GATE 2025 ECE Question Paper with Solutions

Time Allowed : 3 hours	**Maximum Marks :** 100	**Total questions :** 65

General Instructions

Read the following instructions very carefully and strictly follow them:

This question paper is divided into three sections:

1. The total duration of the examination is 3 hours. The question paper contains three sections -

Section A: General Aptitude

Section B: Engineering Mathematics

Section C: Subject Based Questions

2. The total number of questions is **65**, carrying a maximum of **100 marks**.

3. The marking scheme is as follows:

(i) For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.

(ii) For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.

(iii) No negative marking for numerical answer type (NAT) questions.

4. No marks will be awarded for unanswered questions.

5. Follow the instructions provided during the exam for submitting your answers.

(1) Consider three Boolean variables x, y, z. A majority function outputs 1 if the majority of its inputs are 1, otherwise, it outputs 0. Derive the Boolean expression for the majority function and simplify it using Boolean algebra.

(A) $xy + yz + xz$

(B) $x + y + z$

(C) $x(y \oplus z) + yz$

(D) $(x \oplus y) \oplus z$

Correct Answer: (A) $xy + yz + xz$

Solution:

Understanding the Majority Function.

A majority function for three variables x, y, z is defined as:

$$f(x, y, z) = 1 \text{ when at least two inputs are } 1.$$

The sum of minterms where the function evaluates to 1 is:

$$\Sigma(3, 5, 6, 7)$$

Using Boolean algebra, the canonical sum-of-products (SOP) expression for the majority function is:

$$f(x, y, z) = xy + yz + xz$$

We can also derive this using simplifications:

1. Expanding in terms of the majority condition:

$$xy + yz + xz$$

2. Alternative simplification using XOR:

$$f(x, y, z) = x(y \oplus z) + yz$$

Since both derivations are equivalent, the minimal expression is:

$$xy + yz + xz$$

Quick Tip

In logic circuits, the majority function is useful in voting systems and fault-tolerant computing, where outputs depend on the majority of inputs.

(2) Find the next term in the sequence: 3, 9, 19, 33, _

(A) 47

(B) 49

(C) 51

(D) 53

Correct Answer: (C) 51

Solution:

Finding the Pattern in the Sequence.

The given sequence:

$$3, 9, 19, 33, \_$$

Finding first-order differences:

$$9 - 3 = 6, \quad 19 - 9 = 10, \quad 33 - 19 = 14$$

Finding second-order differences:

$$10 - 6 = 4, \quad 14 - 10 = 4$$

Since the second-order difference is constant (4), the sequence follows a quadratic pattern.

The next first-order difference is:

$$18 = 14 + 4$$

Thus, the next term in the sequence is:

$$33 + 18 = 51$$

Quick Tip

A constant second-order difference indicates a quadratic sequence. Use this method to predict future terms efficiently.

(3) Given the matrix:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

If rank(A) is at least 3, then what are the possible values of α, β, γ?

(A) $\alpha \neq 0$, $\gamma \neq 0$, β is arbitrary

(B) $\alpha = 0$, $\gamma = 0$, $\beta \neq 0$

(C) $\alpha = 0$, $\gamma = 0$, $\beta = 0$

(D) $\alpha \neq 0$, $\gamma = 0$, $\beta = 0$

Correct Answer: (A) $\alpha \neq 0$ or $\gamma \neq 0$, β is arbitrary

Solution:

Understanding Rank in an Upper Triangular Matrix.

For a matrix in upper triangular form, its rank is equal to the number of nonzero diagonal elements.

The diagonal elements of A are:

$$2, 6, \alpha, \gamma$$

Since rank(A) must be at least 3, we require at least three of these elements to be nonzero.

- 2 and 6 are already nonzero.

- At least one of α or γ must be nonzero.

- β does not affect rank, so it can be any value.

Thus, the possible values are:

$$\alpha \neq 0 \quad \text{or} \quad \gamma \neq 0, \quad \beta \text{ is arbitrary.}$$

Quick Tip

For an upper triangular matrix, rank is simply the number of nonzero diagonal elements.

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(4) What operation is performed by the given circuit?

(A) B to G Converter

(B) Subtractor

(C) Multiplier

(D) None of the above

Correct Answer: (B) Subtractor

Solution:

Understanding the Circuit.

The given circuit consists of:

- A Full Adder (FA)

- An XOR gate that inverts y (i.e., $y' = y \oplus 1 = \bar{y}$)

- A Carry-in value of 1

Binary Subtraction via Two's Complement:

$$A - B = A + \bar{B} + 1$$

Here:

- x is directly passed to the FA.

- y is complemented via XOR with 1.

- Carry-in = 1.

This is the standard way to perform subtraction using addition, confirming that the circuit acts as a subtractor.

Quick Tip

Subtraction can be implemented using addition by converting the second operand into its Two's Complement form ($\bar{B} + 1$).

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(5) Which of the following best describes the phase and sinusoidal components in the given control system plot?

(A) $\sin \omega_c$, θ is ω_p

(B) R is ω_p, θ is ω_c

(C) θ is ω_c, $\sin$ is ω_p

(D) R is ω_c, $\sin \omega_p$

Correct Answer: (A) $\sin \omega_c$, θ is ω_p

Solution:

Understanding Frequency Domain Parameters.

The given diagram represents a frequency response plot, likely from Nyquist Stability Criterion or Bode Plot Analysis.

1. Sinusoidal Component $\sin \omega_c$:

- Indicates the system's response at critical frequency ω_c.

- This often corresponds to the gain crossover frequency in a Bode plot.

2. Phase Angle θ at ω_p:

- ω_p is the phase crossover frequency, where phase margin is measured.

- It is a key factor in stability determination.

Thus, the correct answer is:

$$\sin \omega_c, \quad \theta \text{ is } \omega_p$$

Quick Tip

In control systems, the gain crossover frequency (ω_c) and phase crossover frequency (ω_p) help analyze stability margins using Bode and Nyquist plots.

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(6) If a complex function $f(z)$ is analytic everywhere inside a closed contour C with anti-clockwise direction, then which of the following statements are correct?

(A) $\oint_C \cos z \, dz = 0$

(B) $\oint_C \sec z \, dz \neq 0$

(C) $\oint_C z^n \, dz = 0$

(D) $\oint_C e^z \, dz = 0$

Correct Answer: (A), (B), (C), (D)

Solution:

Applying Contour Integration Theorems.

By Cauchy's Integral Theorem (CIT):

$$\oint_C f(z)dz = 0, \quad \text{if } f(z) \text{ is analytic inside and on } C.$$

Checking the options:

1. $\cos z$ is entire (analytic everywhere), so $\oint_C \cos z dz = 0$. (A) is correct.

2. $\sec z = \frac{1}{\cos z}$ has poles where $\cos z = 0$ (e.g., $z = \frac{\pi}{2}, -\frac{\pi}{2}$). - If a pole exists inside C, the integral is nonzero. (B) is correct.

3. z^n is analytic for all $n \neq -1$, so $\oint_C z^n dz = 0$ for $n \neq -1$. (C) is correct.

4. e^z is entire (analytic everywhere), so $\oint_C e^z dz = 0$. (D) is correct.

Quick Tip

Cauchy's Integral Theorem states that if a function is analytic inside and on a closed contour, its integral is zero. However, if a function has poles inside the contour, then we apply Residue Theorem.

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(7) Two fair dice are rolled, and the random variable X denotes the sum of the outcomes. What is the expected value of X?

(A) 6.5

(B) 7

(C) 7.5

(D) 8

Correct Answer: (B) 7

Solution:

Expectation of the Sum of Two Dice.

Let X_1 and X_2 be the outcomes of two fair six-sided dice. The sum of outcomes is:

$$X = X_1 + X_2$$

For a single fair die:

$$E[X_1] = E[X_2] = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

By linearity of expectation:

$$E[X] = E[X_1] + E[X_2] = 3.5 + 3.5 = 7$$

Thus, the expected value of X is:

$$\mathbf{7}$$

Quick Tip

For any two independent random variables X and Y, the expectation follows the additive property:

$$E[X+Y] = E[X] + E[Y]$$

This property simplifies expectation calculations in probability theory.

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(8) Given that $Z_0 = 50\Omega$, $Z_L = (50 - j75)\Omega$, and $P_i = 10$ mW, find the power delivered to the load.

(A) 5.2 mW

(B) 6.4 mW

(C) 7.8 mW

(D) 8.6 mW

Correct Answer: (B) 6.4 mW

Solution:

Step 1: Calculate the Reflection Coefficient.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Substituting values:

$$\Gamma = \frac{(50 - j75) - 50}{(50 - j75) + 50} = \frac{-j75}{100 - j75}$$

Multiplying by the complex conjugate:

$$\Gamma = \frac{5625 - j7500}{15625}$$

$$|\Gamma| = \sqrt{\left(\frac{5625}{15625}\right)^2 + \left(\frac{-7500}{15625}\right)^2} = 0.6$$

Step 2: Compute Power Delivered to Load.

The formula for power delivered:

$$P_L = P_i(1 - |\Gamma|^2)$$

$$P_L = 10(1 - 0.36) = 10 \times 0.64 = 6.4 \text{ mW}$$

Thus, the final power delivered is:

$$6.4 \text{ mW}$$

Quick Tip

In transmission line problems, power delivered is calculated using reflection coefficient, which accounts for power loss due to mismatched impedances.

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(9) Given $y(t) = f(\alpha t)$, analyze how Fourier coefficients and time period are affected.

(A) $C_k = d_k \ \forall k$

(B) $C_k = \alpha d_k$

(C) If time period of $f(t)$ is T_0, then the time period of $f(\alpha t)$ is $\frac{T_0}{\alpha}$

(D) If time period of $f(t)$ is T_0, then the time period of $f(\alpha t)$ is αT_0

Correct Answer: (A), (C)

Solution:

Effect of Time Scaling on Fourier Series.

Given $y(t) = f(\alpha t)$, the Fourier series of $f(t)$ is:

$$f(t) = \sum_{n=-\infty}^{\infty} C_k e^{-jn\omega_0 t}$$

Applying time scaling:

$$y(t) = f(\alpha t) = \sum_{n=-\infty}^{\infty} C_k e^{-jn\omega_0(\alpha t)}$$

Rewriting:

$$y(t) = \sum_{n=-\infty}^{\infty} C_k e^{-jn(\alpha\omega_0)t}$$

Comparing with the standard Fourier series form:

$$y(t) = \sum_{n=-\infty}^{\infty} d_k e^{-jn\omega_0' t}$$

We get: - $\omega_0' = \alpha\omega_0$ (frequency scales by α).

- Fourier coefficients remain the same: $d_k = C_k$.

Checking the given options:

1. $C_k = d_k$ (Correct)

2. $C_k = \alpha d_k$ (Incorrect)

3. Time period transformation:

$$T' = \frac{T_0}{\alpha}$$

(Correct)

4. $T' = \alpha T_0$ (Incorrect).

Thus, the correct answers are (A) and (C).

Quick Tip

In time-scaling $y(t) = f(\alpha t)$, - The time period scales as $T' = \frac{T_0}{\alpha}$. - The fundamental frequency scales as $\omega_0' = \alpha\omega_0$. - The Fourier coefficients remain unchanged.

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(10) What is the maximum frequency for Clk in MHz, given the flip-flop and logic gate delays?

(A) 100 MHz

(B) 200 MHz

(C) 250 MHz

(D) 500 MHz

Correct Answer: (B) 200 MHz

Solution:

Step 1: Understanding the Timing Parameters.

The given timing parameters from the circuit:

- Setup Time (t_s) = 2 ns

- Hold Time (t_h) = 0 ns

- Clock-to-Q Propagation Delay (t_{pcq}) = 2 ns

- Logic Gate Propagation Delay = 1 ns

Step 2: Compute the Minimum Clock Period.

The minimum clock period for a sequential circuit is given by:

$$T_{min} = t_{pcq} + t_{pd(logic)} + t_{setup}$$

Substituting values:

$$T_{min} = 2 \text{ ns} + 1 \text{ ns} + 2 \text{ ns} = 5 \text{ ns}$$

Step 3: Compute the Maximum Clock Frequency.

$$f_{max} = \frac{1}{T_{min}} = \frac{1}{5 \times 10^{-9}} \text{ Hz}$$

$$f_{max} = 200 \text{ MHz}$$

Thus, the maximum clock frequency is:

$$\boxed{200 \text{ MHz}}$$

Quick Tip

For a synchronous sequential circuit, the maximum clock frequency is determined by the longest combinational delay path between flip-flops plus setup time constraints.
