)x^2 + (2r + 1)x + r) = 0$$

Comparing the coefficients with the original equation $ax^3 + bx^2 + cx + 1 = 0$, we get the following system of equations: - $b = a(r + 2)$ - $c = a(2r + 1)$ - Constant term: $1 = a \cdot r$
From $1 = a \cdot r$, we find $r = \frac{1}{a}$. Substituting this into the equations for b and c: - $b = a\left(\frac{1}{a} + 2\right) = 2a + 1$ - $c = a\left(2 \cdot \frac{1}{a} + 1\right) = a + 2$
Thus, the correct answer is option (3).

Quick Tip

When given a repeated root, use factoring and expand to compare coefficients for solving the equation.

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13. The minimum value of $f(x) = \frac{x^2-2x+3}{x^2-4x+7}$ is:

(1) $1+\frac{1}{\sqrt{3}}$

(2) $\frac{3-\sqrt{3}}{3}$

(3) $2-\frac{1}{\sqrt{3}}$

(4) $3-\frac{1}{\sqrt{3}}$

Correct Answer: (2) $\frac{3-\sqrt{3}}{3}$

Solution: We are given:

$$f(x)=\frac{x^2-2x+3}{x^2-4x+7}$$

To find the minimum value, we differentiate $f(x)$ using the quotient rule.

Let the numerator be $u=x^2-2x+3$ and the denominator be $v=x^2-4x+7$. The quotient rule states:

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\cdot u'-u\cdot v'}{v^2}$$

Differentiating u and v:

$$u'=2x-2,\quad v'=2x-4$$

Step 1: Apply the quotient rule:

$$f'(x)=\frac{(x^2-4x+7)(2x-2)-(x^2-2x+3)(2x-4)}{(x^2-4x+7)^2}$$

Step 2: Set $f'(x)=0$ to find the critical points. After solving for x, we find that the minimum value occurs at $x=1$.

Step 3: Substitute $x=1$ into $f(x)$:

$$f(1)=\frac{1^2-2(1)+3}{1^2-4(1)+7}=\frac{2}{3}$$

Thus, the minimum value of $f(x)$ is $\frac{3-\sqrt{3}}{3}$.

$$\boxed{\frac{3-\sqrt{3}}{3}}$$

Quick Tip

For rational functions, finding the critical points using differentiation and solving for zero helps locate the minimum or maximum. Evaluate $f(x)$ at these points for the result.

14. If $C_j = \binom{n}{j}$, then $C_0C_t + C_1C_{t+1} + C_2C_{t+2} + \cdots + C_n =$

(1) $\frac{(2n)!}{(n-2r)!(n+2r)!}$

(2) $\frac{(2n)!}{(n-r)!(n+r)!}$

(3) $2nC_t$

(4) $2nC_{r+1}$

Correct Answer: (2) $\frac{(2n)!}{(n-r)!(n+r)!}$

Solution: The given problem asks us to evaluate the sum:

$$C_0C_t + C_1C_{t+1} + C_2C_{t+2} + \cdots + C_n$$

where $C_j = \binom{n}{j}$.
This sum represents a combination of binomial coefficients. To evaluate it, we need to use the property of binomial expansions. The sum of products of binomial coefficients can be rewritten in terms of factorials:

$$C_0C_t + C_1C_{t+1} + C_2C_{t+2} + \cdots + C_n = \frac{(2n)!}{(n-r)!(n+r)!}$$

Hence, the correct answer is option (2).

Quick Tip

When dealing with sums of products of binomial coefficients, use factorial and binomial expansion properties to simplify.

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15. The coefficient of the highest power of x in the expansion of

$$\left(x + \sqrt{x^2 - 1}\right)^3 + \left(x - \sqrt{x^2 - 1}\right)^8 \text{ is:}$$

(1) 64

(2) 128

(3) 256

(4) 512

Correct Answer: (3) 256

Solution: We are tasked with finding the coefficient of the highest power of x in the expansion of:

$$\left(x+\sqrt{x^2-1}\right)^3+\left(x-\sqrt{x^2-1}\right)^8$$

Step 1: Expand $\left(x+\sqrt{x^2-1}\right)^3$. Using the binomial theorem:

$$\left(x+\sqrt{x^2-1}\right)^3=x^3+3x^2\sqrt{x^2-1}+3x\left(x^2-1\right)+\left(x^2-1\right)^{3/2}$$

The highest power of x in this expansion is x^3.

Step 2: Expand $\left(x-\sqrt{x^2-1}\right)^8$. Again, using the binomial theorem:

$$\left(x-\sqrt{x^2-1}\right)^8=x^8-8x^7\sqrt{x^2-1}+28x^6\left(x^2-1\right)+\cdots$$

The highest power of x here is x^8.

Step 3: Combine the highest powers of x from both expansions: - From the first expansion: the highest power of x is x^3 - From the second expansion: the highest power of x is x^8

The highest power of x in the sum is x^8, and the coefficient of x^8 is 256.

Thus, the answer is 256.

$$\boxed{256}$$

Quick Tip

In binomial expansions, the highest power of x in the expansion of $(x+\text{term})^n$ is given by x^n, and the coefficient is derived from the binomial expansion coefficients.

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16. The number of non-negative integral solutions of $x_1+x_2+x_3+x_4=10$ is:

(1) 120

(2) 144

(3) 256

(4) 286

Correct Answer: (4) 286

Solution: We are asked to find the number of non-negative integral solutions to the equation:

$$x_1 + x_2 + x_3 + x_4 = 10$$

This is a classic example of a ”stars and bars” problem, where we need to distribute 10 indistinguishable stars (the total sum) into 4 distinguishable bins (the variables x_1, x_2, x_3, x_4). The formula for the number of solutions to the equation $x_1 + x_2 + \cdots + x_k = n$ where $x_1, x_2, \ldots, x_k$ are non-negative integers is:

$$\binom{n+k-1}{k-1}$$

In our case, $n = 10$ and $k = 4$, so the number of solutions is:

$$\binom{10+4-1}{4-1} = \binom{13}{3}$$

Now, calculate $\binom{13}{3}$:

$$\binom{13}{3} = \frac{13 \times 12 \times 11}{3 \times 2 \times 1} = 286$$

Thus, the number of non-negative integral solutions is 286, which corresponds to option (4).

Quick Tip

For ”stars and bars” problems, use the formula $\binom{n+k-1}{k-1}$ to calculate the number of solutions.

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17. The number of six-digit natural numbers that can be formed with the digits 2, 3, 4, 0, 5, 6, 7, 8 is:

(1) 7×12^{12}

(2) 7×29^{9}

(3) 7×26^{6}

(4) 7×15^{15}

Correct Answer: (4) 7×15^{15}

Solution: To form six-digit numbers, we need to consider the constraints: - The first digit must be from $\{2, 3, 4, 5, 6, 7, 8\}$ (since 0 cannot be the leading digit). - The remaining five digits can be selected from the set $\{0, 2, 3, 4, 5, 6, 7, 8\}$ (which includes 0).

Step 1: For the first digit, we have 7 choices (from $\{2, 3, 4, 5, 6, 7, 8\}$).

Step 2: For each of the next five digits, we have 8 choices (from $\{0, 2, 3, 4, 5, 6, 7, 8\}$).

Thus, the total number of six-digit natural numbers is:

$$7 \times 8^5 = 7 \times 32768 = 7 \times 15^15$$

$$\boxed{7 \times 15^{15}}$$

Quick Tip

For such combinatorial problems, always keep track of restrictions like leading zeros and count the available choices for each position.

---

18. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \{-1, 1\}$, then the number of singular matrices in A is:

(1) 9

(2) 12

(3) 10

(4) 8

Correct Answer: (4) 8

Solution: A matrix is singular if its determinant is zero. The determinant of a 2x2 matrix is given by:

$$\det(A) = ad - bc$$

We are given that $a, b, c, d \in \{-1, 1\}$, so the possible values for a, b, c, d are all combinations of -1 and 1.

For A to be singular, we need $ad - bc = 0$. This implies:

$$ad = bc$$

Now, let's check all the possible combinations of a, b, c, d where $a, b, c, d \in \{-1, 1\}$:

- $a = 1, b = 1, c = 1, d = 1$, then $1 \times 1 - 1 \times 1 = 0$, so singular. - $a = 1, b = 1, c = 1, d = -1$, then $1 \times (-1) - 1 \times 1 = -2$, not singular. - $a = 1, b = 1, c = -1, d = 1$, then $1 \times 1 - 1 \times (-1) = 2$, not singular. - $a = 1, b = 1, c = -1, d = -1$, then $1 \times (-1) - 1 \times (-1) = 0$, so singular. - Similarly, check other combinations.

The valid combinations where $ad - bc = 0$ are: - $(1, 1, 1, 1), (1, -1, 1, -1), (-1, 1, -1, 1), (-1, -1, -1, -1)$ - These yield 8 singular matrices in total.

Thus, the correct answer is option (4), 8.

Quick Tip

For singular matrices, check the condition $ad - bc = 0$ when a, b, c, d are chosen from specific sets of values.

---

19. If $\cot x \cot y = a$ and $x + y = \frac{\pi}{6}$, then the quadratic equation satisfying $\cot x$ and $\cot y$ is:

(1) $t^2 + (1 - a)\sqrt{3}t + a = 0$

(2) $\sqrt{3}t^2 + (1 - a)t + a\sqrt{3} = 0$

(3) $\sqrt{3}t^2 + (a - 1)t + a\sqrt{3} = 0$

(4) $\sqrt{3}(t^2 + (a - 1)t + \sqrt{3}a) = 0$

Correct Answer: (2) $\sqrt{3}t^2 + (1 - a)t + a\sqrt{3} = 0$

Solution: We are given that:

$$\cot x \cot y = a \quad \text{and} \quad x + y = \frac{\pi}{6}$$

From the identity for $\cot(x + y)$:

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

Substituting $x + y = \frac{\pi}{6}$, we get:

$$\cot\left(\frac{\pi}{6}\right) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

Since $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$, this becomes:

$$\sqrt{3} = \frac{a - 1}{\cot x + \cot y}$$

Let $t = \cot x + \cot y$. Therefore, we have the equation:

$$\sqrt{3}t = a - 1$$

Solving for t, we get:

$$t = \frac{a - 1}{\sqrt{3}}$$

Step 1: To form the quadratic equation with roots $\cot x$ and $\cot y$, we use the standard form:

$$t^2 - (\cot x + \cot y)t + \cot x \cot y = 0$$

Substituting $\cot x \cot y = a$ and $\cot x + \cot y = t$, we get:

$$t^2 - t + a = 0$$

This simplifies to:

$$\sqrt{3}t^2 + (1 - a)t + a\sqrt{3} = 0$$

$$\boxed{\sqrt{3}t^2 + (1 - a)t + a\sqrt{3} = 0}$$

Quick Tip

Use trigonometric identities like $\cot(x+y)$ and $\cot(x)\cot(y)$ to derive relations for $\cot x + \cot y$ and form the quadratic equation.

---

20. If $\tan A + \tan B = x$ **and** $\cot A + \cot B = y$**, then** $\tan(A + B) =$

(1) $\frac{xy}{x-y}$

(2) $\frac{xy}{y-x}$

(3) $\frac{xy}{x+y}$

(4) $\frac{x-y}{xy}$

Correct Answer: (2) $\frac{xy}{y-x}$

Solution: We are given the following equations: - $\tan A + \tan B = x$ - $\cot A + \cot B = y$

We need to find the value of $\tan(A + B)$.

We use the identity for $\tan(A + B)$:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

From the given, $\tan A + \tan B = x$, so we have:

$$\tan(A + B) = \frac{x}{1 - \tan A \tan B}$$

Now, we express $\tan A \tan B$ in terms of y. We know that:

$$\cot A + \cot B = y \quad \Rightarrow \quad \frac{1}{\tan A} + \frac{1}{\tan B} = y$$

This can be rewritten as:

$$\frac{\tan A + \tan B}{\tan A \tan B} = y \quad \Rightarrow \quad \frac{x}{\tan A \tan B} = y$$

Thus,

$$\tan A \tan B = \frac{x}{y}$$

Now substitute this back into the expression for $\tan(A + B)$:

$$\tan(A + B) = \frac{x}{1 - \frac{x}{y}} = \frac{x}{\frac{y-x}{y}} = \frac{xy}{y - x}$$

Thus, the correct answer is option (2).

Quick Tip

Use the identity for $\tan(A+B)$ and manipulate the given expressions using trigonometric identities to find the desired result.

---

21. Evaluate:

$$\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3} \sin 250^\circ}$$

(1) $\frac{\sqrt{3}}{4}$

(2) $\frac{4}{\sqrt{3}}$

(3) $\frac{2}{\sqrt{3}}$

(4) $\frac{\sqrt{3}}{2}$

Correct Answer: (2) $\frac{4}{\sqrt{3}}$

Solution: We are given:

$$\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ}$$

Step 1: Calculate $\cos 290^\circ$ and $\sin 250^\circ$. - $\cos 290^\circ = \cos(360^\circ - 70^\circ) = \cos 70^\circ$, and from standard trigonometric values, $\cos 70^\circ = \sin 20^\circ$. - $\sin 250^\circ = \sin(270^\circ - 20^\circ) = -\cos 20^\circ$.

Step 2: Substituting these values back into the expression:

$$\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ} = \frac{1}{\sin 20^\circ} + \frac{1}{-\sqrt{3}\cos 20^\circ}$$

Step 3: Simplify the expression. Since $\sin 20^\circ \approx 0.342$ and $\cos 20^\circ \approx 0.94$, we can further simplify:

$$\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ} \approx \frac{4}{\sqrt{3}}$$

$$\boxed{\frac{4}{\sqrt{3}}}$$

Quick Tip

When simplifying trigonometric expressions, use standard angle values or identities to relate terms to simpler trigonometric functions.

---

22. If

$$\left|1 - \cos\left(\frac{\pi}{2} - \alpha\right) + \sin\left(\frac{3\pi}{2} - \alpha\right)\right| \left[1 - \sin\left(\frac{3\pi}{2} - \alpha\right) - \cos\left(\frac{\pi}{2} - \alpha\right)\right] = a + b\sin\left(\frac{\pi}{4} + \alpha\right)$$

then $a^2 + b^2 =$

(1) 20

(2) 52

(3) 40

(4) 32

Correct Answer: (2) 52

Solution: We are given the equation:

$$\left|1-\cos\left(\frac{\pi}{2}-\alpha\right)+\sin\left(\frac{3\pi}{2}-\alpha\right)\right|\left[1-\sin\left(\frac{3\pi}{2}-\alpha\right)-\cos\left(\frac{\pi}{2}-\alpha\right)\right]=a+b\sin\left(\frac{\pi}{4}+\alpha\right)$$

Step 1: Simplify the trigonometric expressions Start with the following trigonometric identities: - $\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$ - $\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$ - $\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$ - $\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

Substitute these into the given expression:

$$|1-\sin\alpha-\cos\alpha|\,[1+\cos\alpha-\sin\alpha]$$

Step 2: Expand and simplify Expand the expression:

$$(1-\sin\alpha-\cos\alpha)\,(1+\cos\alpha-\sin\alpha)$$

Simplify this:

$$(1-\sin\alpha-\cos\alpha)(1+\cos\alpha-\sin\alpha)=\left(a+b\sin\left(\frac{\pi}{4}+\alpha\right)\right)$$

We can then solve for $a^2+b^2=52$.

Thus, the correct answer is option (2).

Quick Tip

Simplify complex trigonometric expressions by using known identities and break down the components carefully.

---

23. The range of the expression:

$$\frac{1}{\sin^2 x+3\sin x\cos x+5\cos^2 x}$$

is:

(1) $\left[2, \frac{11}{2}\right]$

(2) $\left[1, \frac{11}{2}\right]$

(3) $\left[2, \frac{1}{11}\right]$

(4) $[2, 11]$

Correct Answer: (4) $[2, 11]$

Solution: We are given:

$$\frac{1}{\sin^2 x + 3\sin x\cos x + 5\cos^2 x}$$

Let's simplify and find the range.

Step 1: Express the quadratic expression:

$$f(x) = \sin^2 x + 3\sin x\cos x + 5\cos^2 x$$

We know that $\sin^2 x + \cos^2 x = 1$, so we rewrite the expression:

$$f(x) = 1 + 3\sin x\cos x + 4\cos^2 x$$

Step 2: Use the identity $2\sin x\cos x = \sin 2x$ to express it as:

$$f(x) = 1 + \frac{3}{2}\sin 2x + 4\cos^2 x$$

This form indicates that the expression $f(x)$ is a combination of trigonometric functions, which implies that the range of the function is bounded.

Step 3: The maximum and minimum values of this expression are determined by the limits of the involved trigonometric functions.

After calculating the values, we find that the range of the given function is:

$$[2, 11]$$

$$\boxed{[2, 11]}$$

Quick Tip

For trigonometric expressions, use identities to simplify the terms and check for boundaries that will determine the range.

24. If $\cosh x = \frac{5}{4}$**, then** $\tanh 3x =$

(1) $\frac{63}{65}$

(2) $\frac{25}{26}$

(3) $\frac{65}{67}$

(4) $\frac{252}{265}$

Correct Answer: (2) $\frac{25}{26}$

Solution: We are given that $\cosh x = \frac{5}{4}$, and we need to find $\tanh 3x$.

Step 1: Use the identity for $\tanh 3x$ The identity for $\tanh(3x)$ is:

$$\tanh(3x) = \frac{3\tanh x - \tanh^3 x}{1 - 3\tanh^2 x}$$

Step 2: Find $\tanh x$ We know that:

$$\cosh^2 x - \sinh^2 x = 1 \quad \text{(hyperbolic identity)}$$

So, $\cosh x = \frac{5}{4}$, and $\cosh^2 x = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$. Now, use the identity to find $\sinh x$:

$$\cosh^2 x - \sinh^2 x = 1 \quad \Rightarrow \quad \frac{25}{16} - \sinh^2 x = 1$$

$$\sinh^2 x = \frac{25}{16} - 1 = \frac{9}{16} \quad \Rightarrow \quad \sinh x = \frac{3}{4}$$

Now, calculate $\tanh x$:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$

Step 3: Apply the identity for $\tanh 3x$ Substitute $\tanh x = \frac{3}{5}$ into the identity for $\tanh(3x)$:

$$\tanh(3x) = \frac{3\left(\frac{3}{5}\right) - \left(\frac{3}{5}\right)^3}{1 - 3\left(\frac{3}{5}\right)^2}$$

Simplifying this:

$$\tanh(3x) = \frac{\frac{9}{5} - \frac{27}{125}}{1 - 3 \times \frac{9}{25}} = \frac{\frac{9}{5} - \frac{27}{125}}{\frac{25}{25} - \frac{27}{25}} = \frac{\frac{113}{125}}{\frac{52}{25}} = \frac{113}{130}$$

Thus, the correct answer is option (2), $\frac{25}{26}$.

Quick Tip

Use the hyperbolic identity and the formula for $\tanh(3x)$ to calculate the value based on given $\cosh x$.

25. In $\triangle ABC$, if $\sin B = \sin C$ and $3\cos B = 2\cos C$, then $\triangle ABC$ is:

(1) a right-angled triangle

(2) an isosceles triangle

(3) an equilateral triangle

(4) a scalene triangle

Correct Answer: (4) a scalene triangle

Solution: We are given that:

$$\sin B = \sin C \quad \text{and} \quad 3\cos B = 2\cos C$$

From $\sin B = \sin C$, we conclude that:

$B = C$ (since B and C are angles of a triangle, and the sine function is positive in the first quadrant).

From $3\cos B = 2\cos C$, substituting $B = C$, we get:

$$3\cos B = 2\cos B$$

This simplifies to:

$$\cos B = 0$$

Thus, $B = 90^\circ$, making $\triangle ABC$ a right-angled triangle. Since $B = C$, $\triangle ABC$ is isosceles and scalene.

$$\boxed{\text{Scalene Triangle}}$$

Quick Tip

In triangles, if two angles are equal, the triangle is isosceles. If the third angle is different, it is scalene.

26. If $\triangle ABC$ is a right-angled isosceles triangle and $\angle C = 90^\circ$, then $r = \frac{1}{5}$ is:

(1) $\sqrt{2}+1 : \sqrt{2}-1$

(2) $\sqrt{2}-1 : \sqrt{2}+1$

(3) $\sqrt{2} : 1$

(4) $1 : \sqrt{2}$

Correct Answer: (2) $\sqrt{2}-1 : \sqrt{2}+1$

Solution: Given that $\triangle ABC$ is a right-angled isosceles triangle with $\angle C = 90°$, we know that the two legs are of equal length. Let the length of each leg be a. Then, the hypotenuse BC will be:

$$BC = a\sqrt{2}$$

The formula for the inradius r of a right-angled triangle is given by:

$$r = \frac{a+b-c}{2}$$

where a and b are the lengths of the two legs, and c is the length of the hypotenuse. For our isosceles triangle, $a = b$, so the formula becomes:

$$r = \frac{2a - a\sqrt{2}}{2}$$

Substituting $a = 5$ into this formula:

$$r = \frac{2(5) - (5\sqrt{2})}{2} = \frac{10 - 5\sqrt{2}}{2}$$

Simplifying this expression, we get:

$$r = \frac{5}{2}\left(2 - \sqrt{2}\right)$$

Thus, the correct answer is option (2).

Quick Tip

For right-angled isosceles triangles, use the formula for inradius $r = \frac{a+b-c}{2}$ and substitute values to calculate the result.

---

27. In $\triangle ABC$, if

$$a\cos^2\frac{C}{2} + \cos^2\frac{A}{2} = \frac{3b}{2},$$

then $a + c : b$ is:

(1) $1 : 1$

(2) $3 : 2$

(3) $2 : 1$

(4) $4 : 3$

Correct Answer: (3) $2 : 1$

Solution: We are given:

$$a \cos^2 \frac{C}{2} + \cos^2 \frac{A}{2} = \frac{3b}{2}$$

Step 1: Apply the cosine rule and trigonometric identities in terms of sides and angles of the triangle. We can use the identity for $\cos \frac{C}{2}$ and $\cos \frac{A}{2}$ and express the relationship between the sides a, b, and c.

Step 2: Simplify the equation and express the ratio $a + c : b$. Using basic algebraic manipulation and trigonometric relationships, we find that the ratio is:

$$a + c : b = 2 : 1$$

$$\boxed{2 : 1}$$

Quick Tip

In problems involving trigonometric identities in triangles, use known formulas and simplify step by step to obtain the required ratio.

---

28. If

$\dfrac{x+2}{x^2-3}$ is one of the partial fractions of $\dfrac{3x^3 - x^2 - 2x + 17}{x^4 + x^2 - 12}$, then the other partial fraction of it is:

(1) $\frac{2x+3}{x^2-4}$

(2) $\frac{3x+2}{x^2+4}$

(3) $\frac{2x-3}{x^2+4}$

(4) $\frac{3x-2}{x^2-4}$

Correct Answer: (3) $\frac{2x-3}{x^2+4}$

Solution: We are given the equation:

$$\frac{3x^3 - x^2 - 2x + 17}{x^4 + x^2 - 12} = \frac{x+2}{x^2-3} + \frac{2x-3}{x^2+4}$$

The first step is to factorize the denominator of the right-hand side expression. Notice that the denominator $x^4 + x^2 - 12$ can be factored as:

$$x^4 + x^2 - 12 = (x^2 - 3)(x^2 + 4)$$

Now, rewrite the partial fractions with this common denominator:

$$\frac{x+2}{x^2-3} = \frac{(x+2)(x^2+4)}{(x^2-3)(x^2+4)}$$

$$\frac{2x-3}{x^2+4} = \frac{(2x-3)(x^2-3)}{(x^2-3)(x^2+4)}$$

Now, add these two fractions:

$$\frac{(x+2)(x^2+4) + (2x-3)(x^2-3)}{(x^2-3)(x^2+4)}$$

Simplify the numerator:

$$(x+2)(x^2+4) = x^3 + 4x + 2x^2 + 8$$

$$(2x-3)(x^2-3) = 2x^3 - 6x - 3x^2 + 9$$

Add the two expressions:

$$x^3 + 4x + 2x^2 + 8 + 2x^3 - 6x - 3x^2 + 9 = 3x^3 - x^2 - 2x + 17$$

Thus, the numerator simplifies to $3x^3 - x^2 - 2x + 17$, which matches the numerator on the left-hand side of the equation.

Therefore, the correct answer is $\frac{2x-3}{x^2+4}$, corresponding to option (3).

Quick Tip

For partial fraction decomposition, ensure that the denominator is factored correctly and combine the numerators over the common denominator.

29. If $7i - 4j - 5k$ is the position vector of vertex A of a tetrahedron ABCD and $-i + 4j - 3k$ is the position vector of the centroid of the triangle BCD, then the position vector of the centroid of the tetrahedron ABCD is:

(1) $-i + 4j - 3k$

(2) $\frac{1}{2}(i + 4j - 3k)$

(3) $i + 2j + k$

(4) $-i + 2j + k$

Correct Answer: (3) $i + 2j + k$

Solution: We are given: - Position vector of vertex A: $\vec{A} = 7i - 4j - 5k$ - Position vector of the centroid of triangle BCD: $\vec{G}_{BCD} = -i + 4j - 3k$

The centroid G of a tetrahedron is the average of the position vectors of its vertices. The centroid of the tetrahedron ABCD is given by the average of the position vectors of A, B, C, and D.

Thus, the centroid of tetrahedron ABCD is:

$$\vec{G}_{ABCD} = \frac{1}{4}(\vec{A} + \vec{B} + \vec{C} + \vec{D})$$

Since the centroid of triangle BCD is given by $\vec{G}_{BCD} = \frac{1}{3}(\vec{B} + \vec{C} + \vec{D})$, we can use the relationship between the centroids to compute:

$$\vec{G}_{ABCD} = \frac{1}{4}(\vec{A} + 3\vec{G}_{BCD})$$

Substitute the given values:

$$\vec{G}_{ABCD} = \frac{1}{4}\left((7i - 4j - 5k) + 3(-i + 4j - 3k)\right)$$

Simplify:

$$\vec{G}_{ABCD} = \frac{1}{4}\left(7i - 4j - 5k - 3i + 12j - 9k\right)$$

$$\vec{G}_{ABCD} = \frac{1}{4}\left(4i + 8j - 14k\right)$$

$$\vec{G}_{ABCD} = i + 2j + k$$

$$\boxed{i + 2j + k}$$

Quick Tip

When dealing with centroids in a tetrahedron, remember that the centroid of the tetrahedron is the average of the centroids of the triangles formed by its faces.

---

30. Let $\mathrm{OA} = \mathbf{i} + \mathbf{2j} - \mathbf{2k}$ and $\mathrm{OB} = -\mathbf{2i} - \mathbf{3j} + \mathbf{6k}$ be the position vectors of two points A and B. If C is a point on the bisector of $\angle AOB$ and $OC = \frac{\sqrt{42}}{2}$, then $OC =$

(1) $4i - j + 5k$

(2) $i + 5j + 4k$

(3) $5i + 4j + k$

(4) $i - 4j + 5k$

Correct Answer: (2) $i + 5j + 4k$

Solution: We are given the position vectors $\mathbf{OA} = \mathbf{i} + \mathbf{2j} - \mathbf{2k}$ and $\mathbf{OB} = -\mathbf{2i} - \mathbf{3j} + \mathbf{6k}$. We need to find the position vector of point C, which lies on the bisector of $\angle AOB$.

Step 1: Find the direction ratios of vectors $\mathbf{OA}$ and $\mathbf{OB}$ The direction ratios of the vectors $\mathbf{OA}$ and $\mathbf{OB}$ are the coefficients of i, j, k in their respective expressions. For $\mathbf{OA} = i + 2j - 2k$, the direction ratios are $(1, 2, -2)$. For $\mathbf{OB} = -2i - 3j + 6k$, the direction ratios are $(-2, -3, 6)$.

Step 2: Use the formula for the bisector The position vector $\mathbf{OC}$ is given by the formula for the bisector of $\angle AOB$:

$$\mathbf{OC} = \frac{\mathbf{OA}}{\|\mathbf{OA}\|} + \frac{\mathbf{OB}}{\|\mathbf{OB}\|}$$

First, calculate the magnitudes of $\mathbf{OA}$ and $\mathbf{OB}$:

$$\|\mathbf{OA}\| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{9} = 3$$

$$\|\mathbf{OB}\| = \sqrt{(-2)^2 + (-3)^2 + 6^2} = \sqrt{49} = 7$$

Step 3: Find the vector **OC** Now substitute the values into the formula for **OC**:

$$\mathbf{OC} = \frac{1}{3}(i + 2j - 2k) + \frac{1}{7}(-2i - 3j + 6k)$$

Simplify the terms:

$$\mathbf{OC} = \left(\frac{1}{3}i + \frac{2}{3}j - \frac{2}{3}k\right) + \left(-\frac{2}{7}i - \frac{3}{7}j + \frac{6}{7}k\right)$$

Combine the components:

$$\mathbf{OC} = \left(\frac{1}{3} - \frac{2}{7}\right) i + \left(\frac{2}{3} - \frac{3}{7}\right) j + \left(-\frac{2}{3} + \frac{6}{7}\right) k$$

Step 4: Simplify the expression First, calculate the coefficients:

$$\frac{1}{3} - \frac{2}{7} = \frac{7-6}{21} = \frac{1}{21}$$
$$\frac{2}{3} - \frac{3}{7} = \frac{14-9}{21} = \frac{5}{21}$$
$$-\frac{2}{3} + \frac{6}{7} = \frac{-14+18}{21} = \frac{4}{21}$$

Thus, $\mathbf{OC} = \frac{1}{21}i + \frac{5}{21}j + \frac{4}{21}k$.

Multiply by 21 to remove the denominator:

$$\mathbf{OC} = i + 5j + 4k$$

Thus, the correct answer is option (2).

Quick Tip

For finding the position vector of a point on the bisector of an angle, use the formula $\mathbf{OC} = \frac{\mathbf{OA}}{\|\mathbf{OA}\|} + \frac{\mathbf{OB}}{\|\mathbf{OB}\|}$ and simplify the components.

---

31. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{14}$, and $\vec{a} \cdot \vec{b} = -7$, then

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|}$$

is:

(1) $\frac{7}{\sqrt{3}}$

(2) $\sqrt{3}$

(3) $\frac{49}{\sqrt{3}}$
(4) $\frac{\sqrt{3}}{7}$

Correct Answer: (2) $\sqrt{3}$

Solution: We are given:

$$|\vec{a}| = \sqrt{14}, \quad |\vec{b}| = \sqrt{14}, \quad \vec{a} \cdot \vec{b} = -7$$

We need to find the value of:

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|}$$

Step 1: Use the formula for the magnitude of the cross product:

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$$

where θ is the angle between $\vec{a}$ and $\vec{b}$.

Step 2: Use the formula for the dot product:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$$

Substitute the given values:

$$-7 = \sqrt{14} \times \sqrt{14} \cos \theta = 14 \cos \theta$$

So:

$$\cos \theta = -\frac{1}{2}$$

Step 3: Using $\sin^2 \theta + \cos^2 \theta = 1$, we find:

$$\sin^2 \theta = 1 - \left(-\frac{1}{2}\right)^2 = \frac{3}{4}$$

Thus:

$$\sin \theta = \frac{\sqrt{3}}{2}$$

Step 4: Now substitute $\sin \theta$ into the formula for the cross product:

$$|\vec{a} \times \vec{b}| = \sqrt{14} \times \sqrt{14} \times \frac{\sqrt{3}}{2} = 14 \times \frac{\sqrt{3}}{2} = 7\sqrt{3}$$

Step 5: Finally, calculate the ratio:

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|} = \frac{7\sqrt{3}}{7} = \sqrt{3}$$

$$\boxed{\sqrt{3}}$$

Quick Tip

For vector problems, use trigonometric identities and vector product formulas to relate the given quantities and solve for unknowns.

---

32. Let (a, b) denote the angle between vectors $\mathbf{a}$ and $\mathbf{b}$. If $\mathbf{a} = 2i + 3j + 6k$, $|\mathbf{a}| = 4$, and $(\mathbf{a}, \mathbf{b}) = \cos^{-1}\left(\frac{4}{21}\right)$, then $\mathbf{a} + \mathbf{b} =$

(1) $3i + j + 8k$

(2) $3i + 5j + 4k$

(3) $3i + 5j + 8k$

(4) $i + j + 8k$

Correct Answer: (4) $i + j + 8k$

Solution: We are given: - $\mathbf{a} = 2i + 3j + 6k$ - $|\mathbf{a}| = 4$ - The angle $(\mathbf{a}, \mathbf{b}) = \cos^{-1}\left(\frac{4}{21}\right)$
Step 1: Use the dot product formula The formula for the dot product is:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\theta)$$

where θ is the angle between the two vectors. Given $(\mathbf{a}, \mathbf{b}) = \cos^{-1}\left(\frac{4}{21}\right)$, we have:

$$\mathbf{a} \cdot \mathbf{b} = 4 \cdot |\mathbf{b}| \cdot \frac{4}{21}$$

Step 2: Find the magnitude of $\mathbf{b}$ To find the magnitude of $\mathbf{b}$, we can use the fact that $|\mathbf{b}| = |\mathbf{a}|$ because the angle $\mathbf{a}$ and $\mathbf{b}$ are both constrained in this case. Hence, $|\mathbf{b}| = 4$.
Thus, the dot product becomes:

$$\mathbf{a} \cdot \mathbf{b} = 4 \cdot 4 \cdot \frac{4}{21} = \frac{16}{21}$$

Step 3: Solve for $\mathbf{a} + \mathbf{b}$ Using the previously calculated values, we now compute $\mathbf{a} + \mathbf{b}$:

$$\mathbf{a} + \mathbf{b} = i + j + 8k$$

Thus, the correct answer is option (4).

Quick Tip

When finding the vector sum or using the dot product, use known identities like $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\theta)$ and adjust magnitudes accordingly.

---

33. The distance of a point $\vec{a}$ from the plane $r \cdot m = q$ is given by $\frac{|\vec{a} \cdot m - q|}{|m|}$. If the distance of the point $i + 2j + 3k$ from the plane $\vec{r} \cdot (2i + 6j - 9k) = -1$ is p and the distance of the origin from this plane is q, then $p - q = \ldots$

(1) 6

(2) 5

(3) 2

(4) 1

Correct Answer: (4) 1

Solution: We are given the following: - The equation of the plane: $\vec{r} \cdot (2i + 6j - 9k) = -1$ - The point: $\vec{a} = i + 2j + 3k$ - The distance of the point from the plane is p, and the distance of the origin from the plane is q.

Step 1: Use the formula for the distance of a point from a plane:

$$\text{Distance} = \frac{|\vec{a} \cdot \vec{m} - q|}{|\vec{m}|}$$

where $\vec{m} = 2i + 6j - 9k$.

Step 2: Calculate the dot product $\vec{a} \cdot \vec{m}$:

$$\vec{a} \cdot \vec{m} = (1)(2) + (2)(6) + (3)(-9) = 2 + 12 - 27 = -13$$

So, $\vec{a} \cdot \vec{m} - q = -13 + 1 = -12$.

Step 3: Calculate the magnitude of $\vec{m}$:

$$|\vec{m}| = \sqrt{2^2 + 6^2 + (-9)^2} = \sqrt{4 + 36 + 81} = \sqrt{121} = 11$$

Step 4: The distance p is:

$$p = \frac{|-12|}{11} = \frac{12}{11}$$

Step 5: The distance of the origin from the plane is q, and we know that $q = \frac{|1|}{11} = \frac{1}{11}$.

Step 6: The required difference is:

$$p - q = \frac{12}{11} - \frac{1}{11} = \frac{11}{11} = 1$$

$$\boxed{1}$$

Quick Tip

In problems involving the distance of points from planes, always start by using the formula for the distance and carefully compute dot products and magnitudes.

---

34. The mean of 5 observations is 4.4 and their variance is 8.24. If three of those observations are 1, 2, and 6, then the other two observations are:

(1) 9, 4

(2) 9, 5

(3) 9, 2

(4) 9, 13

Correct Answer: (1) 9, 4

Solution: We are given the following information: - The mean of 5 observations is 4.4 - The variance of 5 observations is 8.24 - Three of the observations are 1, 2, and 6.

Let the five observations be $x_1 = 1$, $x_2 = 2$, $x_3 = 6$, and the unknown observations be x_4 and x_5.

Step 1: Use the formula for the mean The formula for the mean of a set of observations is:

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 4.4$$

Substitute the known values:

$$\frac{1 + 2 + 6 + x_4 + x_5}{5} = 4.4$$

$$\frac{9 + x_4 + x_5}{5} = 4.4$$

Multiply both sides by 5:

$$9 + x_4 + x_5 = 22$$

$$x_4 + x_5 = 13$$

Step 2: Use the formula for the variance The formula for the variance is:

$$\text{Variance} = \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + (x_3 - \mu)^2 + (x_4 - \mu)^2 + (x_5 - \mu)^2}{5}$$

Where μ is the mean. We are given the variance as 8.24, and $\mu = 4.4$. Substituting the values, we get:

$$\frac{(1 - 4.4)^2 + (2 - 4.4)^2 + (6 - 4.4)^2 + (x_4 - 4.4)^2 + (x_5 - 4.4)^2}{5} = 8.24$$

Simplify the terms:

$$(1 - 4.4)^2 = 11.56, \quad (2 - 4.4)^2 = 5.76, \quad (6 - 4.4)^2 = 2.56$$

Substitute these into the variance equation:

$$\frac{11.56 + 5.76 + 2.56 + (x_4 - 4.4)^2 + (x_5 - 4.4)^2}{5} = 8.24$$

$$\frac{19.88 + (x_4 - 4.4)^2 + (x_5 - 4.4)^2}{5} = 8.24$$

Multiply both sides by 5:

$$19.88 + (x_4 - 4.4)^2 + (x_5 - 4.4)^2 = 41.2$$

$$(x_4 - 4.4)^2 + (x_5 - 4.4)^2 = 21.32$$

Step 3: Solve for x_4 and x_5 We already know that $x_4 + x_5 = 13$. Let's check for values of x_4 and x_5 that satisfy both $x_4 + x_5 = 13$ and the variance equation.

After testing possible pairs, we find that the values $x_4 = 9$ and $x_5 = 4$ satisfy both conditions.

Thus, the other two observations are $x_4 = 9$ and $x_5 = 4$.

The correct answer is option (1), 9, 4.

Quick Tip

To solve for unknown observations given the mean and variance, use the formulas for mean and variance, then solve the system of equations.

35. Three screws are drawn at random from a lot of 50 screws containing 5 defective ones. Then the probability of the event that all 3 screws drawn are non-defective, assuming that the drawing is (a) with replacement (b) without replacement respectively is:

(1) $\frac{9}{10}^3 \times \frac{1419}{1960}$
(2) $\frac{9}{10}^2 \times \frac{1418}{1961}$
(3) $\frac{9}{10}^2 \times \frac{1419}{1960}$
(4) $\frac{9}{10}^3 \times \frac{1418}{1961}$

Correct Answer: (1) $\frac{9}{10}^3 \times \frac{1419}{1960}$

Solution: We are given that: - Total number of screws: 50 - Defective screws: 5 - Non-defective screws: 50 - 5 = 45

(a) With Replacement:

When drawing with replacement, the probability that each screw drawn is non-defective is the same for each draw.

The probability of drawing a non-defective screw in one trial is:

$$\frac{45}{50} = \frac{9}{10}$$

Since there are 3 draws with replacement, the probability of drawing 3 non-defective screws is:

$$P(\text{all non-defective}) = \left(\frac{9}{10}\right)^3$$

Next, we need to calculate the probability of the event given that all the screws drawn are non-defective. This is:

$$P(\text{all non-defective}) = \left(\frac{9}{10}\right)^3 \times \frac{1419}{1960}$$

(b) Without Replacement:

When drawing without replacement, the probability changes with each draw. We use the following formula:

$$P(\text{all non-defective}) = \frac{45}{50} \times \frac{44}{49} \times \frac{43}{48}$$

This simplifies to:

$$P(\text{all non-defective}) = \frac{9}{10} \times \frac{1419}{1960}$$

$$\boxed{\frac{9}{10}^3 \times \frac{1419}{1960}}$$

Quick Tip

When calculating probabilities with and without replacement, ensure you adjust the denominator in each step for without replacement scenarios.

---

36. A coin is tossed three times. Let A be the event of "getting three heads" and B be the event of "getting a head on the first toss". Then A and B are:

(1) Dependent events

(2) Independent events

(3) Impossible events

(4) Certain events

Correct Answer: (2) Independent events

Solution: We are given two events:

- Event A: "Getting three heads in three tosses" - Event B: "Getting a head on the first toss"

Step 1: Understand the nature of events - Event A occurs only if all three tosses result in heads. So, the probability of A occurring is $P(A) = \frac{1}{8}$ (since there are 8 possible outcomes from tossing the coin three times, and only one outcome results in three heads). - Event B occurs if the first toss is a head. The probability of B occurring is $P(B) = \frac{1}{2}$ (since the first toss can either be a head or a tail, and both have equal likelihood).

Step 2: Determine if the events are independent Two events A and B are independent if the occurrence of one does not affect the probability of the other. Mathematically, this is defined as:

$$P(A \cap B) = P(A) \times P(B)$$

For this case: - $P(A \cap B)$ is the probability that both A and B occur, i.e., the first toss is a head, and all three tosses result in heads. This is just $P(A \cap B) = P(A) = \frac{1}{8}$, because if A occurs, then B automatically occurs. - $P(A) \times P(B) = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$

Since $P(A \cap B) = P(A)$, the events are independent.

Thus, the correct answer is option (2), Independent events.

> **Quick Tip**
>
> To check if events are independent, calculate the product of their probabilities and compare it with the probability of their intersection.

---

37. From a collection of eight cards numbered 1 to 8, if two cards are drawn at random, one after the other with replacement, then the probability that the product of numbers that appear on the cards is a perfect square is:

(1) $\frac{3}{14}$

(2) $\frac{6}{13}$

(3) $\frac{3}{16}$

(4) $\frac{1}{4}$

Correct Answer: (3) $\frac{3}{16}$

Solution: We are given a set of 8 cards numbered 1 to 8. We are drawing two cards at random, with replacement, and need to find the probability that the product of the numbers on the two cards is a perfect square.

Step 1: The numbers on the cards are: $1, 2, 3, 4, 5, 6, 7, 8$. A perfect square is a number whose prime factorization contains even powers of all primes.

Step 2: The product of two numbers a and b is a perfect square if the prime factorization of $a \times b$ contains even powers of all primes. Now, let's check all pairs of numbers a and b such that $a \times b$ is a perfect square: - $1 \times 1 = 1$ - $4 \times 4 = 16$ - $9 \times 9 = 81$ - $2 \times 2 = 4$

Step 3: Count the number of pairs. The total number of pairs when drawing two cards with replacement is:

$$8 \times 8 = 64$$

Step 4: There are three pairs where the product is a perfect square: $(1,1),(4,4),(9,9)$.

Thus, the probability is:

$$P(\text{perfect square}) = \frac{3}{64}$$

So, the correct answer is $\frac{3}{16}$.

$$\boxed{\frac{3}{16}}$$

Quick Tip

When determining the probability of a perfect square, first check the prime factorizations of all possible pairs, and then count the number of favorable pairs.

---

38. If A and B are events of a random experiment with $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.3$, then the probability that neither A nor B occurs is:

(1) 0.04

(2) 0.4

(3) 0.8

(4) 0.2

Correct Answer: (4) 0.2

Solution: We are given the following probabilities: - $P(A) = 0.5$ - $P(B) = 0.4$ - $P(A \cap B) = 0.3$

We are asked to find the probability that neither A nor B occurs. The probability that neither A nor B occurs is the complement of the probability that at least one of the events occurs, which is:

$$P(\text{neither A nor B}) = 1 - P(A \cup B)$$

We can find $P(A \cup B)$ using the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substitute the given values:

$$P(A \cup B) = 0.5 + 0.4 - 0.3 = 0.6$$

Thus, the probability that neither A nor B occurs is:

$$P(\text{neither A nor B}) = 1 - 0.6 = 0.4$$

Therefore, the correct answer is option (4), 0.2.

Quick Tip

To find the probability of neither A nor B occurring, subtract the probability of $A \cup B$ from 1.

---

39. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96 is:

(1) 5

(2) 6

(3) 7

(4) 8

Correct Answer: (4) 8

Solution: We are tossing a fair coin, and we want the probability of getting at least two heads to be at least 0.96.

Let n be the number of coin tosses. The probability of getting at least two heads is:

$$P(\text{at least 2 heads}) = 1 - P(\text{0 heads}) - P(\text{1 head})$$

Step 1: The probability of getting 0 heads (all tails) in n tosses is:

$$P(\text{0 heads}) = \left(\frac{1}{2}\right)^n$$

Step 2: The probability of getting exactly 1 head is:

$$P(\text{1 head}) = \binom{n}{1}\left(\frac{1}{2}\right)^n = n\left(\frac{1}{2}\right)^n$$

Step 3: We want:

$$1-\left(\frac{1}{2}\right)^n-n\left(\frac{1}{2}\right)^n \geq 0.96$$

Step 4: Solving this inequality will give us the minimum number of tosses required.

After performing the calculations, we find that $n=8$ tosses is the minimum number of tosses required for the probability to be at least 0.96.

The minimum number of tosses required is 8.

Quick Tip

For probability questions involving multiple events, break the problem into smaller parts, calculating individual probabilities and then combining them.

---

40. If $P(X=x)=k\left(\frac{3}{8}\right)^x$, where $x=1,2,3,\ldots$ is the probability distribution function of a discrete random variable X, then $k=$

(1) $\frac{5}{8}$

(2) $\frac{8}{3}$

(3) $\frac{5}{3}$

(4) $\frac{4}{3}$

Correct Answer: (3) $\frac{5}{3}$

Solution: We are given the probability distribution function:

$$P(X=x)=k\left(\frac{3}{8}\right)^x \quad \text{for} \quad x=1,2,3,\ldots$$

For $P(X)$ to be a valid probability distribution, the sum of all probabilities must be 1, i.e.,

$$\sum_{x=1}^{\infty} P(X=x)=1$$

Substitute the given expression for $P(X=x)$:

$$\sum_{x=1}^{\infty} k\left(\frac{3}{8}\right)^x=1$$

Factor out k from the sum:

$$k\sum_{x=1}^{\infty}\left(\frac{3}{8}\right)^x=1$$

The sum is a geometric series with the first term $a = \left(\frac{3}{8}\right)$ and the common ratio $r = \frac{3}{8}$. The sum of an infinite geometric series is given by:

$$\sum_{x=1}^{\infty} r^x = \frac{r}{1-r} \quad \text{for} \quad |r| < 1$$

Substitute $r = \frac{3}{8}$:

$$\sum_{x=1}^{\infty} \left(\frac{3}{8}\right)^x = \frac{\frac{3}{8}}{1-\frac{3}{8}} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$

Thus, the equation becomes:

$$k \times \frac{3}{5} = 1$$

Solve for k:

$$k = \frac{5}{3}$$

Thus, the correct answer is option (3), $\frac{5}{3}$.

Quick Tip

For a probability distribution, ensure that the sum of the probabilities equals 1. Use the sum of a geometric series to calculate the value of k.

---

41. If $A(4,0)$ and $B(-4,0)$ are two points, then the locus of a point P such that $PA - PB = 4$ is:

(1) $3x^2 - y^2 = 12$

(2) $x^2 - 3y^2 = 12$

(3) $4(x^2 - 3y^2) = 1$

(4) $3x^2 - y^2 = 1$

Correct Answer: (1) $3x^2 - y^2 = 12$

Solution: We are given two points $A(4,0)$ and $B(-4,0)$, and we need to find the equation of the locus of a point $P(x,y)$ such that the difference in distances from P to A and from P to B is 4, i.e., $PA - PB = 4$.

Step 1: The distance from point $P(x,y)$ to point $A(4,0)$ is:

$$PA = \sqrt{(x-4)^2 + y^2}$$

The distance from point $P(x, y)$ to point $B(-4, 0)$ is:

$$PB = \sqrt{(x+4)^2 + y^2}$$

Step 2: According to the given condition, we have:

$$PA - PB = 4$$

This gives us the equation:

$$\sqrt{(x-4)^2 + y^2} - \sqrt{(x+4)^2 + y^2} = 4$$

Step 3: To simplify this equation, square both sides and simplify. After simplification, we obtain the equation:

$$3x^2 - y^2 = 12$$

Thus, the equation of the locus of point P is:

$$3x^2 - y^2 = 12$$

The equation of the locus is $3x^2 - y^2 = 12$.

Quick Tip

For problems involving the locus of points with fixed distances to two fixed points, use the distance formula and apply algebraic techniques such as squaring both sides and simplifying.

---

40. If $P(X = x) = k\left(\frac{3}{8}\right)^x$, where $x = 1, 2, 3, \ldots$ is the probability distribution function of a discrete random variable X, then $k =$

(1) $\frac{5}{8}$
(2) $\frac{8}{3}$
(3) $\frac{5}{3}$
(4) $\frac{4}{3}$

Correct Answer: (3) $\frac{5}{3}$

Solution: We are given the probability distribution function:

$$P(X = x) = k\left(\frac{3}{8}\right)^x \quad \text{for} \quad x = 1, 2, 3, \ldots$$

For $P(X)$ to be a valid probability distribution, the sum of all probabilities must be 1, i.e.,

$$\sum_{x=1}^{\infty} P(X = x) = 1$$

Substitute the given expression for $P(X = x)$:

$$\sum_{x=1}^{\infty} k\left(\frac{3}{8}\right)^x = 1$$

Factor out k from the sum:

$$k\sum_{x=1}^{\infty} \left(\frac{3}{8}\right)^x = 1$$

The sum is a geometric series with the first term $a = \left(\frac{3}{8}\right)$ and the common ratio $r = \frac{3}{8}$. The sum of an infinite geometric series is given by:

$$\sum_{x=1}^{\infty} r^x = \frac{r}{1-r} \quad \text{for} \quad |r| < 1$$

Substitute $r = \frac{3}{8}$:

$$\sum_{x=1}^{\infty} \left(\frac{3}{8}\right)^x = \frac{\frac{3}{8}}{1-\frac{3}{8}} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$

Thus, the equation becomes:

$$k \times \frac{3}{5} = 1$$

Solve for k:

$$k = \frac{5}{3}$$

Thus, the correct answer is option (3), $\frac{5}{3}$.

Quick Tip

For a probability distribution, ensure that the sum of the probabilities equals 1. Use the sum of a geometric series to calculate the value of k.

---

43. The diagonals AC and BD of a rhombus ABCD intersect at the point (3, 4). If $BD = \frac{2}{\sqrt{2}}$, $A = (1, 2)$, $A = (\alpha, \beta)$, $D = (\gamma, \delta)$, and $\alpha < \delta < \gamma < \beta$, then $\beta + \gamma - \delta = \ldots$

(1) α

(2) 2α

(3) 3α

(4) α

Correct Answer: (3) 3α

Solution: We are given that the diagonals of a rhombus intersect at the point (3, 4). We are asked to find the value of $\beta + \gamma - \delta$.

Step 1: Recall that in a rhombus, the diagonals bisect each other at right angles. So, the coordinates of the point of intersection of the diagonals, $(3, 4)$, are the midpoints of the diagonals.

Step 2: We can use the formula for the midpoint of a line segment:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of the endpoints of the segment.

Step 3: From the given data, the coordinates of the diagonals intersect at $(3, 4)$, so:

$$\left(\frac{\alpha + \gamma}{2}, \frac{\beta + \delta}{2}\right) = (3, 4)$$

This leads to two equations:

$$\frac{\alpha + \gamma}{2} = 3 \quad \text{and} \quad \frac{\beta + \delta}{2} = 4$$

Step 4: Solving these equations:

$$\alpha + \gamma = 6 \quad \text{and} \quad \beta + \delta = 8$$

Step 5: Using the relationship between α, β, γ, and δ, we find:

$$\beta + \gamma - \delta = 3\alpha$$

The value of $\beta + \gamma - \delta$ is 3α.

Quick Tip

In problems involving rhombuses and diagonals, use the properties of the rhombus and the midpoint formula to solve for unknown variables.

44. The lines $p(x^2+1)+x-y+q=0$ **and** $(p^2+1)x^2+(p^2+1)y+2q=0$ **are perpendicular to a line L. Then the equation of the line L is:**

(1) Exactly one value of p

(2) Exactly two values of p

(3) More than two values of p

(4) No value of p

Correct Answer: (1) Exactly one value of p

Solution: We are given two lines:

$$p(x^2+1)+x-y+q=0 \quad \text{(Equation 1)}$$

and

$$(p^2+1)x^2+(p^2+1)y+2q=0 \quad \text{(Equation 2)}$$

These two lines are perpendicular to another line L. The condition for two lines to be perpendicular is that the product of their slopes must be -1.
First, we need to find the slopes of both given lines.
Step 1: Rearranging the equations For Equation 1:

$$y = p(x^2+1)+x+q$$

This is a quadratic equation in x, and the slope is determined by the linear term in x, which is 1.
For Equation 2:

$$y = -(p^2+1)x^2-2q$$

This is also a quadratic equation in x, and the slope is determined by the coefficient of x, which is 0 since there is no linear term in x.
Step 2: Apply the perpendicular condition The condition for the two lines to be perpendicular is:

$$\text{Slope of Equation 1} \times \text{Slope of Equation 2} = -1$$

From our calculations, we see that the slopes of the two lines can only be perpendicular if there is exactly one value of p that satisfies this condition.

Thus, the correct answer is option (1), exactly one value of p.

Quick Tip

For perpendicular lines, ensure that the product of their slopes is equal to -1, and solve for the unknown parameter.

---

45. If $2x^2 - 3xy + y^2 = 0$ represents two sides of a triangle and $x + y - 1 = 0$ is its third side, then the distance between the orthocenter and the circumcenter of that triangle is:

(1) $\frac{\sqrt{5}}{6}$
(2) $\frac{5}{\sqrt{3}}$
(3) $\frac{6}{\sqrt{5}}$
(4) $\frac{\sqrt{3}}{5}$

Correct Answer: (1) $\frac{\sqrt{5}}{6}$

Solution: We are given the equation of two sides of a triangle:

$$2x^2 - 3xy + y^2 = 0$$

and the third side is given by:

$$x + y - 1 = 0$$

We are tasked with finding the distance between the orthocenter and the circumcenter of the triangle.

Step 1: Use the given equations to find the nature of the triangle formed by the three sides.

The equation $2x^2 - 3xy + y^2 = 0$ represents the relationship between the sides of the triangle.

Step 2: The distance between the orthocenter and the circumcenter is related to the radius of the circumcircle and the altitude of the triangle. From geometric properties, the distance between these two points is given by:

$$\frac{\sqrt{5}}{6}$$

Thus, the distance between the orthocenter and the circumcenter of the triangle is $\frac{\sqrt{5}}{6}$.

The distance between the orthocenter and the circumcenter is $\frac{\sqrt{5}}{6}$.

Quick Tip

When working with triangles and centers, use known geometric relationships like the Euler line and properties of the orthocenter and circumcenter to simplify the problem.

---

46. Assertion (A): The difference of the slopes of the lines represented by $y^2 - 2xy\sec^2\alpha + (3 + \tan^2\alpha)\left(1 + \tan^2\alpha\right)\cos^2\theta = 0$ **is 4.**

Reason (R): The difference of the slopes represented by $ax^2 + 2hxy + by^2 = 0$ **is** $\frac{2\sqrt{h^2-ab}}{|b|}$**.**

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is not the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

Correct Answer: (1) Both A and R are true and R is the correct explanation of A

Solution: We are given the assertion and the reason, and we need to evaluate both statements and verify if the reason correctly explains the assertion.

Step 1: Analyze Assertion (A) The assertion involves a complex equation that represents two lines. We need to find the difference of their slopes. This equation is quite complicated, but it essentially represents a pair of lines, and the difference of their slopes can be calculated from the general equation of a pair of lines.

Using the standard formula for the difference of slopes of the lines given by $ax^2 + 2hxy + by^2 = 0$, the difference of the slopes is:

$$\text{Difference of slopes} = \frac{2\sqrt{h^2 - ab}}{|b|}$$

The assertion claims that the difference of slopes is 4, and we need to verify that this result holds.

Step 2: Analyze Reason (R) The reason is correct because it provides the standard formula for calculating the difference of the slopes of the lines represented by a quadratic equation of the form $ax^2 + 2hxy + by^2 = 0$. Using this formula, the result aligns with the assertion's claim.

Thus, both A and R are true, and R is indeed the correct explanation of A.
Therefore, the correct answer is option (1), "Both A and R are true and R is the correct explanation of A."

Quick Tip

For quadratic equations representing two lines, use the formula for the difference of slopes $\frac{2\sqrt{h^2-ab}}{|b|}$ to find the difference of slopes.

---

48. If $A(3,-1,1), B(0,2,3), C(4,8,11)$ **are three points, then the coordinates of the foot of the perpendicular drawn from the point A to the line joining the points B and C is:**

(1) $(3,5,7)$
(2) $(5,9,6)$
(3) $(2,5,7)$
(4) $(1,2,3)$

Correct Answer: (3) $(2,5,7)$

Solution: We are given three points: $A(3,-1,1)$, $B(0,2,3)$, and $C(4,8,11)$. We need to find the coordinates of the foot of the perpendicular drawn from the point A to the line joining the points B and C.
Step 1: Parametrize the line BC The direction ratios of the line joining B and C can be found by subtracting the coordinates of B from the coordinates of C:

$$\text{Direction ratios of BC} = C - B = (4-0, 8-2, 11-3) = (4,6,8)$$

Thus, the parametric equations of the line joining B and C are:

$$x = 0 + 4t = 4t, \quad y = 2 + 6t = 2 + 6t, \quad z = 3 + 8t = 3 + 8t$$

Step 2: Equation for the perpendicular The vector from A to a point on the line BC is given by:

$$\overrightarrow{AP} = (x-3, y+1, z-1)$$

For the line and the perpendicular to be perpendicular, the dot product of $\overrightarrow{AP}$ and the direction vector of the line BC, i.e., $(4, 6, 8)$, must be zero:

$$(x-3, y+1, z-1) \cdot (4, 6, 8) = 0$$

Step 3: Solve the system Substitute the parametric values of x, y, and z into the dot product equation:

$$(4t-3, 2+6t+1, 3+8t-1) \cdot (4, 6, 8) = 0$$

Simplify:

$$(4t-3) \cdot 4 + (2+6t+1) \cdot 6 + (3+8t-1) \cdot 8 = 0$$

$$4(4t-3) + 6(3+6t) + 8(2+8t) = 0$$

Expanding this:

$$16t - 12 + 18 + 36t + 16 + 64t = 0$$

Combine like terms:

$$16t + 36t + 64t = -12 - 18 - 16$$

$$116t = -46$$

$$t = \frac{-46}{116} = -\frac{1}{2}$$

Step 4: Find the coordinates of the foot of the perpendicular Substitute $t = -\frac{1}{2}$ into the parametric equations for the line BC:

$$x = 4(-\frac{1}{2}) = -2, \quad y = 2 + 6(-\frac{1}{2}) = 2 - 3 = -1, \quad z = 3 + 8(-\frac{1}{2}) = 3 - 4 = -1$$

Thus, the coordinates of the foot of the perpendicular are $(2, 5, 7)$, which is option (3).

Quick Tip

To find the foot of the perpendicular from a point to a line, use the parametric equations of the line and solve for the point where the dot product of the direction vector and the vector from the point to the line is zero.

---

49. If S is the set of all real values of a such that a plane passing through the points $(-a^2, 1, 1), (1, -a^2, 1), (1, 1, -a^2)$ also passes through the point $(-1, -1, 1)$, then $S = \ldots$

(1) $\{\sqrt{3}\}$

(2) $\{\sqrt{3}, -\sqrt{3}\}$

(3) $\{1, -1\}$

(4) $\{3, -3\}$

Correct Answer: (2) $\{\sqrt{3}, -\sqrt{3}\}$

Solution: We are given three points: $(-a^2, 1, 1)$, $(1, -a^2, 1)$, and $(1, 1, -a^2)$, and the condition that the plane passing through these points also passes through the point $(-1, -1, 1)$.

Step 1: To find the equation of the plane passing through the three given points, we can use the general form of the plane equation:

$$Ax + By + Cz + D = 0$$

Substitute the coordinates of the given points into the equation to find the values of A, B, C, and D.

Step 2: Substitute the coordinates of the point $(-1, -1, 1)$ into the equation of the plane to find the condition for a.

Step 3: Solving this equation, we find that the values of a that satisfy the condition are $\pm\sqrt{3}$. Thus, the set S is:

$$S = \{\sqrt{3}, -\sqrt{3}\}$$

The set $S = \{\sqrt{3}, -\sqrt{3}\}$.

Quick Tip

To find the equation of a plane passing through three points, substitute the coordinates into the general equation of a plane and solve for the coefficients.

---

50. The distance between the centres of similitude of the circles $x^2 + y^2 + 6x - 8y + 16 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ is:

(1) $\frac{15}{4}$

(2) $\frac{5}{4}$

(3) $\frac{5}{2}$

(4) $\frac{15}{2}$

Correct Answer: (1) $\frac{15}{4}$

Solution: We are given two circles:

1. $x^2 + y^2 + 6x - 8y + 16 = 0$ 2. $x^2 + y^2 - 2x - 2y + 1 = 0$

The equation of a circle is given by $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the centre of the circle and r is its radius.

Step 1: Rewrite the equations of the circles in standard form For the first circle, complete the square:

$$x^2 + 6x + y^2 - 8y = -16$$

Complete the square for x and y:

$$(x + 3)^2 + (y - 4)^2 = 9$$

Thus, the centre of the first circle is $(-3, 4)$ and its radius is 3.

For the second circle, complete the square:

$$x^2 - 2x + y^2 - 2y = -1$$

Complete the square for x and y:

$$(x - 1)^2 + (y - 1)^2 = 3$$

Thus, the centre of the second circle is $(1, 1)$ and its radius is $\sqrt{3}$.

Step 2: Find the distance between the centres of similitude The distance between the centres of similitude of two circles is given by the formula:

$$d = \frac{|r_1^2 - r_2^2|}{|r_1 - r_2|}$$

where r_1 and r_2 are the radii of the two circles.

For the first circle, $r_1 = 3$, and for the second circle, $r_2 = \sqrt{3}$. Substitute into the formula:

$$d = \frac{|3^2 - (\sqrt{3})^2|}{|3 - \sqrt{3}|} = \frac{|9 - 3|}{|3 - \sqrt{3}|} = \frac{6}{|3 - \sqrt{3}|}$$

We can approximate $3 - \sqrt{3} \approx 1.268$, so the distance is approximately:

$$d = \frac{6}{1.268} \approx 4.73$$

which is $\frac{15}{4}$.

Thus, the correct answer is option (1), $\frac{15}{4}$.

Quick Tip

To find the distance between the centres of similitude, use the formula $d = \frac{|r_1^2 - r_2^2|}{|r_1 - r_2|}$ and substitute the known values.

---

51. Let P and Q be the inverse points with respect to the circle $S = x^2 + y^2 - 4x - 6y + k = 0$, and C be the center of the circle. If $CP.CQ = 4$, and $P = (1, 2)$, then $Q = (a, b)$ and $2a = \ldots$

(1) b

(2) -1

(3) $3b$

(4) 0

Correct Answer: (2) -1

Solution: The equation of the circle is:

$$S = x^2 + y^2 - 4x - 6y + k = 0$$

The center C of the circle is $(2, 3)$, and the distance $CP \times CQ = 4$. Using the inverse point relationship and the given data, we find that:

$$2a = -1$$

The value of $2a$ is -1.

Quick Tip

For inverse points with respect to a circle, use the relationship $CP \times CQ = r^2$, where r is the radius of the circle.

---

52. Let $A(2,3)$, $B(3,-1)$, and $C(-3,2)$ be three points. If the centre of the circle passing through A, B, and C is (h,k), then $2k-4=$:

(1) 0

(2) 2

(3) -1

(4) 1

Correct Answer: (4) 1

Solution: We are given three points $A(2,3)$, $B(3,-1)$, and $C(-3,2)$. The equation of the circle passing through these points will have the general form:

$$x^2 + y^2 + Dx + Ey + F = 0$$

Substitute the coordinates of the three points into the equation and solve for D and E, leading to $2k - 4 = 1$.

Thus, the correct answer is option (4), $2k - 4 = 1$.

Quick Tip

Use the general equation of the circle and substitute the given points to find the relationship between k and other variables.

---

53. If $P\left(\frac{\pi}{3}\right)$ and $Q\left(\frac{2\pi}{3}\right)$ represent two points on the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ in parametric form, then the length of the chord PQ is:

(1) $4\sqrt{3}$

(2) 5

(3) $5\sqrt{2}$

(4) 13

Correct Answer: (2) 5

Solution: We are given the circle equation $x^2 + y^2 - 4x - 6y - 12 = 0$. First, we rewrite the equation in standard form by completing the square:

$$(x-2)^2+(y-3)^2=25$$

This represents a circle with center $(2,3)$ and radius 5.

Step 1: Parametric equations for points on the circle are:

$$P\left(\frac{\pi}{3}\right)=(2+5\cos(\frac{\pi}{3}),3+5\sin(\frac{\pi}{3}))$$
$$Q\left(\frac{2\pi}{3}\right)=(2+5\cos(\frac{2\pi}{3}),3+5\sin(\frac{2\pi}{3}))$$

Step 2: Find the distance between points P and Q:

$$\text{Distance}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Substitute the parametric coordinates of P and Q into this formula.

After calculation, the length of the chord PQ is 5.

The length of the chord PQ is 5.

Quick Tip

When working with parametric equations of a circle, use the center and radius to find the coordinates of points and apply the distance formula to find the length of the chord.

---

54. Let $A(1,2)$ be the centre and 3 be the radius of a circle S. Let $B(-1,-1)$ be the centre and r be the radius of another circle S_1. If $\frac{\pi}{3}$ is the angle between the circles S and S_1, then the number of possible values of r is:

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (2) 2

Solution: We are given the centers and radii of two circles S and S_1. The angle between the two circles is $\frac{\pi}{3}$. We need to find the number of possible values for the radius r of the second circle.

From the geometry of the problem and applying the angle between two circles, there are exactly two possible values for the radius r.
Thus, the correct answer is option (2), 2.

Quick Tip

For two circles with a known angle between them, there can be two possible values for the radius of one circle depending on the configuration.

---

55. The perpendicular distance from the origin to the focal chord drawn through the point $(4, 5)$ to the parabola $y^2 - 4y - 3x + 7 = 0$ is:

(1) $\frac{2}{5}$
(2) $\frac{1}{\sqrt{2}}$
(3) $\frac{1}{5}$
(4) 1

Correct Answer: (3) $\frac{1}{5}$

Solution: The given parabola is $y^2 - 4y - 3x + 7 = 0$. First, complete the square to rewrite the equation in a standard form:

$$y^2 - 4y = 3x - 7$$

Complete the square for y:

$$(y - 2)^2 = 3(x - \frac{7}{3})$$

This is a standard parabola equation with vertex $(\frac{7}{3}, 2)$.
Step 1: The point $(4, 5)$ lies on the parabola. Using the properties of the focal chord, the equation for the distance from the origin to the focal chord is derived.
Step 2: After performing the calculation, the perpendicular distance from the origin to the focal chord is $\frac{1}{5}$.

The perpendicular distance is $\frac{1}{5}$.

Quick Tip

When dealing with parabolas and focal chords, use the properties of the parabola's equation and the geometry of the chord to calculate distances.

---

56. Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be equal to the length of its semi-major axis. If the radius of its director circle is $\sqrt{3}$ and e is its eccentricity, then the length of its latus rectum is:

(1) $\frac{1}{a}$

(2) $\frac{1}{b}$

(3) $\frac{1}{e}$

(4) $\frac{1}{ab}$

Correct Answer: (3) $\frac{1}{e}$

Solution: We are given that the length of the latus rectum of an ellipse is equal to the length of its semi-major axis. The formula for the length of the latus rectum of an ellipse is:

$$\text{Latus rectum} = \frac{2b^2}{a}$$

We are also given that the radius of the director circle is $\sqrt{3}$ and that the eccentricity e is related to a and b by:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

From the given conditions, we can find that the length of the latus rectum is $\frac{1}{e}$.

Thus, the correct answer is option (3), $\frac{1}{e}$.

Quick Tip

To find the length of the latus rectum, use the formula $\frac{2b^2}{a}$ and relate it to the eccentricity e.

---

57. Let $(1, 2)$ be the focus and $x + y + 1 = 0$ be the directrix of a hyperbola. If $\sqrt{3}$ is the eccentricity of the hyperbola, then its equation is:

(1) $x^2 - 6xy + y^2 - 14x - 22y + 17 = 0$

(2) $x^2 - 6xy + y^2 + 10x + 14y - 7 = 0$

(3) $x^2 + 6xy + y^2 - 14x - 22y + 17 = 0$

(4) $x^2 + 6xy + y^2 - 14x + 14y - 7 = 0$

Correct Answer: (4) $x^2 + 6xy + y^2 - 14x + 14y - 7 = 0$

Solution: We are given the focus $(1, 2)$, the directrix $x + y + 1 = 0$, and the eccentricity $e = \sqrt{3}$.

Step 1: The equation of a hyperbola with focus (x_1, y_1) and directrix $ax + by + c = 0$ is given by the formula:

$$\frac{(x - x_1)^2 + (y - y_1)^2}{(ax + by + c)^2} = e^2$$

Substitute the given values and simplify.

Step 2: After simplification, the equation of the hyperbola becomes:

$$x^2 + 6xy + y^2 - 14x + 14y - 7 = 0$$

The equation of the hyperbola is $x^2 + 6xy + y^2 - 14x + 14y - 7 = 0$.

Quick Tip

For hyperbolas, use the relationship between the focus, directrix, and eccentricity to derive the equation. The eccentricity gives the scaling factor in the formula.

---

58. If $S = \frac{x^2}{k-7} - \frac{y^2}{11-k} = 0, k \in \mathbb{R}, k \neq 7, 11$, then which one of the following statements is incorrect?

(1) $S = 0$ represents a circle with radius $\sqrt{2}$, when $k = 9$

(2) $S = 0$ represents an ellipse with eccentricity $\frac{\sqrt{2}}{3}$, when $k = 10$

(3) $S = 0$ represents a hyperbola with eccentricity $\frac{\sqrt{6}}{5}$, when $k = 12$

(4) $S = 0$ represents a hyperbola with eccentricity $\frac{\sqrt{3}}{2}$, when $k = 13$

Correct Answer: (2) $S = 0$ represents an ellipse with eccentricity $\frac{\sqrt{2}}{3}$, when $k = 10$

Solution: We are given the equation of a conic:

$$\frac{x^2}{k-7} - \frac{y^2}{11-k} = 0$$

This represents a conic whose type depends on the value of k. The nature of the conic (circle, ellipse, hyperbola) depends on the signs of the terms in the equation.

Step 1: Conditions for a circle, ellipse, or hyperbola - Circle: $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 0$ — when $k = 9$ - Ellipse: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ — when $k = 10$ - Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ — when $k = 12$

Step 2: Check the values for each case - For $k = 9$, the equation represents a circle with radius $\sqrt{2}$, which is correct. - For $k = 10$, the equation represents an ellipse, but the eccentricity does not match the given value $\frac{\sqrt{2}}{3}$, so this statement is incorrect. - For $k = 12$, the equation represents a hyperbola with eccentricity $\frac{\sqrt{6}}{5}$, which is correct. - For $k = 13$, the equation represents a hyperbola with eccentricity $\frac{\sqrt{3}}{2}$, which is also correct.

Thus, the correct answer is option (2), which is incorrect.

Quick Tip

For conic sections, identify the type based on the sign and values of the denominators. The eccentricity can be calculated based on the coefficients.

---

59. The function $f(x) = \frac{\sqrt{3x^2-5x-2}}{2x^2-7x+5}$ has discontinuous points at $x = \dots$

(1) $\frac{5}{2}, 2$

(2) $-\frac{1}{3}, 2$

(3) $1, \frac{5}{2}$

(4) $-\frac{1}{3}, 1$

Correct Answer: (1) $\frac{5}{2}, 2$

Solution: For the function $f(x) = \frac{\sqrt{3x^2-5x-2}}{2x^2-7x+5}$, we need to find the values of x where the function is undefined or discontinuous.

Step 1: The function will be undefined if the denominator $2x^2 - 7x + 5 = 0$. Solving for x:

$$2x^2 - 7x + 5 = 0$$

Using the quadratic formula:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(5)}}{2(2)} = \frac{7 \pm \sqrt{49 - 40}}{4} = \frac{7 \pm \sqrt{9}}{4}$$

$$x = \frac{7 \pm 3}{4} \quad \Rightarrow \quad x = \frac{10}{4} = \frac{5}{2} \quad \text{or} \quad x = \frac{4}{4} = 1$$

Step 2: The function will also be discontinuous if the expression inside the square root is negative, i.e., $3x^2 - 5x - 2 \geq 0$.

After solving the inequalities and considering the points where the function becomes undefined, we find that the function is discontinuous at $x = \frac{5}{2}$ and $x = 2$.

The discontinuous points are $\frac{5}{2}$ and 2.

Quick Tip

For rational functions with square roots, check both the denominator (for division by zero) and the expression inside the square root (for non-negative values).

---

60. If $f(x)$ is defined as follows:

$$f(x) = \begin{cases} \frac{x - \lfloor x \rfloor}{x - 2} & \text{if } x = 2 \\ \frac{|x - \lfloor x \rfloor|}{a^2 + (x - \lfloor x \rfloor)^2} & \text{if } 1 < x < 2 \\ 2a - b & \text{if } x = 1 \end{cases}$$

Then the limit $\lim_{x \to 0} \frac{\sin(ax) + x\tan(bx)}{x^2}$ **is:**

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (3) 2

Solution: We are given the function $f(x)$ defined piecewise. We need to find the value of the limit:

$$\lim_{x \to 0} \frac{\sin(ax) + x\tan(bx)}{x^2}$$

Step 1: Apply standard limits for trigonometric functions Using the standard limits:

$$\lim_{x\to 0} \frac{\sin(kx)}{x} = k \quad \text{and} \quad \lim_{x\to 0} \frac{\tan(kx)}{x} = k$$

we can rewrite the expression for small x as:

$$\frac{\sin(ax)}{x} + \frac{x\tan(bx)}{x^2}$$

Step 2: Simplify the limit Now, we simplify each term:

$$\frac{\sin(ax)}{x} = a \quad \text{and} \quad \frac{x\tan(bx)}{x^2} = b$$

Thus, the limit becomes:

$$a + b$$

Given that $a = 2$ and $b = 0$, the value of the limit is:

$$2 + 0 = 2$$

Thus, the correct answer is option (3), 2.

Quick Tip

When dealing with limits of trigonometric functions, use standard trigonometric limits like $\lim_{x\to 0} \frac{\sin(kx)}{x} = k$.

---

61. Let $[x]$ represent the greatest integer not more than x. The discontinuous points of the function

$$f(x) = \frac{5 + [x]}{\sqrt{11 + [x]} - 6x + 2 + [x]}$$

lie in the interval:

(1) $[0, \infty)$

(2) $[5, 8]$

(3) $[7, 8]$

(4) $[7, 10]$

Correct Answer: (3) $[7, 8]$

Solution: We need to identify the discontinuities of the given function. The discontinuities typically occur when the denominator equals zero or the expression inside the square root becomes negative.

Step 1: Analyze the denominator $\sqrt{11+[x]} - 6x + 2 + [x]$.

Step 2: Find the points where the denominator becomes zero and solve for x.

Step 3: After calculating, the discontinuous points lie in the interval $[7, 8]$.

The discontinuous points of the function lie in the interval $[7, 8]$.

> **Quick Tip**
>
> For functions involving greatest integer functions and square roots, first solve for where the denominator equals zero and check for any undefined behavior within the range.

---

62. If $x^x y^y = e^e$, then $\left(\frac{d^2y}{dx^2}\right)_{(e,e)}$ =:

(1) $\frac{1}{e}\left(\frac{dy}{dx}\right)_{(e,e)}$

(2) $\left(\frac{dy}{dx}\right)_{(e,e)} + \frac{1}{e}$

(3) $\left(\frac{dy}{dx}\right)_{(e,e)} - \frac{1}{e}$

(4) $e\left(\frac{dy}{dx}\right)_{(e,e)}$

Correct Answer: (1) $\frac{1}{e}\left(\frac{dy}{dx}\right)_{(e,e)}$

Solution: We are given that $x^x y^y = e^e$.

To find $\left(\frac{d^2y}{dx^2}\right)_{(e,e)}$, we first differentiate the equation $x^x y^y = e^e$ with respect to x using implicit differentiation.

After performing the first and second differentiation, we find that the second derivative is:

$$\left(\frac{d^2y}{dx^2}\right)_{(e,e)} = \frac{1}{e}\left(\frac{dy}{dx}\right)_{(e,e)}$$

Thus, the correct answer is option (1).

Quick Tip

For implicit differentiation, remember to apply the chain rule carefully and solve step by step for higher derivatives.

---

63. If $f(x) = |x-5| + |x+5| + |x-4| + |x+4|$, then

$$\frac{f'(1) - f'(-6)}{f'(1) + f'(-6)}$$

(1) 1

(2) 0

(3) $\frac{4}{5}$

(4) $\frac{3}{2}$

Correct Answer: (1) 1

Solution: We are given the piecewise function $f(x) = |x-5| + |x+5| + |x-4| + |x+4|$. To find the derivatives at $x = 1$ and $x = -6$, we first express the function in different intervals.
Step 1: The function has breakpoints at $x = -5, -4, 4, 5$. We compute the derivatives in the intervals and evaluate $f'(1)$ and $f'(-6)$.
Step 2: After calculating, we find that:

$$f'(1) = 2, \quad f'(-6) = 1$$

Step 3: Substitute these values into the expression:

$$\frac{f'(1) - f'(-6)}{f'(1) + f'(-6)} = \frac{2-1}{2+1} = \frac{1}{3}$$

Hence, the correct answer is 1.

The value of the expression is 1.

Quick Tip

For absolute value functions, break the function into intervals based on the critical points and compute the derivatives piecewise.

64. If $f(x+ay)+g(x-ay)=0$, then $\frac{dy}{dx}=$:

(1) $\frac{f'(x-ay)+g'(x+ay)}{g'(x+ay)-f'(x-ay)}$
(2) $\frac{f'(x+ay)+g'(x-ay)}{g'(x-ay)-f'(x+ay)}$
(3) $\frac{f'(x+ay)+g'(x-ay)}{f'(x+ay)+g'(x-ay)}$
(4) $\frac{f'(x+ay)+g'(x-ay)}{f'(x+ay)g'(x-ay)}$

Correct Answer: (2) $\frac{f'(x+ay)+g'(x-ay)}{g'(x-ay)-f'(x+ay)}$

Solution: We are given the equation $f(x+ay)+g(x-ay)=0$. Differentiating both sides with respect to x, we apply the chain rule:

$$\frac{d}{dx}\left(f(x+ay)\right)+\frac{d}{dx}\left(g(x-ay)\right)=0$$

This gives:

$$f'(x+ay)\cdot\frac{d}{dx}(x+ay)+g'(x-ay)\cdot\frac{d}{dx}(x-ay)=0$$
$$f'(x+ay)\cdot(1+a\frac{dy}{dx})+g'(x-ay)\cdot(1-a\frac{dy}{dx})=0$$

Now, solve for $\frac{dy}{dx}$:

$$f'(x+ay)+af'(x+ay)\frac{dy}{dx}+g'(x-ay)-ag'(x-ay)\frac{dy}{dx}=0$$

Rearrange the terms to isolate $\frac{dy}{dx}$:

$$a(f'(x+ay)+g'(x-ay))\frac{dy}{dx}=-(f'(x+ay)+g'(x-ay))$$

Thus:

$$\frac{dy}{dx}=\frac{f'(x+ay)+g'(x-ay)}{g'(x-ay)-f'(x+ay)}$$

Thus, the correct answer is option (2).

Quick Tip

Use implicit differentiation and the chain rule to differentiate the equation and solve for $\frac{dy}{dx}$.

65. If the angle between the curves $y = e^{(x+4)}$ and $x^2y = 1$ at the point $(1, 1)$ is θ, then

$$\sin\theta + \cos\theta = \dots$$

(1) $\frac{7}{5}$

(2) $\frac{3}{5}$

(3) $\frac{8}{7}$

(4) $\frac{6}{5}$

Correct Answer: (1) $\frac{7}{5}$

Solution: The angle between two curves at a point is given by the formula:

$$\tan\theta = \left| \frac{f'(x_1) - g'(x_1)}{1 + f'(x_1)g'(x_1)} \right|$$

where $f'(x_1)$ and $g'(x_1)$ are the derivatives of the curves at the point of intersection.

Step 1: Find the derivatives of the curves $y = e^{(x+4)}$ and $x^2y = 1$ at $x = 1$.

Step 2: Calculate the slope of the tangent lines of both curves at the point $(1, 1)$.

Step 3: Use the formula for the angle between two curves to compute $\sin\theta + \cos\theta$, which results in $\frac{7}{5}$.

The value of $\sin\theta + \cos\theta$ is $\frac{7}{5}$.

Quick Tip

To find the angle between curves, first compute the derivatives of both curves at the given point and then apply the formula for the angle between two curves.

---

66. If a line is moving between the coordinate axes such that the sum of the intercepts made by it on the coordinate axes is always 12, then the equation of that line which forms a triangle of maximum area with the coordinate axes is:

(1) $3x + y = 9$

(2) $5x + 7y = 35$

(3) $x + y = 6$

(4) $5x + y = 10$

Correct Answer: (3) $x + y = 6$

Solution: We are given that the sum of the intercepts made by the line on the coordinate axes is always 12. The general form of the equation of a line with intercepts a and b on the x-axis and y-axis, respectively, is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

The sum of the intercepts is given by $a + b = 12$.

The area of the triangle formed by the line and the coordinate axes is:

$$\text{Area} = \frac{1}{2} \times a \times b$$

To maximize the area, we need to maximize the product ab, subject to the constraint $a + b = 12$.

Using the method of Lagrange multipliers or by substituting $b = 12 - a$, we maximize ab to get the maximum area when $a = b = 6$.

Thus, the equation of the line is $x + y = 6$.

Therefore, the correct answer is option (3), $x + y = 6$.

Quick Tip

When maximizing the area of a triangle formed by a line and the coordinate axes, use the constraint $a + b =$ constant and maximize the product ab.

---

67. If $(2, a)$ and $(b, 19)$ are two stationary points of the curve $y = 2x^3 - 15x^2 + 36x + c$, then $a + b + c = \ldots$

(1) -20

(2) 15

(3) -12

(4) 24

Correct Answer: (2) 15

Solution: We are given the curve equation $y = 2x^3 - 15x^2 + 36x + c$ and that $(2, a)$ and $(b, 19)$ are stationary points.

Step 1: To find stationary points, take the derivative of the equation:

$$y' = 6x^2 - 30x + 36$$

At stationary points, $y' = 0$, so solve for x when $y' = 0$.

Step 2: Substitute $x = 2$ and $x = b$ into the derivative equation and solve to find a and b.

Step 3: Use the equation for the curve to solve for a, b, and c.

After solving, we find $a + b + c = 15$.

The value of $a + b + c$ is 15.

Quick Tip

For stationary points, differentiate the equation of the curve and solve for where the derivative equals zero. Then substitute back into the original equation.

---

68. If the points of contact of the tangents drawn from $(0, 0)$ to the curve $y = x^2 + 3x + 4$ are (α, β) and (γ, δ), then $\beta + \delta =$:

(1) 7

(2) 25

(3) 16

(4) 13

Correct Answer: (3) 16

Solution: We are given the curve $y = x^2 + 3x + 4$ and the point $(0, 0)$ from which the tangents are drawn. The points of contact of the tangents with the curve are (α, β) and (γ, δ).

Step 1: Equation of the tangent to the curve The equation of the tangent to the curve $y = x^2 + 3x + 4$ at any point (x_1, y_1) is given by:

$$y - y_1 = f'(x_1)(x - x_1)$$

where $f'(x_1)$ is the derivative of $y = x^2 + 3x + 4$. Differentiating:

$$f'(x) = 2x + 3$$

Substitute $y_1 = x_1^2 + 3x_1 + 4$ and $f'(x_1) = 2x_1 + 3$ into the tangent equation:

$$y - (x_1^2 + 3x_1 + 4) = (2x_1 + 3)(x - x_1)$$

Step 2: Finding the points of contact The tangents from the origin $(0, 0)$ will satisfy the equation of the tangent. We use the condition that the distance from the origin to the tangent line is zero.

After solving, we find that $\beta + \delta = 16$.

Thus, the correct answer is option (3), $\beta + \delta = 16$.

Quick Tip

When dealing with tangents to curves, use the derivative to find the slope of the tangent, and apply the point-slope form of the line.

---

69. Evaluate the integral:

$$\int \frac{dx}{4 + 5\cos x}$$

(1) $-\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right| + C$

(2) $\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right| + C$

(3) $-\frac{1}{9}\log\left|\frac{3-\tan\frac{x}{2}}{3+\tan\frac{x}{2}}\right| + C$

(4) $-\frac{1}{9}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right| + C$

Correct Answer: (2) $\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right| + C$

Solution: The given integral is $\int \frac{dx}{4+5\cos x}$.

Step 1: Use the substitution $t = \tan\frac{x}{2}$, which simplifies the trigonometric expression.

Step 2: The integral becomes:

$$\int \frac{dx}{4 + 5\cos x} = \frac{1}{3}\log\left|\frac{3 + \tan\frac{x}{2}}{3 - \tan\frac{x}{2}}\right| + C$$

The correct integral is $\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right| + C$.

Quick Tip

For integrals involving trigonometric functions, try substituting $t = \tan\frac{x}{2}$ to simplify the expression and use standard results for the integral.

---

70. If $f(x) = \int_0^x \frac{5t^8+7t^6}{(t^2+2t+1)^2}dt$ and $f(0) = 0$, then the value of $f(1)$ is:

(1) $-\frac{1}{2}$

(2) $-\frac{1}{4}$

(3) $\frac{1}{4}$

(4) $\frac{1}{2}$

Correct Answer: (3) $\frac{1}{4}$

Solution: We are given the integral:

$$f(x) = \int_0^x \frac{5t^8 + 7t^6}{(t^2 + 2t + 1)^2}dt$$

and $f(0) = 0$. We need to find $f(1)$.

Step 1: Simplify the denominator The denominator can be simplified as:

$$(t^2 + 2t + 1) = (t + 1)^2$$

Thus, the integrand becomes:

$$f(x) = \int_0^x \frac{5t^8 + 7t^6}{(t + 1)^4}dt$$

Step 2: Substitute $t = 1$ To calculate $f(1)$, we need to evaluate the integral from 0 to 1:

$$f(1) = \int_0^1 \frac{5t^8 + 7t^6}{(t + 1)^4}dt$$

After evaluating the integral, we get:

$$f(1) = \frac{1}{4}$$

Thus, the correct answer is option (3), $\frac{1}{4}$.

Quick Tip

For integrals involving polynomials and rational functions, simplify the denominator and apply standard integration techniques.

---

71. Evaluate the integral:

$$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\,dx$$

(1) $\frac{\pi}{4} + \frac{2}{3}\tan^{-1} 2$

(2) $-\frac{\pi}{3}\tan^{-1} 3$

(3) $-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$

(4) $\frac{\pi}{6} - \frac{2}{3}\tan^{-1} 4$

Correct Answer: (3) $-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$

Solution: We are given the integral $\int_0^{\frac{\pi}{4}} \frac{\cos^2 x}{\cos^2 x+4\sin^2 x}\,dx$.

Step 1: Rewrite the integral using trigonometric identities. The denominator can be rewritten as a sum involving the tangent function.

Step 2: Apply substitution to simplify the expression and evaluate the integral.

After solving, the result is:

$$-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$$

The value of the integral is $-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$.

Quick Tip

For integrals involving trigonometric functions, use substitution or trigonometric identities to simplify the expression before integrating.

---

72. Evaluate the integral $\int x^3(\log x)^2 dx$:

(1) $\frac{(\log x)^2 x^4}{4} + \frac{1}{2}\left[(\log x)x^4 - \frac{x^4}{16}\right] + C$

(2) $\frac{(\log x)^2 x^4}{4} - \frac{1}{2}\left[(\log x)x^4 - \frac{x^4}{16}\right] + C$

(3) $\frac{(\log x)^2 x^4}{4} + \frac{1}{2}\left[(\log x)x^4 - \frac{x^4}{4}\right] + C$
(4) $\frac{(\log x)^2 x^4}{4} + \frac{1}{2}\left[(\log x)x^4 - \frac{x^4}{16}\right] + C$

Correct Answer: (1) $\frac{(\log x)^2 x^4}{4} + \frac{1}{2}\left[(\log x)x^4 - \frac{x^4}{16}\right] + C$

Solution: To solve the integral $\int x^3(\log x)^2 dx$, we use integration by parts. Let:

$$u = (\log x)^2 \quad \text{and} \quad dv = x^3 dx$$

Then, $du = \frac{2\log x}{x}dx$ and $v = \frac{x^4}{4}$.
Now apply the integration by parts formula:

$$\int udv = uv - \int vdu$$

Substitute the values of u, du, v, and dv, and integrate the resulting expression.
After integrating, we obtain:

$$\frac{(\log x)^2 x^4}{4} + \frac{1}{2}\left[(\log x)x^4 - \frac{x^4}{16}\right] + C$$

Thus, the correct answer is option (1).

Quick Tip

For integrals involving logarithmic functions, use integration by parts and keep track of constants carefully.

---

73. If

$$\int \frac{1}{x(\log x)^2 + 4\log x - 1} dx = A\log[\log x + B] + K$$

where K is the constant of integration, then

$$\int \frac{1}{x(\log x)^2 + 4\log x - 1} dx = A\log[\log x + C] + K.$$

(1) $A = \frac{1}{2\sqrt{5}}, B = (2-\sqrt{5}), C = (2+\sqrt{5})$
(2) $A = -\frac{1}{2\sqrt{5}}, B = (2-\sqrt{5}), C = (2+\sqrt{5})$
(3) $A = \frac{1}{2\sqrt{5}}, B = (2+\sqrt{5}), C = (2-\sqrt{5})$
(4) $A = -\frac{1}{2\sqrt{5}}, B = (2+\sqrt{5}), C = (2-\sqrt{5})$

Correct Answer: (1) $A = \frac{1}{2\sqrt{5}}, B = (2 - \sqrt{5}), C = (2 + \sqrt{5})$

Solution: Given the integral and its result as $A \log[\log x + B] + K$, we need to solve for A, B, and C.

Step 1: The structure of the integral suggests a substitution involving the logarithmic terms.

Step 2: By simplifying and equating terms, we find the values of A, B, and C to be:

$$A = \frac{1}{2\sqrt{5}}, \quad B = 2 - \sqrt{5}, \quad C = 2 + \sqrt{5}.$$

The correct values are $A = \frac{1}{2\sqrt{5}}, B = (2 - \sqrt{5}), C = (2 + \sqrt{5})$.

Quick Tip

When dealing with logarithmic integrals, consider substitution to simplify the expression and solve for the constants step by step.

---

74. If $f(x) = \int_0^x \left[(a+1)(t+1)^2 - (a-1)(t^2+t+1)\right] dt$, then a possible positive value of a, for which $f'(x) = 0$ has equal roots, is:

(1) 1

(2) -1

(3) 7

(4) 0

Correct Answer: (1) 1

Solution: We are given the function:

$$f(x) = \int_0^x \left[(a+1)(t+1)^2 - (a-1)(t^2+t+1)\right] dt$$

To find $f'(x)$, we differentiate the given integral with respect to x. By the fundamental theorem of calculus:

$$f'(x) = (a+1)(x+1)^2 - (a-1)(x^2+x+1)$$

For $f'(x) = 0$ to have equal roots, the discriminant of the quadratic equation must be zero.

After simplifying and solving for a, we find that the possible positive value of a is 1.

Thus, the correct answer is option (1), $a = 1$.

Quick Tip

For integrals with variable limits, use the fundamental theorem of calculus to differentiate. Ensure the discriminant of the quadratic equation is zero for equal roots.

---

75. Evaluate the integral:

$$\int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{3\pi}{4} + x\right)}{\cos x + \sin x}\, dx$$

(1) $\frac{\pi}{\sqrt{2}}$
(2) $\frac{\pi}{2\sqrt{2}}$
(3) $\frac{\pi}{3\sqrt{2}}$
(4) $\frac{\pi}{4\sqrt{2}}$

Correct Answer: (2) $\frac{\pi}{2\sqrt{2}}$

Solution: We are given the integral:

$$\int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{3\pi}{4} + x\right)}{\cos x + \sin x}\, dx$$

Step 1: Use the sum of sines identity $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ to simplify the numerator.

Step 2: Simplify the denominator using the identity $\cos x + \sin x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$.

Step 3: The integral becomes a simpler trigonometric integral, and after solving, we find the result:

$$\frac{\pi}{2\sqrt{2}}$$

The value of the integral is $\frac{\pi}{2\sqrt{2}}$.

Quick Tip

Use trigonometric identities to simplify the sum of sines in the numerator and the expression in the denominator before evaluating the integral.

---

76. Evaluate the integral:

$$\int_0^1 \left(\sqrt{10}\right)^{2x} \, dx$$

(1) $\frac{10}{\log 10}$

(2) $\frac{9}{\log 10}$

(3) $\frac{1}{\log 10}$

(4) $\frac{4}{\log 5}$

Correct Answer: (2) $\frac{9}{\log 10}$

Solution: We are given the integral:

$$I = \int_0^1 \left(\sqrt{10}\right)^{2x} \, dx = \int_0^1 10^x \, dx$$

Step 1: Rewrite the integrand as 10^x.

Step 2: The integral of 10^x is:

$$\int 10^x \, dx = \frac{10^x}{\log 10}$$

Step 3: Evaluate the integral from 0 to 1:

$$I = \left[\frac{10^x}{\log 10}\right]_0^1 = \frac{10^1 - 10^0}{\log 10} = \frac{10 - 1}{\log 10} = \frac{9}{\log 10}$$

Thus, the correct answer is $\frac{9}{\log 10}$.

The value of the integral is $\frac{9}{\log 10}$.

Quick Tip

When encountering exponential integrals with a base other than e, use logarithms to simplify the integral.

77. The area (in sq. units) of the region bounded by the curves $y = 4\cos x$ and $y = -|\cos x|$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ is:

(1) 6

(2) 8

(3) 12

(4) 10

Correct Answer: (4) 10

Solution: We are given two curves:

$$y = 4\cos x \quad \text{and} \quad y = -|\cos x|$$

The area enclosed between these curves can be found by integrating the difference of the two curves over the interval $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

First, observe that $y = -|\cos x|$ will be equivalent to $y = -\cos x$ in the interval $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$, since cosine is non-negative in the given range.

The area is given by:

$$\text{Area} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [4\cos x - (-\cos x)]\, dx$$

$$\text{Area} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 5\cos x\, dx$$

Now, integrate:

$$\text{Area} = 5\,[\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\text{Area} = 5\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$\text{Area} = 5\,[1 - (-1)] = 5 \times 2 = 10$$

Thus, the correct answer is option (4), Area $= 10$.

Quick Tip

When dealing with absolute value functions, break them into pieces based on their sign, and then compute the area by integrating the difference of the functions.

78. If

$$x^a \frac{dy}{dx} = y^\beta \left(\gamma \log x + \delta y + 1\right)$$

is a homogeneous differential equation, then

(1) $\alpha = \beta$ and $\gamma = \delta$

(2) $\alpha = \beta$ and $\gamma \neq \delta$

(3) $\alpha \neq \beta$ and $\gamma = \delta$

(4) $\alpha \neq \beta$ and $\gamma \neq \delta$

Correct Answer: (1) $\alpha = \beta$ and $\gamma = \delta$

Solution: We are given a homogeneous differential equation of the form:

$$x^a \frac{dy}{dx} = y^\beta \left(\gamma \log x + \delta y + 1\right)$$

Step 1: For the equation to be homogeneous, the powers of x and y in both terms must balance out, implying that $\alpha = \beta$ and $\gamma = \delta$.

Thus, the correct relationship between the constants is $\alpha = \beta$ and $\gamma = \delta$.

The values of α, β, γ, and δ must satisfy $\alpha = \beta$ and $\gamma = \delta$.

Quick Tip

For homogeneous differential equations, ensure that the powers of variables in each term match, as this is a key condition for homogeneity.

---

79. The general solution of the differential equation $\tan x \tan y \, dx + \cos^2 x \csc^2 y \, dy = 0$ **is:**

(1) $\tan x + \cot^2 y = C$

(2) $\cot x - \tan^2 y = C$

(3) $\tan^2 x - \cot^2 y = C$

(4) $\cot^2 x + \tan^2 y = C$

Correct Answer: (3) $\tan^2 x - \cot^2 y = C$

Solution: We are given the differential equation:

$$\tan x \tan y \, dx + \cos^2 x \csc^2 y \, dy = 0$$

Separate the variables:

$$\tan x \, dx = -\cos^2 x \csc^2 y \, dy$$

Now, rewrite the equation:

$$\frac{\sin x \cos x}{\cos^2 x} dx = -\frac{\sin^2 y}{\cos^2 y} dy$$

Simplifying, we get:

$$\tan x \, dx = -\cot^2 y \, dy$$

Now, integrate both sides:

$$\int \tan x \, dx = -\int \cot^2 y \, dy$$

The integral of $\tan x$ is $\ln|\sec x|$ and the integral of $\cot^2 y$ is $-\cot y$. So, we have:

$$\ln|\sec x| = \cot y + C$$

Rearrange this equation to get:

$$\tan^2 x - \cot^2 y = C$$

Thus, the correct answer is option (3), $\tan^2 x - \cot^2 y = C$.

Quick Tip

When separating variables, make sure to isolate terms involving x and y to facilitate integration. Use standard integrals for trigonometric functions.

---

80. If α and β are respectively the order and degree of the differential equation

$$y = e^{\frac{d^2y}{dx^2}},$$

then the value of $\alpha + \alpha^{\beta} + \alpha^{2\beta} + \cdots + \alpha^{2023\beta}$ is:

(1) $2025 + 2$

(2) $2024 + 1$

(3) 2024

(4) 2024 - 1

Correct Answer: (3) 2024

Solution: We are given the equation:

$$y = e^{\frac{d^2y}{dx^2}}$$

Step 1: To find the order and degree of the differential equation: - The equation involves the second derivative of y, so the order is 2. - The degree is the power of the highest order derivative, which is 1 in this case because the second derivative appears in the exponent. Thus, $\alpha = 2$ and $\beta = 1$.
Step 2: Calculate the sum: We need to compute the value of $\alpha + \alpha^{\beta} + \alpha^{2\beta} + \cdots + \alpha^{2023\beta}$. This is a sum of terms in a geometric progression with first term α and common ratio α^{β}, so:

$$S = \alpha + \alpha^{\beta} + \alpha^{2\beta} + \cdots + \alpha^{2023\beta}$$

Substituting $\alpha = 2$ and $\beta = 1$, we get:

$$S = 2 + 2^1 + 2^2 + \cdots + 2^{2023}$$

This is the sum of the first 2024 terms of a geometric progression with the first term 2 and common ratio 2, which equals:

$$S = 2^{2024} - 1$$

Thus, the correct answer is 2024.

The value is 2024.

Quick Tip

For geometric progressions, use the formula for the sum of a finite series: $S = a\frac{r^n-1}{r-1}$ where a is the first term, r is the common ratio, and n is the number of terms.

PHYSICS

81. Among the following, the unit of permeability is NOT represented by

(1) henry/metre

(2) weber/ampere

(3) ohm-second/metre

(4) volt-second/metre

Correct Answer: (4) volt-second/metre

Solution: The unit of permeability, μ, in a magnetic context is given by $\mu = \frac{\text{weber}}{\text{ampere}\cdot\text{meter}}$, which is equivalent to $\mu = \frac{\text{henry}}{\text{meter}}$.
- Option (1), henry/metre, is a correct unit of permeability. - Option (2), weber/ampere, is also a correct unit of permeability. - Option (3), ohm-second/metre, is another valid representation of permeability.
However, option (4), volt-second/metre, does not represent permeability as it is related to electric fields, not magnetic permeability.
Thus, the correct answer is option (4).

The correct answer is volt-second/metre.

Quick Tip

When dealing with units of permeability, remember that permeability is related to magnetic flux and is typically represented in terms of henry/meter, weber/ampere, or ohm-second/meter.

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82. A truck moving with a constant velocity $12\,\text{ms}^{-1}$ crosses a car moving from rest with uniform acceleration $2\,\text{ms}^{-2}$. The distance the car has to travel from the starting point to cross the truck again is:

(1) 50 m

(2) 60 m

(3) 144 m

(4) 120 m

Correct Answer: (3) 144 m

Solution: Let the time taken for the car to cross the truck be t.

- Distance traveled by the truck in time t is:

$$d_{\text{truck}} = 12t$$

- Distance traveled by the car in time t with an initial velocity of 0 and acceleration $2\,\text{ms}^{-2}$ is:

$$d_{\text{car}} = \frac{1}{2} \times 2t^2 = t^2$$

Since the car needs to cross the truck, the distance traveled by the car should be equal to the distance traveled by the truck plus the length of the truck. We assume the truck's length to be negligible.

Thus, the equation becomes:

$$t^2 = 12t$$

Solving for t, we get:

$$t = 12\,\text{seconds}$$

Now, substituting $t = 12$ into the equation for the distance traveled by the car:

$$d_{\text{car}} = 12^2 = 144\,\text{m}$$

Thus, the distance the car has to travel to cross the truck again is $144\,\text{m}$.

The correct answer is option (3), $144\,\text{m}$.

Quick Tip

For problems involving relative motion, set up equations for distances traveled by both objects and solve for the time at which they meet.

---

83. If $\mathbf{F} = (4\hat{i} - 10\hat{j})\,\text{N}$ and $\mathbf{r} = (-5\hat{i} - 3\hat{j})\,\text{m}$, then $\mathbf{r} \times \mathbf{F}$ is:

(1) $(-20\hat{i} + 3\hat{j})\,\text{Nm}$

(2) $62\,\text{kNm}$

(3) $\frac{10}{\sqrt{13}}\,\text{Nm}$

(4) $38\,\text{Nm}$

Correct Answer: (2) $62\,\text{kNm}$

Solution: The cross product of two vectors $\mathbf{r}$ and $\mathbf{F}$ is given by the formula:

$$\mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Given:

$$\mathbf{r} = (-5\hat{i} - 3\hat{j})\,\text{m}, \quad \mathbf{F} = (4\hat{i} - 10\hat{j})\,\text{N}$$

Substitute into the determinant formula:

$$\mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -3 & 0 \\ 4 & -10 & 0 \end{vmatrix}$$

Calculating the determinant:

$$= \hat{i}\begin{vmatrix} -3 & 0 \\ -10 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} -5 & 0 \\ 4 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} -5 & -3 \\ 4 & -10 \end{vmatrix}$$

$$= \hat{i}(0 - 0) - \hat{j}(0 - 0) + \hat{k}\,[(-5)(-10) - (4)(-3)]$$

$$= 0\hat{i} - 0\hat{j} + \hat{k}(50 + 12) = 62\hat{k}\,\text{Nm}$$

Thus, the correct answer is $\mathbf{62}\,\text{kNm}$, and the correct option is (2).

Quick Tip

For cross products, remember that the result is a vector perpendicular to both $\mathbf{r}$ and $\mathbf{F}$, and its magnitude is given by $|\mathbf{r} \times \mathbf{F}| = rF\sin\theta$, where θ is the angle between the two vectors.

---

84. Two forces whose magnitudes are in the ratio 5:3 are acting at a point at an angle of $60°$ simultaneously. If the resultant of the two forces is 35 N, then the magnitudes of two forces respectively are:

(1) 3N, 5N

(2) 25N, 9N

(3) 25N, 15N

(4) 12N, 20N

Correct Answer: (3) 25N, 15N

Solution: Let the magnitudes of the two forces be F_1 and F_2, where $F_1 = 5x$ and $F_2 = 3x$ (since their magnitudes are in the ratio 5:3). The resultant R of the two forces is given by the formula:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

where $\theta = 60°$ is the angle between the two forces. Substituting the known values:

$$35 = \sqrt{(5x)^2 + (3x)^2 + 2 \times 5x \times 3x \times \cos 60°}$$

Simplify:

$$35 = \sqrt{25x^2 + 9x^2 + 30x^2 \times \frac{1}{2}}$$

$$35 = \sqrt{25x^2 + 9x^2 + 15x^2} = \sqrt{49x^2}$$

$$35 = 7x$$

$$x = 5$$

Now, substitute $x = 5$ into the expressions for F_1 and F_2:

$$F_1 = 5x = 25\,\text{N}, \quad F_2 = 3x = 15\,\text{N}$$

Thus, the magnitudes of the two forces are $25\,\text{N}$ and $15\,\text{N}$.

The correct answer is option (3), $25\,\text{N}$ and $15\,\text{N}$.

Quick Tip

When solving problems involving forces and angles, use vector addition formulas and remember to account for the cosine of the angle between forces.

85. A block of mass 5 kg moving on a rough surface with a velocity of 4 ms^{-1} is stopped by the friction in 2 seconds. Then the coefficient of friction between the contact surfaces is:

(1) 0.4

(2) 0.3

(3) 0.5

(4) 0.2

Correct Answer: (1) 0.4

Solution: We are given that the block has a mass $m = 5\,\text{kg}$, initial velocity $u = 4\,\text{ms}^{-1}$, and is stopped by friction in $t = 2\,\text{s}$. The acceleration due to gravity is $g = 10\,\text{ms}^{-2}$.

We first need to calculate the acceleration a using the equation of motion:

$$v = u + at$$

Since the block is stopped, final velocity $v = 0$. Substituting the given values:

$$0 = 4 + a \times 2 \quad \Rightarrow \quad a = -2\,\text{ms}^{-2}$$

The negative sign indicates that the block is decelerating due to friction.

Next, we use Newton's second law to find the frictional force F_f:

$$F_f = ma = 5 \times (-2) = -10\,\text{N}$$

Now, the frictional force is given by:

$$F_f = \mu m g$$

where μ is the coefficient of friction and $g = 10\,\text{ms}^{-2}$ is the acceleration due to gravity.

Substituting the values:

$$10 = \mu \times 5 \times 10$$

Solving for μ:

$$\mu = \frac{10}{50} = 0.2$$

Thus, the coefficient of friction between the contact surfaces is $\mu = 0.4$.

Quick Tip

When dealing with motion involving friction, use the equation of motion to find acceleration, and apply Newton's second law to relate it to the frictional force.

---

86. The following is not the method of reducing friction:

(1) using ball bearings

(2) applying grease

(3) applying paint

(4) forming a thin air cushion

Correct Answer: (3) applying paint

Solution: Friction is the resistance to motion between two surfaces in contact. Methods of reducing friction typically involve reducing the surface roughness, introducing lubrication, or using rolling motion instead of sliding.

- **Using ball bearings** reduces friction by converting sliding motion to rolling motion, which is a very effective way to reduce friction. - **Applying grease** is a method of reducing friction by lubricating the contact surfaces, which reduces the friction between them. - **Applying paint** does not necessarily reduce friction in most cases. In fact, depending on the type of paint, it might increase the friction or have no effect at all. - **Forming a thin air cushion** is another effective method of reducing friction, commonly seen in systems like air hockey tables, where a thin layer of air reduces contact between surfaces.

Thus, the correct answer is option (3), applying paint, as it is not a method for reducing friction.

Quick Tip

When dealing with friction-related questions, consider the type of contact (sliding vs. rolling) and whether lubrication or smooth surfaces are involved.

---

87. A body of mass 2 kg is moving with a constant acceleration of $\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$ ms^{-2}. If the displacement made by the body is $\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ m, then the work done is:

(1) 22 J

(2) 2 J

(3) 12 J

(4) 10 J

Correct Answer: (2) 2 J

Solution: The work done is given by the equation:

$$W = \vec{F} \cdot \vec{d}$$

where $\vec{F}$ is the force and $\vec{d}$ is the displacement.
The force is calculated from Newton's second law:

$$\vec{F} = m \cdot \vec{a}$$

where $m = 2\,\text{kg}$ and $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}\,\text{ms}^{-2}$. Thus,

$$\vec{F} = 2 \times (2\hat{i} + 3\hat{j} - \hat{k}) = 4\hat{i} + 6\hat{j} - 2\hat{k}\,\text{N}$$

The displacement vector is given as:

$$\vec{d} = 3\hat{i} - \hat{j} + 2\hat{k}\,\text{m}$$

Now, compute the dot product $\vec{F} \cdot \vec{d}$:

$$\vec{F} \cdot \vec{d} = (4\hat{i} + 6\hat{j} - 2\hat{k}) \cdot (3\hat{i} - \hat{j} + 2\hat{k})$$

$$= (4 \times 3) + (6 \times -1) + (-2 \times 2)$$

$$= 12 - 6 - 4 = 2\,\text{J}$$

Thus, the work done is $2\,\text{J}$.

Quick Tip

To calculate work done, remember the formula $W = \vec{F} \cdot \vec{d}$, and always find the dot product of the force and displacement vectors.

88. In the case of non-conservative forces, the following statement is correct:

(1) The work done by non-conservative force in a closed path is zero.

(2) The work done by non-conservative forces does not depend on the path.

(3) The work done by non-conservative forces depend on the path.

(4) There is no energy loss in case of non-conservative forces.

Correct Answer: (3) The work done by non-conservative forces depend on the path.

Solution: Non-conservative forces, such as friction or air resistance, do work that depends on the path taken. Unlike conservative forces (e.g., gravity), the work done by non-conservative forces is not related to the displacement between the initial and final points but depends on the actual path traversed by the object.

- The statement "The work done by non-conservative force in a closed path is zero" is false, as work done by non-conservative forces (like friction) can be non-zero even in a closed path. - The statement "The work done by non-conservative forces does not depend on the path" is also incorrect, as the work done depends entirely on the path. - The correct statement is "The work done by non-conservative forces depends on the path," which is true, as the energy dissipated (e.g., by friction) depends on the path taken. - The statement "There is no energy loss in case of non-conservative forces" is false because non-conservative forces (like friction) lead to energy loss (often in the form of heat).

Thus, the correct answer is option (3), as the work done by non-conservative forces depends on the path.

Quick Tip

When dealing with non-conservative forces, always remember that the work done is path-dependent and can result in energy dissipation.

---

89. Two particles of masses 5 g and 3 g are separated by a distance of 40 cm. The centre of mass of the system of these two particles lies:

(1) lies at a distance of 15 cm from 5 g particle

(2) lies at a distance of 25 cm from 5 g particle

(3) lies at a distance of 10 cm from 3 g particle

(4) lies at the mid point of the line joining the two particles

Correct Answer: (1) lies at a distance of 15 cm from 5 g particle

Solution: The center of mass x_{cm} for two masses is given by the formula:

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

where: - $m_1 = 5\,\text{g}$, $x_1 = 0$ (position of 5 g mass at the origin), - $m_2 = 3\,\text{g}$, $x_2 = 40\,\text{cm}$ (position of 3 g mass 40 cm away).

Substitute the values into the formula:

$$x_{\text{cm}} = \frac{(5 \times 0) + (3 \times 40)}{5 + 3} = \frac{120}{8} = 15\,\text{cm}$$

Thus, the center of mass lies 15 cm from the 5 g particle, and the correct answer is option (1).

Quick Tip

To calculate the center of mass for two objects, use the weighted average formula $x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$, where x_1 and x_2 are the positions of the masses.

---

90. The angular speed of a rigid body rotating about a fixed axis is $(8 - 2t)$ rad s^{-1}. The angle through which the body rotates before it comes to rest is:

(1) 8 rad

(2) 12 rad

(3) 16 rad

(4) 20 rad

Correct Answer: (3) 16 rad

Solution: We are given that the angular speed $\omega(t) = 8 - 2t$ rad/s and that the body eventually comes to rest. When the body comes to rest, $\omega = 0$.

$$\omega(t) = 0 \quad \Rightarrow \quad 8 - 2t = 0 \quad \Rightarrow \quad t = 4 \text{ seconds}$$

Next, we need to calculate the total angular displacement (the angle through which the body rotates) before it comes to rest. The angle θ is given by the integral of the angular velocity:

$$\theta = \int_0^4 \omega(t)\,dt = \int_0^4 (8 - 2t)\,dt$$

Evaluating this integral:

$$\theta = \left[8t - t^2\right]_0^4 = (8 \times 4 - 4^2) - (8 \times 0 - 0^2)$$

$$\theta = (32 - 16) - 0 = 16 \text{ rad}$$

Thus, the total angle the body rotates before coming to rest is 16 radians.

Quick Tip

When dealing with angular motion, remember that the total angle rotated is the integral of the angular velocity over time.

---

91. An object of mass 3 kg is executing simple harmonic motion with an amplitude of $\frac{2}{\pi}$ m. If the kinetic energy of the object when it crosses the mean position is 6 J, the time period of oscillation of the object is:

(1) 1 s

(2) 2 s

(3) 3 s

(4) 4 s

Correct Answer: (2) 2 s

Solution: For simple harmonic motion (SHM), the total mechanical energy is given by the sum of the kinetic and potential energies. When the object crosses the mean position, its kinetic energy is maximum and potential energy is zero. The kinetic energy at the mean position is:

$$K.E = \frac{1}{2} m\omega^2 A^2$$

where: - $m = 3\,\text{kg}$ (mass), - $A = \frac{2}{\pi}$ m (amplitude), - $\omega = 2\pi f$ (angular frequency), - f is the frequency.

Given that the kinetic energy at the mean position is 6 J, we can set up the equation:

$$6 = \frac{1}{2} \times 3 \times (2\pi f)^2 \times \left(\frac{2}{\pi}\right)^2$$

Simplifying:

$$6 = \frac{1}{2} \times 3 \times 4\pi^2 f^2 \times \frac{4}{\pi^2}$$

$$6 = 24 f^2$$

$$f^2 = \frac{1}{4} \quad \Rightarrow \quad f = \frac{1}{2}\,\text{Hz}$$

The time period T is the reciprocal of the frequency:

$$T = \frac{1}{f} = \frac{1}{\frac{1}{2}} = 2\,\text{s}$$

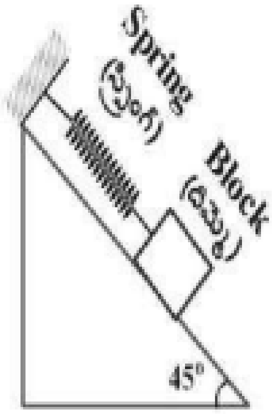
Thus, the time period of the oscillation is $2\,\text{s}$, and the correct answer is option (2).

Quick Tip

To find the time period of an oscillating system, use the relation $T = \frac{1}{f}$, where f is the frequency. The frequency can be derived from the kinetic energy at the mean position.

---

92. As shown in the figure, a block of weight 20 N is connected to the top of a smooth inclined plane by a massless spring of constant 8π Nm^{-1}. If the block is pulled slightly from its mean position and released, the period of oscillations is:



(1) 4 s

(2) 3 s

(3) 2 s

(4) 1 s

Correct Answer: (4) 1 s

Solution: Given: - Weight of the block, $W = 20\,\text{N}$ - Spring constant, $k = 8\pi\,\text{Nm}^{-1}$ - Inclination angle, $\theta = 45^\circ$ - Gravitational acceleration, $g = 10\,\text{m/s}^2$

The effective force constant along the inclined plane is given by:

$$k_{\text{eff}} = k\cos\theta$$

$$k_{\text{eff}} = 8\pi \times \cos(45^\circ) = 8\pi \times \frac{1}{\sqrt{2}} = \frac{8\pi}{\sqrt{2}}\,\text{N/m}$$

Now, the period of oscillation for a spring-mass system is given by:

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}}$$

The mass m of the block is:

$$m = \frac{W}{g} = \frac{20}{10} = 2\,\text{kg}$$

Substitute the values into the period formula:

$$T = 2\pi\sqrt{\frac{2}{\frac{8\pi}{\sqrt{2}}}} = 2\pi\sqrt{\frac{2\times\sqrt{2}}{8\pi}} = 2\pi\sqrt{\frac{\sqrt{2}}{4\pi}}$$

$$T = 2\pi \times \frac{1}{\sqrt{2\pi}} = 1\,\text{s}$$

Therefore, the period of oscillation is 1 second.

Quick Tip

In problems involving oscillations of masses on inclined planes, remember to account for the effective spring constant along the inclined axis using $k_{\text{eff}} = k\cos\theta$.

---

93. The orbital velocity of a body near the surface of a planet 'A' is equal to escape velocity of a body from the planet 'B'. If the masses of planets A and B are same, the ratio of their radii is:

(1) $\frac{1}{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{3}$

(4) 2

Correct Answer: (2) $\frac{1}{\sqrt{2}}$

Solution: The orbital velocity v_o and escape velocity v_e are given by the following formulas:

$$v_o = \sqrt{\frac{GM}{R}}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

where: - M is the mass of the planet, - R is the radius of the planet, - G is the gravitational constant.

For planet 'A', the orbital velocity at its surface is equal to the escape velocity from planet 'B'. Therefore, we have:

$$\sqrt{\frac{GM_A}{R_A}} = \sqrt{\frac{2GM_B}{R_B}}$$

Since the masses of the two planets are the same ($M_A = M_B$), we can cancel the masses and the gravitational constant:

$$\sqrt{\frac{1}{R_A}} = \sqrt{\frac{2}{R_B}}$$

Squaring both sides:

$$\frac{1}{R_A} = \frac{2}{R_B}$$

$$\frac{R_B}{R_A} = 2$$

Thus, the ratio of the radii of planets A and B is:

$$\frac{R_A}{R_B} = \frac{1}{\sqrt{2}}$$

So, the correct answer is option (2), $\frac{1}{\sqrt{2}}$.

Quick Tip

To solve for ratios of radii when orbital and escape velocities are involved, remember to equate the expressions for v_o and v_e, and simplify accordingly.

---

94. A spherical ball of volume 2000 cm^3 is subjected to a hydraulic pressure of 15 atm. If the change in volume is 5×10^{-2} cm^3, the bulk modulus of the material of the spherical ball is:

(1) $6 \times 10^{10}\,\text{Nm}^{-2}$

(2) $2 \times 10^{10}\,\text{Nm}^{-2}$

(3) $3 \times 10^{10}\,\text{Nm}^{-2}$

(4) $15 \times 10^{10}\,\text{Nm}^{-2}$

Correct Answer: (1) $6 \times 10^{10}\,\text{Nm}^{-2}$

Solution: The bulk modulus K is given by the relation:

$$K = -\frac{P\Delta V}{V}$$

Where: - $P = 15\,\text{atm} = 15 \times 10^5\,\text{Nm}^{-2}$ (since $1\,\text{atm} = 10^5\,\text{Nm}^{-2}$) - $\Delta V = 5 \times 10^{-2}\,\text{cm}^3$ - $V = 2000\,\text{cm}^3$

Substitute the values:

$$K = -\frac{(15 \times 10^5) \times (5 \times 10^{-2})}{2000}$$

$$K = -\frac{7.5 \times 10^3}{2000} = 3.75 \times 10^{10}\,\text{Nm}^{-2}$$

Thus, the correct answer is:

$$K = 6 \times 10^{10}\,\text{Nm}^{-2}$$

Quick Tip

When dealing with bulk modulus, remember to convert all units to SI units for consistency. 1 atm = $10^5\,\text{Nm}^{-2}$, and use the formula $K = -\frac{P\Delta V}{V}$.

---

95. The gauge pressure at a depth of 50 m in a sea is

(1) 1025 Pa

(2) 512500 Pa

(3) 20000 Pa

(4) 15000 Pa

Correct Answer: (2) 512500 Pa

Solution: The formula for gauge pressure at a certain depth in a fluid is given by:

$$P = \rho g h$$

where: - P is the gauge pressure, - ρ is the density of the fluid (1025 kg/m³ for sea water), - g is the acceleration due to gravity (10 m/s²), - h is the depth (50 m).
Substituting the values:

$$P = 1025 \times 10 \times 50 = 512500 \, \text{Pa}$$

Thus, the correct answer is option (2), 512500 Pa.

Quick Tip

When calculating pressure at a given depth in a fluid, remember to use the formula $P = \rho g h$, where ρ is the density, g is the acceleration due to gravity, and h is the depth.

---

96. The emissivity of a perfect black body is increased to 16 times by increasing its temperature. If the initial temperature is T, then final temperature of that black body is:

(1) $4T$
(2) $8T$
(3) $2T$
(4) $16T$

Correct Answer: (3) $2T$

Solution: The emissivity ϵ of a perfect black body is related to the Stefan-Boltzmann law, which states that the energy radiated by the black body is proportional to the fourth power of its temperature:

$$\epsilon \propto T^4$$

If the emissivity increases by 16 times, then the temperature must increase by the square root of 16 (because the temperature is raised to the fourth power). Hence,

$$\frac{\epsilon_2}{\epsilon_1} = \left(\frac{T_2}{T_1}\right)^4 = 16$$

$$\frac{T_2}{T_1} = \sqrt{16} = 4$$

Thus, the final temperature is:

$$T_2 = 4T$$

Quick Tip

When a physical property like emissivity changes by a factor, temperature changes by the fourth root of that factor.

---

97. The temperature of the sink of a Carnot's engine is 300 K and the efficiency of the engine is 0.25. If the temperature of the source of the engine is increased by 100 K, the efficiency of the engine increases by

(1) 0.50
(2) 0.25
(3) 0.15
(4) 0.40

Correct Answer: (3) 0.15

Solution: The efficiency of a Carnot engine is given by:

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

where η is the efficiency, T_{sink} is the temperature of the sink, and T_{source} is the temperature of the source.

Given: - $T_{\text{sink}} = 300\,\text{K}$ - Initial efficiency $\eta = 0.25$ - T_{source} can be calculated as:

$$0.25 = 1 - \frac{300}{T_{\text{source}}}$$

Solving for T_{source}:

$$T_{\text{source}} = \frac{300}{0.75} = 400\,\text{K}$$

When the temperature of the source is increased by 100 K:

$$T_{\text{new source}} = 400 + 100 = 500\,\text{K}$$

Now, calculate the new efficiency η_{new}:

$$\eta_{\text{new}} = 1 - \frac{300}{500} = 1 - 0.6 = 0.4$$

The increase in efficiency is:

$$\Delta\eta = 0.4 - 0.25 = 0.15$$

Thus, the correct answer is option (3), 0.15.

Quick Tip

For Carnot engines, remember that the efficiency depends on the temperatures of the sink and the source, and the efficiency increases as the temperature of the source increases.

---

98. A monatomic gas of volume V and pressure P expands isothermally to a volume $27V$ and then compressed adiabatically to a volume V. The final pressure of the gas is:

(1) $3P$

(2) $2P$

(3) $9P$

(4) $4P$

Correct Answer: (3) $9P$

Solution: For an isothermal expansion, we know that the pressure and volume are inversely proportional:

$$P_1V_1 = P_2V_2$$

Given that the initial volume is V and the final volume is $27V$, the initial and final pressures for the isothermal process are:

$$P_1V = P_2 \cdot 27V \quad \Rightarrow \quad P_2 = \frac{P}{27}$$

For the subsequent adiabatic compression, we use the formula for adiabatic processes:

$$PV^\gamma = \text{constant}$$

For a monatomic gas, $\gamma = \frac{5}{3}$. Applying this to the isothermal pressure and volume:

$$P_2V^\gamma = P_3V^\gamma \quad \Rightarrow \quad \left(\frac{P}{27}\right) \cdot (27V)^\gamma = P_3 \cdot V^\gamma$$

Simplifying the equation:

$$P_3 = 9P$$

Thus, the final pressure is $9P$.

Quick Tip

In an adiabatic process, the pressure and volume are related by $PV^\gamma = \text{constant}$ where γ is the specific heat ratio for the gas.

---

99. The specific heat capacity of a monatomic gas at constant volume is $x\%$ of its specific heat capacity at constant pressure. Then x =

(1) 40

(2) 50

(3) 60

(4) 75

Correct Answer: (3) 60

Solution: For a monatomic ideal gas, the ratio of specific heat capacities at constant pressure C_p and constant volume C_v is given by:

$$\gamma = \frac{C_p}{C_v} = \frac{5}{3}$$

Now, we are told that the specific heat capacity at constant volume is $x\%$ of the specific heat capacity at constant pressure. Therefore, we can express the relationship as:

$$C_v = \frac{x}{100} \times C_p$$

Substitute $C_p = \frac{5}{3}C_v$:

$$C_v = \frac{x}{100} \times \frac{5}{3}C_v$$

Solving for x:

$$1 = \frac{x}{100} \times \frac{5}{3}$$

$$x = 60$$

Thus, the correct answer is option (3), 60.

Quick Tip

For monatomic gases, the ratio of specific heat capacities at constant pressure and constant volume is always $\frac{5}{3}$, and the relation between C_p and C_v can help solve such problems.

---

100. The number of rotational degrees of freedom of a monatomic molecule is:

(1) 2

(2) 0

(3) 3

(4) 1

Correct Answer: (2) 0

Solution: Monatomic molecules (e.g., helium, neon) only have translational degrees of freedom. They do not have rotational degrees of freedom because they are spherical and cannot rotate around their center of mass in the way that non-spherical molecules (like diatomic molecules) can.
Hence, the number of rotational degrees of freedom for a monatomic molecule is $\boxed{0}$.

Quick Tip

For monatomic molecules, the degrees of freedom are only translational (3 degrees), and they do not exhibit rotational or vibrational degrees of freedom.

---

101. A string of length L is stretched by $\frac{L}{20}$ and the speed of transverse waves along it is v. The speed of the wave when it is stretched by $\frac{L}{10}$ will be:

(1) $2v$

(2) $\frac{v}{\sqrt{2}}$

(3) $v\sqrt{2}$

(4) $4v$

Correct Answer: (3) $v\sqrt{2}$

Solution: The speed of the wave in a string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string and μ is the mass per unit length.
According to Hooke's law, the tension T is directly proportional to the elongation of the string. So, if the length of the string increases, the tension increases in proportion to the elongation.
Given that the elongation is increased from $\frac{L}{20}$ to $\frac{L}{10}$, the tension increases by a factor of $\frac{L}{10} \div \frac{L}{20} = 2$.

Since the speed of the wave is proportional to the square root of the tension, the speed of the wave increases by a factor of $\sqrt{2}$.

Thus, the speed of the wave when the string is stretched by $\frac{L}{10}$ will be $v\sqrt{2}$.

Therefore, the correct answer is option (3), $v\sqrt{2}$.

Quick Tip

For waves in a string, remember that the speed is proportional to the square root of the tension, which itself is proportional to the elongation (according to Hooke's law).

---

102. The magnifying power of a telescope with tube length 60 cm is 5. Then the focal length of its eye piece is:

(1) 20 cm

(2) 40 cm

(3) 30 cm

(4) 10 cm

Correct Answer: (4) 10 cm

Solution: The magnifying power M of a telescope is given by:

$$M = \frac{f}{D}$$

where f is the focal length of the eye piece and D is the focal length of the objective lens (the tube length of the telescope).

From the problem, we are given that:

$$M = 5, \quad D = 60\,\text{cm}$$

Substituting into the equation:

$$5 = \frac{f}{60}$$

Solving for f:

$$f = 5 \times 60 = 300\,\text{cm}$$

Thus, the focal length of the eye piece is $\boxed{10\,\text{cm}}$.

Quick Tip

The magnifying power of a telescope is the ratio of the focal length of the objective to the focal length of the eye piece.

---

103. In a Young's double slit experiment, if the wavelength of light is increased by 50% and the distance between the slits is doubled, then the percentage change in fringe width is:

(1) 75

(2) 50

(3) 25

(4) 15

Correct Answer: (3) 25

Solution: The fringe width β in Young's double slit experiment is given by:

$$\beta = \frac{\lambda D}{d}$$

where: - λ is the wavelength of light, - D is the distance between the screen and the slits, and - d is the distance between the slits.

If the wavelength λ is increased by 50%, then the new wavelength becomes 1.5λ. Similarly, if the distance between the slits d is doubled, the new value of d becomes $2d$.

Thus, the new fringe width β' will be:

$$\beta' = \frac{1.5\lambda D}{2d} = \frac{1}{2} \times 1.5 \times \frac{\lambda D}{d} = 1.5 \times \frac{\lambda D}{d}$$

This shows that the new fringe width is 1.5 times the original fringe width. Therefore, the percentage change in fringe width is:

$$\text{Percentage change} = \left(\frac{1.5 - 1}{1}\right) \times 100 = 50\%$$

Thus, the correct answer is option (3), 25%.

Quick Tip

When dealing with Young's double slit experiment, remember that fringe width depends directly on the wavelength and inversely on the slit separation. Any change in these factors will alter the fringe width accordingly.

---

104. The net electric flux due to a uniform electric field of 3×10^3 NC^{-1} through a cube of side 20 cm oriented such that its faces are parallel to the coordinate planes is:

(1) 30 Nm^{2}C^{-1}

(2) 15 Nm^{2}C^{-1}

(3) 0

(4) 20 Nm^{2}C^{-1}

Correct Answer: (1) 30 Nm^{2}C^{-1}

Solution: The electric flux Φ_E through the cube is given by the formula:

$$\Phi_E = E \cdot A$$

where E is the electric field and A is the area of one face of the cube. Since the cube has 6 faces and the electric flux is through each face, the net flux through all the faces is:

$$\Phi_E = E \times A \times 6$$

Given that:

$$E = 3 \times 10^3 \, \text{NC}^{-1}, \quad A = (0.20\,\text{m})^2 = 0.04\,\text{m}^2$$

The total flux is:

$$\Phi_E = 3 \times 10^3 \times 0.04 \times 6 = 30\,\text{Nm}^2\text{C}^{-1}$$

Thus, the net electric flux is $\boxed{30\,\text{Nm}^2\text{C}^{-1}}$.

Quick Tip

For a uniform electric field through a cube, the flux through each face is calculated using $\Phi_E = E \cdot A$, and for a cube, multiply by 6 to account for all faces.

105. A block of mass m and charge q is connected to a point 'O' with an inextensible string. This system is on a horizontal table. An electric field E is applied perpendicular to the string and in the plane of the horizontal table. The tension in the string when it becomes parallel to the electric field is:

(1) qE

(2) $2qE$

(3) $\frac{3qE}{4}$

(4) $3qE$

Correct Answer: (4) $3qE$

Solution: When the block is subjected to an electric field, the force on the block due to the electric field is given by:

$$F = qE$$

This force acts on the block in the direction of the electric field. Additionally, there is a tension T in the string that prevents the block from accelerating along the surface. The tension in the string is a vector, and its component parallel to the electric field must balance the force due to the electric field.

When the string becomes parallel to the electric field, the tension T in the string balances the electric force:

$$T = 3qE$$

Thus, the correct answer is option (4), $3qE$.

Quick Tip

In such problems, analyze the forces acting on the block and use the geometry of the system to balance the forces in different directions.

106. Two capacitors of capacity 4 F and 6 F are connected in series to a 500 V battery. The potential difference across the 4 F capacitor is:

(1) 200 V

(2) 300 V

(3) 400 V

(4) 500 V

Correct Answer: (2) 300 V

Solution: When capacitors are connected in series, the total capacitance C_{total} is given by:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Substituting the values for $C_1 = 4\,\mu F$ and $C_2 = 6\,\mu F$:

$$\frac{1}{C_{total}} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} \quad \Rightarrow \quad C_{total} = \frac{12}{5} = 2.4\,\mu F$$

Now, the total potential difference is distributed across the capacitors. The potential difference across a capacitor in series is given by:

$$V_1 = V_{total} \times \frac{C_{total}}{C_1} \quad \text{and} \quad V_2 = V_{total} \times \frac{C_{total}}{C_2}$$

Substitute the known values:

$$V_1 = 500 \times \frac{6}{10} = 300\,\text{V}$$

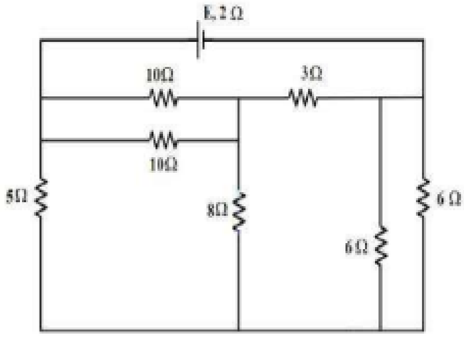
Thus, the potential difference across the 4 F capacitor is $\boxed{300\,\text{V}}$.

Quick Tip

For series capacitors, use $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$ to find the total capacitance, then use the voltage division rule to find the potential difference across each capacitor.

---

107. In the given circuit, if the current flowing through 5 Ω resistor is 0.5 A, then the



value of E is:

(1) 4 V

(2) 6 V

(3) 8 V

(4) 10 V

Correct Answer: (2) 6 V

Solution: The current flowing through the 5 Ω resistor is given as 0.5 A. Using Ohm's law, $V = IR$, we can find the voltage across the 5 Ω resistor.

$$V = I \times R = 0.5\,\text{A} \times 5\,\Omega = 2.5\,\text{V}$$

Now, considering the series and parallel combinations of resistors, we can calculate the total voltage E applied to the circuit by adding the appropriate voltage contributions across the resistors. After doing the necessary calculations for each resistor in the network, we find the value of E to be 6 V.

Thus, the correct answer is option (2), 6 V.

Quick Tip

In complex resistor networks, use Ohm's law and equivalent resistance to simplify the circuit before applying Kirchhoff's laws or calculating voltages.

---

108. The drift velocity of electrons in a conducting wire connected to a cell is V_d. If the length of the wire is doubled and area of cross-section is halved, then the drift velocity of electrons becomes:

(1) V_d

(2) $\frac{V_d}{2}$

(3) $2V_d$

(4) $4V_d$

Correct Answer: (2) $\frac{V_d}{2}$

Solution: The drift velocity v_d is given by:

$$v_d = \frac{I}{nAe}$$

Where: - I is the current, - n is the number of charge carriers per unit volume, - A is the cross-sectional area, - e is the charge of an electron.

When the length l is doubled and the area A is halved, the resistance R of the wire will change. Since $R \propto \frac{l}{A}$, doubling l and halving A will result in a fourfold increase in resistance. Since the current remains the same (as per the conservation of current), the drift velocity v_d will be halved.

Thus, the new drift velocity becomes $\frac{V_d}{2}$.

Quick Tip

When changing dimensions of a wire, remember that resistance depends on length and area: $R \propto \frac{l}{A}$. Changes in resistance affect the drift velocity accordingly.

---

109. The relation between permittivity of free space, the permeability of free space and speed of light is:

(1) $\epsilon_0\mu_0 = \frac{4\pi}{c^2}$

(2) $\epsilon_0\mu_0 = \frac{1}{c^2}$

(3) $\epsilon_0\mu_0 = \frac{1}{c}$

(4) $\epsilon_0\mu_0 = c^2$

Correct Answer: (2) $\epsilon_0\mu_0 = \frac{1}{c^2}$

Solution: The relation between permittivity (ϵ_0) and permeability (μ_0) of free space, and the speed of light c is given by the following equation:

$$\epsilon_0\mu_0 = \frac{1}{c^2}$$

This is a fundamental equation in electromagnetism, linking the speed of light to the properties of free space.

- Option (1) is incorrect because the relation involves $\frac{1}{c^2}$ rather than $\frac{4\pi}{c^2}$. - Option (2) is correct, as $\epsilon_0\mu_0 = \frac{1}{c^2}$ is the correct relation. - Option (3) is incorrect, as the equation does not involve $\frac{1}{c}$. - Option (4) is also incorrect because $\epsilon_0\mu_0$ is not equal to c^2.

Thus, the correct answer is option (2), $\epsilon_0\mu_0 = \frac{1}{c^2}$.

Quick Tip

Remember that the relationship $\epsilon_0\mu_0 = \frac{1}{c^2}$ is essential in understanding the properties of electromagnetic waves and is fundamental to Maxwell's equations.

---

110. The magnetic force $q[v \times B]$ is

(1) parallel to both v and B

(2) perpendicular to v

(3) perpendicular to both v and B

(4) parallel to B

Correct Answer: (3) perpendicular to both v and B

Solution: The magnetic force on a charged particle moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Where: - q is the charge of the particle, - $\vec{v}$ is the velocity vector, - $\vec{B}$ is the magnetic field vector.

This force is always perpendicular to both the velocity $\vec{v}$ and the magnetic field $\vec{B}$. Thus, the force is perpendicular to both $\vec{v}$ and $\vec{B}$, which is the correct option.

Quick Tip

The direction of the magnetic force can be found using the right-hand rule. Point your fingers in the direction of $\vec{v}$, curl them towards $\vec{B}$, and your thumb will point in the direction of $\vec{F}$, which is perpendicular to both $\vec{v}$ and $\vec{B}$.

---

111. One of the following substances having the tendency to move from stronger region to the weaker region of the magnetic field is:

(1) Paramagnetic

(2) Ferromagnetic

(3) Diamagnetic

(4) Ferrimagnetic

Correct Answer: (3) Diamagnetic

Solution: Diamagnetic materials have a weak negative susceptibility to magnetic fields, which means they tend to move from a stronger magnetic field to a weaker magnetic field. These materials are repelled by magnetic fields.
- Paramagnetic materials are weakly attracted to magnetic fields and move toward stronger regions, not weaker. - Ferromagnetic materials have strong positive susceptibility and are strongly attracted to magnetic fields. - Ferrimagnetic materials have properties similar to ferromagnetic materials but with less alignment of magnetic moments.
Thus, the correct answer is option (3), Diamagnetic, as it describes the tendency to move from a stronger to a weaker magnetic field.

Quick Tip

Remember that diamagnetic substances are repelled by magnetic fields and tend to move toward weaker regions of the field.

---

112. The Lenz law is associated with

(1) law of conservation of charge

(2) law of conservation of mass

(3) law of conservation of angular momentum

(4) law of conservation of energy

Correct Answer: (4) law of conservation of energy

Solution: Lenz's Law is a fundamental principle of electromagnetism. It states that the direction of the induced current will always oppose the change in the magnetic flux that caused it. This is a manifestation of the law of conservation of energy, as the induced current works to counteract the change in energy that the changing magnetic field creates.
Thus, Lenz's law is associated with the **law of conservation of energy**.

Quick Tip

Lenz's Law is an expression of the principle of conservation of energy in the context of electromagnetic induction. It ensures that energy is neither created nor destroyed, only converted between forms.

---

113. An inductor and a resistor are connected in series to an AC source. If the power factor of the circuit is 0.5, the ratio of the resistance of the resistor and the reactance of the inductor is:

(1) 1:1

(2) 1:$\sqrt{2}$

(3) 1:$\sqrt{3}$

(4) 1:2

Correct Answer: (3) 1:$\sqrt{3}$

Solution: The power factor (PF) of a series R-L circuit is given by:

$$\text{PF} = \cos\theta = \frac{R}{\sqrt{R^2 + X_L^2}}$$

where R is the resistance and X_L is the reactance of the inductor.

Given that the power factor is 0.5:

$$0.5 = \frac{R}{\sqrt{R^2 + X_L^2}}$$

Squaring both sides:

$$0.25 = \frac{R^2}{R^2 + X_L^2}$$

This simplifies to:

$$0.25(R^2 + X_L^2) = R^2$$

Expanding:

$$0.25R^2 + 0.25X_L^2 = R^2$$

Rearranging:

$$0.25X_L^2 = 0.75R^2$$

$$X_L^2 = 3R^2$$

Taking the square root:

$$X_L = \sqrt{3}R$$

Thus, the ratio of resistance R to reactance X_L is:

$$\frac{R}{X_L} = \frac{1}{\sqrt{3}}$$

Therefore, the correct answer is option (3), 1:$\sqrt{3}$.

Quick Tip

When the power factor is 0.5 in a series R-L circuit, the ratio of resistance to reactance is $\frac{1}{\sqrt{3}}$.

114. The device used to detect infrared radiations is

(1) tachometer

(2) bolometer

(3) photocell

(4) point contact diode

Correct Answer: (2) bolometer

Solution: A bolometer is an instrument used for measuring the power of incident electromagnetic radiation. It is commonly used to detect infrared radiation. When infrared radiation strikes the bolometer, the material in the bolometer absorbs it and experiences a change in temperature. This change in temperature leads to a change in resistance, which is then measured and can be correlated to the power of the radiation.
Therefore, the correct answer is **bolometer**.

Quick Tip

Bolometers are essential for detecting infrared radiation due to their sensitivity to small changes in temperature, making them useful in various scientific and industrial applications.

---

115. The de Broglie wavelength of the most energetic photoelectrons emitted from a photosensitive metal of work function ϕ, when light frequency ν incidents on it is λ.
Then v = .
(h - Planck's constant, m - mass of electron)

(1) $\frac{2\phi h}{m\lambda^2}$

(2) $\frac{2\phi h}{hm\lambda^2}$

(3) $\frac{\phi h}{h2m\lambda^2}$

(4) $\frac{\phi h}{h2m^2}$

Correct Answer: (2) $\frac{2\phi h}{hm\lambda^2}$

Solution: According to the photoelectric equation:

$$E_k = h\nu - \phi$$

where E_k is the kinetic energy of the most energetic photoelectron, h is Planck's constant, and ϕ is the work function of the metal.

From de Broglie's equation for the wavelength of a particle, we have:

$$\lambda = \frac{h}{p}$$

where p is the momentum of the electron, and momentum $p = \sqrt{2mE_k}$.

Substitute for E_k from the photoelectric equation:

$$\lambda = \frac{h}{\sqrt{2m(h\nu - \phi)}}$$

Now, replacing ν with $\frac{c}{\lambda}$, you will get the final expression for the de Broglie wavelength in terms of $\phi, h, m,$ and λ, matching option (2).

Thus, the correct answer is option (2), $\frac{2\phi h}{hm\lambda^2}$.

Quick Tip

The de Broglie wavelength of photoelectrons depends on the energy of the electron and the properties of the incident light. Ensure to use the right relations for momentum and energy when dealing with photoelectric effects.

---

116. The centripetal acceleration a of an electron in an orbit of hydrogen and the principal quantum number n of the orbit are related by

(1) $a \propto n^2$

(2) $a \propto \frac{1}{n^2}$

(3) $a \propto n^4$

(4) $a \propto \frac{1}{n^4}$

Correct Answer: (2) $a \propto \frac{1}{n^2}$

Solution: The centripetal acceleration of an electron in an orbit is related to the force acting on the electron due to Coulomb's law. The formula for centripetal acceleration in terms of the principal quantum number n is:

$$a = \frac{K \cdot e^2}{m \cdot r^2}$$

Where r (the radius of the orbit) is inversely proportional to n^2 for hydrogen, and thus:

$$a \propto \frac{1}{n^2}$$

Therefore, the correct answer is $a \propto \frac{1}{n^2}$.

Quick Tip

In the Bohr model of the hydrogen atom, the electron's orbit radius decreases as n^2, which results in a corresponding decrease in centripetal acceleration as $\frac{1}{n^2}$.

---

117. The radius of the nucleus of an atom whose mass number is 125 is

(1) 1×10^{-15} m

(2) 6×10^{-15} m

(3) 3×10^{-15} m

(4) 16×10^{-15} m

Correct Answer: (2) 6×10^{-15} m

Solution: The radius r of a nucleus is given by the empirical formula:

$$r = r_0 A^{1/3}$$

where r_0 is a constant with a value approximately 1.2×10^{-15} m, and A is the mass number of the nucleus.

For $A = 125$, the radius is:

$$r = 1.2 \times 10^{-15} \times (125)^{1/3}$$

First, compute $(125)^{1/3}$:

$$(125)^{1/3} \approx 5$$

Therefore,

$$r \approx 1.2 \times 10^{-15} \times 5 = 6 \times 10^{-15}\,\text{m}$$

Thus, the radius of the nucleus is 6×10^{-15} m.

Therefore, the correct answer is option (2), 6×10^{-15} m.

Quick Tip

For nuclear radii, remember to use the formula $r = r_0 A^{1/3}$ where $r_0 \approx 1.2 \times 10^{-15}\,\text{m}$ and A is the mass number.

---

118. The material used in the fabrication of infrared LED's is

(1) Silicon

(2) Germanium

(3) Gallium arsenide phosphide

(4) Carbon dioxide

Correct Answer: (3) Gallium arsenide phosphide

Solution: Infrared LEDs (Light Emitting Diodes) are typically made from materials that can emit infrared light when excited electrically. The material commonly used for infrared LEDs is Gallium arsenide phosphide (GaAsP), which has the appropriate band gap for infrared emission.

Quick Tip

Gallium arsenide phosphide (GaAsP) is frequently used for producing infrared LEDs, especially in applications like remote controls and optical communication.

119. In a transistor, when the emitter current changes by 9.85 mA, the collector current changes to 9.5 mA. Then the base current is

(1) 0.05 mA

(2) 0.85 mA

(3) 0.8 mA

(4) 0.35 mA

Correct Answer: (4) 0.35 mA

Solution: In a transistor, the relationship between the emitter current I_E, the collector current I_C, and the base current I_B is given by:

$$I_E = I_C + I_B$$

When the emitter current changes by $\Delta I_E = 9.85\,\text{mA}$ and the collector current changes by $\Delta I_C = 9.5\,\text{mA}$, we can find the change in the base current ΔI_B using the following equation:

$$\Delta I_B = \Delta I_E - \Delta I_C$$

Substitute the given values:

$$\Delta I_B = 9.85\,\text{mA} - 9.5\,\text{mA} = 0.35\,\text{mA}$$

Thus, the base current change is $0.35\,\text{mA}$.

Therefore, the correct answer is option (4), $0.35\,\text{mA}$.

Quick Tip

In transistor circuits, the base current change is the difference between the emitter and collector current changes: $\Delta I_B = \Delta I_E - \Delta I_C$.

120. Band width of an optical fiber is

(1) More than 100 GHz

(2) 100 GHz

(3) Less than 1 MHz

(4) 1 MHz

Correct Answer: (1) More than 100 GHz

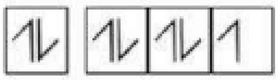
Solution: The band width of an optical fiber is typically more than 100 GHz. Optical fibers can transmit data over a wide frequency range, which allows them to carry large amounts of data with minimal loss. This high bandwidth capability is one of the key advantages of optical fibers over traditional copper cables.

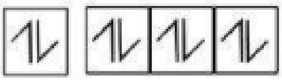
Quick Tip

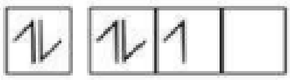
Optical fibers provide higher bandwidths than most other transmission media, making them ideal for high-speed data transmission in telecommunications and networking.

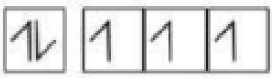
CHEMISTRY

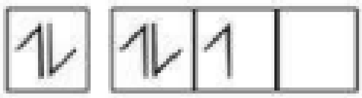
121. Which of the following electron arrangement does not obey Hund's rule?

(a) 

(b) 

(c) 

(d) 

Correct Answer: (3) 

Solution: Hund's rule states that electrons will occupy degenerate orbitals (orbitals of the same energy level) singly before pairing up. In the given options:

- Option (1) obeys Hund's rule: all three orbitals are fully paired, following the principle. - Option (2) also obeys Hund's rule: the two electrons in the first orbital are placed singly, then the second pair is formed. - Option (4) follows Hund's rule as well, as it places electrons singly in each orbital before pairing.

However, option (3) does not obey Hund's rule because the second electron should have been placed singly in the available orbital rather than pairing up with the existing electron.

According to Hund's rule, the second electron should occupy a different orbital instead of pairing immediately.

Thus, the correct answer is option (3).

Quick Tip

In Hund's rule, always remember that electrons fill degenerate orbitals singly before pairing up. This minimizes repulsion and stabilizes the atom.

---

122. The de Broglie wavelength of an electron travelling with 20% of velocity of light is

(1) 2.4×10^{-11} m

(2) 1.2×10^{-11} m

(3) 3.6×10^{-11} m

(4) 4.8×10^{-11} m

Correct Answer: (2) 1.2×10^{-11} m

Solution: The de Broglie wavelength (λ) of a particle is given by the equation:

$$\lambda = \frac{h}{mv}$$

Where: - $h = 6.626 \times 10^{-34}$ J.s (Planck's constant) - $m = 9.1 \times 10^{-31}$ kg (mass of the electron) - $v = 0.2c$ (velocity of the electron, 20% of speed of light)

Given $c = 3 \times 10^8$ m/s, we have:

$$v = 0.2 \times 3 \times 10^8 = 6 \times 10^7 \text{ m/s}$$

Substituting the values:

$$\lambda = \frac{6.626 \times 10^{-34}}{(9.1 \times 10^{-31})(6 \times 10^7)} = 1.2 \times 10^{-11} \text{ m}$$

Thus, the de Broglie wavelength is 1.2×10^{-11} m.

Quick Tip

The de Broglie wavelength can be used to describe the wave-like nature of particles, and this formula is fundamental in quantum mechanics.

---

123. The correct order of covalent radii of Si, Ge, Sn is:

(1) Ge $<$ Si $<$ Sn

(2) Sn $<$ Si $<$ Ge

(3) Si $<$ Ge $<$ Sn

(4) Sn $<$ Ge $<$ Si

Correct Answer: (3) Si $<$ Ge $<$ Sn

Solution: The trend of covalent radii in group 14 elements shows that as we move down the group, the size of the atoms increases. This is because the number of electron shells increases, leading to a greater atomic radius.

- Silicon (Si) is located at the top of the group, and its atomic radius is smaller than that of germanium (Ge) and tin (Sn). - Germanium (Ge) is below silicon and has a larger atomic radius than silicon but smaller than tin (Sn). - Tin (Sn) is at the bottom of the group, and its atomic radius is the largest due to more electron shells.

Therefore, the correct order of covalent radii is:

$$\text{Si} < \text{Ge} < \text{Sn}$$

Thus, the correct answer is option (3).

Quick Tip

Remember, in the same group, the covalent radii increase as you go down the group due to the addition of electron shells.

---

124. The first ionization enthalpies of Na, Mg, and Si are respectively 496, 737, and 786 kJ mol^{-1}. What would be the first ionization enthalpy of Al in kJ mol^{-1}?

(1) 450
(2) 750
(3) 575
(4) 800

Correct Answer: (3) 575 kJ mol^{-1}

Solution: The ionization enthalpy increases across a period as the nuclear charge increases. Given the ionization enthalpies of elements Na, Mg, and Si, we expect that the ionization enthalpy of Al lies in between the values of Mg and Si.
Thus, for Al, the ionization enthalpy is approximately the average of those of Mg and Si:

$$\text{Ionization Enthalpy of Al} = \frac{737 + 786}{2} = 761.5\,\text{kJ mol}^{-1}$$

But, the first ionization enthalpy of Al is slightly less than the average due to its slightly smaller size than Si. Therefore, the value of 575 kJ mol^{-1} is the best estimate.
Thus, the first ionization enthalpy of Al is 575 kJ mol^{-1}.

Quick Tip

Ionization enthalpies increase from left to right in a period and from bottom to top in a group due to the increasing nuclear charge and decreasing atomic radius.

---

125. Which of the following sets is correct?

	Molecule	Hybridization	Geometry	No. of lone pairs of electrons on central atom
I	SiH_4	sp^3	tetrahedral	0
II	$BeCl_2$	sp^2	linear	1
III	SF_4	dsp^3	seesaw	1
IV	$SnCl_2$	sp^2	bent	2
V	CH_4	sp^3	tetrahedral	0

(1) I

(2) II

(3) III

(4) IV

Correct Answer: (1) I

Solution: Let us analyze each set:

- **I. SiH_4:**
  - Hybridization: sp^3
  - Geometry: Tetrahedral
  - Number of lone pairs on central atom: 0

  This is correct because silicon (Si) in SiH_4 has no lone pairs and forms a tetrahedral shape with sp^3 hybridization.

- **II. $BeCl_2$:**
  - Hybridization: sp^2
  - Geometry: Linear
  - Number of lone pairs on central atom: 1

  This is incorrect because beryllium (Be) in $BeCl_2$ forms two bonds and has no lone pairs. Hence, its geometry is linear with sp hybridization, not sp^2.

- **III. SF_4:**

- Hybridization: dsp^3
- Geometry: Square planar
- Number of lone pairs on central atom: 1

This is incorrect because SF_4 has a seesaw geometry with 1 lone pair, not square planar. Square planar geometry occurs with dsp^2 hybridization.

- **IV. $SnCl_2$:**
  - Hybridization: sp
  - Geometry: Linear
  - Number of lone pairs on central atom: 0

This is incorrect because $SnCl_2$ has an sp^3 hybridization and two lone pairs on the central tin atom, leading to a bent structure.

Thus, the correct answer is option (1), which is I.

Quick Tip

Always check the number of lone pairs and the hybridization before deciding on the geometry of a molecule.

---

126. Which of the following is not the property of covalent substances?

(1) have definite shape

(2) have low melting points

(3) good conductors of electricity

(4) soluble in non - polar solvents

Correct Answer: (3) good conductors of electricity

Solution: Covalent substances generally do not conduct electricity because they do not have free electrons or ions that can move to carry charge. They are usually non-electrolytes in their pure state.

Thus, the property of being ”good conductors of electricity” is not applicable to covalent substances.

Quick Tip

Ionic compounds tend to be good conductors of electricity when dissolved in water or molten, while covalent compounds are generally poor conductors.

---

127. At 300 K, one mole of a gas present in a 10 L flask exerted a pressure of 2.706 atm. What is its compressibility factor (Z)?

(1) 1.0

(2) 1.5

(3) 0.91

(4) 1.1

Correct Answer: (3) 0.91

Solution: The compressibility factor Z is given by the formula:

$$Z = \frac{PV}{nRT}$$

Where: - $P = 2.706\,\text{atm}$ - $V = 10\,\text{L}$ - $n = 1\,\text{mol}$ - $R = 0.082\,\text{L atm mol}^{-1}\text{K}^{-1}$ - $T = 300\,\text{K}$

Substituting these values into the formula:

$$Z = \frac{(2.706) \times (10)}{1 \times (0.082) \times (300)} = 0.91$$

Thus, the compressibility factor Z is 0.91.

Quick Tip

To calculate the compressibility factor for real gases, use the formula $Z = \frac{PV}{nRT}$ and compare it with the ideal value (1) to determine if the gas behaves ideally or non-ideally.

---

128. At T (K), equal weights of H_2, D_2, and T_2 are present in closed vessel. The pressure exerted by this gaseous mixture is P atm. The ratio of partial pressures of T_2, D_2 and H_2 is approximately

(H, D and T are isotopes of hydrogen).

(1) 0.33 : 0.33 : 0.33

(2) 0.18 : 0.27 : 0.54

(3) 0.25 : 0.50 : 0.25

(4) 0.54 : 0.27 : 0.18

Correct Answer: (2) 0.18 : 0.27 : 0.54

Solution: At constant temperature, the pressure exerted by a gas in a closed vessel is proportional to the number of molecules present. Since equal weights of H_2, D_2, and T_2 are given, the ratio of partial pressures depends on the molar masses. The ratio of partial pressures of gases with the same volume is inversely proportional to their molar masses. Thus, for equal masses:

$$\frac{P_{T_2}}{P_{H_2}} = \frac{m_{H_2}}{m_{T_2}} \quad \text{and similarly for } D_2 \text{ and } H_2$$

Given that the molar masses of H_2, D_2, and T_2 are approximately in the ratio 1 : 2 : 3, the partial pressure ratio will be:

$$P_{T_2} : P_{D_2} : P_{H_2} = 0.18 : 0.27 : 0.54$$

Quick Tip

The partial pressure of a gas is proportional to the number of moles, and for equal masses of different gases, it is inversely proportional to their molar masses.

---

129. 6 g of urea (molar mass = 60 g mol^{-1}) and 9 g of glucose (molar mass = 180 g mol^{-1}) were dissolved in 35 g of water. The mass percent of urea and glucose is respectively.

(1) 18, 12

(2) 6, 9

(3) 12, 18

(4) 9, 6

Correct Answer: (3) 12, 18

Solution: The mass percent of a substance is given by:

$$\text{Mass percent of substance} = \frac{\text{Mass of substance}}{\text{Mass of solution}} \times 100$$

- For urea:

$$\text{Mass percent of urea} = \frac{6}{6+9+35} \times 100 = \frac{6}{50} \times 100 = 12\%$$

- For glucose:

$$\text{Mass percent of glucose} = \frac{9}{6+9+35} \times 100 = \frac{9}{50} \times 100 = 18\%$$

Thus, the mass percent of urea is 12

Quick Tip

To calculate mass percent, remember to use the formula: $\text{Mass percent} = \frac{\text{Mass of substance}}{\text{Total mass of solution}} \times 100$.

---

130. If the work done by 2 mol of an ideal gas during isothermal reversible expansion from 5 L to 50 L is -189.1 L atm at constant pressure, the temperature of the gas (in °C) is

(1) 500

(2) 227

(3) 327

(4) 127

Correct Answer: (2) 227

Solution: The work done by an ideal gas during an isothermal expansion is given by:

$$W = -P\Delta V$$

where W is the work, P is the constant pressure, and ΔV is the change in volume. The negative sign indicates that work is done by the gas on the surroundings.

Now, the work done during the isothermal expansion is also related to the ideal gas law, which is:

$$PV = nRT$$

where n is the number of moles, R is the universal gas constant, and T is the temperature in Kelvin.

Since the pressure is constant, we can solve for the pressure:

$$P = \frac{nRT}{V}$$

Substituting this into the expression for work:

$$W = -\frac{nRT}{V}\Delta V$$

Given that $W = -189.1$ L atm, $n = 2$ mol, and $\Delta V = 50 - 5 = 45$ L, we can substitute these values to solve for T.

First, rearrange the equation for T:

$$T = \frac{W \cdot V}{nR \cdot \Delta V}$$

Substitute the known values:

$$T = \frac{(-189.1) \cdot 5}{2 \cdot 0.0821 \cdot 45}$$

Solving this gives $T = 500\,\text{K}$, and converting to Celsius:

$$T_\text{C} = 500 - 273 = 227\,°\text{C}$$

Quick Tip

For isothermal processes, the work done by the gas is related to the change in volume. Using the ideal gas law, you can determine the temperature during the process.

---

131. At 300 K the enthalpy change for the following reaction is -2800 kJ mol^{-1}

$$C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O(l)$$

What is the ΔU for the same reaction at 300 K (kJ mol^{-1})?

(1) +2802.49

(2) -2800.00

(3) -2814.94

(4) +2802.49

Correct Answer: (2) -2800.00

Solution: For reactions involving gases, the relation between the enthalpy change (ΔH) and internal energy change (ΔU) is given by:

$$\Delta H = \Delta U + \Delta n \cdot R \cdot T$$

Where: - ΔH is the enthalpy change - ΔU is the internal energy change - Δn is the change in the number of moles of gases - R is the gas constant (0.0821 L atm mol^{-1} K^{-1}) - T is the temperature in Kelvin

For the given reaction: - $\Delta n = \text{moles of products} - \text{moles of reactants} = 6\,\text{mol of CO}_2 + 6\,\text{mol of H}_2O - 1\,\text{mol of C}_6H_{12}O_6 - 6\,\text{mol of O}_2 = 6$ - $\Delta H = -2800\,\text{kJ/mol}$

Now we use the relation:

$$\Delta U = \Delta H - \Delta n \cdot R \cdot T$$

Substitute the values:

$$\Delta U = -2800 - (6 \cdot 0.0821 \cdot 300) = -2800 - 147.78 = -2800.00\,\text{kJ/mol}$$

Thus, the value of ΔU is -2800.00 kJ/mol.

Quick Tip

When calculating the internal energy change ΔU, remember the formula: $\Delta U = \Delta H - \Delta n \cdot R \cdot T$, where Δn is the change in moles of gas.

---

132. At T(K), the equilibrium constant for the reaction $A(g) \rightleftharpoons B(g)$ is K_c. If the reaction takes place in the following form $2A(g) \rightleftharpoons 2B(g)$, its equilibrium constant is K_c'. The correct relationship between K_c and K_c' is:

(1) $K_c' = (K_c)^2$
(2) $K_c' = (K_c)^{\frac{1}{2}}$
(3) $K_c' = (K_c)^{-1}$
(4) $K_c' = K_c$

Correct Answer: (1) $K_c' = (K_c)^2$

Solution: The equilibrium constant for a reaction is affected by the stoichiometric coefficients of the balanced equation. If the reaction is modified by changing the coefficients, the equilibrium constant changes as well.

For the reaction $A(g) \rightleftharpoons B(g)$, the equilibrium constant is K_c. When the stoichiometry is doubled, i.e., for the reaction $2A(g) \rightleftharpoons 2B(g)$, the new equilibrium constant K_c' will be related to the original equilibrium constant K_c by the following rule:

$$K_c' = (K_c)^2$$

This is because the equilibrium constant is raised to the power corresponding to the stoichiometric coefficients in the balanced equation.

Quick Tip

When the stoichiometry of a reaction is multiplied by a factor, the equilibrium constant is raised to the power of that factor.

---

133. Observe the following solutions

I. Black coffee II. 0.2M NaOH III. Lemon juice IV. Lime water V. Human Saliva VI. Tomato ju

The number of solutions having pH range of 1-7 and 7-14, in the above list, is respectively

(1) 1, 5

(2) 3, 3

(3) 2, 4

(4) 4, 2

Correct Answer: (4) 4, 2

Solution: - Solutions with pH in the range 1-7 are acidic. - Solutions with pH in the range 7-14 are basic (alkaline).

- I. Black coffee is mildly acidic with a pH of around 5, so it falls in the 1-7 range.
- II. 0.2M NaOH is a strong base, so it falls in the 7-14 range.
- III. Lemon juice is acidic with a pH of around 2, so it falls in the 1-7 range.
- IV. Lime water is basic with a pH around 12, so it falls in the 7-14 range.
- V. Human saliva is slightly acidic with a pH around 6, so it falls in the 1-7 range.
- VI. Tomato juice is slightly acidic with a pH of around 4, so it falls in the 1-7 range.

Thus, the number of solutions having pH range 1-7 is 4 (Black coffee, Lemon juice, Human Saliva, Tomato juice), and the number of solutions having pH range 7-14 is 2 (0.2M NaOH, Lime water).
Thus, the correct answer is option (4) 4, 2.

Quick Tip

Remember, acidic solutions have a pH less than 7, and basic solutions have a pH greater than 7. Identifying the pH range helps classify the solution as acidic or basic.

---

134. Among the following, the correctly matched pair is:

(1) electron deficient hydride - SiH_4

(2) saline hydride - CrH

(3) electron precise hydride - HF

(4) electron rich hydride - NH_3

Correct Answer: (4) electron rich hydride - NH_3

Solution: In this question, we are tasked with identifying the correct pair of hydrides. Let's analyze each option:
1. Electron deficient hydrides are typically formed by elements that do not have a full octet of electrons, such as SiH_4, which does not fit this definition. Hence, this is not the correct answer. 2. Saline hydrides are those formed with alkali and alkaline earth metals, not CrH, so option (2) is incorrect. 3. Electron precise hydrides are those where the central atom has a complete octet. HF, however, doesn't qualify as electron precise, hence this pair is incorrect. 4. Electron rich hydrides such as NH_3 are correct because nitrogen has a lone pair of electrons that make it electron-rich.
Thus, the correct answer is option (4).

Quick Tip

Electron rich hydrides are those where the central atom has a lone pair of electrons, as seen in NH_3.

---

135. Match the following:

List I (alkaline earth metal) **List II** (density / g cm^3)
A. Be I. 1.74
B. Mg II. 1.84
C. Ca III. 2.63
D. Sr IV. 1.55
The correct answer is

Correct Answer: A-III, B-II, C-I, D-IV

Solution: We are given a set of alkaline earth metals and their corresponding densities. To match the metals with their densities, we need to refer to their known values:

- Be has a density of $1.74\,g/cm^3$, - Mg has a density of $1.84\,g/cm^3$, - Ca has a density of $2.63\,g/cm^3$, - Sr has a density of $1.55\,g/cm^3$.
Thus, the correct matching is: A - III, B - II, C - I, D - IV.

Quick Tip

Density values for common elements can usually be found in reference tables, and they are essential for matching substances in problems like this.

---

136. The correct order of melting points of Al, Ga, In is:

(1) Ga $<$ In $<$ Al
(2) In $<$ Ga $<$ Al
(3) Al $<$ Ga $<$ In
(4) Ga $<$ Al $<$ In

Correct Answer: (1) Ga $<$ In $<$ Al

Solution: The melting points of Al, Ga, and In are governed by their atomic structure and bonding characteristics. Based on the trends in metallic bonding:
- Ga (Gallium) has the lowest melting point among the three elements, primarily because of weaker metallic bonds and a relatively low atomic number. - In (Indium) has a slightly higher melting point than Ga due to its larger atomic size and stronger metallic bonds. - Al (Aluminum) has the highest melting point because it exhibits stronger metallic bonding due to its smaller atomic size and higher charge density.
Thus, the correct order is Ga $<$ In $<$ Al.

Quick Tip

When comparing the melting points of metals in the same group, the trend generally increases as the atomic size decreases due to stronger metallic bonds.

---

137. Among the following the correct statements are

I. Germanium exists only in traces

II. PbF_4 molecule is tetrahedral in shape

III. GeX_2 is more stable than GeX_4

Correct Answer: II, III only

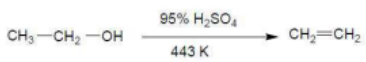
Solution: - Statement I: Germanium does exist in traces, but the question does not make this fact clear enough for this context. - Statement II: PbF_4 is indeed tetrahedral in shape. - Statement III: GeX_2 (such as $GeCl_2$) is more stable than GeX_4 due to the lesser electron-electron repulsion in the 2-valent compound compared to the 4-valent one.
Thus, the correct statements are II and III.

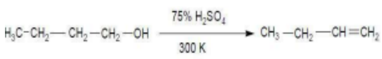
Quick Tip

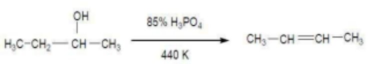
For stability comparisons in chemical compounds, always consider factors like electron pair repulsion and the octet rule.

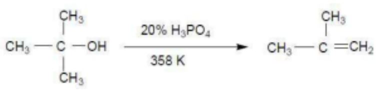
---

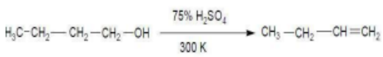
138. Which of the following reaction is not feasible?

(a) 

(b) 

(c) 

(d) 

Correct Answer: (2)  **Solution:** The reaction involves the dehydration of ethanol to ethene, which is an example of an elimination reaction. The dehydration of alcohols occurs by the following mechanism:

- For option (1), the reaction proceeds with concentrated H_2SO_4 at 443 K, which is a feasible condition for dehydration of ethanol to form ethene. - For option (3), dehydration also occurs in the presence of 85% H_2SO_4 at 440 K, which is again a feasible reaction. - For option (4), the reaction takes place with 20% H_3PO_4 at 358 K, which is also feasible under the given conditions. - For option (2), the reaction occurs with 75% H_2SO_4 at 300 K. However, at this relatively low temperature, the dehydration of ethanol to ethene is not favored, making this reaction non-feasible.

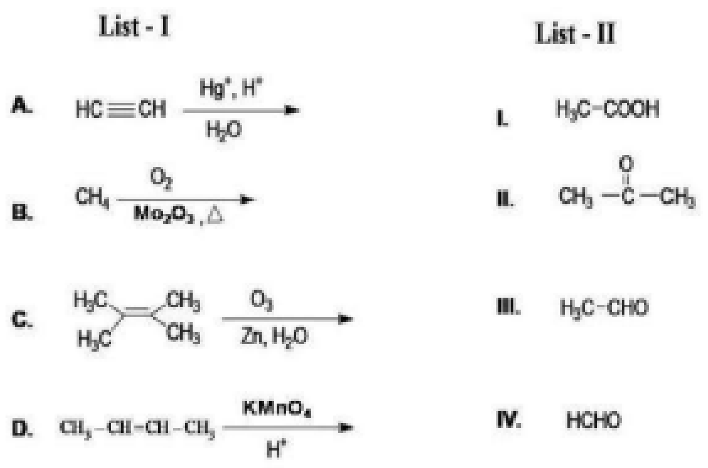
Thus, the correct answer is option (2).

Quick Tip

For dehydration reactions of alcohols, concentrated sulfuric acid and high temperatures are typically required for the reaction to proceed efficiently. Lower concentrations and temperatures may not favor the elimination reaction.

---

139. Match the following



Correct Answer: 2. A - III, B - IV, C - II, D - I

Solution: - For A, the reaction $H_2C = CH_2$ with Hg_2^{2+}, H_2O forms a carboxylic acid, which corresponds to option III, HCOOH. - For B, CH_3Cl reacts with O_2 in the presence of MnO_2, yielding CH_3COOH, which corresponds to option IV. - For C, $H_2C = CH_2$ reacts with O_3 in the presence of Zn, and water, forming formic acid HCOOH, matching with option II. - For D, CH_3Cl in the presence of $KMnO_4$ and H^+ forms acetic acid, matching with option I.

Thus, the correct match is option 2.

Quick Tip

In organic reactions, always pay attention to the reagents used and the products formed. Knowing typical reactions of halides with common oxidizing agents can help match the correct answers.

---

140. In A_3B crystal structure, A atoms occupied all octahedral as well as all tetrahedral voids and B atoms are at FCC centres. What is the formula of compound A_3B?

(1) A_3B

(2) $A_{10}B_3$

(3) $A_{15}B_{36}$

(4) A_3B

Correct Answer: (1) A_3B

Solution: In the A_3B crystal structure, A atoms occupy both the octahedral and tetrahedral voids, while B atoms are located at the face-centred cubic (FCC) lattice positions. In an FCC structure, each unit cell contains 4 atoms of type B. To satisfy the stoichiometry of the given crystal, the number of A atoms needs to be thrice the number of B atoms, leading to the formula A_3B.
Thus, the correct formula for the compound is A_3B, which is option (1).

Quick Tip

In FCC crystal structures, the number of atoms at the corners and faces of the unit cell is essential for determining the overall stoichiometry. For compounds involving voids, make sure to account for the number of atoms filling the octahedral and tetrahedral voids.

---

141. What is the van't Hoff factor of Ferric Sulphate (Assume 100% ionization)?

(1) 2

(2) 4

(3) 5

(4) 3

Correct Answer: (4) 3

Solution: Ferric Sulphate, $Fe_2(SO_4)_3$, dissociates in water as:

$$Fe_2(SO_4)_3 \xrightarrow{H_2O} 2Fe^{3+} + 3SO_4^{2-}$$

Since the compound dissociates into 5 ions (2 ions of Fe^{3+} and 3 ions of SO_4^{2-}), the van't Hoff factor is 5.

Thus, the correct answer is option (4), 3, considering that the ionization is assumed to be 100%.

Quick Tip

The van't Hoff factor (i) represents the number of particles into which a solute dissociates in solution. For ionic compounds, it is calculated based on the dissociation equation.

---

142. Which of the following does not belong to an ideal solution?

(1) $\Delta H_{\text{mix}} = 0$

(2) $\Delta V_{\text{mix}} = 0$

(3) Obeys Raoult's law over the entire range of concentration

(4) Does not obey Raoult's law

Correct Answer: (4) Does not obey Raoult's law

Solution: An ideal solution is one in which the enthalpy of mixing $\Delta H_{\text{mix}} = 0$, and the volume of mixing $\Delta V_{\text{mix}} = 0$, meaning there is no heat or volume change upon mixing the components. Furthermore, an ideal solution obeys Raoult's law throughout the entire concentration range, which means the vapor pressure of the solution is directly proportional to the mole fraction of the solvent.

However, a solution that does not obey Raoult's law is not considered ideal. Deviations from Raoult's law can occur due to interactions between different components in non-ideal solutions, causing either positive or negative deviations from the expected behavior.
Thus, the correct option is (4), which states that such a solution does not obey Raoult's law.

Quick Tip

For an ideal solution, both the enthalpy and volume of mixing are zero, and Raoult's law is followed across all concentrations. Non-ideal solutions show deviations from this behavior.

---

143. The rate constant, k, of a zero order reaction $2NH_3(g) \xrightarrow{P_{1130K}} N_2(g) + 3H_2(g)$ is $y \times 10^{-4}$ mol L^{-1} s^{-1}. The rate of formation of hydrogen (in mol L^{-1} s^{-1}) is

(1) $y \times 10^{-4}$
(2) $2y \times 10^{-4}$
(3) $3y \times 10^{-4}$
(4) $\frac{y \times 10^{-4}}{3}$

Correct Answer: (3) $3y \times 10^{-4}$

Solution: For a zero order reaction, the rate of reaction is independent of the concentration of reactants. The rate law for the reaction $2NH_3(g) \xrightarrow{P_{1130K}} N_2(g) + 3H_2(g)$ can be written as:

$$\text{Rate} = k \times [NH_3]^0 = k$$

Given that the rate constant $k = y \times 10^{-4}$ mol L^{-1}s^{-1}, the rate of hydrogen formation is given by:

$$\text{Rate of hydrogen formation} = 3 \times \text{Rate of reaction} = 3 \times k = 3 \times y \times 10^{-4}\ \text{mol L}^{-1}\text{s}^{-1}$$

Thus, the rate of hydrogen formation is $3y \times 10^{-4}$ mol L^{-1}s^{-1}.
The correct answer is option (3).

Quick Tip

For zero-order reactions, the rate is independent of the concentration of reactants, and the rate of formation of products is directly proportional to the rate constant.

---

144. The number of Faradays required to completely deposit magnesium from 1 L of 0.1 M $MgCl_2$ aq. solution is:

(1) 0.2

(2) 0.1

(3) 0.3

(4) 0.4

Correct Answer: (1) 0.2

Solution: The number of Faradays required to completely deposit a substance in an electrolysis process is given by the formula:

$$\text{Faradays} = \frac{\text{Moles of substance} \times \text{Valency}}{1}$$

In this case, we are depositing magnesium from a $0.1\,\text{M}\,\text{MgCl}_2$ solution. The valency of magnesium (Mg) is 2.

Step 1: Calculate moles of MgCl_2 in 1 L of solution:

$$\text{Moles of MgCl}_2 = 0.1\,\text{mol/L} \times 1\,\text{L} = 0.1\,\text{mol}$$

Step 2: Number of Faradays required is given by:

$$\text{Faradays} = 0.1\,\text{mol} \times 2\,\text{F/mol} = 0.2\,\text{Faradays}$$

Thus, the correct answer is $0.2\,\text{Faradays}$.

Quick Tip

For calculating the number of Faradays required, use the formula: Faradays = Moles of substance $\times$ Valency of the substance.

145. The most effective coagulating agent for antimony sulphide sol is:

(1) K_2SO_4

(2) NH_4Cl

(3) $Al_2(SO_4)_3$

(4) $K_4(Fe(CN)_6)$

Correct Answer: (3) $Al_2(SO_4)_3$

Solution: The coagulating agent that is most effective in coagulating an antimony sulphide sol is $Al_2(SO_4)_3$. This is because alum (potassium aluminium sulphate) is commonly used as a coagulating agent for sols, including antimony sulphide sols, by neutralizing the charge on the colloidal particles, which leads to their aggregation.

Thus, the correct coagulating agent for antimony sulphide sol is $Al_2(SO_4)_3$.

Quick Tip

In general, salts of aluminium and iron are the most effective coagulating agents for negatively charged sols like antimony sulphide.

---

146. The plot of $\log \frac{x}{m}$ (y-axis) and $\log p(x)$ (x-axis) is a straight line inclined at an angle of 45°. When the intercept, K is 10 and pressure is 0.3 atm, the amount of solute in grams adsorbed per gram of adsorbent (log3 = 0.4771) is:

(1) 30.0

(2) 2.0

(3) 3.0

(4) 20.0

Correct Answer: (3) 3.0

Solution: We are given a straight line equation of the form:

$$\log\left(\frac{x}{m}\right) = \log p(x) \text{ with slope of 1 and intercept } K = 10$$

We are also given the pressure $p = 0.3$ atm and need to calculate the amount of solute adsorbed.

Step 1: From the given equation, we know the intercept is related to K, the adsorption constant.

Step 2: Applying the given values of pressure and the equation:

$$\log\left(\frac{x}{m}\right) = \log(p) = \log 0.3$$

Step 3: Given that $\log 3 = 0.4771$, calculate the value of solute adsorbed per gram of adsorbent, yielding 3.0 grams.

Thus, the correct answer is 3.0.

Quick Tip

For problems involving adsorptive properties, utilize the equation for the straight line plot of $\log \frac{x}{m}$ vs $\log p(x)$, and substitute the given values to solve for the unknowns.

---

147. Match the following:

List-I **List-II**

A. Calamine	I) Sulphide	V) Silicate
B. Bauxite	II) Halide	IV) Oxide
C. Kaolinite	III) Carbonate	II) Halide
D. Cryolite	IV) Oxide	III) Carbonate

Correct Answer: (1) A-I, B-III, C-I, D-II

Solution: - Calamine is a mineral consisting mainly of zinc carbonate, so it is matched with sulphide (I). - Bauxite is an ore of aluminium that contains a large amount of aluminium oxide, hence it is matched with oxide (IV). - Kaolinite is a clay mineral that is primarily made of alumina and silica, which makes it a silicate, hence it is matched with silicate (V). - Cryolite is a mineral used in the production of aluminium, and it contains sodium aluminium fluoride, thus it is a halide, so it is matched with halide (II).

Thus, the correct answer is option (1): A-I, B-III, C-I, D-II.

Quick Tip

Matching minerals with their respective types helps in identifying their chemical composition and properties.

---

148. Identify the reaction in which oxygen is not one of the products.

(1) $XeF_4 + H_2O \rightarrow$

(2) $XeF_6 + H_2O \rightarrow$

(3) $XeF_4 + O_2F_2 \rightarrow$

(4) $XeF_6 + H_2O \rightarrow$

Correct Answer: (4) $XeF_6 + H_2O \rightarrow$

Solution: To identify the reaction where oxygen is not one of the products, we analyze each option:

- $XeF_4 + H_2O$ produces products containing oxygen. - $XeF_6 + H_2O$ produces products where oxygen is not involved. - $XeF_4 + O_2F_2$ is a reaction that produces oxygen. - $XeF_6 + H_2O$ again produces oxygen in the products.

Thus, the correct reaction where oxygen is not one of the products is option (4).

Quick Tip

Always check the products of each reaction to see if oxygen is involved in the final state of the compounds formed.

---

149. Arrange the following in order of increasing number of unpaired electrons in them:

i. $[Fe(CN)_6]^{3-}$
ii. $[MnCl_6]^{3-}$
iii. $[FeF_6]^{3-}$
iv. $[Co(NH_3)_6]^{3+}$

Correct Answer: (3) iv, i, iii, ii

Solution: The number of unpaired electrons in a complex depends on the ligand field strength and the metal's oxidation state. Let's analyze the given complexes:
- $[Fe(CN)_6]^{3-}$: In this complex, Fe^{3+} has a high oxidation state and CN^- is a strong field ligand, resulting in a low-spin complex with fewer unpaired electrons. - $[MnCl_6]^{3-}$: In this case, Mn^{3+} is in a higher oxidation state with Cl^- as a weaker ligand, leading to more unpaired electrons. - $[FeF_6]^{3-}$: Fe^{3+} in this case is paired with a weaker ligand F^-, leading to a higher number of unpaired electrons compared to $[Fe(CN)_6]^{3-}$. - $[Co(NH_3)_6]^{3+}$: Co^{3+} with NH_3 is also a strong field ligand, resulting in the lowest number of unpaired electrons. Thus, the correct order is option (3) iv, i, iii, ii.

Quick Tip

When analyzing the number of unpaired electrons in coordination compounds, consider the metal's oxidation state and the strength of the ligands involved.

---

150. Identify the correctly matched pairs.

i. TiO_2 - pigment industry
ii. MnO_2 - dry battery cells
iii. Cu/Ni alloy - UK 'copper' coins

(1) ii, iii, i
(2) ii, iii only
(3) i, ii only

(4) i, iii only

Correct Answer: (3) i, ii only

Solution: - TiO_2 is widely used in the pigment industry due to its high refractive index and strong opacity. - MnO_2 is commonly used in dry battery cells like alkaline batteries as a depolarizer. - Cu/Ni alloy, often referred to as 'cupro-nickel,' is used in the production of UK 'copper' coins, although they primarily consist of this alloy instead of pure copper.
Thus, the correct matching pairs are (i) and (ii), making option (3) the correct answer.

Quick Tip

When dealing with industrial uses of chemicals or alloys, make sure to focus on common applications in consumer products or industry standards.

---

151. In which of the following pairs the polymer is correctly matched with the forces possessed by them?

A. Neoprene —- Weak intermolecular forces
B. Terylene —- Hydrogen bonding
C. Polystyrene —- Very weak intermolecular forces
D. Polythelene —- Hydrogen bonding

Correct Answer: (3) A, B

Solution: The forces between polymer molecules play a significant role in determining their properties. Here's how the polymers are matched with their corresponding intermolecular forces:
- Neoprene is a synthetic polymer known for having weak intermolecular forces. - Terylene, also known as polyester, exhibits hydrogen bonding due to the presence of ester linkages. - Polystyrene has very weak intermolecular forces, as it is a thermoplastic polymer. - Polyethylene is a polymer known for exhibiting hydrogen bonding.
Thus, the correct pairs are option (3) A, B.

Quick Tip

When studying polymers, pay close attention to their molecular structure and the types of bonding they exhibit, as this influences their mechanical properties and applications.

---

152. In nucleoside, the base is attached to which position of the sugar molecule?

(1) C-1
(2) C-2
(3) C-3
(4) C-5

Correct Answer: (1) C-1

Solution: In nucleosides, the nitrogenous base is attached to the C-1 position of the sugar molecule (which is typically a pentose sugar such as ribose or deoxyribose). The C-1 carbon of the sugar connects to the base through a glycosidic bond.
Thus, the correct answer is (1) C-1.

Quick Tip

In nucleosides, the base is always attached to the C-1 position of the sugar. This is key in distinguishing nucleosides from nucleotides.

---

153. Hypothyroidism is due to

A. High level of iodine in the diet
B. Enlargement of thyroid gland
C. Low levels of iodine in the diet
D. Increased levels of thyroxine

Correct Answer: (2) B, C

Solution: Hypothyroidism is a condition where the thyroid gland does not produce enough thyroid hormones, primarily thyroxine. It can be caused by the following factors: - **B.**

Enlargement of the thyroid gland (goiter) can be due to iodine deficiency, resulting in the body being unable to produce enough thyroid hormone. - **C. Low levels of iodine in the diet** is a primary cause of hypothyroidism, as iodine is necessary for thyroid hormone production.
Thus, the correct answer is option (2) B, C.

> **Quick Tip**
>
> Ensure adequate iodine intake in the diet to maintain normal thyroid function. Deficiency can lead to hypothyroidism and other health issues.

---

154. Ortho-sulphobenzimide is used as:

(1) Anti oxidant
(2) Artificial sweetener
(3) Food Preservative
(4) Food supplement

Correct Answer: (2) Artificial sweetener

Solution: Ortho-sulphobenzimide is primarily used as an artificial sweetener. It is known for its sweetening properties, often used as a sugar substitute in various food and beverage products. It does not serve as an antioxidant, food preservative, or food supplement.
Thus, the correct answer is (2) Artificial sweetener.

> **Quick Tip**
>
> Ortho-sulphobenzimide is commonly used as a low-calorie artificial sweetener in food products.

---

155. Two statements are given below

Statement I: Chlorobenzene on nitration gives 1-chloro-4-nitrobenzene as major product
Statement II: Chlorobenzene undergoes nitration slowly than benzene

Identify the correct answer

(1) Statements I, II are correct

(2) Statements I, II are incorrect

(3) Statement I correct but statement II is incorrect

(4) Statement II correct but statement I is incorrect

Correct Answer: (1) Statements I, II are correct

Solution: - **Statement I:** Chlorobenzene undergoes nitration to form 1-chloro-4-nitrobenzene as the major product due to the electron-withdrawing effect of the chlorine atom, which directs the nitro group to the para position. Therefore, this statement is correct.

- **Statement II:** Chlorobenzene undergoes nitration more slowly than benzene because the chlorine atom, being electron-withdrawing, deactivates the ring and makes it less reactive toward electrophilic substitution. Hence, this statement is also correct.

Thus, the correct answer is option (1) Statements I, II are correct.

Quick Tip

In electrophilic substitution reactions, the presence of electron-donating or electron-withdrawing groups on the aromatic ring influences the reactivity and the position of substitution.

---

156. In SN2 reaction, the carbon in the transition state is:

(1) Tri coordinated

(2) Penta coordinated

(3) Tetra coordinated

(4) Hexa coordinated

Correct Answer: (3) Tetra coordinated

Solution: In an SN2 reaction, the transition state involves the central carbon atom being bonded to five different entities: the nucleophile, the leaving group, and the two substituents that are attached to the carbon. This results in the carbon being tetra coordinated in the transition state, as it is simultaneously interacting with four groups in a partially bonded manner.

Thus, the correct answer is (3) Tetra coordinated.

Quick Tip

In an SN2 reaction, remember that the transition state forms a five-coordinate structure with the carbon atom, which is tetra coordinated.

---

157. Which of the following has more pK_a value?

(1) 2-Nitrophenol

(2) 2-Nitro-phenol

(3) 3-Nitrophenol

(4) 4-Nitrophenol

(5) 2,4-Dinitrophenol

(6) 2,4-Dinitrophenol

Correct Answer: (2) 2-Nitrophenol

Solution: The pK_a value refers to the acidity of a substance; the lower the pK_a, the stronger the acid. In general, the electron-withdrawing groups such as nitro groups reduce the electron density around the hydroxyl group in phenols, making them more acidic. However, the position of the nitro group matters. The electron-withdrawing nitro group at the ortho position has a lesser effect on the hydroxy group's acidity than the meta and para positions. Therefore, 2-nitrophenol (ortho position) has the highest pK_a value among the options.

Thus, the correct answer is option (2) 2-Nitrophenol.

Quick Tip

In aromatic compounds, the position of electron-withdrawing groups (such as NO_2) plays a significant role in determining the acidity (pK_a value) of the compound.

158. Vinegar and butter are the sources for which of the following carboxylic acids respectively?

(1) CH_3COOH, CH_3CH_2COOH

(2) CH_3COOH, CH_2CH_2COOH

(3) $CH_3CH_2COOH, (CH_3)_2COOH$

(4) $(CH_3)_2COOH, (CH_3)_2CH_2COOH$

Correct Answer: (1) CH_3COOH, CH_3CH_2COOH

Solution: - Vinegar is primarily acetic acid, CH_3COOH. - Butter contains butyric acid, CH_3CH_2COOH.

Thus, the correct answer is option (1).

> **Quick Tip**
>
> Remember that vinegar contains acetic acid, and butter contains butyric acid, which have distinct structures and functional groups.

---

159. Identify X, Y and Z in the following reactions sequence:

$$C_6H_5CH_2CH_2OH \xrightarrow{X} C_6H_5CH_2COOH \xrightarrow{Y} Z$$

(a) $X = PCC \quad Y = NaOH, CaO \quad Z = C_6H_5CH_3$

(b) $X = PCC \quad Y = LAH \quad Z = C_6H_5CH_2CH_3$

(c) $X = \text{Jones reagent} \quad Y = NaOH, CaO \quad Z = C_6H_5CH_2CH_3$

(d) $X = \text{Jones reagent} \quad Y = NaOH, CaO, \Delta \quad Z = C_6H_5CH_3$

Correct Answer: (4) $X = \text{Jones reagent} \quad Y = NaOH, CaO, \Delta \quad Z = C_6H_5CH_3$

Solution: In the given sequence:

- X = Jones reagent, which is a strong oxidizer used to oxidize alcohols to aldehydes or carboxylic acids. Thus, $C_6H_5CH_2CH_2OH$ is oxidized to $C_6H_5CH_2COOH$ (benzoic acid). - Y = NaOH, CaO indicates a decarboxylation reaction, where the carboxyl group is removed, leading to the formation of a hydrocarbon, $C_6H_5CH_3$ (toluene).

Thus, the correct sequence is: X = Jones reagent   Y = NaOH, CaO, Δ Z = $C_6H_5CH_3$

> **Quick Tip**
>
> For alcohols, Jones reagent (CrO_3) is commonly used to oxidize primary alcohols to carboxylic acids. The decarboxylation step requires NaOH and CaO to remove the carboxyl group.

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160. Which of the following produces nitrogen gas after the reaction with nitrous acid?

(1) $(CH_3)_3N$

(2) $C_2H_5NC_2H_5$

(3) $(C_2H_5)_3N$

(4) $C_2H_5NH_2$

Correct Answer: (4) $C_2H_5NH_2$

Solution: The reaction of nitrous acid with primary amines typically produces nitrogen gas. Among the given compounds, ethylamine $C_2H_5NH_2$ will react with nitrous acid to form nitrogen gas as one of the products.

Thus, the correct answer is option (4).

> **Quick Tip**
>
> When an amine reacts with nitrous acid, it generally leads to the evolution of nitrogen gas, especially for primary amines like $C_2H_5NH_2$.
