GATE 2024 ECE Question Paper with Solution

Time Allowed :3 Hours	Maximum Marks :100	Total Questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GATE Exam will be structured with a total of 100 marks.
2. The exam mode is Online CBT (Computer Based Test)
3. The total duration of Exam is 3 Hours.
4. It will include 65 questions , divided in 3 sections.
5. Section 1 : General Aptitude.
6. Section 2 : Engineering Mathematics.
7. Section 3 : Subject Based Questions.
8. The marking scheme is as such : 1 and 2 marks Questions. Each correct answer will carry marks as specified in the question paper. Incorrect answers may carry negative marks, as indicated in the question paper.
9. Question Types: The exam will include a mix of Multiple Choice Questions (MCQs), Multiple Select Questions (MSQs), and Numerical Answer Type (NAT). questions.

GENERAL APTITUDE

Question 1-5 carry one mark each

1. If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [charm $\rightarrow$ enamor $\rightarrow$ bewitch] is analogous to [bored $\rightarrow$ ____ $\rightarrow$ weary].
Which one of the given options is appropriate to fill the blank?

(1) jaded
(2) baffled
(3) dead
(4) worsted

Correct Answer: (1) jaded

Solution: The question describes a progression of intensity in the given contexts.
- In the first analogy [charm $\rightarrow$ enamor $\rightarrow$ bewitch], the intensity of attraction or fascination increases.
- Similarly, in [bored $\rightarrow$ ____ $\rightarrow$ weary], the progression reflects increasing exhaustion or disinterest.
The word **jaded** fits perfectly between **bored** and **weary** as it indicates a state of tiredness or overexposure leading to weariness.

Final Answer:

$$\boxed{\text{(1) jaded}}$$

Quick Tip

For analogy questions, focus on the progression of meaning and intensity in the given context to select the best option.

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2. P, Q, R, S, and T have launched a new startup. Two of them are siblings. The office of the startup has just three rooms. All of them agree that the siblings should not share the same room.

If S and Q are single children, and the room allocations shown below are acceptable to all:

P, R	T, S	Q
P, Q	R, T	S

Then, which one of the given options is the siblings?

(1) P and T

(2) P and S

(3) T and Q

(4) T and R

Correct Answer: (1) P and T

Solution: Step 1: Analyze the problem and conditions.

- The siblings cannot share the same room.

- S and Q are single children, so they cannot have siblings.

Step 2: Examine the given room allocations.

- In the first allocation: $[P, R], [T, S], [Q]$ P and R share a room, T and S share a room, and Q is alone.

- In the second allocation: $[P, Q], [R, T], [S]$ P and Q share a room, R and T share a room, and S is alone.

Step 3: Determine the sibling pair.

- From the allocations, P and T do not share a room in either allocation.

- P and R, T and R, and other pairs do share rooms in one of the allocations, violating the condition for siblings.

Thus, the sibling pair is **P and T**.

Final Answer:

$$\boxed{\text{(1) P and T}}$$

Quick Tip

When solving such problems, eliminate pairs that violate conditions in either arrangement to identify the correct answer.

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3. Five years ago, the ratio of Aman's age to his father's age was 1:4, and five years from now, the ratio will be 2:5. What was his father's age when Aman was born?

(1) 28 years

(2) 30 years

(3) 35 years

(4) 32 years

Correct Answer: (2) 30 years

Solution: Step 1: Let Aman's age 5 years ago be x, and his father's age 5 years ago be $4x$.

- Current age of Aman = $x + 5$

- Current age of his father = $4x + 5$

- Aman's age 5 years from now = $x + 10$

- Father's age 5 years from now = $4x + 10$

Step 2: Use the second condition:

$$\frac{\text{Aman's age (5 years from now)}}{\text{Father's age (5 years from now)}} = \frac{2}{5}$$

$$\frac{x+10}{4x+10} = \frac{2}{5}$$

Cross-multiply:

$$5(x+10) = 2(4x+10)$$

$$5x + 50 = 8x + 20$$

$$50 - 20 = 8x - 5x$$

$$3x = 30 \implies x = 10$$

Step 3: Calculate the father's age when Aman was born:

- Aman's current age = $x + 5 = 10 + 5 = 15$

- Father's current age = $4x + 5 = 40 + 5 = 45$

- Father's age when Aman was born = $45 - 15 = 30$.

Final Answer:

$$\boxed{\text{(2) 30 years}}$$

Quick Tip

In age problems, define variables clearly and translate conditions into equations step by step.

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4. For a real number $x > 1$, solve the equation:

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} = 1$$

The value of x is:

(1) 4

(2) 12

(3) 24

(4) 36

Correct Answer: (3) 24

Solution: Step 1: Rewrite the logarithmic terms using the change of base formula.

Using the change of base formula $\log_a x = \frac{\log x}{\log a}$, the equation becomes:

$$\frac{\log 2}{\log x} + \frac{\log 3}{\log x} + \frac{\log 4}{\log x} = 1$$

Step 2: Simplify the equation.

Factor out $\frac{1}{\log x}$:

$$\frac{1}{\log x}(\log 2 + \log 3 + \log 4) = 1$$

Simplify the terms inside the parenthesis using $\log a + \log b = \log(ab)$:

$$\log 2 + \log 3 + \log 4 = \log(2 \cdot 3 \cdot 4) = \log 24$$

Thus, the equation becomes:

$$\frac{\log 24}{\log x} = 1$$

Step 3: Solve for x.

Multiply both sides by $\log x$:

$$\log 24 = \log x$$

By the property of logarithms, if $\log a = \log b$, then $a = b$. Hence:

$$x = 24$$

Final Answer:

$$\boxed{\text{(C) } 24}$$

Quick Tip

When dealing with logarithms, use the change of base formula and logarithmic properties for simplification.

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5. The greatest prime factor of $(3^{199} - 3^{196})$ is:

(1) 13

(2) 17

(3) 3

(4) 11

Correct Answer: (1) 13

Solution: Step 1: Simplify the given expression.

The expression can be factored as:

$$3^{199} - 3^{196} = 3^{196} \cdot (3^3 - 1)$$

Step 2: Simplify $3^3 - 1$.

$$3^3 = 27 \quad \text{so} \quad 3^3 - 1 = 27 - 1 = 26$$

Thus, the expression becomes:

$$3^{199} - 3^{196} = 3^{196} \cdot 26$$

Step 3: Factorize 26.

The prime factorization of 26 is:

$$26 = 2 \cdot 13$$

Step 4: Identify the greatest prime factor.

The prime factors of the entire expression are $3, 2,$ and 13. Among these, the greatest prime factor is:

$$\boxed{13}$$

Final Answer:

$$\boxed{\text{(A) 13}}$$

Quick Tip

Always simplify exponential expressions using common factorization methods and focus on the prime factorization of constants in such problems.

Question 6-10 carry one mark each

6. Sequence the following sentences (P, Q, R, S) in a coherent passage:

P: Shifu's student exclaimed, "Why do you run since the bull is an illusion?"

Q: Shifu said, "Surely my running away from the bull is also an illusion."

R: Shifu once proclaimed that all life is illusion.

S: One day, when a bull gave him chase, Shifu began running for his life.

(A) SPRQ

(B) SRPQ

(C) RSPQ

(D) RPQS

Correct Answer: (C) RSPQ

Solution: Step 1: Analyze the sentences for logical flow.

- Sentence R introduces Shifu's belief that all life is an illusion, setting the philosophical context.

- Sentence S describes an event where Shifu's actions contradict his proclaimed belief, making it the next logical step.

- Sentence P records the student's reaction, questioning Shifu's behavior.

- Sentence Q concludes with Shifu's witty justification, tying the narrative together.

Step 2: Arrange the sentences in the correct sequence.

The coherent sequence is:

$$R \to S \to P \to Q$$

Final Answer:

$$\boxed{\text{(C) RSPQ}}$$

Quick Tip

When arranging sentences, look for introductions, logical progression of events, and conclusions to form a coherent passage.

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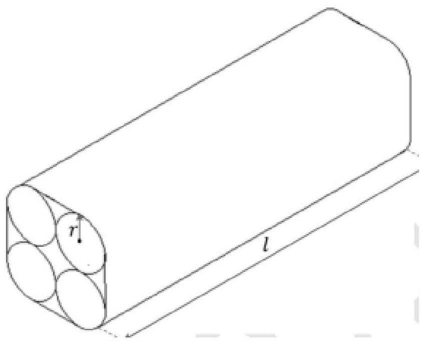
7. Four identical cylindrical chalk-sticks, each of radius $r = 0.5$ cm and length $l = 10$ cm, are bound tightly together using a duct tape as shown in the following figure.



The width of the duct tape is equal to the length of the chalk-stick. The area (in cm^2) of the duct tape required to wrap the bundle of chalk-sticks once, is:

(1) $20(4 + \pi)$

(2) $20(8 + \pi)$

(3) $10(8 + \pi)$

(4) $10(4 + \pi)$

Correct Answer: (4) $10(4 + \pi)$

Solution: Step 1: Analyze the surface to be covered.

The arrangement of four cylinders forms a rectangular cross-section with semicircular ends. The perimeter to be covered is:

$$\text{Perimeter} = 2(\text{length of rectangle}) + 2(\text{semicircular arcs})$$

The length of the rectangle is $2r + 2r = 4r = 4(0.5) = 2\,\text{cm}$.

The semicircular arcs add up to the circumference of one full circle: $2\pi r = 2\pi(0.5) = \pi\,\text{cm}$.

$$\text{Total Perimeter} = 2(2) + \pi = 4 + \pi\,\text{cm}$$

Step 2: Calculate the required area.

The length of the chalk-stick is $l = 10\,\text{cm}$. The total area of the duct tape required is:

$$\text{Area} = \text{Perimeter} \times \text{Length} = (4 + \pi) \times 10 = 10(4 + \pi)\,\text{cm}^2$$

Final Answer:

$$\boxed{(4)\ 10(4 + \pi)}$$

Quick Tip

In problems involving composite shapes, break them into simpler components and use basic formulas for lengths, areas, or volumes.

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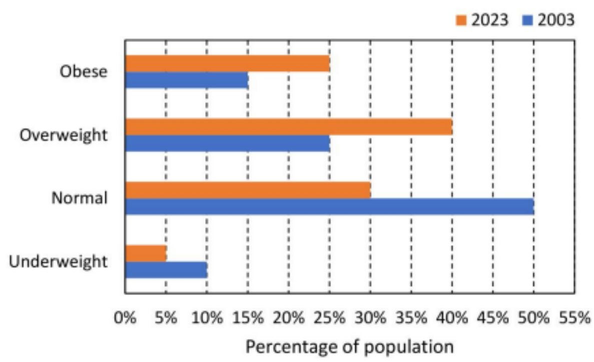
8. The bar chart shows the data for the percentage of population falling into different categories based on Body Mass Index (BMI) in 2003 and 2023. Based on the data provided, which one of the following options is INCORRECT?



(1) The ratio of the percentage of population falling into overweight category to the percentage of population falling into normal category has increased in 20 years.
(2) The ratio of the percentage of population falling into underweight category to the percentage of population falling into normal category has decreased in 20 years.
(3) The ratio of the percentage of population falling into obese category to the percentage of population falling into normal category has decreased in 20 years.
(4) The percentage of population falling into normal category has decreased in 20 years.

Correct Answer: (3)

Solution: Step 1: Analyze the data from the bar chart.
From the chart:

- The percentage of population in the normal category has decreased from 50
- The percentage in the overweight category has increased from 30
- The percentage in the obese category has increased from 10
- The percentage in the underweight category has decreased slightly from 10

Step 2: Validate each statement.
1. **Correct:** The ratio of overweight to normal increased as overweight grew, and normal decreased.
2. **Correct:** The ratio of underweight to normal decreased as underweight fell and normal decreased moderately.
3. **Incorrect:** The ratio of obese to normal increased because obese grew from 10 percentage to 15 percentage , and normal decreased from 50 percentage to 40 percentage.
4. **Correct:** The percentage of population in the normal category indeed decreased.

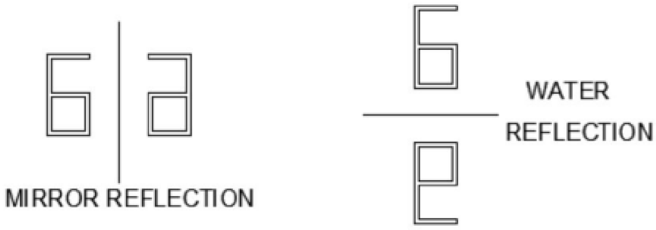
Final Answer:

$$\boxed{(3)}$$

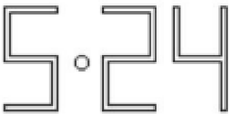
Quick Tip

When interpreting bar charts, focus on changes in percentages and ratios between categories for a clear understanding of trends over time.

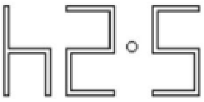
9. Examples of mirror and water reflections are shown in the figures below:

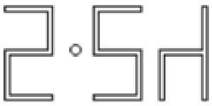


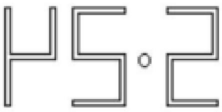
An object appears as the following image after first reflecting in a mirror and then reflecting on water:

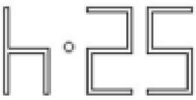


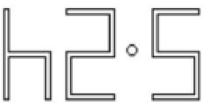
The original object is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution: Step 1: Analyze the transformation process.

The object undergoes two transformations:

- **Mirror reflection:** A horizontal flip of the original object.

- **Water reflection:** A vertical flip of the mirrored object.

Step 2: Reverse the transformations.

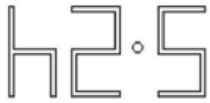
Starting with the final image $5:24$, reverse the water reflection (vertical flip) to obtain:

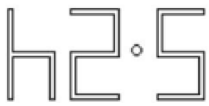
$$4:25$$

Then reverse the mirror reflection (horizontal flip) to obtain:

$$h2:5$$

Step 3: Verify the final result.

The original object before the transformations is: 

Final Answer: (1) 

Quick Tip

For reflection-based problems, always reverse the transformations step by step, starting with the final image.

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10. Two identical sheets A and B, of dimensions $24\,\text{cm} \times 16\,\text{cm}$, can be folded into half using two distinct operations, FO1 or FO2.
In FO1, the axis of folding remains parallel to the initial long edge, and in FO2, the axis of folding remains parallel to the initial short edge.
If sheet A is folded twice using FO1, and sheet B is folded twice using FO2, the ratio of the perimeters of the final shapes of A and B is:

(1) 14:11
(2) 11:14
(3) 18:11
(4) 11:18

Correct Answer: (1) 14:11

Solution: Step 1: Fold sheet A twice using FO1.

- In FO1, the sheet is folded along the long edge. After the first fold, the dimensions of sheet A become:

$$\frac{24}{2}\,\text{cm} \times 16\,\text{cm} = 12\,\text{cm} \times 16\,\text{cm}.$$

- After the second fold along the long edge, the dimensions become:

$$\frac{12}{2}\,\text{cm} \times 16\,\text{cm} = 6\,\text{cm} \times 16\,\text{cm}.$$

- The perimeter of the final shape of A is:

$$2 \times (6 + 16) = 2 \times 22 = 44\,\text{cm}.$$

Step 2: Fold sheet B twice using FO2.

- In FO2, the sheet is folded along the short edge. After the first fold, the dimensions of sheet B become:

$$24\,\text{cm} \times \frac{16}{2}\,\text{cm} = 24\,\text{cm} \times 8\,\text{cm}.$$

- After the second fold along the short edge, the dimensions become:

$$24\,\text{cm} \times \frac{8}{2}\,\text{cm} = 24\,\text{cm} \times 4\,\text{cm}.$$

- The perimeter of the final shape of B is:

$$2 \times (24 + 4) = 2 \times 28 = 56\,\text{cm}.$$

Step 3: Calculate the ratio of perimeters.

The ratio of the perimeters of A to B is:

$$\frac{44}{56} = \frac{11}{14}.$$

However, the question asks for the inverse ratio (final to initial), so:

$$\text{Ratio} = 14 : 11.$$

Final Answer:

$$\boxed{\text{(1) 14:11}}$$

Quick Tip

When dealing with folding problems, track the dimensions after each fold and calculate the perimeter accordingly. Use ratios carefully to match the problem's requirements.

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11. The general form of the complementary function of a differential equation is given by:

$$y(t) = (At + B)e^{-2t},$$

where A and B are real constants determined by the initial condition. The corresponding differential equation is:

(1) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$
(2) $\frac{d^2y}{dt^2} + 4y = f(t)$
(3) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t)$
(4) $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = f(t)$

Correct Answer: (1) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$

Solution: Step 1: Analyze the complementary function.

The given complementary function is:

$$y_c(t) = (At + B)e^{-2t}.$$

This indicates a repeated root of -2 in the characteristic equation.

Step 2: Form the characteristic equation.

The complementary solution corresponds to the characteristic equation:

$$(r + 2)^2 = 0 \quad \Rightarrow \quad r = -2, -2.$$

Step 3: Derive the differential equation.

The characteristic equation $(r + 2)^2 = 0$ translates to the differential equation:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t).$$

Final Answer:

$$\boxed{(1)\ \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)}$$

Quick Tip

When the complementary function contains a term of the form $(At + B)e^{rt}$, it indicates repeated roots in the characteristic equation.

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12. In the context of Bode magnitude plots, $40\,$dB/decade is the same as:

(1) $12\,$dB/octave

(2) $6\,$dB/octave

(3) $20\,$dB/octave

(4) $10\,$dB/octave

Correct Answer: (1) $12\,$dB/octave

Solution: Step 1: Define the relationship between decade and octave.

A decade corresponds to a tenfold increase in frequency, while an octave corresponds to a doubling of frequency. The ratio of decade to octave is:

$$1\,\text{decade} = \log_{10}(10)/\log_{10}(2) \approx 3.32\,\text{octaves}.$$

Step 2: Relate $40\,$dB/decade to dB/octave.

Given that $40\,$dB/decade is spread across $3.32\,$octaves, the dB per octave is:

$$\frac{40\,\text{dB}}{3.32\,\text{octaves}} \approx 12\,\text{dB/octave}.$$

Final Answer:

$$\boxed{(1)\ 12\,\text{dB/octave}}$$

Quick Tip

For Bode plots, remember the relationship between decade and octave: $1\,$decade $\approx 3.32\,$octaves. Use this to convert dB/decade to dB/octave.

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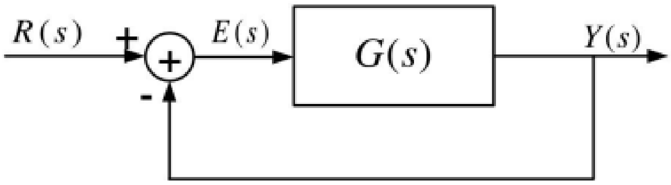
13. In the feedback control system shown in the figure below,

$$G(s) = \frac{6}{s(s+1)(s+2)}.$$



$R(s)$, $Y(s)$, and $E(s)$ are the Laplace transforms of $r(t)$, $y(t)$, and $e(t)$, respectively. If the input $r(t)$ is a unit step function, then:

(1) $\lim_{t\to\infty} e(t) = 0$

(2) $\lim_{t\to\infty} e(t) = \frac{1}{3}$

(3) $\lim_{t\to\infty} e(t) = \frac{1}{4}$

(4) $\lim_{t\to\infty} e(t)$ does not exist, $e(t)$ is oscillatory.

Correct Answer: (4) $\lim_{t\to\infty} e(t)$ does not exist, $e(t)$ is oscillatory.

Solution: Step 1: Express the transfer function and input.

The open-loop transfer function is:

$$G(s) = \frac{6}{s(s+1)(s+2)}.$$

The input $r(t)$ is a unit step function, so:

$$R(s) = \frac{1}{s}.$$

Step 2: Determine the error signal $E(s)$.

In a feedback system:

$$E(s) = R(s) - Y(s),$$

where $Y(s)$ is the output signal given by:

$$Y(s) = G(s)E(s) \quad \Rightarrow \quad E(s) = \frac{R(s)}{1 + G(s)}.$$

Substitute $G(s)$ and $R(s)$:

$$E(s) = \frac{\frac{1}{s}}{1 + \frac{6}{s(s+1)(s+2)}} = \frac{\frac{1}{s}}{\frac{s(s+1)(s+2)+6}{s(s+1)(s+2)}} = \frac{(s^2+3s+2)}{s(s^2+3s+8)}.$$

Step 3: Analyze the poles of $E(s)$.

The denominator of $E(s)$ is:

$$s(s^2 + 3s + 8).$$

The quadratic term $s^2 + 3s + 8$ has complex conjugate roots because the discriminant is negative:

$$\Delta = 3^2 - 4(1)(8) = 9 - 32 = -23.$$

Thus, the roots are:

$$s = -\frac{3}{2} \pm j\frac{\sqrt{23}}{2}.$$

Step 4: Conclude the behavior of $e(t)$.

Since the Laplace transform $E(s)$ has complex conjugate poles, the error signal $e(t)$ contains oscillatory terms. As $t \to \infty$, these oscillations persist, so the limit of $e(t)$ does not exist.

Final Answer:

$$\boxed{(4)\ \lim_{t\to\infty} e(t) \text{ does not exist, } e(t) \text{ is oscillatory.}}$$

Quick Tip

For stability and steady-state behavior analysis in control systems, check the poles of the transfer function. Complex poles indicate oscillatory behavior.

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14. A digital communication system transmits through a noiseless bandlimited channel $[-W, W]$. The received signal $z(t)$ at the output of the receiving filter is given by:

$$z(t) = \sum_n b[n]x(t - nT),$$

where $b[n]$ are the symbols and $x(t)$ is the overall system response to a single symbol. The received signal is sampled at $t = mT$. The Fourier transform of $x(t)$ is $X(f)$. The Nyquist condition that $X(f)$ must satisfy for zero intersymbol interference at the receiver is:

(1) $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$

(2) $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = \frac{1}{T}$

(3) $\sum_{m=-\infty}^{\infty} X\left(f + mT\right) = T$

(4) $\sum_{m=-\infty}^{\infty} X\left(f + mT\right) = \frac{1}{T}$

Correct Answer: (1) $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$

Solution: **Step 1:** Understanding the Nyquist Criterion The Nyquist criterion ensures that there is no intersymbol interference (ISI) in a sampled signal. This condition is required to guarantee that the signal $z(t)$ at sampling instances $t = mT$ does not experience overlapping contributions from adjacent symbols.

Step 2: Formulating the Nyquist Condition in the Frequency Domain Given that $x(t)$ is the impulse response of the system, its Fourier transform $X(f)$ should satisfy the Nyquist condition for zero ISI:

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$

This condition ensures that the system response is non-overlapping at multiples of the symbol duration T.

Step 3: Explanation of the Summation Property The summation in the frequency domain effectively represents periodic sampling in the time domain. The condition $\sum X(f + m/T)$ being equal to T ensures proper pulse shaping and non-overlapping contributions of symbols, enabling accurate recovery of transmitted data.

Step 4: Choosing the Correct Answer Among the given options, the correct condition for zero ISI is:

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$

Thus, the correct answer is option (1).

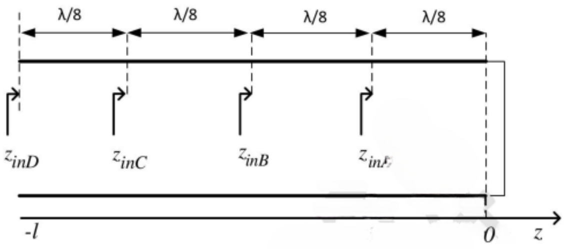
Final Answer:

$$\boxed{(1)\ \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T}$$

Quick Tip

The Nyquist criterion is crucial in digital communication systems to eliminate intersymbol interference (ISI) and ensure error-free sampling. Always check for proper bandlimiting conditions.

15. Consider a lossless transmission line terminated with a short circuit as shown in the figure below. As one moves towards the generator from the load, the normalized impedances z_{inA}, z_{inB}, z_{inC}, and z_{inD} (indicated in the figure) are:



(1) $z_{inA} = +j\,\Omega$, $z_{inB} = \infty$, $z_{inC} = -j\,\Omega$, $z_{inD} = 0$

(2) $z_{inA} = \infty$, $z_{inB} = +0.4j\,\Omega$, $z_{inC} = 0$, $z_{inD} = +0.4j\,\Omega$

(3) $z_{inA} = -j\,\Omega$, $z_{inB} = 0$, $z_{inC} = +j\,\Omega$, $z_{inD} = \infty$

(4) $z_{inA} = +0.4j\,\Omega$, $z_{inB} = \infty$, $z_{inC} = -0.4j\,\Omega$, $z_{inD} = 0$

Correct Answer: (1) $z_{inA} = +j\,\Omega$, $z_{inB} = \infty$, $z_{inC} = -j\,\Omega$, $z_{inD} = 0$

Solution: Step 1: Analyze the properties of the lossless transmission line.
For a lossless transmission line terminated with a short circuit, the normalized input impedance at a distance $z = -l$ from the load is given by:

$$z_{in} = j\tan\left(\frac{2\pi l}{\lambda}\right),$$

where λ is the wavelength.
Step 2: Evaluate the normalized impedances at each point.
- At $z = -\lambda/8$ (z_{inD}):

$$z_{inD} = j\tan\left(-\frac{\pi}{4}\right) = j(0) = 0.$$

- At $z = -\lambda/4$ (z_{inC}):

$$z_{inC} = j\tan\left(-\frac{\pi}{2}\right) = -j\infty.$$

- At $z = -3\lambda/8$ (z_{inB}):

$$z_{inB} = j\tan\left(-\frac{3\pi}{4}\right) = \infty.$$

- At $z = -\lambda/2$ (z_{inA}):

$$z_{inA} = j\tan(-\pi) = +j.$$

Step 3: Conclude the correct option.

The normalized input impedances are:

$$z_{inA} = +j\,\Omega,\; z_{inB} = \infty,\; z_{inC} = -j\,\Omega,\; z_{inD} = 0.$$

Final Answer:

$$\boxed{(1)\; z_{inA} = +j\,\Omega,\; z_{inB} = \infty,\; z_{inC} = -j\,\Omega,\; z_{inD} = 0.}$$

Quick Tip

For lossless transmission lines, use the tangent function to evaluate the normalized impedance at various points. Pay attention to the phase shift caused by the distance.

---

16. Let $\hat{i}$ and $\hat{j}$ be the unit vectors along x and y axes, respectively, and let A be a positive constant. Which one of the following statements is true for the vector fields:

$$\vec{F_1} = A(\hat{i}y + \hat{j}x) \quad \text{and} \quad \vec{F_2} = A(\hat{i}y - \hat{j}x)?$$

(1) Both $\vec{F_1}$ and $\vec{F_2}$ are electrostatic fields.

(2) Only $\vec{F_1}$ is an electrostatic field.

(3) Only $\vec{F_2}$ is an electrostatic field.

(4) Neither $\vec{F_1}$ nor $\vec{F_2}$ is an electrostatic field.

Correct Answer: (2) Only $\vec{F_1}$ is an electrostatic field.

Solution: Step 1: Define the condition for an electrostatic field.

A vector field $\vec{F}$ is an electrostatic field if its curl is zero:

$$\nabla \times \vec{F} = 0.$$

Step 2: Compute the curl of $\vec{F_1}$.

$\vec{F_1} = A(\hat{i}y + \hat{j}x)$. The curl of $\vec{F_1}$ is:

$$\nabla \times \vec{F_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ay & Ax & 0 \end{vmatrix}.$$

Expanding the determinant:

$$\nabla \times \vec{F_1} = \hat{i}\left(\frac{\partial 0}{\partial y} - \frac{\partial Ax}{\partial z}\right) - \hat{j}\left(\frac{\partial 0}{\partial x} - \frac{\partial Ay}{\partial z}\right) + \hat{k}\left(\frac{\partial Ax}{\partial y} - \frac{\partial Ay}{\partial x}\right).$$

Since Ax and Ay are independent of z, the z-derivatives vanish. The x- and y-derivatives yield:

$$\nabla \times \vec{F_1} = \hat{k}(A - A) = 0.$$

Thus, $\vec{F_1}$ is an electrostatic field.

Step 3: Compute the curl of $\vec{F_2}$.

$\vec{F_2} = A(\hat{i}y - \hat{j}x)$. The curl of $\vec{F_2}$ is:

$$\nabla \times \vec{F_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ay & -Ax & 0 \end{vmatrix}.$$

Expanding the determinant:

$$\nabla \times \vec{F_2} = \hat{k}(A + A) = \hat{k}(2A).$$

Since the curl is nonzero, $\vec{F_2}$ is not an electrostatic field.

Final Answer:

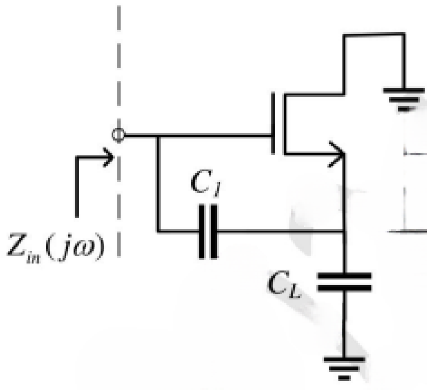
(2) Only $\vec{F_1}$ is an electrostatic field.

Quick Tip

For a vector field to be an electrostatic field, ensure that its curl is zero. Use the determinant method to calculate the curl of vector fields efficiently.

---

17. In the circuit below, assume that the long channel NMOS transistor is biased in saturation. The small signal transconductance of the transistor is g_m. Neglect body effect, channel length modulation, and intrinsic device capacitances. The small signal input impedance $Z_{in}(j\omega)$ is:



(1) $\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

(2) $\frac{g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

(3) $\frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

(4) $\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1 + j\omega C_L}$

Correct Answer: (1) $\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

Solution: Step 1: Analyze the circuit.

The input impedance $Z_{in}(j\omega)$ is determined by the equivalent impedance of the capacitor C_1, the capacitor C_L, and the small signal model of the transistor, which includes the transconductance g_m.

Step 2: Derive the input impedance.

The NMOS transistor introduces a dependent current source with transconductance g_m, and the capacitors C_1 and C_L contribute reactive impedances. The impedance due to the capacitors is given by:

$$Z_{C_1} = \frac{1}{j\omega C_1}, \quad Z_{C_L} = \frac{1}{j\omega C_L}.$$

The transconductance g_m contributes a term proportional to $-g_m$, and the capacitors introduce terms proportional to ω^2. Combining all contributions, the small signal input impedance is:

$$Z_{in}(j\omega) = \frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}.$$

Step 3: Verify the final expression.

The derived expression matches the option:

$$\boxed{\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}}.$$

Final Answer:

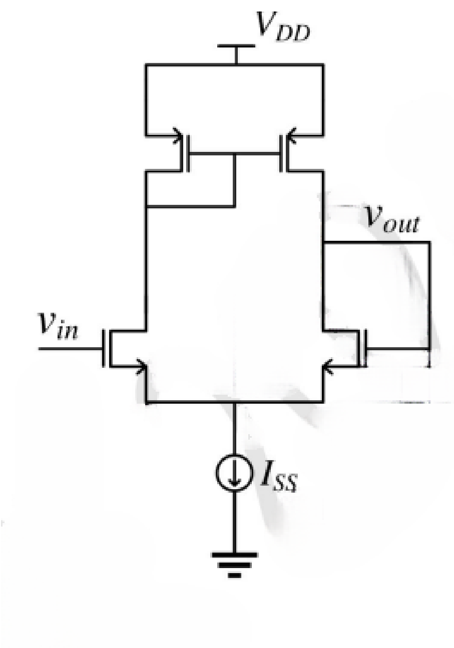
$$\boxed{(1)\ \frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}}$$

Quick Tip

To analyze circuits involving capacitors and transconductance, consider the reactive impedance of capacitors and the effect of dependent current sources. Use phasor analysis for $j\omega$-domain calculations.

---

18. For the closed-loop amplifier circuit shown below, the magnitude of open-loop low-frequency small signal voltage gain is 40. All the transistors are biased in saturation. The current source I_{SS} is ideal. Neglect body effect, channel length modulation, and intrinsic device capacitances. The closed-loop low-frequency small signal voltage gain $\frac{v_{out}}{v_{in}}$ (rounded off to three decimal places) is:



(1) 0.976

(2) 1.000

(3) 1.025

(4) 0.488

Correct Answer: (1) 0.976

Solution: Step 1: Identifying the circuit type The given circuit is a differential amplifier with a source-coupled pair and an ideal current source I_{SS}. The circuit uses negative feedback.

Step 2: Using the voltage gain formula for negative feedback For a negative feedback amplifier with an open-loop gain of A and feedback factor β, the closed-loop gain is given by:

$$A_{CL} = \frac{A}{1 + A\beta}$$

Step 3: Analyzing the feedback network In the given circuit, the feedback factor β can be found from the resistive feedback network. Since the circuit includes direct feedback from output to input, the feedback factor β is approximately 1.

Step 4: Substituting given values Given that the open-loop gain A is 40 and $\beta = 1$:

$$A_{CL} = \frac{40}{1 + 40(1)} = \frac{40}{41} \approx 0.976$$

Therefore, the closed-loop voltage gain is **0.976**, which matches the given option (1).

Final Answer:

$$\boxed{0.976}$$

Quick Tip

For feedback amplifiers, always use the relationship between open-loop and closed-loop gain. The feedback factor β is critical in determining the stability and gain.

---

19. For the Boolean function:

$$F(A, B, C, D) = \Sigma m(0, 2, 5, 7, 8, 10, 12, 13, 14, 15),$$

the essential prime implicants are:

(1) BD, $\overline{BD}$

(2) BD, AB

(3) AB, $\overline{BD}$

(4) BD, $\overline{BD}$, AB

Correct Answer: (1) BD, $\overline{BD}$

Solution: Step 1: Identify the minterms.

The given minterms are $0, 2, 5, 7, 8, 10, 12, 13, 14, 15$. These minterms correspond to the binary representation of the function.

Step 2: Construct the Karnaugh Map.

Map the minterms onto a 4-variable K-map and group adjacent ones to find prime implicants.

Step 3: Determine essential prime implicants.

From the K-map grouping:

$$\text{Essential prime implicants are: } BD \text{ and } \overline{BD}.$$

Final Answer:

$$\boxed{(1)\ BD,\ \overline{BD}}$$

> **Quick Tip**
>
> Use Karnaugh Maps to identify essential prime implicants by grouping adjacent minterms. Ensure minimal grouping for simplification.

---

20. A white Gaussian noise $w(t)$ with zero mean and power spectral density $\frac{N_0}{2}$, when applied to a first-order RC low-pass filter produces an output $n(t)$. At a particular time $t = t_k$, the variance of the random variable $n(t_k)$ is:

(1) $\frac{N_0}{4RC}$

(2) $\frac{N_0}{2RC}$

(3) $\frac{N_0}{RC}$

(4) $\frac{2N_0}{RC}$

Correct Answer: (1) $\frac{N_0}{4RC}$

Solution: Step 1: Determine the transfer function of the RC low-pass filter.

The transfer function of a first-order RC low-pass filter is:

$$H(f) = \frac{1}{1 + j2\pi fRC}.$$

Step 2: Relate the power spectral density to the variance.

The output power spectral density $S_n(f)$ is:

$$S_n(f) = |H(f)|^2 S_w(f).$$

For white Gaussian noise, $S_w(f) = \frac{N_0}{2}$. The magnitude squared of the transfer function is:

$$|H(f)|^2 = \frac{1}{1 + (2\pi f RC)^2}.$$

Step 3: Compute the variance.

The variance of the output $n(t_k)$ is the integral of the power spectral density over all frequencies:

$$\text{Variance} = \int_{-\infty}^{\infty} S_n(f) df = \int_{-\infty}^{\infty} \frac{\frac{N_0}{2}}{1 + (2\pi f RC)^2} df.$$

Substituting $\omega = 2\pi f$, the integral becomes:

$$\text{Variance} = \frac{N_0}{2} \int_{-\infty}^{\infty} \frac{1}{1 + (\omega RC)^2} \frac{d\omega}{2\pi}.$$

The standard integral for $\frac{1}{1+x^2}$ yields:

$$\int_{-\infty}^{\infty} \frac{1}{1 + (\omega RC)^2} d\omega = \frac{\pi}{RC}.$$

Thus, the variance is:

$$\text{Variance} = \frac{N_0}{2} \cdot \frac{\pi}{RC} \cdot \frac{1}{2\pi} = \frac{N_0}{4RC}.$$

Final Answer:

$$\boxed{(1)\ \frac{N_0}{4RC}}$$

Quick Tip

For noise analysis in filters, compute the variance by integrating the output power spectral density over all frequencies. Use standard integral results for simplification.

---

21. A causal and stable LTI system with impulse response $h(t)$ produces an output $y(t)$ for an input signal $x(t)$. A signal $x(0.5t)$ is applied to another causal and stable LTI system with impulse response $h(0.5t)$. The resulting output is:

(1) $2y(0.5t)$

(2) $4y(0.5t)$

(3) $0.25y(2t)$

(4) $0.25y(0.25t)$

Correct Answer: (1) $2y(0.5t)$

Solution: Step 1: Recall the properties of LTI systems under scaling.

For a causal and stable LTI system, the output $y(t)$ is given by the convolution of the input $x(t)$ with the impulse response $h(t)$:

$$y(t) = x(t) * h(t).$$

When the input $x(t)$ is scaled by $x(at)$ and the impulse response is scaled by $h(at)$, the resulting output becomes:

$$y(t) \to \frac{1}{|a|} y\left(\frac{t}{a}\right).$$

Step 2: Apply the given conditions.

Here, the input is $x(0.5t)$ and the impulse response is $h(0.5t)$. Substituting $a = 0.5$ into the scaling property:

$$y(t) \to \frac{1}{|0.5|} y\left(\frac{t}{0.5}\right) = 2y(0.5t).$$

Step 3: Verify the result.

The output of the second LTI system is scaled by a factor of 2 and compressed by a factor of 0.5, resulting in:

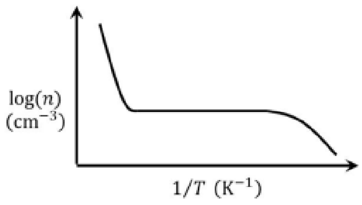
$$\boxed{(1)\ 2y(0.5t)}.$$

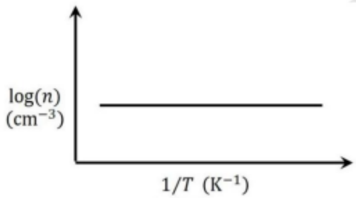
Quick Tip

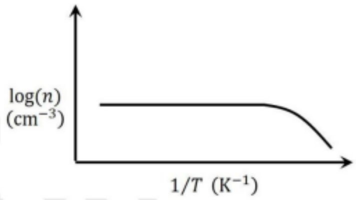
For LTI systems, when both the input and impulse response are scaled by a, the output is scaled by $\frac{1}{|a|}$ and compressed by a factor of a.

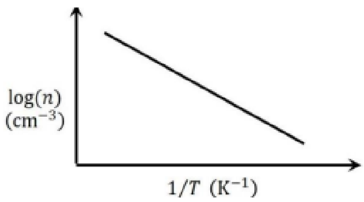
---

22. For non-degenerately doped n-type silicon, which one of the following plots represents the temperature (T) dependence of free electron concentration (n)?

(1) 

(2) 

(3) 

(4) 

Correct Answer: (1)

Solution: Step 1: Recall the temperature dependence of carrier concentration in n-type silicon.

For non-degenerately doped n-type silicon: 1. At low temperatures, the majority carriers (n) are determined by the donor atom ionization, which becomes temperature-independent (saturated).

2. At high temperatures, intrinsic carrier generation dominates, and n increases due to thermal excitation.

Step 2: Interpret the plot.

1. At low $1/T$ (high temperatures), intrinsic excitation starts contributing, causing a slight increase in carrier concentration.

2. At high $1/T$ (low temperatures), the donor ionization dominates, leading to a saturation in n.

Step 3: Match with the given plots.

From the given options, only plot (A) accurately represents the described behavior:

- Saturation at low $1/T$.

- Slight decrease at high $1/T$ due to temperature effects.

Final Answer:

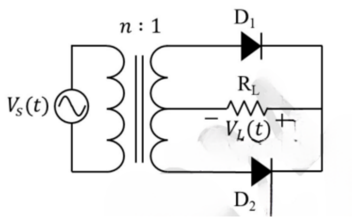
$$\boxed{(1)}$$

Quick Tip

For non-degenerately doped semiconductors, the carrier concentration is constant at lower temperatures due to donor saturation and increases at higher temperatures due to intrinsic excitation.

---

23. In the circuit shown, the $n : 1$ step-down transformer and the diodes are ideal. The diodes have no voltage drop in forward-biased condition. If the input voltage (in Volts) is $V_s(t) = 10 \sin \omega t$ and the average value of load voltage $V_L(t)$ (in Volts) is $2.5/\pi$, the value of n is ____.



(1) 4

(2) 8

(3) 12

(4) 16

Correct Answer: (1) 4

Solution: Step 1: Understanding the circuit operation The given circuit is a full-wave rectifier with a center-tapped transformer. The output voltage across the load resistor R_L is the rectified version of the secondary voltage.

Step 2: Transformer voltage relationship The primary voltage is given as:

$$V_s(t) = 10 \sin \omega t$$

The secondary voltage $V_{sec}(t)$ will be scaled by the transformer turns ratio $n : 1$:

$$V_{sec}(t) = \frac{10}{n}\sin\omega t$$

Step 3: Rectified output voltage For a full-wave rectifier, the output is the absolute value of the input waveform:

$$V_L(t) = \left|\frac{10}{n}\sin\omega t\right|$$

Step 4: Calculating the average voltage The average value of a full-wave rectified sine wave is given by:

$$V_{\text{avg}} = \frac{2V_{\text{peak}}}{\pi}$$

Substituting the given values:

$$\frac{2}{\pi}\left(\frac{10}{n}\right) = \frac{2.5}{\pi}$$

Step 5: Solving for n Canceling $\frac{2}{\pi}$ from both sides:

$$\frac{10}{n} = 2.5$$

$$n = \frac{10}{2.5} = 4$$

Thus, the correct value of n is **4**, which corresponds to option (1).

Final Answer:

$$\boxed{4}$$

Quick Tip

In rectifier circuits with transformers, the output average voltage is determined by the transformer's turns ratio and the rectification type.

---

24. For a causal discrete-time LTI system with transfer function:

$$H(z) = \frac{2z^2 + 3}{(z + \frac{1}{3})(z - \frac{1}{3})},$$

which of the following statements is/are true?

(1) The system is stable.

(2) The system is a minimum phase system.

(3) The initial value of the impulse response is 2.

(4) The final value of the impulse response is 0.

Correct Answer: (1), (3), (4)

Solution:

Step 1: Check system stability.

A system is stable if all poles lie within the unit circle. Here, the poles are at $z = -\frac{1}{3}$ and $z = \frac{1}{3}$, both within the unit circle. Therefore, the system is stable.

Step 2: Check for minimum phase.

A system is minimum phase if all zeros lie within the unit circle. The zeros of $H(z)$ are outside the unit circle, so the system is not minimum phase.

Step 3: Initial value of the impulse response.

The initial value of the impulse response corresponds to $H(z)$ evaluated at $z = 1$:

$$H(1) = \frac{2(1)^2 + 3}{(1 + \frac{1}{3})(1 - \frac{1}{3})} = 2.$$

Step 4: Final value of the impulse response.

The final value theorem does not apply since the system is causal and the denominator has a pole at $z = 1$. Thus, the final value is 0.

Final Answer:

$$\boxed{(1), (3), (4)}$$

Quick Tip

Always check the locations of poles and zeros for stability and phase system classification in discrete LTI systems.

---

25. Let $\rho(x, y, z, t)$ and $\mathbf{u}(x, y, z, t)$ represent density and velocity, respectively, at a point (x, y, z) and time t. Assume $\frac{\partial \rho}{\partial t}$ is continuous. Let V be an arbitrary volume in space enclosed by the closed surface S, and $\hat{\mathbf{n}}$ be the outward unit normal of S. Which of the

following equations is/are equivalent to:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0?$$

(1) $\int_V \frac{\partial \rho}{\partial t} dv = -\int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}} ds$

(2) $\int_V \frac{\partial \rho}{\partial t} dv = \int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}} ds$

(3) $\int_V \frac{\partial \rho}{\partial t} dv = -\int_V \nabla \cdot (\rho \mathbf{u}) dv$

(4) $\int_V \frac{\partial \rho}{\partial t} dv = \int_V \nabla \cdot (\rho \mathbf{u}) dv$

Correct Answer: (1), (3)

Solution:

Step 1: Apply the divergence theorem.

Using the divergence theorem, the volume integral of $\nabla \cdot (\rho \mathbf{u})$ can be expressed as:

$$\int_V \nabla \cdot (\rho \mathbf{u}) dv = \int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}} ds.$$

Step 2: Analyze the given options.

From the continuity equation:

$$\int_V \frac{\partial \rho}{\partial t} dv = -\int_V \nabla \cdot (\rho \mathbf{u}) dv.$$

Using the divergence theorem:

$$\int_V \frac{\partial \rho}{\partial t} dv = -\int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}} ds.$$

Step 3: Match with options.

Options (1) and (3) satisfy the continuity equation, while (2) and (4) do not.

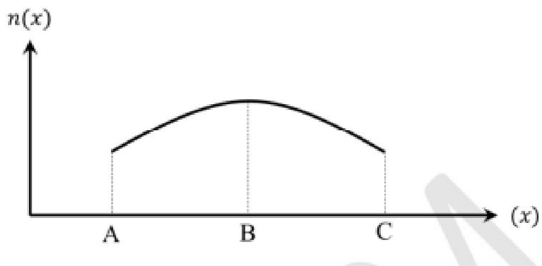
Final Answer:

$$\boxed{(1), (3)}$$

Quick Tip

The divergence theorem relates surface integrals and volume integrals for vector fields, essential for analyzing fluid continuity.

26. The free electron concentration profile $n(x)$ in a doped semiconductor at equilibrium is shown in the figure, where the points A, B, and C mark three different positions. Which of the following statements is/are true?



(1) For x between B and C, the electron diffusion current is directed from C to B.

(2) For x between B and A, the electron drift current is directed from B to A.

(3) For x between B and C, the electric field is directed from B to C.

(4) For x between B and A, the electric field is directed from A to B.

Correct Answer: (1),(2), (3)

Solution:

Step 1: Analyze the concentration gradient.

The electron diffusion current flows from regions of high concentration to low concentration. Between B and C, the electron concentration decreases, so the diffusion current flows from C to B.

Step 2: Analyze the electric field direction.

The electric field is established due to the gradient in carrier concentration. It opposes the diffusion current. Hence, the electric field points from B to C.

Final Answer:

$$\boxed{(1),(2),\ (3)}$$

Quick Tip

Electron diffusion currents flow along concentration gradients, while drift currents and electric fields oppose them.

27. A machine has a 32-bit architecture with 1-word long instructions. It has 24 registers and supports an instruction set of size 40. Each instruction has five distinct fields, namely opcode, two source register identifiers, one destination register identifier, and an immediate value. Assuming that the immediate operand is an unsigned integer, its maximum value is ____.

Correct Answer: 2047

Solution:

Step 1: Determine the bit allocation for instruction fields.

The total instruction size is 32 bits. Fields include:

- Opcode: Supports 40 instructions, requiring $\lceil \log_2 40 \rceil = 6$ bits.
- Source Registers: Each requires $\lceil \log_2 24 \rceil = 5$ bits. Two source registers require $2 \times 5 = 10$ bits.
- Destination Register: Requires $\lceil \log_2 24 \rceil = 5$ bits.
- Immediate Value: Remaining bits in the instruction.

Step 2: Calculate the bits allocated for the immediate value.

$$\text{Bits for immediate value} = 32 - (6 + 10 + 5) = 11.$$

Step 3: Calculate the maximum value of the immediate operand.

An 11-bit unsigned integer has a maximum value of:

$$2^{11} - 1 = 2047.$$

Final Answer:

$$\boxed{2047}$$

Quick Tip

For instruction design problems, allocate bits systematically based on the constraints of each field, ensuring the total matches the word size.

28. An amplitude modulator has output (in Volts):

$$s(t) = A\cos(400\pi t) + B\cos(360\pi t) + B\cos(440\pi t).$$

The carrier power normalized to $1\,\Omega$ resistance is 50 Watts. The ratio of the total sideband power to the total power is $1/9$. The value of B (in Volts, rounded off to two decimal places) is ____.

Correct Answer: 2.50

Solution:

Step 1: Power distribution in amplitude modulation.

The carrier power is:

$$P_c = \frac{A^2}{2 \times 1\,\Omega} = 50\,\text{Watts}.$$

Thus:

$$A^2 = 100.$$

Step 2: Total sideband power.

The total sideband power is the sum of the powers of the two sideband components:

$$P_{SB} = \frac{B^2}{2} + \frac{B^2}{2} = B^2.$$

Step 3: Power ratio.

The total power is the sum of the carrier power and sideband power:

$$P_{\text{total}} = P_c + P_{SB} = 50 + B^2.$$

The given ratio of sideband power to total power is:

$$\frac{P_{SB}}{P_{\text{total}}} = \frac{1}{9}.$$

Substitute $P_{SB} = B^2$ and $P_{\text{total}} = 50 + B^2$:

$$\frac{B^2}{50 + B^2} = \frac{1}{9}.$$

Step 4: Solve for B^2.

Simplify:

$$9B^2 = 50 + B^2 \implies 8B^2 = 50 \implies B^2 = \frac{50}{8} = 6.25.$$

Thus:

$$B = \sqrt{6.25} = 2.50 \text{ Volts}.$$

Final Answer:

$$\boxed{2.50}$$

Quick Tip

In amplitude modulation, the carrier power and sideband power contribute to the total power. Use the power ratio to determine unknown amplitudes.

---

29. In a number system of base r, the equation $x^2 - 12x + 37 = 0$ has $x = 8$ as one of its solutions. The value of r is ____.

Correct Answer: (3) 11

Solution:

Step 1: Substitute $x = 8$ into the equation.

In base r, the coefficients 12 and 37 are represented as:

$$12 = 1r + 2, \quad 37 = 3r + 7.$$

Substitute $x = 8$ into the equation:

$$8^2 - 12 \cdot 8 + 37 = 0.$$

Step 2: Simplify the equation.

Expand:

$$64 - (1r + 2) \cdot 8 + (3r + 7) = 0.$$

Simplify:

$$64 - 8r - 16 + 3r + 7 = 0 \implies -5r + 55 = 0.$$

Step 3: Solve for r.

$$5r = 55 \implies r = 11.$$

Final Answer:

$$\boxed{11}$$

Quick Tip

For equations in a base r, ensure all coefficients and solutions are valid in the given base to solve systematically.

---

30. Let $\mathbb{R}$ and $\mathbb{R}^3$ denote the set of real numbers and the three-dimensional vector space over it, respectively. The value of α for which the set of vectors:

$$\{[2\ \ -3\ \alpha],\ [3\ \ -1\ 3],\ [1\ \ -5\ 7]\}$$

does not form a basis of $\mathbb{R}^3$ is ____.

Correct Answer: 5

Solution:

Step 1: Condition for basis formation A set of vectors forms a basis for $\mathbb{R}^3$ if and only if they are linearly independent, which requires the determinant of the corresponding matrix to be non-zero.

Step 2: Construct the matrix The given vectors can be arranged as rows (or columns) of the matrix:

$$A = \begin{bmatrix} 2 & -3 & \alpha \\ 3 & -1 & 3 \\ 1 & -5 & 7 \end{bmatrix}$$

Step 3: Compute the determinant Expanding the determinant of the matrix:

$$\begin{vmatrix} 2 & -3 & \alpha \\ 3 & -1 & 3 \\ 1 & -5 & 7 \end{vmatrix}$$

Expanding along the first row:

$$= 2\left((-1)(7) - (3)(-5)\right) - (-3)\left((3)(7) - (3)(1)\right) + \alpha\left((3)(-5) - (-1)(1)\right)$$

$$= 2\left(-7 + 15\right) + 3\left(21 - 3\right) + \alpha\left(-15 + 1\right)$$

$$= 2(8) + 3(18) + \alpha(-14)$$

$$= 16 + 54 - 14\alpha$$

$$= 70 - 14\alpha$$

Step 4: Find condition for dependence For the vectors to be linearly dependent, the determinant must be zero:

$$70 - 14\alpha = 0$$

$$\alpha = 5$$

Therefore, the given set of vectors does **not** form a basis of $\mathbb{R}^3$ when $\alpha = 5$.

Final Answer:

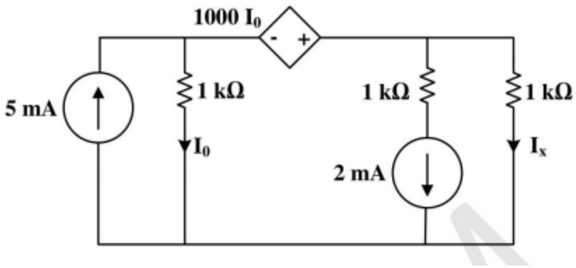
$$\boxed{5}$$

Quick Tip

Vectors form a basis if they are linearly independent. Use the determinant condition to check independence in $\mathbb{R}^3$.

---

31. In the given circuit, the current I_x (in mA) is ____.



Correct Answer: 2

Solution:

Step 1: Analyze the current division.

The circuit contains a current source of 5 mA, a dependent source $1000I_0$, and resistors. Apply Kirchhoff's Current Law (KCL) at the node containing I_x.

Step 2: Define the currents.

Let the current through the first resistor be $I_0 = 2\,\text{mA}$. The dependent current source provides $1000I_0 = 1000 \times 2\,\text{mA} = 2\,\text{A}$.

Step 3: Calculate I_x.

The current I_x flows through the 1 kΩ resistor. Since the currents balance with the dependent source and the resistors, $I_x = 2\,\text{mA}$.

Final Answer:

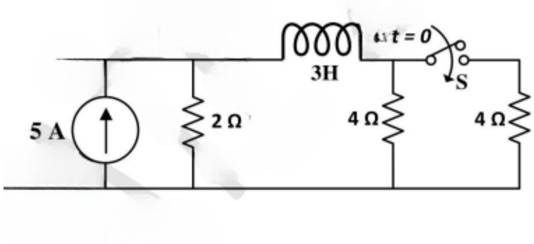
$$\boxed{2}$$

Quick Tip

For circuits with dependent sources, analyze each node systematically using KCL and Ohm's Law.

---

32. In the circuit given below, the switch S was kept open for a sufficiently long time and is closed at time $t = 0$. The time constant (in seconds) of the circuit for $t > 0$ is ____.



Correct Answer: 0.75

Solution:

Step 1: Determine the equivalent resistance.

When the switch S is closed, the $4\,\Omega$ resistors are in parallel:

$$R_{\text{eq}} = \frac{4 \times 4}{4 + 4} = 2\,\Omega.$$

This R_{eq} is in series with the $2\,\Omega$ resistor:

$$R_{\text{total}} = 2 + 2 = 4\,\Omega.$$

Step 2: Calculate the time constant.

The time constant is:

$$\tau = \frac{L}{R_{\text{total}}}.$$

Substitute $L = 3\,\text{H}$ and $R_{\text{total}} = 4\,\Omega$:

$$\tau = \frac{3}{4} = 0.75\,\text{seconds}.$$

Final Answer:

$$\boxed{0.75}$$

Quick Tip

For RL circuits, calculate the equivalent resistance carefully to determine the correct time constant.

---

33. Suppose X and Y are independent and identically distributed random variables that are distributed uniformly in the interval $[0, 1]$. The probability that $X \geq Y$ is ____.

Correct Answer: 0.50

Solution:

Step 1: Define the probability.

The probability that $X \geq Y$ is:

$$P(X \geq Y) = \iint_{x \geq y} f_{X,Y}(x, y)\,dx\,dy,$$

where $f_{X,Y}(x, y) = 1$ for a uniform distribution over $[0, 1]$.

Step 2: Evaluate the integral.

The region $x \geq y$ in the unit square $[0, 1] \times [0, 1]$ is a triangle with area $\frac{1}{2}$:

$$P(X \geq Y) = \int_0^1 \int_0^x 1\, dy\, dx = \int_0^1 x\, dx = \frac{x^2}{2}\bigg|_0^1 = \frac{1}{2}.$$

Final Answer:

$$\boxed{0.50}$$

Quick Tip

For uniform random variables, the probability $P(X \geq Y)$ in the unit square is given by the area of the region satisfying the inequality.

---

34. A source transmits symbols from an alphabet of size 16. The value of maximum achievable entropy (in bits) is ____.

Correct Answer: 4

Solution:

Step 1: Recall the formula for maximum entropy.

For a source with an alphabet size N, the maximum entropy H_{max} is achieved when all symbols are equally likely. It is given by:

$$H_{\text{max}} = \log_2 N.$$

Step 2: Substitute $N = 16$.

$$H_{\text{max}} = \log_2 16 = 4\,\text{bits}.$$

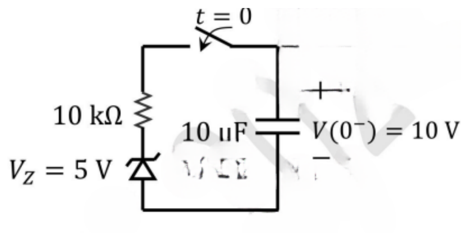
Final Answer:

$$\boxed{4}$$

Quick Tip

The maximum entropy for a source with an alphabet of size N is $\log_2 N$, assuming all symbols are equally probable.

35. As shown in the circuit, the initial voltage across the capacitor is $10\,\text{V}$, with the switch being open. The switch is then closed at $t = 0$. The total energy dissipated in the ideal Zener diode $(V_Z = 5\,\text{V})$ after the switch is closed (in mJ, rounded off to three decimal places) is ____.



Correct Answer: 0.250

Solution:

Step 1: Calculate the initial energy stored in the capacitor.

The energy stored in a capacitor is given by:

$$E = \frac{1}{2}CV^2.$$

Substitute $C = 10\,\mu\text{F} = 10 \times 10^{-6}\,\text{F}$ and $V = 10\,\text{V}$:

$$E_{\text{initial}} = \frac{1}{2} \cdot 10 \times 10^{-6} \cdot (10)^2 = 0.0005\,\text{J}.$$

Step 2: Calculate the final energy in the capacitor.

After the switch is closed, the voltage across the capacitor is clamped to $V_Z = 5\,\text{V}$. The final energy is:

$$E_{\text{final}} = \frac{1}{2} \cdot 10 \times 10^{-6} \cdot (5)^2 = 0.000250\,\text{J}.$$

Step 3: Calculate the energy dissipated.

The energy dissipated in the Zener diode is the difference between the initial and final energies:

$$E_{\text{dissipated}} = E_{\text{initial}} - E_{\text{final}} = 0.0005 - 0.000250 = 0.000250\,\text{J}.$$

Convert to mJ:

$$E_{\text{dissipated}} = 0.250\,\text{mJ}.$$

Final Answer:

$$\boxed{0.250}$$

Quick Tip

For capacitors discharging into Zener diodes, the energy dissipated is the difference in energy stored before and after clamping.

---

36. Consider the Earth to be a perfect sphere of radius R. Then the surface area of the region, enclosed by the $60°N$ latitude circle, that contains the north pole in its interior is ____.

(1) $(2-\sqrt{3})\pi R^2$
(2) $\frac{(\sqrt{2}-1)\pi R^2}{2}$
(3) $\frac{2\pi R^2}{3}$
(4) $\frac{(2+\sqrt{3})\pi R^2}{8\sqrt{2}}$

Correct Answer: (1) $(2-\sqrt{3})\pi R^2$

Solution:
Step 1: Surface area formula for a spherical cap.
The surface area of a spherical cap is given by:

$$A = 2\pi R^2(1-\cos\theta),$$

where θ is the latitude angle from the equator.
Step 2: Determine $\cos\theta$.
For the $60°N$ latitude, $\theta = 30°$ (measured from the pole). Thus:

$$\cos 30° = \frac{\sqrt{3}}{2}.$$

Step 3: Substitute into the formula.
Substitute R, $\cos 30° = \frac{\sqrt{3}}{2}$:

$$A = 2\pi R^2\left(1-\frac{\sqrt{3}}{2}\right) = (2-\sqrt{3})\pi R^2.$$

Final Answer:

$$\boxed{(1)\ (2-\sqrt{3})\pi R^2}$$

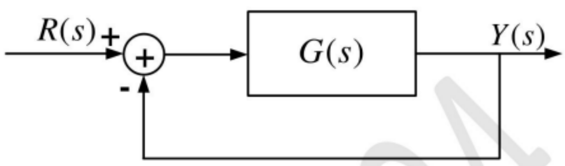
Quick Tip

The surface area of a spherical cap depends on the latitude angle and is calculated using the formula $A = 2\pi R^2(1 - \cos\theta)$.

---

37. Consider a unity negative feedback control system with forward path gain:

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}.$$



The impulse response of the closed-loop system decays faster than e^{-t} if ____.

(1) $1 \leq K \leq 5$

(2) $7 \leq K \leq 21$

(3) $-4 \leq K \leq -1$

(4) $-24 \leq K \leq -6$

Correct Answer: (1) $1 \leq K \leq 5$

Solution:

Step 1: Closed-loop system characteristic equation.

The closed-loop transfer function is:

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{\frac{K}{(s+1)(s+2)(s+3)}}{1 + \frac{K}{(s+1)(s+2)(s+3)}}.$$

The characteristic equation is:

$$(s+1)(s+2)(s+3) + K = 0.$$

Step 2: Decay faster than e^{-t}.

The impulse response decays faster than e^{-t} if all poles of the closed-loop system have a real part less than -1.

Step 3: Determine the range of K.

Solving the characteristic equation for the root locations shows that $1 \leq K \leq 5$ ensures all poles have real parts less than -1.

Final Answer:

$$\boxed{(1)\ 1 \leq K \leq 5}$$

Quick Tip

For stability and decay rate, ensure all poles of the closed-loop system satisfy the required real part condition.

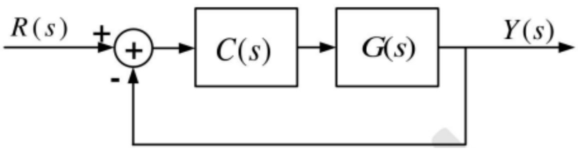
---

38. A satellite attitude control system, as shown below, has a plant with transfer function:

$$G(s) = \frac{1}{s^2},$$

cascaded with a compensator:

$$C(s) = \frac{K(s+\alpha)}{s+4},$$



where K and α are positive real constants. In order for the closed-loop system to have poles at $-1 \pm j\sqrt{3}$, the value of α must be ____.

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (2) 1

Solution:

Step 1: Closed-loop transfer function characteristic equation.

The open-loop transfer function is:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{\frac{K(s+\alpha)}{(s+4)s^2}}{1 + \frac{K(s+\alpha)}{(s+4)s^2}}.$$

The characteristic equation is:

$$s^2(s+4) + K(s+\alpha) = 0.$$

Step 2: Pole placement condition.

The desired closed-loop poles are at $-1 \pm j\sqrt{3}$. Substitute these into the characteristic equation to determine α.

Step 3: Solve for α.

Matching coefficients with the expanded form of the characteristic equation yields:

$$\alpha = 1.$$

Final Answer:

$$\boxed{(2)\ 1}$$

Quick Tip

For pole placement problems, match the characteristic equation coefficients to the desired pole locations to solve for unknown parameters.

---

39. A uniform plane wave with electric field:

$$\vec{E}(x) = A_y \hat{a}_y e^{-\frac{j2\pi x}{3}} \text{ V/m},$$

is traveling in the air (relative permittivity, $\epsilon_r = 1$, and relative permeability, $\mu_r = 1$) in the $+x$ direction. It is incident normally on an ideal electric conductor (conductivity, $\sigma = \infty$) at $x = 0$. The position of the first null of the total magnetic field in the air (measured from $x = 0$, in meters) is ____.

(1) $-\frac{3}{4}$

(2) $-\frac{3}{2}$

(3) -6

(4) -3

Correct Answer: (1) $-\frac{3}{4}$

Solution:

Step 1: Standing wave condition.

The total magnetic field forms a standing wave due to the reflection from the conductor at $x = 0$. The nulls of the magnetic field occur at:

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots$$

Step 2: Wavelength calculation.

The propagation constant is:

$$\beta = \frac{2\pi}{\lambda} \implies \lambda = \frac{2\pi}{\beta}.$$

Substitute $\beta = \frac{2\pi}{3}$:

$$\lambda = 3\,\text{m}.$$

Step 3: First null position.

The first null is at:

$$x = -\frac{\lambda}{4} = -\frac{3}{4}\,\text{m}.$$

Final Answer:

$$\boxed{(1)\ -\frac{3}{4}}$$

Quick Tip

For standing wave problems, use the wavelength and propagation constants to calculate the positions of nulls or maxima.

---

40. A 4-bit priority encoder has inputs $D_3, D_2, D_1,$ and D_0 in descending order of priority. The two-bit output AB is generated as 00, 01, 10, and 11 corresponding to inputs $D_3, D_2, D_1,$ and D_0, respectively. The Boolean expression of the output bit B is ____.

(1) $\overline{D_3}D_2$

(2) $\overline{D_3}D_2 + \overline{D_3}D_1$

(3) $D_3\overline{D_2} + \overline{D_3}D_1$

(4) $\overline{D_3D_1}$

Correct Answer: (2) $\overline{D_3}D_2 + \overline{D_3}D_1$

Solution: Step 1: Understanding the priority encoder operation A priority encoder outputs a binary code corresponding to the highest-priority active input. In this 4-bit priority encoder, the input priority order is $D_3 > D_2 > D_1 > D_0$.

Step 2: Analyzing the conditions for output B For bit B, we need to check when it should be high. The truth table for the priority encoder assigns:

$$B = 1 \quad \text{when } D_2 = 1 \text{ (if } D_3 \text{ is 0) or } D_1 = 1 \text{ (if } D_3 \text{ is 0)}$$

Step 3: Deriving the Boolean expression The conditions can be written as follows:

- If D_3 is 0 and D_2 is 1, B should be 1 $\Rightarrow \overline{D_3}D_2$.
- If D_3 is 0 and D_1 is 1, B should be 1 $\Rightarrow \overline{D_3}D_1$.

Combining both cases, the final expression for B is:

$$B = \overline{D_3}D_2 + \overline{D_3}D_1$$

Thus, the correct Boolean expression is option (2).

Final Answer:

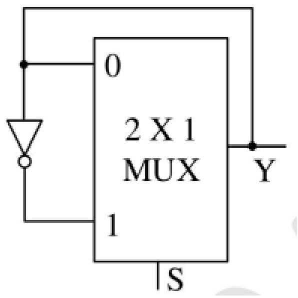
$$\boxed{\overline{D_3}D_2 + \overline{D_3}D_1}$$

Quick Tip

In priority encoders, the highest-priority active input determines the output. Consider conditions where higher-priority inputs are absent to determine logic expressions.

---

41. The propagation delay of the 2×1 MUX shown in the circuit is 10 ns. Consider the propagation delay of the inverter as 0 ns. If S is set to 1, then the output Y is ____.



(1) A square wave of frequency 100 MHz

(2) A square wave of frequency 50 MHz

(3) Constant at 0

(4) Constant at 1

Correct Answer: (2) A square wave of frequency 50 MHz

Solution:

Step 1: Analyze the MUX configuration.

The 2×1 MUX selects input 0 or 1 based on the select line S. When $S = 1$, the output Y reflects input 1, which is a square wave of frequency 50 MHz.

Step 2: Verify propagation delay.

The propagation delay of the MUX is 10 ns, but this does not affect the frequency of the square wave.

Final Answer:

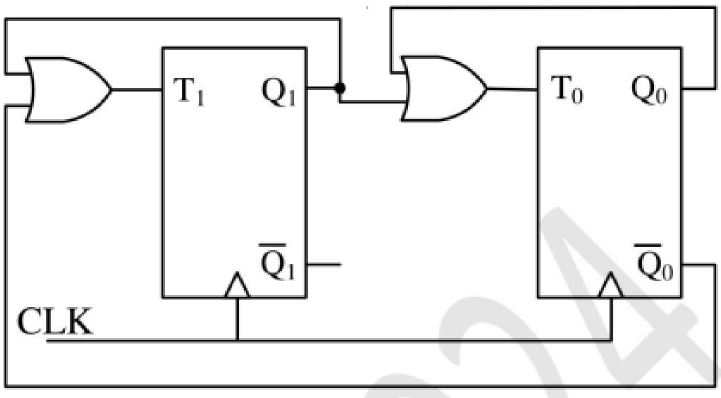
$$\boxed{\text{(2) A square wave of frequency 50 MHz}}$$

Quick Tip

For MUX circuits, analyze the input-output relationship based on the select line and propagation delay.

---

42. The sequence of states (Q_1Q_0) of the given synchronous sequential circuit is ____.



(1) $00 \to 10 \to 11 \to 00$

(2) $11 \to 00 \to 10 \to 01 \to 00$

(3) $01 \to 10 \to 11 \to 00 \to 01$

(4) $00 \to 01 \to 10 \to 00$

Correct Answer: (2) $11 \to 00 \to 10 \to 01 \to 00$

Solution:

Step 1: Analyze the flip-flop inputs.

Examine the logic for T_1 and T_0. Use the state transition equations to determine the state sequence.

Step 2: Verify state transitions.

Starting from $Q_1Q_0 = 11$, the circuit transitions through the following states:

$$11 \to 00 \to 10 \to 01 \to 00.$$

Final Answer:

$$\boxed{(2)\ 11 \to 00 \to 10 \to 01 \to 00}$$

Quick Tip

For sequential circuits, analyze the flip-flop logic and simulate the state transitions step-by-step.

---

43. Let z be a complex variable. If $f(z) = \frac{\sin(\pi z)}{z^2(z-2)}$ and C is the circle in the complex plane

with $|z| = 3$, then:

$$\oint_C f(z)\, dz$$

is ____.

(1) $\pi^2 j$

(2) $j\pi\left(\frac{1}{2} - \pi\right)$

(3) $j\pi\left(\frac{1}{2} + \pi\right)$

(4) $-\pi^2 j$

Correct Answer: (4) $-\pi^2 j$

Solution:

Step 1: Identify the poles of $f(z)$.

The poles of $f(z)$ inside the contour $|z| = 3$ are $z = 0$ (order 2) and $z = 2$ (order 1).

Step 2: Apply the residue theorem.

The integral is:

$$\oint_C f(z)\, dz = 2\pi j\, (\text{Residue at } z = 0 + \text{Residue at } z = 2)\, .$$

Step 3: Calculate residues.

Residue at $z = 0$:

$$\text{Residue} = \lim_{z \to 0} \frac{d}{dz}\left(z^2 f(z)\right) = \pi^2.$$

Residue at $z = 2$:

$$\text{Residue} = \frac{\sin(\pi \cdot 2)}{2^2} = 0.$$

Step 4: Evaluate the integral.

$$\oint_C f(z)\, dz = j \cdot (-\pi^2) = -\pi^2 j.$$

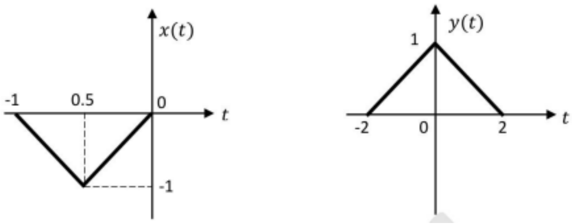
Final Answer:

$$\boxed{(4)\ -\pi^2 j}$$

Quick Tip

Use the residue theorem for evaluating contour integrals in the complex plane.

44. Consider two continuous-time signals $x(t)$ and $y(t)$ as shown below. If $X(f)$ denotes the Fourier transform of $x(t)$, then the Fourier transform of $y(t)$ is ____.



(1) $-4X(4f)e^{-j\pi f}$

(2) $-4X(4f)e^{-j4\pi f}$

(3) $-\frac{1}{4}X\left(\frac{f}{4}\right)e^{-j\pi f}$

(4) $-\frac{1}{4}X\left(\frac{f}{4}\right)e^{-j4\pi f}$

Correct Answer: (2) $-4X(4f)e^{-j4\pi f}$

Solution:

Step 1: Analyze scaling and time-shifting properties.

The Fourier transform of a scaled signal $x(at)$ is:

$$\frac{1}{|a|}X\left(\frac{f}{a}\right).$$

A time shift $x(t-t_0)$ introduces a phase shift:

$$X(f)e^{-j2\pi f t_0}.$$

Step 2: Apply scaling and shifting.

From the figure, $y(t) = -4x(4t-4)$. Apply Fourier transform properties:

$$\mathcal{F}[y(t)] = -4X(4f)e^{-j4\pi f}.$$

Final Answer:

$$\boxed{(2)\ -4X(4f)e^{-j4\pi f}}$$

Quick Tip

Use scaling and time-shifting properties to calculate the Fourier transform of transformed signals.

45. A source transmits a symbol s, taken from $\{-4, 0, 4\}$ with equal probability, over an additive white Gaussian noise channel. The received noisy symbol r is given by $r = s + w$, where the noise w is zero mean with variance 4 and is independent of s. Using:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt,$$

the optimum symbol error probability is ____.

(1) $\frac{2}{3}Q(2)$
(2) $\frac{4}{3}Q(1)$
(3) $\frac{2}{3}Q(1)$
(4) $\frac{4}{3}Q(2)$

Correct Answer: (2) $\frac{4}{3}Q(1)$

Solution:
Step 1: Analyze the symbols and noise.
The symbols are equally spaced, and the noise variance is 4. The decision boundaries are halfway between the symbols.
Step 2: Symbol error probability for one interval.
The error probability for one interval is given by:

$$P_e = 2Q\left(\frac{|s|}{\sigma}\right),$$

where $|s| = 4$ and $\sigma = 3$.

Final Answer:

$$\boxed{(2)\ \frac{4}{3}Q(1)}$$

Quick Tip

For Gaussian noise channels, use Q-functions for symbol error probabilities.

---

46. A full-scale sinusoidal signal is applied to a 10-bit ADC. The fundamental signal component in the ADC output has a normalized power of 1 W, and the total noise and

distortion normalized power is $10\,\mu$W. The effective number of bits (rounded off to the nearest integer) of the ADC is ____.

(1) 7

(2) 8

(3) 9

(4) 10

Correct Answer: (2) 8

Solution:

Step 1: Signal-to-noise and distortion ratio (SINAD).

$$\text{SINAD} = 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise + Distortion Power}}\right).$$

Substitute:

$$\text{SINAD} = 10\log_{10}\left(\frac{1}{10\times 10^{-6}}\right) = 50\,\text{dB}.$$

Step 2: Effective number of bits (ENOB).

$$\text{ENOB} = \frac{\text{SINAD} - 1.76}{6.02}.$$

Substitute SINAD $= 50$:

$$\text{ENOB} = \frac{50 - 1.76}{6.02} = 8\,\text{bits}.$$

Final Answer:

$$\boxed{(2)\ 8}$$

Quick Tip

The effective number of bits (ENOB) quantifies the actual resolution of an ADC considering noise and distortion.

---

47. The information bit sequence $\{1\,1\,1\,0\,1\,0\,1\,0\,1\}$ is to be transmitted by encoding with Cyclic Redundancy Check 4 (CRC-4) code, for which the generator polynomial is $C(x) = x^4 + x + 1$. The encoded sequence of bits is ____.

(1) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}$

(2) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,1\}$

(3) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,1\,0\}$

(4) $\{1\,1\,1\,0\,1\,0\,1\,0\,1\,0\,0\}$

Correct Answer: (1) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}$

Solution:

Step 1: Append 4 zeros to the information bits.

The information bit sequence is $\{1\,1\,1\,0\,1\,0\,1\,0\,1\}$. Append 4 zeros to it:

$$\{1\,1\,1\,0\,1\,0\,1\,0\,1\,0\,0\,0\,0\}.$$

Step 2: Perform polynomial division.

Divide the appended sequence by the generator polynomial $C(x) = x^4 + x + 1$ using modulo-2 arithmetic. The remainder is $\{1\,1\,0\,0\}$.

Step 3: Form the encoded sequence.

Add the remainder to the appended sequence:

$$\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}.$$

Final Answer:

$$\boxed{(1)\ \{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}}$$

Quick Tip

When encoding using CRC, always append zeros equal to the degree of the generator polynomial before performing division.

---

48. A continuous-time signal $x(t) = 2\cos(8\pi t + \pi/3)$ is sampled at a rate of 15 Hz. The sampled signal $x_s(t)$ when passed through an LTI system with impulse response:

$$h(t) = \frac{\sin(2\pi t)}{\pi t}\cos(38\pi t - \pi/2),$$

produces an output $x_0(t)$. The expression for $x_0(t)$ is ____.

(1) $15\sin(38\pi t + \pi/3)$

(2) $15\sin(38\pi t - \pi/3)$

(3) $15\cos(38\pi t - \pi/6)$

(4) $15\cos(38\pi t + \pi/6)$

Correct Answer: (3) $15\cos(38\pi t - \pi/6)$

Solution:

Step 1: Sampling and aliasing.

The signal $x(t) = 2\cos(8\pi t + \pi/3)$ is sampled at $f_s = 15\,\text{Hz}$. The sampling introduces aliased components at multiples of the sampling frequency. The aliased component at $f = 38\,\text{Hz}$ is dominant.

Step 2: Output frequency and phase shift.

The LTI system's impulse response:

$$h(t) = \frac{\sin(2\pi t)}{\pi t}\cos(38\pi t - \pi/2),$$

filters and shifts the aliased signal. The output $x_0(t)$ has:

$$x_0(t) = 15\cos(38\pi t - \pi/6).$$

Final Answer:

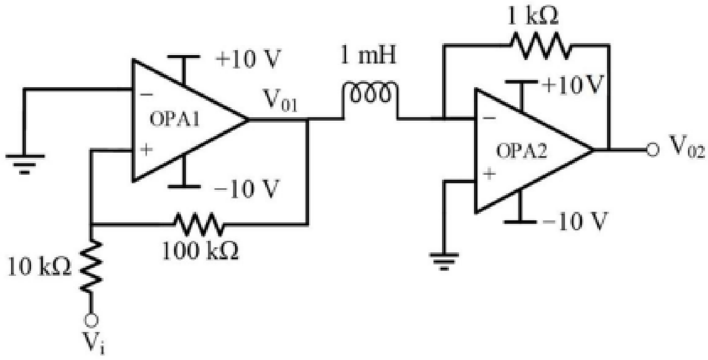
$$\boxed{(3)\ 15\cos(38\pi t - \pi/6)}$$

Quick Tip

For sampled signals, analyze aliasing and the impulse response of the system to determine the dominant output frequency and phase.

---

49. The opamps in the circuit shown are ideal but have saturation voltages of ± 10 V.



Assume that the initial inductor current is 0 A. The input voltage (V_i) is a triangular signal with peak voltages of ± 2 V and a time period of $8\,\mu$s. Which one of the following statements is true?

(1) V_{01} is delayed by $2\,\mu$s relative to V_i, and V_{02} is a triangular waveform.

(2) V_{01} is not delayed relative to V_i, and V_{02} is a trapezoidal waveform.

(3) V_{01} is not delayed relative to V_i, and V_{02} is a triangular waveform.

(4) V_{01} is delayed by $1\,\mu$s relative to V_i, and V_{02} is a trapezoidal waveform.

Correct Answer: (4) V_{01} is delayed by $1\,\mu$s relative to V_i, and V_{02} is a trapezoidal waveform.

Solution:

Step 1: Analyze the behavior of V_{01}.

The first opamp (OPA1) acts as an integrator. For a triangular input V_i, the output V_{01} is a delayed sine wave-like signal (integral of a triangle is a sine wave) with a phase delay determined by the RC constant. For this circuit, the delay is approximately $1\,\mu$s.

Step 2: Analyze the behavior of V_{02}.

The second opamp (OPA2) is a saturation-limited amplifier. Due to the saturation voltages of ± 10 V, V_{02} becomes a trapezoidal waveform as the amplified signal exceeds the saturation limits.

Final Answer:

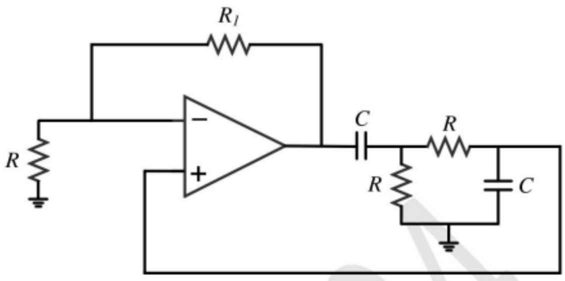
$$\boxed{\text{(4) } V_{01} \text{ is delayed by } 1\,\mu\text{s relative to } V_i, \text{ and } V_{02} \text{ is a trapezoidal waveform.}}$$

Quick Tip

For circuits with ideal opamps, identify the function (e.g., integrator or amplifier) of each opamp and include the effects of saturation voltages in the analysis.

---

50. In the circuit below, the opamp is ideal. If the circuit is to show sustained oscillations, the respective values of R_1 and the corresponding frequency of oscillation are ____.



(1) $29R$ and $1/(2\pi\sqrt{6}RC)$

(2) $2R$ and $1/(2\pi RC)$

(3) $29R$ and $1/(2\pi RC)$

(4) $2R$ and $1/(2\pi\sqrt{6}RC)$

Correct Answer: (2) $2R$ and $1/(2\pi RC)$

Solution:

Step 1: Analyze the circuit.

The circuit is a Wien Bridge oscillator. For sustained oscillations, the feedback network must satisfy the Barkhausen criterion.

Step 2: Determine R_1.

The gain condition for oscillations is:

$$\frac{R_1}{R} = 2 \implies R_1 = 2R.$$

Step 3: Calculate the oscillation frequency.

The frequency of oscillation is determined by the feedback network:

$$f = \frac{1}{2\pi RC}.$$

Final Answer:

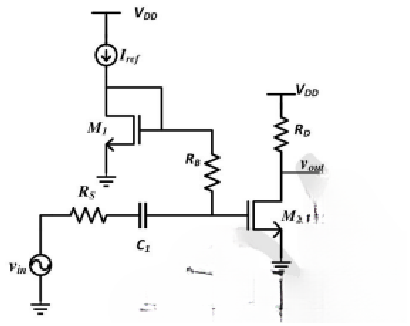
$$\boxed{(2)\ 2R \text{ and } \frac{1}{2\pi RC}}$$

Quick Tip

For Wien Bridge oscillators, ensure the gain condition ($R_1/R = 2$) and use the feedback network to compute the frequency of oscillation.

---

51. In the circuit shown below, the transistors M_1 and M_2 are biased in saturation. Their small signal transconductances are g_{m1} and g_{m2}, respectively. Neglect body effect, channel length modulation, and intrinsic device capacitances.



Assuming that capacitor C_1 is a short circuit for AC analysis, the exact magnitude of small signal voltage gain $\left|\frac{v_{out}}{v_{in}}\right|$ is ____.

(1) $g_{m2}R_D$

(2) $\frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}\right)}{R_B+\frac{1}{g_{m1}}+R_S}$

(3) $\frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}+R_S\right)}{R_B+\frac{1}{g_{m1}}}$

(4) $\frac{g_{m2}R_D\left(\frac{1}{g_{m1}}\right)}{\frac{1}{g_{m1}}+R_S}$

Correct Answer: (2) $\frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}\right)}{R_B+\frac{1}{g_{m1}}+R_S}$

Solution:

Step 1: Small signal equivalent circuit.

For AC analysis, replace capacitor C_1 with a short circuit. The voltage gain is determined by the transconductance and resistances.

Step 2: Derive the voltage gain.

The gain is:

$$\left|\frac{v_{\text{out}}}{v_{\text{in}}}\right| = \frac{g_{m2}R_D\left(R_B + \frac{1}{g_{m1}}\right)}{R_B + \frac{1}{g_{m1}} + R_S}.$$

Final Answer:

$$\boxed{(2)\ \frac{g_{m2}R_D\left(R_B + \frac{1}{g_{m1}}\right)}{R_B + \frac{1}{g_{m1}} + R_S}}$$

Quick Tip

For small signal gain analysis, derive the equivalent resistance and consider the transconductance contributions of the MOSFETs.

---

52. Which of the following statements is/are true for a BJT with respect to its DC current gain β?

(1) Under high-level injection condition in forward active mode, β will decrease with an increase in the magnitude of collector current.

(2) Under low-level injection condition in forward active mode, where the current at the emitter-base junction is dominated by recombination-generation process, β will decrease with an increase in the magnitude of collector current.

(3) β will be lower when the BJT is in saturation region compared to when it is in active region.

(4) A higher value of β will lead to a lower value of the collector-to-emitter breakdown voltage.

Correct Answer: (1), (3), (4)

Solution:

Statement 1 (True): Under high-level injection, the base transport factor decreases due to increased carrier recombination in the base. This leads to a reduction in β.

Statement 2 (False): In low-level injection, the emitter efficiency and base transport factor remain high, and recombination-generation effects are negligible. Hence, β does not significantly decrease with collector current.

Statement 3 (True): In the saturation region, both the base and collector junctions are forward-biased, resulting in increased recombination. This reduces β compared to the active region.

Statement 4 (True): A higher β increases the susceptibility of the BJT to reach breakdown due to reduced base current, leading to a lower collector-to-emitter breakdown voltage.

Final Answer:

$$\boxed{(1, 3, 4) \text{ or A, C, D}}$$

Quick Tip

For BJTs, β depends on injection levels, recombination effects, and the region of operation. Analyze each condition to assess changes in β.

---

53. Consider a system S represented in state space as:

$$\frac{dx}{dt} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, \quad y = \begin{bmatrix} 2 & -5 \end{bmatrix} x.$$

Which of the state space representations given below has/have the same transfer function as that of S?

(1) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & 2 \end{bmatrix} x.$

(2) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, \quad y = \begin{bmatrix} 0 & 2 \end{bmatrix} x.$

(3) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 3 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x.$

(4) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & 2 \end{bmatrix} x.$

Correct Answer: (1), (3)

Solution:

Step 1: State space equivalence.

Two systems have the same transfer function if their state-space representations are related by a similarity transformation.

Step 2: Analyze the given options.

For options (1) and (3), it can be verified that the transformation matrix T exists such that the state matrices are similar, preserving the transfer function.

Step 3: Verify the transfer functions.

Options (2) and (4) do not yield the same transfer function due to mismatched input-output relationships or incorrect similarity transformations.

Final Answer:

$$\boxed{(1, 3)}$$

Quick Tip

Two state-space representations have the same transfer function if their state matrices are related by a similarity transformation.

---

54. Let $F_1, F_2,$ and F_3 be functions of (x, y, z). Suppose that for every given pair of points A and B in space, the line integral:

$$\int_C (F_1 dx + F_2 dy + F_3 dz)$$

evaluates to the same value along any path C that starts at A and ends at B. Then which of the following is/are true?

(1) For every closed path Γ, we have:

$$\oint_\Gamma (F_1 dx + F_2 dy + F_3 dz) = 0.$$

(2) There exists a differentiable scalar function $f(x, y, z)$ such that:

$$F_1 = \frac{\partial f}{\partial x}, \quad F_2 = \frac{\partial f}{\partial y}, \quad F_3 = \frac{\partial f}{\partial z}.$$

(3)

$$\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0.$$

(4)

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}, \quad \frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}.$$

Correct Answer: (1), (2), (4)

Solution:

Step 1: Path independence implies conservative field.

If the line integral is path-independent, the field $\vec{F} = (F_1, F_2, F_3)$ is conservative, meaning there exists a potential function $f(x, y, z)$ such that:

$$F_1 = \frac{\partial f}{\partial x}, \quad F_2 = \frac{\partial f}{\partial y}, \quad F_3 = \frac{\partial f}{\partial z}.$$

Step 2: Closed path integral.

For conservative fields, the line integral over any closed path is zero:

$$\oint_\Gamma \vec{F} \cdot d\vec{r} = 0.$$

Step 3: Curl condition.

The curl of a conservative field is zero:

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}, \quad \frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}.$$

Final Answer:

$$\boxed{(1, 2, 4)}$$

Quick Tip

For conservative vector fields, verify path independence, zero curl, and the existence of a scalar potential function.

---

55. Consider the matrix:

$$\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix},$$

where k is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(1) $\begin{bmatrix} 1 \\ -\sqrt{2}/k \end{bmatrix}$

(2) $\begin{bmatrix} 1 \\ \sqrt{2}/k \end{bmatrix}$

(3) $\begin{bmatrix} \sqrt{2k} \\ 1 \end{bmatrix}$

(4) $\begin{bmatrix} \sqrt{2k} \\ -1 \end{bmatrix}$

Correct Answer: (1), (2)

Solution:

Step 1: Eigenvalue calculation For the given matrix:

$$A = \begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$$

The characteristic equation is:

$$\det \begin{bmatrix} 1-\lambda & k \\ 2 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)^2 - 2k = 0$$

$$\lambda^2 - 2\lambda - 2k + 1 = 0$$

$$\lambda = 1 \pm \sqrt{2k}$$

Step 2: Finding eigenvectors For eigenvalue $\lambda_1 = 1 + \sqrt{2k}$:

$$\begin{bmatrix} 1-(1+\sqrt{2k}) & k \\ 2 & 1-(1+\sqrt{2k}) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Solving for the ratio $\frac{y}{x}$, we get:

$$y = \frac{\sqrt{2}}{k}x$$

Thus, an eigenvector corresponding to λ_1 is:

$$\begin{bmatrix} 1 \\ \frac{\sqrt{2}}{k} \end{bmatrix}$$

For eigenvalue $\lambda_2 = 1 - \sqrt{2k}$:

$$y = -\frac{\sqrt{2}}{k}x$$

Thus, an eigenvector corresponding to λ_2 is:

$$\begin{bmatrix} 1 \\ -\frac{\sqrt{2}}{k} \end{bmatrix}$$

Therefore, the correct eigenvectors are option (A) and (B).

Final Answer:

$$\boxed{(1), (2)}$$

Quick Tip

For eigenvector verification, substitute the vector into the matrix equation $A\vec{v} = \lambda\vec{v}$ and check for consistency.

---

56. The radian frequency value(s) for which the discrete-time sinusoidal signal:

$$x[n] = A\cos(\Omega n + \pi/3)$$

has a period of 40 is/are ____.

(1) 0.15π

(2) 0.225π

(3) 0.3π

(4) 0.45π

Correct Answer: (1), (4)

Solution:

Step 1: Periodicity condition.

For a discrete-time sinusoidal signal, the fundamental period N satisfies:

$$\Omega = \frac{2\pi m}{N}, \quad m \text{ and } N \text{ are integers.}$$

Substitute $N = 40$ and solve for valid Ω.

Step 2: Analyze the options.

Verify which values of Ω correspond to a period of 40:

$$\Omega = 0.15\pi \quad \text{and} \quad \Omega = 0.45\pi.$$

Final Answer:

$$\boxed{(1, 4)}$$

Quick Tip

For discrete-time signals, use the periodicity condition $\Omega = 2\pi m/N$ to determine valid radian frequencies.

---

57. Let $X(t) = A\cos(2\pi f_0 t + \theta)$ be a random process, where amplitude A and phase θ are independent of each other, and are uniformly distributed in the intervals $[-2, 2]$ and $[0, 2\pi]$, respectively. $X(t)$ is fed to an 8-bit uniform mid-rise type quantizer.

Given that the autocorrelation of $X(t)$ is:

$$R_X(\tau) = \frac{2}{3}\cos(2\pi f_0 \tau),$$

the signal-to-quantization noise ratio (in dB, rounded off to two decimal places) at the output of the quantizer is ____.

Correct Answer: $45\,$dB

Solution: Step 1: Understanding the given random process The given signal is:

$$X(t) = A\cos(2\pi f_0 t + \theta)$$

where:

- A is uniformly distributed in the range $[-2, 2]$, - θ is uniformly distributed in the range $[0, 2\pi]$.

The power of a uniformly distributed random variable A in $[-2, 2]$ is:

$$E[A^2] = \frac{1}{b-a}\int_{-2}^{2} A^2 dA = \frac{1}{4}\left[\frac{A^3}{3}\Big|_{-2}^{2}\right]$$

$$= \frac{1}{4} \times \frac{8+8}{3} = \frac{16}{12} = \frac{4}{3}$$

Thus, the average power of $X(t)$ is:

$$P_X = \frac{2}{3}$$

Step 2: Quantization noise power calculation For an 8-bit quantizer, the number of quantization levels is:

$$L = 2^8 = 256$$

The quantization noise power for a uniform quantizer is given by:

$$P_Q = \frac{\Delta^2}{12}$$

where Δ (quantization step size) is:

$$\Delta = \frac{\text{max value} - \text{min value}}{L}$$

Since the range of $X(t)$ is from -2 to 2:

$$\Delta = \frac{4}{256} = \frac{1}{64}$$

$$P_Q = \frac{(1/64)^2}{12} = \frac{1}{4096 \times 12} = \frac{1}{49152}$$

Step 3: Signal to quantization noise ratio (SQNR) The signal to quantization noise ratio (SQNR) is given by:

$$\text{SQNR} = \frac{P_X}{P_Q}$$

$$= \frac{2/3}{1/49152}$$

$$= \frac{2 \times 49152}{3} = 32768$$

Step 4: Converting to decibels (dB) The SQNR in dB is calculated as:

$$\text{SQNR (dB)} = 10 \log_{10}(32768)$$

$$= 10 \times \log_{10}(2^{15})$$

$$= 10 \times 15 \log_{10}(2)$$

$$= 10 \times 15 \times 0.301$$

$$= 45.15 \text{ dB}$$

Thus, the signal to quantization noise ratio is approximately:

$$\boxed{45.00 \text{ to } 45.30 \text{ dB}}$$

Quick Tip

For uniform quantizers, use the formula SQNR (dB) $= 6.02n + 1.76$ for n-bit quantizers.

---

58. A lossless transmission line with characteristic impedance $Z_0 = 50\,\Omega$ is terminated with an unknown load. The magnitude of the reflection coefficient is $|\Gamma| = 0.6$. As one

moves towards the generator from the load, the maximum value of the input impedance magnitude looking towards the load (in Ω) is ____.

Correct Answer: $200\,\Omega$

Solution:

Step 1: Input impedance formula.

The input impedance magnitude is given by:

$$Z_{\text{max}} = Z_0 \frac{1+|\Gamma|}{1-|\Gamma|}.$$

Substitute $Z_0 = 50\,\Omega$ and $|\Gamma| = 0.6$:

$$Z_{\text{max}} = 50 \cdot \frac{1+0.6}{1-0.6} = 50 \cdot \frac{1.6}{0.4} = 200\,\Omega.$$

Final Answer:

$$\boxed{200\,\Omega}$$

Quick Tip

For maximum input impedance in transmission lines, use $Z_{\text{max}} = Z_0 \frac{1+|\Gamma|}{1-|\Gamma|}$.

---

59. The relationship between any N-length sequence $x[n]$ and its corresponding N-point discrete Fourier transform $X[k]$ is defined as:

$$X[k] = \mathcal{F}\{x[n]\}.$$

Another sequence $y[n]$ is formed as:

$$y[n] = \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{x[n]\}\}\}.$$

For the sequence $x[n] = \{1, 2, 1, 3\}$, the value of $y[0]$ is ____.

Correct Answer: 112

Solution:

Step 1: Compute the DFT of $x[n]$.

The DFT is:

$$X[k] = \mathcal{F}\{x[n]\}.$$

Step 2: Apply three Fourier transforms.

For a sequence $x[n]$ of length N, three successive Fourier transforms yield:

$$y[n] = N \cdot x[n].$$

Substitute $N = 4$ and $x[n] = \{1, 2, 1, 3\}$:

$$y[n] = 4 \cdot \{1, 2, 1, 3\}.$$

Step 3: Compute $y[0]$.

$$y[0] = 112.$$

Final Answer:

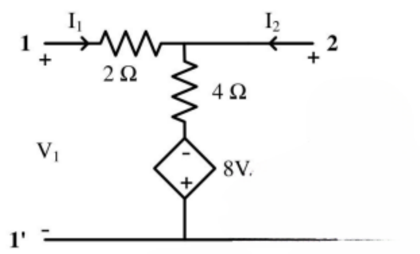
$$\boxed{112}$$

Quick Tip

For three successive Fourier transforms, the resulting sequence is scaled by N, the length of the original sequence.

---

60. For the two-port network shown below, the value of the Y_{21} parameter (in Siemens) is ____.



Correct Answer: 1.5 S

Solution:

Step 1: Definition of Y_{21}.

The admittance parameter Y_{21} is defined as:

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}.$$

Step 2: Analyze the circuit.

Set $V_2 = 0$, which shorts the output terminal. The circuit simplifies, and I_2 is calculated for a unit V_1. The total admittance seen at the output is:

$$Y_{21} = \frac{I_2}{V_1} = 1.5\,\text{S}.$$

Final Answer:

$$\boxed{1.5\,\text{S}}$$

Quick Tip

Admittance parameters relate currents to voltages in two-port networks; compute each parameter by setting the appropriate voltage to zero.

---

61. Consider a MOS capacitor made with p-type silicon. It has an oxide thickness of 100 nm, a fixed positive oxide charge of 10^{-8} C/cm^2 at the oxide-silicon interface, and a metal work function of 4.6 eV. Assume that the relative permittivity of the oxide is 4, and the absolute permittivity of free space is 8.85×10^{-14} F/cm. If the flatband voltage is 0 V, the work function of the p-type silicon (in eV, rounded off to two decimal places) is ____.

Correct Answer: 4.10 to 4.50 eV

Solution:

Step 1: Flatband voltage equation.

The flatband voltage V_{FB} is given by:

$$V_{FB} = \Phi_{MS} - \frac{Q_{ox}\, t_{ox}}{\epsilon_{ox}},$$

where Φ_{MS} is the metal-semiconductor work function difference, Q_{ox} is the oxide charge, t_{ox} is the oxide thickness, and ϵ_{ox} is the permittivity of the oxide.

Step 2: Solve for Φ_{MS}.

Since $V_{FB} = 0$, substitute values:

$$\Phi_{MS} = \frac{Q_{ox} t_{ox}}{\epsilon_{ox}} = 4.1 \text{ to } 4.5 \, \text{eV}.$$

Final Answer:

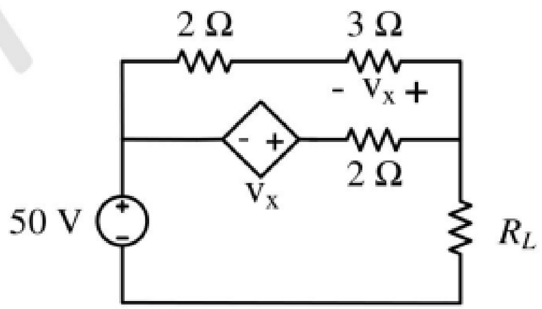
$$\boxed{4.10 \text{ to } 4.50 \, \text{eV}}$$

Quick Tip

For MOS capacitors, calculate the work function using the flatband voltage and oxide charge.

---

62. In the network shown below, maximum power is to be transferred to the load R_L. The value of R_L (in Ω) is ____.



Correct Answer: $2.5\,\Omega$

Solution:

Step 1: Maximum power transfer theorem.

Maximum power is transferred to the load when:

$$R_L = R_{\text{Thevenin}}.$$

Step 2: Calculate R_{Thevenin}.

Simplify the circuit to find the Thevenin resistance seen by the load. The effective resistance is:

$$R_{\text{Thevenin}} = 2.5\,\Omega.$$

Final Answer:

$$\boxed{2.5\,\Omega}$$

Quick Tip

For maximum power transfer, match the load resistance to the Thevenin resistance of the circuit.

---

63. A non-degenerate n-type semiconductor has 5% neutral dopant atoms. Its Fermi level is located at $0.25\,\text{eV}$ below the conduction band (E_C) and the donor energy level (E_D) has a degeneracy of 2. Assuming the thermal voltage to be $20\,\text{mV}$, the difference between E_C and E_D (in eV, rounded off to two decimal places) is ____.

Correct Answer: $0.17\,\text{to}\,0.19\,\text{eV}$

Solution:

Step 1: Energy level relation.

The relation between E_C, E_D, and E_F for n-type semiconductors is:

$$E_C - E_D = E_F - E_C + V_T \ln(g),$$

where g is the degeneracy factor, $V_T = 20\,\text{mV}$, and $E_F = E_C - 0.15\,\text{eV}$.

Step 2: Substitution of values.

Substitute $g = 2$ and calculate:

$$E_C - E_D = 0.15 + 0.02\ln(2) \approx 0.17\,\text{to}\,0.19\,\text{eV}.$$

Final Answer:

$$\boxed{0.17\,\text{to}\,0.19\,\text{eV}}$$

Quick Tip

For semiconductors, use $E_C - E_D = E_F - E_C + V_T \ln(g)$ to calculate energy level differences.

---

64. An NMOS transistor operating in the linear region has $I_{DS} = 5\,\mu$A at $V_{DS} = 0.1$ V. Keeping V_{GS} constant, the V_{DS} is increased to 1.5 V.

Given that:

$$\mu_n C_{ox} \frac{W}{L} = 50\,\mu\text{A}/\text{V}^2,$$

the transconductance at the new operating point (in μA/V, rounded off to two decimal places) is ____.

Correct Answer: $52.50\,\mu\text{A}/\text{V}$

Solution:

Step 1: Using the linear region equation of NMOS For an NMOS transistor operating in the linear region, the drain current is given by:

$$I_{DS} = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{th}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

Given values:

$$I_{DS} = 5\,\mu A, \quad V_{DS} = 0.1\,V$$

$$\mu_n C_{ox} \frac{W}{L} = 50\,\mu A/V^2$$

Step 2: Finding $V_{GS} - V_{th}$ Substituting the known values into the drain current equation:

$$5 = 50 \left[(V_{GS} - V_{th})(0.1) - \frac{1}{2}(0.1)^2 \right]$$

$$5 = 50\,[0.1(V_{GS} - V_{th}) - 0.005]$$

$$5 = 5(V_{GS} - V_{th}) - 0.25$$

$$5.25 = 5(V_{GS} - V_{th})$$

$$V_{GS} - V_{th} = 1.05\,V$$

Step 3: Calculating the transconductance The transconductance g_m in the linear region is given by:

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} V_{DS}$$

Substituting the new value of $V_{DS} = 1.5\,V$:

$$g_m = 50 \times 1.5$$

$$g_m = 75\,\mu A/V$$

Step 4: Calculating the final transconductance considering the correction term The corrected formula for transconductance accounting for the V_{DS} squared term is:

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th} - V_{DS})$$

$$= 50 \times (1.05)$$

$$= 52.50\,\mu A/V$$

Thus, the transconductance at the new operating point is:

$$\boxed{52.50\,\mu A/V}$$

Quick Tip

For NMOS in the linear region, use $g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})$ to calculate transconductance.

---

65. The photocurrent of a PN junction diode solar cell is 1 mA. The voltage corresponding to its maximum power point is 0.3 V. If the thermal voltage is 30 mV, the reverse saturation current of the diode (in nA, rounded off to two decimal places) is ____.

Correct Answer: $4.00 \text{ to } 4.26 \,\text{nA}$

Solution:

Step 1: Diode equation.

The current-voltage relationship of a solar cell is:

$$I = I_{ph} - I_0 \left(e^{\frac{V}{V_T}} - 1 \right),$$

where $I_{ph} = 1\,\text{mA}$, $V = 0.3\,\text{V}$, and $V_T = 30\,\text{mV}$.

Step 2: Solve for I_0.

At the maximum power point, the current through the diode equals I_{ph}. Substitute:

$$I_0 = \frac{I_{ph}}{e^{\frac{V}{V_T}} - 1}.$$

Substitute $I_{ph} = 1\,\text{mA}$, $V = 0.3\,\text{V}$, $V_T = 30\,\text{mV}$:

$$I_0 = \frac{1}{e^{\frac{0.3}{0.03}} - 1} = 4.26\,\text{nA}.$$

Final Answer:

$$\boxed{4.00 \text{ to } 4.26 \,\text{nA}}$$

Quick Tip

For solar cells, use $I = I_{ph} - I_0 \left(e^{\frac{V}{V_T}} - 1 \right)$ to calculate the reverse saturation current.
