General Aptitude (GA)

Q.1 – Q.5 Carry ONE mark Each

Q.1 If '→' denotes increasing order of intensity, then the meaning of the words

[walk → jog → sprint] is analogous to [bothered → ________ → daunted].

Which one of the given options is appropriate to fill the blank?

(A) phased

(B) phrased

(C) fazed

(D) fused

Q.2 Two wizards try to create a spell using all the four elements, *water*, *air*, *fire*, and *earth*. For this, they decide to mix all these elements in all possible orders. They also decide to work independently. After trying all possible combination of elements, they conclude that the spell does not work.

How many attempts does each wizard make before coming to this conclusion, independently?

(A) 24

(B) 48

(C) 16

(D) 12

Q.3 In an engineering college of 10,000 students, 1,500 like neither their core branches nor other branches. The number of students who like their core branches is $1/4^{th}$ of the number of students who like other branches. The number of students who like both their core and other branches is 500.

The number of students who like their core branches is

(A) 1,800

(B) 3,500

(C) 1,600

(D) 1,500

Q.4 For positive non-zero real variables x and y, if

$$\ln\left(\frac{x+y}{2}\right) = \frac{1}{2}[\ln(x) + \ln(y)]$$

then, the value of $\frac{x}{y} + \frac{y}{x}$ is

(A) 1

(B) 1/2

(C) 2

(D) 4

Q.5 In the sequence 6, 9, 14, x, 30, 41, a possible value of x is

(A) 25

(B) 21

(C) 18

(D) 20

Q.6 – Q.10 Carry TWO marks Each

Q.6 Sequence the following sentences in a coherent passage.

P: This fortuitous geological event generated a colossal amount of energy and heat that resulted in the rocks rising to an average height of 4 km across the contact zone.

Q: Thus, the geophysicists tend to think of the Himalayas as an active geological event rather than as a static geological feature.

R: The natural process of the cooling of this massive edifice absorbed large quantities of atmospheric carbon dioxide, altering the earth's atmosphere and making it better suited for life.

S: Many millennia ago, a breakaway chunk of bedrock from the Antarctic Plate collided with the massive Eurasian Plate.

(A) QPSR

(B) QSPR

(C) SPRQ

(D) SRPQ

Q.7 A person sold two different items at the same price. He made 10% profit in one item, and 10% loss in the other item. In selling these two items, the person made a total of

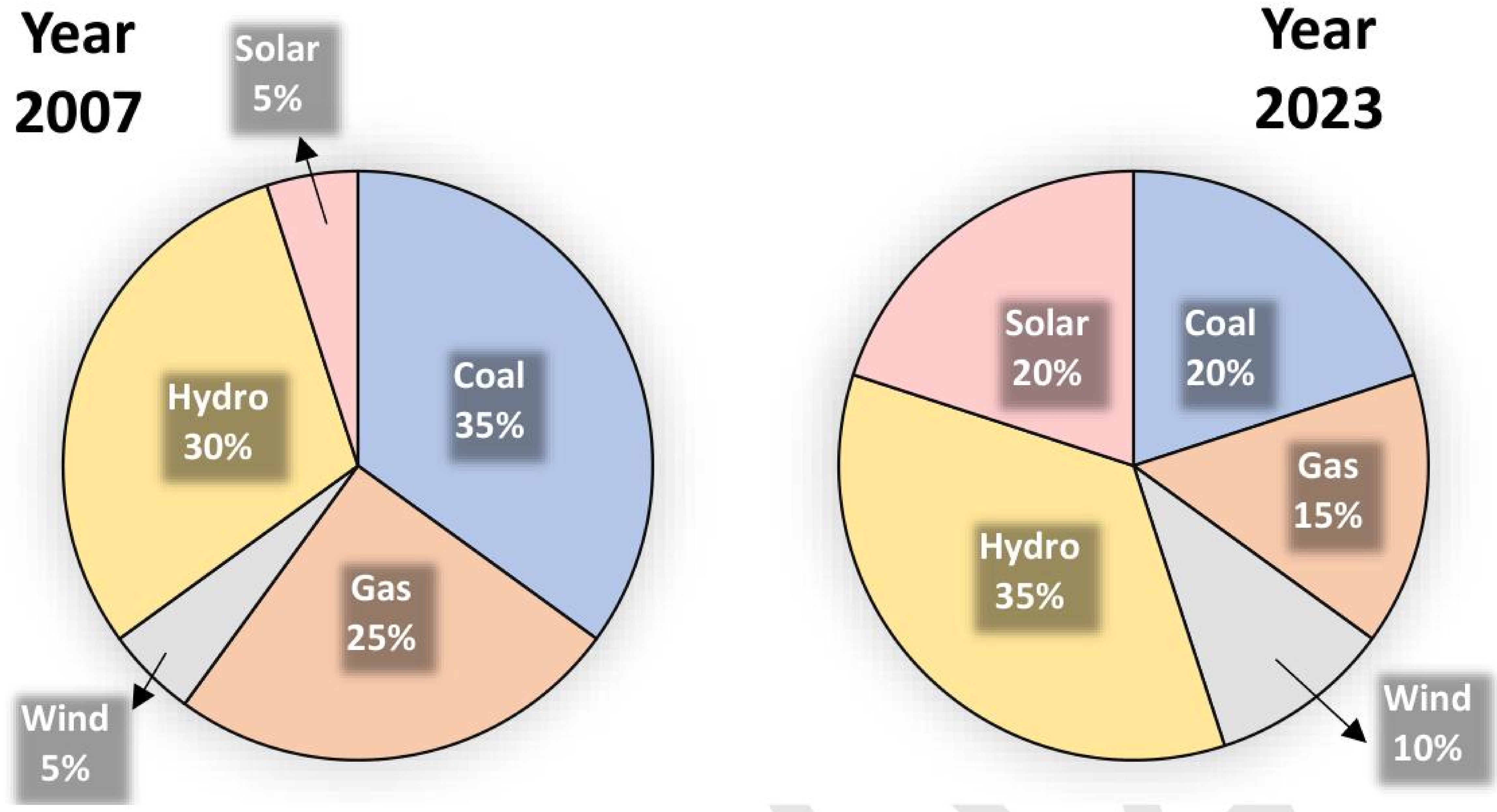
(A) 1% profit

(B) 2% profit

(C) 1% loss

(D) 2% loss

Q.8 The pie charts depict the shares of various power generation technologies in the total electricity generation of a country for the years 2007 and 2023.



The renewable sources of electricity generation consist of Hydro, Solar and Wind. Assuming that the total electricity generated remains the same from 2007 to 2023, what is the percentage increase in the share of the renewable sources of electricity generation over this period?

(A) 25%

(B) 50%

(C) 77.5%

(D) 62.5%

Q.9 A cube is to be cut into 8 pieces of equal size and shape. Here, each cut should be straight and it should not stop till it reaches the other end of the cube.

The minimum number of such cuts required is

(A) 3

(B) 4

(C) 7

(D) 8

Q.10 In the 4 × 4 array shown below, each cell of the first three rows has either a cross (X) or a number.

1	X	4	3
X	5	5	4
3	X	6	X

The number in a cell represents the count of the immediate neighboring cells (left, right, top, bottom, diagonals) NOT having a cross (X). Given that the last row has no crosses (X), the sum of the four numbers to be filled in the last row is

(A) 11

(B) 10

(C) 12

(D) 9

Q.11 – Q.35 Carry ONE mark Each

Q.11 Let D be the region bounded by the line $y = x$ and the parabola $y = 4x - x^2$. Then $\iint_D x \, dx \, dy$ equals

(A) $\frac{27}{4}$

(B) $\frac{29}{4}$

(C) 7

(D) 6

Q.12 Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = \sqrt{6}$ and $a_{n+1} = \sqrt{6 + a_n}$ for $n \geq 1$. Consider the following statements:

(I) $\{a_n\}_{n\geq 1}$ is an increasing sequence.

(II) $\lim_{n\to\infty} a_n = 2$.

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.13 Let A be a 3×3 real matrix and let I_3 be the 3×3 identity matrix. Which one of the following statements is NOT true?

(A) If the row-reduced echelon form of A is I_3, then zero is not an eigenvalue of A

(B) If zero is not an eigenvalue of A, then the row-reduced echelon form of A is I_3

(C) If A has three distinct eigenvalues, then the row-reduced echelon form of A is I_3

(D) If the system of equations $A\boldsymbol{x} = \boldsymbol{b}$ has a solution for every 3×1 real column vector $\boldsymbol{b}$, then the row-reduced echelon form of A is I_3

Q.14 Let $\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}$ be vectors in $\mathbb{R}^4$. Let U be the span of $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ and let V be the span of $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$.

Consider the following statements:

(I) If the dimension of $U \cap V$ is 2 and the dimension of U is 3, then $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ is linearly dependent.

(II) If $U + V = \{\boldsymbol{u} + \boldsymbol{v} \ : \boldsymbol{u} \in U, \boldsymbol{v} \in V\} = \mathbb{R}^4$, then either $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is linearly independent or $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ is linearly independent.

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.15 Consider $\mathbb{R}^2$ with standard inner product. If $\boldsymbol{u} = \begin{bmatrix} a \\ b \end{bmatrix}$ is the vector in $\mathbb{R}^2$ such that the inner product of $\boldsymbol{u}$ with $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is 2 and with $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ is -1, then which one of the following statements is true?

(A) Inner product of $\boldsymbol{u}$ with $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is $\frac{1}{2}$

(B) Inner product $\boldsymbol{u}$ with $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is $\frac{3}{5}$

(C) Inner product of $\boldsymbol{u}$ with $\begin{bmatrix} 2 \\ -\frac{1}{2} \end{bmatrix}$ is $-\frac{6}{5}$

(D) Inner product of $\boldsymbol{u}$ with $\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$ is $\frac{4}{5}$

Q.16 Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$ be a 2×3 real matrix,

where $(a_1, a_2, a_3) \neq (0, 0, 0)$ and $(b_1, b_2, b_3) \neq (0, 0, 0)$. Assume that the rank of A is 1. Define the subspaces $W = \left\{ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : A\boldsymbol{x} = \boldsymbol{0} \right\}$,

$$W_1 = \left\{ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : a_1x_1 + a_2x_2 + a_3x_3 = 0 \right\} \text{ and}$$

$$W_2 = \left\{ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : b_1x_1 + b_2x_2 + b_3x_3 = 0 \right\}.$$

Consider the following statements:

(I) $W = W_1 \cap W_2$

(II) $W_1 = W_2$

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.17 Let X be a random variable taking only two values, 1 and 2. Let $M_X(\cdot)$ be the moment generating function of X. If the expectation of X is $\frac{10}{7}$, then the fourth derivative of $M_X(\cdot)$ evaluated at 0 equals

(A) $\frac{52}{7}$

(B) $\frac{67}{7}$

(C) $\frac{48}{7}$

(D) $\frac{60}{7}$

Q.18 Two fair dice, one having red and another having blue colour, are tossed independently once. Let A be the event that the die having red colour will show 5 or 6. Let B be the event that the sum of the outcomes will be 7 and let C be the event that the sum of the outcomes will be 8. Then which one of the following statements is true?

(A) A and B are independent as well as A and C are independent

(B) A and B are independent, but A and C are not independent

(C) A and C are independent, but A and B are not independent

(D) Neither A and B are independent, nor A and C are independent

Q.19 Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha > 0, \beta > 0$. If $E(X) = \frac{1}{3}$ and $E(X^2) = \frac{1}{6}$, then $\alpha + 3\,\beta$ equals

(A) 7

(B) 5

(C) 4

(D) 8

Q.20 Let X and Y be two random variables with cumulative distribution functions $F_X(\cdot)$ and $F_Y(\cdot)$, respectively. Then which one of the following statements is NOT true?

(A) There exist X and Y such that $F_X(u) = F_Y(u)$ for all $u \in \mathbb{R}$, and $P(X \neq Y) > 0$

(B) There exist X and Y such that $F_X(u) = F_Y(u)$ for all $u \in \mathbb{R}$, and $P(X = Y) = 0$

(C) If X and Y are independent then X^2 and Y^2 are also independent

(D) If X^2 and Y^2 are independent then X and Y are also independent

Q.21 Let $\{F_n\}_{n\geq 1}$ be a sequence of cumulative distribution functions given by

$$F_n(x) = \begin{cases} 0 & \text{if } x < -n \\ \dfrac{x+n}{2n} & \text{if } -n \leq x < n \\ 1 & \text{if } x \geq n. \end{cases}$$

Which one of the following statements is true?

(A) $F_n(x)$ converges for all $x \in \mathbb{R}$ and the limiting function is a cumulative distribution function

(B) $F_n(x)$ converges for all $x \in \mathbb{R}$, but the limiting function is not a cumulative distribution function

(C) $F_n(x)$ does not converge for any $x \in \mathbb{R}$

(D) There exist $x, y \in \mathbb{R}$ such that $F_n(x)$ converges but $F_n(y)$ does not converge

Q.22 Let $\{W(t)\}_{t\geq 0}$ be a standard Brownian motion. Which one of the following statements is NOT true?

(A) $E[W(7)] = 0$

(B) $E[W(5)W(9)] = 7$

(C) $2W(1)$ is normally distributed with mean 0 and variance 4

(D) $E[W(5) \mid W(3) = 3] = 3$

Q.23 Let X_1, X_2, X_3 be three independent and identically distributed binomial random variables with number of trials $n = 100$ and success probability $p\ (0 < p < 1)$, which is an unknown parameter. Let $T_1 = (X_1 + X_2,\ X_3)$ and $T_2 = X_1 + X_2 + X_3$. Consider the following statements:

(I) The distribution of T_2 given $T_1 = t_1$ is independent of p.

(II) The distribution of T_1 given $T_2 = t_2$ is independent of p.

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.24 Let $X_1, X_2, \dots, X_n$ be a random sample of size $n\ (\geq 2)$ from a population having probability density function

$$f(x;\theta) = \begin{cases} \theta(2x)^{\theta-1} & \text{if } 0 < x \leq \frac{1}{2} \\ \theta(2-2x)^{\theta-1} & \text{if } \frac{1}{2} < x \leq 1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter. Then which one of the following is a maximum likelihood estimator of θ?

(A) $n\left[\sum_{\left\{i:\, X_i \leq \frac{1}{2}\right\}} \log_e 2X_i + \sum_{\left\{i:\, X_i > \frac{1}{2}\right\}} \log_e (2-2X_i)\right]^{-1}$

(B) $-n\left[\sum_{\left\{i:\, X_i \leq \frac{1}{2}\right\}} \log_e 2X_i + \sum_{\left\{i:\, X_i > \frac{1}{2}\right\}} \log_e (2-2X_i)\right]^{-1}$

(C) $n\left[\sum_{\{1\leq i\leq n\}} \log_e 2X_i + \sum_{\{1\leq i\leq n\}} \log_e (2-2X_i)\right]^{-1}$

(D) $-n\left[\sum_{\{1\leq i\leq n\}} \log_e 2X_i + \sum_{\{1\leq i\leq n\}} \log_e (2-2X_i)\right]^{-1}$

Q.25 In a testing of hypothesis problem, which one of the following statements is true?

(A) The probability of the Type-I error cannot be higher than the probability of the Type-II error

(B) Type-II error occurs if the test accepts the null hypothesis when the null hypothesis is actually false

(C) Type-I error occurs if the test rejects the null hypothesis when the null hypothesis is actually false

(D) The sum of the probability of the Type-I error and the probability of the Type-II error should be 1

Q.26 A random sample of size 40 is drawn from a population having four distinct categories as $i = 1, 2, 3, 4$. The data are given as

Category	1	2	3	4
Observed Frequency	5	8	12	15

Let θ_i be the probability that an observation comes from the i-th category, $i = 1, 2, 3, 4$. If the chi-square goodness-of-fit test is used to test $H_0: \theta_i = \frac{1}{4}, i = 1, 2, 3, 4$ against $H_1: \theta_i \neq \frac{1}{4}$, for some $i = 1, 2, 3, 4$, then which one of the following statements is true?

(A) Under H_0, the test statistic follows central chi-square distribution with 3 degrees of freedom and the observed value of the test statistic is 5.8

(B) Under H_0, the test statistic follows central chi-square distribution with 3 degrees of freedom and the observed value of the test statistic is 1.4

(C) Under H_0, the test statistic follows central chi-square distribution with 4 degrees of freedom and the observed value of the test statistic is 5.8

(D) Under H_0, the test statistic follows central chi-square distribution with 4 degrees of freedom and the observed value of the test statistic is 1.4

Q.27 Let $X_1, X_2, \dots, X_n$ be a random sample of size $n\ (\geq 11)$ from a population with continuous and strictly increasing cumulative distribution function $F(\cdot)$ with an unknown median M. To test $H_0: M = 10$ against $H_1: M > 10$ at level α, let the statistic T denote the number of observations larger than 10. Let t_0 be the observed value of the test statistic T. Consider the test which rejects H_0 if $T \geq c$. Then the p-value of the test is

(A) $$\sum_{i=t_0}^{n} \frac{n!}{i!\,(n-i)!}\,(0.5)^n$$

(B) $$\sum_{i=10}^{n} \frac{n!}{i!\,(n-i)!}\,(0.5)^n$$

(C) $$\sum_{i=0}^{10} \frac{n!}{i!\,(n-i)!}\,(0.5)^n$$

(D) $$\sum_{i=0}^{t_0} \frac{n!}{i!\,(n-i)!}\,(0.5)^n$$

Q.28 Consider the simple linear regression model

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \;\; i = 1, 2, \dots, n,$$

where β_0 and β_1 are unknown parameters, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. Let $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\widehat{\beta_1}$ be the least squares estimator of β_1. Then which one of the following statements is true?

(A) The covariance between $\bar{y}$ and $\widehat{\beta_1}$ is less than 0

(B) The covariance between $\bar{y}$ and $\widehat{\beta_1}$ is greater than 0

(C) The covariance between $\bar{y}$ and $\widehat{\beta_1}$ is equal to 0

(D) The covariance between $\bar{y}$ and $\widehat{\beta_1}$ does not exist

Q.29 Consider the simple linear regression model

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \;\; i = 1, 2, \dots, n \quad (n \geq 3),$$

where β_0 and β_1 are unknown parameters, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. Let $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$, $i = 1, 2, \dots, n$, where $\hat{\beta}_0$ and $\hat{\beta}_1$ represent least squares estimators of β_0 and β_1, respectively. Let $T_1 = \frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$ and $T_2 = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$. Then which one of the following statements is true?

(A) Both T_1 and T_2 are unbiased estimators of σ^2

(B) T_1 is an unbiased estimator of σ^2, but T_2 is not an unbiased estimator of σ^2

(C) T_1 is not an unbiased estimator of σ^2, but T_2 is an unbiased estimator of σ^2

(D) Neither T_1 nor T_2 is an unbiased estimator of σ^2

Q.30 Consider the power series $\sum_{n=0}^{\infty} a_n x^n$, where $a_{2n+1} = \frac{1}{2^{2n+1}}$ and $a_{2n} = \frac{1}{3^{2n}}$ for $n = 0,1,2,\dots$. The radius of convergence of the power series equals __________ (*in integer*).

Q.31 Let X be a random variable having Poisson distribution with mean $\lambda > 0$ such that $P(X = 4) = 2\,P(X = 5)$. If $p_k = P(X = k)$, $k = 0,1,2,\dots,$ and $p_\alpha = \max_k p_k$, then α equals __________ (*in integer*).

Q.32 Let X_1, X_2, X_3 be independent and identically distributed random variables with common probability density function

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then $P(\min\{X_1,\ X_2,\ X_3\} \geq E(X_1))$ equals __________ (*rounded off to two decimal places*).

Q.33 Let (X, Y) have a bivariate normal distribution with $E(X) = E(Y) = 0$. Denote the conditional variance of X given $Y = 1$ by $Var(X|Y = 1)$ and the conditional variance of Y given $X = 2$ by $Var(Y|X = 2)$.
If $\frac{E(Y|X=2)}{E(X|Y=1)} = 8$ then $\frac{Var(Y|X=2)}{Var(X|Y=1)}$ equals __________ (*in integer*).

Q.34 Let X be a random sample of size one from a population having $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. Let $\Phi(\cdot)$ denote the cumulative distribution function of a standard normal random variable and let $\chi^2_{\nu,\alpha}$ denote the $(1-\alpha)$-th quantile of the central chi-square distribution with ν degrees of freedom. It is given that $\Phi(1.96) = 0.975, \Phi(1.64) = 0.95$, $\chi^2_{1,0.05} = 3.841$, $\chi^2_{2,0.05} = 5.991$. To test $H_0: \sigma^2 = 1$ against $H_1: \sigma^2 = 2$, using the Neyman-Pearson most powerful test of size 0.05, the critical region is given by $\lambda(X) > c$, where $c \geq 0$ is a constant and $\lambda(x) = \frac{f(x;\, \sigma^2=2)}{f(x;\, \sigma^2=1)}$, where $f(x; \sigma^2)$ is the probability density function of a $N(0, \sigma^2)$ distribution. Then the value of c equals __________ (*rounded off to two decimal places*).

Q.35 Consider the linear regression model

$$y_i = \beta_1 x_i + \epsilon_i, \;\; i = 1, 2, \dots, n,$$

where β_1 is an unknown parameter, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. The five data points $(x_1, y_1) = (2, 5)$, $(x_2, y_2) = (1, 6)$, $(x_3, y_3) = (3, 4)$, $(x_4, y_4) = (2, 3)$ and $(x_5, y_5) = (4, 6)$ yield the least squares estimate of β_1 to be equal to __________ (*rounded off to two decimal places*).

Q.36 – Q.65 Carry TWO marks Each

Q.36 Consider the function $f: \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = 108\, xy - 2x^2y - 2xy^2.$$

Which one of the following statements is NOT true?

(A) f has four critical points

(B) f has a local minimum at $(0, 0)$

(C) f has a local maximum at $(18, 18)$

(D) f has two or more saddle points

Q.37 Consider the function $f: \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{otherwise.} \end{cases}$$

If f_x denotes the partial derivative of f with respect to x and f_y denotes the partial derivative of f with respect to y, then which one of the following statements is NOT true?

(A) f is continuous at $(0,0)$

(B) $f_x(0,0) \neq f_y(0,0)$

(C) f_x is continuous at $(0,0)$

(D) f_y is not continuous at $(0,0)$

Q.38 Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{8}(x+1)^2 & \text{if } -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

If $Y = 1 - X^2$, then $P\left(Y \le \frac{3}{4}\right)$ equals

(A) $\frac{19}{32}$

(B) $\frac{9}{16}$

(C) $\frac{15}{32}$

(D) $\frac{5}{8}$

Q.39 Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{c_1}{\sqrt{x}} & \text{if } 0 < x \leq 1 \\ \frac{c_2}{x^2} & \text{if } 1 < x < \infty \\ 0 & \text{otherwise,} \end{cases}$$

where c_1 and c_2 are appropriate real constants. If $P\left(X \in \left[\frac{1}{4}, 4\right]\right) = \frac{5}{8}$, then consider the following statements:

(I) $P(X \in [3,5]) = \frac{1}{12}$

(II) Both X as well as $\frac{1}{X}$ do not have finite expectations.

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.40 At a single-staff checkout counter of a supermarket store, the time taken in minutes to complete the service of a customer is a random variable having probability density function given by

$$f(x) = \begin{cases} \frac{1}{10} e^{-\frac{x}{10}} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

When you arrive at the counter, you observe that there is already one person in service. You are also informed that the person has been in service for 5 minutes. Assuming that the service times of different customers are independent of each other, the probability that your total waiting time (which is the sum of your waiting time in queue and service time) is more than 15 minutes equals

(A) $\frac{5}{2} e^{-\frac{3}{2}}$

(B) $\frac{3}{2} e^{-\frac{3}{2}}$

(C) $\frac{3}{2} e^{-\frac{5}{2}}$

(D) $\frac{5}{2} e^{-\frac{5}{2}}$

Q.41 Let X be a random variable having discrete uniform distribution on $\{1, 3, 5, 7, \dots, 99\}$. Then $E(X \mid X \text{ is not a multiple of } 15)$ equals

(A) $\frac{2365}{47}$

(B) $\frac{2365}{50}$

(C) 50

(D) 47

Q.42 Let $X_1, X_2, \dots, X_n$ be independent and identically distributed random variables having $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$. Consider the following statements:

(I) If $\frac{\sqrt{5}}{\sigma\,(2n+1)} \sum_{i=1}^{n} i\,(X_i - \mu)$ has $N(0,1)$ distribution, then $n = 2$.

(II) $E\left(\left(-\log_e\left(\Phi\left(\frac{X_1-\mu}{\sigma}\right)\right)\right)^3\right) = 6$, where $\Phi(\cdot)$ denotes the cumulative distribution function of $N(0,1)$ random variable.

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.43 Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables having probability density function

$$f(x) = \begin{cases} e^{-(x-\theta)} & \text{if } x \geq \theta \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$. Consider the following statements:

(I) $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to $\frac{\theta+1}{2}$, as $n \to \infty$.

(II) $\lim_{n\to\infty} E(\min\{X_1, X_2, \dots, X_n\}) = \theta$.

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.44 Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables such that X_n has Poisson distribution with mean λ_n, where $\lambda_n = \lambda + \frac{1}{2^n}$, $n \geq 1$, and $\lambda > 0$ is an unknown parameter. Which one of the following statements is true?

(A) $\frac{1}{n}\sum_{i=1}^{n} X_i$ is an unbiased estimator of λ

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i$ is a consistent estimator of λ

(C) $\sum_{i=1}^{n} X_i$ is a consistent estimator of λ

(D) $\frac{1}{n^2}\sum_{i=1}^{n} X_i$ is an unbiased estimator of λ

Q.45 Let $\boldsymbol{X_1}, \boldsymbol{X_2}, \ldots, \boldsymbol{X_{25}}$ be a random sample of size 25 from a population having $N_3(\boldsymbol{\mu}, \Sigma)$ distribution, where $\boldsymbol{\mu}$ and non-singular Σ are unknown parameters. Let $S = \frac{1}{24}\sum_{j=1}^{25}(\boldsymbol{X}_j - \overline{\boldsymbol{X}})(\boldsymbol{X}_j - \overline{\boldsymbol{X}})^T$, where $\overline{\boldsymbol{X}} = \frac{1}{25}\sum_{j=1}^{25}\boldsymbol{X}_j$, and

$B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$. Then which one of the following statements is true?

(A) $24\,BSB^T$ follows a Wishart distribution of order 3 with 24 degrees of freedom

(B) $24\,BSB^T$ follows a Wishart distribution of order 2 with 25 degrees of freedom

(C) $24\,BSB^T$ follows a Wishart distribution of order 2 with 24 degrees of freedom

(D) $24\,BSB^T$ follows a Wishart distribution of order 3 with 25 degrees of freedom

Q.46 Consider the simple linear regression model

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad i = 1, 2, \ldots, n,$$

where β_0, β_1 are unknown parameters, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. Let $\hat{\beta}_i$ be the least squares estimator of $\beta_i, i = 0,1$. Consider the following statements:

(I) A 95% joint confidence region for (β_0, β_1) is the region bounded by an ellipse.

(II) The expression for covariance between $\hat{\beta}_0$ and $\hat{\beta}_1$ does not involve σ^2.

Which of the above statements is/are true?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.47 If $f: [-2,2] \to \mathbb{R}$ is a continuous function, then which of the following statements is/are true?

(A) $F: [0,2] \to \mathbb{R}$ defined by $F(x) = \int_0^x f(t)\, dt$ is differentiable on $(0,2)$

(B) For any $x_1, x_2, \dots, x_{10} \in [-2,2]$ there exists a point $x_0 \in [-2,2]$ such that

$$f(x_0) = \frac{1}{10}\left(f(x_1) + f(x_2) + \cdots + f(x_{10})\right)$$

(C) f is bounded on $[-2,2]$

(D) If f is differentiable at 0 and $f(0) = 0$, then

$$\lim_{x \to 0} \frac{1}{x}\left(f(x) + f\left(\frac{x}{2}\right) + \cdots + f\left(\frac{x}{10}\right)\right) = 10\, f'(0),$$

where $f'(0)$ is the derivative of f at 0

Q.48 Let A be an $n \times n$ real matrix. Which of the following statements is/are true?

(A) If A is a symmetric matrix such that $A + \epsilon I_n$ is positive semi-definite for every $\epsilon > 0$, where I_n is the $n \times n$ identity matrix, then A is positive semi-definite

(B) If n is odd, then $A - A^T$ is not invertible

(C) If A is a symmetric matrix such that the singular values of A are all strictly positive, then A is positive definite

(D) If 1 is the only singular value of A, then A is orthogonal

Q.49 Which of the following statements is/are true?

(A) If A is a 3×3 real matrix with 3 distinct eigenvalues, then A is diagonalizable

(B) If A is a 3×3 real matrix such that A^2 is diagonalizable, then A is diagonalizable

(C) For real numbers $a, b, c,$ if $\begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix}$ is diagonalizable, then $a = b = c = 0$

(D) For real numbers $a, b, c, d, e, f,$ if $A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$ is diagonalizable,

then $AA^T = A^T A$

Q.50 Let $\Omega = \{1, 2, 3, \ldots\}$ and $\mathcal{H}$ be the collection of all subsets of Ω. Let P be a probability measure on $\mathcal{H}$ such that $P(\{k\}) = \frac{1}{2^k}, k \geq 1$. Further, let $X: \Omega \to \mathbb{R}$ be defined by $X(\omega) = \omega$ for all $\omega \in \Omega$. Then which of the following statements is/are true?

(A) There exists $k \in \Omega$ such that $P(X = k) < 10^{-6}$

(B) $\lim_{n\to\infty} P\left(X \geq 4 + \frac{1}{n}\right) = \frac{1}{16}$

(C) $\lim_{n\to\infty} P\left(4 + \frac{1}{n^2} \leq X < 5 - \frac{1}{n}\right) = \frac{1}{16}$

(D) If $x_n = 3 + \frac{(-1)^n}{n}, n \geq 1$, then $\lim_{n\to\infty} P(X \leq x_n) = \frac{7}{8}$

Q.51 Let (X, Y) be a random vector having joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{3}{4} & \text{if } x^2 \le y < 1 \text{ and } -1 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Which of the following statements is/are true?

(A) X has the same distribution as $-X$

(B) $E(Y \mid X = 0) = \frac{1}{2}$

(C) The correlation coefficient between X and Y is 0

(D) X and Y are independent

Q.52 Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables such that the probability density function of X_n is given by

$$f_n(x) = \begin{cases} \frac{1}{\lambda_n} e^{-\frac{x}{\lambda_n}} & \text{if } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda_n = 10 - \sum_{i=1}^{n} \frac{5}{2^{i-1}}$. Which of the following statements is/are true?

(A) $\{X_n\}_{n\geq 1}$ converges in distribution to the zero random variable

(B) $\{X_n\}_{n\geq 1}$ converges in probability to the zero random variable

(C) $\{X_n\}_{n\geq 1}$ converges in distribution to a random variable having Poisson distribution with mean 10

(D) $\{X_n\}_{n\geq 1}$ converges in probability to a random variable having Poisson distribution with mean 10

Q.53 Let $\{X_n\}_{n\geq 1}$ be a time homogeneous discrete time Markov chain with state space $\{0, 1, 2\}$ and transition probability matrix $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$. Which of the following statements is/are true?

(A) 0 and 1 are recurrent states

(B) 2 is a transient state

(C) Markov chain has a unique stationary distribution

(D) Markov chain is irreducible

Q.54 Let $X_1, X_2, \dots, X_n$ be a random sample of size $n\ (\geq 2)$ from a population having Poisson distribution with mean λ, where $\lambda > 0$ is an unknown parameter. If $T_1 = \bar{X}$ and $T_2 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$, where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then which of the following statements is/are true?

(A) T_1 is an unbiased estimator of λ

(B) T_2 is an unbiased estimator of $\sqrt{\lambda}$

(C) T_2^2 is an unbiased estimator of λ

(D) Both T_1 and T_2 are estimators of λ as well as λ^2

Q.55 Let X_1, X_2, X_3 be a random sample of size 3 from a population having Bernoulli distribution with parameter p, where $p \in (0, 1)$ is unknown. Define $T_1(X_i, X_j, X_k) = X_i - X_j(1 - X_k), T_2(X_i, X_j, X_k) = \frac{1}{2}(X_iX_j + X_jX_k)$, for $i, j, k = 1,2,3; i \neq j \neq k$. Let x_1, x_2, x_3 denote realizations from the random sample. Then which of the following statements is/are true?

(A) $T_1(X_1, X_2, X_3)$ has the same distribution as $T_1(X_2, X_3, X_1)$, but $T_1(x_1, x_2, x_3) \neq T_1(x_2, x_3, x_1)$ for some realization x_1, x_2, x_3

(B) $T_2(X_1, X_2, X_3)$ and $T_2(X_3, X_2, X_1)$ are both unbiased estimators for p^2

(C) $T_1(X_1, X_2, X_3)$ and $T_1(X_2, X_3, X_1)$ are both unbiased estimators for p^2 and $T_1(x_1, x_2, x_3) = T_1(x_2, x_3, x_1)$ for all realizations x_1, x_2, x_3

(D) $T_2(x_1, x_2, x_3) = T_2(x_3, x_2, x_1)$ for all realizations x_1, x_2, x_3

Q.56 Let $X_1, X_2, \dots, X_n$ be a random sample of size $n\ (\geq 2)$ from a population having probability density function

$$f(x;\lambda) = \begin{cases} \frac{1}{\lambda} e^{-\frac{x}{\lambda}} & \text{if } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda > 0$ is an unknown parameter. Let $T_1 = \sum_{i=1}^{n} X_i$ and $T_2 = \left(\sum_{i=1}^{n} X_i\right)^{-1}$. For any positive integer ν and any $\alpha \in (0,1)$, let $\chi^2_{\nu;\alpha}$ denote the $(1-\alpha)$-th quantile of the central chi-square distribution with ν degrees of freedom. Consider testing $H_0: \lambda = \lambda_0$ against $H_1: \lambda > \lambda_0$. Then which of the following tests is/are uniformly most powerful test at level α?

(A) H_0 is rejected if $\frac{2}{\lambda_0} T_1 > \chi^2_{2n,\alpha}$

(B) H_0 is rejected if $\frac{2}{\lambda_0} T_1 > \chi^2_{2n,1-\alpha}$

(C) H_0 is rejected if $\frac{\lambda_0}{2} T_2 > \chi^2_{2n,\alpha}$

(D) H_0 is rejected if $\frac{\lambda_0}{2} T_2 > \chi^2_{2n,1-\alpha}$

Q.57 Let $\{1, 6, 5, 3\}$ and $\{11, 7, 15, 4\}$ be realizations of two independent random samples of size 4 from two separate populations having cumulative distribution functions $F(\cdot)$ and $G(\cdot)$, respectively, and probability density functions $f(\cdot)$ and $g(\cdot)$, respectively. To test $H_0: F(t) = G(t)$ for all t, against $H_1: F(t) \geq G(t)$ with strict inequality for some t, let U_{MW} denote the Mann-Whitney U-test statistic. Let, under H_0, $P(U_{MW} > 12) \leq 0.10$, $P(U_{MW} > 14) \leq 0.05$, $P(U_{MW} > 15) \leq 0.025$ and $P(U_{MW} > 16) \leq 0.01$. Then, based on the given data, which of the following statements is/are true?

(A) H_0 is rejected at level 0.10

(B) H_0 is rejected at level 0.05

(C) H_0 is rejected at level 0.025

(D) H_0 is rejected at level 0.01

Q.58 Let (X_1, X_2, X_3) have $N_3(\boldsymbol{\mu}, \Sigma)$ distribution with $\boldsymbol{\mu} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$. For which of the following vectors $\boldsymbol{a}$, X_2 and $X_2 - \boldsymbol{a}^T \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ are independent?

(A) $\boldsymbol{a} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(B) $\boldsymbol{a} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

(C) $\boldsymbol{a} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

(D) $\boldsymbol{a} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Q.59 Let A be a 2×2 real matrix, such that trace of A is 5 and determinant of A is 6. If the characteristic polynomial of $(A + I_2)^{-1}$ is $x^2 - bx + c$, where I_2 is the 2×2 identity matrix, then $\frac{b}{c}$ equals __________ (*in integer*).

Q.60 Let $\{X_n\}_{n \geq 1}$ be a time homogeneous discrete time Markov chain with state space $\{0, 1, 2\}$ and transition probability matrix $\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$.

If $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{4}$, then $32\, E(X_2)$ equals __________ (*in integer*).

Q.61 Let (X, Y) be a random vector having joint moment generating function given by

$$M_{X,Y}(u, v) = \frac{e^{\frac{u^2}{2}}}{(1-2v)^3}, -\infty < u < \infty, -\infty < v < \frac{1}{2}.$$

Then $E\left(\frac{6X^2}{Y}\right)$ equals __________ (*rounded off to two decimal places*).

Q.62 Let $\{N(t)\}_{t\geq 0}$ be a Poisson process with rate λ, where $\lambda > 0$ is an unknown parameter. Staring from the origin, an intercity road has $N(t)$ number of potholes up to a distance of t kilometers. Starting from the origin, potholes are found at the following distances (in kilometers)

$$0.9, 1.3, 1.8, 2.7, 3.4, 4.1, 4.7, 5.5, 6.2, 6.8, 7.4, 8.1, 8.9, 9.2, 9.7.$$

Based on the above data, the method of moment estimate of λ equals __________ (*rounded off to two decimal places*).

Q.63 Let X_1, X_2, X_3, X_4 be a random sample of size 4 from a population having uniform distribution over the interval $(0, \theta)$, where $\theta > 0$ is an unknown parameter. Let $X_{(4)} = \max\{X_1, X_2, X_3, X_4\}$. To test $H_0: \theta = 1$ against $H_1: \theta = 0.1$, consider the critical region that rejects H_0 if $X_{(4)} < 0.3$. Let p be the probability of the Type-I error. Then $100\,p$ equals ____________ (*rounded off to two decimal places*).

Q.64 Let a random sample of size 4 from a normal population with unknown mean μ and variance 1 yield the sample mean of 0.16. Let $\Phi(\cdot)$ be the cumulative distribution function of the standard normal random variable. It is given that $\Phi(2.28) = 0.989, \Phi(1.96) = 0.975, \Phi(1.64) = 0.95$. If the likelihood ratio test of size 0.05 is used to test $H_0: \mu = 0$ against $H_1: \mu \neq 0$, then the power of the test at the sample mean equals __________ (*rounded off to three decimal places*).

Q.65 Consider the multiple linear regression model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i, \;\; i = 1, 2, \dots, 25,$$

where β_0, β_1 and β_2 are unknown parameters, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. Let $F_{\alpha,m,n}$ be such that $P(Y > F_{\alpha,m,n}) = \alpha$, where Y is a random variable having an F-distribution with m and n degrees of freedom. Suppose that testing

$H_0: \beta_1 = \beta_2 = 0$ against H_1: At least one of β_1, β_2 is not 0

involves computing $F_0 = 11 \frac{R^2}{1-R^2}$ and rejecting H_0 if the computed value F_0 exceeds $F_{\alpha,2,22}$. Given that $F_{0.025,2,22} = 4.38$, $F_{0.05,2,22} = 3.44$, the smallest value of R^2 that would lead to rejection of H_0 for $\alpha = 0.05$ equals __________(*rounded off to two decimal places*).