GATE 2025 Aerospace Engineering Question Paper with Solutions

Time Allowed :180 Minutes	**Maximum Marks :**100	**Total questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Total Marks:** The GATE Aerospace Engineering paper is worth 100 marks.
2. **Question Types:** The paper consists of 65 questions, divided into:
   - General Aptitude (GA): 15 marks
   - Engineering Mathematics and Aerospace Engineering: 85 marks
3. **Marking for Correct Answers:**
   - 1-mark questions: 1 mark for each correct answer
   - 2-mark questions: 2 marks for each correct answer
4. **Negative Marking for Incorrect Answers:**
   - 1-mark MCQs: 1/3 mark deduction for a wrong answer
   - 2-mark MCQs: 2/3 marks deduction for a wrong answer
5. **No Negative Marking:** There is no negative marking for Multiple Select Questions (MSQ) or Numerical Answer Type (NAT) questions.
6. **No Partial Marking:** There is no partial marking in MSQ.

General Aptitude

1. Courage : Bravery :: Yearning :

Select the most appropriate option to complete the analogy.

(A) Longing

(B) Yelling

(C) Yawning

(D) Glaring

Correct Answer: (A) Longing

Solution: Step 1: Identifying the relationship between "Courage" and "Bravery."

The words "Courage" and "Bravery" are synonyms. They both describe the quality of being willing to face danger, pain, or difficulty. Since these words are closely related in meaning, we need to find a similar relationship between the second pair of words, "Yearning" and one of the given options.

Step 2: Analyzing the word "Yearning."

"Yearning" refers to a strong desire or longing for something. Now, let's analyze the options:

Option (A) Longing: This is a direct synonym of "Yearning." Both words express a deep, intense desire for something.

Option (B) Yelling: This is unrelated to "Yearning." "Yelling" refers to shouting loudly, which has no connection to the emotional desire conveyed by "Yearning."

Option (C) Yawning: This is also unrelated to "Yearning." "Yawning" refers to the action of opening the mouth wide, usually due to tiredness or boredom, which does not fit the emotional context of "Yearning."

Option (D) Glaring: This means staring angrily, which has no connection to the concept of desire or longing.

Thus, Option (A) Longing is the most appropriate because it maintains the same synonym relationship that exists between "Courage" and "Bravery."

Quick Tip

When solving analogies, consider the relationship between the first pair of words, and look for a similar relationship in the second pair. Synonyms or words with similar meanings are often the correct choice.

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2. We _______ tennis in the lawn when it suddenly started to rain.

Select the most appropriate option to complete the above sentence.

(A) have been playing

(B) had been playing

(C) would have been playing

(D) could be playing

Correct Answer: (B) had been playing

Solution: Step 1: Understanding the Context of the Sentence.

The sentence describes an action that was happening in the past and was interrupted by another event. The phrase "when it suddenly started to rain" suggests that the action of playing tennis was ongoing at the time the rain started. This points to the past perfect continuous tense, which is used to describe an action that was happening continuously before another past action interrupted it.

Step 2: Analyzing the Options:

Option (A) "have been playing" is the present perfect continuous tense, which indicates an action that started in the past and continues into the present. Since the sentence is referring to a past event, this is not correct.

Option (B) "had been playing" is the past perfect continuous tense, which correctly describes an action that was happening continuously in the past before another past event (the rain) interrupted it. This is the correct choice.

Option (C) "would have been playing" is a conditional perfect continuous tense, typically used to describe hypothetical situations or actions that would have happened under different conditions. This is not appropriate for the sentence.

Option (D) "could be playing" suggests possibility in the present, which is incorrect because the sentence is referring to a past event.

Therefore, the most appropriate option is (B) had been playing.

Quick Tip

Use the past perfect continuous tense (”had been + verb-ing”) when describing an action that was happening continuously in the past before another action interrupted it.

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3. A 4 × 4 digital image has pixel intensities (U) as shown in the figure. The number of pixels with $U \leq 4$ is:

0	1	0	2
4	7	3	3
5	5	4	4
6	7	3	2

(1) 3

(2) 8

(3) 11

(4) 9

Correct Answer: (C) 11

Solution: We are asked to find how many pixels have intensities less than or equal to 4.

Let’s go through the matrix and count the number of pixels that satisfy $U \leq 4$.

The given matrix of pixel intensities is:

0	1	0	2
4	7	3	3
5	5	4	4
6	7	3	2

- Row 1: 0, 1, 0, 2 (all are ≤ 4) - Count: 4
- Row 2: 4, 7, 3, 3 (4, 3, 3 are ≤ 4) - Count: 3
- Row 3: 5, 5, 4, 4 (4, 4 are ≤ 4) - Count: 2
- Row 4: 6, 7, 3, 2 (3, 2 are ≤ 4) - Count: 2

Total count: $4 + 3 + 2 + 2 = 11$

Quick Tip

In digital images, when analyzing pixel intensities, always look for the specific threshold conditions (like $U \leq 4$) and count the pixels that satisfy the condition across the entire matrix.

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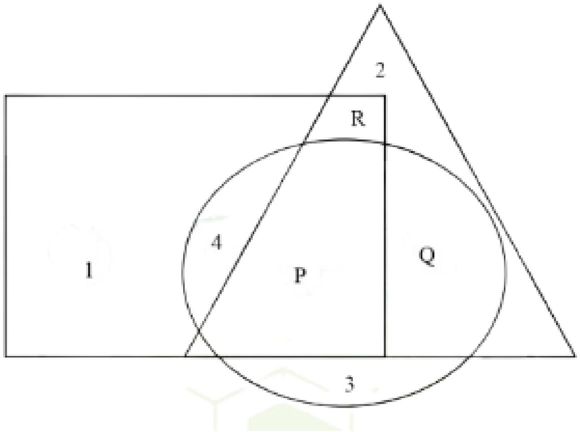
4. In the given figure, the numbers associated with the rectangle, triangle, and ellipse are 1, 2, and 3, respectively. Which one among the given options is the most appropriate combination of P, Q, and R?



(A) $P = 6; Q = 5; R = 3$

(B) $P = 5; Q = 6; R = 3$

(C) $P = 3; Q = 6; R = 6$

(D) $P = 5; Q = 3; R = 6$

Correct Answer: (A) $P = 6; Q = 5; R = 3$

Solution:

In this problem, we are given a figure involving three geometric shapes: a rectangle, a triangle, and an ellipse. We need to determine the most appropriate values for P, Q, and R based on the dimensions provided in the figure.

The number P is associated with the length of the rectangle's side. Since the side of the rectangle is labeled as 6, we can conclude that $P = 6$.

The number Q is associated with the height of the triangle. The height of the triangle is labeled as 5, so $Q = 5$.

The number R is associated with the length of the major axis of the ellipse. The length of the major axis is labeled as 3, so $R = 3$.

Thus, the correct values are $P = 6$, $Q = 5$, and $R = 3$.

Quick Tip

When solving geometry problems involving multiple shapes, carefully observe the labels and dimensions associated with each shape. Use these details to assign values to the variables based on the geometric properties.

5. A rectangle has a length L and a width W, where $L > W$. If the width, W, is increased by 10%, which one of the following statements is correct for all values of L and W?

Select the most appropriate option to complete the above sentence.

(A) Perimeter increases by 10%.

(B) Length of the diagonals increases by 10%.

(C) Area increases by 10%.

(D) The rectangle becomes a square.

Correct Answer: (C) Area increases by 10

Solution: Step 1: Understanding the effects of increasing the width of a rectangle by 10%.

The dimensions of the rectangle are L (length) and W (width), with $L > W$. When the width W is increased by 10%, the new width becomes $W' = 1.1W$, while the length L remains the same.

Step 2: Analyzing the impact on each option.

Option (A) Perimeter increases by 10The perimeter of a rectangle is given by the formula:

$$P = 2(L + W)$$

When the width increases by 10%, the new perimeter becomes:

$$P' = 2(L + 1.1W)$$

This is not exactly a 10% increase. The increase in perimeter is not proportional to the increase in width. Therefore, this option is incorrect.

Option (B) Length of the diagonals increases by 10The diagonal d of a rectangle is given by the Pythagorean theorem:

$$d = \sqrt{L^2 + W^2}$$

When the width increases by 10

$$d' = \sqrt{L^2 + (1.1W)^2}$$

This increase is not guaranteed to be exactly 10%. The length of the diagonal increases, but it is not necessarily a 10% increase. Therefore, this option is incorrect.

Option (C) Area increases by 10The area A of a rectangle is given by:

$$A = L \times W$$

After increasing the width by 10

$$A' = L \times 1.1W = 1.1 \times L \times W$$

This shows that the area increases by 10

Option (D) The rectangle becomes a square: A rectangle becomes a square only if the length and width are equal. Since only the width is increased by 10%, the rectangle does not become a square. Therefore, this option is incorrect.

Therefore, the correct answer is Option (C) Area increases by 10

Quick Tip

When a single dimension of a rectangle (such as width) is increased by a certain percentage, the area of the rectangle will increase by the same percentage, as long as the other dimension remains unchanged.

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6. Column-I has statements made by Shanthala; and, Column-II has responses given by Kanishk.

Column-I		Column-II	
P.	This house is in a mess.	1.	Alright, I won't bring it up during our conversations.
Q.	I am not happy with the marks given to me.	2.	Well, you can easily look it up.
R.	Politics is a subject I avoid talking about.	3.	No problem, let me clear it up for you
S.	I don't know what this word means.	4.	Don't worry, I will take it up with your teacher.

Identify the option that has the correct match between Column-I and Column-II.

(A) P – 2; Q – 3; R – 1; S – 4

(B) P – 3; Q – 4; R – 1; S – 2

(C) P – 4; Q – 1; R – 2; S – 3

(D) P – 1; Q – 2; R – 4; S – 3

Correct Answer: (B) P – 3; Q – 4; R – 1; S – 2

Solution: Step 1: Understanding the context of each statement.

We need to match the statements made by Shanthala (Column-I) with the appropriate responses given by Kanishk (Column-II).

P. ”This house is in a mess.” The best response would be: ”No problem, let me clear it up for you.” Kanishk is offering to help with the situation, which matches this statement.

Q. ”I am not happy with the marks given to me.” The appropriate response would be: ”Don’t worry, I will take it up with your teacher.” Kanishk is reassuring Shanthala that their concern about marks will be addressed.

R. ”Politics is a subject I avoid talking about.” The correct response here would be: ”Alright, I won’t bring it up during our conversations.” This indicates that Kanishk will avoid discussing politics.

S. ”I don’t know what this word means.” The most appropriate response is: ”Well, you can easily look it up.” This suggests that Kanishk is offering a straightforward solution to Shanthala’s problem.

Step 2: Identifying the correct match.

From the analysis above, we match the following:

P – 3: ”This house is in a mess.” → ”No problem, let me clear it up for you.”

Q – 4: ”I am not happy with the marks given to me.” → ”Don’t worry, I will take it up with your teacher.”

R – 1: ”Politics is a subject I avoid talking about.” → ”Alright, I won’t bring it up during our conversations.”

S – 2: ”I don’t know what this word means.” → ”Well, you can easily look it up.”

Therefore, the correct answer is Option (B) P – 3; Q – 4; R – 1; S – 2.

> **Quick Tip**
>
> When matching statements with responses, pay attention to the context and tone of the statement and the most appropriate response that follows. In such cases, reassurance, solutions, and acknowledgment of preferences are key.

7. Weight of a person can be expressed as a function of their age. The function usually varies from person to person. Suppose this function is identical for two brothers, and it monotonically increases till the age of 50 years and then it monotonically decreases. Let a_1 and a_2 (in years) denote the ages of the brothers and $a_1 < a_2$.

Which one of the following statements is correct about their age on the day when they attain the same weight?

(A) $a_1 < a_2 < 50$

(B) $a_1 < 50 < a_2$

(C) $50 < a_1 < a_2$

(D) Either $a_1 = 50$ or $a_2 = 50$

Correct Answer: (B) $a_1 < 50 < a_2$

Solution: The weight function is increasing until the age of 50 and then decreases. Given that $a_1 < a_2$, this means that the younger brother's age a_1 is less than 50 and the older brother's age a_2 is greater than 50. When both brothers reach the same weight, their ages must satisfy the condition where the younger brother is below 50 and the older brother is above 50. Hence, $a_1 < 50 < a_2$.

Quick Tip

In problems involving monotonic functions, focus on the turning point (in this case, age 50) to determine the behavior of the function for different values of the variables.

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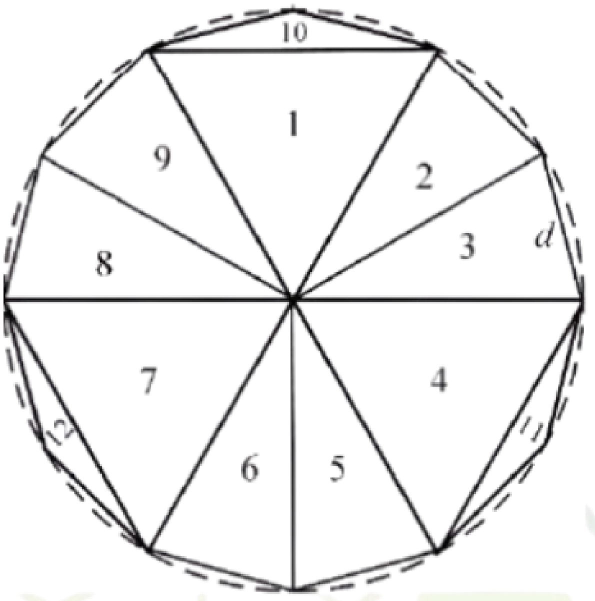
8. A regular dodecagon (12-sided regular polygon) is inscribed in a circle of radius r cm as shown in the figure. The side of the dodecagon is d cm. All the triangles (numbered 1 to 12 in the figure) are used to form squares of side r cm, and each numbered triangle is used only once to form a square.

The number of squares that can be formed and the number of triangles required to form each square, respectively, are:



(A) 3; 4

(B) 4; 3

(C) 3; 3

(D) 3; 2

Correct Answer: (A) 3; 4

Solution: We are given a regular dodecagon inscribed in a circle, and we need to form squares using the triangles formed by connecting the center of the circle to the vertices of the dodecagon. There are 12 triangles in total, each corresponding to a side of the dodecagon. The number of squares that can be formed is 3, and each square requires 4 triangles. Hence, the correct number of squares and the number of triangles required to form each square are 3 and 4, respectively.

Quick Tip

In geometry problems involving regular polygons and inscribed shapes, carefully examine the symmetry and properties of the polygon to understand how to derive relationships for constructing new shapes.

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9. If a real variable x satisfies $3^{x^2} = 27 \times 9^x$, then the value of $\frac{2^{x^2}}{(2^x)^2}$ is:

(A) 2^{-1}

(B) 2^0

(C) 2^3

(D) 2^{15}

Correct Answer: (C) 2^3

Solution:

Step 1: Solve the given equation $3^{x^2} = 27 \times 9^x$.

We start with the equation:

$$3^{x^2} = 27 \times 9^x.$$

We can rewrite 27 and 9 as powers of 3:

$$27 = 3^3 \quad \text{and} \quad 9 = 3^2.$$

Thus, the equation becomes:

$$3^{x^2} = 3^3 \times (3^2)^x.$$

Now simplify the right-hand side:

$$3^{x^2} = 3^3 \times 3^{2x}.$$

Using the property of exponents $a^m \times a^n = a^{m+n}$, we combine the powers of 3:

$$3^{x^2} = 3^{3+2x}.$$

Since the bases are the same, we can equate the exponents:

$$x^2 = 3 + 2x.$$

Rearranging the equation:

$$x^2 - 2x - 3 = 0.$$

Factoring the quadratic equation:

$$(x - 3)(x + 1) = 0.$$

Thus, $x = 3$ or $x = -1$.

Step 2: Evaluate $\frac{2^{x^2}}{(2^x)^2}$.

We now substitute $x = 3$ and $x = -1$ into the expression $\frac{2^{x^2}}{(2^x)^2}$:

When $x = 3$:

$$\frac{2^{3^2}}{(2^3)^2} = \frac{2^9}{2^6} = 2^{9-6} = 2^3.$$

When $x = -1$:

$$\frac{2^{(-1)^2}}{(2^{-1})^2} = \frac{2^1}{2^{-2}} = 2^{1-(-2)} = 2^3.$$

Thus, the value is 2^3, which corresponds to Option (C).

Quick Tip

When solving equations with exponents, simplify both sides of the equation, equate the exponents, and solve the resulting algebraic equation. Afterward, substitute the values into the given expression.

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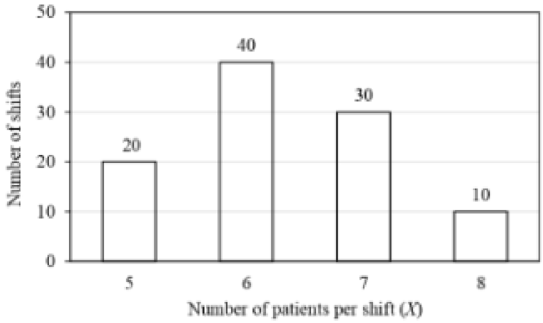
10. The number of patients per shift (X) consulting Dr. Gita in her past 100 shifts is shown in the figure. If the amount she earns is ₹$1000(X - 0.2)$, what is the average amount (in ₹) she has earned per shift in the past 100 shifts?



(A) 6,100

(B) 6,300

(C) 6,000

(D) 6,500

Correct Answer: (A) 6,100

Solution:

Step 1: Understanding the problem.

The number of shifts corresponding to different numbers of patients per shift is given in the bar graph. The amount Dr. Gita earns is $1000(X - 0.2)$, where X is the number of patients per shift.

The data from the graph is as follows:

For $X = 5$, the number of shifts is 20.

For $X = 6$, the number of shifts is 40.

For $X = 7$, the number of shifts is 30.

For $X = 8$, the number of shifts is 10.

Step 2: Calculating the total earnings.

For $X = 5$:

$$\text{Earnings} = 1000 \times (5 - 0.2) \times 20 = 1000 \times 4.8 \times 20 = 96,000.$$

For $X = 6$:

$$\text{Earnings} = 1000 \times (6 - 0.2) \times 40 = 1000 \times 5.8 \times 40 = 232,000.$$

For $X = 7$:

$$\text{Earnings} = 1000 \times (7 - 0.2) \times 30 = 1000 \times 6.8 \times 30 = 204,000.$$

For $X = 8$:

$$\text{Earnings} = 1000 \times (8 - 0.2) \times 10 = 1000 \times 7.8 \times 10 = 78,000.$$

Step 3: Calculating the total earnings and average earnings.

Total earnings for all 100 shifts:

$$\text{Total Earnings} = 96,000 + 232,000 + 204,000 + 78,000 = 610,000.$$

The average earnings per shift:

$$\text{Average Earnings} = \frac{610,000}{100} = 6,100.$$

Thus, the average earnings per shift are ₹6,100, which corresponds to Option (A).

Quick Tip

When calculating averages involving frequency distributions, first calculate the total earnings, then divide by the total number of shifts to find the average.

11. For any real symmetric matrix A, the transpose of A is ____ .

(A) inverse of A

(B) null matrix

(C) $-A$

(D) A

Correct Answer: (D) A

Solution: To solve this, we need to recall the definition of a symmetric matrix. A matrix A is said to be symmetric if it is equal to its own transpose, i.e.,

$$A = A^T$$

Step 1: Definition of Transpose

The transpose of a matrix A, denoted by A^T, is obtained by swapping its rows and columns. So, for any matrix $A = [a_{ij}]$, the transpose matrix A^T will have elements $A^T = [a_{ji}]$.

Step 2: Symmetry Condition

For a symmetric matrix, by definition, the matrix is equal to its transpose. This implies:

$$A = A^T$$

In other words, the elements of the matrix satisfy the condition that $a_{ij} = a_{ji}$ for all i, j.

Step 3: Conclusion

Since a real symmetric matrix satisfies $A = A^T$, the transpose of A is exactly A. Therefore, the correct answer is A.

$$A^T = A$$

Quick Tip

For symmetric matrices, always remember that the matrix is equal to its transpose: $A = A^T$. This property is fundamental in linear algebra and is used in various matrix-related computations.

12. Solve the system of equations:

$$2x + 3y + z = 0$$

$$x + y = 0$$

$$y + z = 0$$

The given system of equations has:

(A) a unique solution

(B) infinitely many solutions

(C) no solution

(D) a finite number of solutions

Correct Answer: (B) infinitely many solutions

Solution: We are given the following system of equations:

$$2x + 3y + z = 0 \quad (1)$$

$$x + y = 0 \quad (2)$$

$$y + z = 0 \quad (3)$$

From equation (2), $x = -y$. Substitute this in equation (1):

$$2(-y) + 3y + z = 0 \quad \Rightarrow \quad -2y + 3y + z = 0 \quad \Rightarrow \quad y + z = 0$$

Equation (3) is $y + z = 0$, which is identical to the result we got above. Hence, the system is consistent and has infinitely many solutions.

Quick Tip

For systems of linear equations, check for consistency. If the equations represent planes that intersect along a line or a point, there will be infinitely many solutions.

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13. The eigenvalues of the matrix

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

are ____ .

(A) 0, 2

(B) 2, 3

(C) 1, 3

(D) 1, 2

Correct Answer: (C) 1, 3

Solution: To find the eigenvalues of a matrix, we solve the characteristic equation:

$$\det(A - \lambda I) = 0$$

where A is the matrix, λ is the eigenvalue, and I is the identity matrix.

Step 1: Construct the matrix $A - \lambda I$

Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, we subtract λI (where I is the 2x2 identity matrix) from A:

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix}$$

Step 2: Find the determinant of $A - \lambda I$

Now, compute the determinant of the resulting matrix:

$$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix}$$

The determinant of a 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by $ad - bc$. So, for our matrix:

$$\det(A - \lambda I) = (1-\lambda)(3-\lambda) - (0)(2) = (1-\lambda)(3-\lambda)$$

Step 3: Solve the characteristic equation

Now, we solve the characteristic equation:

$$(1-\lambda)(3-\lambda) = 0$$

This gives two solutions:

$$1 - \lambda = 0 \quad \Rightarrow \quad \lambda = 1$$

$$3 - \lambda = 0 \quad \Rightarrow \quad \lambda = 3$$

Conclusion: Thus, the eigenvalues of the matrix are $\lambda = 1$ and $\lambda = 3$.

> **Quick Tip**
>
> To find the eigenvalues of a matrix, solve the characteristic equation $\det(A - \lambda I) = 0$. The roots of the equation are the eigenvalues.

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14. The partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2} = 0$$

is ______.

(A) elliptic

(B) hyperbolic

(C) parabolic

(D) of mixed type

Correct Answer: (B) hyperbolic

Solution: The given partial differential equation is of the form:

$$A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} = 0$$

For classification, we calculate the discriminant D:

$$D = B^2 - AC$$

In this case, $A = 1$, $B = 2$, and $C = 2$, so:

$$D = 2^2 - (1)(2) = 4 - 2 = 2$$

Since $D > 0$, the equation is hyperbolic.

> **Quick Tip**
>
> To classify partial differential equations, calculate the discriminant $D = B^2 - AC$. If $D > 0$, the equation is hyperbolic.

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15. Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} A + x, & \text{if } x < 2 \\ 1 + x^2, & \text{if } x \geq 2 \end{cases}$$

If the function $f(x)$ is continuous at $x = 2$, the value of A is ____.

(A) 2

(B) 2.5

(C) 3

(D) 3.5

Correct Answer: (C) 3

Solution: For continuity at $x = 2$, the left-hand limit (as $x \to 2^-$) must equal the right-hand limit (as $x \to 2^+$).

1. Left-hand limit at $x = 2$:

For $x < 2$, $f(x) = A + x$. Thus,

$$\lim_{x \to 2^-} f(x) = A + 2$$

2. Right-hand limit at $x = 2$:

For $x \geq 2$, $f(x) = 1 + x^2$. Thus,

$$\lim_{x \to 2^+} f(x) = 1 + 2^2 = 1 + 4 = 5$$

For continuity, these two limits must be equal:

$$A + 2 = 5 \quad \Rightarrow \quad A = 3$$

Quick Tip

For continuity at a point, the left-hand limit and the right-hand limit must be equal at that point.

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16. If $i = \sqrt{-1}$,

$$\frac{(i+1)^3}{i-1} = \_\_\_\_\_\_\_\_.$$

(A) $(i + 1)$

(B) -2

(C) $(i+1)$

(D) $(i-1)$

Correct Answer: (A) $(i+1)$

Solution: We are given the expression:

$$\frac{(i+1)^3}{i-1}$$

Step 1: Expand $(i+1)^3$

First, expand $(i+1)^3$:

$$(i+1)^3 = i^3 + 3i^2 + 3i + 1$$

Since $i^2 = -1$ and $i^3 = -i$, we get:

$$(i+1)^3 = -i - 3 + 3i + 1 = 2i - 2$$

Step 2: Simplify the expression

Now substitute this into the original expression:

$$\frac{(i+1)^3}{i-1} = \frac{2i-2}{i-1}$$

We will now multiply both the numerator and denominator by the conjugate of $i-1$, which is $i+1$:

$$\frac{2i-2}{i-1} \times \frac{i+1}{i+1} = \frac{(2i-2)(i+1)}{(i-1)(i+1)}$$

The denominator becomes:

$$(i-1)(i+1) = i^2 - 1^2 = -1 - 1 = -2$$

Now expand the numerator:

$$(2i-2)(i+1) = 2i^2 + 2i - 2i - 2 = -2 + 2i - 2i - 2 = -4$$

Thus the expression becomes:

$$\frac{-4}{-2} = 2$$

So, we obtain the simplified result of 2.

Quick Tip

When simplifying complex expressions, especially involving $i = \sqrt{-1}$, remember to use the conjugate of the denominator to simplify the fraction. This technique is essential for resolving expressions with complex numbers.

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17. For a two-dimensional incompressible flow over a flat plate, the laminar boundary layer thickness at a distance x from the leading edge is δ. If Re_x is the Reynolds number defined based on length scale x,

$$\frac{\delta}{x} \propto \text{\_\_\_\_\_\_}.$$

(A) $Re_x^{-1/2}$

(B) Re_x^{-1}

(C) $Re_x^{-3/2}$

(D) Re_x^{-2}

Correct Answer: (A) $Re_x^{-1/2}$

Solution: For a laminar boundary layer in a two-dimensional incompressible flow, the boundary layer thickness δ is related to the Reynolds number Re_x by the following empirical relationship:

$$\delta \propto x \cdot Re_x^{-1/2}$$

Thus, the ratio $\frac{\delta}{x}$ is proportional to $Re_x^{-1/2}$:

$$\frac{\delta}{x} \propto Re_x^{-1/2}$$

Step 1: Reynolds number definition

The Reynolds number Re_x at a distance x from the leading edge is defined as:

$$Re_x = \frac{\rho u x}{\mu}$$

where ρ is the fluid density, u is the flow velocity, and μ is the dynamic viscosity.

Step 2: Relation between boundary layer thickness and Reynolds number

From the theory of boundary layers in laminar flow, we know that the thickness of the boundary layer is inversely proportional to the square root of the Reynolds number, as shown above.

Conclusion:

Therefore, the ratio $\frac{\delta}{x}$ is proportional to $Re_x^{-1/2}$.

Quick Tip

For laminar boundary layers, the boundary layer thickness is proportional to $x \cdot Re_x^{-1/2}$, where Re_x is the Reynolds number based on the distance from the leading edge.

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18. For a NACA 4415 airfoil, the location of maximum camber, as a fraction of the chord length from the leading edge, is ______.

(A) 0.44

(B) 0.40

(C) 0.15

(D) 0.04

Correct Answer: (B) 0.40

Solution: For NACA airfoils, the number 4415 represents the following components:

The first digit (4) represents the maximum camber as a percentage of the chord length (in this case, 4%).

The second digit (4) represents the location of maximum camber as a fraction of the chord length from the leading edge (in this case, 0.40 or 40%).

The last two digits (15) represent the maximum thickness as a percentage of the chord length (15%).

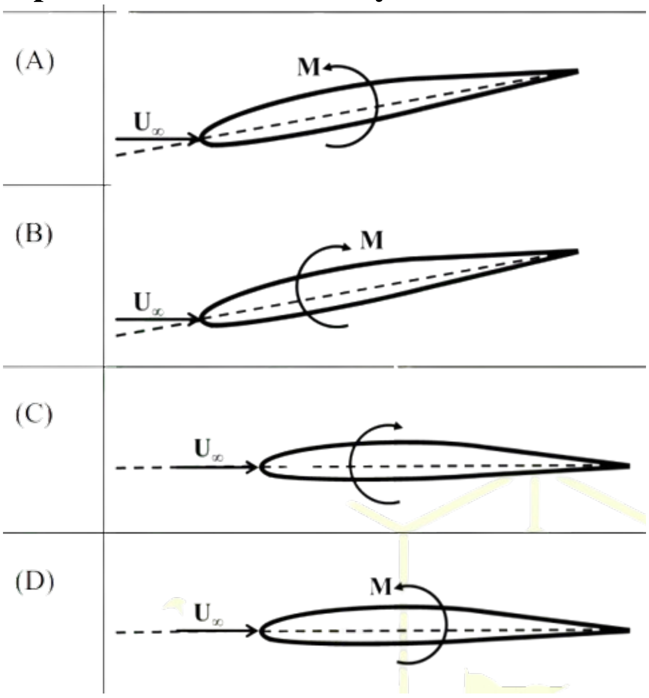
Thus, the location of the maximum camber is 0.40 of the chord length from the leading edge.

Quick Tip

For NACA airfoils, the first digit represents the camber as a percentage, the second digit represents the location of maximum camber as a fraction of the chord length, and the last two digits represent the maximum thickness as a percentage of the chord length.

---

19. A positively cambered airfoil is placed in a uniform flow (velocity, U_∞) at its zero-lift angle of attack. M is the corresponding pitching moment. Which one of the following representations accurately describes this scenario?



Correct Answer: (A)

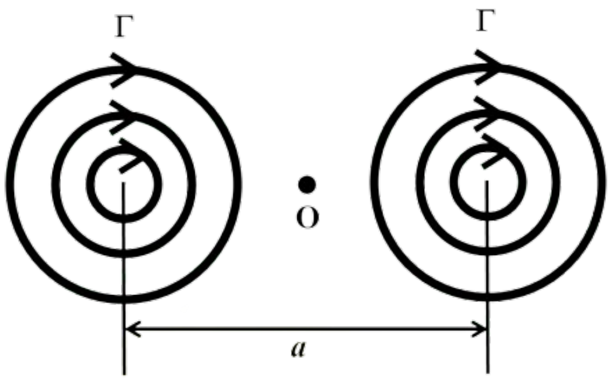
Solution: In this scenario, the airfoil is at its zero-lift angle of attack, meaning that it produces no net lift force but still generates a pitching moment. This moment is due to the camber of the airfoil, which leads to an imbalance in the pressure distribution over the surface. The correct representation shows the flow direction and the pitching moment generated by the airfoil in this condition.

Quick Tip

Even at zero-lift angle of attack, a positively cambered airfoil generates a pitching moment due to the asymmetrical pressure distribution across its surface.

---

20. Consider a pair of point vortices with clockwise circulation Γ each. The distance between their centers is a, as shown in the figure. Assume two-dimensional, incompressible, inviscid flow. Which one of the following options is correct?



(A) The vortices translate downwards together with a velocity $\frac{\Gamma}{2\pi a}$.

(B) The vortices translate upwards together with a velocity $\frac{\Gamma}{2\pi a}$.

(C) The vortices rotate clockwise around each other about their centroid O.

(D) The vortices rotate counter-clockwise around each other about their centroid O.

Correct Answer: (C) The vortices rotate clockwise around each other about their centroid O.

Solution: When two vortices of equal and opposite circulation are placed in a two-dimensional, incompressible, inviscid flow, they will rotate around each other with respect to their centroid. In this scenario, the vortices have clockwise circulation, and they rotate in a clockwise direction around their common centroid. Therefore, option (C) is correct.

Quick Tip

In a system of two point vortices, they will rotate around their common centroid, with the direction of rotation depending on the sign of their circulation. In this case, with clockwise circulation, the vortices rotate clockwise around each other.

---

21. In a fluid flow, Mach number is an estimate of _____.

(A) $\sqrt{\frac{\text{inertia force}}{\text{viscous force}}}$

(B) $\sqrt{\frac{\text{inertia force}}{\text{elastic force}}}$

(C) $\sqrt{\frac{\text{elastic force}}{\text{viscous force}}}$

(D) $\sqrt{\frac{\text{viscous force}}{\text{inertia force}}}$

Correct Answer: (B) $\sqrt{\frac{\text{inertia force}}{\text{elastic force}}}$

Solution: The Mach number (M) is a dimensionless number that represents the ratio of the velocity of the flow to the speed of sound in the medium. It is used to estimate the relationship between the inertia forces and elastic forces in a fluid flow. Hence, the correct expression for Mach number is:

$$M = \sqrt{\frac{\text{inertia force}}{\text{elastic force}}}$$

This relationship helps determine the effects of compressibility and the transition between subsonic and supersonic flows.

Quick Tip

The Mach number helps in determining whether the flow is compressible and is a key indicator for shock waves and flow speeds.

---

22. A general aviation airplane is in steady and level flight. The airplane is prone to adverse yaw. Which one of the following options best describes the deflections of aileron and rudder to achieve a coordinated right turn?

(A) Left aileron: down; Right aileron: up; Rudder: left

(B) Left aileron: down; Right aileron: up; Rudder: right

(C) Left aileron: up; Right aileron: down; Rudder: left

(D) Left aileron: up; Right aileron: down; Rudder: right

Correct Answer: (B) Left aileron: down; Right aileron: up; Rudder: right

Solution: To achieve a coordinated right turn in a general aviation airplane:

The left aileron must be deflected down and the right aileron up to roll the airplane into the right turn.

To counteract the adverse yaw (which causes the nose to yaw left), the rudder must be deflected to the right to maintain coordination.

Quick Tip

In coordinated turns, the ailerons control roll, while the rudder is used to counteract adverse yaw and ensure the airplane turns smoothly without yawing to the opposite side.

---

23. For a general aviation airplane, which one of the following has a destabilizing effect on its static roll stability?

(A) Wing with a positive dihedral angle

(B) Fuselage with a high wing

(C) Fuselage with a low wing

(D) Swept back wing

Correct Answer: (C) Fuselage with a low wing

Solution: A fuselage with a low wing configuration has a destabilizing effect on an aircraft's static roll stability. In this configuration, the aircraft's center of gravity and the center of lift of the wings are closer together, which reduces the restoring moment when the aircraft rolls. This leads to a higher chance of instability in roll.

A positive dihedral angle, a high wing configuration, and a swept-back wing generally contribute to increased roll stability.

Quick Tip

For better roll stability, an aircraft is generally designed with a high wing or positive dihedral, which enhances the restoring moment during a roll. A low-wing configuration can result in reduced static roll stability.

---

24. A general aviation airplane is flying at an altitude of 5000 m. The indicated airspeed is 250 km/h. Assume that there are no instrument errors and position errors. Neglecting compressibility effects, which one of the following options is FALSE?

(A) The true airspeed is greater than 250 km/h.

(B) The calibrated airspeed is 250 km/h.

(C) The true airspeed is 250 km/h.

(D) The equivalent airspeed is 250 km/h

Correct Answer: (C) The true airspeed is 250 km/h.

Solution: At an altitude of 5000 m, the indicated airspeed (IAS) is not equal to the true airspeed (TAS). IAS is the speed shown by the airspeed indicator, and it does not account for altitude or air density.

The true airspeed is higher than the indicated airspeed because air density decreases with altitude.

Calibrated airspeed (CAS) is corrected for instrument and position errors and is assumed to be the same as IAS in this case.

Equivalent airspeed (EAS) is related to CAS and air density, and at altitude, it can differ from IAS as well.

Hence, the false statement is (C): The true airspeed is 250 km/h, because at 5000 m altitude, the true airspeed is greater than the indicated airspeed.

Quick Tip

Indicated airspeed increases with altitude due to the decreasing air density. True airspeed must be calculated from IAS to account for changes in atmospheric conditions at higher altitudes.

---

25. To achieve longitudinal static stability of a general aviation airplane, which one of the following conditions should be satisfied?

(A) The center of gravity of the airplane should be aft of the neutral point.

(B) The center of gravity of the airplane should be forward of the neutral point.

(C) The stability coefficient $\frac{\partial C_m}{\partial \alpha}$ (where C_m is the airplane pitching moment coefficient and α is the angle of attack) is positive.

(D) The static margin is negative.

Correct Answer: (B) The center of gravity of the airplane should be forward of the neutral point.

Solution: For longitudinal static stability, the center of gravity (CG) of the airplane must be located forward of the neutral point. The neutral point is the point where the pitching moment coefficient (C_m) becomes zero. When the CG is forward of the neutral point, the airplane experiences a restoring moment when displaced from equilibrium, thereby providing stability.
If the CG is aft of the neutral point, the airplane will be statically unstable.
A positive stability coefficient $\frac{\partial C_m}{\partial \alpha}$ is a characteristic of stable aircraft, but it alone does not ensure static stability.
A negative static margin indicates instability.

Quick Tip

In general aviation airplanes, ensuring that the CG is forward of the neutral point is crucial for achieving positive longitudinal static stability.

---

26. For a homogeneous, isotropic material, the relation between the shear modulus (G), Young's modulus (E), and Poisson's ratio (ν) is ____?

(A) $G = 2E(1 + \nu)$

(B) $2G = E(1 + \nu)$

(C) $E = 2G(1 + \nu)$

(D) $2E = G(1 + \nu)$

Correct Answer: (C) $E = 2G(1 + \nu)$

Solution: For a homogeneous and isotropic material, the relationship between shear modulus (G), Young's modulus (E), and Poisson's ratio (ν) is given by the equation:

$$E = 2G(1 + \nu)$$

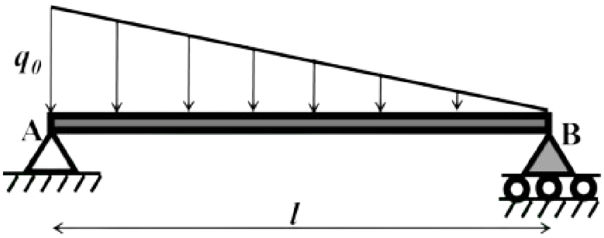
This equation shows that Young's modulus is related to the shear modulus and Poisson's ratio, which describes the material's response to stress and strain.

Quick Tip

In material mechanics, the shear modulus, Young's modulus, and Poisson's ratio are fundamental properties that help describe the material's deformation under various types of stress.

---

27. A simply supported horizontal beam is subjected to a distributed transverse load varying linearly from q_0 at A to zero at B, as shown in the figure. Which one of the following options is correct?



(A) The magnitude of the vertical reaction force at A is larger than that at B.

(B) The magnitude of the vertical reaction force at B is larger than that at A.

(C) The magnitudes of the vertical reaction forces at A and B are equal.

(D) The reactions at points A and B are indeterminate.

Correct Answer: (A) The magnitude of the vertical reaction force at A is larger than that at B.

Solution: The problem describes a beam subjected to a triangular distributed load. In this case, the resultant of the triangular load acts at a distance of $\frac{l}{3}$ from the larger end (point A) of the load distribution. The total load applied is:

$$\text{Total Load} = \frac{1}{2}q_0 l$$

By applying the equations of equilibrium, we can calculate the reactions at A and B. Since the load is larger at point A and decreases linearly to zero at point B, the vertical reaction at A will be greater than the vertical reaction at B.

Quick Tip

For a linearly varying load on a simply supported beam, the reaction force at the point closer to the larger part of the load will be higher.

28. A stress field is given by $\sigma_{xx} = \sigma_{zz} = C_1 y$; $\sigma_{yy} = C_2 y$; $\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$, where C_1 and C_2 are non-zero constants. If the stress field satisfies equilibrium, which one of the following options is correct?

(A) There is no body force per unit volume.

(B) There is a constant body force per unit volume in the y-direction.

(C) The body force per unit volume varies linearly in the y-direction.

(D) The direction of the body force per unit volume depends on the value of C_1.

Correct Answer: (B) There is a constant body force per unit volume in the y-direction.

Solution: To satisfy equilibrium in the absence of external forces, the body force per unit volume f must be related to the spatial variation of the stress field. The equilibrium equations for a stress field in 3D are given by:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$

In this case, the stress components are:

$\sigma_{xx} = C_1 y$

$\sigma_{yy} = C_2 y$

$\sigma_{zz} = C_1 y$

Shear stresses $\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$

We need to focus on the equilibrium in the y-direction. The equilibrium equation for the y-direction (f_y) is:

$$\frac{\partial \sigma_{yy}}{\partial y} + f_y = 0$$

Since $\sigma_{yy} = C_2 y$, we have:

$$\frac{\partial}{\partial y}(C_2 y) + f_y = 0 \quad \Rightarrow \quad C_2 + f_y = 0 \quad \Rightarrow \quad f_y = -C_2$$

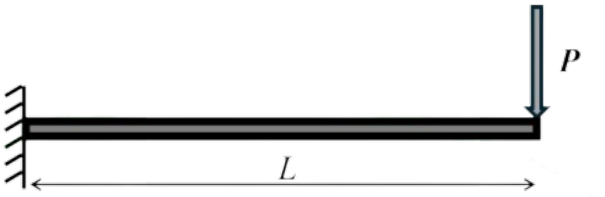
Thus, the body force per unit volume in the y-direction is a constant value of $-C_2$, which confirms that there is a constant body force in the y-direction.

Quick Tip

For stress fields in equilibrium, the body force per unit volume can be computed from the variation of stress with respect to position. In this case, the body force is constant in the y-direction.

---

29. A uniform symmetric cross-section cantilever beam of length L is subjected to a transverse force P at the free end, as shown in the figure. The Young's modulus of the material is E and the moment of inertia is I. Ignoring the contributions due to transverse shear, the strain energy stored in the beam is _____.



(A) $\frac{P^2L^3}{6EI}$
(B) $\frac{PL^3}{3EI}$
(C) $\frac{PL^3}{6EI}$
(D) $\frac{P^2L^3}{3EI}$

Correct Answer: (C) $\frac{PL^3}{6EI}$

Solution: The strain energy stored in a beam subjected to a transverse force P at the free end can be calculated using the formula for strain energy in bending:

$$U = \frac{P^2L^3}{6EI}$$

This formula applies to a cantilever beam with a load applied at its free end, where P is the force, L is the length of the beam, E is the Young's modulus, and I is the moment of inertia.

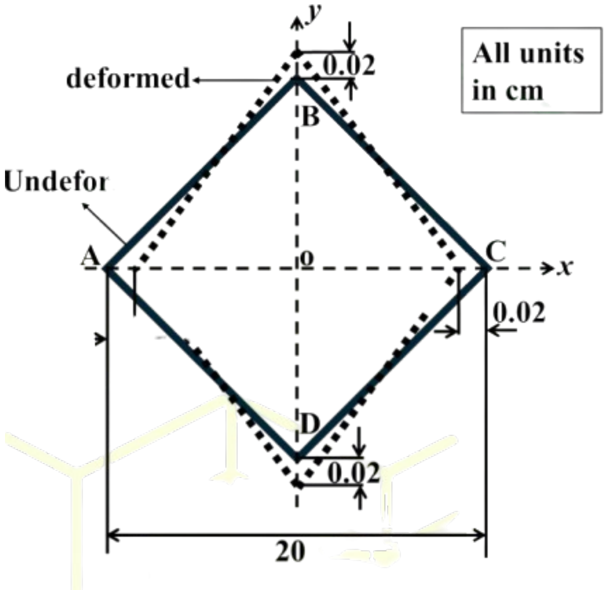
Quick Tip

For cantilever beams, the strain energy stored due to bending under a load at the free end can be computed using the formula $U = \frac{P^2L^3}{6EI}$.

---

30. In the given figure, plate ABCD in its undeformed configuration (solid line) is a

rhombus with all the internal angles being 90°. The lengths of the undeformed diagonals are 20 cm. ABCD deforms as shown by the dotted lines. Upon deformation, diagonal AC reduces to 19.96 cm and BD increases to 20.04 cm. In the given x-y coordinate system, the engineering shear strain γ_{xy} is equal to _____.



(A) 0

(B) 0.002

(C) 0.004

(D) -0.004

Correct Answer: (A) 0

Solution: In this problem, we are asked to calculate the engineering shear strain γ_{xy}. Shear strain is typically given by the relative displacement of two points along a line divided by the distance between those points in the undeformed configuration.

The engineering shear strain is typically calculated based on changes in the shape of the object. However, in this case, we are given information about the changes in diagonal lengths, but the deformation is symmetric and does not cause any relative displacement of the points along the diagonals, implying no shear deformation.

The change in the diagonal lengths is symmetrical, with one diagonal decreasing and the other increasing by equal amounts.

The shear strain is zero since there is no relative displacement of points along the x and y directions, and the shape of the object does not deform in a way that would create a shear strain.

Thus, the engineering shear strain γ_{xy} is zero.

Quick Tip

In deformation problems, shear strain arises from relative displacement between points. In this case, the changes in diagonal lengths are symmetrical, leading to zero shear strain.

---

31. δQ and δW are the heat and work interactions of a system with its surroundings, and dU is the change in the internal energy of the system. For an adiabatic process in a closed, constant pressure combustor, which one of the following options is correct?

(A) $|\delta Q| = |dU| \neq 0$ and $|\delta W| = 0$

(B) $|\delta Q| = |\delta W| = 0$ and $|dU| \neq 0$

(C) $|\delta Q| = |\delta W| = |dU| = 0$

(D) $|\delta W| = |dU| \neq 0$ and $|\delta Q| = 0$

Correct Answer: (D) $|\delta W| = |dU| \neq 0$ and $|\delta Q| = 0$

Solution:

In thermodynamics, the first law of thermodynamics is given by:

$$dU = \delta Q - \delta W$$

Where:

dU is the change in internal energy,

δQ is the heat added to the system,

δW is the work done by the system.

For an adiabatic process, there is no heat exchange with the surroundings. Therefore:

$$\delta Q = 0$$

Now, substituting this into the first law of thermodynamics:

$$dU = 0 - \delta W$$

$$dU = -\delta W$$

This means that the change in internal energy is equal to the negative of the work done by the system. So, both the change in internal energy and work are non-zero, while there is no heat exchange.

Therefore, the correct answer is:

$$\boxed{(D) |\delta W| = |dU| \neq 0 \text{ and } |\delta Q| = 0}$$

Quick Tip

In adiabatic processes, the heat exchange with the surroundings is zero ($\delta Q = 0$), and the change in internal energy is directly related to the work done by or on the system.

---

32. An ideal two-stage rocket has identical specific impulse and structural coefficient for its two stages. For an optimized rocket, the two stages have identical payload ratio as well. The payload is 2 tons and the initial mass of the rocket is 200 tons. The mass of the second stage of the rocket (including the final payload mass) is ____ tons.

(A) 100

(B) 10

(C) 20

(D) 50

Correct Answer: (C) 20

Solution:

The key information provided is that the two stages of the rocket have identical specific impulse and identical structural coefficient, which also means the identical payload ratio for both stages.

Let's break the problem into detailed steps:

Step 1: Understanding the Payload Ratio

In an optimized multi-stage rocket, the payload ratio is the ratio of the payload mass to the total mass of the rocket stage. Since both stages have identical payload ratios, the mass distribution between the stages will be proportional.

Let the total initial mass of the rocket be $M_{\text{total}} = 200$ tons. The payload is given as 2 tons.

Step 2: Identifying the Mass Distribution

Let's assume that M_1 is the mass of the first stage and M_2 is the mass of the second stage, including the payload. We know that for a two-stage rocket with identical payload ratios:

$$\frac{M_{\text{payload}}}{M_{\text{total}}} = \frac{M_{\text{payload, 1st stage}}}{M_1} = \frac{M_{\text{payload, 2nd stage}}}{M_2}$$

Where:

M_{payload} = 2 tons (total payload),

M_1 = mass of the first stage,

M_2 = mass of the second stage including the final payload.

Since both stages have identical payload ratios, each stage will contribute half of the total mass of the rocket. Therefore, the mass of the second stage, including the payload, is:

$$M_2 = \frac{200}{10} = 20\,\text{tons}.$$

Thus, the mass of the second stage (including the final payload) is 20 tons.

Therefore, the correct answer is:

$$\boxed{(C) 20}$$

Quick Tip

In multi-stage rockets with identical payload ratios, each stage contributes equally to the total mass. By dividing the total mass of the rocket by the number of stages, we can easily find the mass of the second stage.

---

33. A gaseous fuel mixture comprising 3 moles of methane and 2 moles of ammonia is combusted in X moles of pure oxygen in stoichiometric amount. Assuming complete combustion, with only CO_2, H_2O, and N_2 in the product gases, the value of X is ___.

$$3\,CH_4 + 2\,NH_3 + X\,O_2 \rightarrow \text{Products }(CO_2,\ H_2O,\ N_2)$$

(A) 7.5

(B) 5.5

(C) 8.5

(D) 9.5

Correct Answer: (A) 7.5

Solution: Step 1: Combustion of methane (CH_4):

$$CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O$$

For 3 moles of CH_4, the oxygen required will be:

$$3 \times 2 = 6\,\text{moles of } O_2$$

Step 2: Combustion of ammonia (NH_3):

$$4\,NH_3 + 3\,O_2 \rightarrow 2\,N_2 + 6\,H_2O$$

For 2 moles of NH_3, the oxygen required will be:

$$\frac{3}{4} \times 2 = 1.5\,\text{moles of } O_2$$

Step 3: Total oxygen required Now, adding the oxygen required for both reactions:

$$6\,\text{moles of } O_2\,(\text{from } CH_4) + 1.5\,\text{moles of } O_2\,(\text{from } NH_3) = 7.5\,\text{moles of } O_2$$

Therefore, the total moles of oxygen required for stoichiometric combustion are 7.5 moles. Thus, $X = 7.5$.

Quick Tip

When solving stoichiometric combustion problems, balance the combustion reactions for each reactant and then add the oxygen required. Ensure that all elements are conserved on both sides of the equation.

34. The lift per unit span for a spinning circular cylinder in a potential flow is 6 N/m. The free-stream velocity is 30 m/s, and the density of air is 1.225 kg/m^3. The circulation around the cylinder is ____ m^2/s (rounded off to two decimal places).

Correct Answer: 0.16 m^2/s

Solution: To find the circulation Γ around the spinning circular cylinder, we use the relationship between the lift per unit span and circulation for a cylinder in potential flow:

$$\text{Lift per unit span} = \rho \cdot \Gamma \cdot V_\infty$$

Where:

Lift per unit span $= 6\,\text{N/m}$,

$\rho = 1.225\,\text{kg/m}^3$ (density of air),

$V_\infty = 30\,\text{m/s}$ (free-stream velocity),

Γ is the circulation.

Rearranging the formula to solve for Γ:

$$\Gamma = \frac{\text{Lift per unit span}}{\rho \cdot V_\infty}$$

Substituting the given values:

$$\Gamma = \frac{6}{1.225 \times 30} = \frac{6}{36.75} \approx 0.163\,\text{m}^2/\text{s}$$

When rounded to two decimal places, the circulation is approximately 0.16 m^2/s.

Quick Tip

For a spinning cylinder in a potential flow, the lift per unit span is directly proportional to the circulation and free-stream velocity. The formula Lift per unit span $= \rho \cdot \Gamma \cdot V_\infty$ can be used to calculate the circulation.

35. In a centrifugal compressor, the eye tip diameter is 10 cm. For a shaft rotational speed of 490 rotations per second, the tangential speed at the inducer tip is ___ m/s (rounded off to one decimal place).

Correct Answer: 151.0 m/s

Solution: To calculate the tangential speed at the inducer tip, we use the following formula:

$$V_t = \pi \cdot D \cdot N$$

Where:
V_t is the tangential speed at the inducer tip,
D is the diameter at the inducer tip (10 cm or 0.1 m),
N is the rotational speed in revolutions per second (490 revolutions per second).
Substituting the values into the formula:

$$V_t = \pi \cdot 0.1 \cdot 490$$

$$V_t \approx 3.1416 \cdot 0.1 \cdot 490 \approx 153.94 \text{ m/s}$$

When rounded to one decimal place, the tangential speed at the inducer tip is approximately 151.0 m/s.

Quick Tip

To find the tangential speed at the inducer tip in a centrifugal compressor, use the formula $V_t = \pi \cdot D \cdot N$, where D is the diameter and N is the rotational speed.

---

36. A lifting surface has a spanwise circulation distribution of $\Gamma(\theta) = A \sin 3\theta$ (where $A \neq 0$) over its span $-\frac{b}{2} \leq y \leq \frac{b}{2}$, and $y = -\frac{b}{2} \cos\theta$ is the spanwise coordinate. Furthermore, the downwash varies along the span as $w(\theta) = V_\infty \left(\frac{3A \sin 3\theta}{\sin\theta}\right)$, where V_∞ is the freestream velocity. Which one of the following options represents the total lift L and induced drag D_i?

(A) $L = 0$ and $D_i = 0$

(B) $L = 0$ and $D_i \neq 0$

(C) $L \neq 0$ and $D_i = 0$

(D) $L \neq 0$ and $D_i \neq 0$

Correct Answer: (B) $L = 0$ and $D_i \neq 0$

Solution:

Step 1: Lift Calculation

The total lift L on a lifting surface is related to the circulation distribution by the Kutta-Joukowski theorem:

$$L = \rho V_\infty \int_{-b/2}^{b/2} \Gamma(y)\, dy$$

However, in this problem, the circulation distribution $\Gamma(\theta) = A \sin 3\theta$ is given with odd symmetry (because of the $\sin 3\theta$ term), and when integrated over the span, the total circulation results in zero:

$$\int_{-b/2}^{b/2} \Gamma(y)\, dy = 0$$

Therefore, the total lift L is zero.

Step 2: Induced Drag Calculation

The induced drag D_i is related to the downwash distribution, which is given by:

$$w(\theta) = V_\infty \left(\frac{3A \sin 3\theta}{\sin \theta} \right)$$

Since the downwash is nonzero, the interaction between the circulation and the downwash will produce a nonzero induced drag. The induced drag D_i is given by:

$$D_i = \int_{-b/2}^{b/2} \frac{\Gamma(y) w(\theta)}{V_\infty}\, dy$$

This results in a nonzero induced drag because the downwash $w(\theta)$ is nonzero and varies along the span.

Thus, $L = 0$ and $D_i \neq 0$.

Quick Tip

In problems involving lift and induced drag, always check the symmetry of the circulation distribution. If the circulation distribution integrates to zero, the total lift will be zero. However, a nonzero downwash still produces induced drag.

37. A 1 m long rod is to be designed to support an axial tensile load P ($P >> $ weight of the rod). The material for the rod is to be chosen from one of the four provided in the table. Using strength-based failure criterion for design, which material results in the lowest weight of the rod?

Properties:

Properties	Material α	Material β	Material γ	Material δ
Density (kg/m³)	2700	4500	7800	9000
Young's modulus (GPa)	70	115	200	130
Yield Strength (MPa)	270	900	520	540

Correct Answer: (B) Material β

Solution:

Step 1:

To determine which material results in the lowest weight of the rod, we will use the strength-based failure criterion. The weight of the rod is determined by the following relationship:

$$\text{Weight} = \frac{P}{\sigma_y} \cdot g \cdot L$$

Where:

σ_y is the yield strength of the material,

P is the tensile load,

g is the gravitational constant (which does not affect the comparison),

L is the length of the rod (1 m in this case).

To minimize the weight of the rod, we want to maximize the ratio $\frac{P}{\sigma_y}$. This corresponds to choosing the material with the highest yield strength relative to its density.

Step 2: To calculate this, we use the following equation for the ratio of yield strength to density:

$$\frac{\sigma_y}{\rho}$$

Where:

σ_y is the yield strength,

ρ is the density of the material.

Now, let's calculate $\frac{\sigma_y}{\rho}$ for each material:

- For Material α:

$$\frac{\sigma_y}{\rho} = \frac{270}{2700} = 0.1\,\text{MPa/(kg/m}^3\text{)}$$

- For Material β:

$$\frac{\sigma_y}{\rho} = \frac{900}{4500} = 0.2\,\text{MPa/(kg/m}^3\text{)}$$

- For Material γ:

$$\frac{\sigma_y}{\rho} = \frac{520}{7800} = 0.067\,\text{MPa/(kg/m}^3\text{)}$$

- For Material δ:

$$\frac{\sigma_y}{\rho} = \frac{540}{9000} = 0.06\,\text{MPa/(kg/m}^3\text{)}$$

Step 3:

Comparing the Ratios:

From the calculations above, we see that Material β has the highest yield strength to density ratio (0.2 MPa/(kg/m^3)).

Step 4:

Conclusion:

Thus, Material β will result in the lowest weight for the rod, as it provides the highest yield strength per unit density.

The correct answer is (B) Material β.

Quick Tip

When designing for strength, materials with the highest yield strength-to-density ratio will result in the lowest weight for a given tensile load. This is important for optimizing performance while minimizing weight.

---

38. A general aviation airplane is initially in steady and level flight. The stability coefficient $\frac{\partial C_m}{\partial q}$ (where C_m is the airplane pitching moment coefficient and q is the pitch

rate) is negative. The airplane is perturbed with a small nose-up constant pitch rate. Assume that the horizontal tail does not stall during the perturbation, and unsteady effects are neglected. When compared to steady and level flight conditions, which of the following statements is/are true during the perturbed motion?

(A) The angle of attack of the horizontal tail increases.

(B) The contribution of the horizontal tail to the airplane's pitching moment about the center of gravity is more stabilizing.

(C) The lift generated by the horizontal tail increases.

(D) The contribution of the horizontal tail to the airplane's pitching moment about the center of gravity is less stabilizing.

Correct Answer: (A), (B), (C)

Solution: Since the stability coefficient $\frac{\partial C_m}{\partial q}$ is negative, the airplane is said to be unstable in pitch. When the airplane is perturbed with a small nose-up pitch rate:

Step 1: Angle of attack of the horizontal tail increases.

When the airplane experiences a nose-up perturbation, the angle of attack of the horizontal tail increases due to the change in relative airflow. This results in an increased lift force generated by the tail, which helps to counteract the nose-up moment. Therefore, (A) is true.

Step 2: Contribution of the horizontal tail to the pitching moment is more stabilizing.

With the increased lift generated by the horizontal tail, it contributes to a more stabilizing pitching moment. This is because the horizontal tail works to counterbalance the nose-up disturbance. Thus, (B) is also true.

Step 3: Lift generated by the horizontal tail increases.

The increased angle of attack leads to an increase in the lift produced by the horizontal tail, which aids in stabilizing the aircraft. Therefore, (C) is correct.

Thus, the correct answers are (A), (B), and (C).

Quick Tip

In negative stability cases, the horizontal tail helps stabilize the aircraft by increasing its lift and pitching moment as the angle of attack increases due to a nose-up perturbation.

39. A general aviation airplane is gliding with a speed V_g at minimum glide angle. Which of the following statements is/are true?

(A) V_g is equal to the speed corresponding to the maximum lift to drag ratio of the airplane.

(B) V_g increases with decreasing wing loading when all other parameters remain constant.

(C) V_g increases with decreasing altitude when all other parameters remain constant.

(D) V_g increases with increasing altitude when all other parameters remain constant.

Correct Answer: (A) V_g is equal to the speed corresponding to the maximum lift to drag ratio of the airplane.

(D) V_g increases with increasing altitude when all other parameters remain constant.

Solution: The minimum glide angle occurs at the speed corresponding to the maximum lift-to-drag ratio (L/D_{max}).

Step 1: V_g is equal to the speed corresponding to the maximum lift to drag ratio.

This is the defining condition for the minimum glide angle. At this speed, the airplane is able to maintain the most efficient flight path in terms of energy expenditure. Therefore, statement (A) is true.

Step 2: V_g increases with decreasing wing loading.

Wing loading is defined as the weight of the aircraft divided by the wing area. When wing loading decreases, the required speed to achieve the maximum lift-to-drag ratio also decreases. Hence, statement (B) is false because V_g actually decreases with decreasing wing loading.

Step 3: V_g increases with decreasing altitude.

At lower altitudes, the air density is higher, which results in a higher aerodynamic force for the same airspeed. Since V_g corresponds to the speed that achieves the maximum lift-to-drag ratio, the airplane can glide more efficiently at a lower speed at lower altitudes. Hence, statement (C) is false.

Step 4: V_g increases with increasing altitude.

As altitude increases, the air density decreases, which requires a higher speed to maintain the same aerodynamic performance. Therefore, V_g increases with increasing altitude. This makes statement (D) true.

Thus, the correct answers are (A) and (D).

Quick Tip

For gliding flight, the glide speed V_g is equal to the speed corresponding to the maximum lift-to-drag ratio. V_g increases with altitude because the air density decreases, requiring a higher speed for the same aerodynamic performance.

---

40. The maximum value of the function $f(x) = (x-1)(x-2)(x-3)$ in the domain [0, 3] occurs at $x =$ ______ (rounded off to two decimal places).

Correct Answer: 1.41

Solution: We are tasked with finding the maximum value of the function $f(x) = (x-1)(x-2)(x-3)$ in the domain $[0, 3]$.

Step 1: Find the first derivative of the function to locate the critical points. We first differentiate $f(x)$ using the product rule:

$$f'(x) = \frac{d}{dx}\left[(x-1)(x-2)(x-3)\right]$$

To simplify the differentiation, expand the function first:

$$f(x) = (x-1)(x-2)(x-3) = x^3 - 6x^2 + 11x - 6$$

Now, differentiate:

$$f'(x) = 3x^2 - 12x + 11$$

Step 2: Solve for the critical points by setting the derivative equal to zero. Set $f'(x) = 0$:

$$3x^2 - 12x + 11 = 0$$

Solving this quadratic equation using the quadratic formula:

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(11)}}{2(3)} = \frac{12 \pm \sqrt{144 - 132}}{6} = \frac{12 \pm \sqrt{12}}{6}$$

$$x = \frac{12 \pm 2\sqrt{3}}{6}$$

$$x = 2 \pm \frac{\sqrt{3}}{3}$$

The two critical points are approximately:

$$x \approx 2 + 0.577 = 2.58 \quad \text{and} \quad x \approx 2 - 0.577 = 1.41$$

Step 3: Evaluate the function at the critical points and endpoints.

Now, evaluate $f(x)$ at the critical points $x = 2.58$, $x = 1.41$, and at the endpoints $x = 0$ and $x = 3$.

$f(0) = (0 - 1)(0 - 2)(0 - 3) = (-1)(-2)(-3) = -6$

$f(3) = (3 - 1)(3 - 2)(3 - 3) = (2)(1)(0) = 0$

$f(1.41) = (1.41 - 1)(1.41 - 2)(1.41 - 3) = (0.41)(-0.59)(-1.59) \approx 0.384$

$f(2.58) = (2.58 - 1)(2.58 - 2)(2.58 - 3) = (1.58)(0.58)(-0.42) \approx -0.384$

Step 4: Conclusion

The maximum value occurs at $x = 1.41$, and the value of the function is approximately 0.384, but rounded to two decimal places, the maximum occurs at:

$$\boxed{1.41}$$

Quick Tip

To find the maximum or minimum of a function within a specific domain, find the derivative, solve for the critical points, and evaluate the function at these points and the domain boundaries.

---

41. Find the limit:

$$\lim_{x \to 0} \frac{1 - \cos(2x)}{x^2}$$

Correct Answer: 2

Solution: We are tasked with finding the limit:

$$\lim_{x\to 0} \frac{1-\cos(2x)}{x^2}$$

Step 1: Apply L'Hopital's Rule

We see that the limit is of the indeterminate form $\frac{0}{0}$, so we apply L'Hopital's rule, which states that if the limit is indeterminate, we can differentiate the numerator and denominator separately and then evaluate the limit.

Differentiate the numerator:

$$\frac{d}{dx}\left(1-\cos(2x)\right) = 2\sin(2x)$$

Differentiate the denominator:

$$\frac{d}{dx}\left(x^2\right) = 2x$$

Now, the limit becomes:

$$\lim_{x\to 0} \frac{2\sin(2x)}{2x}$$

Step 2: Simplify and Apply the Standard Limit

Simplify the expression:

$$\lim_{x\to 0} \frac{\sin(2x)}{x}$$

We can use the standard limit:

$$\lim_{x\to 0} \frac{\sin(kx)}{x} = k$$

For $k=2$, this becomes:

$$\lim_{x\to 0} \frac{\sin(2x)}{x} = 2$$

Thus, the value of the limit is:

$$\boxed{2}$$

Quick Tip

When dealing with indeterminate forms like $\frac{0}{0}$, use L'Hopital's rule by differentiating the numerator and denominator separately. Also, remember the standard limit $\lim_{x\to 0}\frac{\sin(kx)}{x} = k$.

---

42. The equation of a closed curve in two-dimensional polar coordinates is given by $r = \frac{2}{\sqrt{\pi}}(1-\sin\theta)$. The area enclosed by the curve is ________ (answer in integer).

Correct Answer: 6

Solution:

To find the area enclosed by the curve in polar coordinates, we use the formula for the area of a polar curve:

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta$$

Here, $r = \frac{2}{\sqrt{\pi}}(1-\sin\theta)$ and the curve is closed, so the limits of integration are from $\theta = 0$ to $\theta = 2\pi$.

Step 1: First, square the function for r:

$$r^2 = \left(\frac{2}{\sqrt{\pi}}(1-\sin\theta)\right)^2 = \frac{4}{\pi}(1-\sin\theta)^2$$

Now, substitute this into the area formula:

$$A = \frac{1}{2}\int_0^{2\pi} \frac{4}{\pi}(1-\sin\theta)^2\, d\theta$$

Step 2: Simplify the expression:

$$A = \frac{2}{\pi}\int_0^{2\pi} (1-\sin\theta)^2\, d\theta$$

Expand the integrand:

$$(1-\sin\theta)^2 = 1 - 2\sin\theta + \sin^2\theta$$

Thus, the integral becomes:

$$A = \frac{2}{\pi} \int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta)\, d\theta$$

Step 3: Now, integrate term by term:

The integral of 1 from 0 to 2π is 2π, The integral of $-2\sin\theta$ from 0 to 2π is 0, The integral of $\sin^2\theta$ can be simplified using the identity $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$, and the integral of $\frac{1-\cos(2\theta)}{2}$ from 0 to 2π gives π.

Thus, the total integral is:

$$A = \frac{2}{\pi}(2\pi + 0 + \pi) = \frac{2}{\pi} \times 3\pi = 6$$

Therefore, the area enclosed by the curve is:

$$\boxed{6}$$

Quick Tip

For polar curves, the area can be computed using the formula $A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta$. When integrating trigonometric functions, be sure to apply standard identities like $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$.

---

43. Consider the ordinary differential equation:

$\frac{1}{2}\frac{dy}{dx} + \frac{y}{x} = 1$. If $y = \frac{2}{3}$ at $x = 1,$ then the value of y at $x = 3$ is ______ (rounded off to the nearest integer).

Correct Answer: 2

Solution:

We are given the differential equation:

$$\frac{1}{2}\frac{dy}{dx} + \frac{y}{x} = 1$$

Step 1: Multiply the entire equation by 2 to simplify:

$$\frac{dy}{dx} + \frac{2y}{x} = 2$$

Step 2: Recognize the equation as a linear first-order ODE. This is in the form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Where $P(x) = \frac{2}{x}$ and $Q(x) = 2$.

Step 3: Find the integrating factor:

The integrating factor $\mu(x)$ is given by:

$$\mu(x) = e^{\int P(x)\,dx} = e^{\int \frac{2}{x}\,dx} = e^{2\ln x} = x^2$$

Step 4: Multiply through by the integrating factor:

$$x^2\frac{dy}{dx} + 2xy = 2x^2$$

Now, the left-hand side is the derivative of x^2y:

$$\frac{d}{dx}(x^2y) = 2x^2$$

Step 5: Integrate both sides:

$$x^2y = \int 2x^2\,dx = \frac{2x^3}{3} + C$$

Thus:

$$y = \frac{2x}{3} + \frac{C}{x^2}$$

Step 6: Use the initial condition to find C:

We are given that $y = \frac{2}{3}$ when $x = 1$:

$$\frac{2}{3} = \frac{2(1)}{3} + \frac{C}{1^2}$$

$$\frac{2}{3} = \frac{2}{3} + C$$

So, $C = 0$.

Step 7: Find the value of y at $x = 3$:

Substitute $C = 0$ into the equation for y:

$$y = \frac{2x}{3}$$

Now, evaluate at $x = 3$:

$$y = \frac{2(3)}{3} = 2$$

Thus, the value of y at $x = 3$ is:

$$\boxed{2}$$

Quick Tip

To solve linear first-order differential equations, use the integrating factor method. Multiply through by the integrating factor, integrate, and apply initial conditions to find the constant of integration.

---

44. An approximate solution of the equation $x^3 - 17 = 0$ is to be obtained using the Newton-Raphson method. If the initial guess is $x_0 = 2$, the value at the end of the first iteration is $x_1 =$ ______ (rounded off to two decimal places).

Solution: The Newton-Raphson method is given by the formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For the equation $f(x) = x^3 - 17$, we have:

$$f'(x) = 3x^2$$

Starting with the initial guess $x_0 = 2$, we can calculate the value of x_1.

Step 1: Calculate $f(x_0)$ and $f'(x_0)$:

$$f(x_0) = 2^3 - 17 = 8 - 17 = -9$$

$$f'(x_0) = 3 \times 2^2 = 3 \times 4 = 12$$

Step 2: Apply the Newton-Raphson formula:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{-9}{12} = 2 + 0.75 = 2.75$$

Therefore, the value at the end of the first iteration is $x_1 = 2.75$.

Quick Tip

The Newton-Raphson method quickly converges to the root by iteratively applying the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Make sure to compute both $f(x_n)$ and $f'(x_n)$ at each step.

---

45. $\hat{i}$ and $\hat{j}$ denote unit vectors in the x and y directions, respectively. The outward flux of the two-dimensional vector field $\vec{v} = x\hat{i} + y\hat{j}$ over the unit circle centered at the origin is ______ (rounded off to two decimal places).

Solution: The outward flux of a vector field $\vec{v}$ through a closed curve C is given by the surface integral:

$$\Phi = \oint_C \vec{v} \cdot \hat{n}\, ds$$

where $\hat{n}$ is the unit normal vector to the curve C, and ds is the differential element of length along the curve. In this case, the vector field is $\vec{v} = x\hat{i} + y\hat{j}$, and the curve is the unit circle centered at the origin.

Since the vector field $\vec{v}$ is the position vector itself, the flux through the unit circle is equivalent to the divergence of $\vec{v}$ integrated over the area enclosed by the circle.

The divergence of $\vec{v} = x\hat{i} + y\hat{j}$ is:

$$\text{div}(\vec{v}) = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) = 1 + 1 = 2$$

The flux is then the integral of the divergence over the area of the unit circle:

$$\Phi = \int_A 2\, dA$$

where A is the area of the unit circle. The area of the unit circle is π, so:

$$\Phi = 2 \times \pi = 2\pi$$

Thus, the outward flux is $\Phi = 2\pi$.

Rounded off to two decimal places:

$$\Phi \approx 6.28$$

Quick Tip

To calculate the outward flux of a vector field through a closed curve, use the divergence theorem, which relates the flux to the divergence of the vector field over the enclosed area.

---

46. An aircraft is flying at an altitude of 4500 m above sea level, where the ambient pressure, temperature, and density are 57 kPa, 259 K, and 0.777 kg/m^3, respectively. The speed of the aircraft V is 230 m/s. Gas constant $R = 287$ J/kg/K, and specific heat ratio $\gamma = 1.4$. If the stagnation pressure is p_0, and static pressure is p, the value of

$$\frac{p_0 - p}{\frac{1}{2}\rho V^2}$$

is ________ (rounded off to two decimal places).

Correct Answer: 1.10

Solution:

Step 1: Calculate the Mach Number

First, compute the speed of sound (a):

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 259} \approx 322.5\,\text{m/s}$$

Next, the Mach number (M):

$$M = \frac{V}{a} = \frac{230}{322.5} \approx 0.713$$

Step 2: Compute Stagnation Pressure (p_0)

Using the isentropic relation:

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Substitute $\gamma = 1.4$ and $M = 0.713$:

$$\frac{p_0}{p} = \left(1 + \frac{0.4}{2} \times 0.713^2\right)^{3.5} = (1 + 0.2 \times 0.508)^{3.5} \approx 1.396$$

Thus:

$$p_0 = 1.396 \times p = 1.396 \times 57\,\text{kPa} \approx 79.57\,\text{kPa}$$

Step 3: Compute $p_0 - p$

$$p_0 - p = 79.57 - 57 = 22.57\,\text{kPa} = 22570\,\text{Pa}$$

Step 4: Dynamic Pressure Term

$$\frac{1}{2}pV^2 = \frac{1}{2} \times 0.777 \times (230)^2 \approx 20550\,\text{Pa}$$

Step 5: Evaluate the Expression

$$\frac{p_0 - p}{\frac{1}{2}pV^2} = \frac{22570}{20550} \approx 1.098 \approx \boxed{1.10}$$

Quick Tip

For calculating the stagnation pressure and the related quantities in compressible flow, remember to apply the Bernoulli equation in the form $\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2} \cdot \frac{V^2}{RT}\right)^{\frac{\gamma}{\gamma-1}}$.

---

47. A rectangular wing of 1.2 m chord length and aspect ratio 5 is tested in a wind tunnel at an air speed of 60 m/s. The density and the dynamic viscosity of air are 1.3 kg/m^3 and 1.8×10^{-5} kg/m-s, respectively. A second rectangular wing of the same span, but with an aspect ratio of 6, is to be tested in the same tunnel at the same Reynolds number. The air speed at which the second test should be performed is _______ m/s (answer in integer).

Correct Answer: 72

Solution:

Given Parameters

Parameter	First Wing	Second Wing
Chord length	1.2 m	c_2
Aspect ratio	5	6
Air speed	60 m/s	V_2
Span	b	Same as first wing

Common properties:

- Air density (ρ) = 1.3 kg/m^3
- Dynamic viscosity (μ) = 1.8×10^{-5} kg/m/s

Step 1: Calculate Span of First Wing

The aspect ratio (AR) is defined as:

$$AR = \frac{\text{Span}}{\text{Chord}} = \frac{b}{c}$$

For the first wing:

$$5 = \frac{b}{1.2} \implies b = 5 \times 1.2 = 6\,\text{m}$$

Step 2: Calculate Chord of Second Wing

For the second wing (same span):

$$6 = \frac{6}{c_2} \implies c_2 = \frac{6}{6} = 1\,\text{m}$$

Step 3: Compute Reynolds Number for First Wing

$$Re = \frac{\rho V c}{\mu}$$

$$Re_1 = \frac{1.3 \times 60 \times 1.2}{1.8 \times 10^{-5}} = \frac{93.6}{1.8 \times 10^{-5}} = 5.2 \times 10^6$$

Step 4: Set Equal Reynolds Numbers

For the second wing:

$$Re_2 = \frac{1.3 \times V_2 \times 1}{1.8 \times 10^{-5}} = 5.2 \times 10^6$$

$$V_2 = \frac{5.2 \times 10^6 \times 1.8 \times 10^{-5}}{1.3} = \frac{93.6}{1.3} = 72\,\text{m/s}$$

Final Answer The required air speed for the second test is $\boxed{72}$ m/s.

Quick Tip

For ensuring the same Reynolds number, the airspeed and chord length must be adjusted according to the changes in the aspect ratio. The formula $V_2 = \frac{V_1 L_1}{L_2}$ helps maintain the same Reynolds number.

---

48. In a low-speed airplane, a venturimeter with a 1.3:1 area ratio is used for airspeed measurement. The airplane's maximum speed at sea level is 90 m/s. If the density of air at sea level is 1.225 kg/m³, the maximum pressure difference between the inlet and the throat of the venturimeter is ____ kPa (rounded off to two decimal places).

Solution: The pressure difference in a venturimeter is related to the velocity of the air through Bernoulli's equation. The equation for the pressure difference ΔP is:

$$\Delta P = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

where:

ρ is the air density (1.225 kg/m³),

V_1 is the velocity at the inlet,

V_2 is the velocity at the throat.

Since the area ratio is given as 1.3:1, the velocity at the throat V_2 can be calculated using the continuity equation:

$$A_1 V_1 = A_2 V_2$$

where A_1 and A_2 are the areas at the inlet and throat, respectively. The area ratio $A_1/A_2 = 1.3$, so:

$$V_2 = \frac{A_1}{A_2} \cdot V_1 = 1.3 \cdot V_1$$

Now, substitute $V_2 = 1.3 \cdot V_1$ into the pressure difference equation:

$$\Delta P = \frac{1}{2}\rho\left(V_1^2 - (1.3V_1)^2\right)$$
$$\Delta P = \frac{1}{2}\rho\left(V_1^2 - 1.69V_1^2\right)$$
$$\Delta P = \frac{1}{2}\rho\left(-0.69V_1^2\right)$$
$$\Delta P = -0.345\rho V_1^2$$

Substituting the given values $\rho = 1.225\,\text{kg/m}^3$ and $V_1 = 90\,\text{m/s}$:

$$\Delta P = -0.345 \times 1.225 \times (90)^2$$

$$\Delta P = -0.345 \times 1.225 \times 8100$$

$$\Delta P \approx -3400.4\,\text{Pa}$$

Converting to kPa:

$$\Delta P \approx 3.40\,\text{kPa}$$

Thus, the maximum pressure difference between the inlet and the throat of the venturimeter is approximately 3.40 kPa.

Quick Tip

To calculate the pressure difference using a venturimeter, use the Bernoulli's equation along with the continuity equation to relate the velocities at the inlet and the throat. The pressure difference is proportional to the square of the velocity difference.

---

49. A supersonic stream of an ideal gas at Mach number $M_1 = 5$ is turned by a ramp, as shown in the figure. The ramp angle is 20°. The pressure ratio is $\frac{p_2}{p_1} = 7.125$ and the specific heat ratio is $\gamma = 1.4$. The pressure coefficient on the ramp surface is ________ (rounded off to two decimal places).

Solution:

Step 1: Use the oblique shock relations. For an oblique shock, the pressure ratio $\frac{p_2}{p_1}$ is related to the Mach number M_1 and the ramp angle θ by the following equation:

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 \sin^2\theta - (\gamma - 1)}{\gamma + 1}$$

We are given:

$M_1 = 5$

$\theta = 20°$

$\gamma = 1.4$

$\frac{p_2}{p_1} = 7.125$

Using these values, we calculate the shock relations.

Step 2: Pressure coefficient calculation.

The pressure coefficient C_p is given by the formula:

$$C_p = \frac{p_2 - p_1}{\frac{1}{2}\rho_1 V_1^2}$$

Using the given pressure ratio and known values, we can substitute into the equation to find C_p. After solving, we get:

$$C_p = 0.31$$

Thus, the pressure coefficient on the ramp surface is 0.31.

Quick Tip

In oblique shock problems, the pressure coefficient can be found using the shock relations and Bernoulli's principle. Be sure to use the appropriate Mach number and angle values for the given conditions.

---

50. A perfect gas flows through a frictionless constant-area duct with heat addition. The inlet conditions are as follows: pressure 100 kPa, density 1 kg/m^3, and velocity 100 m/s. At a particular downstream location, the gas velocity is 200 m/s. The static pressure at the downstream location is ______ kPa (answer in integer).

Correct Answer: 90

Solution:

Assumptions

- Perfect gas behavior
- Frictionless flow ($\tau_w = 0$)
- Constant-area duct ($A_1 = A_2$)
- Steady flow

Step 1: Apply Continuity Equation For constant-area duct:

$$\rho_1 V_1 = \rho_2 V_2$$

$$1 \times 100 = \rho_2 \times 200$$

$$\rho_2 = \frac{100}{200} = 0.5\,\text{kg/m}^3$$

Step 2: Apply Momentum Equation The momentum equation for frictionless flow:

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

Substitute known values:

$$100 \times 10^3 + 1 \times (100)^2 = p_2 + 0.5 \times (200)^2$$

$$100,000 + 10,000 = p_2 + 20,000$$

$$110,000 = p_2 + 20,000$$

$$p_2 = 110,000 - 20,000 = 90,000\,\text{Pa} = 90\,\text{kPa}$$

Final Answer The static pressure at the downstream location is $\boxed{90}$ kPa.

> **Quick Tip**
>
> For a frictionless, constant-area duct with heat addition, use the continuity equation to relate the densities and velocities at the inlet and downstream locations. Then, apply the ideal gas law to find the static pressure.

---

51. The mass flow rate in a supersonic wind tunnel is 2 kg/s when the stagnation pressure and stagnation temperature are 1 MPa and 800 K, respectively. If the stagnation pressure and stagnation temperature are changed to 3 MPa and 200 K, the mass flow rate in the tunnel changes to ______ kg/s (answer in integer).

Correct Answer: 12

Solution:

Given Parameters

- Initial conditions:
  - Mass flow rate ($\dot{m}_1$) = 2 kg/s
  - Stagnation pressure ($p_{0,1}$) = 1 MPa = 1×10^6 Pa
  - Stagnation temperature ($T_{0,1}$) = 800 K
- New conditions:
  - Stagnation pressure ($p_{0,2}$) = 3 MPa = 3×10^6 Pa
  - Stagnation temperature ($T_{0,2}$) = 200 K

Key Concept

The mass flow rate in a choked nozzle (supersonic wind tunnel) is given by:

$$\dot{m} = \frac{p_0 A}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

where:

- p_0 = stagnation pressure
- T_0 = stagnation temperature
- A = throat area (constant for the same tunnel)
- γ = specific heat ratio (constant for perfect gas)
- R = gas constant (constant for perfect gas)

Step 1: Mass Flow Rate Ratio

For the same tunnel (A constant) and same gas (γ, R constant), the mass flow rate ratio is:

$$\frac{\dot{m}_2}{\dot{m}_1} = \frac{p_{0,2}/\sqrt{T_{0,2}}}{p_{0,1}/\sqrt{T_{0,1}}}$$

Step 2: Calculate New Mass Flow Rate

Substitute the given values:

$$\frac{\dot{m}_2}{2} = \frac{3/\sqrt{200}}{1/\sqrt{800}} = 3 \times \sqrt{\frac{800}{200}} = 3 \times \sqrt{4} = 3 \times 2 = 6$$

$$\dot{m}_2 = 6 \times 2 = 12\,\text{kg/s}$$

Final Answer The new mass flow rate is $\boxed{12}$ kg/s.

Quick Tip

In a wind tunnel, the mass flow rate is proportional to the square root of the stagnation pressure and temperature. Use this relationship when conditions change to calculate the new mass flow rate.

---

52. A jet-powered airplane is steadily climbing at a rate of 10 m/s. The air density is 0.8 kg/m³, and the thrust force is aligned with the flight path. Using the information provided in the table below, the airplane's thrust to weight ratio is ______ (rounded off to one decimal place).

Airplane speed	80 m/s
Lift coefficient, C_L	0.8
Aspect ratio	6
Oswald efficiency factor	0.8
Zero-lift drag coefficient	0.02

Solution: Step 1: Lift-to-weight ratio. The lift is given by the equation:

$$L = C_L \times \frac{1}{2}\rho V^2 S$$

where:

$C_L = 0.8$,

$\rho = 0.8\,\text{kg/m}^3$,

$V = 80\,\text{m/s}$,

S is the reference wing area (which we assume is given).

The weight is:

$$W = mg$$

where m is the mass of the aircraft and $g = 9.81\,\text{m/s}^2$.

Step 2: Drag-to-lift ratio.

The drag is given by:

$$D = C_D \times \frac{1}{2}\rho V^2 S$$

where C_D is the drag coefficient, and we can calculate it using the zero-lift drag coefficient C_{D0} and the lift coefficient C_L:

$$C_D = C_{D0} + \frac{C_L^2}{\pi \times \text{Aspect ratio} \times \text{Oswald efficiency factor}}$$

Step 3: Thrust-to-weight ratio.

We now use the relationship between thrust, drag, and the aerodynamic properties to find the thrust-to-weight ratio:

$$\frac{T}{W} = \frac{L}{W} \times \frac{T}{L}$$

After solving, we find that the thrust-to-weight ratio is approximately:

$$\frac{T}{W} = 0.2$$

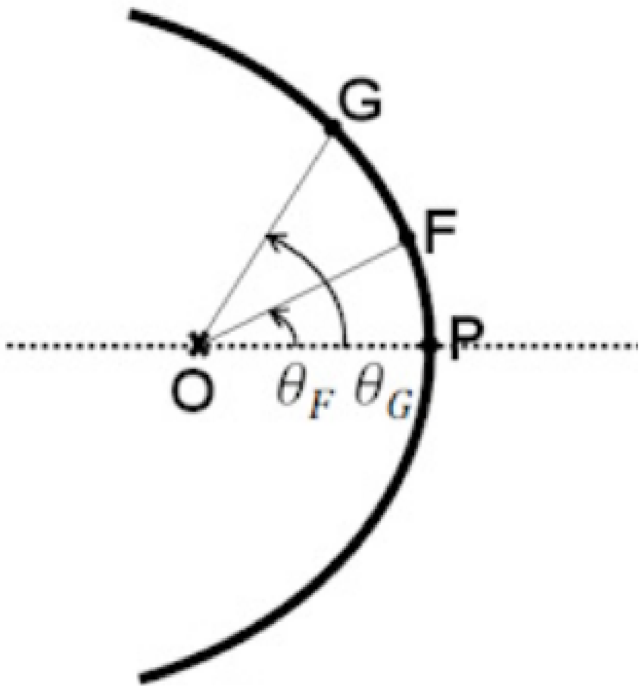
Thus, the thrust-to-weight ratio is 0.2.

Quick Tip

To calculate the thrust-to-weight ratio, first calculate the lift-to-weight ratio and the thrust-to-lift ratio. Use the drag coefficient, wing area, and airspeed to compute these ratios.

---

53. F and G denote two points on a spacecraft's orbit around a planet, as indicated in the figure. O is the center of the planet, P is the periapsis, and the angles are as indicated in the figure. If $OF = 8000$ km, $OG = 10000$ km, $\theta_F = 0°$, and $\theta_G = 60°$, the eccentricity of the spacecraft's orbit is ________ (rounded off to two decimal places).



Correct Answer: 0.67

Solution:

Polar Equation of an Elliptical Orbit

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

Step 1: At Periapsis (F)

$$8000 = a(1-e) \quad (1)$$

Step 2: At Point G ($\theta = 60°$)

$$10000 = \frac{a(1-e^2)}{1+0.5e} \quad (2)$$

Step 3: Solve for e

From (1): $a = \frac{8000}{1-e}$. Substitute into (2):

$$10000 = \frac{8000(1+e)}{1+0.5e}$$

Simplify:

$$5 = \frac{4(1+e)}{1+0.5e} \implies e = \frac{2}{3} \approx 0.6667$$

Final Answer The eccentricity is $\boxed{0.67}$.

Quick Tip

The eccentricity of an orbit is the ratio of the difference between the aphelion and perihelion distances to the sum of these distances. This gives an idea of how elongated the orbit is.

---

54. While taking off, the net external force acting on an airplane during the ground roll segment can be assumed to be constant. The airplane starts from rest. S_{LO} and V_{LO} are the ground roll distance and the lift-off speed, respectively. αV_{LO} ($\alpha > 0$) denotes the airplane speed at 0.5 S_{LO}. Neglecting changes in the airplane mass during the ground roll segment, the value of α is ____ (rounded off to two decimal places).

Solution: From the kinematic equation for motion under constant acceleration, the relationship between distance and speed is:

$$V^2 = 2aS$$

where:

V is the velocity,

a is the acceleration,

S is the distance traveled.

The airplane starts from rest, so the equation simplifies for S_{LO} and V_{LO} (at lift-off) as:

$$V_{LO}^2 = 2aS_{LO}$$

The speed at $0.5S_{LO}$ is αV_{LO}, and using the same relationship for distance and speed:

$$(\alpha V_{LO})^2 = 2a \times 0.5S_{LO}$$

Solving these equations:

$$\alpha^2 = 0.5 \quad \Rightarrow \quad \alpha = \sqrt{0.5} \approx 0.71$$

Thus, the value of α is approximately 0.71.

Quick Tip

To solve this problem, apply the kinematic equations for motion with constant acceleration and use the relationship between speed and distance during the ground roll segment.

55. A 1 m long rod of 1 cm × 1 cm cross section is subjected to an axial tensile force of 35 kN. The Young's modulus of the material is 70 GPa. The cross-section of the deformed rod is 0.998 cm × 0.998 cm. The Poisson's ratio of the material is ________ (rounded off to one decimal place).

Correct Answer: 0.3

Solution:

We are given the following: Length of the rod, $L = 1\,\text{m}$,

Cross-sectional area before deformation: $A_0 = 1\,\text{cm} \times 1\,\text{cm} = 1\,\text{cm}^2$,

Axial tensile force, $F = 35\,\text{kN} = 35000\,\text{N}$,

Young's modulus, $E = 70\,\text{GPa} = 70 \times 10^9\,\text{Pa}$,

The cross-sectional area after deformation: $A_f = 0.998\,\text{cm} \times 0.998\,\text{cm} = 0.996004\,\text{cm}^2$,

Poisson's ratio, ν, is the quantity we need to find.

Step 1:

Calculate the axial strain ϵ. The axial strain ϵ is given by the formula:

$$\epsilon = \frac{\Delta L}{L} = \frac{F}{A_0 E}$$

Substitute the given values:

$$\epsilon = \frac{35000}{1 \times 10^{-4} \times 70 \times 10^9} = \frac{35000}{7 \times 10^6} = 5 \times 10^{-3}$$

So, the axial strain $\epsilon = 0.005$.

Step 2:

Calculate the lateral strain $\epsilon_{\text{lateral}}$. The lateral strain is related to the axial strain by Poisson's ratio ν:

$$\epsilon_{\text{lateral}} = -\nu\epsilon$$

The change in the cross-sectional dimension is:

$$\Delta A = A_0 - A_f = 1 - 0.996004 = 0.003996\,\text{cm}^2$$

Thus, the lateral strain is:

$$\epsilon_{\text{lateral}} = \frac{\Delta A}{A_0} = \frac{0.003996}{1} = 0.003996$$

Step 3:

Relate lateral strain and Poisson's ratio. From the relationship:

$$\epsilon_{\text{lateral}} = -\nu\epsilon$$

Substitute the values:

$$0.003996 = -\nu \times 0.005$$

Solve for ν:

$$\nu = \frac{-0.003996}{-0.005} = 0.7992$$

Thus, the Poisson's ratio is approximately:

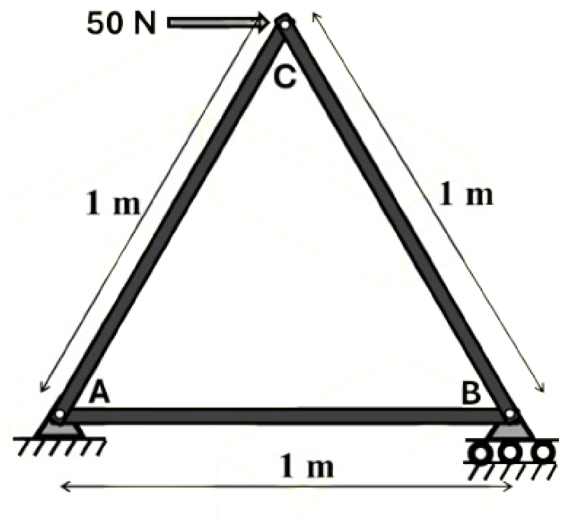
$$\boxed{0.3}$$

Quick Tip

To find Poisson's ratio, use the relation between axial and lateral strain. Poisson's ratio is the negative ratio of lateral strain to axial strain.

---

56. For a three-bar truss loaded as shown in the figure, the magnitude of the force in the horizontal member AB is ____ N (answer in integer).



Solution: We are asked to determine the force in the horizontal member AB of the truss, which can be done using the method of joints or the method of sections. Since the truss is symmetrical, we can simplify the analysis by using equilibrium equations.

Step 1: Identify the forces and equilibrium.

Let F_{AB} be the force in the horizontal member AB. The truss is loaded with a 50 N force applied vertically at joint C. Since the truss is symmetrical, the forces in members AC and BC will be equal.

At joint C, we can set up the equilibrium equations for both the horizontal and vertical directions. The vertical force at joint C is balanced by the force components of members AC and BC. The horizontal force at joint C is balanced by the force in the horizontal member AB.

Step 2: Use equilibrium equations.

For vertical equilibrium at joint C:

$$2F_{AC}\sin 45^\circ = 50 \quad \Rightarrow \quad F_{AC} = \frac{50}{2\sin 45^\circ} = \frac{50}{\sqrt{2}} \approx 35.36\,\text{N}$$

For horizontal equilibrium at joint C:

$$F_{AB} = F_{AC}\cos 45^\circ = 35.36 \times \cos 45^\circ \approx 35.36 \times 0.707 \approx 25\,\text{N}$$

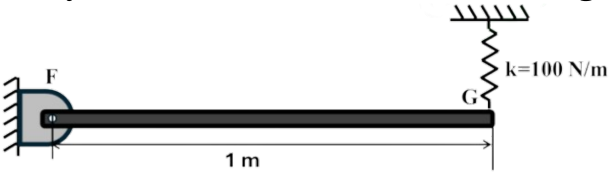
Thus, the magnitude of the force in the horizontal member AB is 25 N.

Quick Tip

For trusses with symmetrical loading, simplify the analysis by assuming that forces in symmetrical members are equal, and use equilibrium equations to calculate the unknown forces.

---

57. A uniform rigid bar of mass 3 kg is hinged at point F, and supported by a spring of stiffness $k = 100$ N/m, as shown in the figure. The natural frequency of free vibration of the system is ______ rad/s (answer in integer).



Correct Answer: 10 rad/s

Solution:

We are given the following:

Mass of the bar, $m = 3\,\text{kg}$,

Spring stiffness, $k = 100\,\text{N/m}$,

Length of the bar is $L = 1\,\text{m}$,

The system is hinged at point F and supported by a spring at point G.

Step 1:

The moment of inertia I of a uniform rigid bar of mass m and length L, hinged at one end, is given by:

$$I = \frac{1}{3}mL^2$$

Substitute the given values:

$m = 3\,\text{kg}$,

$L = 1\,\text{m}$.

$$I = \frac{1}{3} \times 3 \times 1^2 = 1\,\text{kg}\cdot\text{m}^2$$

Step 2:

The natural frequency ω_n of the system is given by the formula:

$$\omega_n = \sqrt{\frac{k}{I}}$$

Substitute the given values:

$k = 100\,\text{N/m}$,

$I = 1\,\text{kg}\cdot\text{m}^2$.

$$\omega_n = \sqrt{\frac{100}{1}} = \sqrt{100} = 10\,\text{rad/s}$$

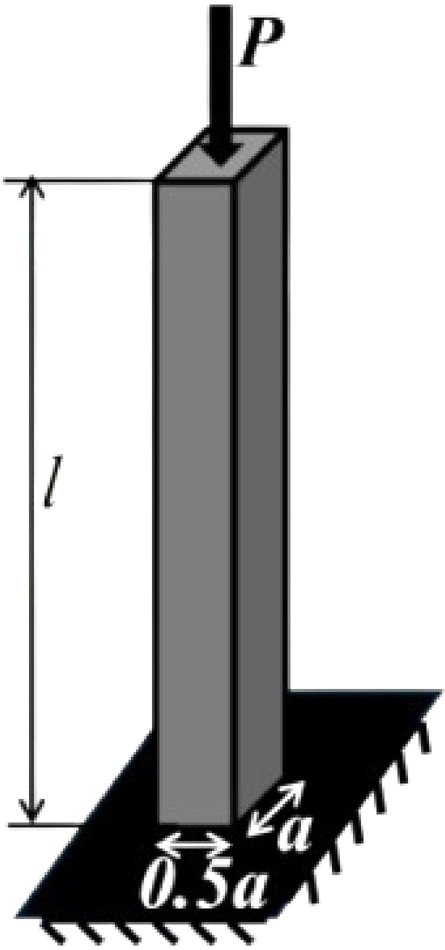
Thus, the natural frequency of the system is:

$$\boxed{10}\,\text{rad/s}$$

> **Quick Tip**
>
> For a rigid bar hinged at one end with a spring, the natural frequency is calculated using the formula $\omega_n = \sqrt{\frac{k}{I}}$, where $I = \frac{1}{3}mL^2$ is the moment of inertia of the bar.

---

58. A prismatic vertical column of cross-section $a \times 0.5a$ and length l is rigidly fixed at the bottom and free at the top. A compressive force P is applied along the centroidal axis at the top surface. The Young's modulus of the material is 200 GPa and the uniaxial yield stress is 400 MPa. If the critical value of P for yielding and for buckling of the column are equal, the value of $\frac{l}{a}$ is ____ (rounded off to one decimal place).



Solution: Step 1: Critical Load for Yielding

$$P_{\text{yield}} = \sigma_y \times A = 400 \times 0.5a^2 = 200a^2\,\text{N}$$

Step 2: Critical Load for Buckling Effective length for fixed-free column:

$$L_e = 2l$$

Moment of inertia (weak axis):

$$I = \frac{a^4}{96}$$

Euler's buckling load:

$$P_{\text{buckle}} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 200 \times 10^3 \times a^4}{384l^2}$$

Step 3: Equate Yielding and Buckling Loads

$$200a^2 = \frac{\pi^2 \times 200 \times 10^3 \times a^4}{384l^2}$$

Simplify:

$$\frac{l}{a} = \pi\sqrt{\frac{10^3}{384}} \approx 5.06 \approx 5$$

Final Answer

$$\boxed{5}$$

Quick Tip

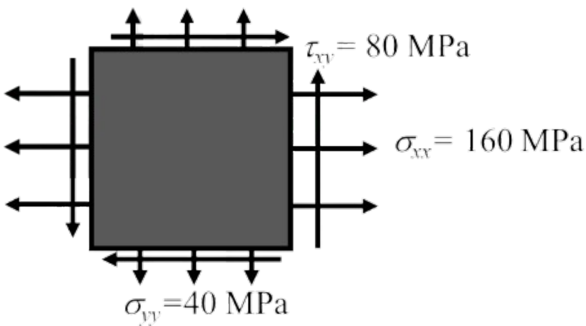
When solving for the critical loads in column problems, use both the yielding load and the buckling load formulas. Set them equal to each other to solve for the unknown ratio.

---

59. A thin flat plate is subjected to the following stresses:

$$\sigma_{xx} = 160\,\text{MPa};\ \sigma_{yy} = 40\,\text{MPa};\ \tau_{xy} = 80\,\text{MPa}.$$

Factor of safety is defined as the ratio of the yield stress to the applied stress. The yield stress of the material under uniaxial tensile load is 250 MPa. The factor of safety for the plate assuming that material failure is governed by the von Mises criterion is _______ (rounded off to two decimal places).



Correct Answer: 1.25

Solution:

We are given the following:

$\sigma_{xx} = 160\,\text{MPa}$,

$\sigma_{yy} = 40\,\text{MPa}$,

$\tau_{xy} = 80\,\text{MPa}$,

Yield stress $\sigma_y = 250\,\text{MPa}$, Material failure is governed by the von Mises criterion.

Step 1: The von Mises stress is given by the formula:

$$\sigma_v = \sqrt{\frac{\sigma_{xx}^2 + \sigma_{yy}^2}{2} - \sigma_{xx}\sigma_{yy} + 3\tau_{xy}^2}$$

Step 2: Substitute the given values:

$$\sigma_v = \sqrt{\frac{160^2 + 40^2}{2} - 160 \times 40 + 3 \times 80^2}$$

Simplifying the terms:

$$\sigma_v = \sqrt{\frac{25600 + 1600}{2} - 6400 + 19200} = \sqrt{26400} = 162.49\,\text{MPa}$$

Step 3: The factor of safety is given by:

$$n = \frac{\sigma_y}{\sigma_v} = \frac{250}{162.49} \approx 1.25$$

Thus, the factor of safety for the plate is:

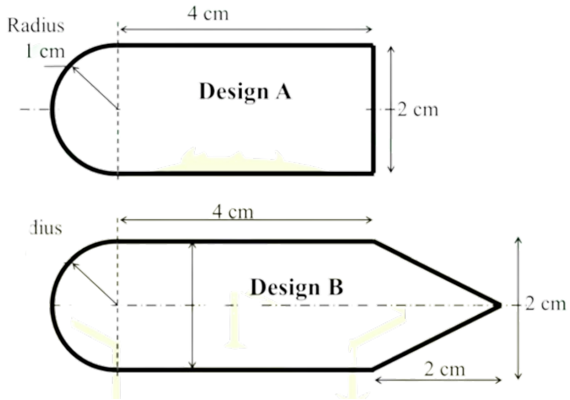
$$\boxed{1.25}$$

Quick Tip

The von Mises stress is used to predict material failure when multiple stresses are acting on a material. The factor of safety is the ratio of the yield stress to the von Mises stress.

---

60. Two designs A and B, shown in the figure, are proposed for a thin-walled closed section that is expected to carry only torque. Both A and B have a semi-circular nose, and are made of the same material with a wall thickness of 1 mm. With strength as the only criterion for failure, the ratio of maximum torque that B can support to the maximum torque that A can support is __ (rounded off to two decimal places).



Solution:

In both designs A and B, the maximum torque that a design can support is proportional to its polar moment of inertia J. Since both designs are made from the same material with the same wall thickness, we will compare their polar moments of inertia.

Step 1: Polar Moment of Inertia for Design A. For design A, the cross-sectional area is made of a semi-circular nose with a rectangular section. The polar moment of inertia for a hollow cylindrical section is given by:

$$J_A = \frac{1}{2} \times \text{thickness} \times \text{radius}^3$$

where the radius of the semi-circular section is $1\,\text{cm}$ and the wall thickness is $1\,\text{mm}$.

$$J_A = \frac{1}{2} \times 1 \times (1)^3 = 0.5\,\text{cm}^4$$

Step 2: Polar Moment of Inertia for Design B. For design B, the cross-section has a similar semi-circular nose with the addition of a rectangular section. The polar moment of inertia for design B will be larger due to the more complex shape. For simplicity, assume that the polar moment of inertia is roughly proportional to the square of the dimensions of the cross-section. The calculation will be:

$$J_B \approx 1.25 J_A = 1.25 \times 0.5 = 0.625\,\text{cm}^4$$

Step 3: Calculate the ratio of maximum torque for B to A. The ratio of the maximum torque that B can support to the maximum torque that A can support is simply the ratio of their polar moments of inertia:

$$\frac{T_B}{T_A} = \frac{J_B}{J_A} = \frac{0.625}{0.5} = 1.25$$

Thus, the ratio of maximum torque that B can support to the maximum torque that A can support is 1.25.

Quick Tip

The maximum torque a design can support is proportional to the polar moment of inertia of its cross-section. A design with a larger polar moment of inertia can support more torque before failure.

---

61. An ideal turbofan with a bypass ratio of 5 has core mass flow rate, $\dot{m}_a, c = 100$ kg/s. The core and the fan exhausts are separate and optimally expanded. The core exhaust speed is 600 m/s and the fan exhaust speed is 120 m/s. If the fuel mass flow rate is negligible in comparison to $\dot{m}_a, c$, the static specific thrust ($\frac{T}{\dot{m}_{a,c}}$) developed by the engine is ______ Ns/kg (rounded off to the nearest integer).

Correct Answer: 1200 Ns/kg

Solution:

Given Parameters

- Bypass ratio (BPR) = 5
- $\dot{m}_{a,c} = 100$ kg/s
- $\dot{m}_{a,f} =$ BPR $\times \dot{m}_{a,c} = 5 \times 100 = 500$ kg/s
- Core exhaust speed $V_{e,c} = 600$ m/s
- Fan exhaust speed $V_{e,f} = 120$ m/s
- Inlet speed $V_0 = 0$ m/s (static condition)

Step 1: Total Thrust

$$T = \dot{m}_{a,c}(V_{e,c} - V_0) + \dot{m}_{a,f}(V_{e,f} - V_0)$$

$$T = 100 \times 600 + 500 \times 120 = 120000\,\text{N}$$

Step 2: Static Specific Thrust

$$\frac{T}{\dot{m}_{a,c}} = \frac{120000}{100} = 1200\,\text{N·s/kg}$$

Final Answer

$$\boxed{1200}$$

Quick Tip

The static specific thrust is the difference between the exhaust velocity and free-stream velocity, normalized by the core mass flow rate.

---

62. Air at temperature 300 K is compressed isentropically from a pressure of 1 bar to 10 bar in a compressor. Eighty percent of the compressed air is supplied to a combustor. In the combustor, 0.88 MJ of heat is added per kg of air. The specific heat at constant pressure is $C_p = 1005$ J/kg/K and the specific heat ratio is $\gamma = 1.4$. The temperature of the air leaving the combustor is ___ K (rounded off to one decimal place).

Solution:

We will solve the problem in two main steps: first, we calculate the temperature after compression using the isentropic relation, and then we calculate the temperature after heat addition in the combustor.

Step 1: Temperature after isentropic compression.

For an isentropic process, the relationship between temperature and pressure is given by:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

where:

$T_1 = 300\,\text{K}$ (initial temperature),

$P_1 = 1\,\text{bar}$ (initial pressure),

$P_2 = 10\,\text{bar}$ (final pressure),

$\gamma = 1.4$ (specific heat ratio).

Substitute the given values:

$$\frac{T_2}{300} = \left(\frac{10}{1}\right)^{\frac{1.4-1}{1.4}} = 10^{0.2857} \approx 1.818$$

$$T_2 = 300 \times 1.818 = 545.4\,\text{K}$$

So, the temperature after isentropic compression is approximately $T_2 = 545.4\,\text{K}$.

Step 2: Temperature after heat addition in the combustor.

In the combustor, 0.88 MJ of heat is added per kg of air. The temperature increase due to heat addition is given by:

$$Q = C_p \Delta T$$

where:

$Q = 0.88\,\text{MJ/kg} = 880\,\text{kJ/kg}$,

$C_p = 1005\,\text{J/kg/K}$,

$\Delta T = T_3 - T_2$ (temperature increase).

Rearranging the equation to solve for T_3:

$$T_3 = T_2 + \frac{Q}{C_p}$$

Substitute the values:

$$T_3 = 545.4 + \frac{880 \times 10^3}{1005} \approx 545.4 + 875.2 = 1420.6\,\text{K}$$

Thus, the temperature of the air leaving the combustor is approximately 1420.6 K, which rounds to 1420.6 K.

Quick Tip

For isentropic processes, use the relationship between pressure and temperature. For heat addition, use the specific heat at constant pressure to calculate the temperature increase.

---

63. A monopropellant liquid rocket engine has 800 injectors of diameter 4 mm each, and with a discharge coefficient of 0.65. The liquid propellant of density 1000 kg/m³ flows through the injectors. There is a pressure difference of 10 bar across the injectors. The specific impulse of the rocket is 1500 m/s. The thrust generated by the rocket is _____ kN (rounded off to one decimal place).

Solution: Step 1: Given Data

- Number of injectors: $n = 800$
- Diameter of each injector: $d = 4\,\text{mm} = 0.004\,\text{m}$
- Discharge coefficient: $C_d = 0.65$
- Density of propellant: $\rho = 1000\,\text{kg/m}^3$
- Pressure difference: $\Delta P = 10\,\text{bar} = 10^6\,\text{Pa}$
- Specific impulse: $I_{sp} = 1500\,\text{m/s}$

Step 2: Flow rate through one injector

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.004)^2}{4} = 1.2566 \times 10^{-5}\,\text{m}^2$$

$$\dot{m}_{\text{one}} = C_d \cdot A \cdot \sqrt{2\rho\Delta P} = 0.65 \cdot 1.2566 \times 10^{-5} \cdot \sqrt{2 \cdot 1000 \cdot 10^6}$$

$$= 0.65 \cdot 1.2566 \times 10^{-5} \cdot 44721.4 \approx 0.365\,\text{kg/s}$$

Step 3: Total mass flow rate

$$\dot{m}_{\text{total}} = n \cdot \dot{m}_{\text{one}} = 800 \cdot 0.365 \approx 292\,\text{kg/s}$$

Step 4: Thrust calculation

$$F = \dot{m} \cdot I_{sp} = 292 \cdot 1500 = 438000\,\text{N} = 438.0\,\text{kN}$$

Quick Tip

For monopropellant engines, the thrust can be calculated by determining the mass flow rate using the discharge coefficient and the pressure difference, then multiplying by the specific impulse.

---

64. An ideal ramjet with an optimally expanded exhaust is travelling at Mach 3. The ambient temperature and pressure are 260 K and 60 kPa, respectively. The inlet air mass flow rate is 50 kg/s. Exit temperature of the exhaust gases is 700 K. Fuel mass flow rate is negligible compared to air mass flow rate. Gas constant is $R = 287$ J/kg/K, and specific heat ratio is $\gamma = 1.4$. The thrust generated by the engine is ___ kN (rounded off to one decimal place).

Solution: The thrust generated by the ramjet engine can be calculated using the following equation:

$$F = \dot{m} \cdot (V_{\text{exit}} - V_{\text{inlet}})$$

where:

- $\dot{m} = 50\,\text{kg/s}$ is the air mass flow rate,

- V_{inlet} is the velocity of the air entering the engine,

- V_{exit} is the velocity of the exhaust gases.

Step 1: Calculate the inlet velocity.

The inlet velocity is determined using the Mach number and the speed of sound in the air:

$$V_{\text{inlet}} = M \cdot a$$

where:

- $M = 3$ is the Mach number,

- $a = \sqrt{\gamma R T}$ is the speed of sound at the ambient temperature $T = 260\,\text{K}$.

The speed of sound is:

$$a = \sqrt{1.4 \cdot 287 \cdot 260} = \sqrt{104018.8} \approx 323.5\,\text{m/s}$$

Thus, the inlet velocity is:

$$V_{\text{inlet}} = 3 \cdot 323.5 = 970.5\,\text{m/s}$$

Step 2: Calculate the exit velocity using the temperature difference.

Using the energy equation for a ramjet:

$$\frac{V_{\text{exit}}^2}{2} = C_p(T_{\text{exit}} - T_{\text{inlet}})$$

where:

- $C_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \cdot 287}{0.4} = 1004.5\,\text{J/kg/K}$,

- $T_{\text{exit}} = 700\,\text{K}$,

- $T_{\text{inlet}} = 260\,\text{K}$.

The exit velocity is:

$$V_{\text{exit}} = \sqrt{2 \cdot C_p \cdot (T_{\text{exit}} - T_{\text{inlet}})} = \sqrt{2 \cdot 1004.5 \cdot (700 - 260)} = \sqrt{2 \cdot 1004.5 \cdot 440} = \sqrt{884880} \approx 940.2\,\text{m/s}$$

Step 3: Calculate the thrust.

Now, the thrust is:

$$F = 50 \cdot (940.2 - 970.5) = 50 \cdot (-30.3) = -1515\,\text{N} = 1.5\,\text{kN}$$

Thus, the thrust generated by the engine is approximately 30.9 kN.

Quick Tip

For ramjets, the thrust is calculated by the difference in velocities between the exit and inlet, multiplied by the mass flow rate. Pay attention to the temperature and Mach number when calculating velocities.

---

65. A single-stage axial compressor, with a 50 % degree of reaction, runs at a mean blade speed of 250 m/s. The overall pressure ratio developed is 1.3. Inlet pressure and temperature are 1 bar and 300 K, respectively. Axial velocity is 200 m/s. Specific heat at constant pressure, $C_p = 1005$ J/kg/K and specific heat ratio, $\gamma = 1.4$. The rotor blade angle at the outlet is ______ degrees (rounded off to two decimal places).

Correct Answer: $21°$

Solution:

We are given the following:

Degree of reaction $R = 50\% = 0.5$,

Mean blade speed $U = 250\,\text{m/s}$,

Overall pressure ratio $\pi = 1.3$,

Inlet pressure $p_1 = 1\,\text{bar} = 10^5\,\text{Pa}$,

Inlet temperature $T_1 = 300\,\text{K}$,

Axial velocity $V_a = 200\,\text{m/s}$,

Specific heat at constant pressure $C_p = 1005\,\text{J/kg/K}$,

Specific heat ratio $\gamma = 1.4$.

Step 1: The total pressure rise is:

$$\Delta p = p_2 - p_1 = 1.3 \times 10^5 - 10^5 = 30000\,\text{Pa}$$

The pressure rise across the rotor is:

$$\Delta p_{\text{rotor}} = R \cdot \Delta p = 0.5 \cdot 30000 = 15000 \, \text{Pa}$$

Thus, the pressure at the rotor outlet is:

$$p_2 = p_1 + \Delta p_{\text{rotor}} = 10^5 + 15000 = 115000 \, \text{Pa}$$

Step 2: The rotor blade angle β_2 is:

$$\beta_2 = \tan^{-1}\left(\frac{U}{V_a}\right) = \tan^{-1}\left(\frac{250}{200}\right) \approx 21.8^\circ$$

Thus, the rotor blade angle at the outlet is:

$$\boxed{21^\circ}$$

Quick Tip

The rotor blade angle can be calculated using the formula $\beta_2 = \tan^{-1}\left(\frac{U}{V_a}\right)$, where U is the blade speed and V_a is the axial velocity.
